14th Jul. 2023 @ Waseda U.

Stochastic Approach to the Inflationary Universe

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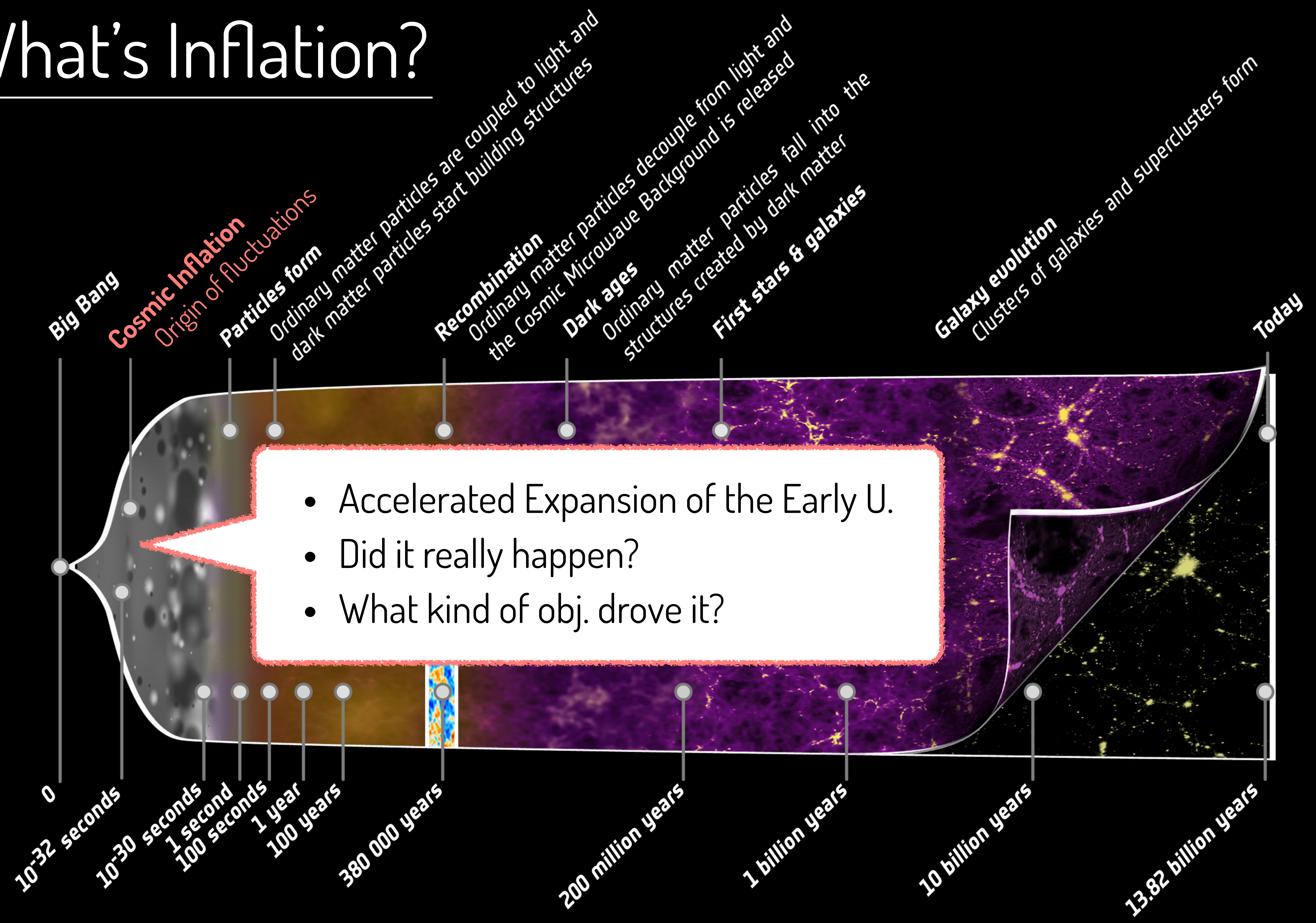
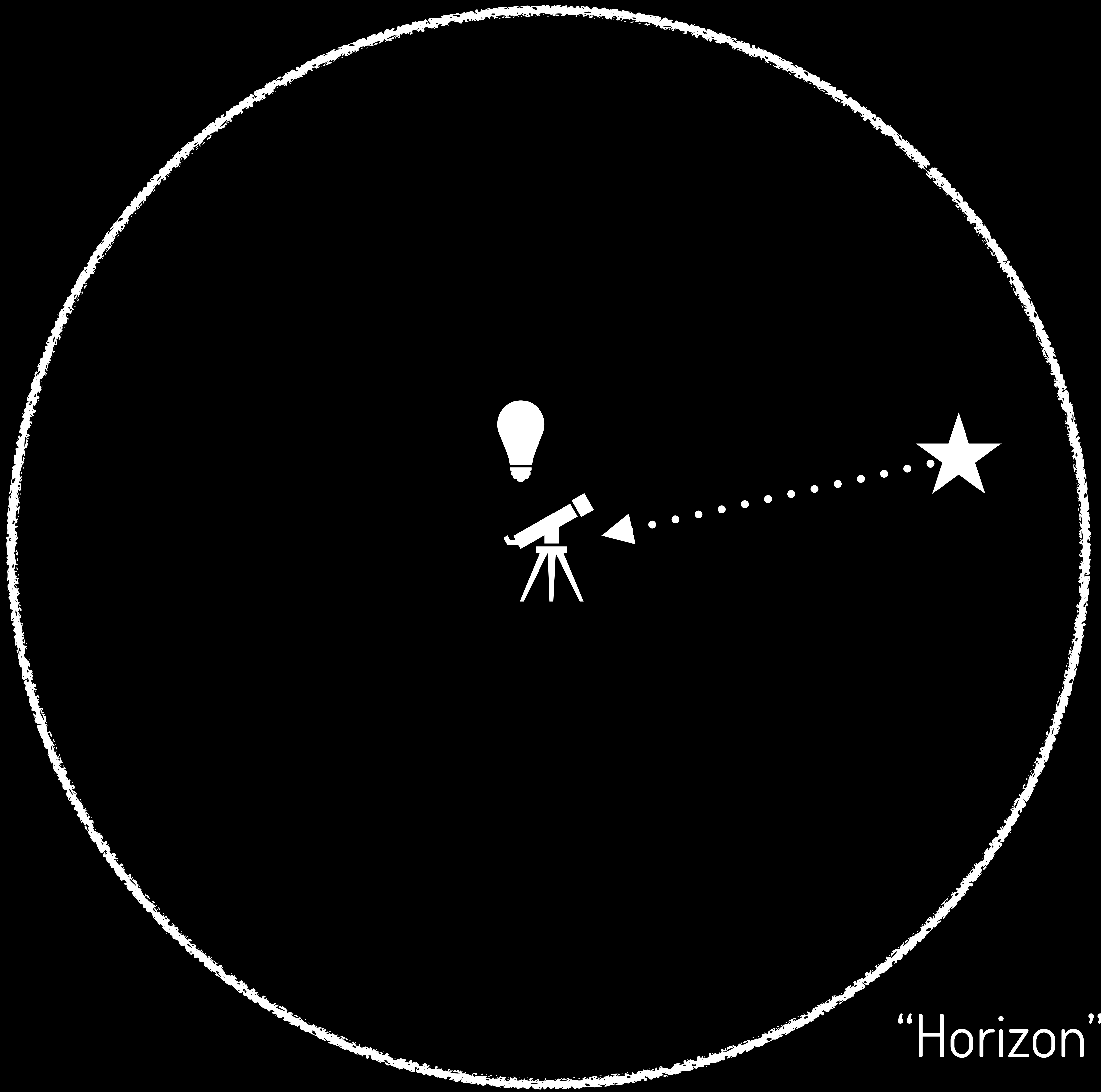
Content

What's Inflation?

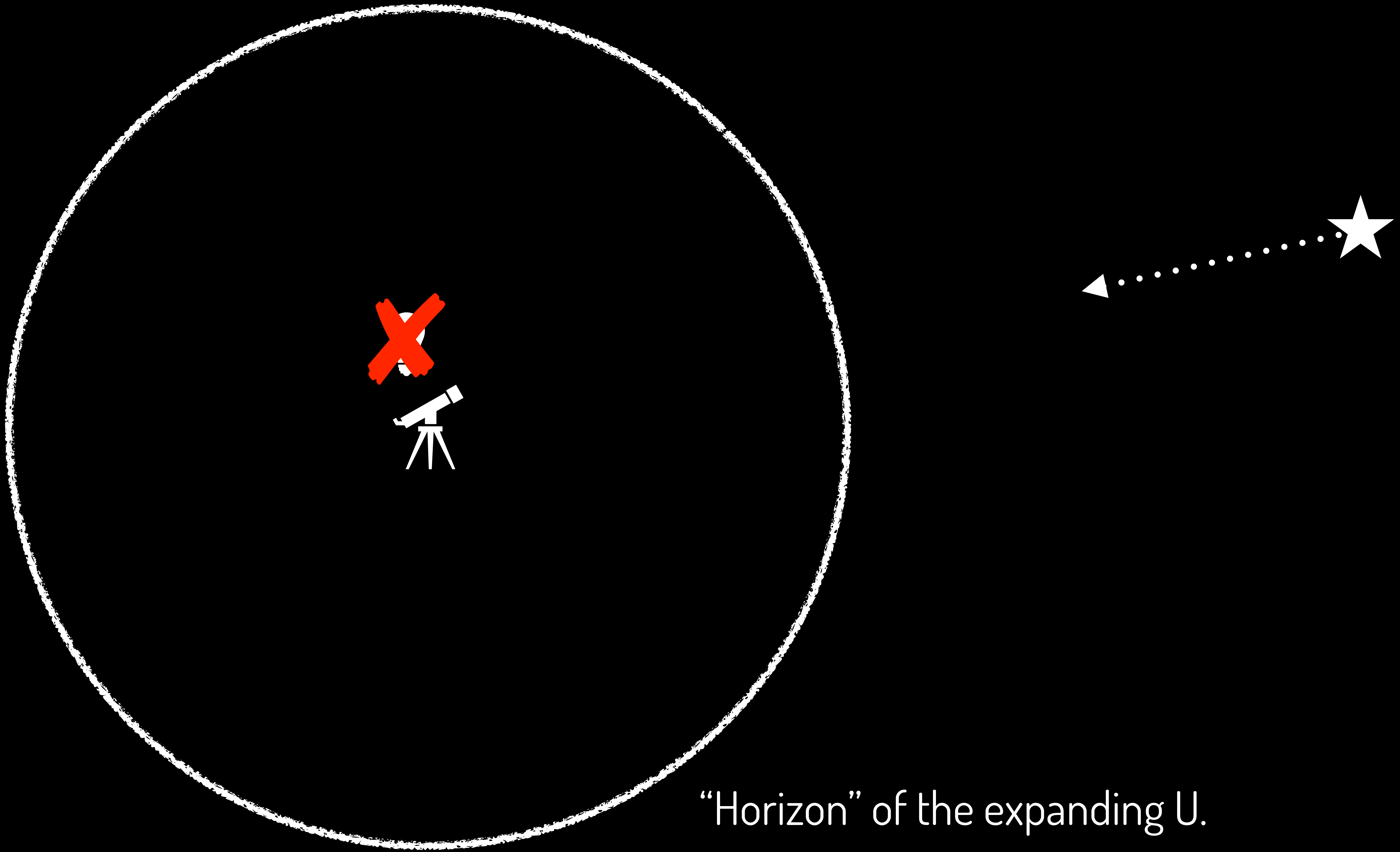


- Accelerated Expansion of the Early U.
- Did it really happen?
- What kind of obj. drove it?

Generation of PTB

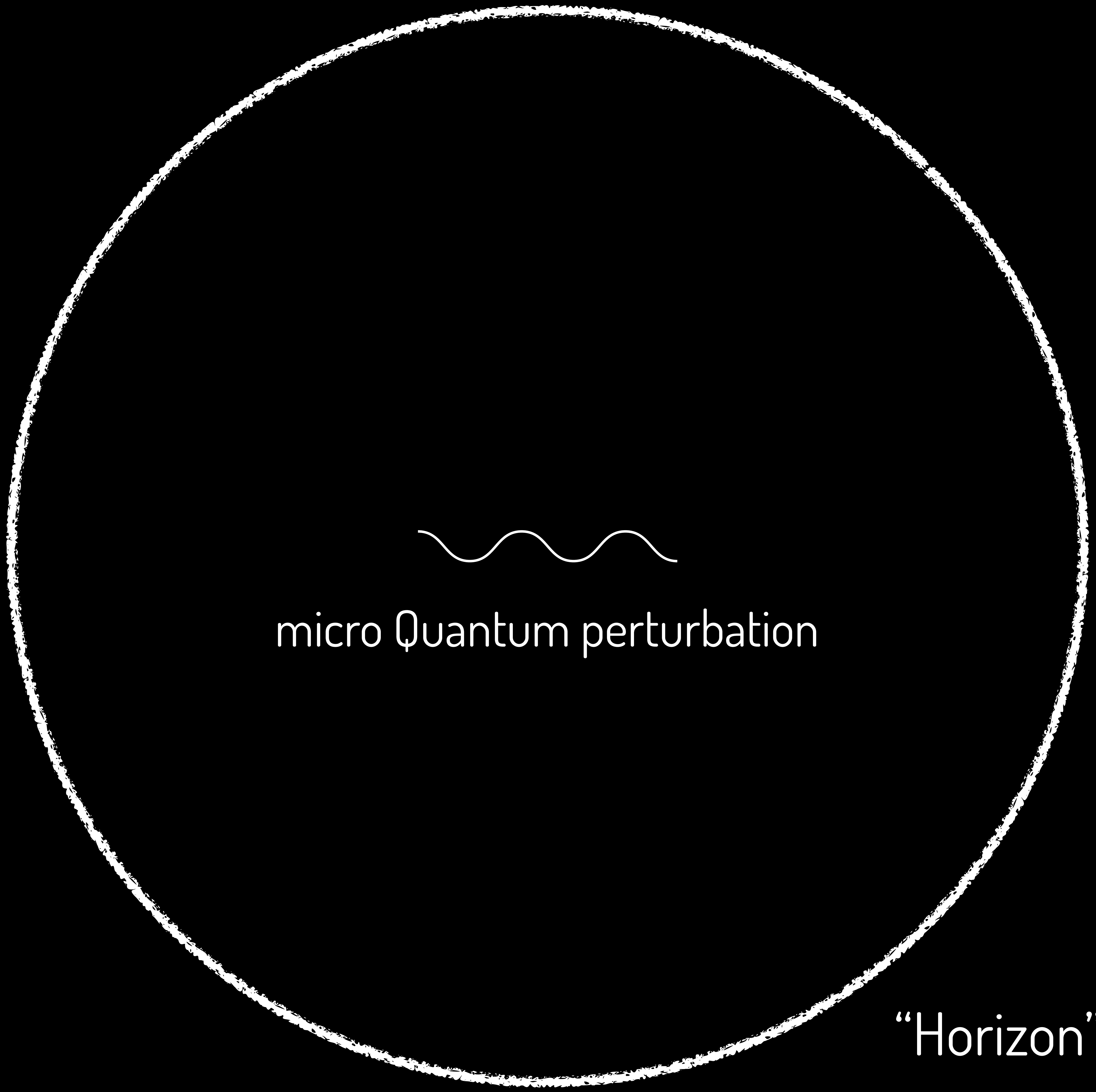


Generation of PTB

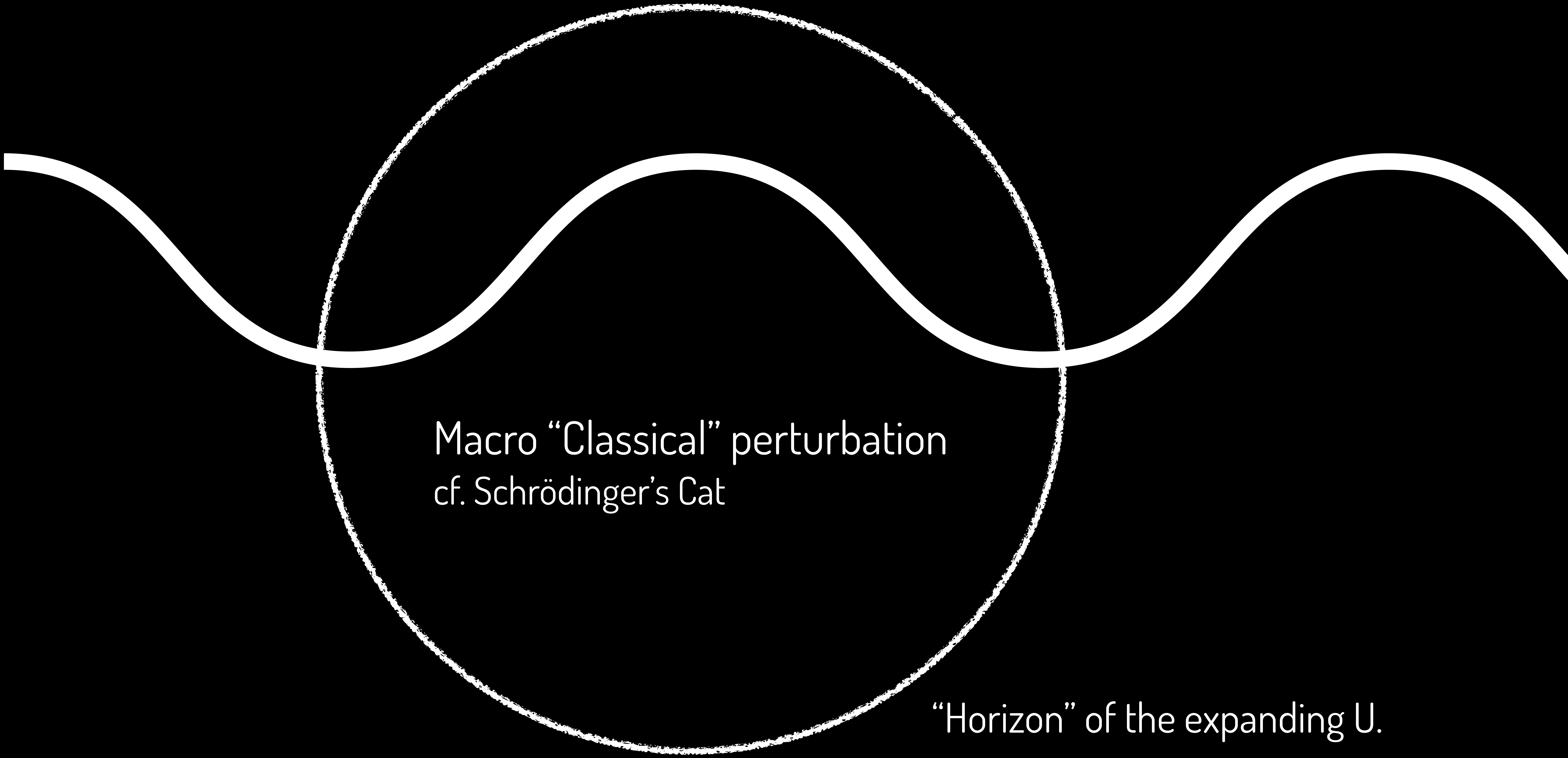


“Horizon” of the expanding U.

Generation of PTB



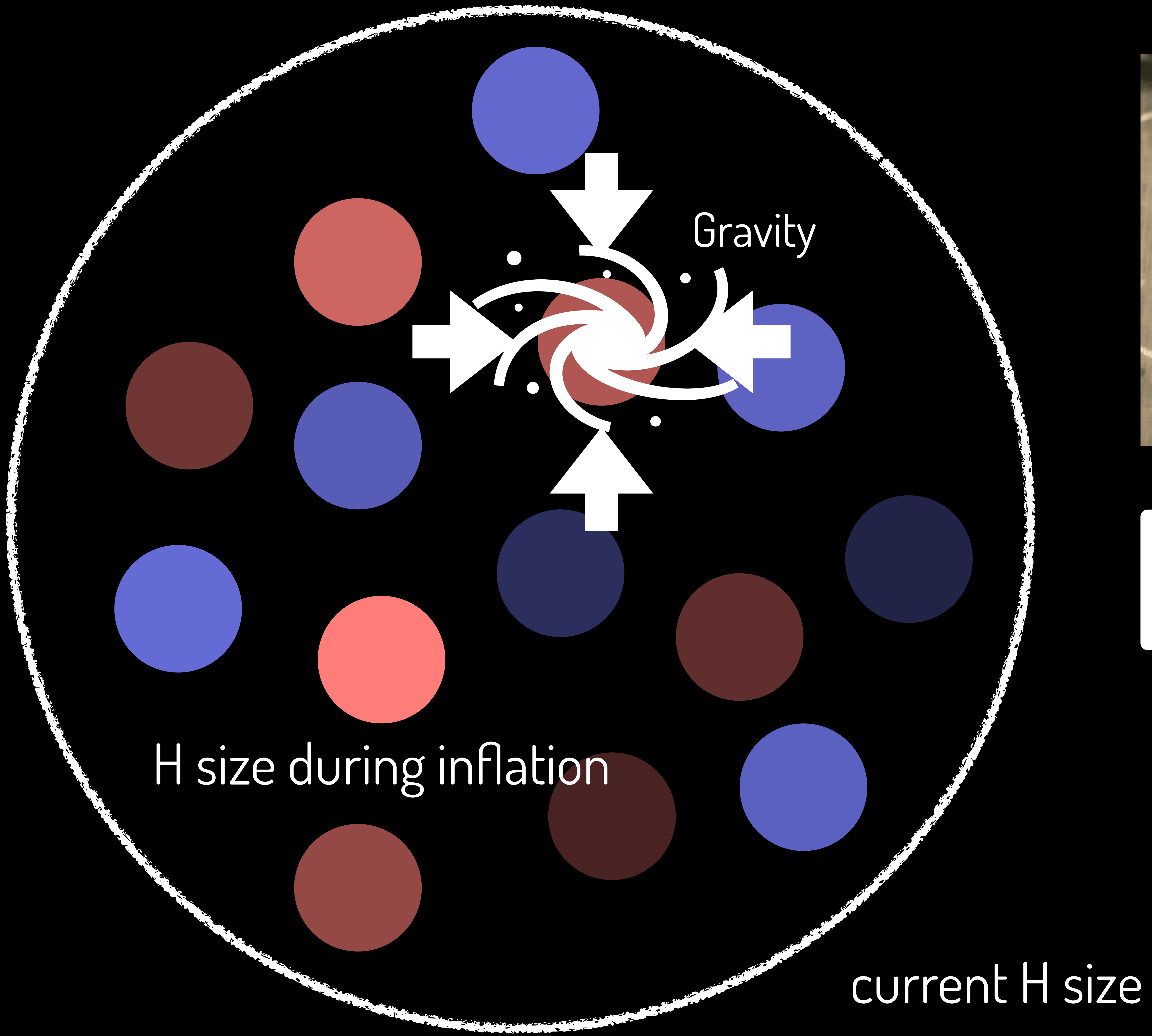
Generation of PTB

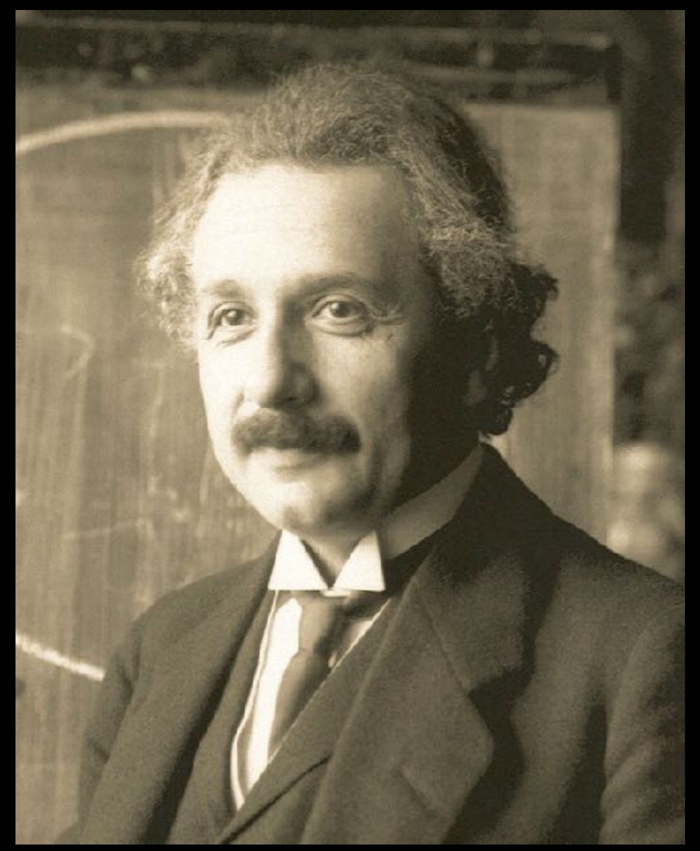


Generation of PTB

"Horizon" of the expanding U.

Generation of PTB

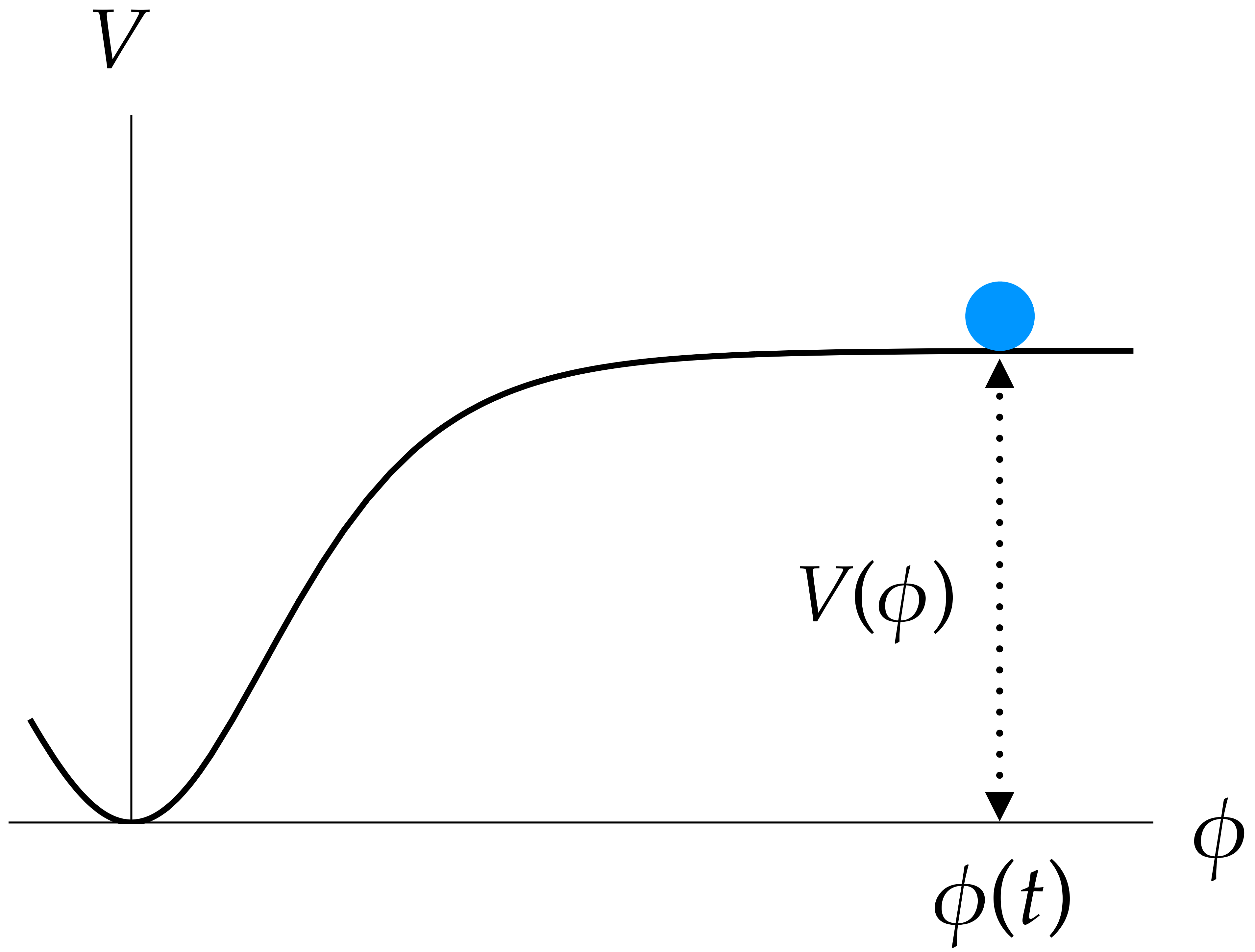




Energy = Mass

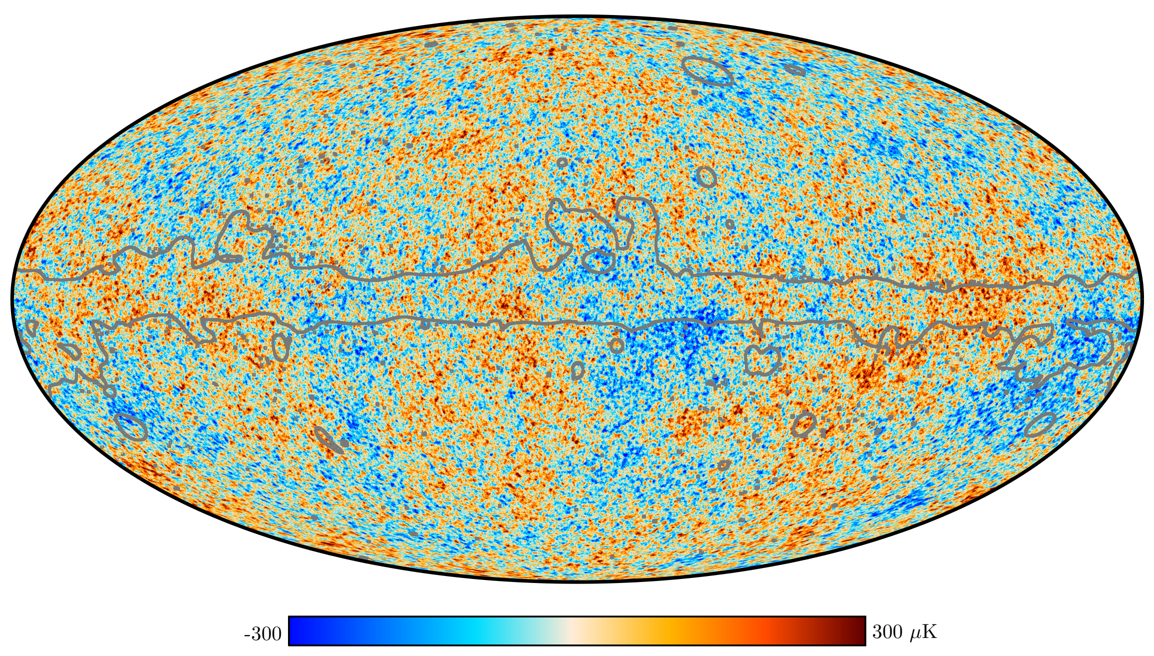
Inflation Theories

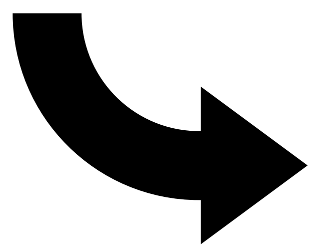
Just to realise an Accelerated Expansion (= Dark Energy = almost const. energy), you only need homogeneous VEV of some scalar $\phi(t)$ with the potential $V(\phi)$.



"Inflation of the # of inflation theories"
by T. Matsubara

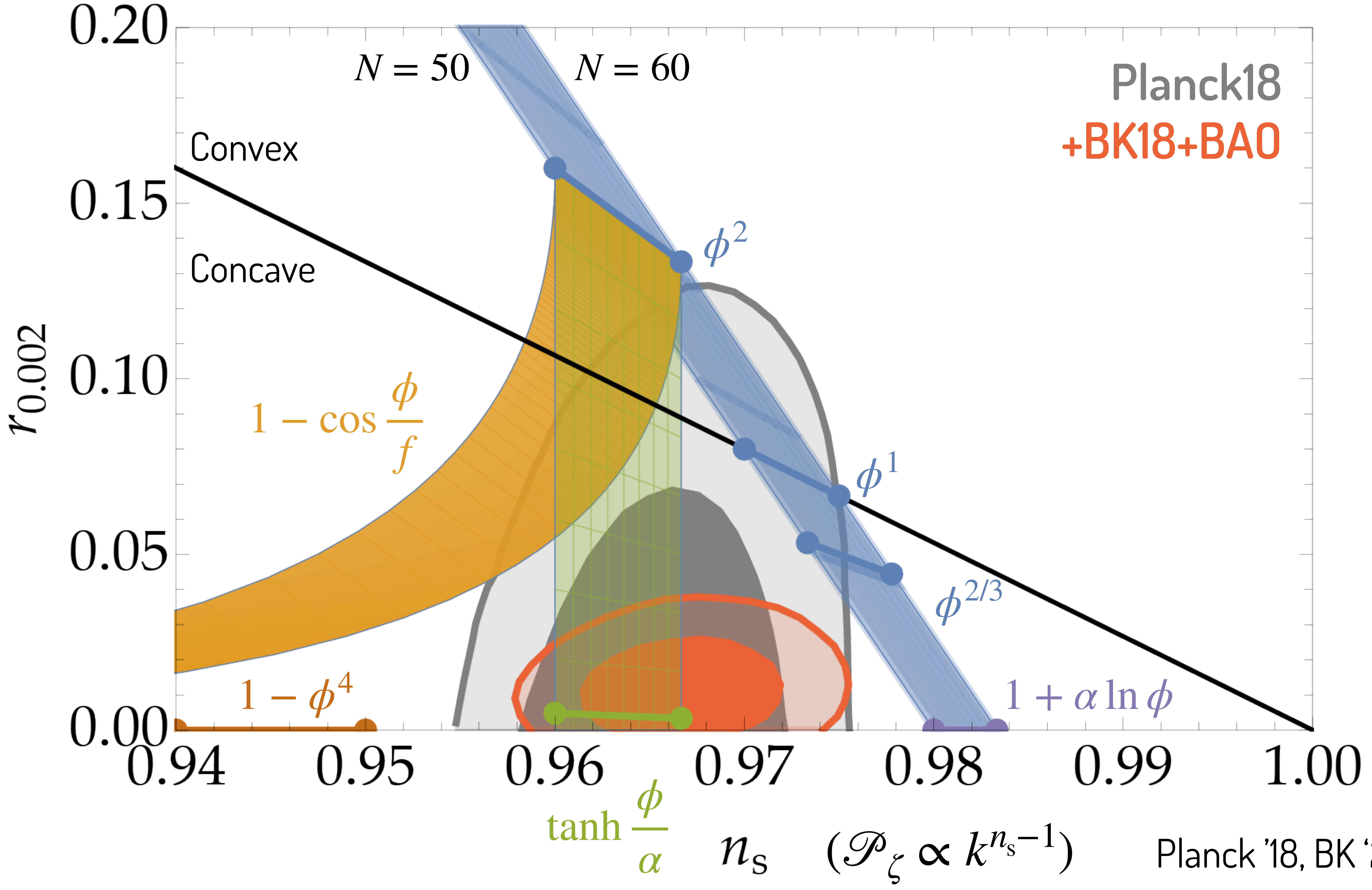
Latest CMB const.





