2024.05.27 @ UTokyo

A Complete Guide to Primordial Black Holes

Yuichiro TADA   Nagoya U. IAR

Escrivà, Kühnel , YT 2211.05767
Mizuguchi, Murata, YT 2405.10692
YT, Terada, Tokuda 2308.04732
Escrivà, YT, Yoo 2311.17760, +Inui 2404.12591 ...

Contents

1. Inflation & Dark Matter

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'12–'17 PhD, UTokyo H. Murayama (IPMU), M. Kawasaki (ICRR)

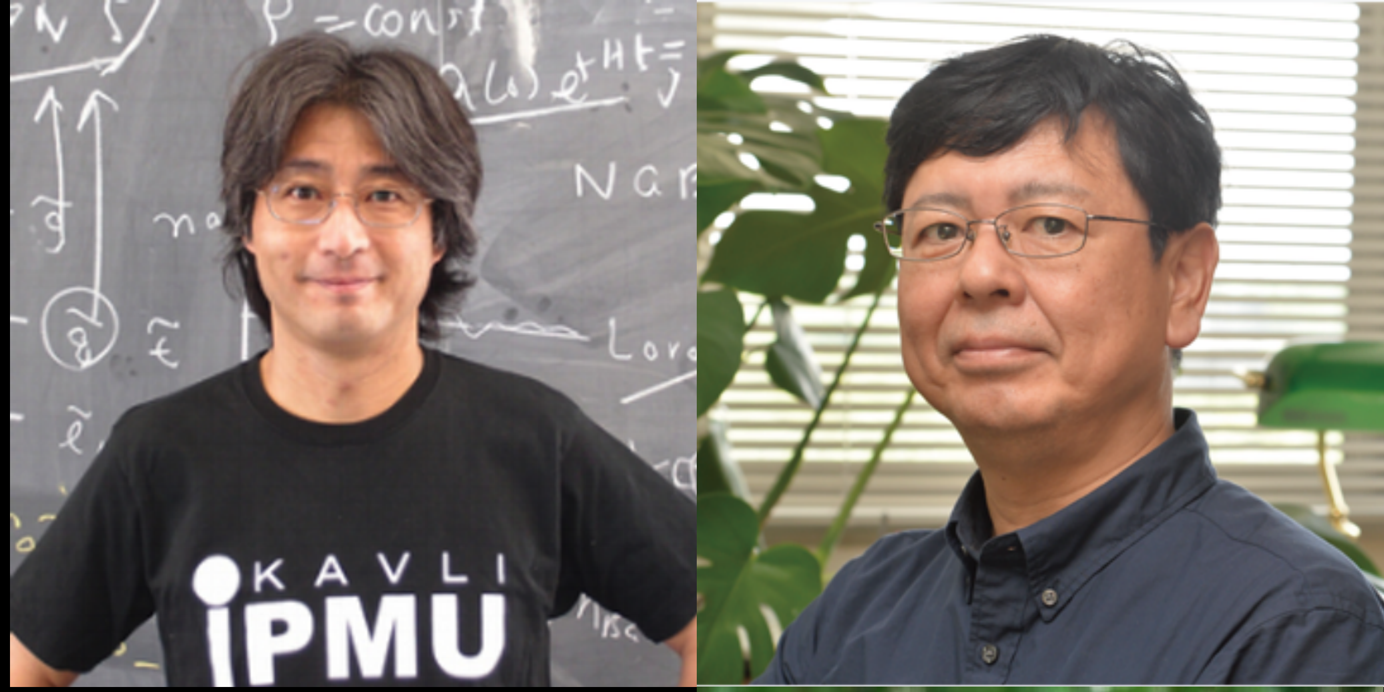
'17–'18 PD, IAP S. Renaux-Petel

'18–'21 JSPS PD, Nagoya U. N. Sugiyama

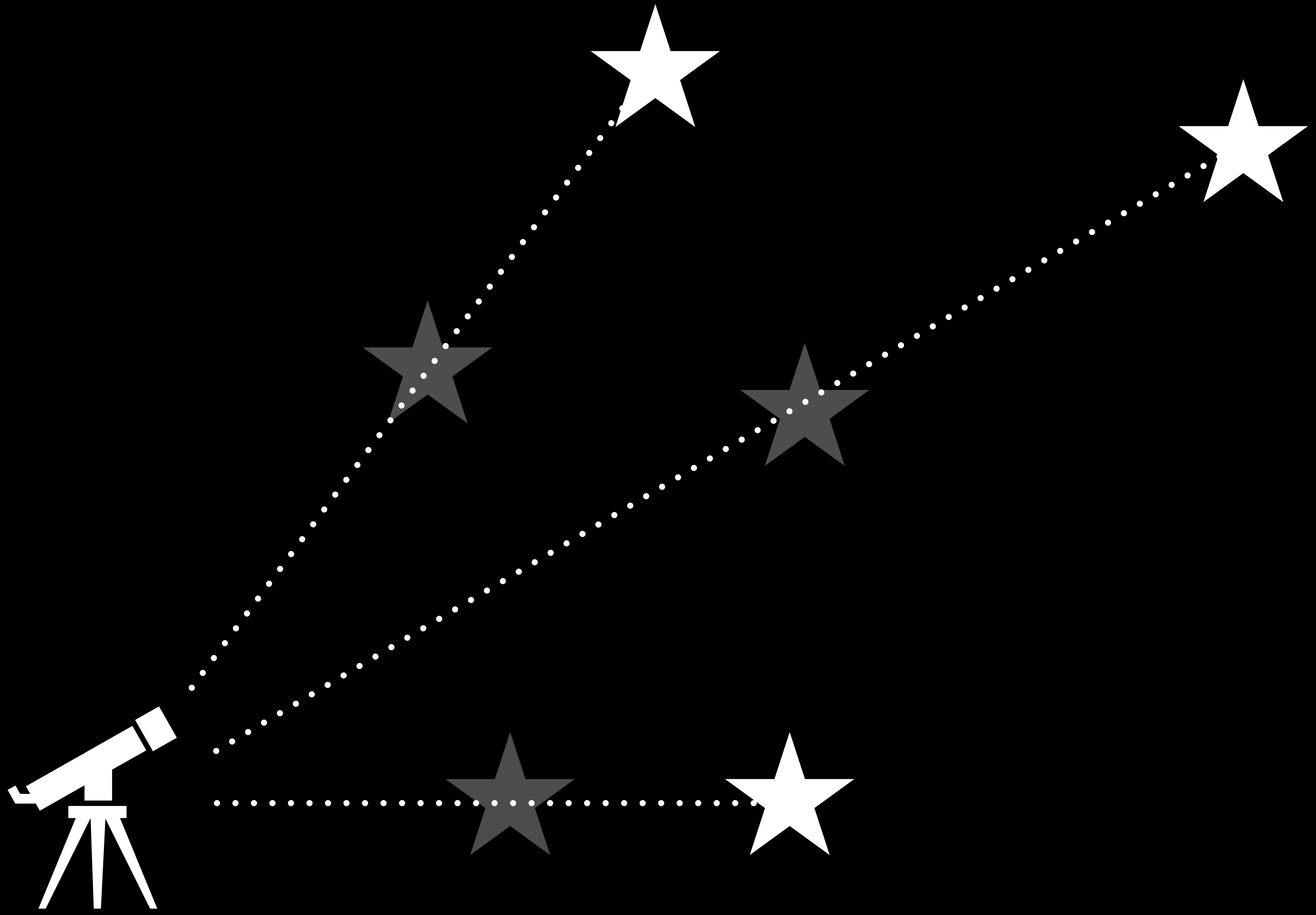
'21- YLC assistant prof., Nagoya U. K. Ichiki

- Theoretical approach to Early Universe

  ↔ Observation = Inflation, PBH, GW, ...




Expanding U.

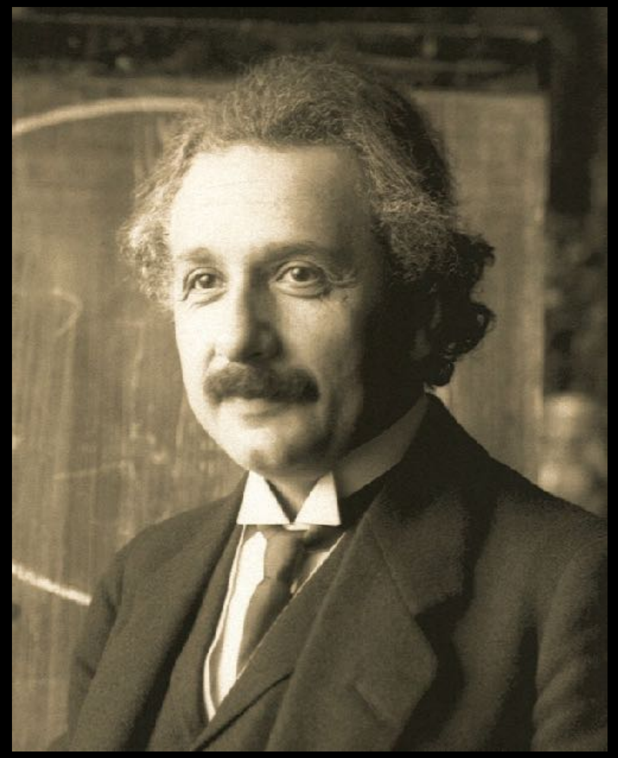


E. Hubble G. Lemaitre

Whole Universe Expanding!!