$T = 2.725\,\mathrm{K} \pm 18\,\mu\mathrm{K}$

$\Rightarrow \mathscr{P}_\zeta = 2.1 \times 10^{-9}$



Planck '18, BK '21

Other Info?

- CMB $\gtrsim 1\,\mathrm{Mpc}$

  → How about smaller scales?

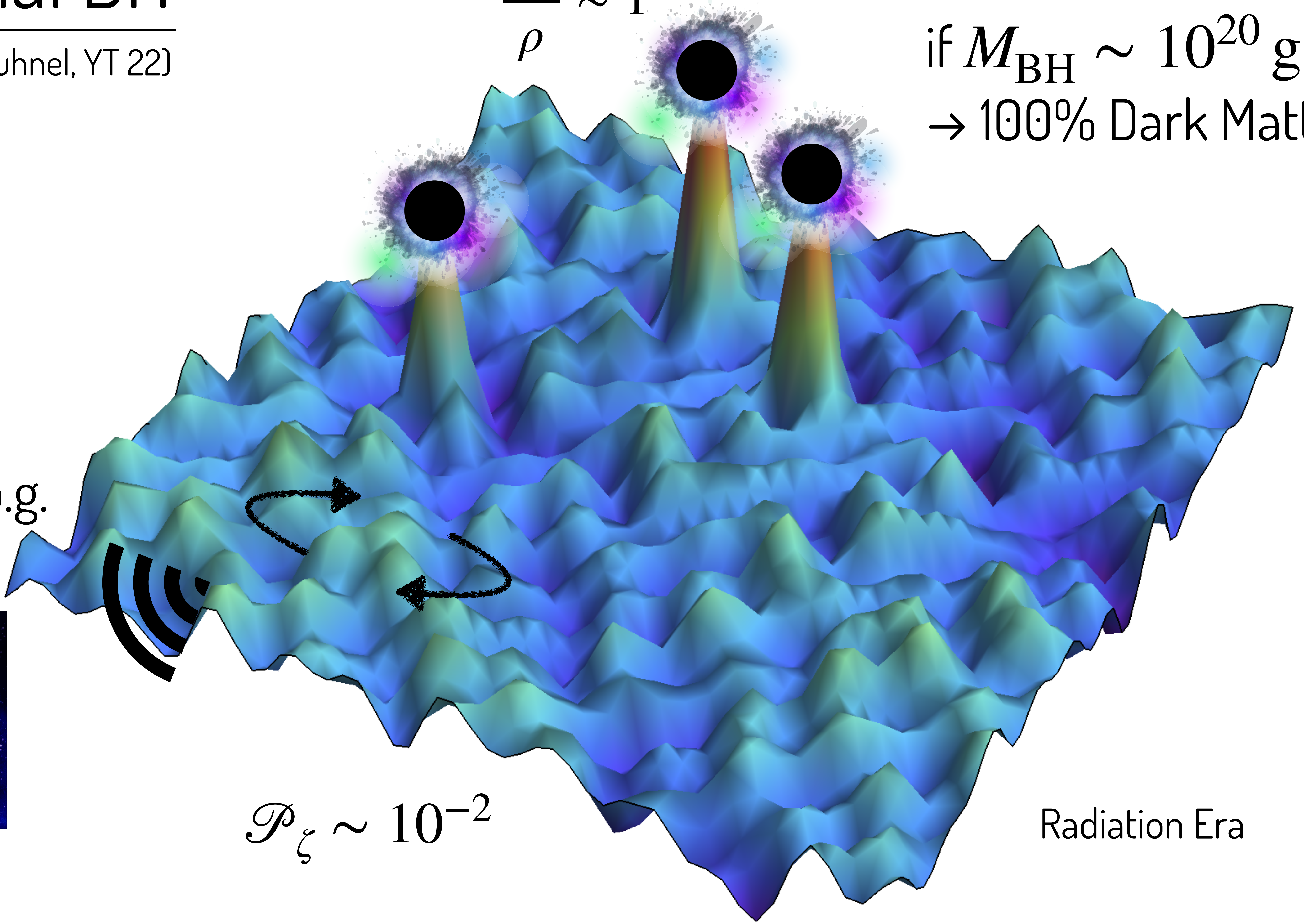
If perturbations get enhanced on small scales...

Primordial BH

Carr & Hawking '74 (cf. Escrivà, Kuhnel, YT 22)

$\frac{\delta\rho}{\rho} \sim 1$

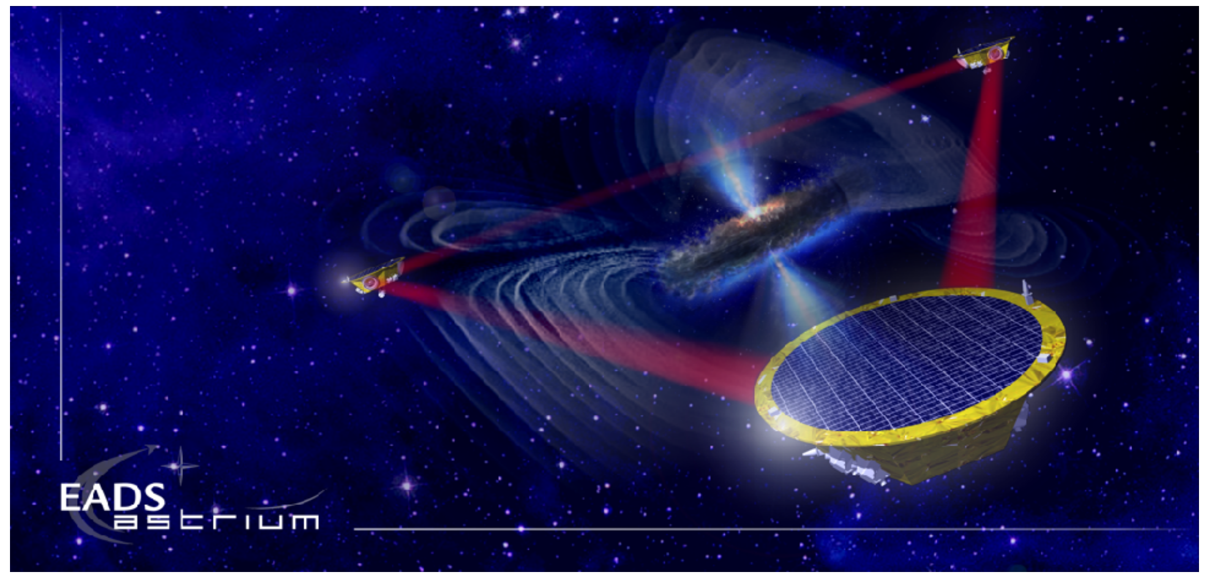
if $M_{\rm BH} \sim 10^{20}\,\rm g$
→ 100% Dark Matter



induced GW b.g.

$\mathscr{P}_\zeta \sim 10^{-2}$

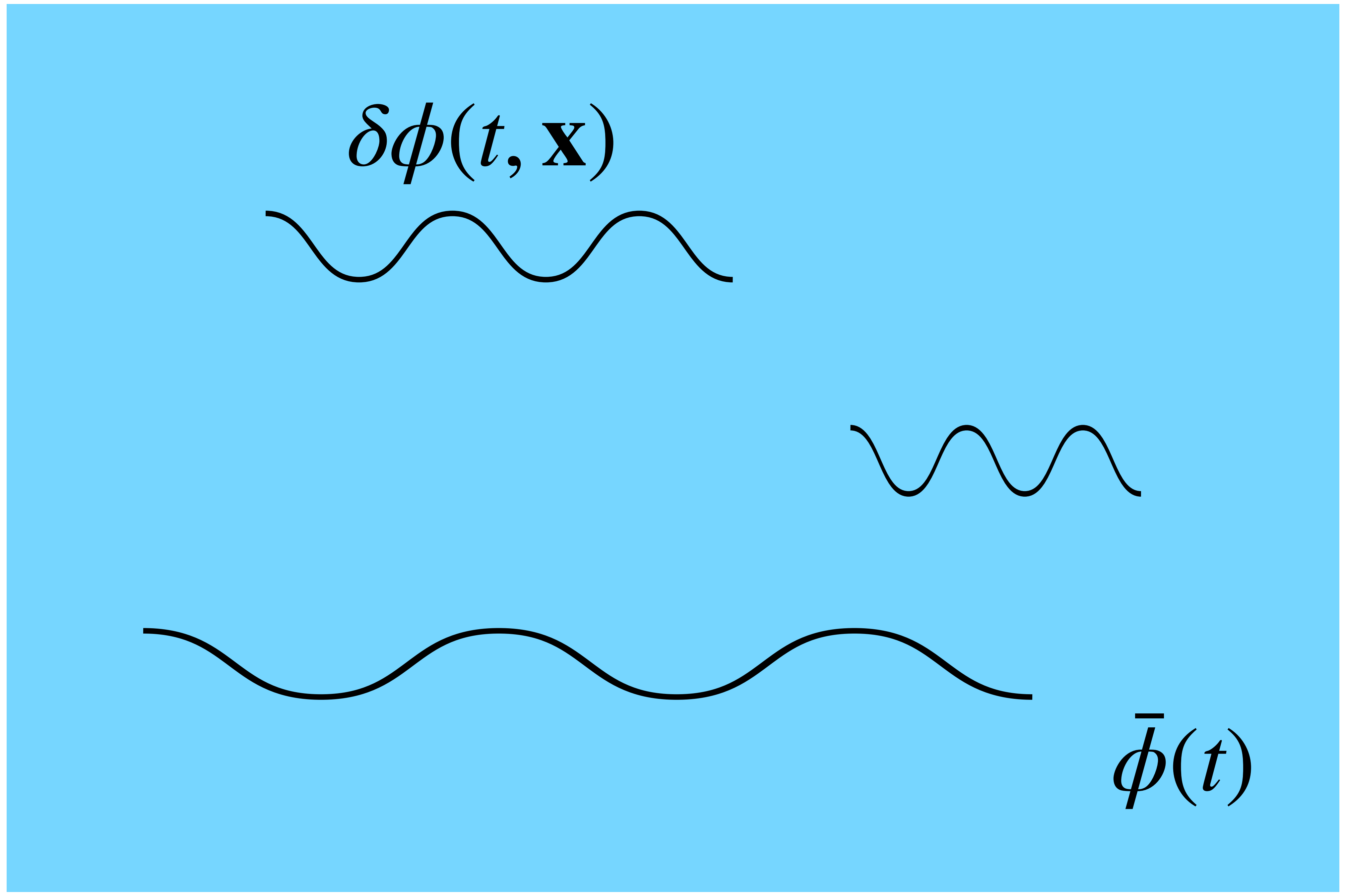
Radiation Era



LISA

Large PTB?

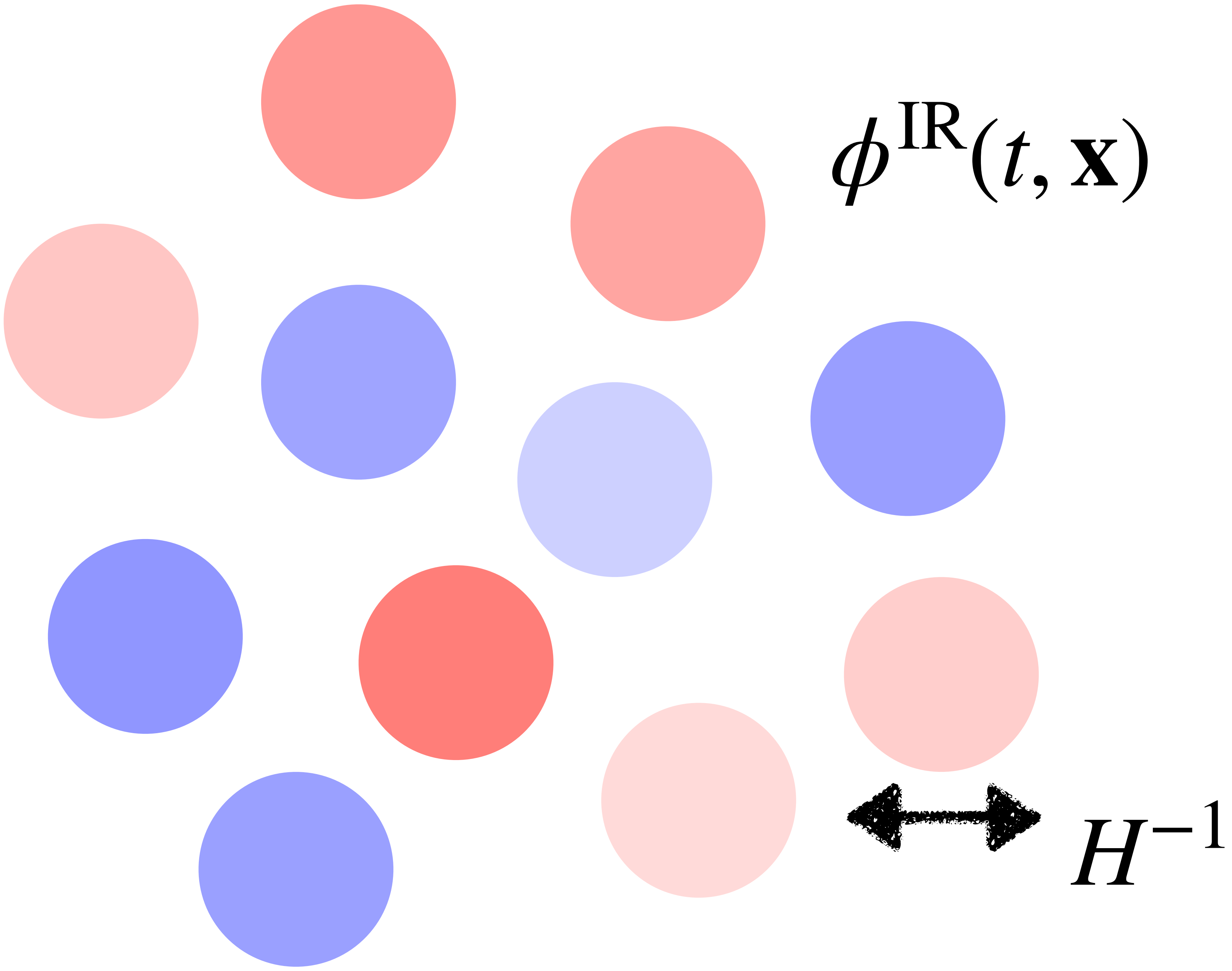
- PTB theory for small PTB



$$\begin{cases} \phi(t, \mathbf{x}) = \bar{\phi}(t) + \delta\phi(t, \mathbf{x}) \\ g_{\mu\nu}(t, \mathbf{x}) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t, \mathbf{x}) \end{cases}$$

- even for large PTB
- only for superH

- Stochastic approach



$$\begin{cases} \phi(t, \mathbf{x}) = \phi^{\mathrm{IR}}(t, \mathbf{x}) + \cancel{\phi^{\mathrm{UV}}(t, \mathbf{x})} \\ g_{\mu\nu}(t, \mathbf{x}) = g^{\mathrm{IR}}_{\mu\nu}(t, \mathbf{x}) + \cancel{g^{\mathrm{UV}}_{\mu\nu}(t, \mathbf{x})} \end{cases}$$

Integrate out

2. Stochastic-δN formalism

Stochastic approach

Starobinsky '86

locally FLRW $\mathrm{d}s_{\mathrm{IR}}^2 = -\frac{1}{H^2(t,\mathbf{x})}\mathrm{d}N^2 + \mathrm{e}^{2N}\mathrm{d}\mathbf{x}^2$

$$\phi(t,\mathbf{x}) = \phi^{\mathrm{IR}}(t,\mathbf{x}) + \phi^{\mathrm{UV}}(t,\mathbf{x}), \quad g_{\mu\nu}(t,\mathbf{x}) = g_{\mu\nu}^{\mathrm{IR}}(t,\mathbf{x}) + g_{\mu\nu}^{\mathrm{UV}}(t,\mathbf{x})$$

$$Z = \int_C \mathscr{D}\phi^{\mathrm{IR}}\mathscr{D}\phi^{\mathrm{UV}}\mathscr{D}g_{\mu\nu}^{\mathrm{UV}}\mathrm{e}^{i(S^{[0]}+S^{[1]}+S^{[2]})}$$

UV free prop.

Horizon exit

UV → IR

const. eq. such as

$$3M_{\mathrm{Pl}}^2H^2(t,\mathbf{x}) = \rho\left(\phi^{\mathrm{IR}}(t,\mathbf{x})\right)$$

$$= \int_C \mathscr{D}\phi^{\mathrm{IR}}\exp\left[iS^{[0]}[\phi^{\mathrm{IR}}] - \phi^{\mathrm{IR}}\cdot\Pi^{\mathrm{UV}}\cdot\phi^{\mathrm{IR}}\right]$$

IR IR

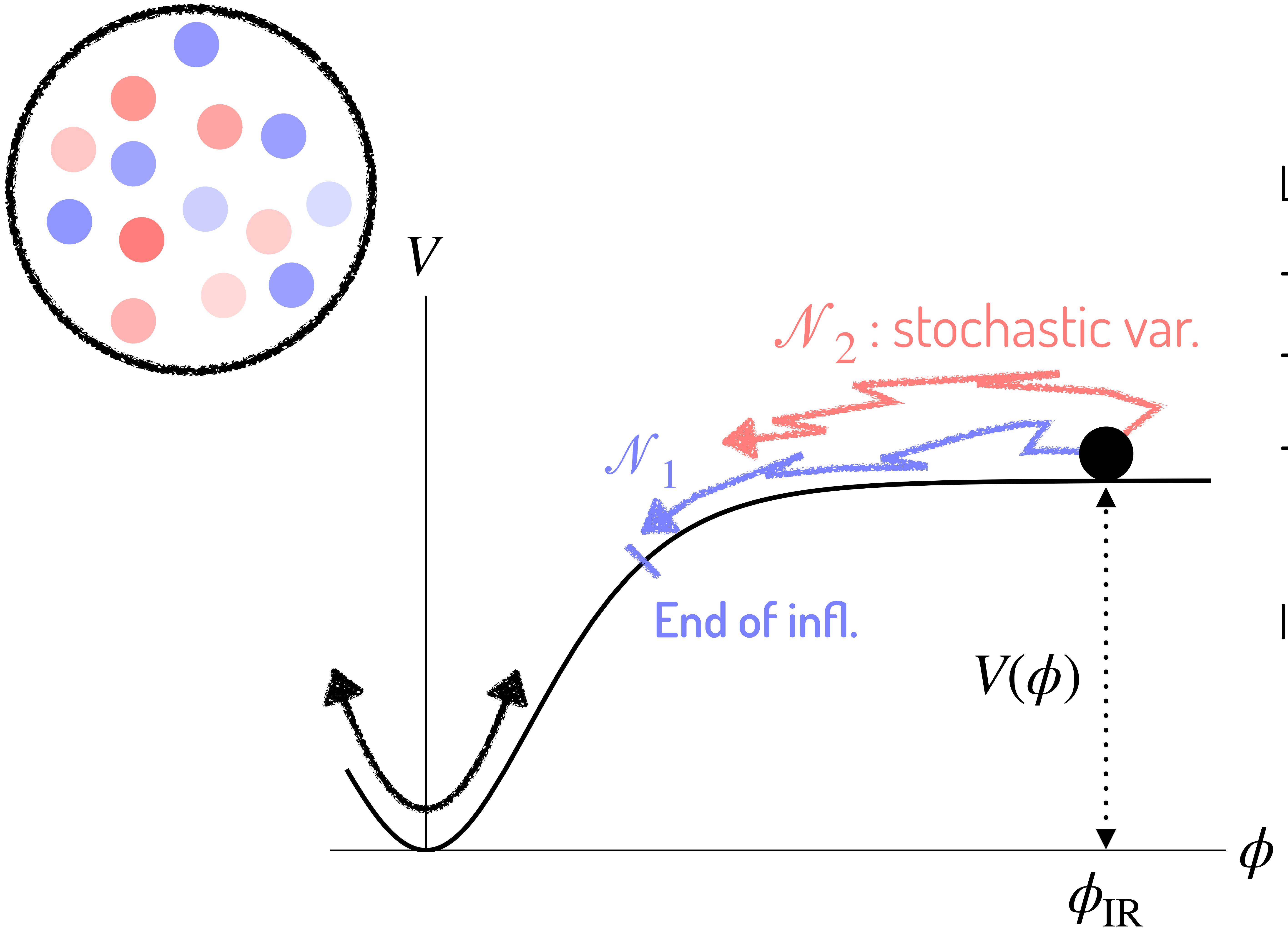
$$= \int_C \mathscr{D}\xi P_{\mathrm{G}}[\xi]\int_C \mathscr{D}\phi^{\mathrm{IR}}\exp\left[iS^{[0]}[\phi^{\mathrm{IR}}] + i\int \mathrm{d}^4x\phi^{\mathrm{IR}}(x)\xi(x)\right]$$

Stochastic Differential Equation

$$\frac{\mathrm{d}\phi^{\mathrm{IR}}}{\mathrm{d}N}(N,\mathbf{x}) \overset{\mathrm{SR}}{=} -\frac{V'(\phi^{\mathrm{IR}}(N,\mathbf{x}))}{3H^2(N,\mathbf{x})} + \xi(N,\mathbf{x}), \qquad \langle\xi(N,\mathbf{x})\xi(N',\mathbf{x}')\rangle = \left(\frac{H}{2\pi}\right)^2\delta(N-N')\frac{\sin(aHr)}{aHr}$$

Stochastic-δN

Kawasaki, YT '15 + Fujita '14 + Takesako '13



Lyth, Malik, Sasaki '05 : the time difference δN is ...

- conserved on superH after inflation
- $\delta\rho \simeq -\frac{\dot\rho}{H}\delta N$
- equivalent to the curv. ptb. ζ

In the stochastic form.

$$\zeta_{H_{\rm inf}^{-1}}(\mathbf{x}) = \delta\mathcal{N}(\mathbf{x}) = \mathcal{N}(\mathbf{x}) - \langle\mathcal{N}\rangle$$

Adjoint FP

Vennin & Starobinsky '15

Langevin eq. : $\dfrac{\mathrm{d}\phi}{\mathrm{d}N} = -\dfrac{V'}{3H^2} + \dfrac{H}{2\pi}\xi$

$\Leftrightarrow$ Fokker–Planck eq. : $\partial_N P(\phi \mid N) = \mathscr{L}_{\mathrm{FP}} \cdot P(\phi \mid N) = \partial_\phi \left[\dfrac{V'}{3H^2} P(\phi \mid N)\right] + \partial_\phi^2 \left[\dfrac{1}{2}\left(\dfrac{H}{2\pi}\right)^2 P(\phi \mid N)\right]$

with the absorption b.c. $P(\phi = \phi_{\mathrm{f}} \mid N) = 0$ at the end of inflation ϕ_{f}

$\Leftrightarrow$ Adjoint FP eq. : $\partial_{\mathcal{N}} P_{\mathrm{FPT}}(\mathcal{N} \mid \phi) = \mathscr{L}_{\mathrm{FP}}^\dagger \cdot P_{\mathrm{FPT}}(\mathcal{N} \mid \phi) = -\dfrac{V'}{3H^2}\partial_\phi P_{\mathrm{FPT}}(\mathcal{N} \mid \phi) + \dfrac{1}{2}\left(\dfrac{H}{2\pi}\right)^2 \partial_\phi^2 P_{\mathrm{FPT}}(\mathcal{N} \mid \phi)$

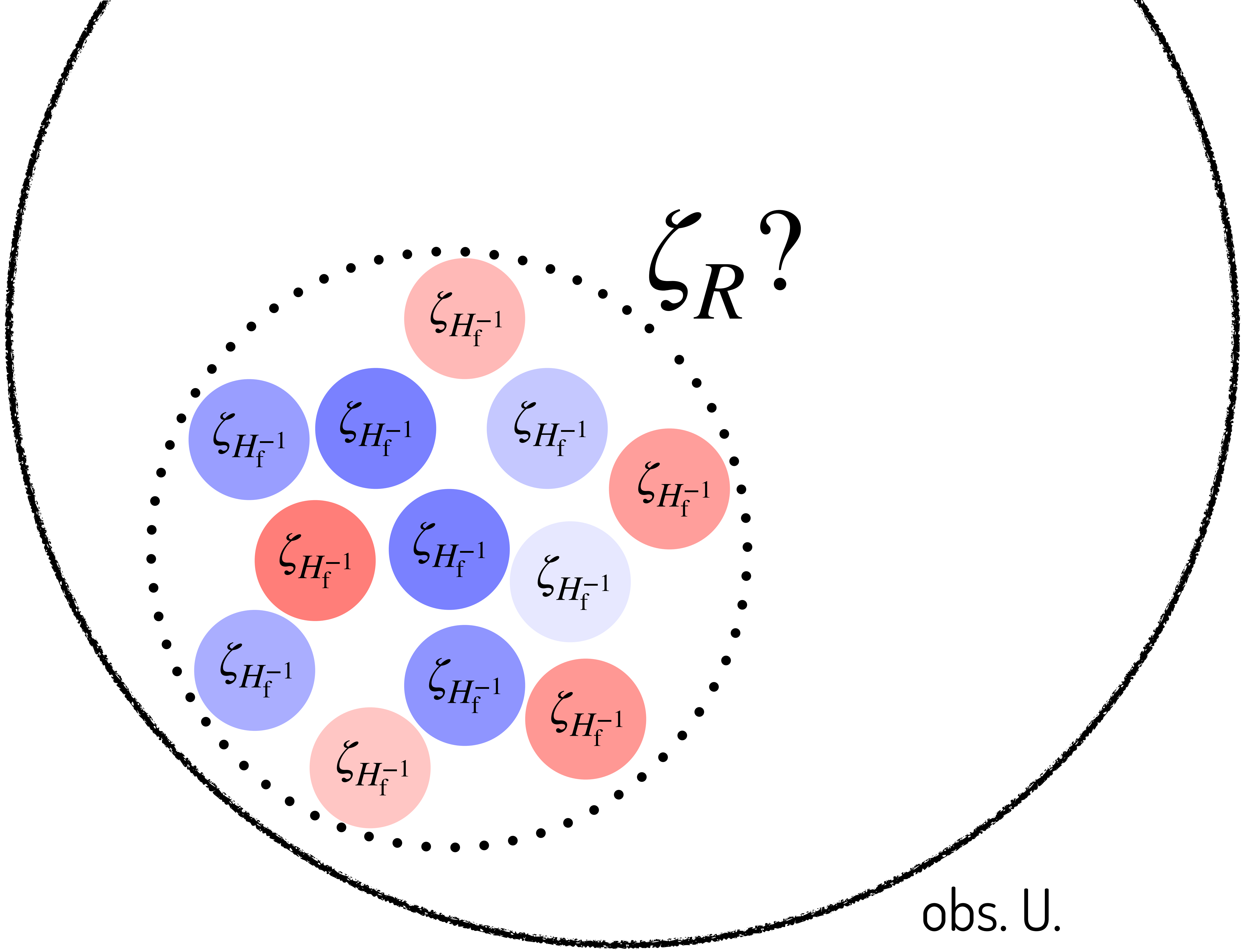
with the b.c. $P_{\mathrm{FPT}}(\mathcal{N} \mid \phi = \phi_{\mathrm{f}}) = \delta(\mathcal{N})$

$\Leftrightarrow$ Series of PDE : $\mathscr{L}_{\mathrm{FP}}^\dagger \cdot \langle \mathcal{N}^n(\phi)\rangle = -n\langle \mathcal{N}^{n-1}(\phi)\rangle, \quad \mathscr{L}_{\mathrm{FP}}^\dagger \cdot \langle \delta\mathcal{N}^2(\phi)\rangle = -\left(\dfrac{H}{2\pi}\right)^2 \Big(\partial_\phi \langle \mathcal{N}(\phi)\rangle\Big)^2$

with the b.c. $\langle \mathcal{N}^n(\phi_{\mathrm{f}})\rangle = 0$

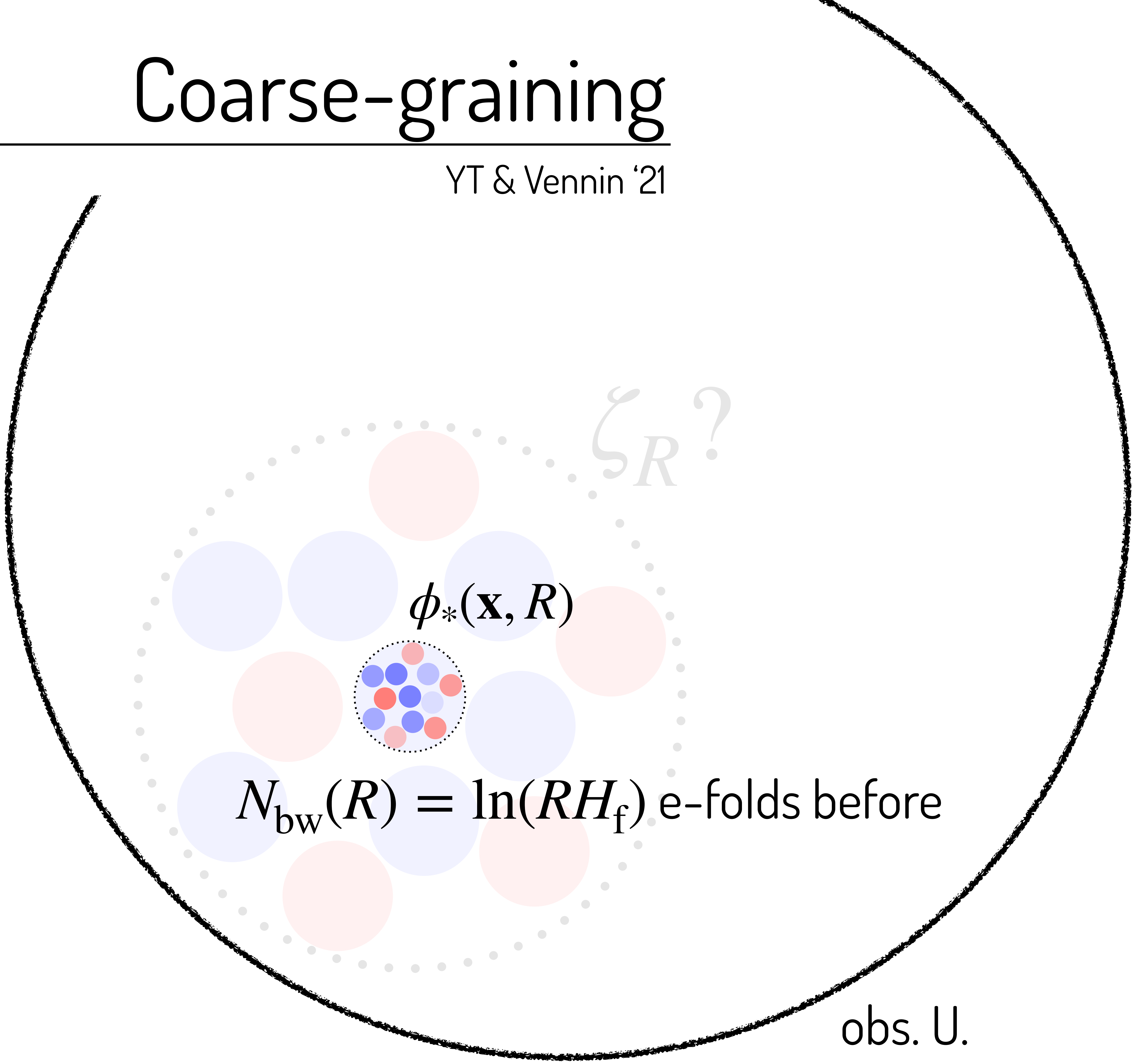
Coarse-graining

YT & Vennin '21



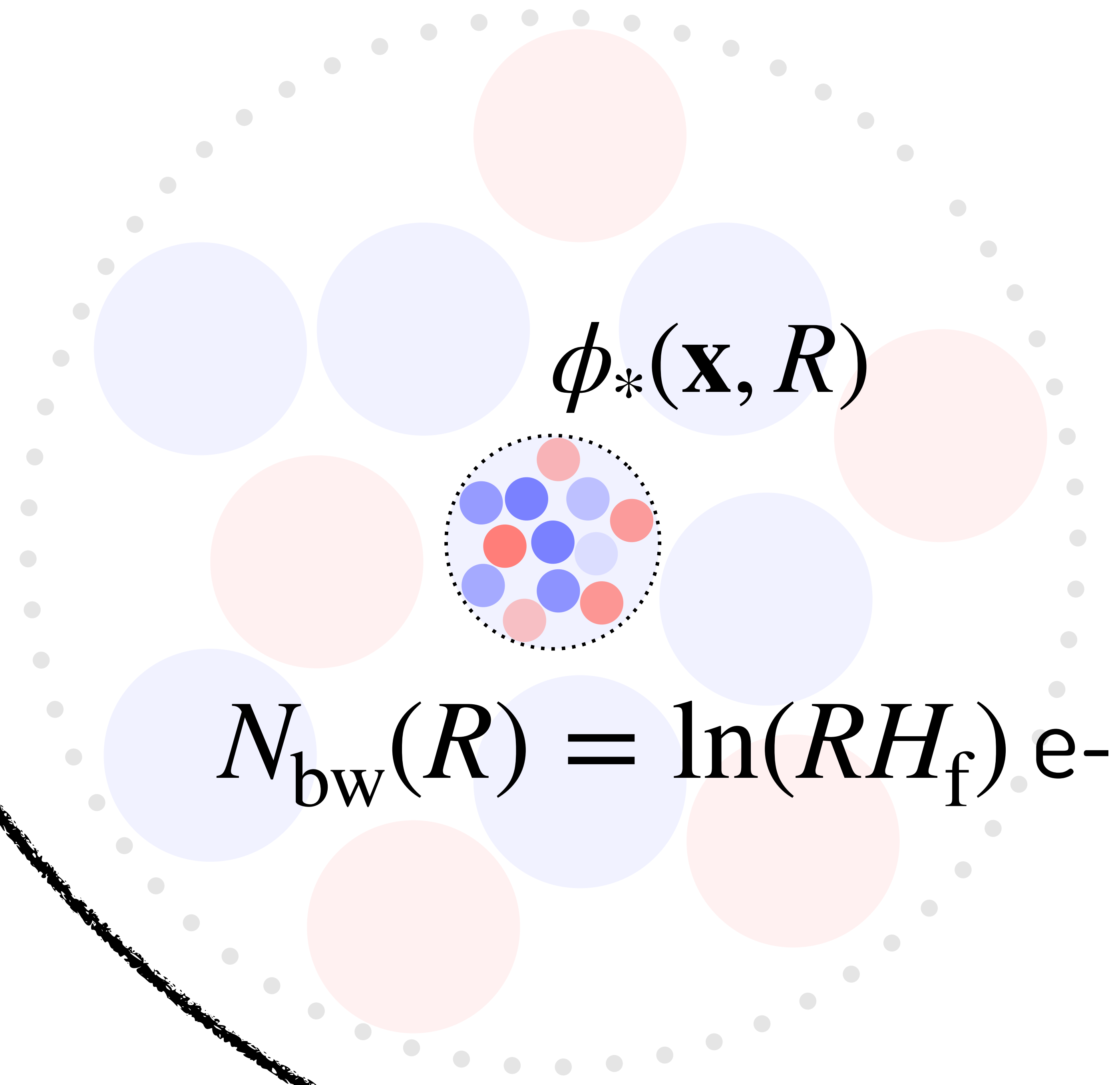
Coarse-graining

YT & Vennin '21



Coarse-graining

YT & Vennin '21



same dyn. until ϕ_*

$$\zeta_R(\mathbf{x}) = \mathcal{N}(\phi_{\mathrm{i}} \to \phi_*(\mathbf{x}, R)) + \langle \mathcal{N}(\phi_*(\mathbf{x}, R)) \rangle - \langle \mathcal{N}(\phi_{\mathrm{i}}) \rangle$$

independent after ϕ_*

$$P(\zeta_R) = \int \mathrm{d}\phi_*\, \mathbb{P}\left(\mathcal{N}_{\mathrm{i}\to *} = \langle \mathcal{N}_{\mathrm{i}} \rangle - \langle \mathcal{N}_* \rangle + \zeta_R\right) \times \mathbb{P}\left(\phi = \phi_* @ - N_{\mathrm{bw}}(R)\right)$$

obs. U.

Power spectrum

Ando & Vennin '20

$$\int^{-\ln\frac{R}{a}} \mathrm{d}\ln k\,\mathcal{P}_\zeta(k) = \langle\zeta_R^2\rangle = \int \mathrm{d}\zeta_R\,\zeta_R^2 P(\zeta_R)$$

$$\Rightarrow \mathcal{P}_\zeta(k) = -\left.\frac{\mathrm{d}\langle\zeta_R^2\rangle}{\mathrm{d}\ln R}\right|_{R=a/k} = -\int \mathrm{d}\phi_*\mathrm{d}\zeta_R\,\zeta_R^2\,\mathbb{P}\left(\mathcal{N}_{\mathrm{i}\to *} = \langle\mathcal{N}_\mathrm{i}\rangle - \langle\mathcal{N}_*\rangle + \zeta_R\right) \times \left.\frac{\partial\mathbb{P}\left(\phi = \phi_*@-N_\mathrm{bw}\right)}{\partial N_\mathrm{bw}}\right|_{N_\mathrm{bw}=\ln\frac{a_\mathrm{f}H_\mathrm{f}}{k}}$$

$$= -\int \mathrm{d}\phi_*\left\langle\left(\mathcal{N}_{\mathrm{i}\to *} - \langle\mathcal{N}_\mathrm{i}\rangle + \langle\mathcal{N}_*\rangle\right)^2\right\rangle \left.\frac{\partial\mathbb{P}\left(\phi = \phi_*@-N_\mathrm{bw}\right)}{\partial N_\mathrm{bw}}\right|_{N_\mathrm{bw}=\ln\frac{a_\mathrm{f}H_\mathrm{f}}{k}}$$

Power spectrum

Renaux-Petal, YT, Vennin in prep.
YT & Yamada '23b

$$\mathscr{P}_\zeta(k) = -\int \mathrm{d}\phi_* \left\langle \left(\mathscr{N}_{\mathrm{i}\to *} - \langle \mathscr{N}_\mathrm{i} \rangle + \langle \mathscr{N}_* \rangle \right)^2 \right\rangle \frac{\partial \mathbb{P}\left(\phi = \phi_* @ -N_\mathrm{bw}\right)}{\partial N_\mathrm{bw}} \Bigg|_{N_\mathrm{bw} = \ln \frac{a_\mathrm{f} H_\mathrm{f}}{k}}$$

$$\mathbb{P}\left(\phi = \phi_* @ -N_\mathrm{bw}\right) \approx \frac{1}{S} \sum_{i=1}^{S} \delta\left(\phi_i(\mathscr{N}_i - N_\mathrm{bw}) - \phi_*\right)$$

$$\frac{\partial \mathbb{P}\left(\phi = \phi_* @ -N_\mathrm{bw}\right)}{\partial N_\mathrm{bw}} \approx \frac{1}{S \times \Delta N} \sum_{i=1}^{S} \left[\delta\left(\phi_i^- - \phi_*\right) - \delta\left(\phi_i^+ - \phi_*\right) \right]$$

$$\phi_i^\pm = \phi_i(\mathscr{N}_i - N_\mathrm{bw} \pm \Delta N)$$

$$\mathscr{P}_\zeta(k) \approx \frac{1}{S \times \Delta N} \sum_{i=1}^{S} \left[\left\langle \left(\mathscr{N}_{\mathrm{i}\to i^+} - \langle \mathscr{N}_\mathrm{i} \rangle + \langle \mathscr{N}_{i^+} \rangle \right)^2 \right\rangle - \left\langle \left(\mathscr{N}_{\mathrm{i}\to i^-} - \langle \mathscr{N}_\mathrm{i} \rangle + \langle \mathscr{N}_{i^-} \rangle \right)^2 \right\rangle \right]$$

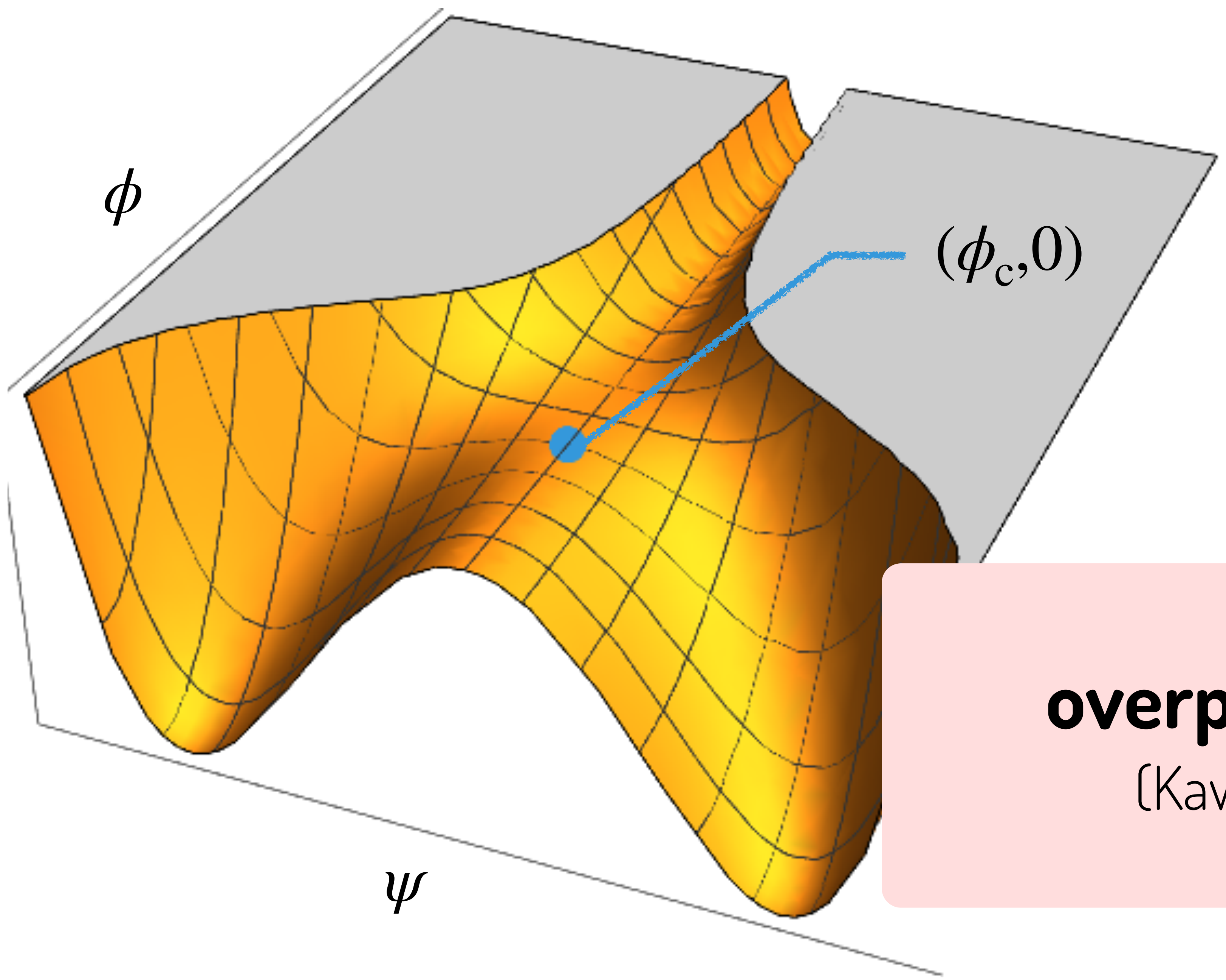
$$\approx \frac{1}{S \times \Delta N} \sum_{i=1}^{S} \left[\left\langle \delta\mathscr{N}^2(\phi_i^-) \right\rangle - \left\langle \delta\mathscr{N}^2(\phi_i^+) \right\rangle \right]$$

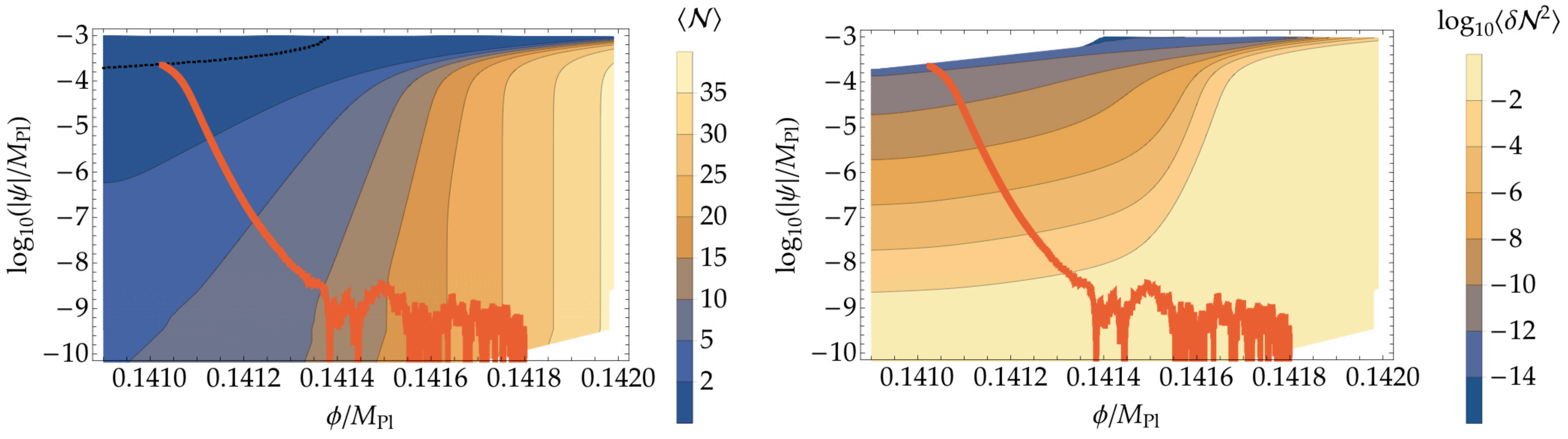
3. Application

Hybrid inflation

Kawasaki & YT '15

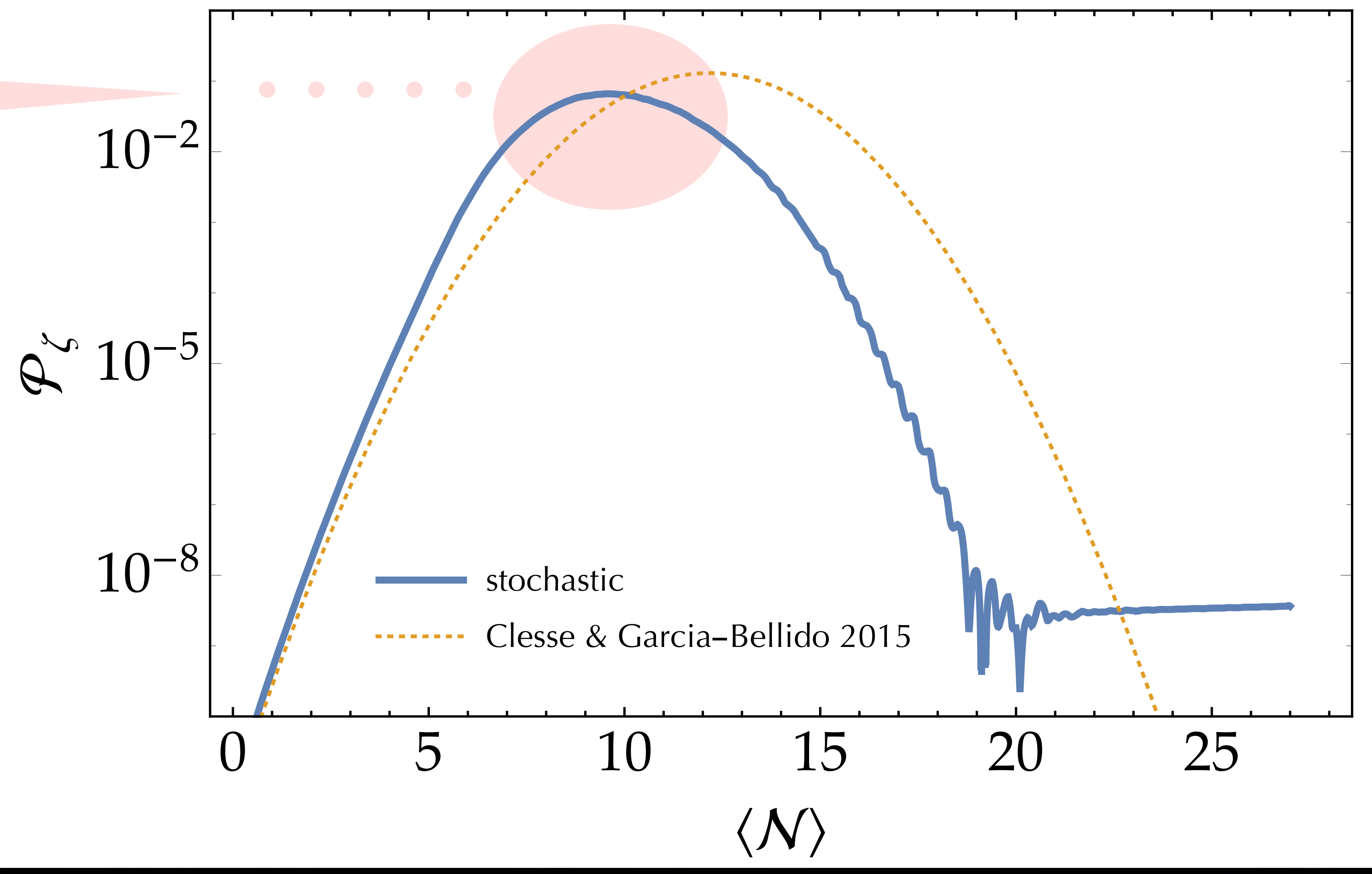
$$V(\phi,\psi)=\Lambda^4\left[\left(1-\frac{\psi^2}{M^2}\right)^2+2\frac{\phi^2\psi^2}{\phi_c^2M^2}+\frac{\phi-\phi_c}{\mu_1}-\frac{(\phi-\phi_c)^2}{\mu_2^2}\right]$$





overproduce PBHs

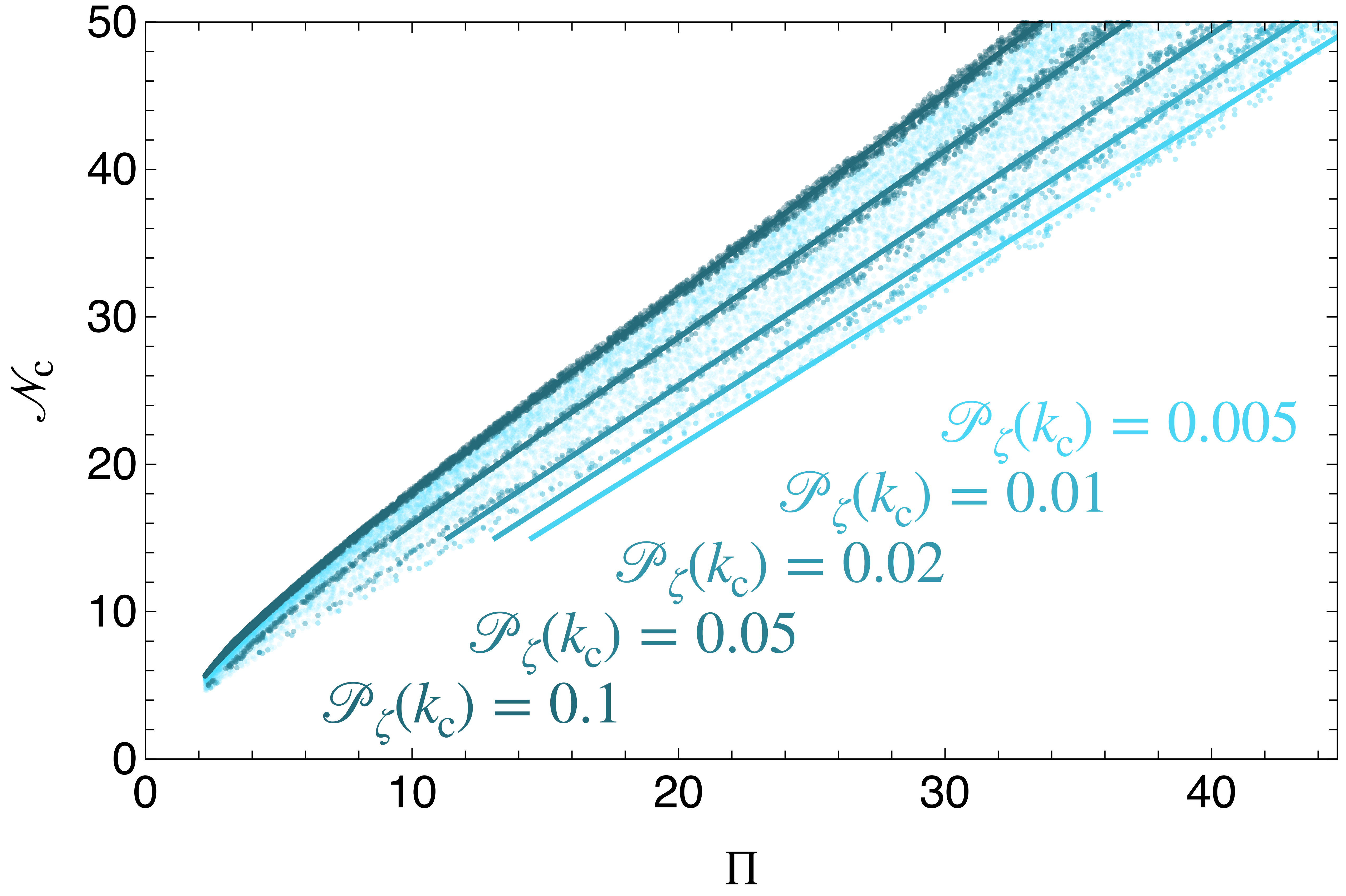
(Kawasaki & YT '15)



~~No-Go~~ 1: ϕ^3

YT & Yamada '23a

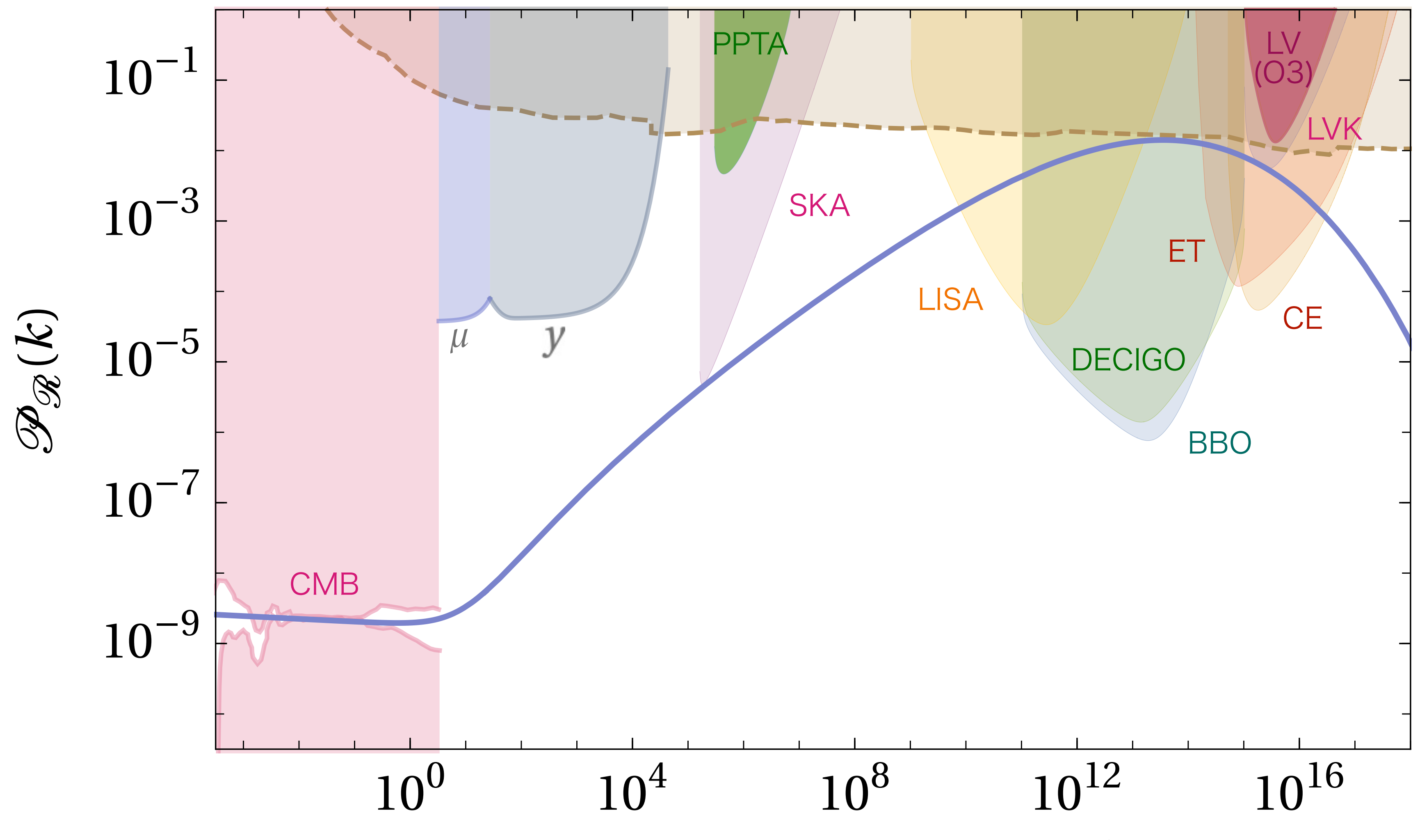
$$V(\phi,\psi)=\Lambda^4\left[\left(1-\frac{\psi^2}{M^2}\right)^2+2\frac{\phi^2\psi^2}{\phi_c^2M^2}+\frac{\phi-\phi_c}{\mu_1}-\frac{(\phi-\phi_c)^2}{\mu_2^2}+\boxed{\frac{(\phi-\phi_c)^3}{\mu_3^3}}\right]$$

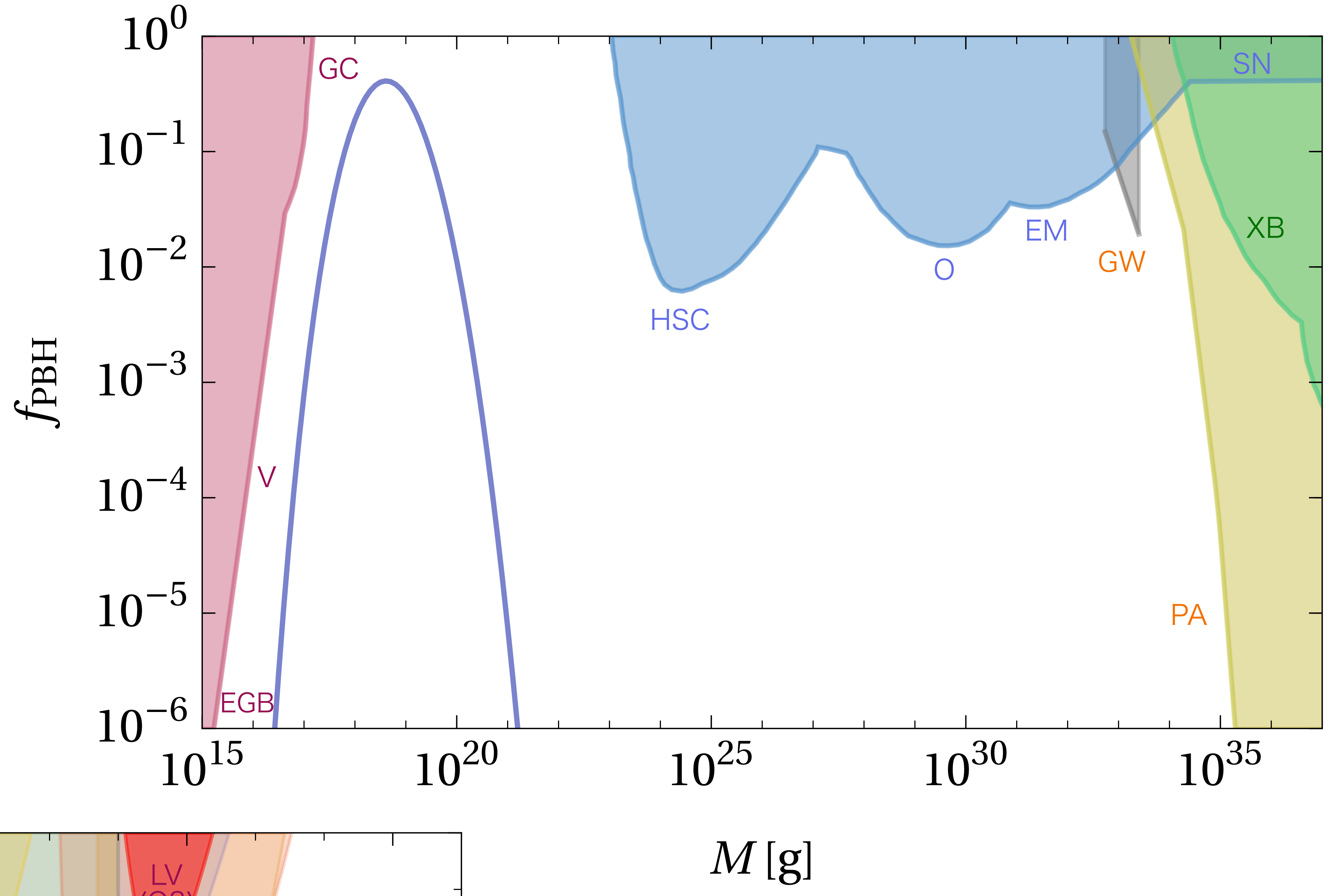


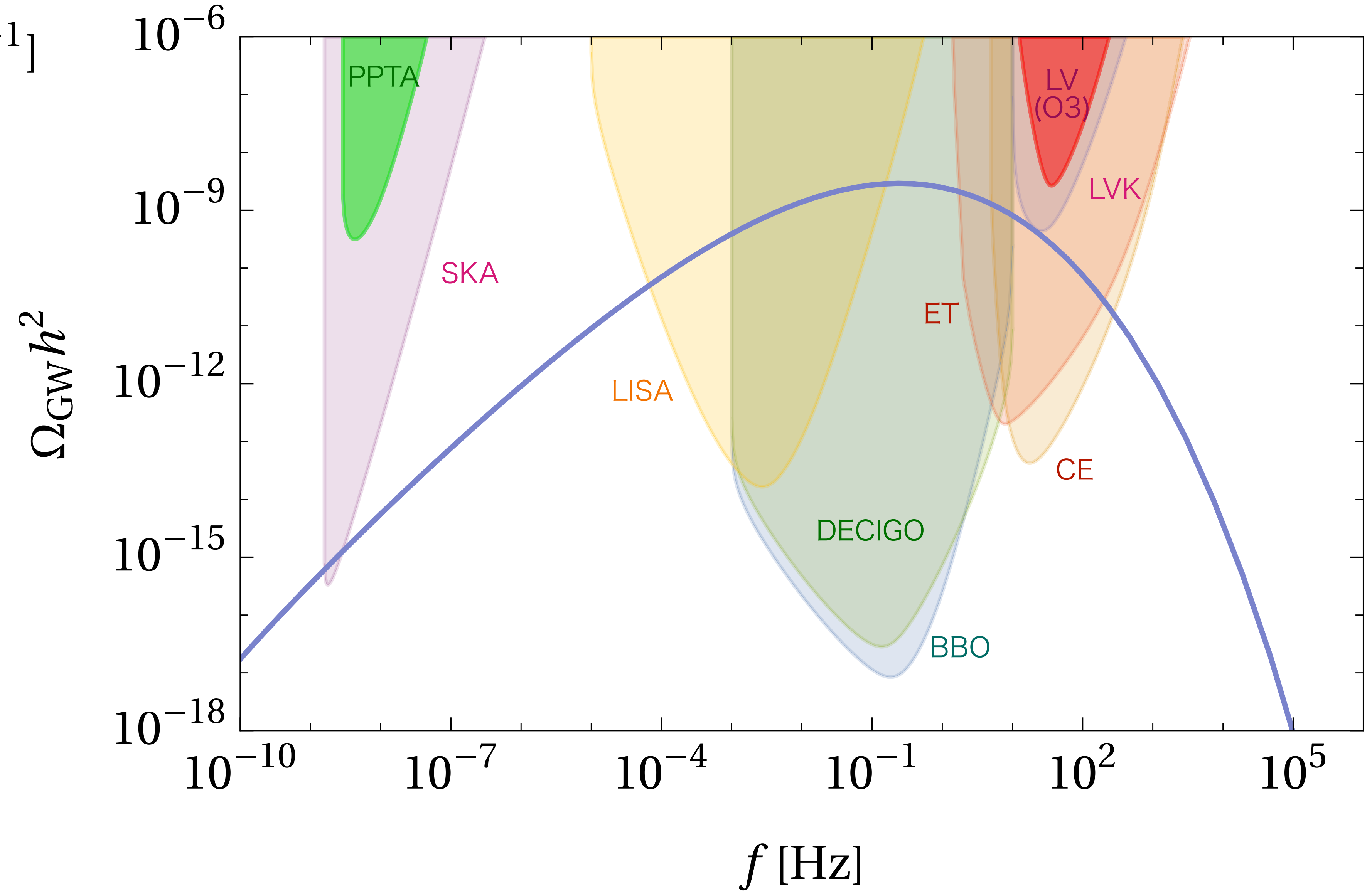
$$\left|\frac{\mathrm{d}\ln\mathcal{P}_\zeta(k_c)}{\mathrm{d}\ln\mu_3}\right|\sim 1$$ mild tuning

~~No Go~~ 1 : ϕ^3

YT & Yamada ‘23a







$$M = \phi_c/\sqrt{2} = M_{\mathrm{Pl}}/10$$

$$\Pi^2 = 185$$

$$\mu_2 = 4.21 M_{\mathrm{Pl}}$$

$$\mu_3 = 0.182 M_{\mathrm{Pl}}$$

~~No Go~~ 2 : multi-ψ

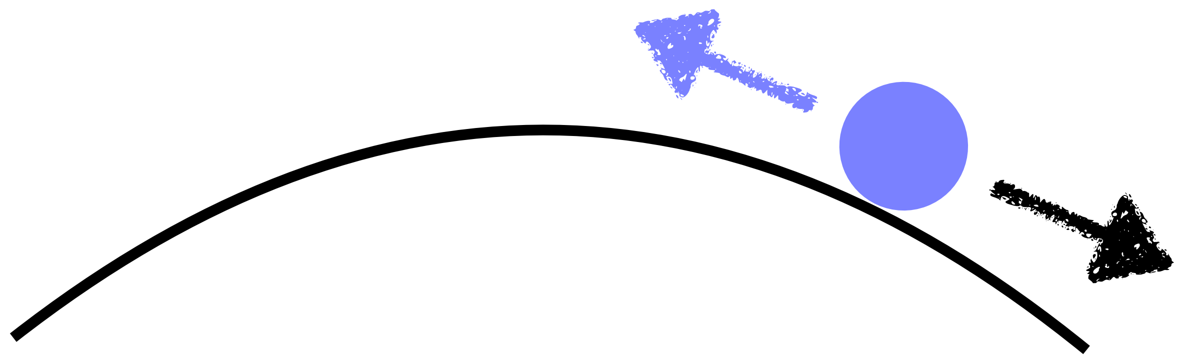
Halpern+ '14
YT & Yamada '23b

$$\psi \to \overrightarrow{\psi} = (\psi_1, \psi_2, \cdots, \psi_{\mathscr{D}})$$

$$\mathscr{L}^{\dagger}_{\mathrm{FP}} \cdot P_{\mathrm{FPT}}(\mathcal{N} \mid \overrightarrow{\psi}) = -M_{\mathrm{Pl}}^2 \frac{V_i}{V} \partial_i P_{\mathrm{FPT}}(\mathcal{N} \mid \overrightarrow{\psi}) + \frac{1}{2}\left(\frac{H}{2\pi}\right)^2 \partial_i^2 P_{\mathrm{FPT}}(\mathcal{N} \mid \overrightarrow{\psi})$$