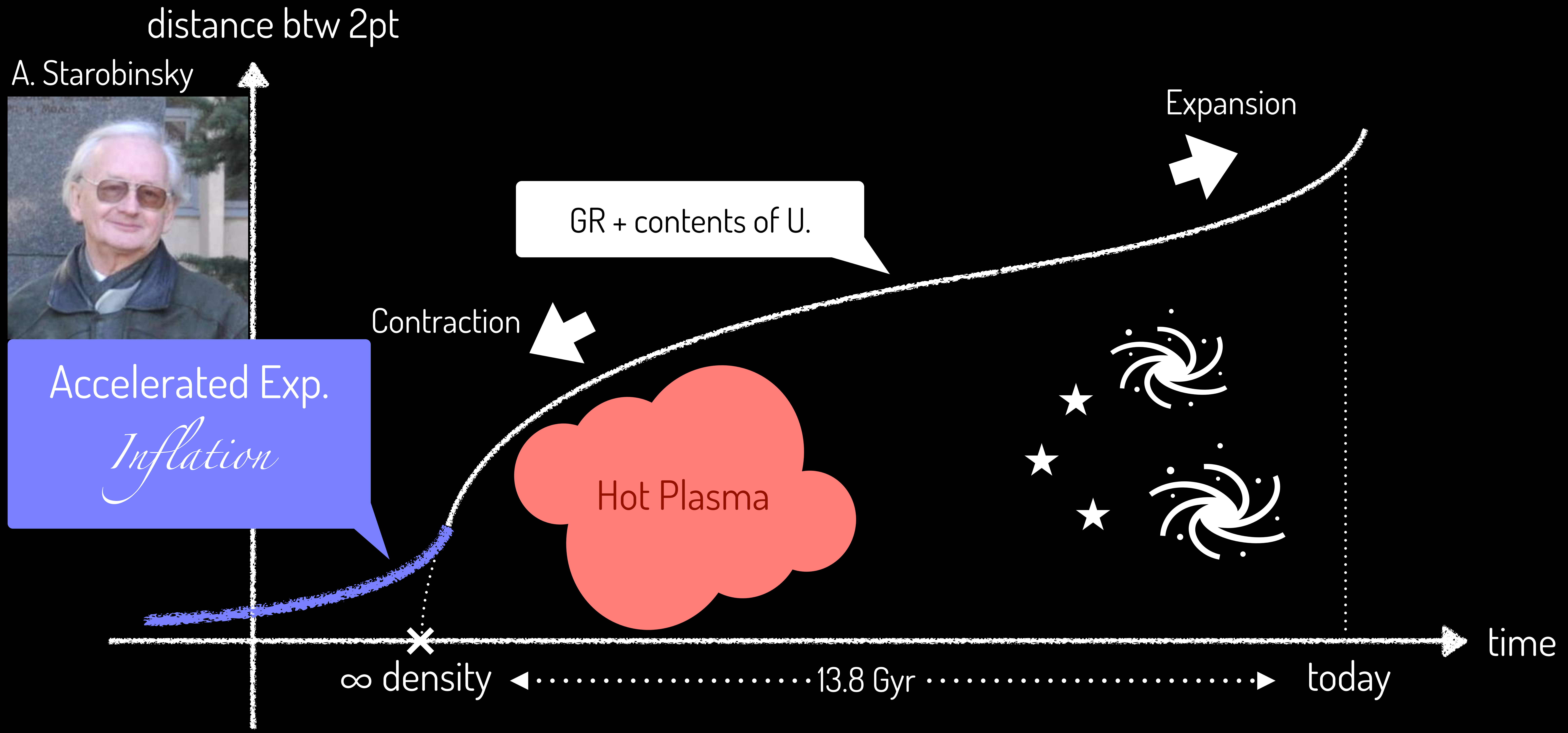
$\Uparrow$

General Relativity

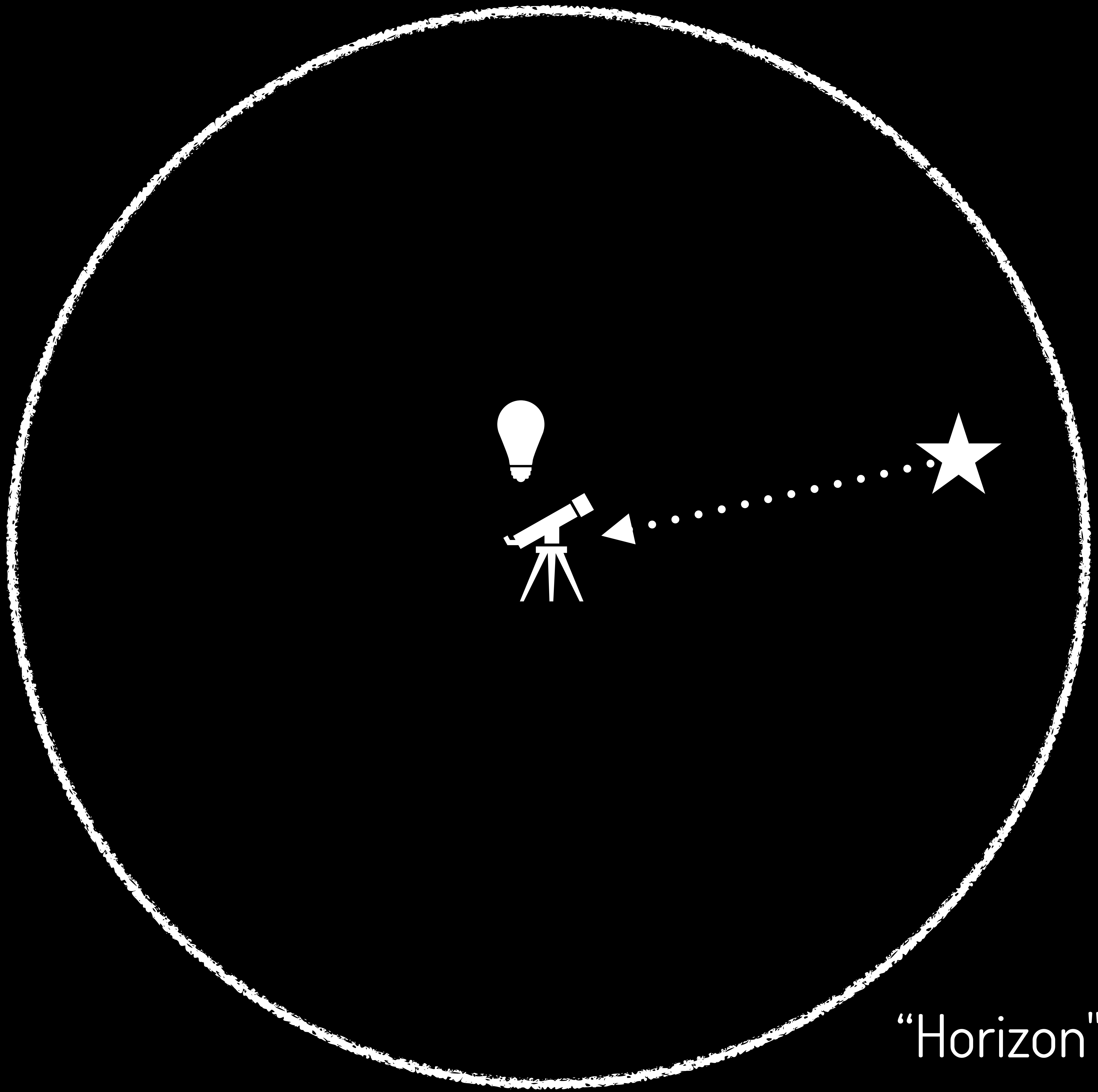


A. Einstein

Expanding U.