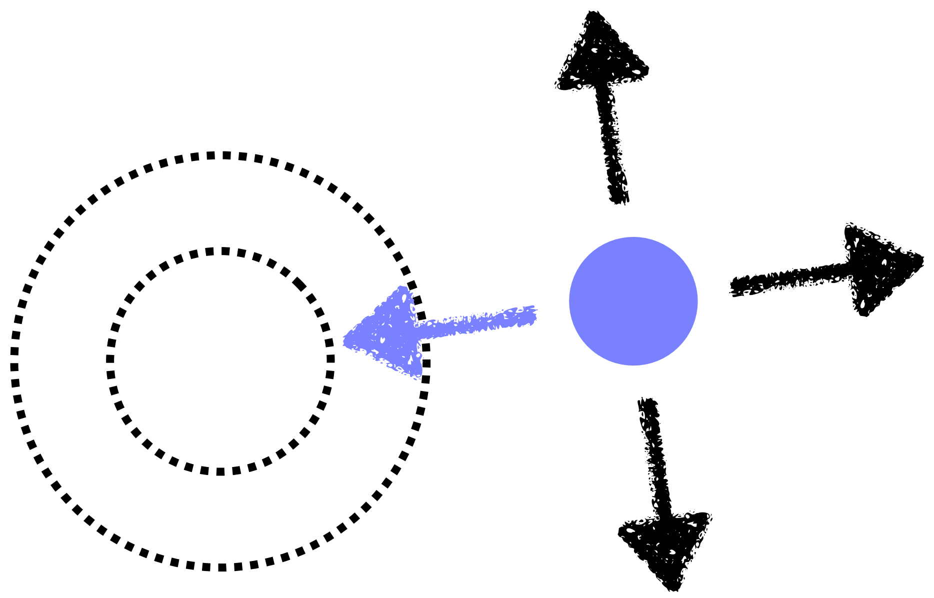
$$= \left[-M_{\mathrm{Pl}}^2 \frac{V_i}{V} + \underbrace{\frac{1}{2}\left(\frac{H}{2\pi}\right)^2 \frac{\mathscr{D}-1}{\psi_r}}_{\text{Centrifugal force}}\right] \partial_{\psi_r} P_{\mathrm{FPT}}(\mathcal{N} \mid \psi_r) + \frac{1}{2}\left(\frac{H}{2\pi}\right)^2 \partial_{\psi_r}^2 P_{\mathrm{FPT}}(\mathcal{N} \mid \psi_r)$$

$\mathscr{D} = 1:$



$p = \frac{1}{2}$

$\mathscr{D} = 2:$



$p = \frac{1}{4}$

$$\Rightarrow \quad \langle \psi_r^2 \rangle_{\mathrm{c}} = \mathscr{D} \, \langle \psi^2 \rangle_{\mathrm{c}} \Big|_{\mathscr{D}=1}$$

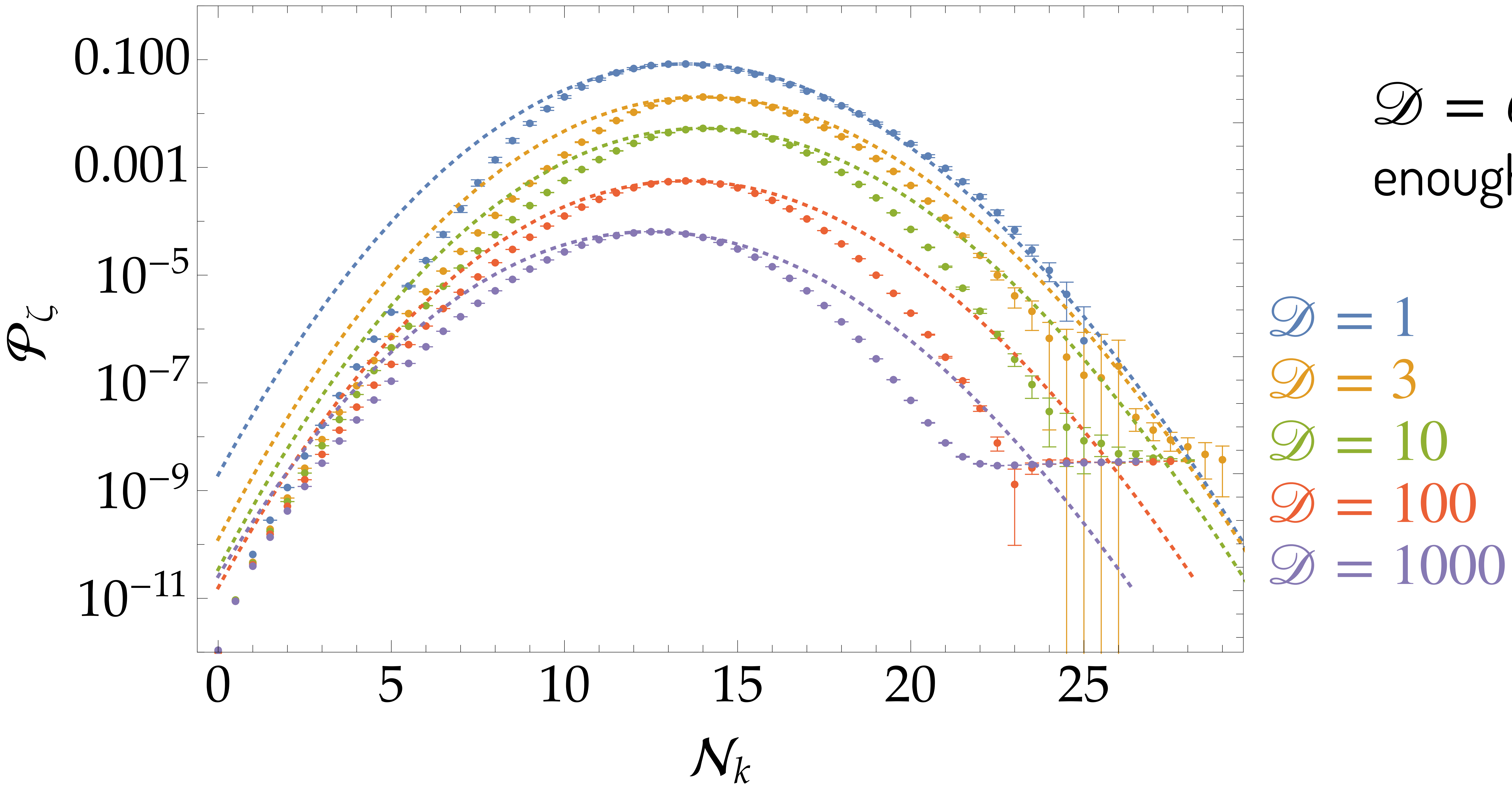
$$\Rightarrow \quad \mathscr{P}_\zeta^{\max}\Big|_{\mathscr{D}} = \frac{1}{\mathscr{D}} \mathscr{P}_\zeta^{\max}\Big|_{\mathscr{D}=1}$$

~~No Go~~ 2 : multi-ψ

YT & Yamada '23b

$$\psi \to \overrightarrow{\psi} = (\psi_1, \psi_2, \cdots, \psi_{\mathscr{D}})$$

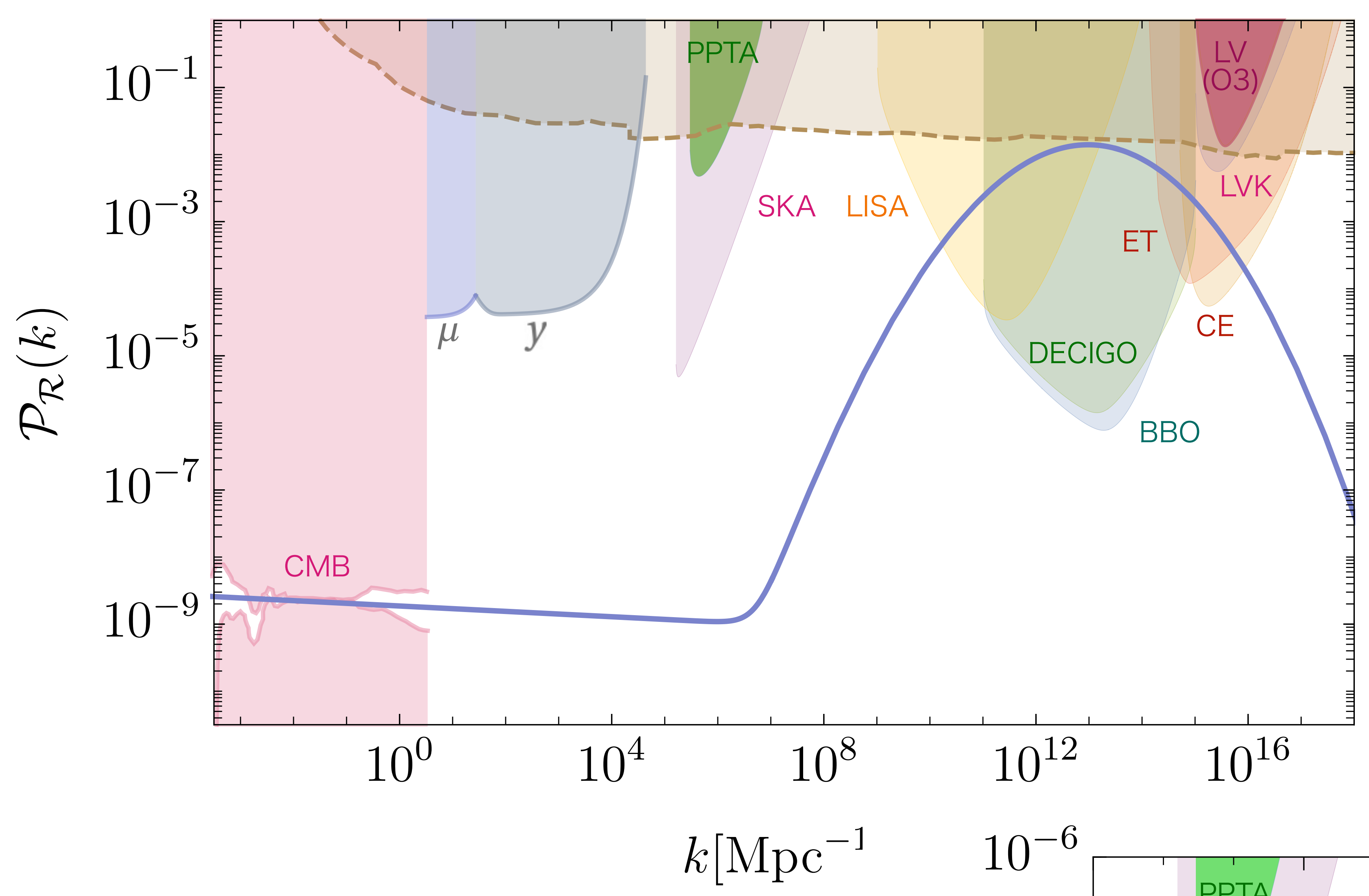
$\Pi^2 = 100$

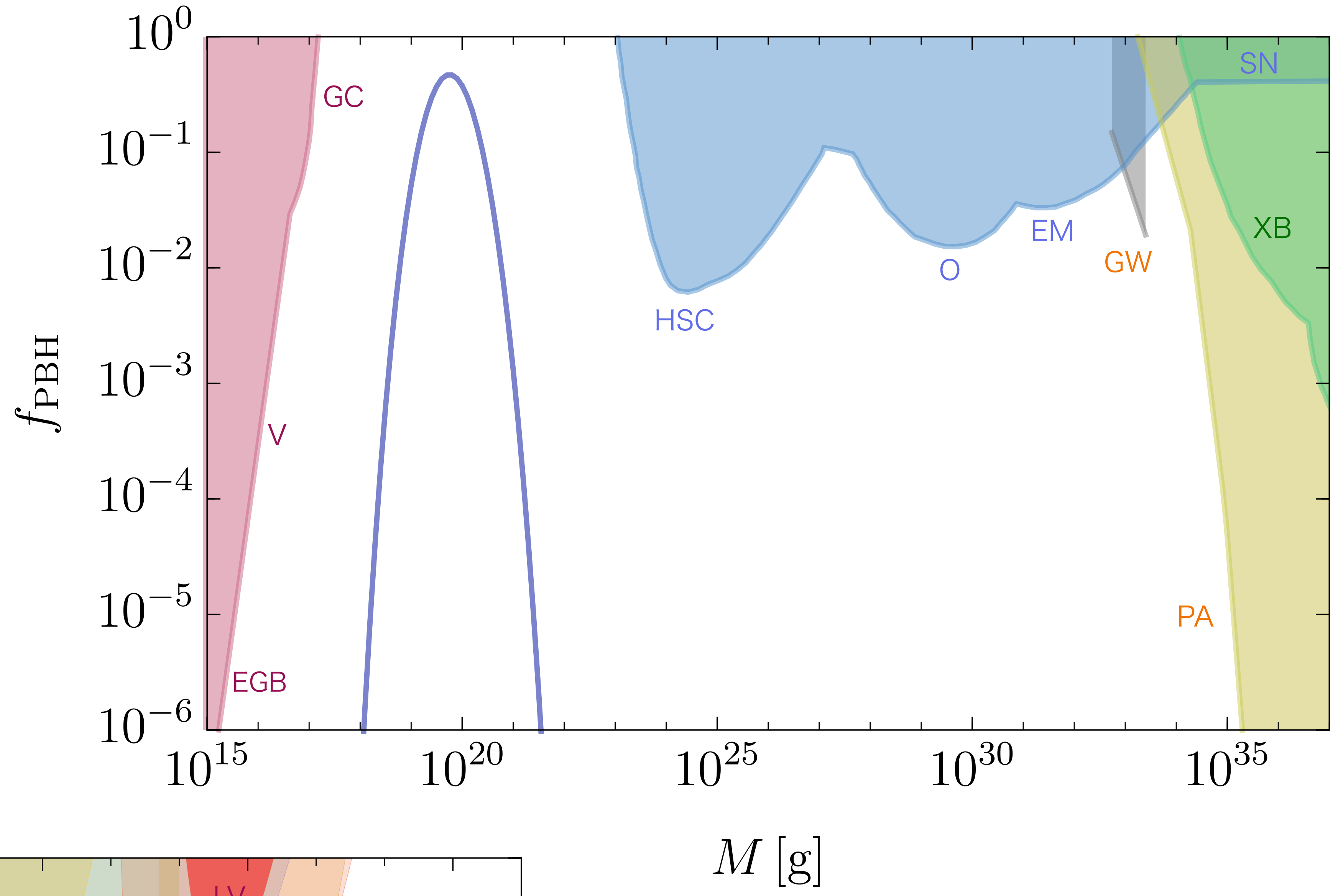


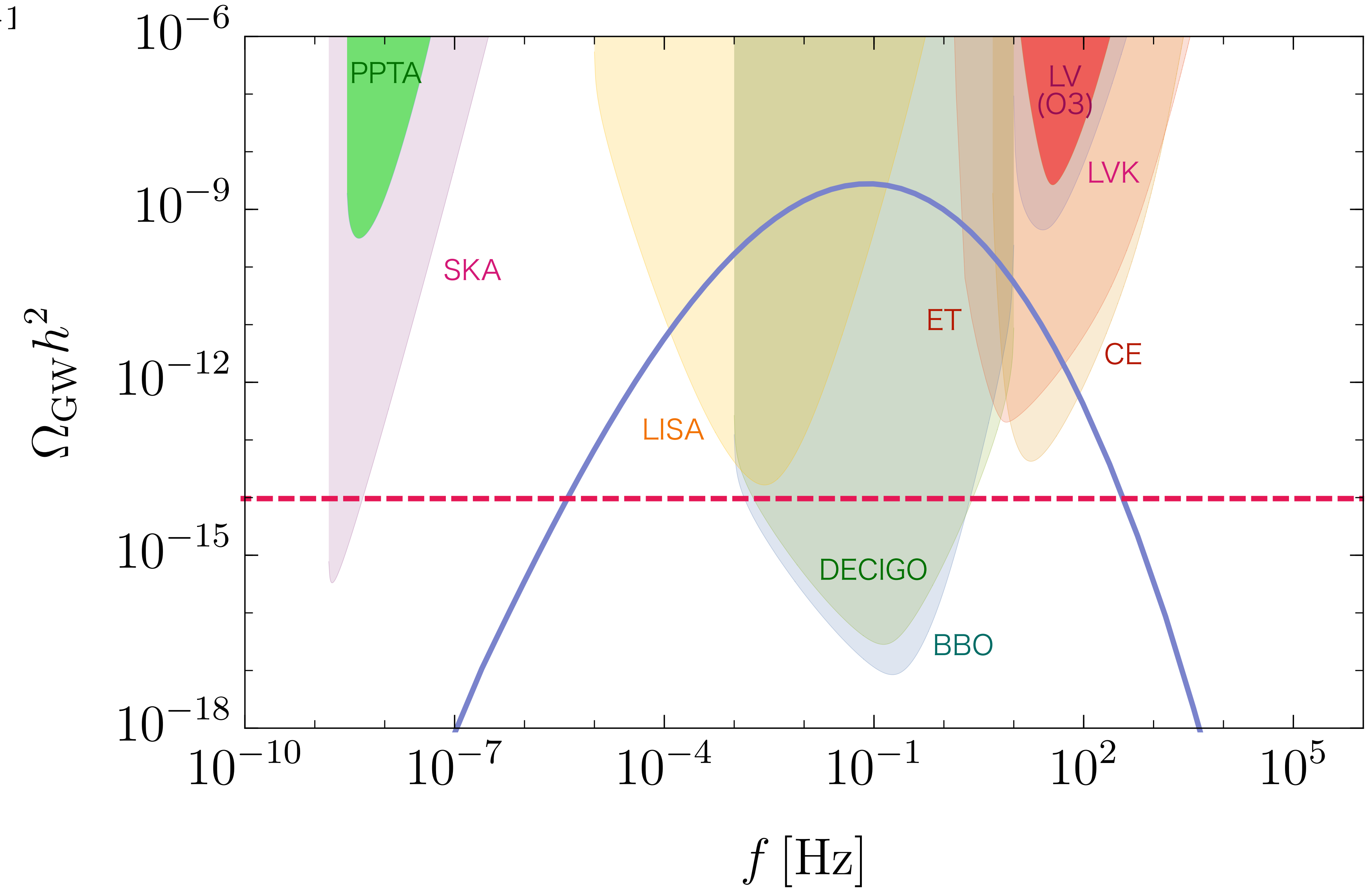
$\mathscr{D} = \mathcal{O}(1)$ would be enough for PBHs

~~No-Go~~ 2 : multi-ψ

YT & Yamada '23b







$M = \phi_c/\sqrt{2} = 10^{16}\,\mathrm{GeV}$

$\Pi^2 = 100$

$\mu_2 = 10M_\mathrm{Pl}$

$\mathscr{D} = 5$

δ as coarse-shelled ζ

YT & Vennin '21

- (linear) density contrast

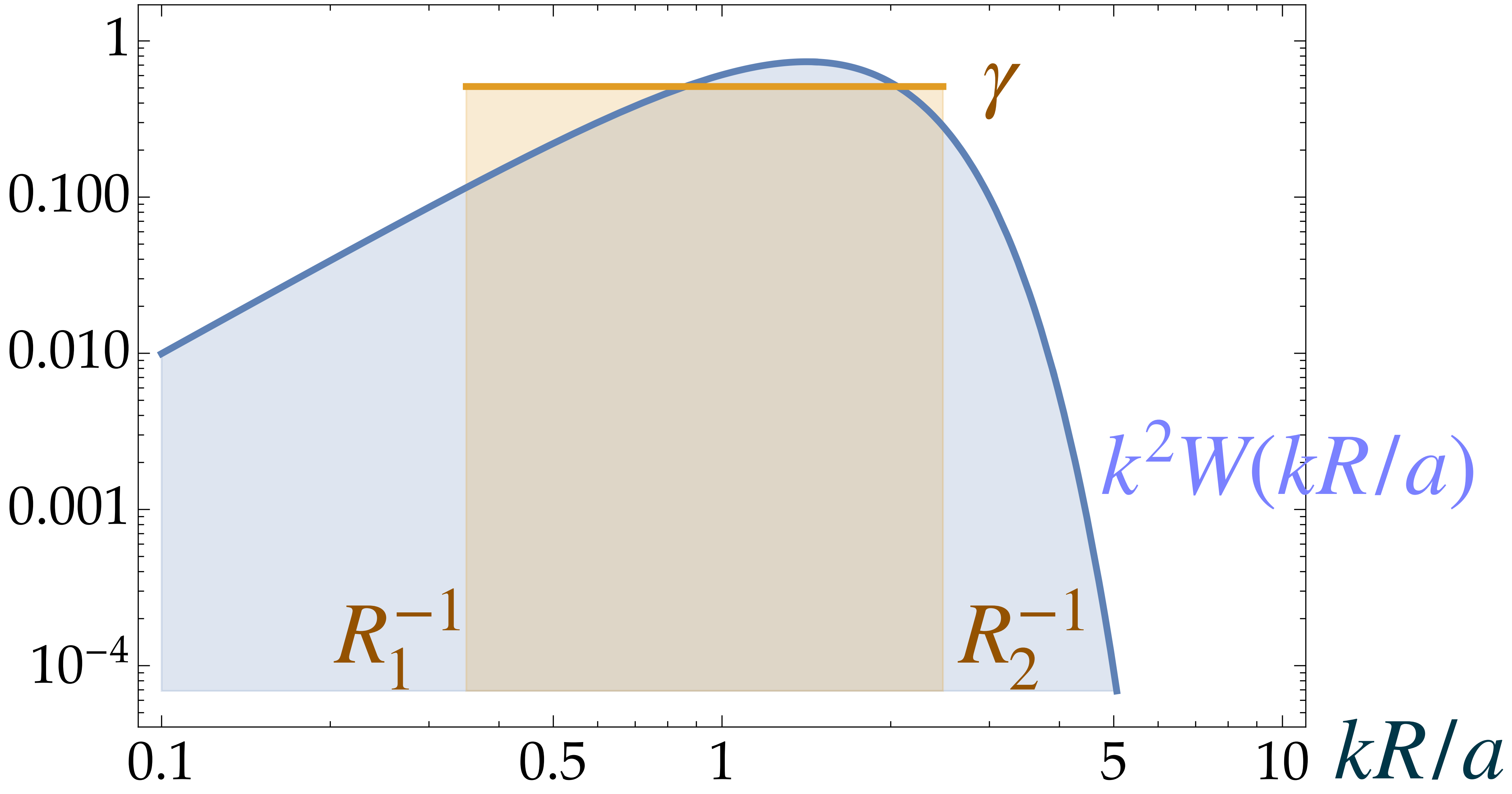
$$\delta = -\frac{9}{4}\frac{1}{a^2H^2}\nabla^2\zeta$$

- (nonlinear) compaction func.

$$\mathscr{C}(r) = \frac{2}{3}\left[1-\left(1+r\zeta'(r)\right)^2\right]$$

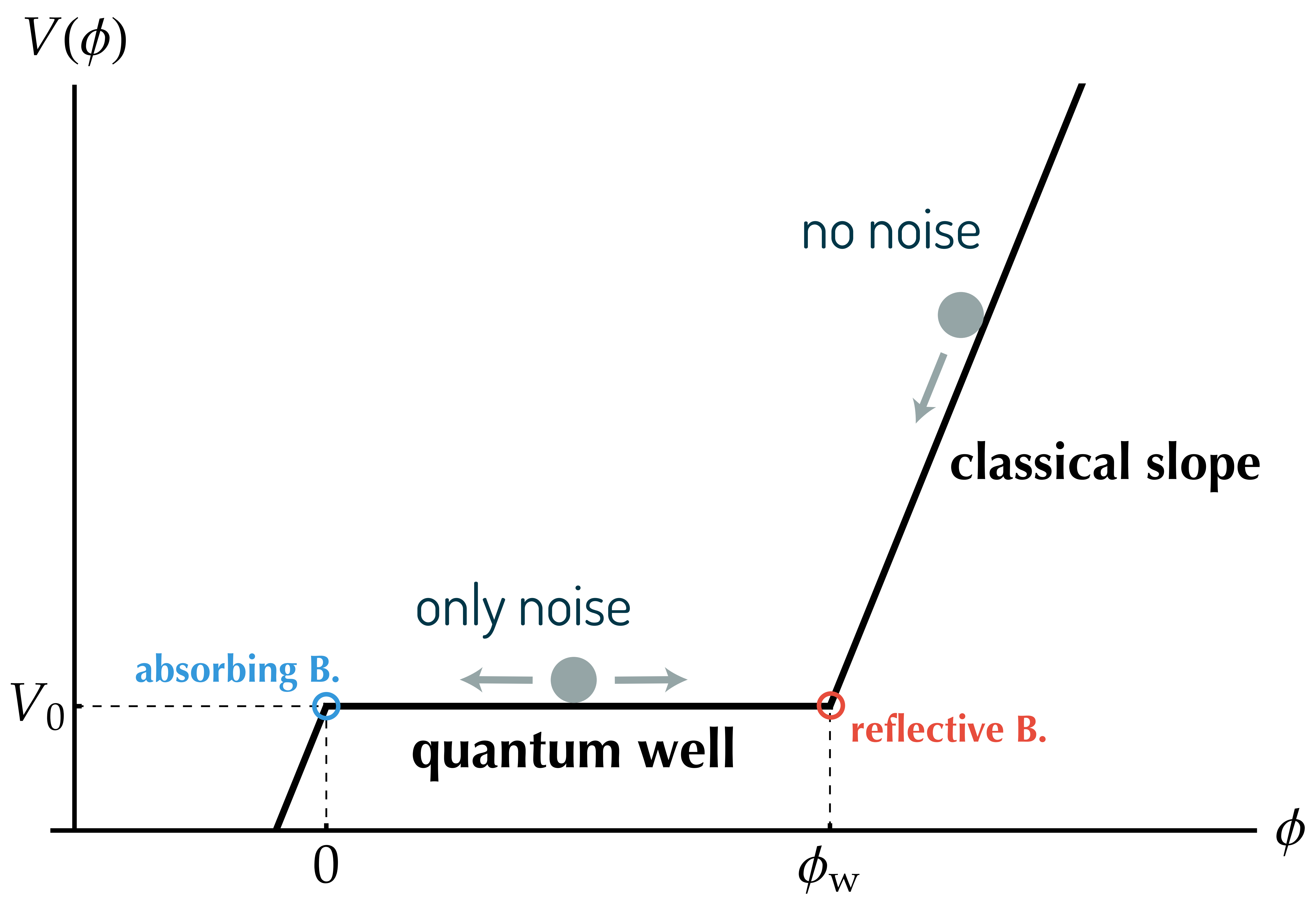
$$\boxed{(-\nabla^2\zeta_R)(\mathbf{k}) = k^2 W\left(\frac{kR}{a}\right)\zeta(\mathbf{k}) \approx \gamma\left(\zeta_{R_2}-\zeta_{R_1}\right) =: \Delta\zeta}$$

w/ appropriate R_1, R_2, γ



Quantum well

YT & Vennin '21

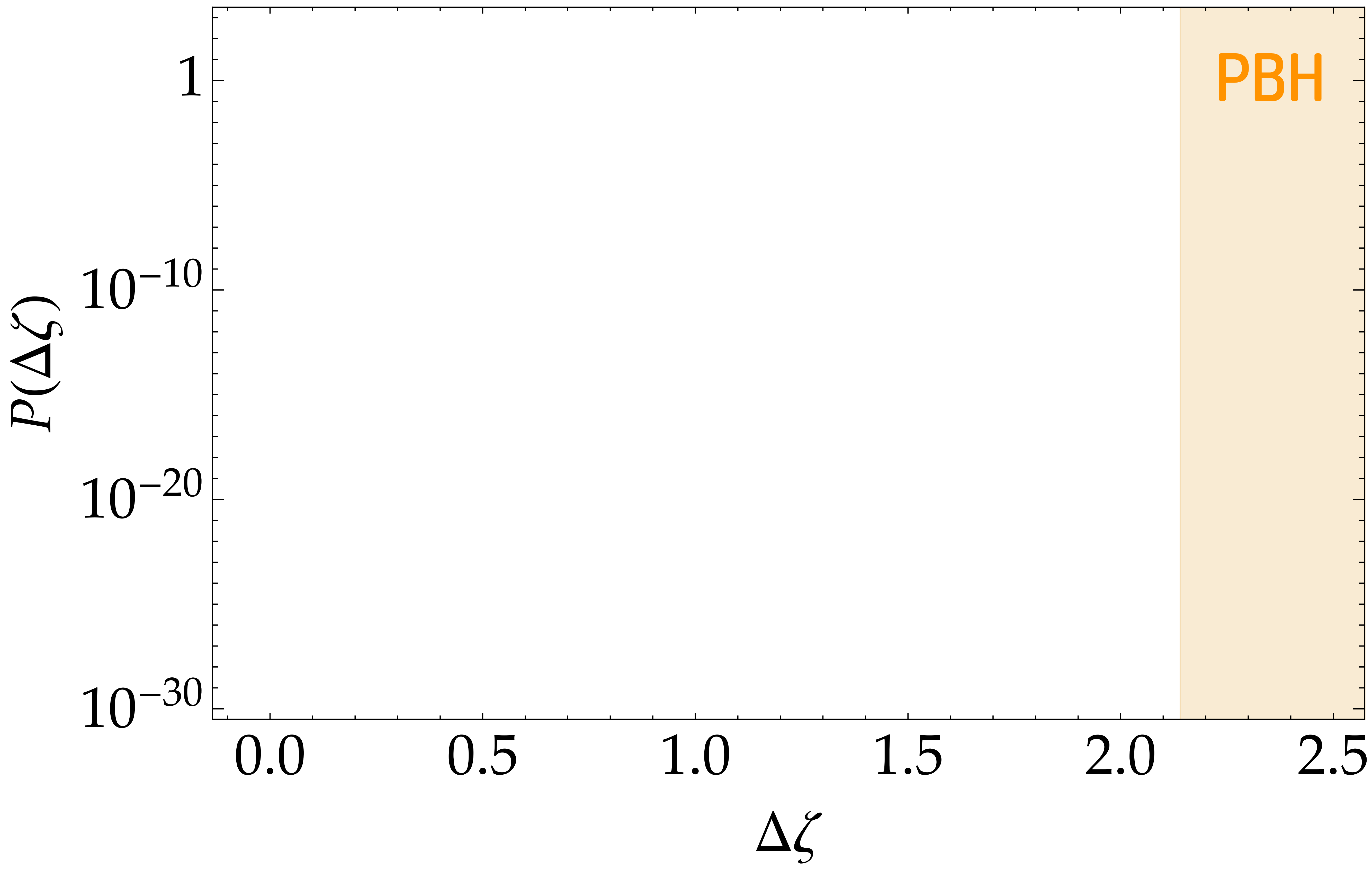


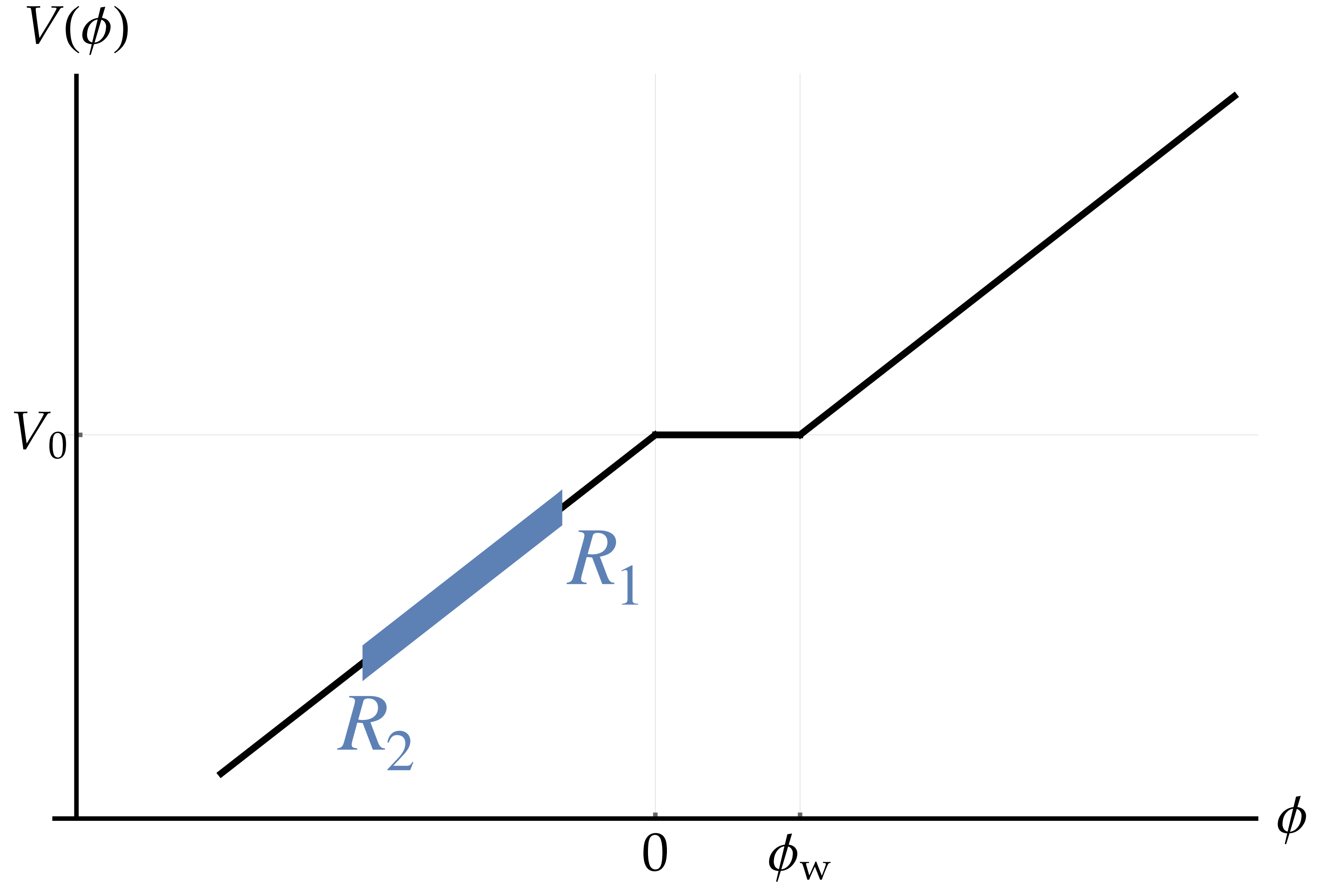
$$\mu = \sqrt{2}\frac{\phi_{\rm w}}{H/2\pi}$$

Quantum well

YT & Vennin '21

$$\mu = \frac{1}{\sqrt{6}}$$



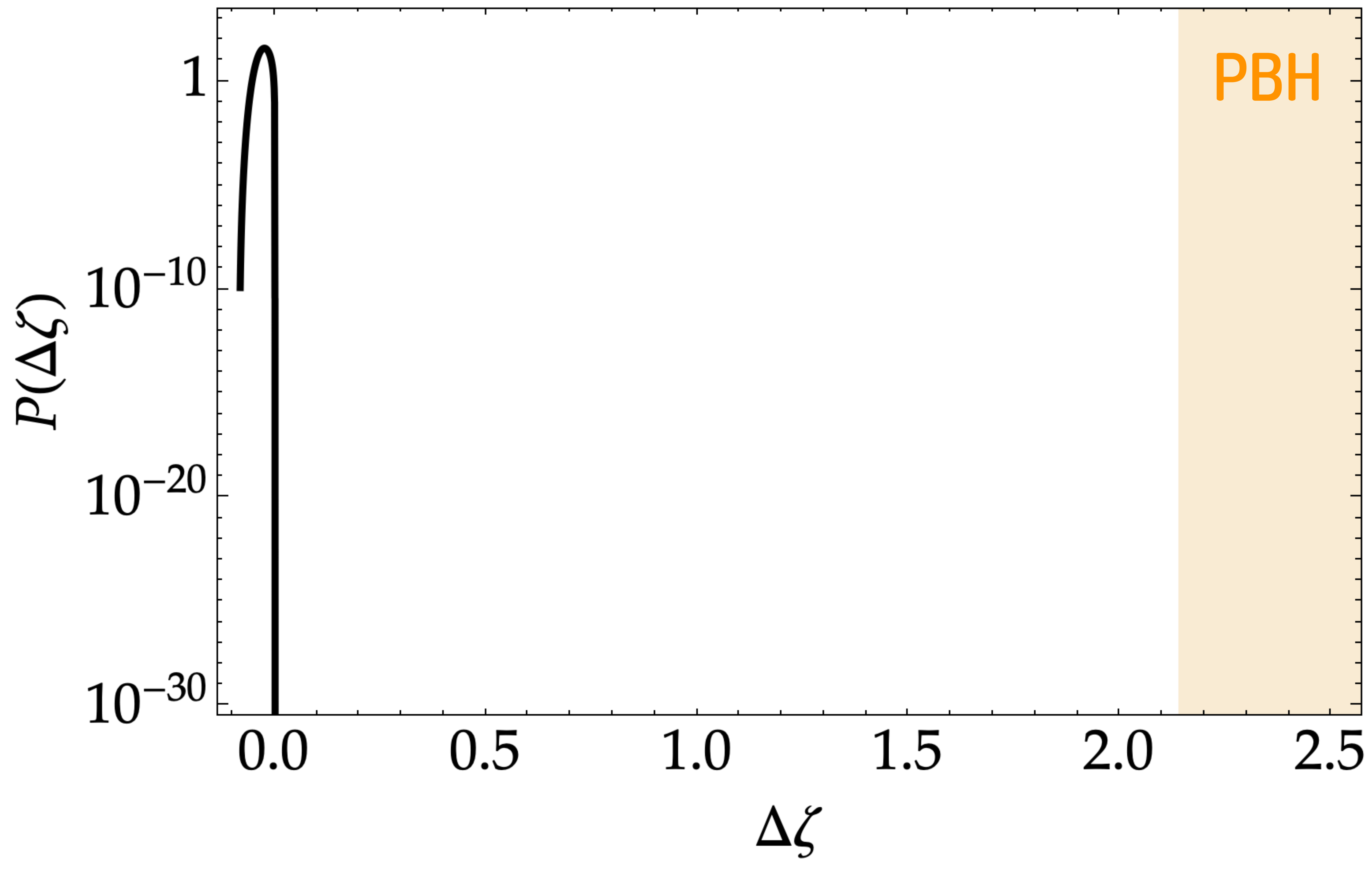


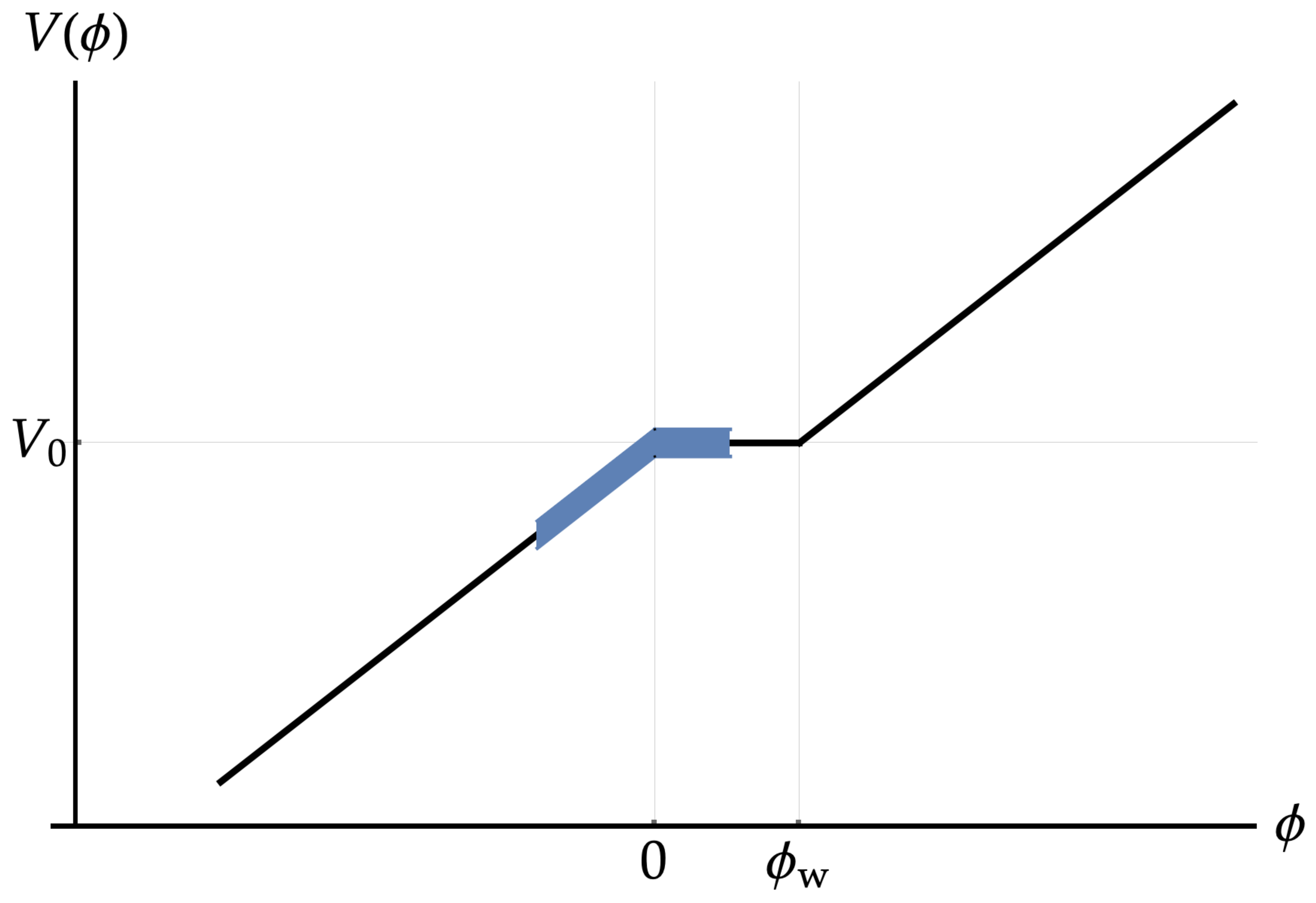
$$\mu = \sqrt{2}\frac{\phi_{\rm w}}{H/2\pi}$$

Quantum well

YT & Vennin '21

$$\mu = \frac{1}{\sqrt{6}}$$





$$\mu = \sqrt{2}\frac{\phi_{\mathrm{w}}}{H/2\pi}$$

Quantum well

YT & Vennin '21

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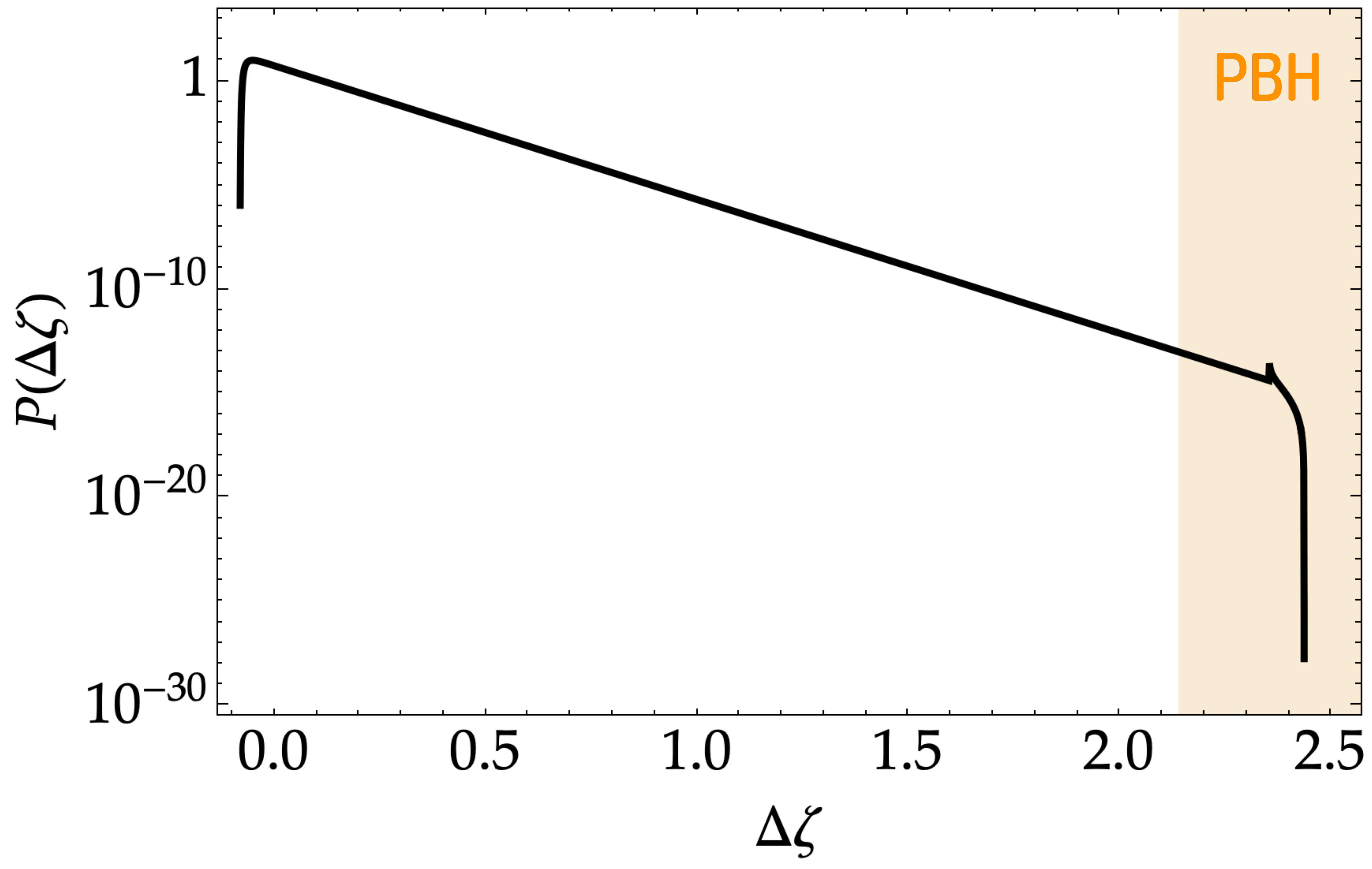

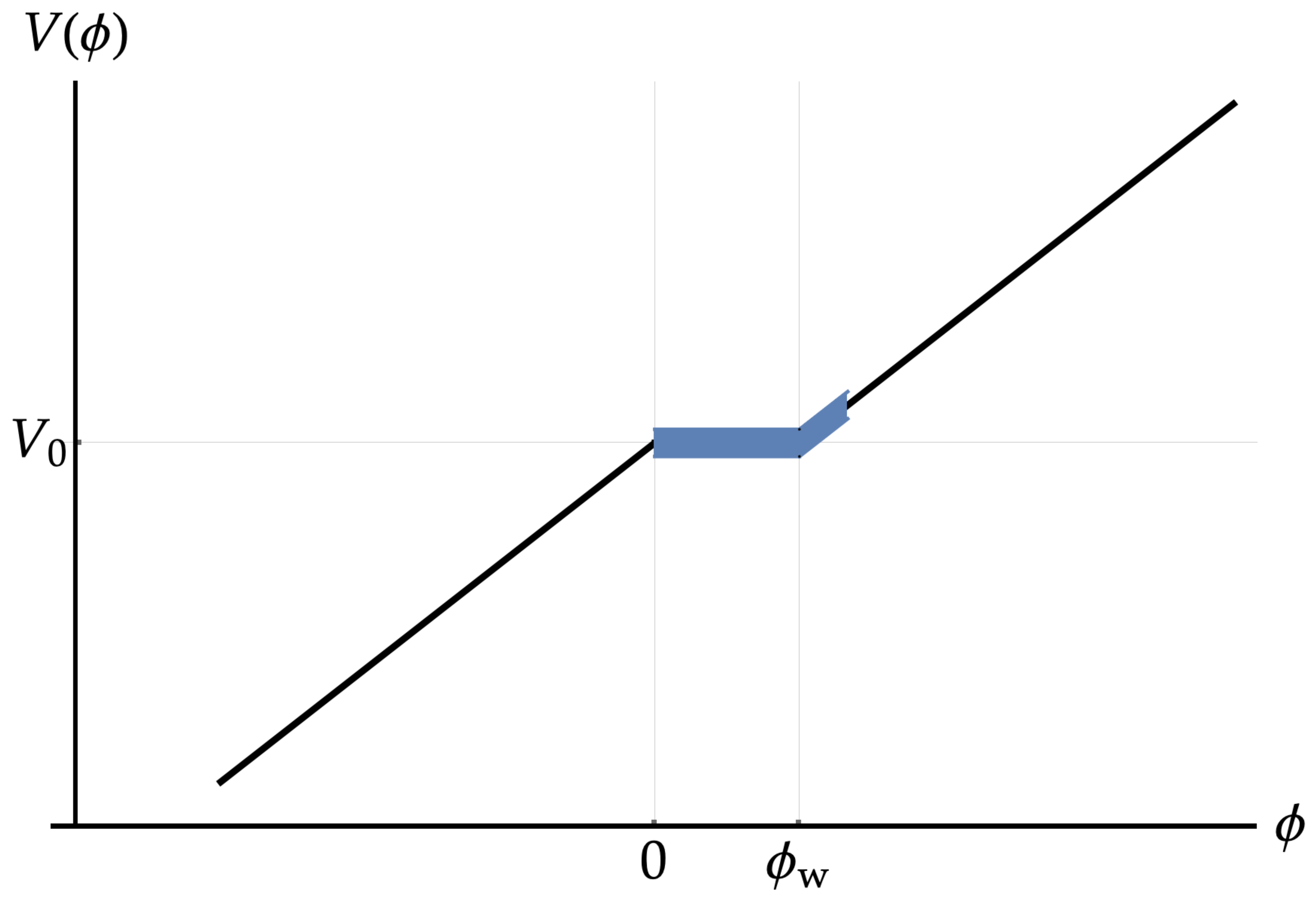

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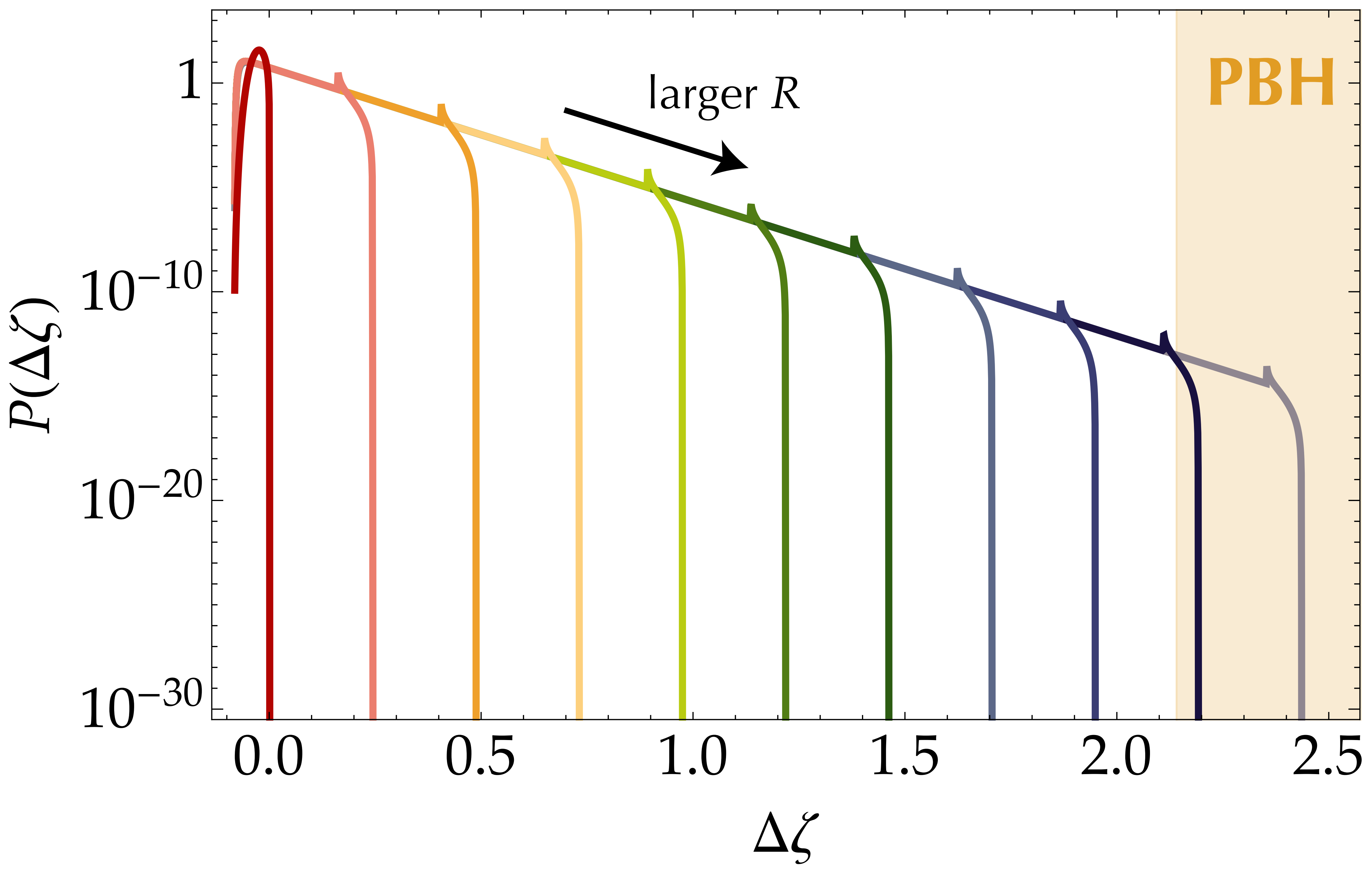
$$\mu = \frac{1}{\sqrt{6}},\ N_{\rm bw}^{(2)} < 0$$



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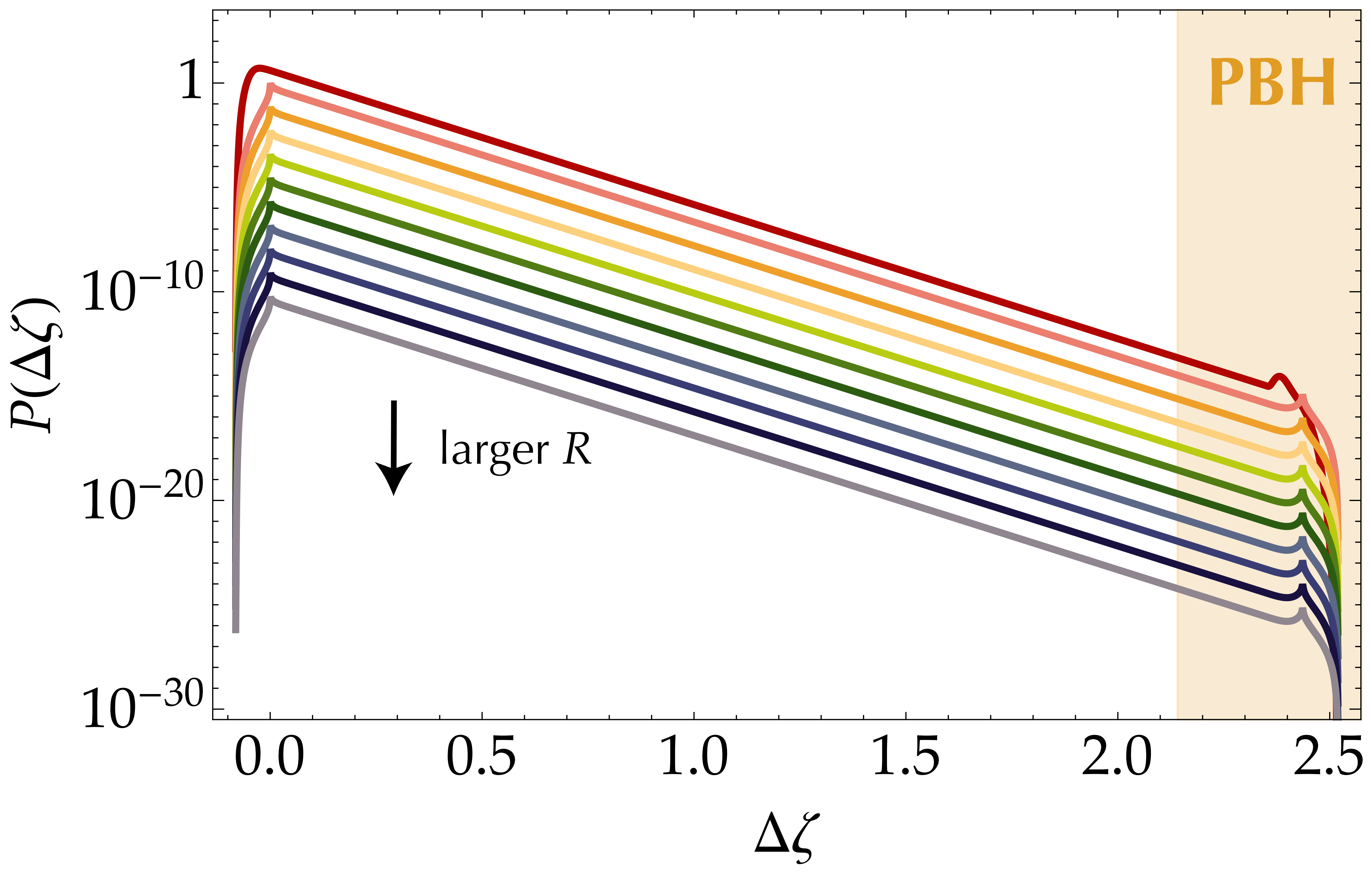

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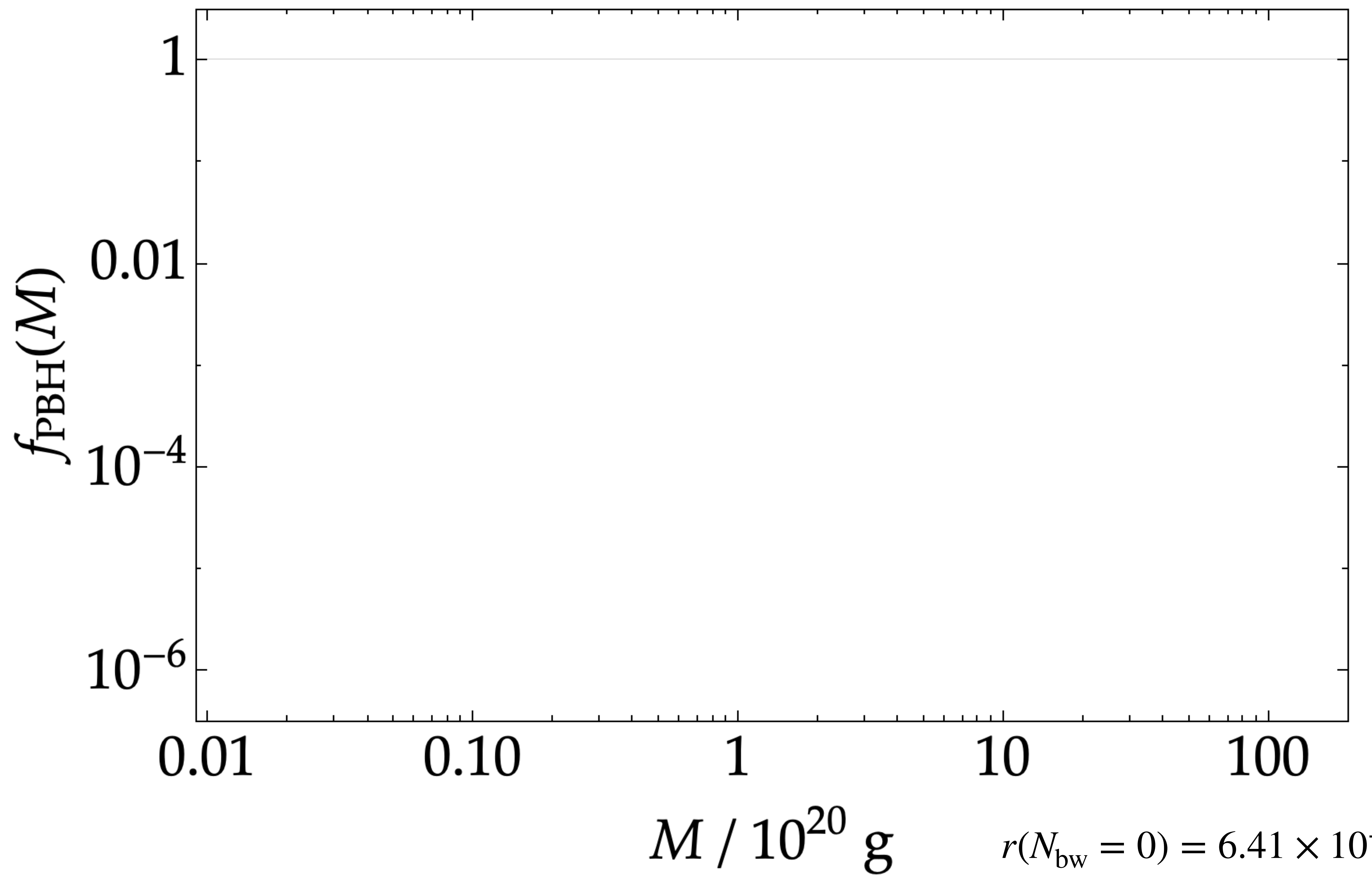
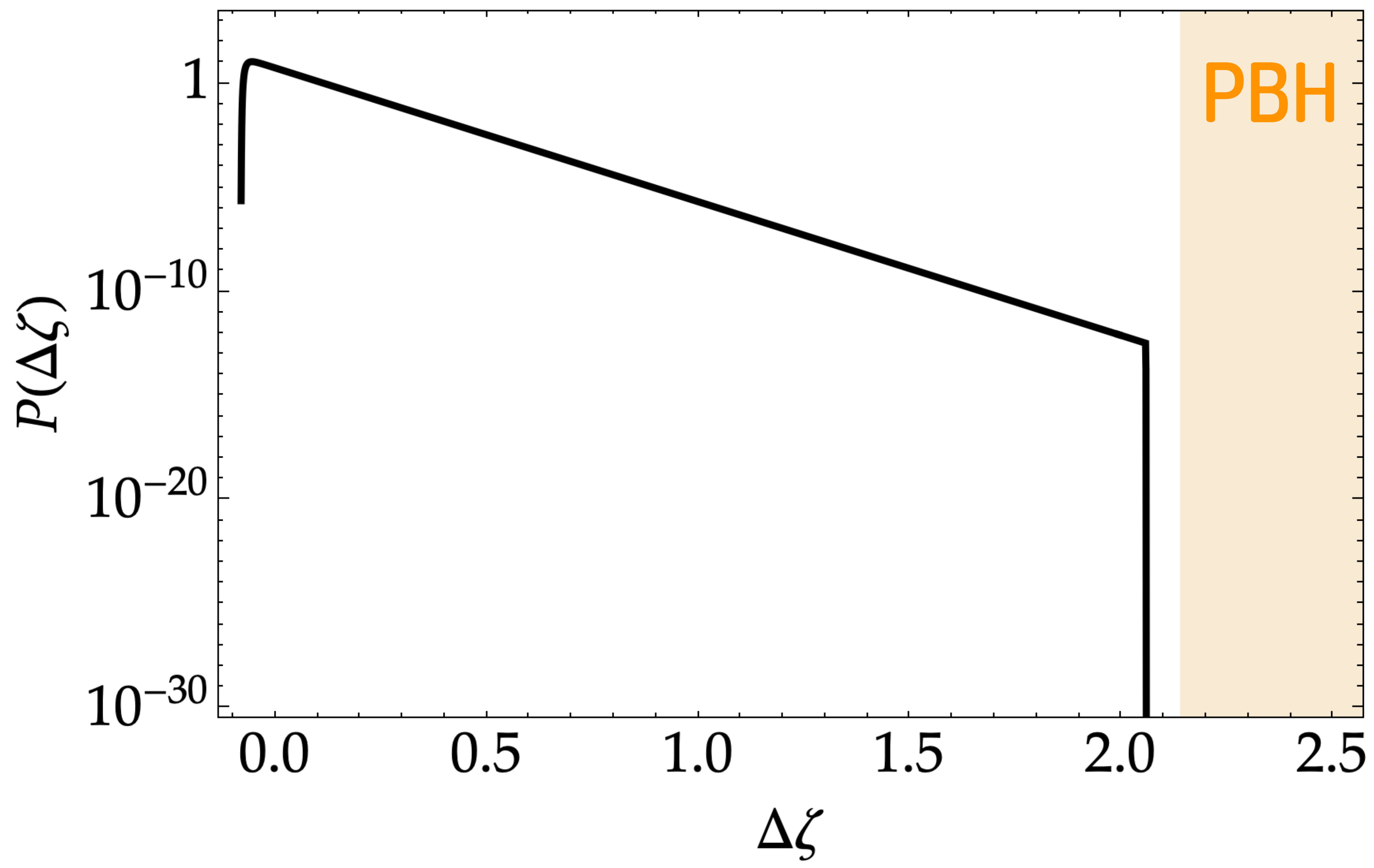


$r(N_{\rm bw}=0) = 6.41\times 10^{-14}\,\mathrm{Mpc}$

$$\mu = \frac{1}{\sqrt{6}}$$



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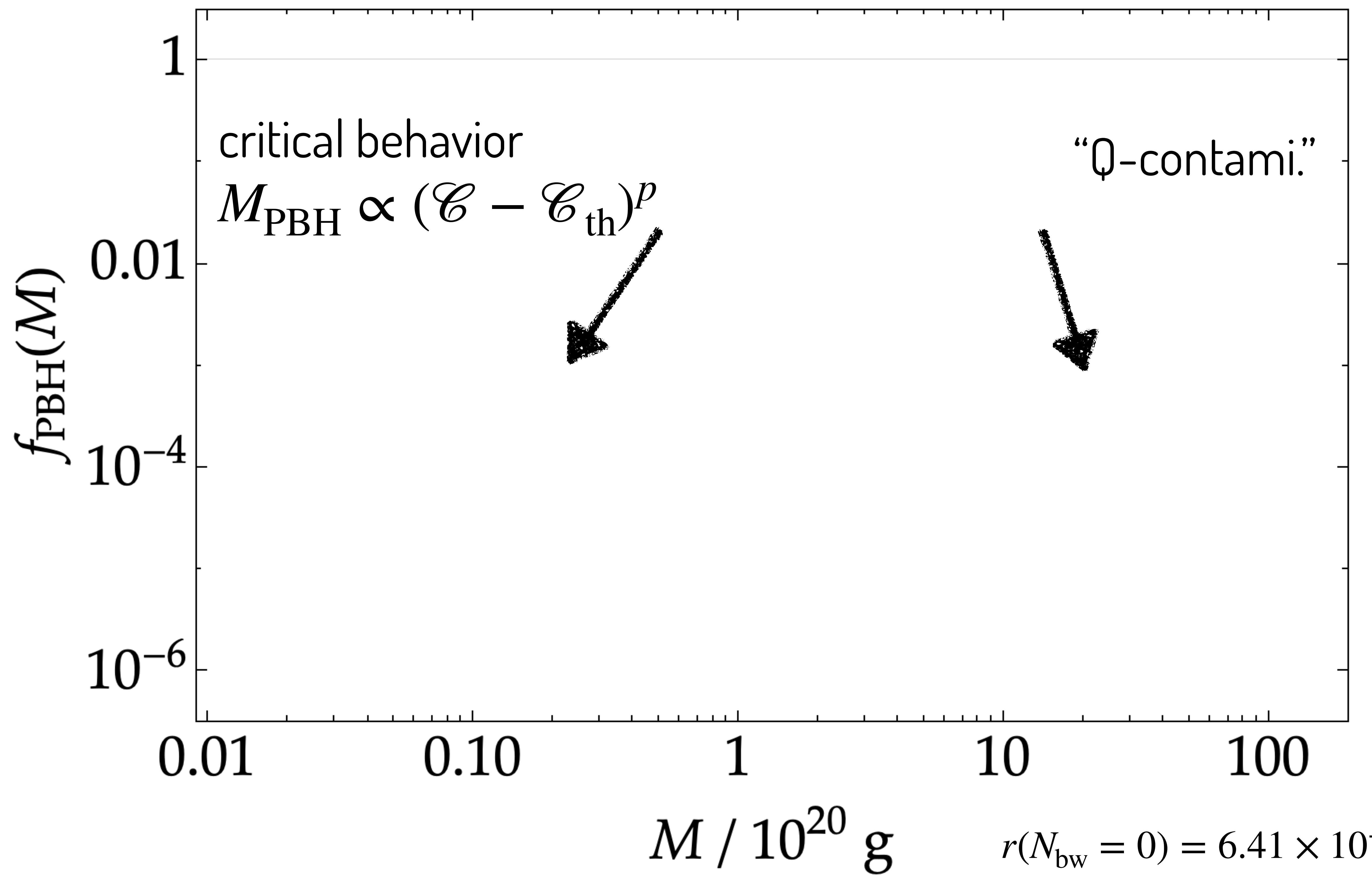