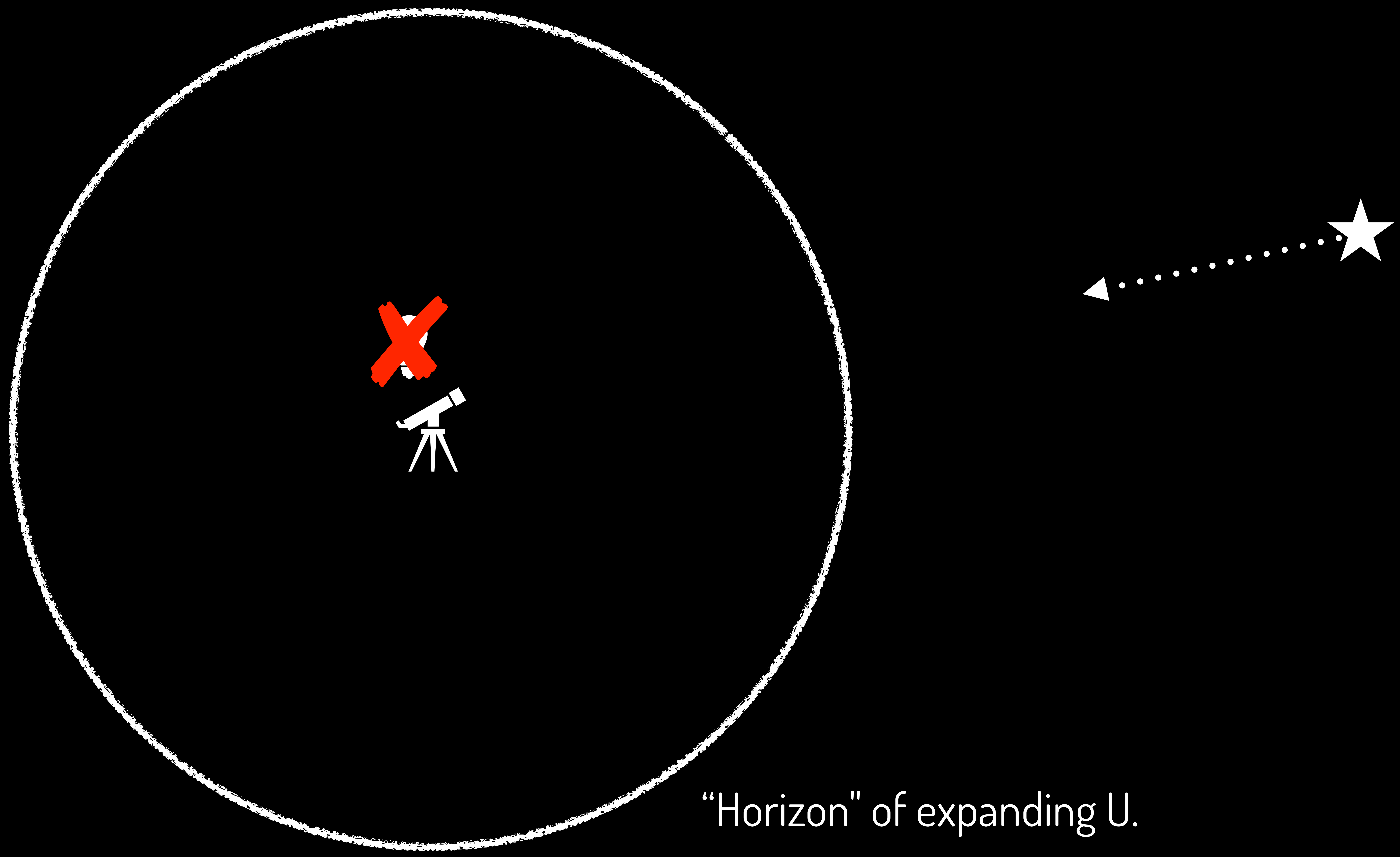

distance btw 2pt
A. Starobinsky
Expansion
GR + contents of U.
Contraction
Accelerated Exp.
Inflation
Hot Plasma
time
∞ density
13.8 Gyr
today

Generation of PTB

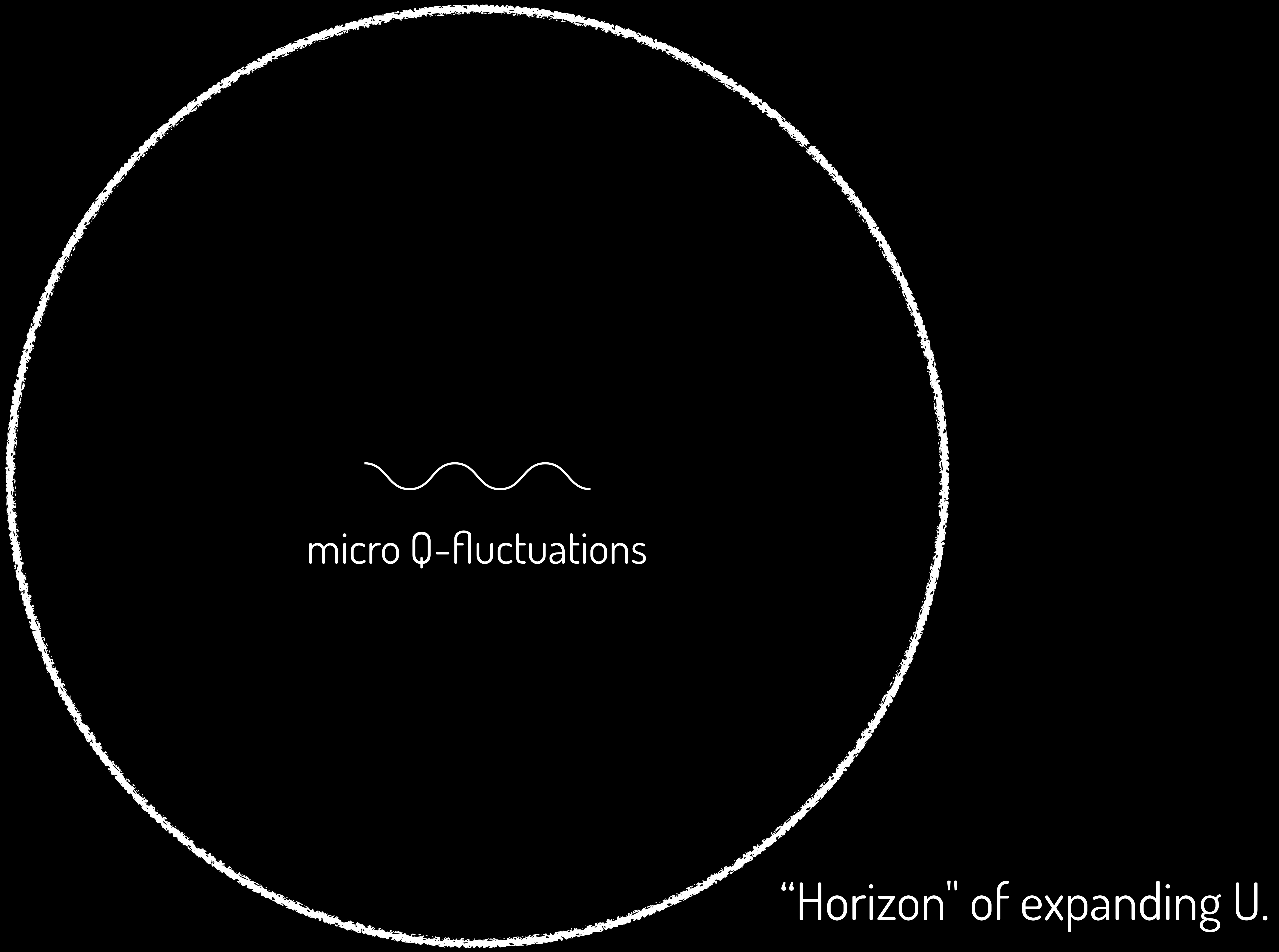


"Horizon" of expanding U.