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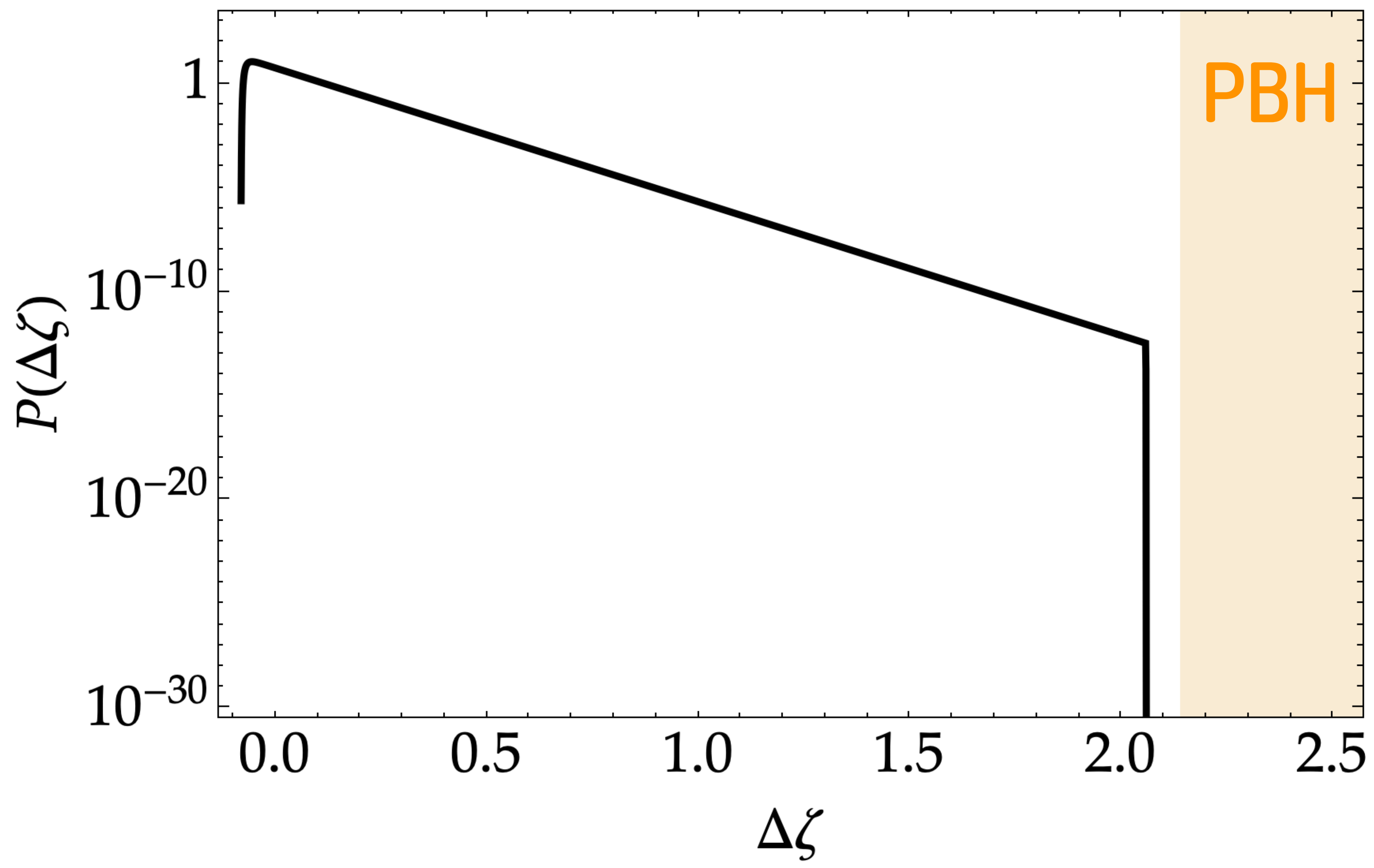

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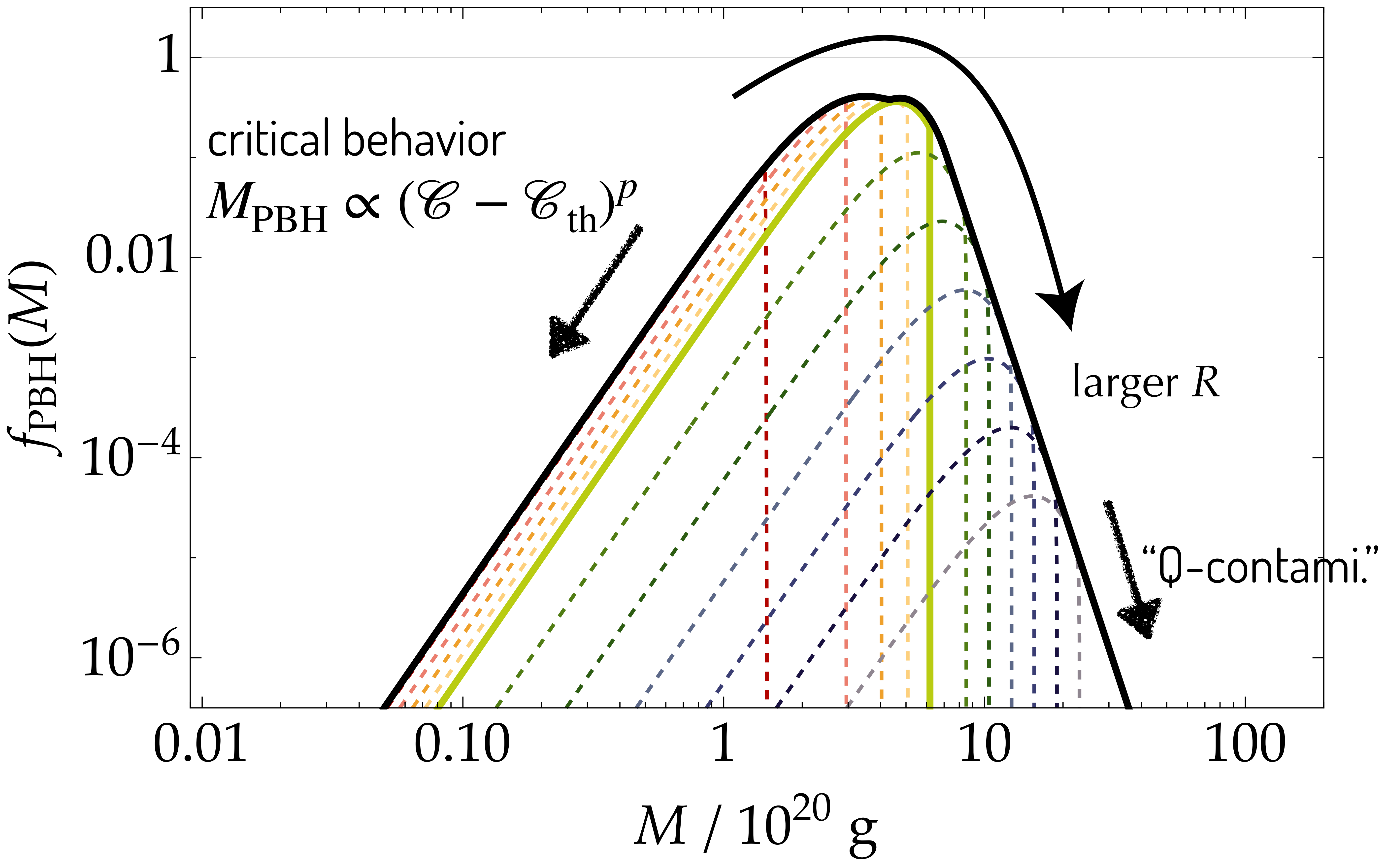
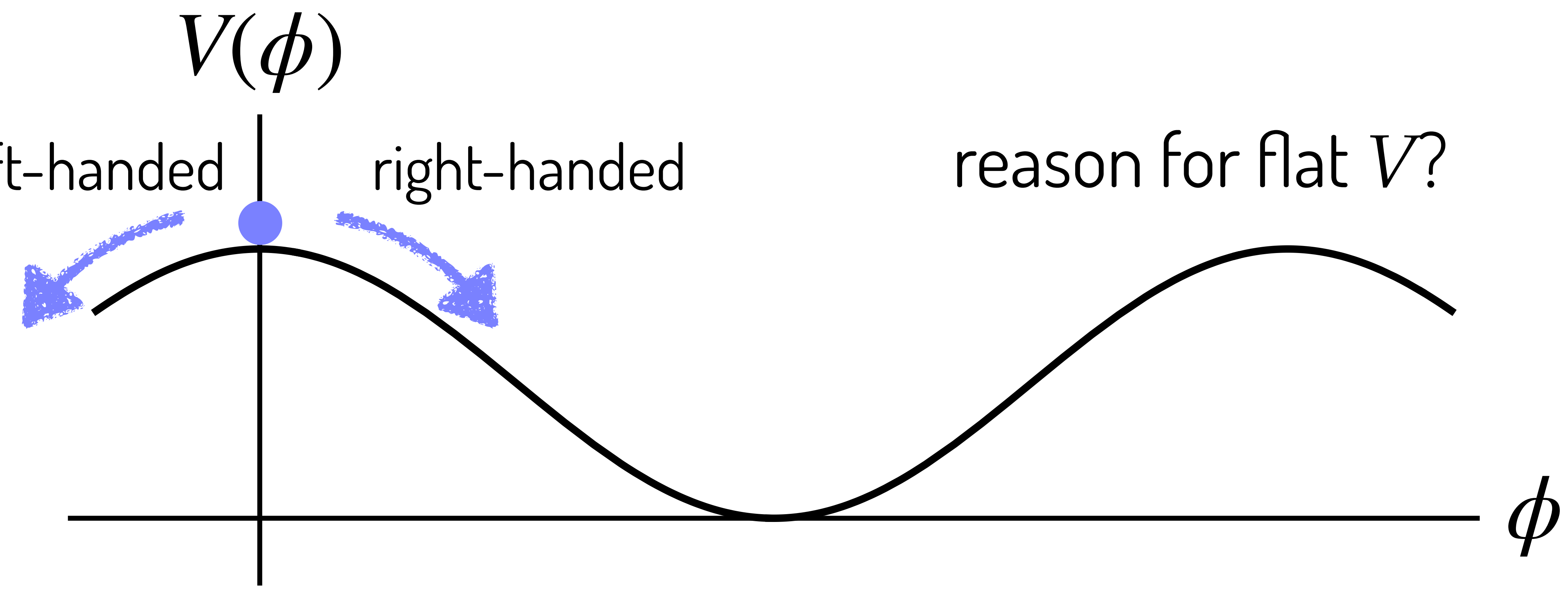


4. Gauge field, Lattice sim. ...

Axion inflation

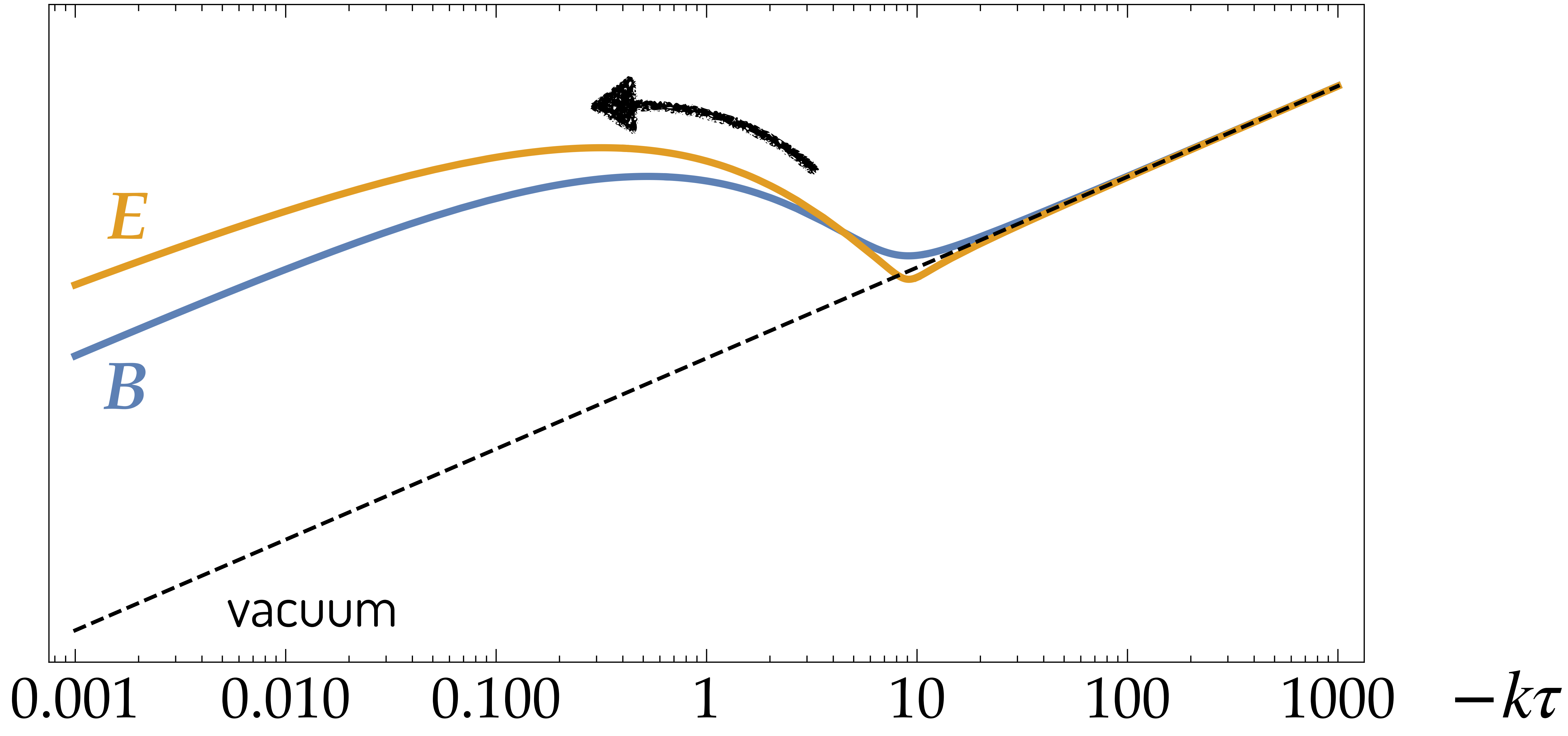
Pseudo NG boson : $V(\phi) = V(\phi + 2\pi f)$



- Chern—Simons coupling

$$\mathscr{L} \supset -\frac{1}{4f}\underbrace{\phi}_{\cancel{CP}} \underbrace{F_{\mu\nu}\tilde{F}^{\mu\nu}}_{\cancel{CP}} \quad \rightarrow \quad \left[\partial_\tau^2 + k^2 \pm 2k\frac{\xi}{\tau}\right] A_\pm(\tau, k) = 0$$

$$\xi = \frac{\dot{\phi}}{2fH}$$

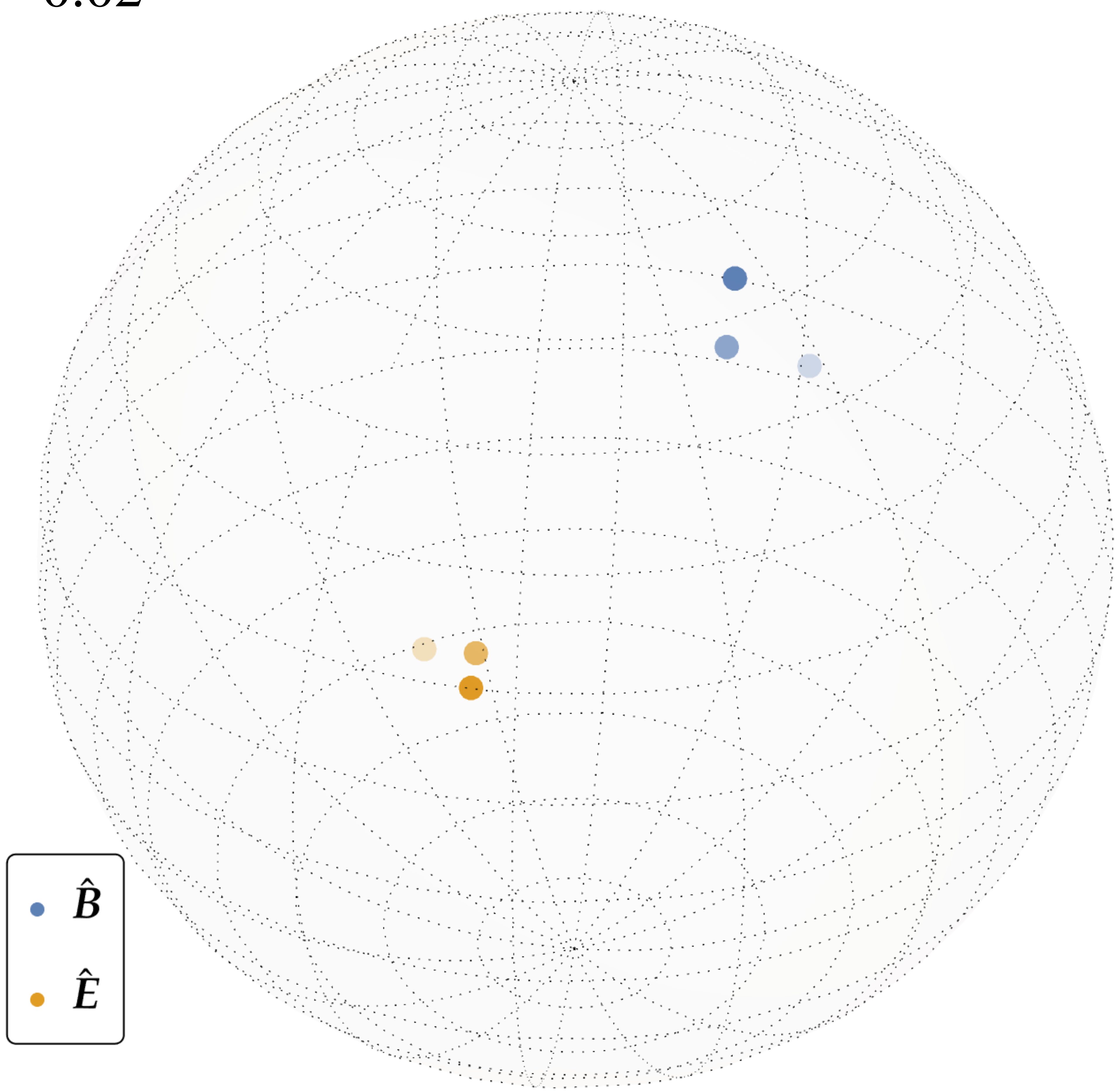


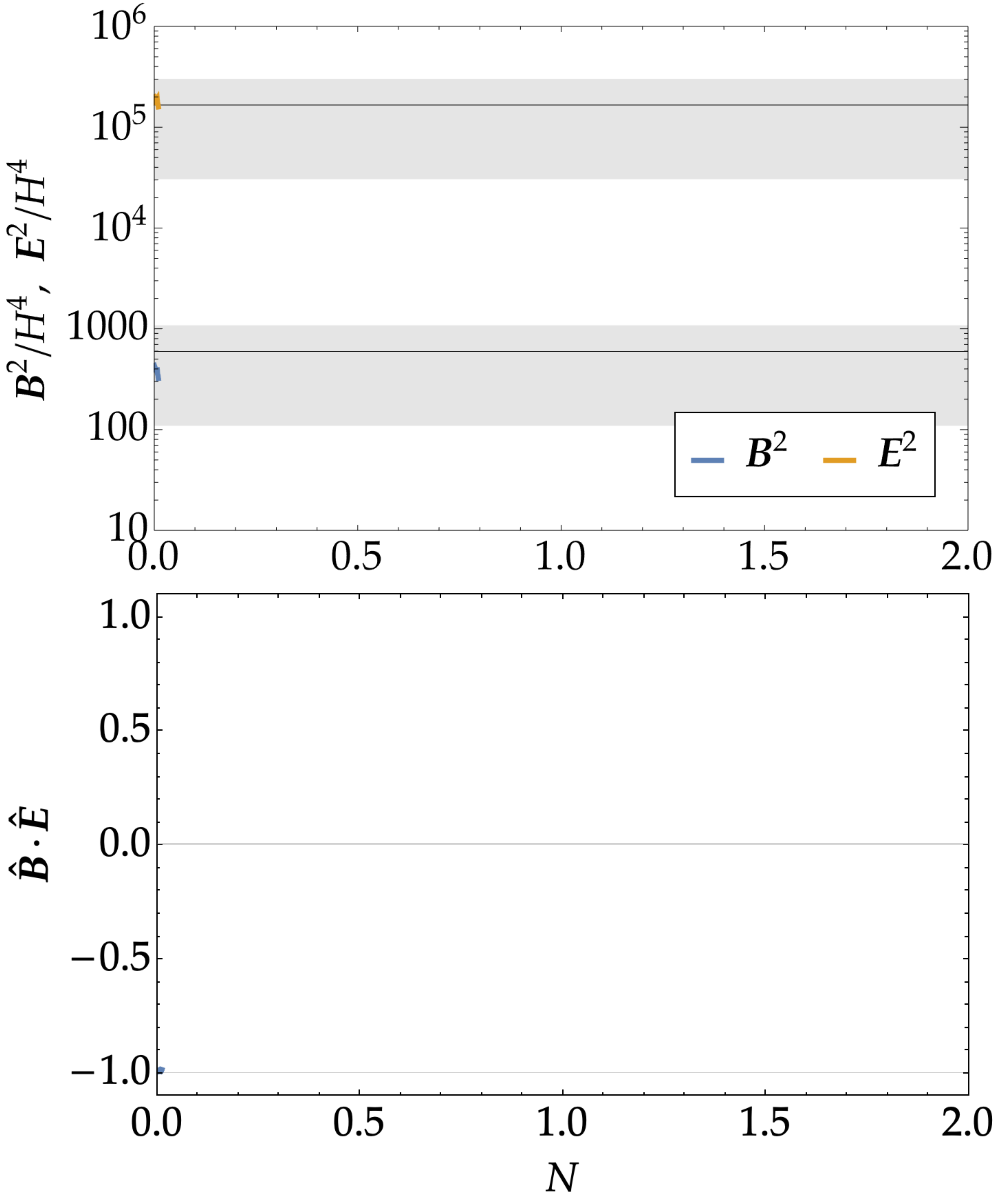
Example

Fujita, Mukaida, YT '22

$H = 10^{-5} M_{\rm Pl}$
$\xi = 5$
$k_{\rm IR}/aH = 0.02$

$N = 0.01$



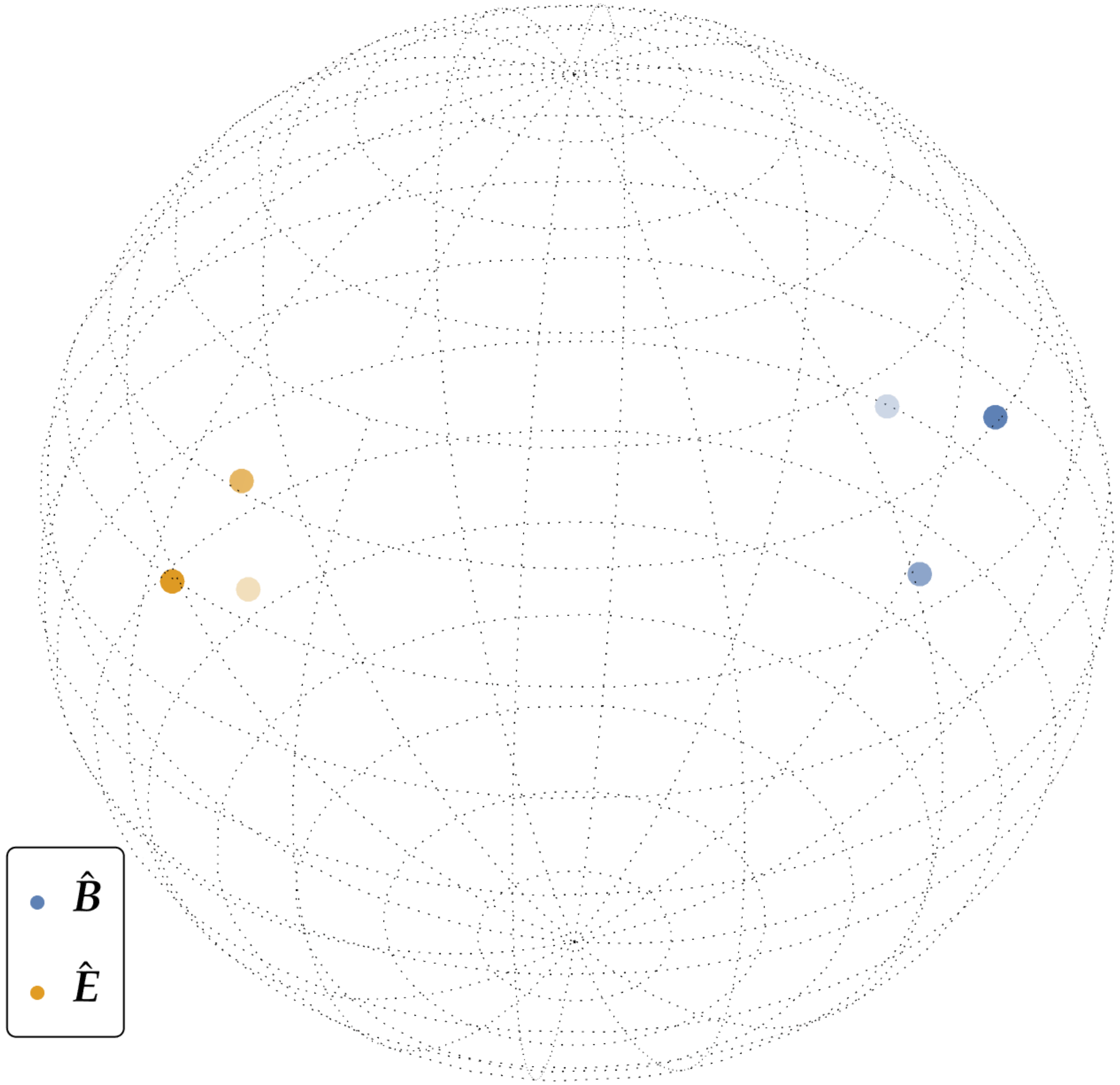


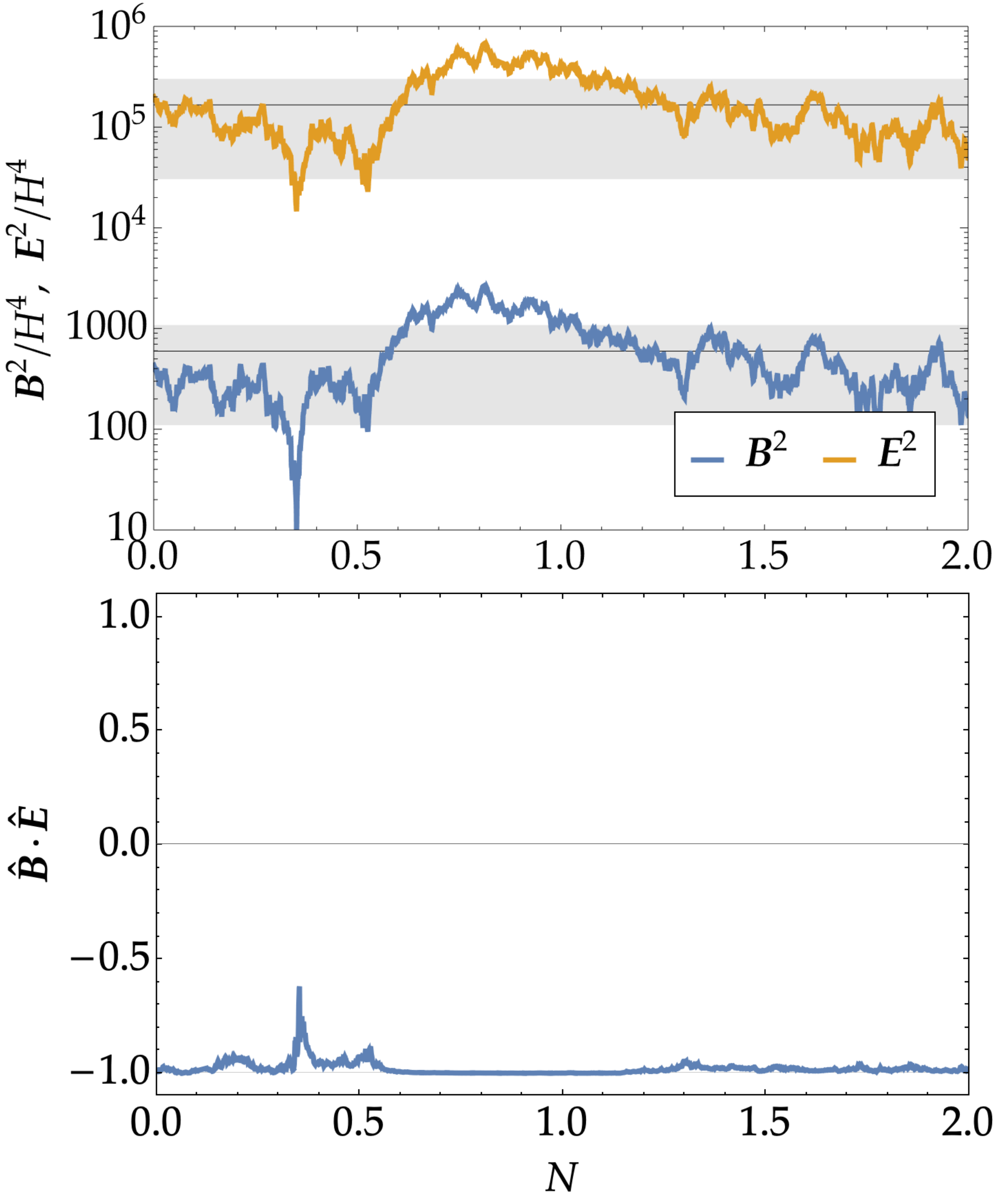
Example

Fujita, Mukaida, YT '22

$H = 10^{-5} M_{\rm Pl}$
$\xi = 5$
$k_{\rm IR}/aH = 0.02$

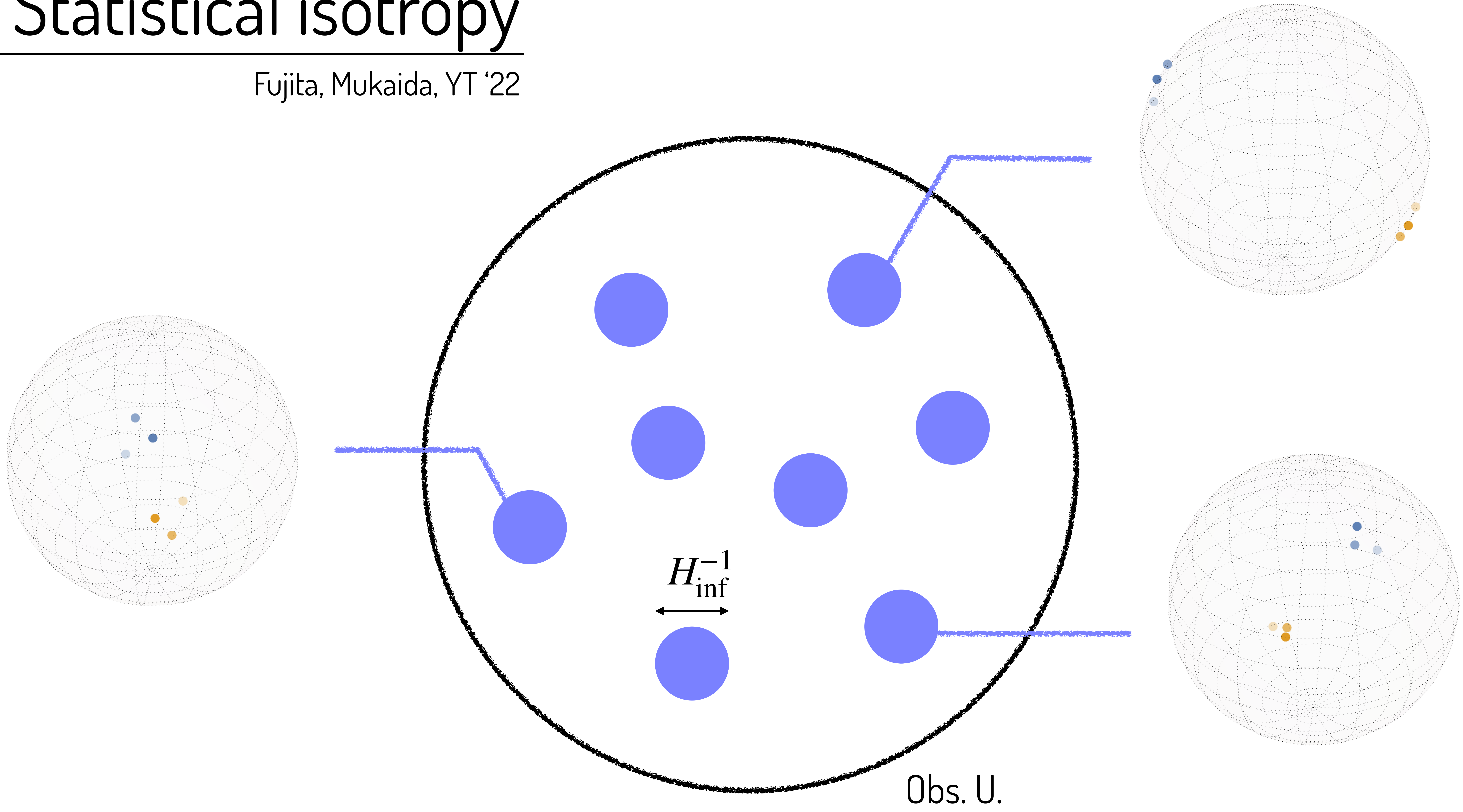
$N = 2.$





Statistical isotropy

Fujita, Mukaida, YT ‘22



STOLAS

— stochastic lattice simulation —

Mizuguchi, Murata, YT in prep.