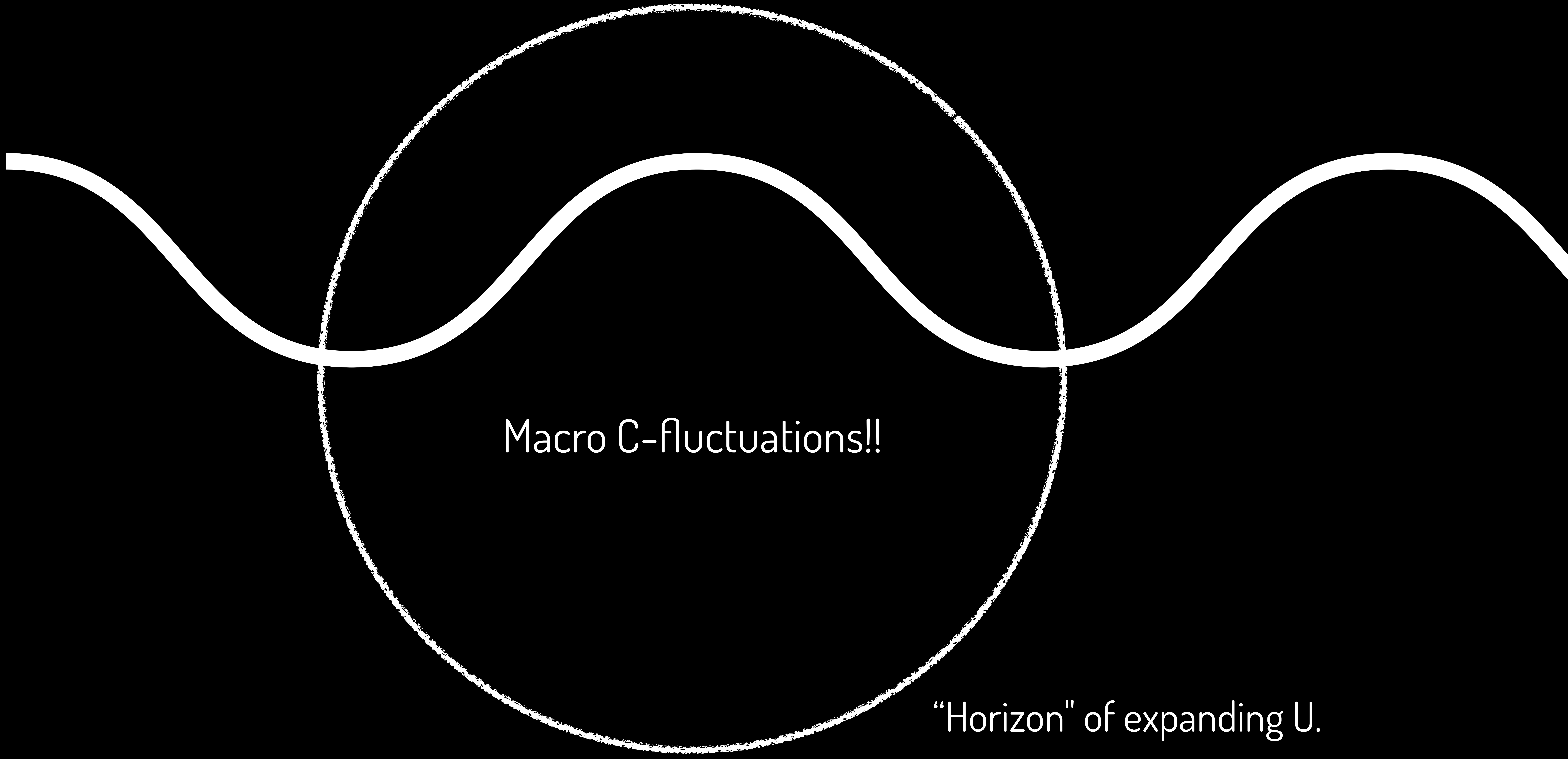
Generation of PTB



Generation of PTB

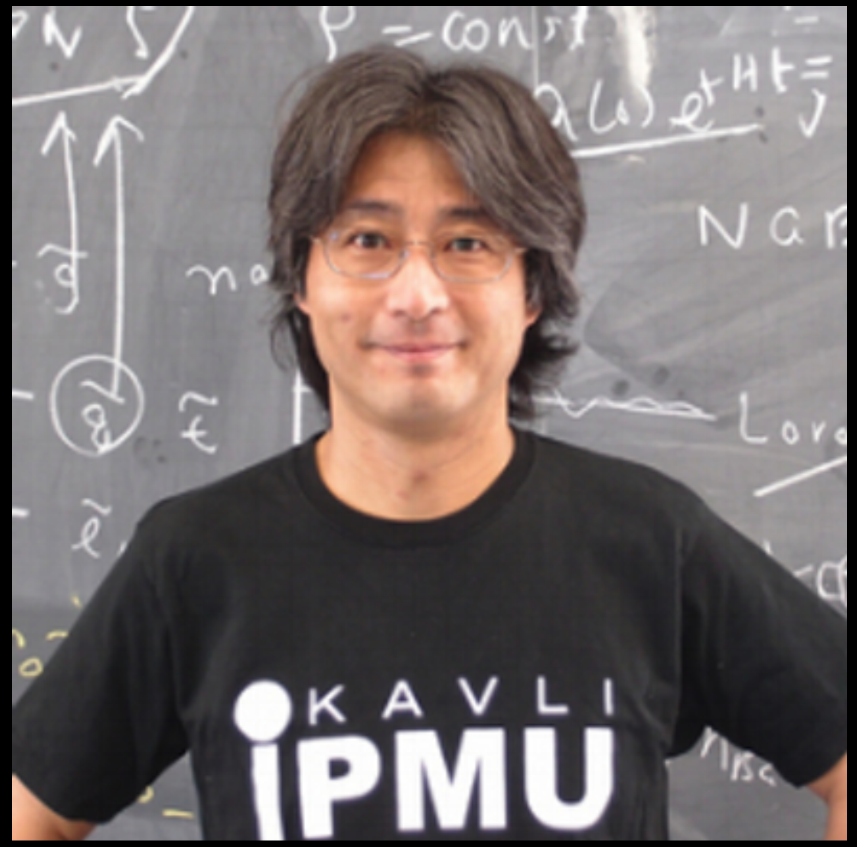


Generation of PTB



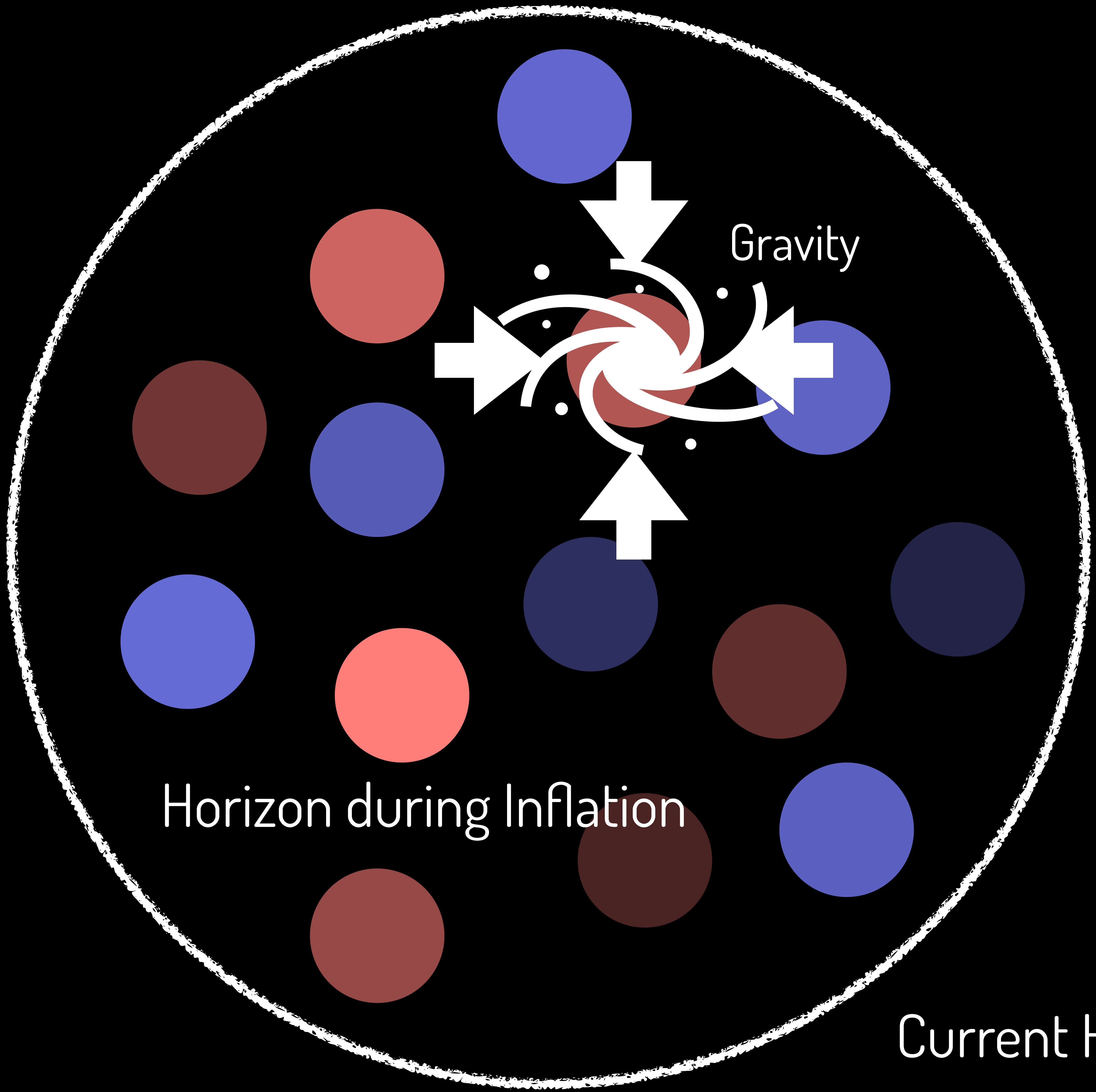
Generation of PTB

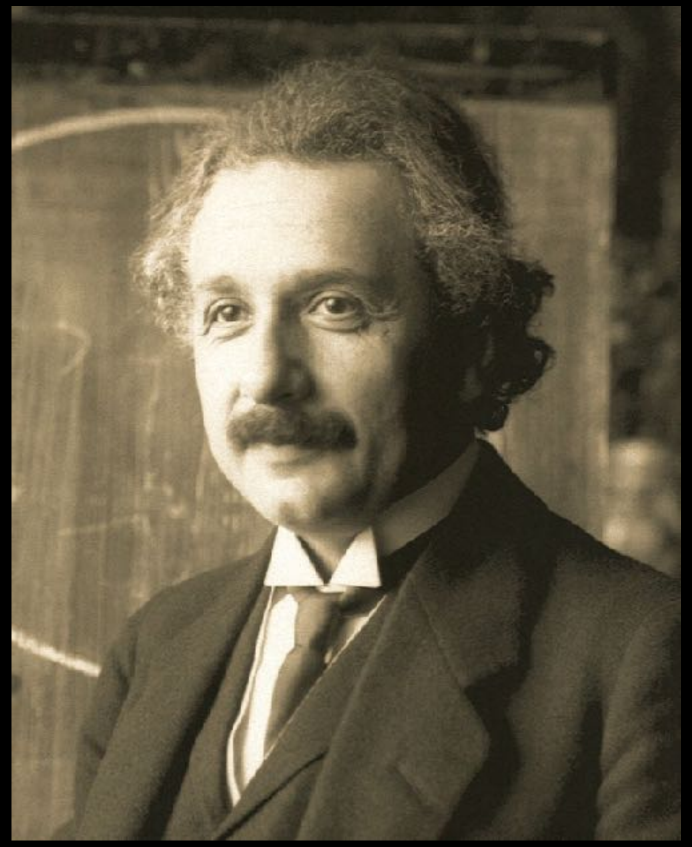
Generation of PTB



Inflation is our “father”

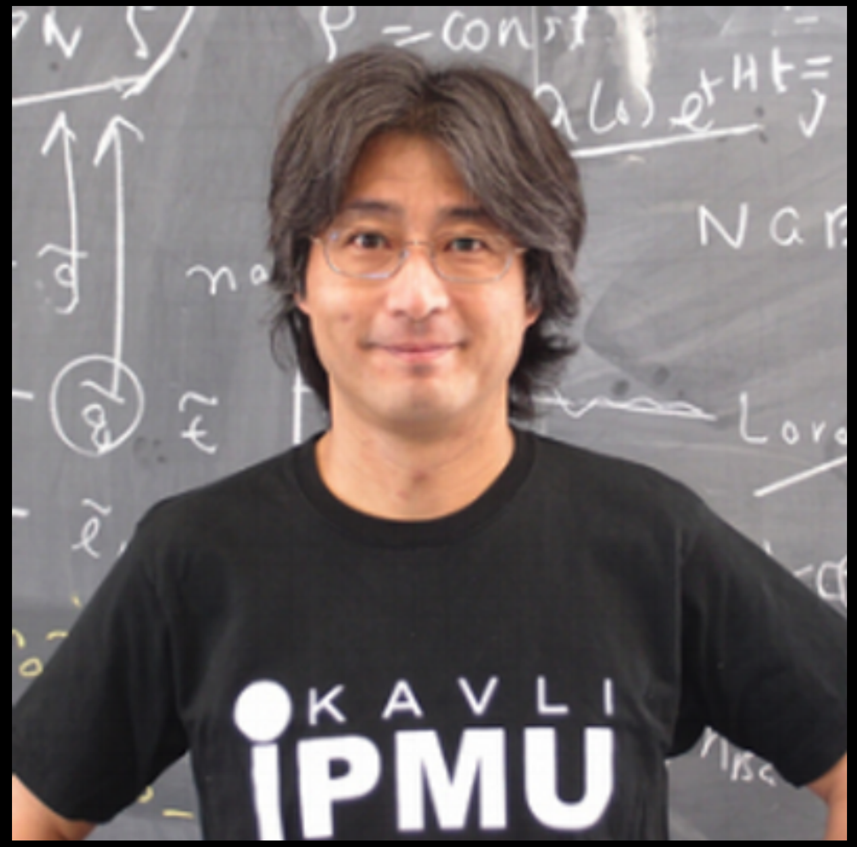
朝日新聞連載





Energy = Mass

Dark Matter



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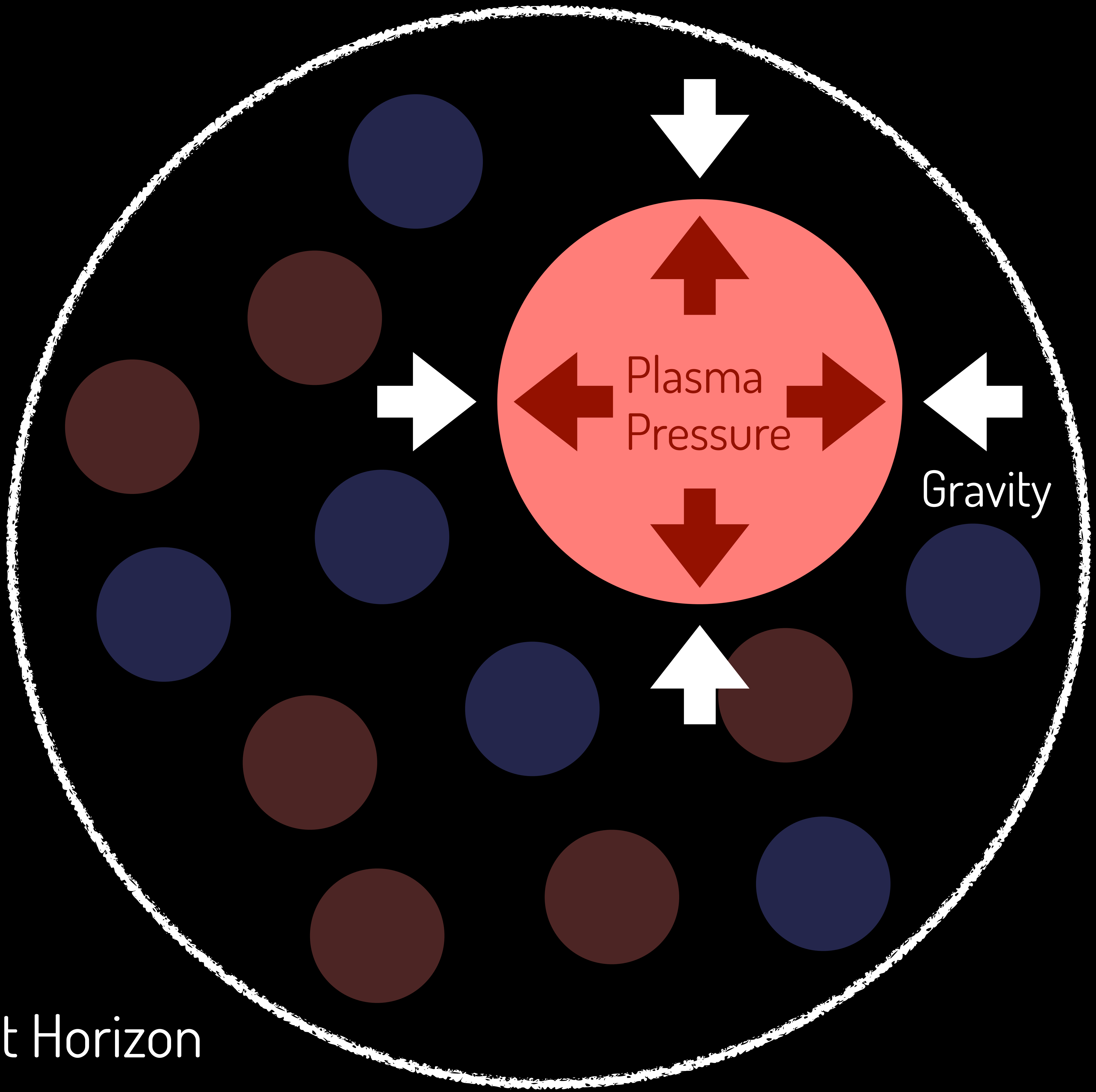


Gathering after Cooling?

↓

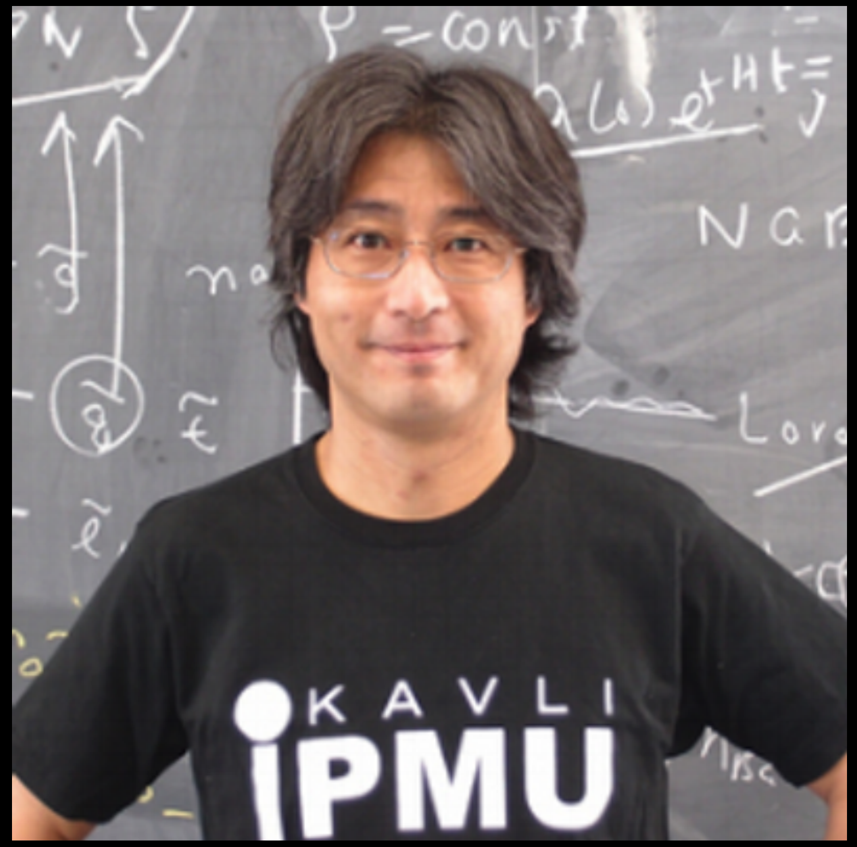
13.8 Gyr is NOT enough!

c.f. 吉田直紀「宇宙のダークマター」
高校生のための東京大学オープンキャンパス2018

Dark Matter

Dark (insensitive to plasma pressure) Massive Matter

Dark Matter



Inflation is our “father”