Gaussian random noise $N(0, \mathrm{d}N)$

$$
\begin{cases}
\mathrm{d}\phi(N,\mathbf{x}) & = \dfrac{\pi(N,\mathbf{x})}{H(N,\mathbf{x})}\mathrm{d}N + \dfrac{H(N,\mathbf{x})}{2\pi} g_{\mathbf{x}\alpha}(N)\boxed{\mathrm{d}W_\alpha}, \\
\mathrm{d}\pi(N,\mathbf{x}) & = \left(-3\pi(N,\mathbf{x}) - \dfrac{V'(\phi(N,\mathbf{x}))}{H(N,\mathbf{x})}\right)\mathrm{d}N, \\
3M_{\mathrm{Pl}}^2 H^2(N,\mathbf{x}) & = \dfrac{1}{2}\pi^2(N,\mathbf{x}) + V(\phi(N,\mathbf{x})), \\
g_{\mathbf{x}\alpha}(N) g_{\mathbf{y}\alpha}(N) & = \dfrac{\sin k_\sigma(N)\,|\,\mathbf{x}-\mathbf{y}\,|}{k_\sigma(N)\,|\,\mathbf{x}-\mathbf{y}\,|}
\end{cases}
$$

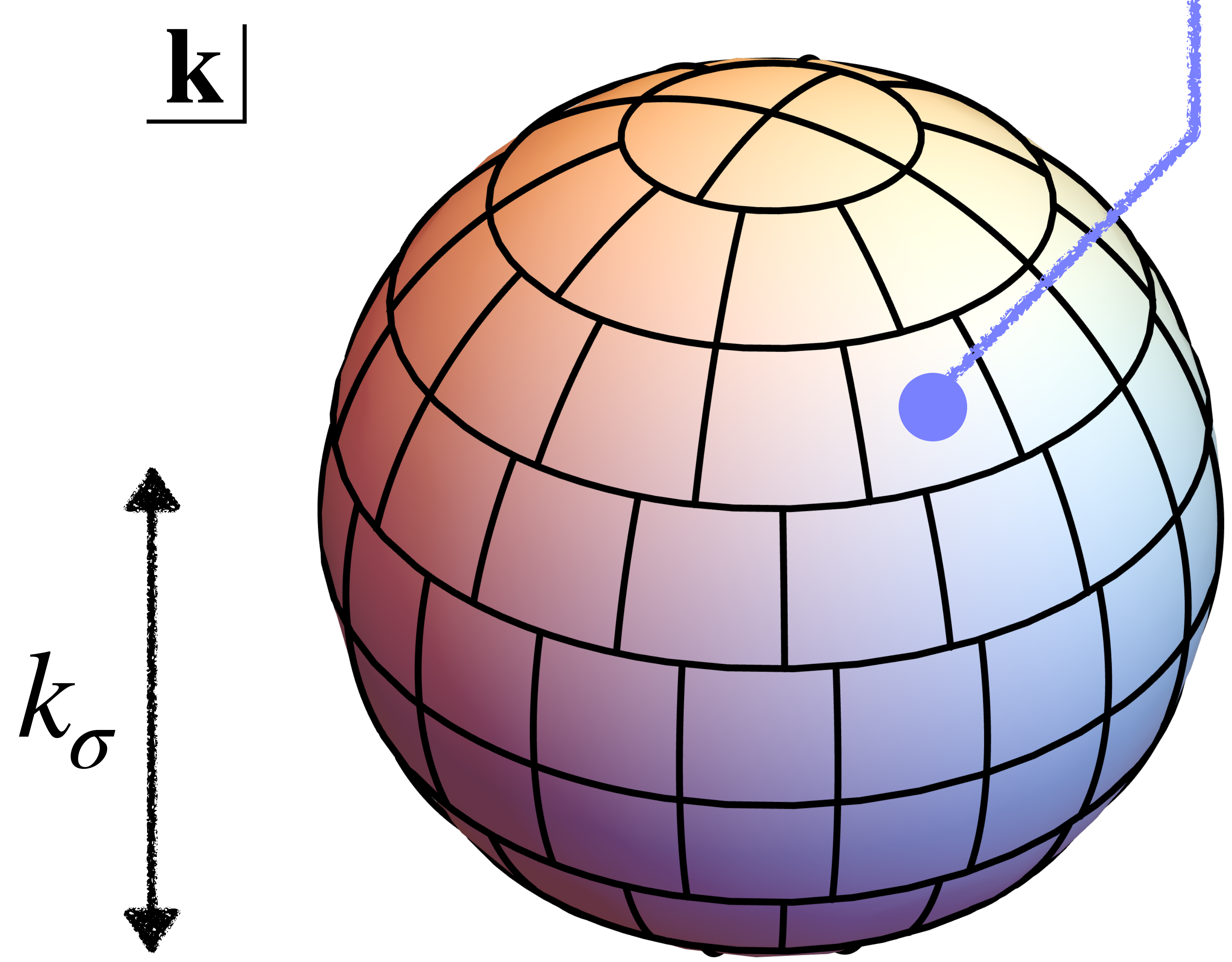
Spatial correlation?

Salopek & Bond '91

✗ $$g_{\mathbf{x}\alpha}(N)g_{\mathbf{y}\alpha}(N) = \frac{\sin k_\sigma(N)\,|\,\mathbf{x}-\mathbf{y}\,|}{k_\sigma(N)\,|\,\mathbf{x}-\mathbf{y}\,|} = C_{\mathbf{xy}}\ :\ 17^3 \times 17^3 \text{ matrix}$$

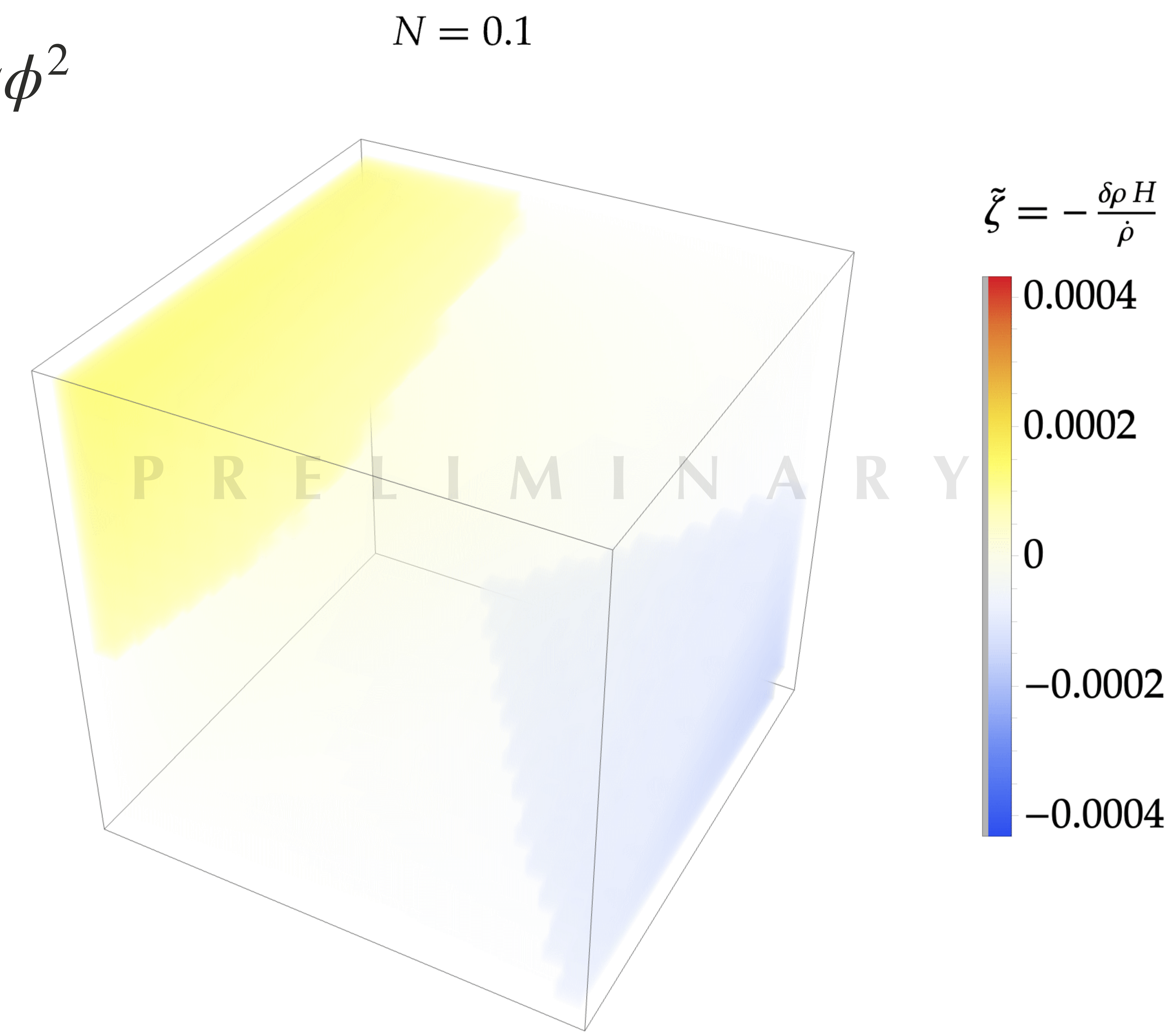
✓ $$g_{\mathbf{x}\alpha}\mathrm{d}W_\alpha = \sum_\Omega \frac{1}{2\sqrt{\pi}}(\cos \mathbf{k}_\sigma \cdot \mathbf{x} - \sin \mathbf{k}_\sigma \cdot \mathbf{x})\boxed{\sqrt{\Delta\Omega}\,\mathrm{d}W_\Omega}$$

inverse Fourier



Ex. 1 : Chaotic

$$V = \frac{1}{2} m^2 \phi^2$$



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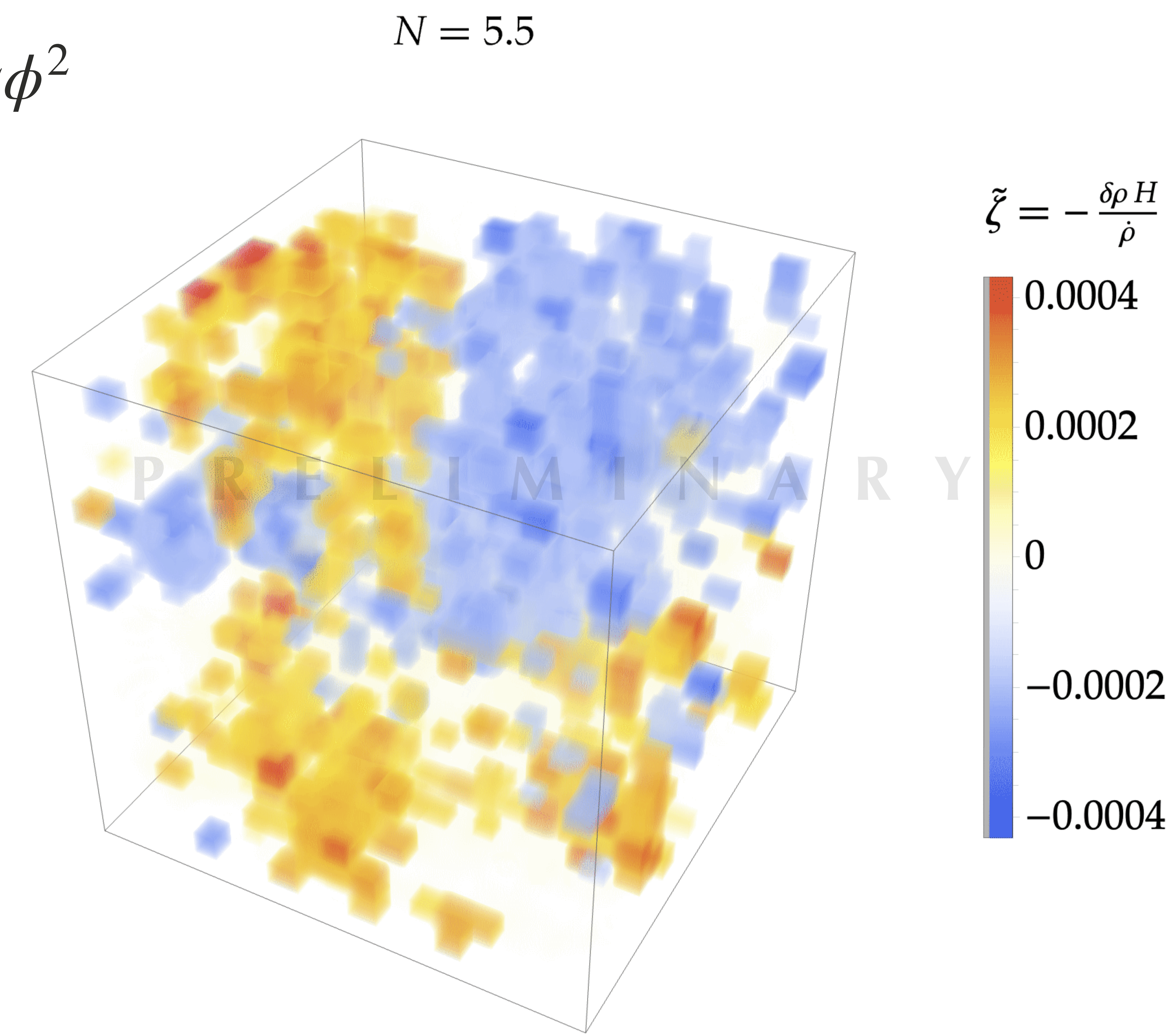
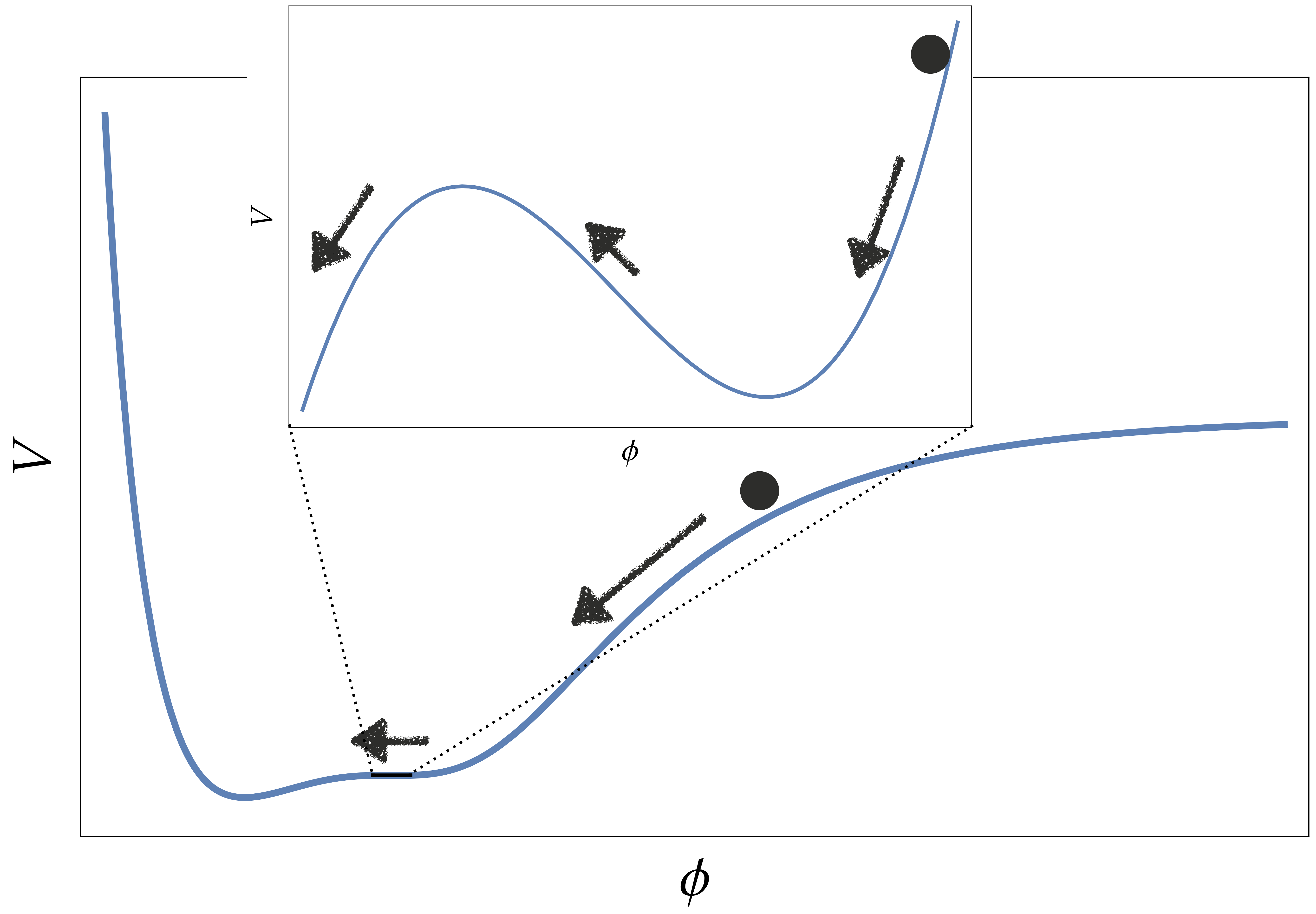


Ex. 2 : Inflection

$$V = \frac{W_0^2}{\mathscr{V}^3}\left[\frac{c_{\rm up}}{\sqrt[3]{\mathscr{V}}} + \frac{a_{\rm w}}{e^{\frac{\phi}{\sqrt{3}}} - b_{\rm w}} - \frac{c_{\rm w}}{e^{\frac{\phi}{\sqrt{3}}}} + \frac{e^{\frac{2\phi}{\sqrt{3}}}}{\mathscr{V}}\left(d_{\rm w} - \frac{g_{\rm w}}{r_{\rm w} e^{\sqrt{3}\phi/\mathscr{V}} + 1}\right)\right]$$

Cicoli+ '18, Biagetti+ '18



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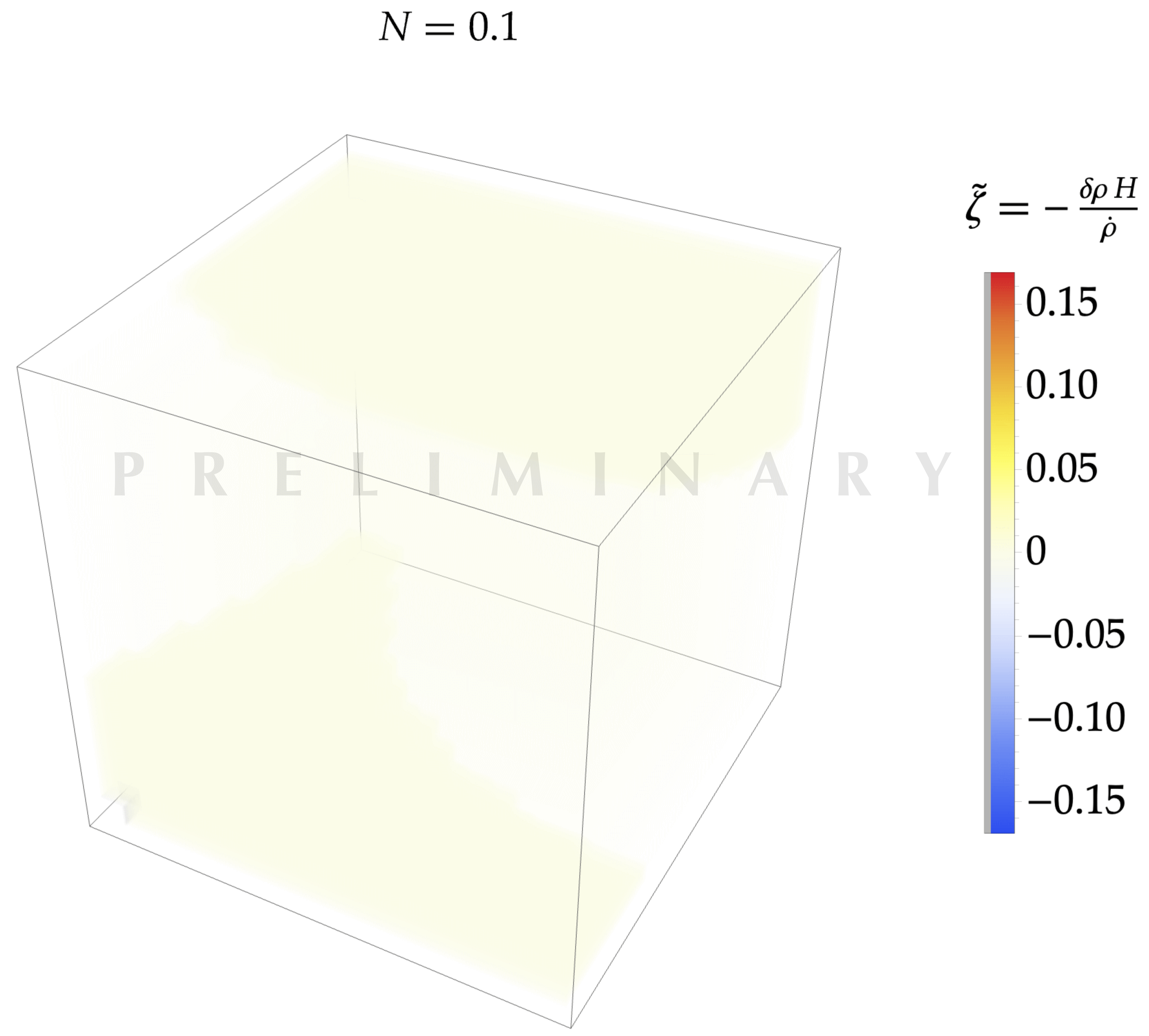

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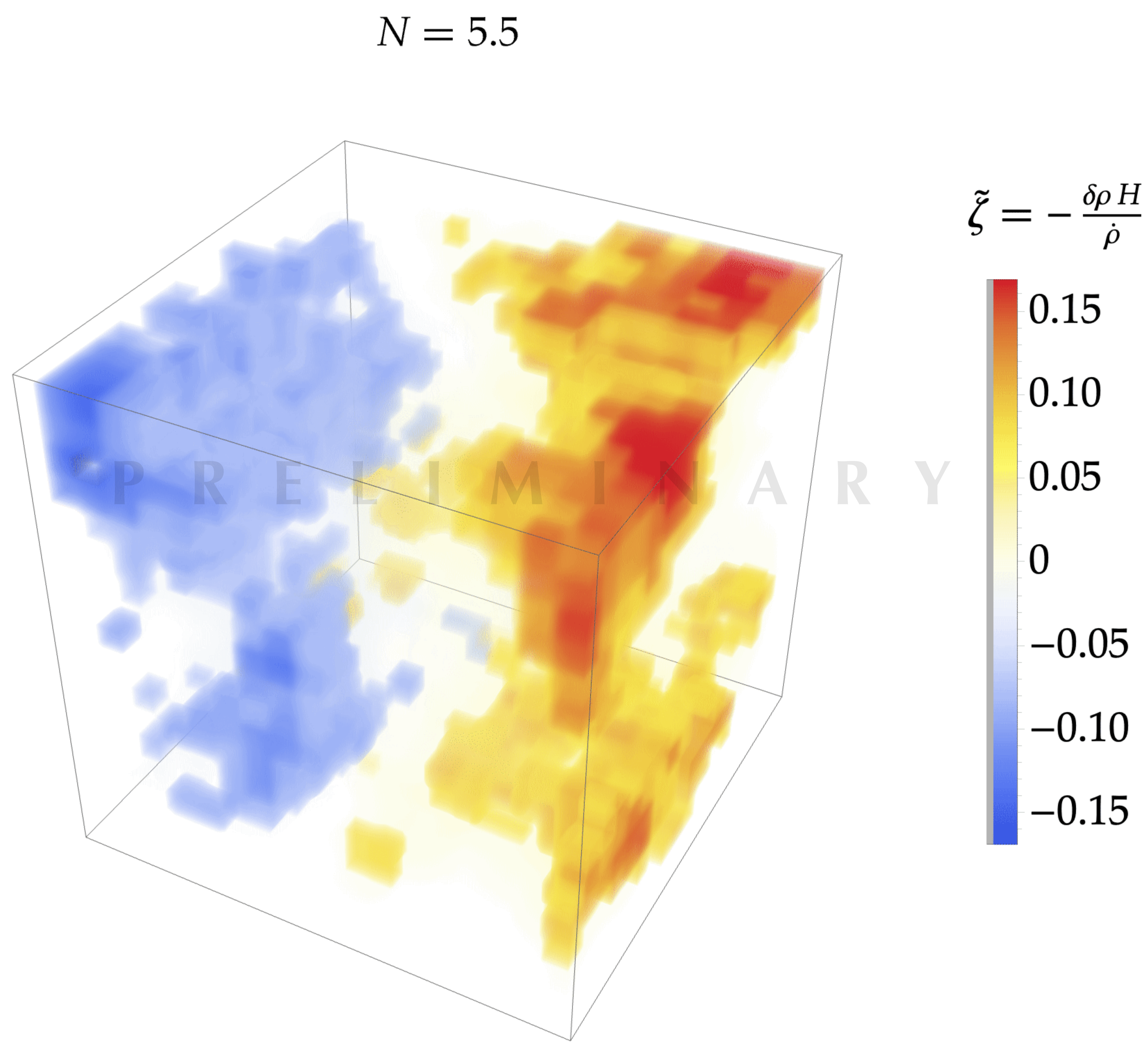

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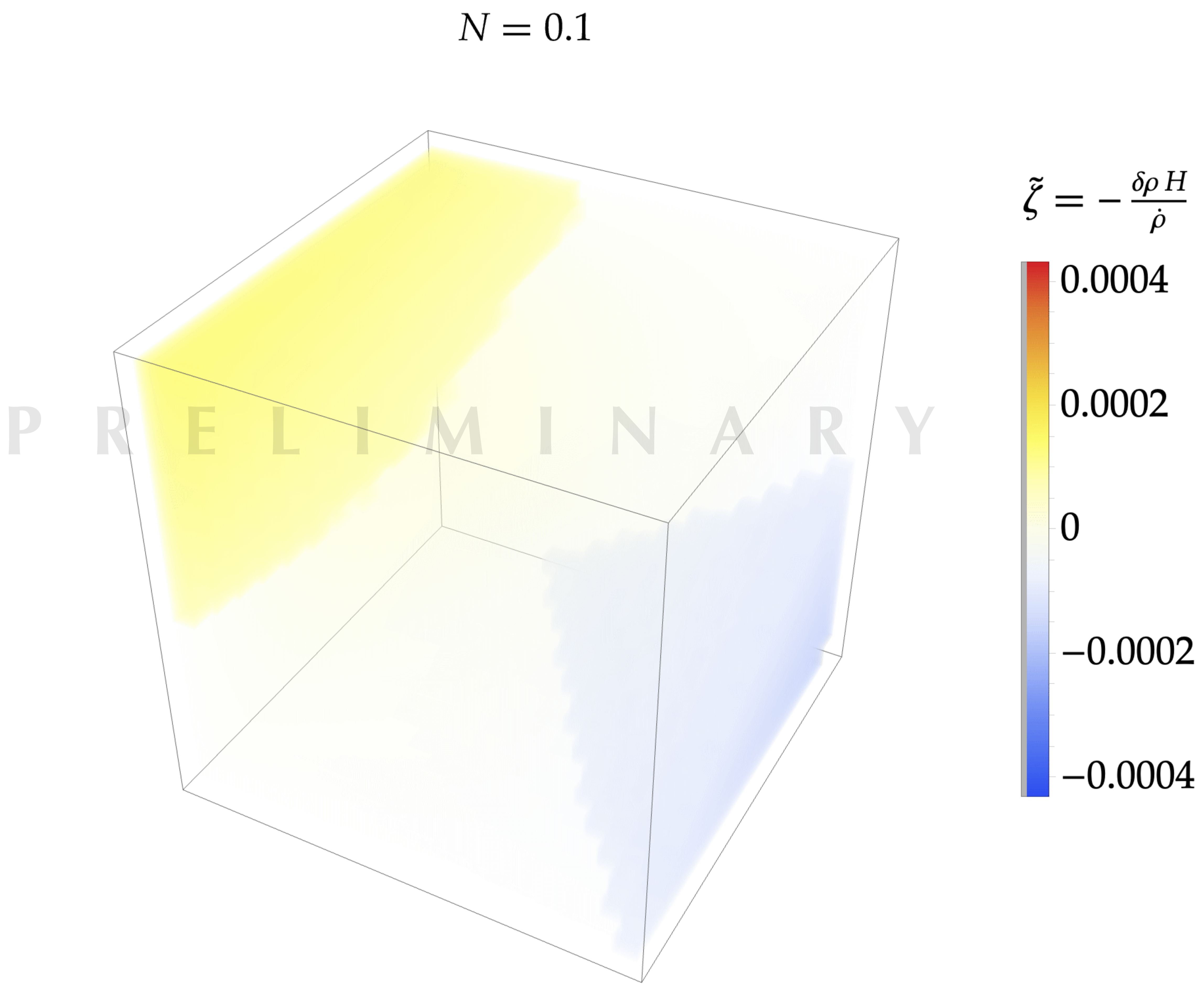

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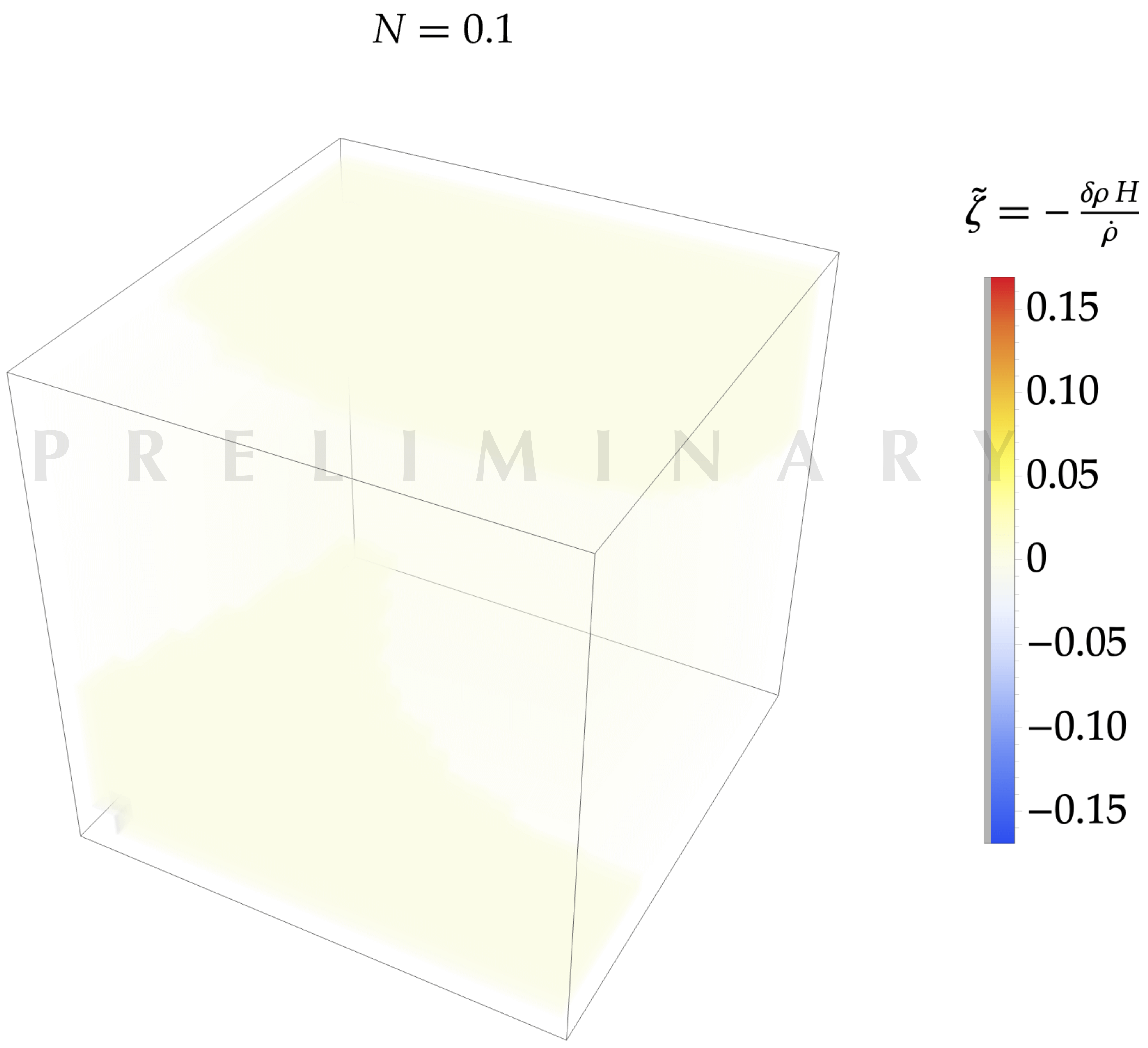

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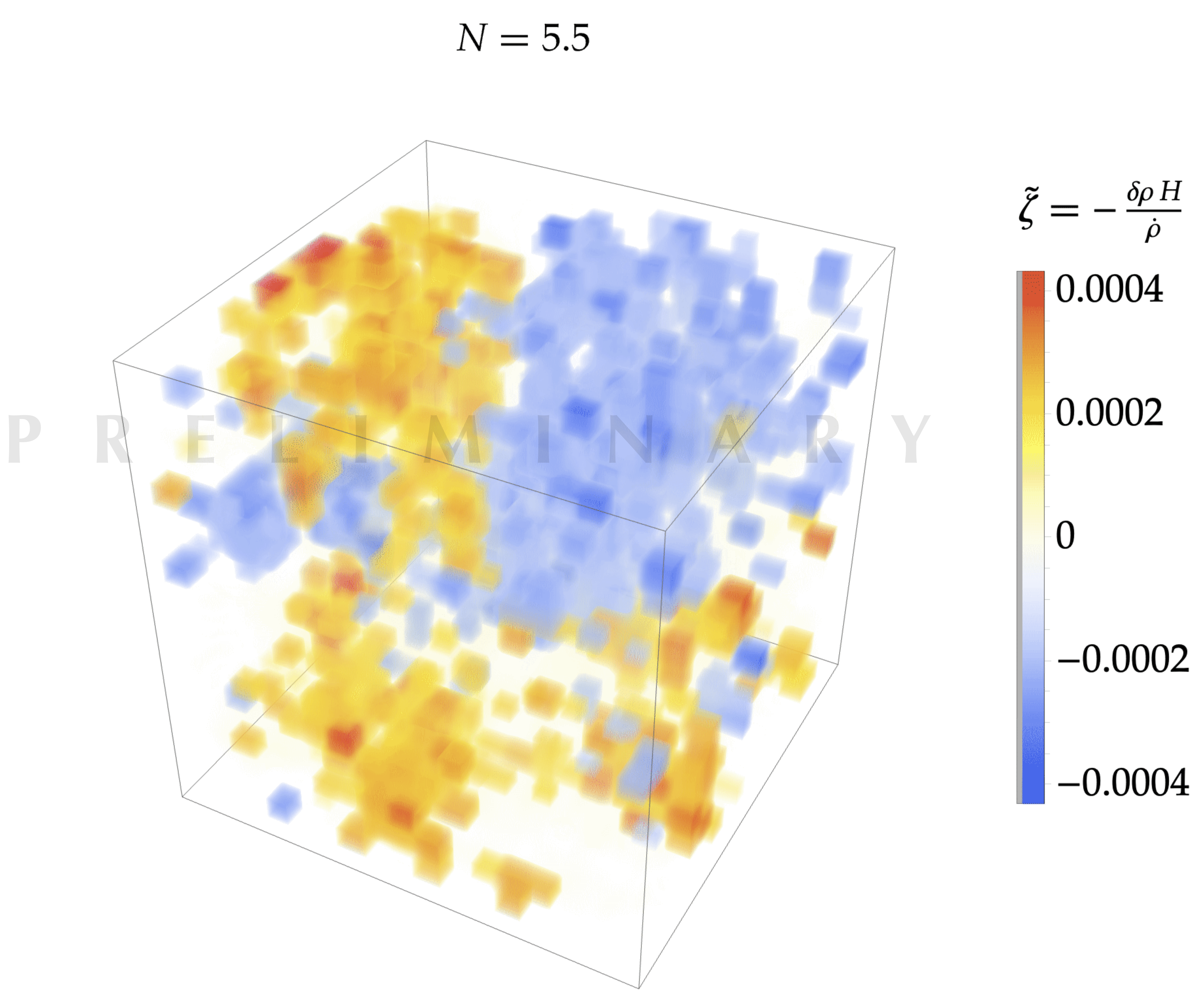