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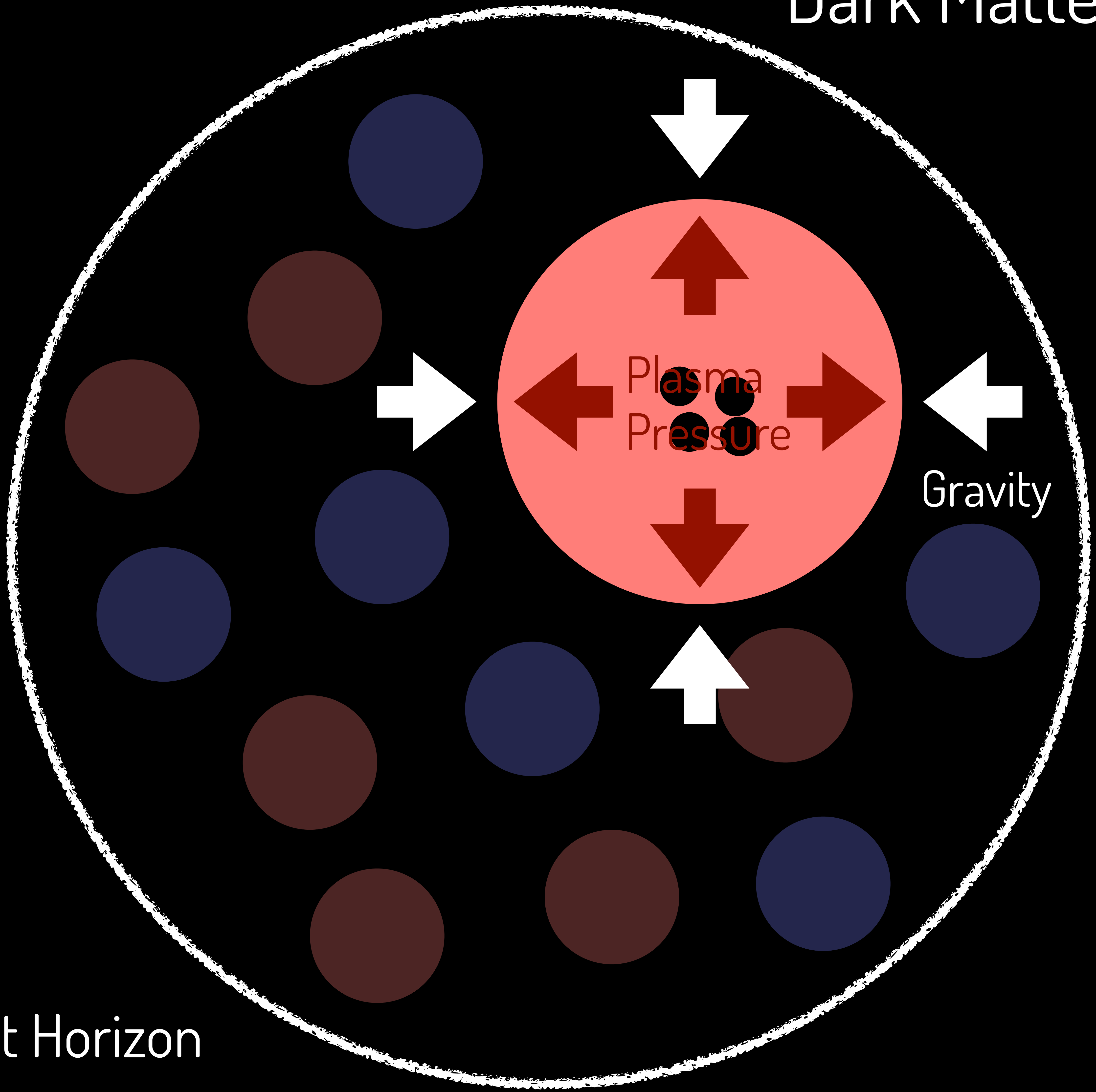
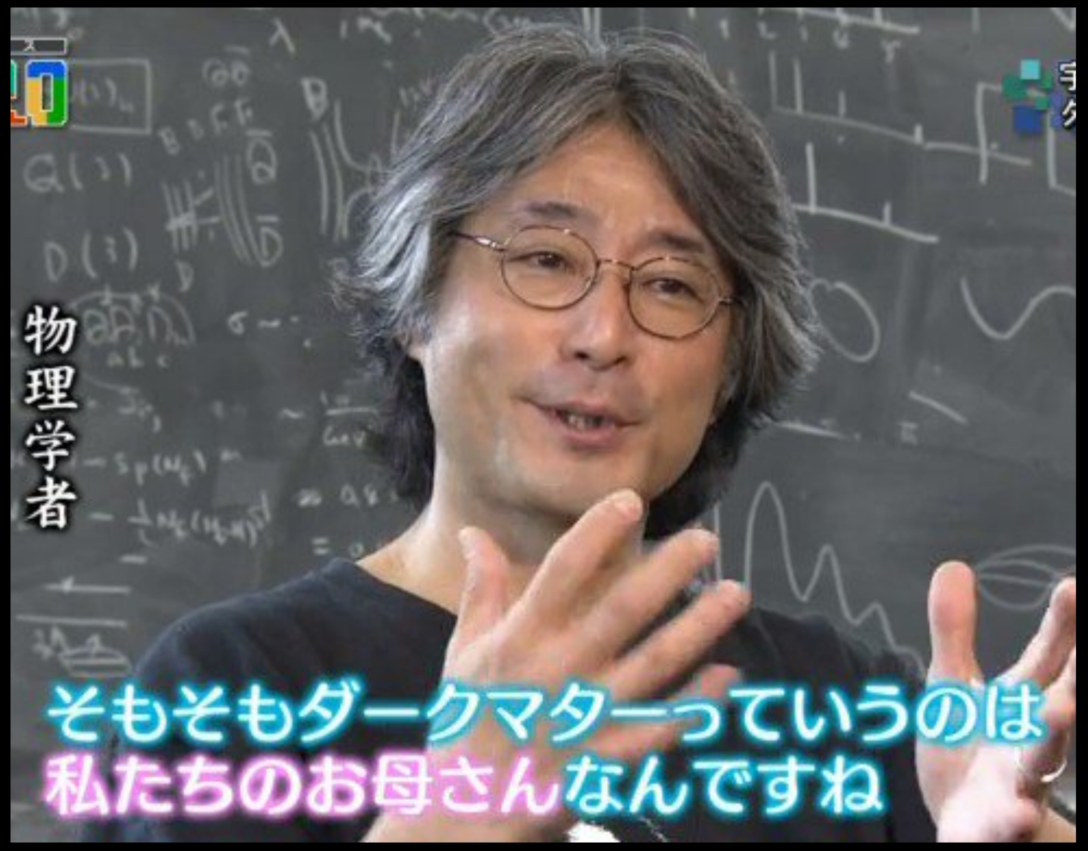


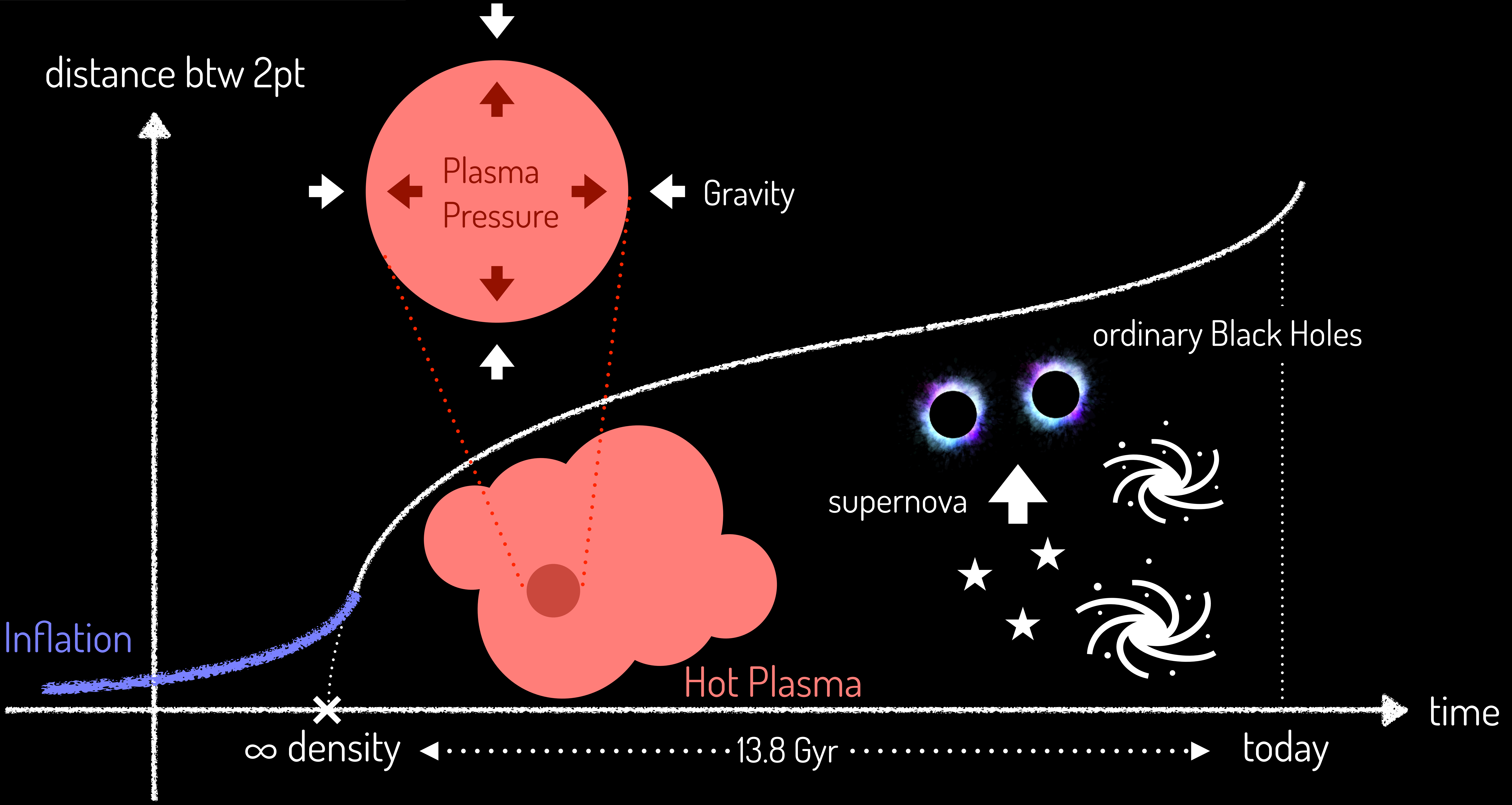


Dark Matter is our “mother”

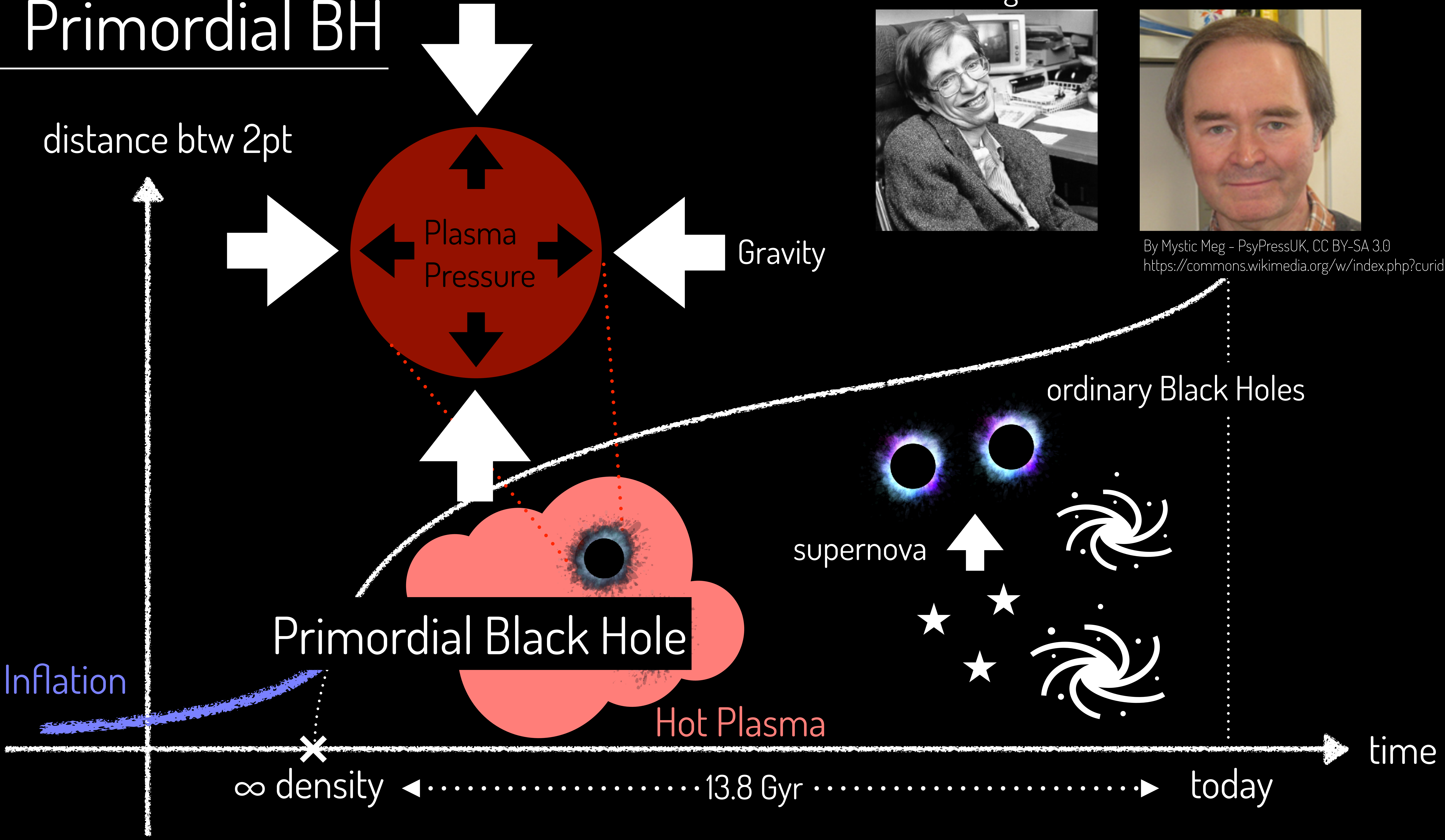
サイエンスZERO


2. Primordial Black Hole

Primordial BH



Primordial BH



S. Hawking

B. Carr

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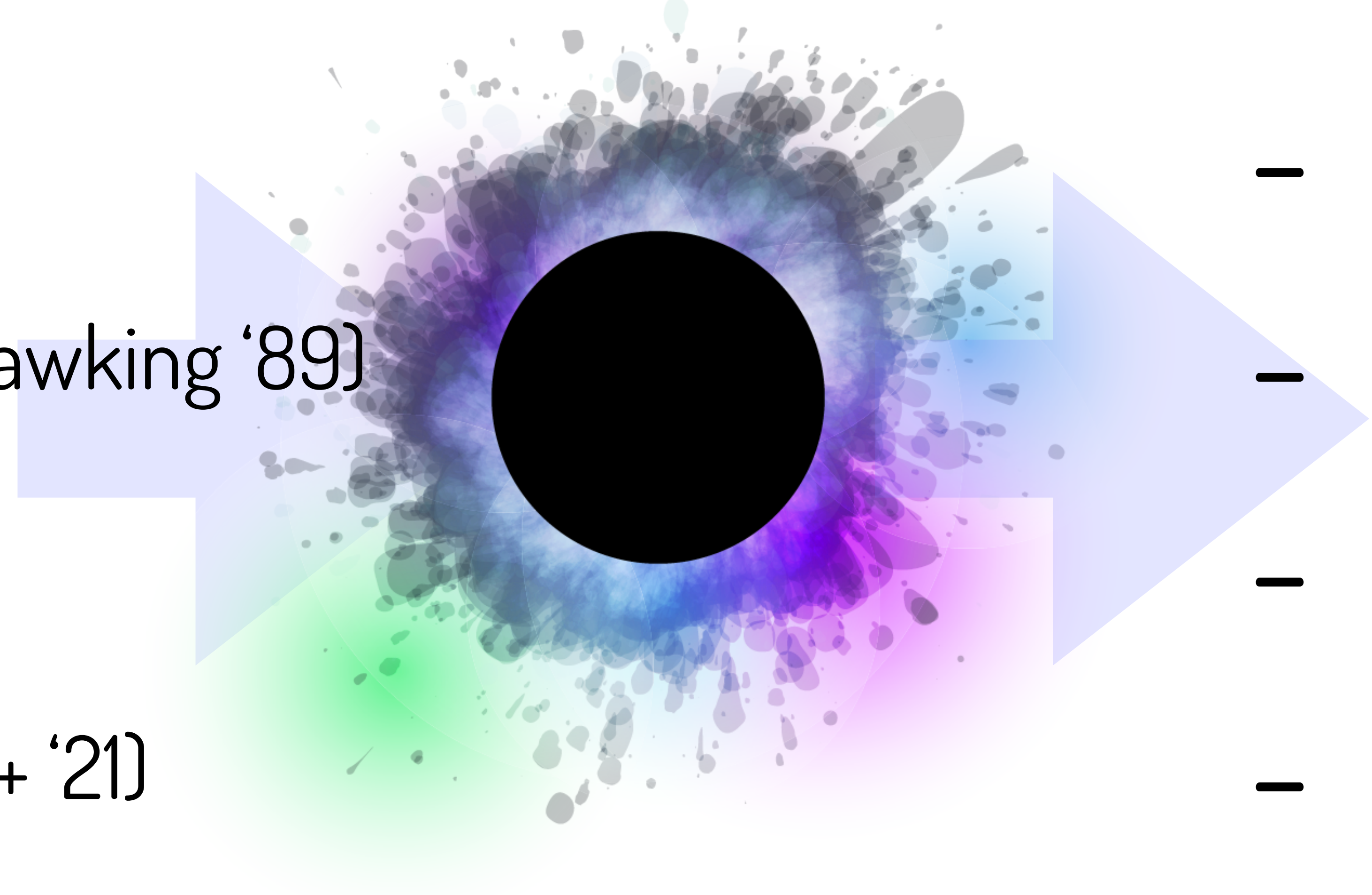
https://commons.wikimedia.org/w/index.php?curid=16321736

Primordial BH

Carr & Hawking '74

- $\sim \mathcal{O}(1)$ overdensity (Carr '75)
- Isocurvature (Dolgov & Silk '93)
- Quark Confinement (Dvali+ '21)
- Collapse of topological defect (Hawking '89)
- Bubble collision (Hawking+ '82)
- Particle trapping in bubble (Baker+ '21)
- Asynchronous 1st PT (Liu+ '21, Lewicki+ '24)
- Scalar 5th force (Flores & Kusenko '20)

⋮



before Star Formation

Primordial Black Hole

- Dark Matter (Chapline '75)
- LVK merger GW? (Sasaki+ '16)
- SMBH seeds? (Düchting '04)
- OGLE lensing obj.? (Niikura+ '19)
- Planet 9? (Scholtz & Unwin '19)
- Trigger of r-process? (Fuller+ '17)
- Baryogenesis? (Baumann+ '07)
- JWST luminous gals? (Hutsi+ '22)

⋮

Primordial BH

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- $\sim \mathcal{O}(1)$ overdensity (Carr
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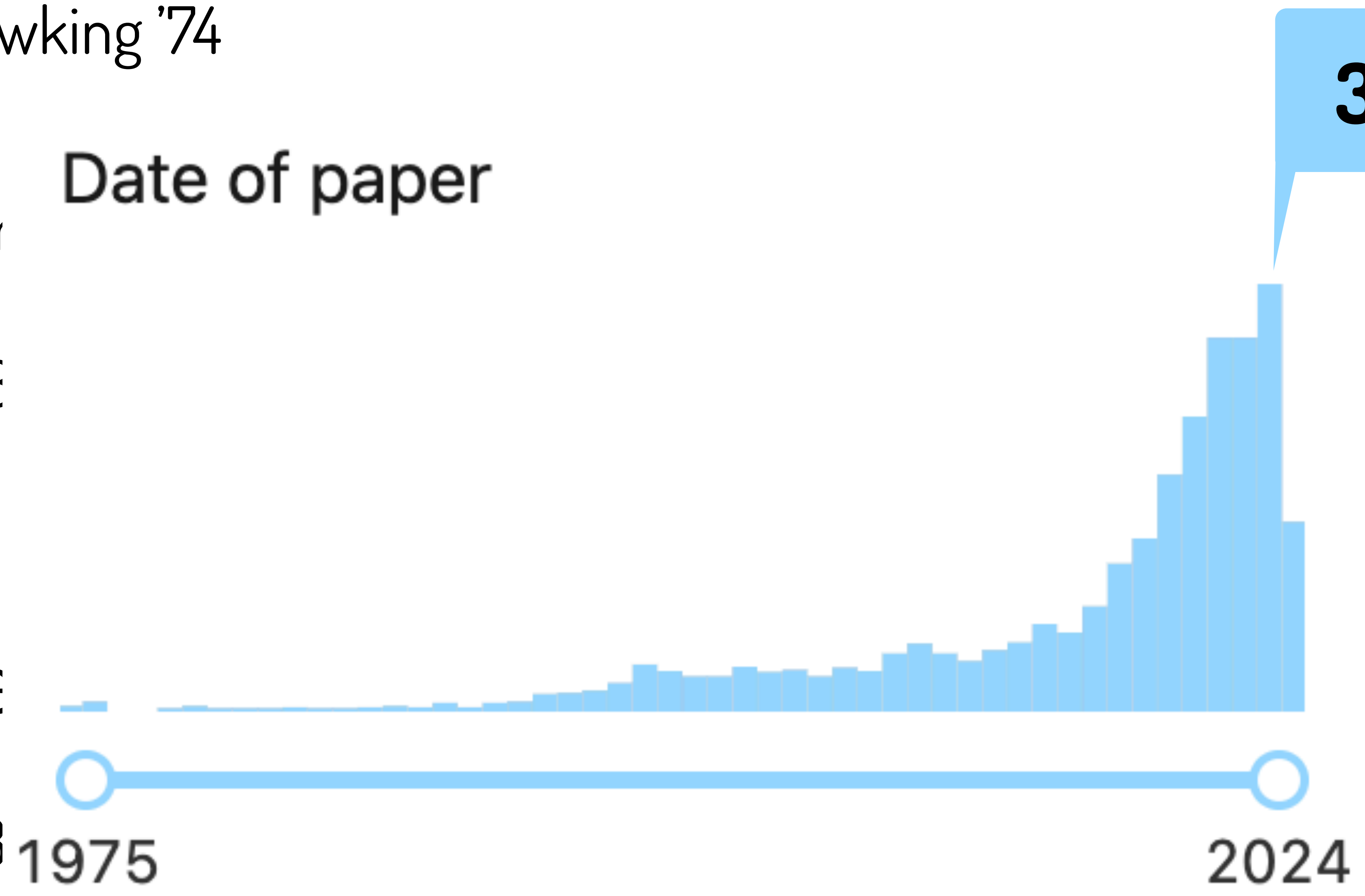
⋮