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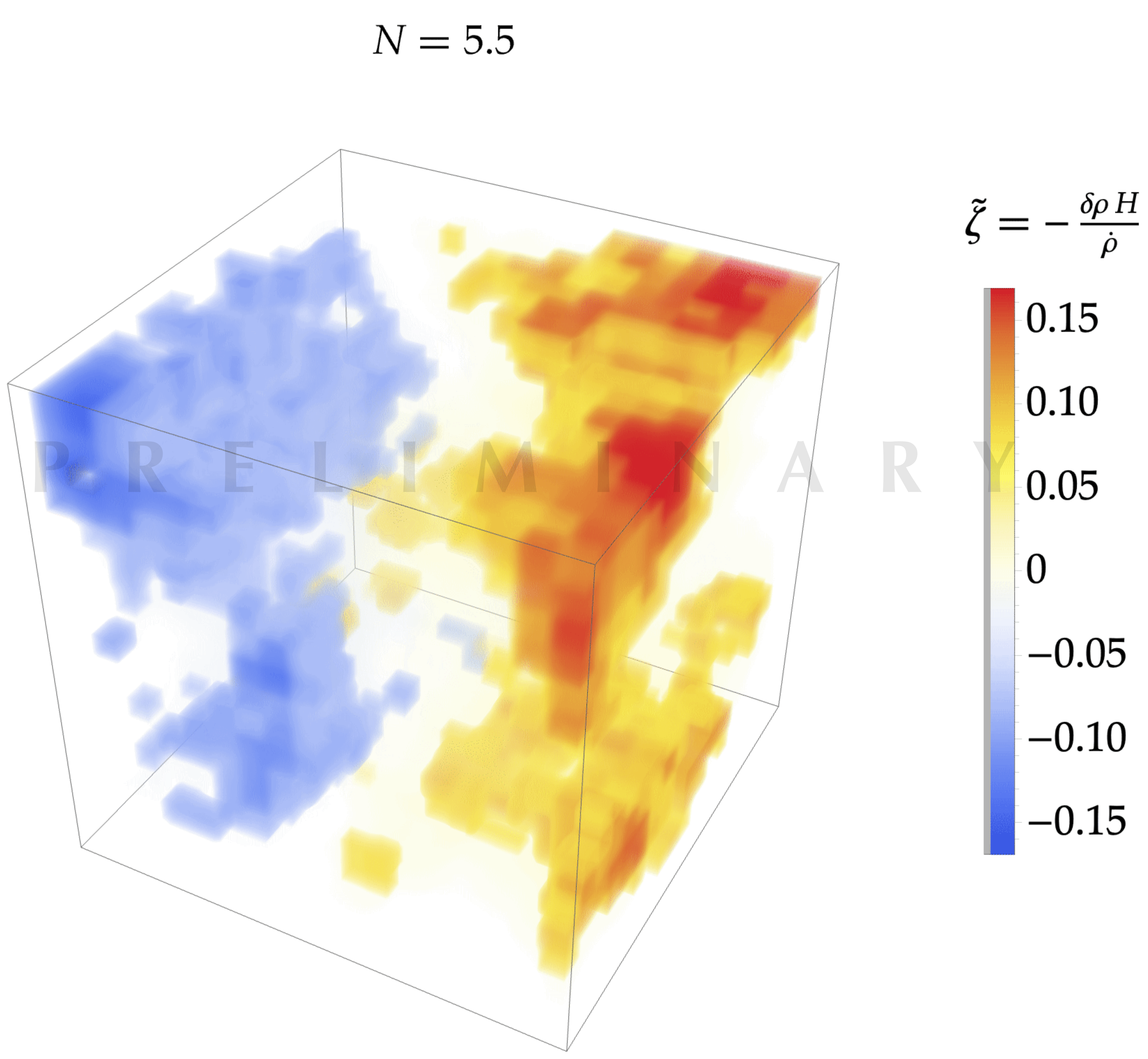

Importance Sampling

Intentionally large noise @ $N = 3$

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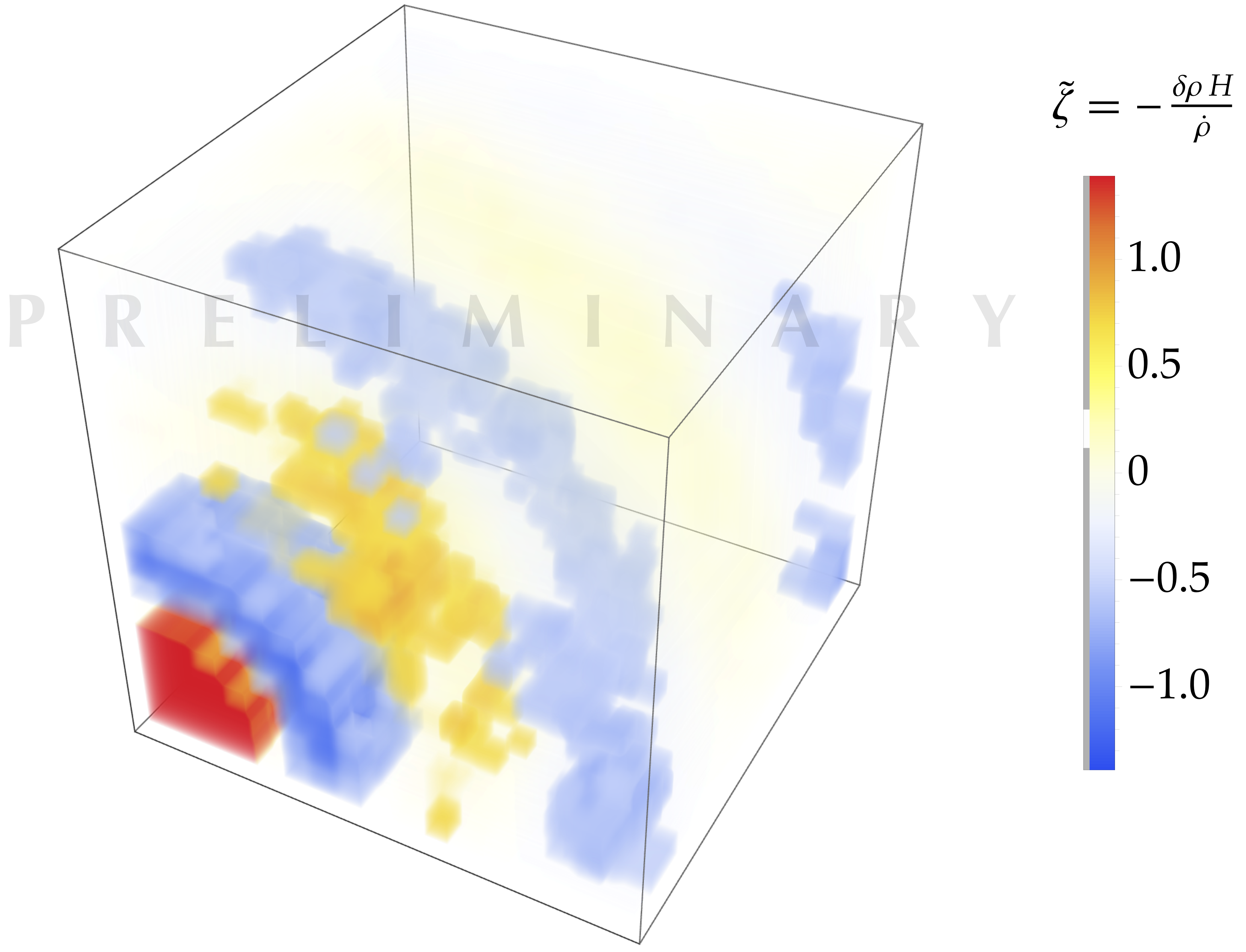

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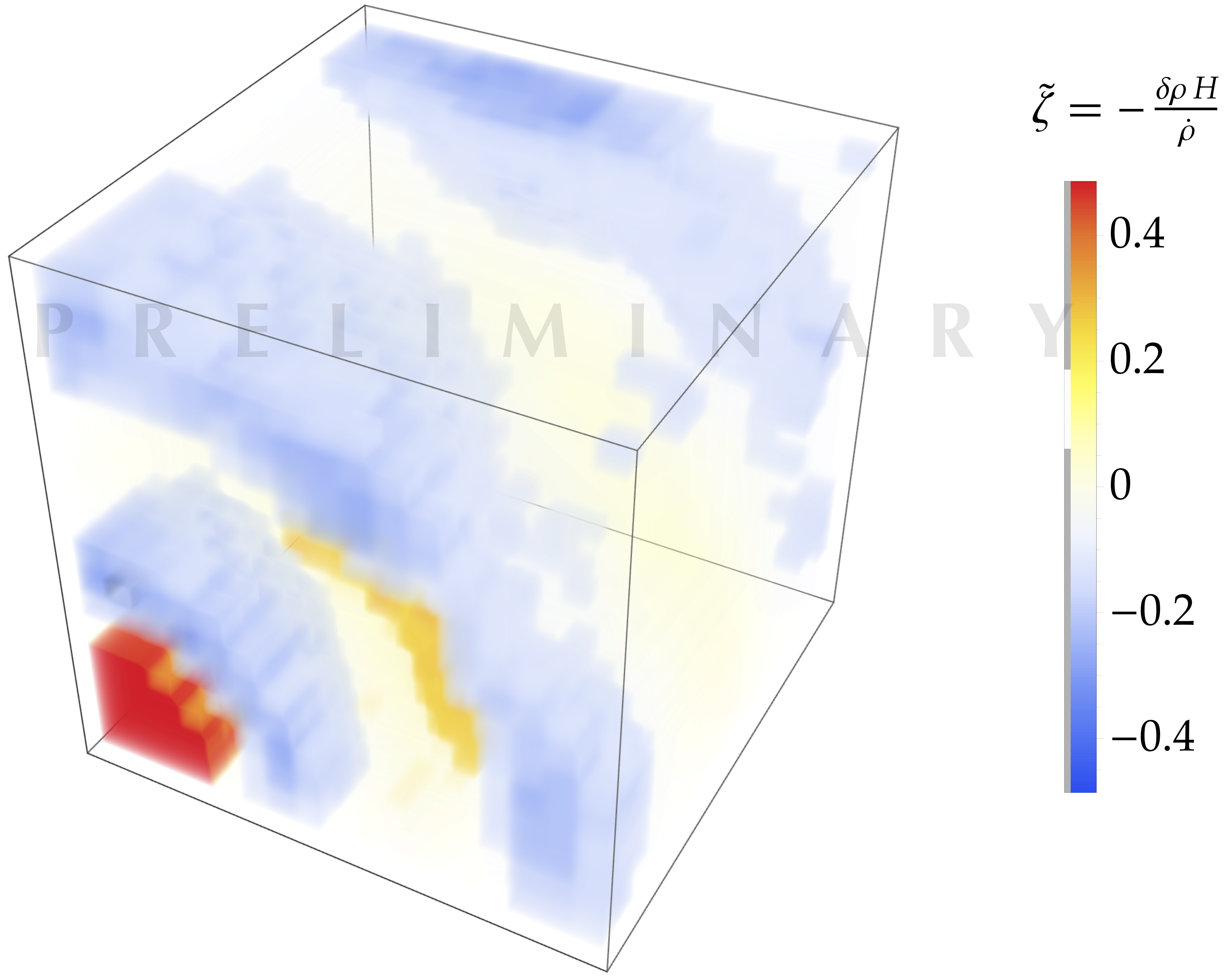

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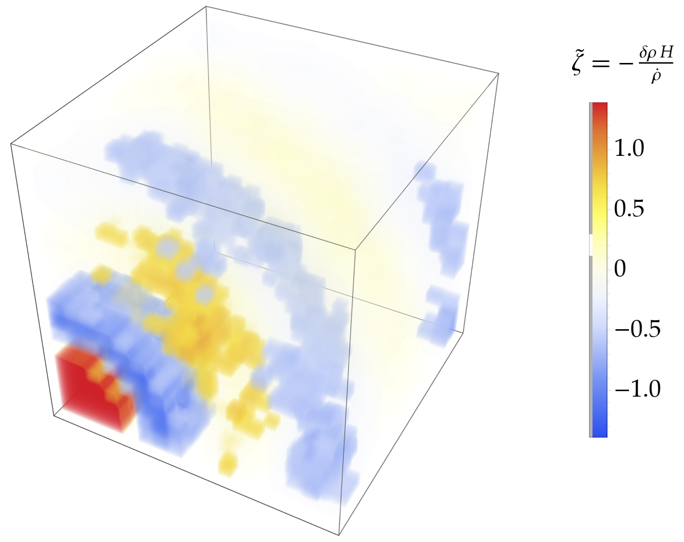

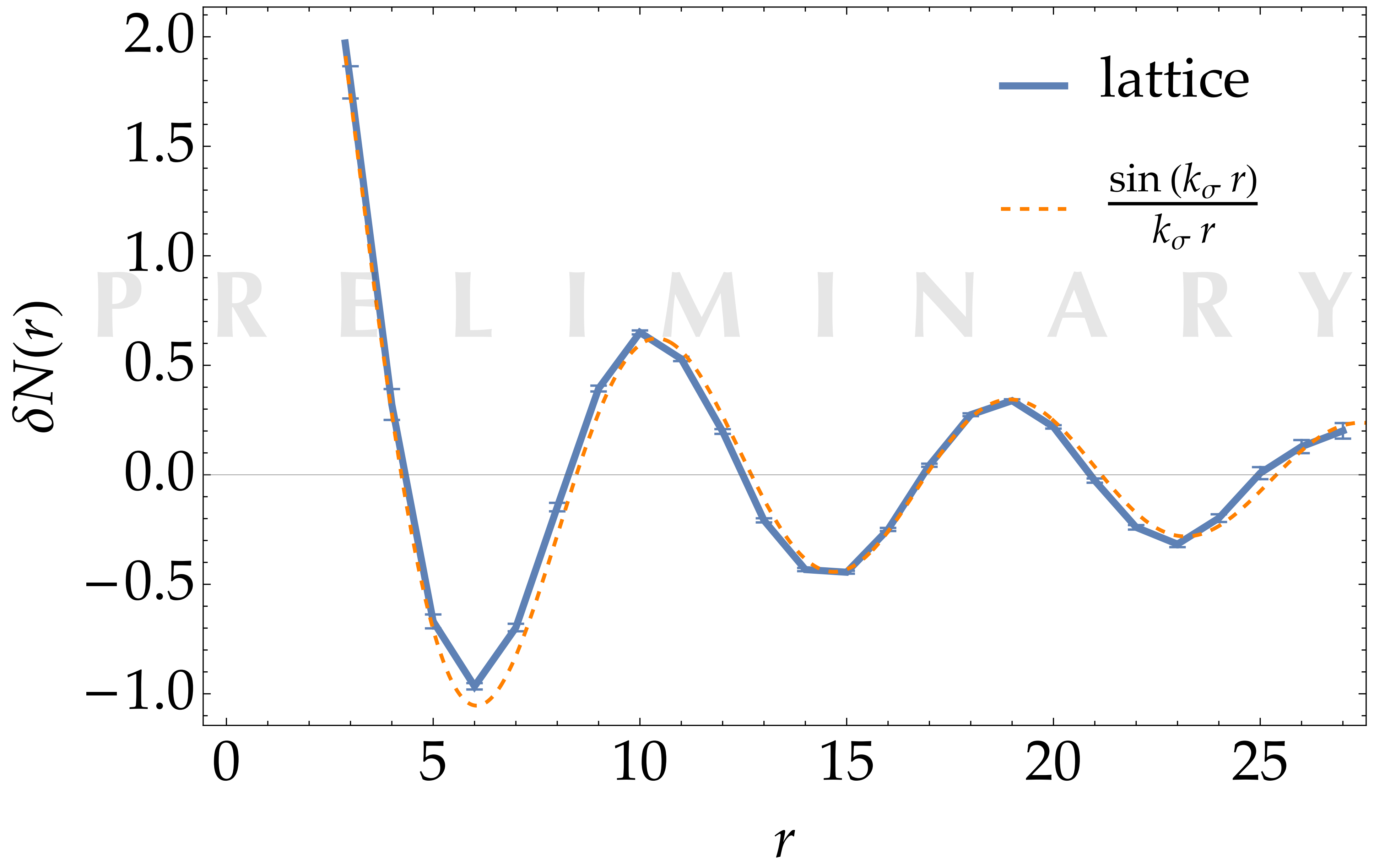

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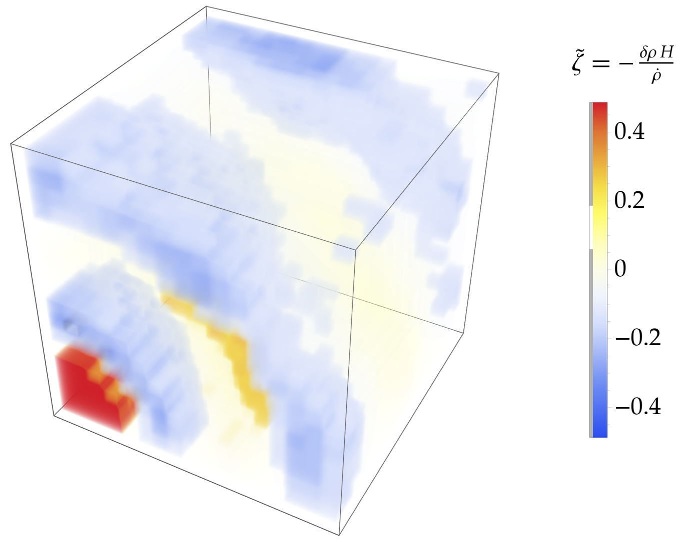
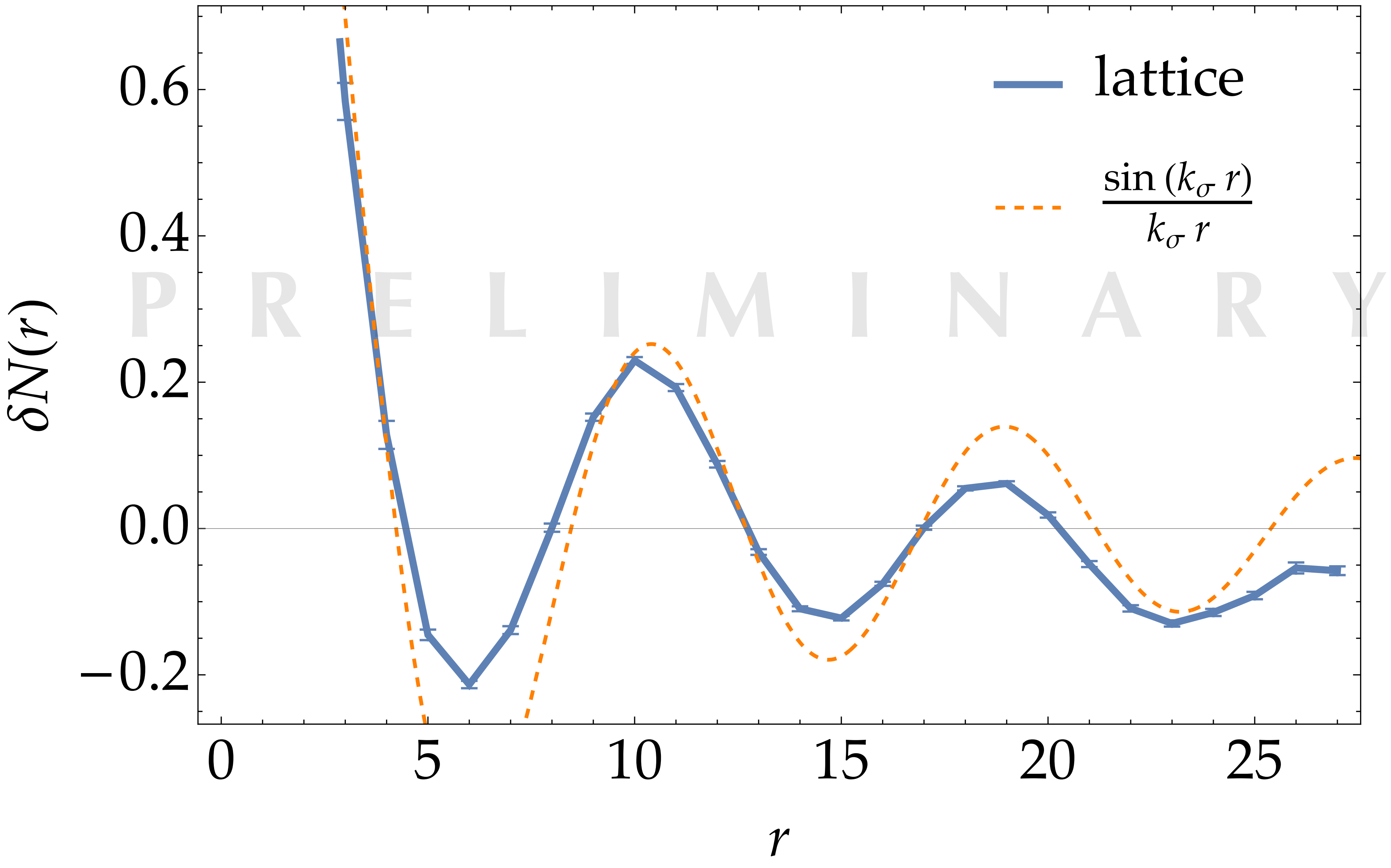




Summary

Stochastic formalism, EFT of superH fields, is a powerful tool

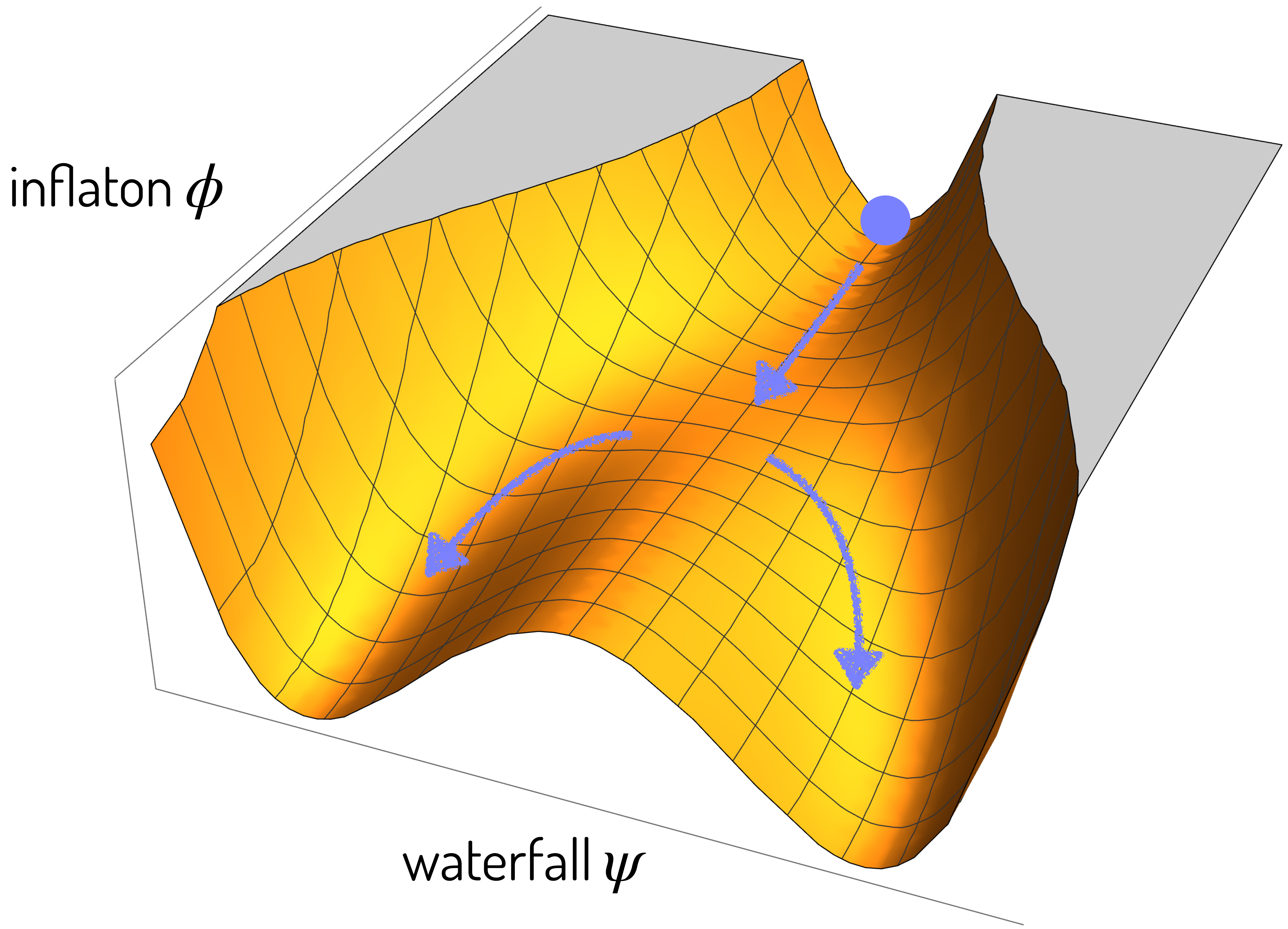
- w/ δN-formalism $\rightarrow$ stats. of ζ
- Non-ptb. analysis of phase transitions
- Graphical simulation of inflatons, gauge fields, ...

Appendices

Hybrid Inflation

Linde '94

$$V(\phi,\psi) = \frac{1}{4\lambda}(M^2 - \lambda\psi^2)^2 + \frac{g^2}{2}\phi^2\psi^2 + V(\phi)$$



- ψ's ptb. determines the whole dynamics during/after phase trs.
- Flat potential can extend the waterfall phase $\mathcal{N}_{\rm water} \simeq \mathcal{N}_{\rm BH}$

Realise Massive PBH??

García-Berido, Linde, Wands '96

Clesse & García-Berido '15

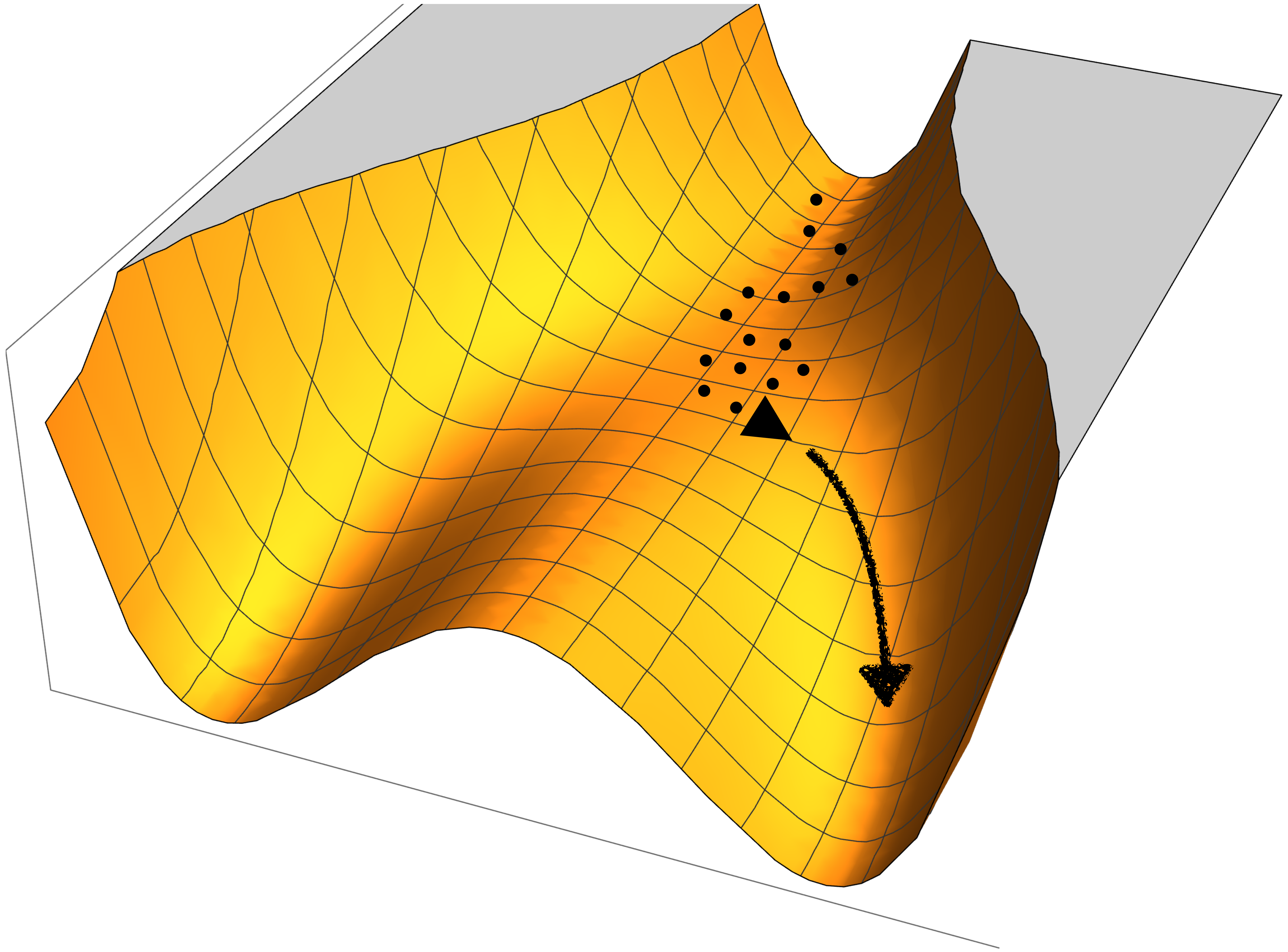
No-Go on Massive PBHs

Kawasaki & YT '15

$$V(\phi,\psi) = \boxed{\Lambda^4}\left[\left(1-\frac{\psi^2}{M^2}\right)^2 + 2\frac{\phi^2\psi^2}{\phi_c^2 M^2} + \frac{\phi-\phi_c}{\mu_1} - \frac{(\phi-\phi_c)^2}{\boxed{\mu_2^2}}\right]$$

$$\mathscr{P}_\zeta(k_{\rm CMB}) \simeq \frac{\Lambda^4}{24\pi^2\epsilon} \simeq 2\times 10^{-9}$$

$$n_s \simeq 1 + 2\frac{V_{\phi\phi}}{V} \simeq 0.96$$



- Valley phase

$$\frac{d\langle\psi^2\rangle}{dN} \simeq -\frac{V_{\psi\psi}}{V}(N)\bigg|_{\psi=0}\langle\psi^2\rangle + \left(\frac{H}{2\pi}\right)^2$$

$$\phi \simeq \phi_c - \frac{N-N_c}{\mu_1}$$

- Waterfall phase

$(\phi_{\rm b.g.},\psi_{\rm b.g.})$ from $\phi=\phi_c,\quad \psi = \sqrt{\langle\psi^2\rangle}\Big|_{\phi_c}$

linear ptb. around $(\phi_{\rm b.g.},\psi_{\rm b.g.})$

No-Go on Massive PBHs

Kawasaki & YT '15

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$$\mathcal{P}_\zeta(k_{\mathrm{CMB}}) \simeq \frac{\Lambda^4}{24\pi^2\epsilon} \simeq 2\times 10^{-9}$$

$$n_s \simeq 1 + 2\frac{V_{\phi\phi}}{V} \simeq 0.96$$

Clesse & García-Bellido '15

$$\begin{cases} \mathcal{N}_{\mathrm{BH}} \simeq \mathcal{N}_c \propto \Pi & \Pi = M\sqrt{\phi_c \mu_1} \\ \mathcal{P}_{\zeta,\max} \simeq \mathcal{P}_\zeta(k_c) \propto \Pi & \end{cases}$$

- Valley phase

$$\frac{\mathrm{d}\langle\psi^2\rangle}{\mathrm{d}N} \simeq -\frac{V_{\psi\psi}}{V}(N)\bigg|_{\psi=0} \langle\psi^2\rangle + \left(\frac{H}{2\pi}\right)^2$$

$$\phi \simeq \phi_c - \frac{N - N_c}{\mu_1}$$

- Waterfall phase

$(\phi_{\mathrm{b.g.}}, \psi_{\mathrm{b.g.}})$ from $\phi = \phi_c$, $\quad \psi = \sqrt{\langle\psi^2\rangle}\Big|_{\phi_c}$

linear ptb. around $(\phi_{\mathrm{b.g.}}, \psi_{\mathrm{b.g.}})$

$$\therefore\ \mathcal{P}_\zeta(k_c) \simeq 0.01\,\mathcal{N}_{\mathrm{BH}}$$

e.g. $\mathcal{P}_\zeta(k_c) \simeq \mathcal{O}(0.1)$ for $\mathcal{N}_{\mathrm{BH}} = \mathcal{O}(10)$

Massive PBHs will be inevitably overproduced!