before Star Formation

Primordial Black Hole

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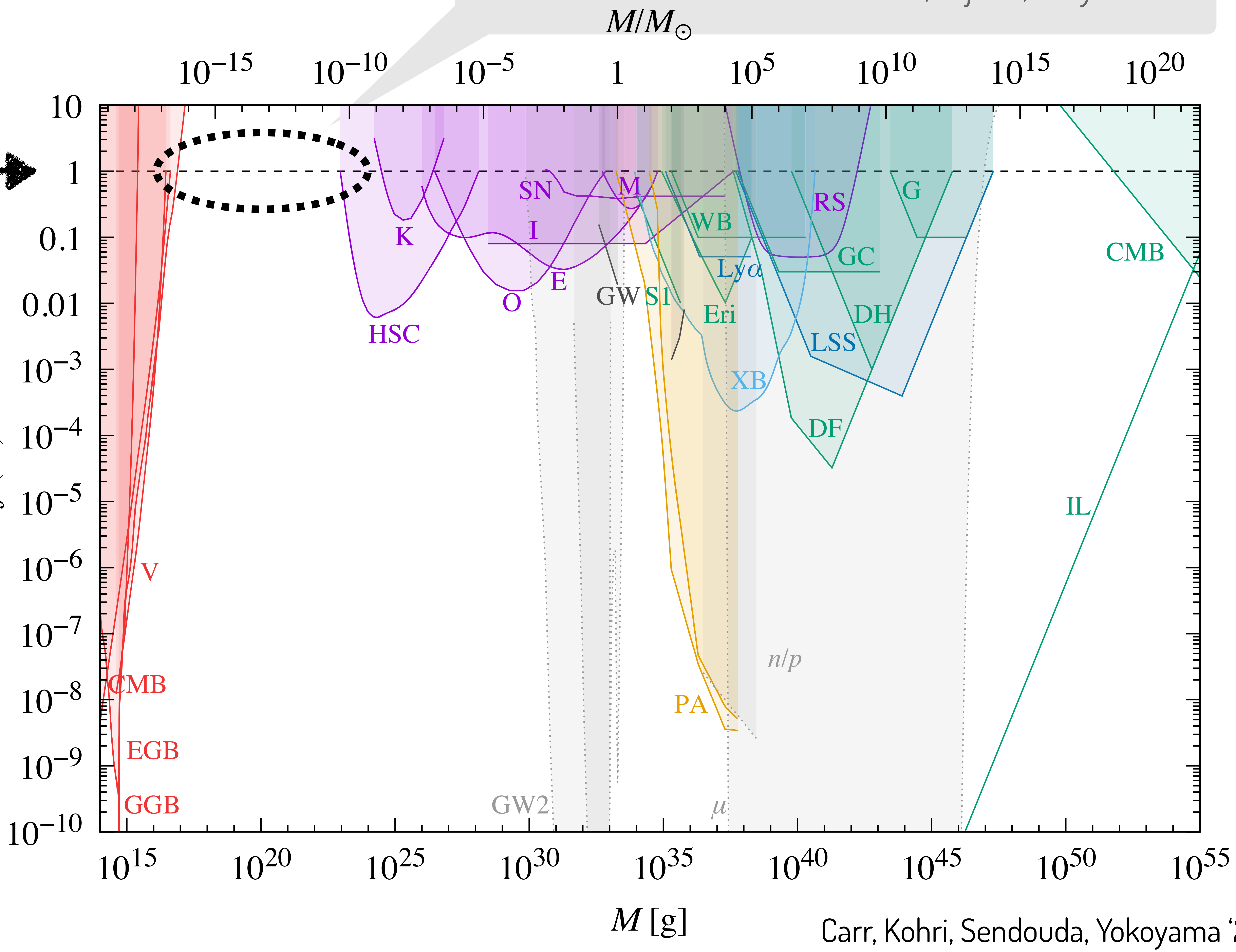
⋮

Obs. Consts.

Stellar Mass Function in Ultra-Faint Dwarfs...?
Esser, Rijcke, Tinyakov '23

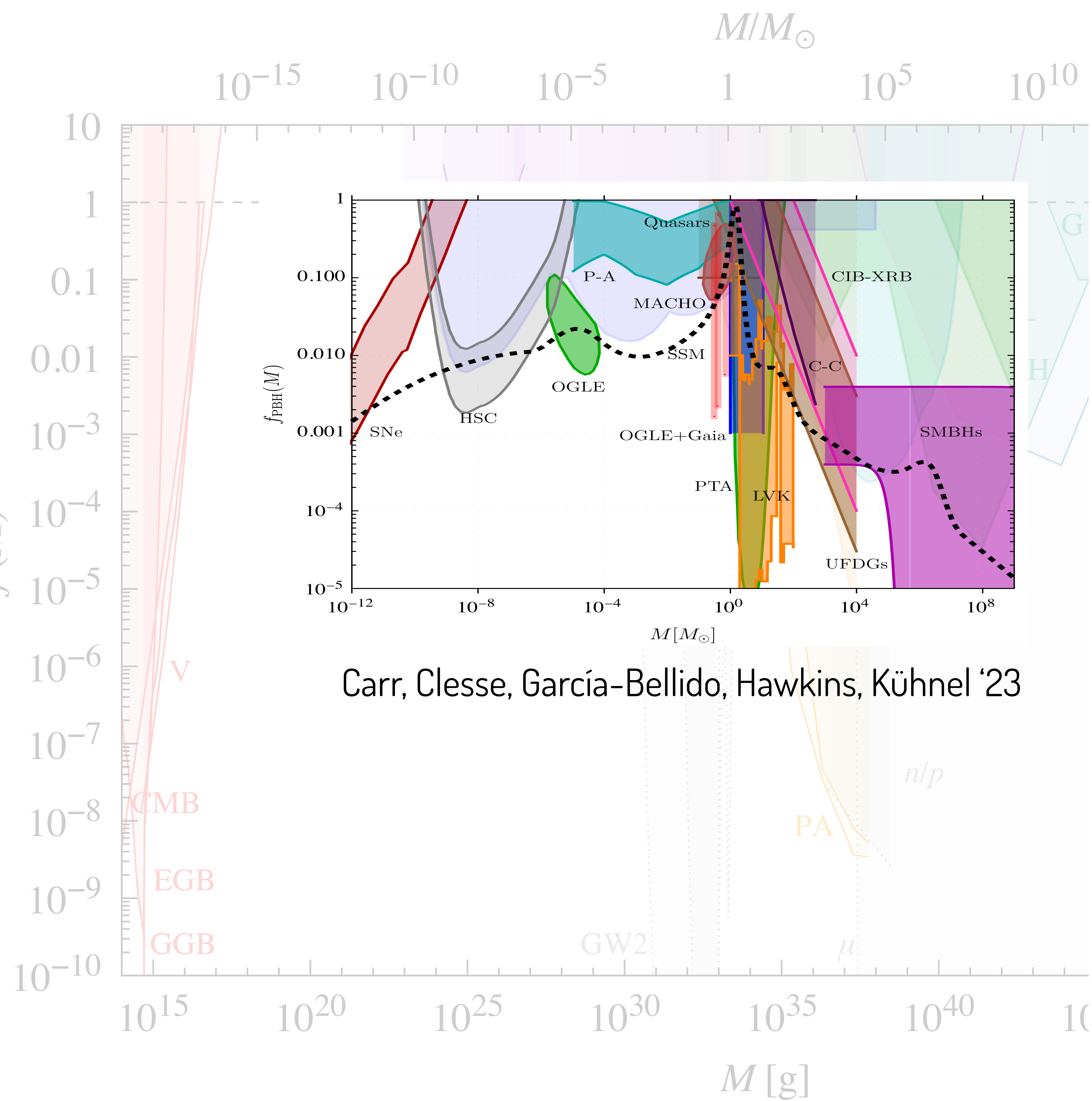
All DM = PBH

PBH / All DM

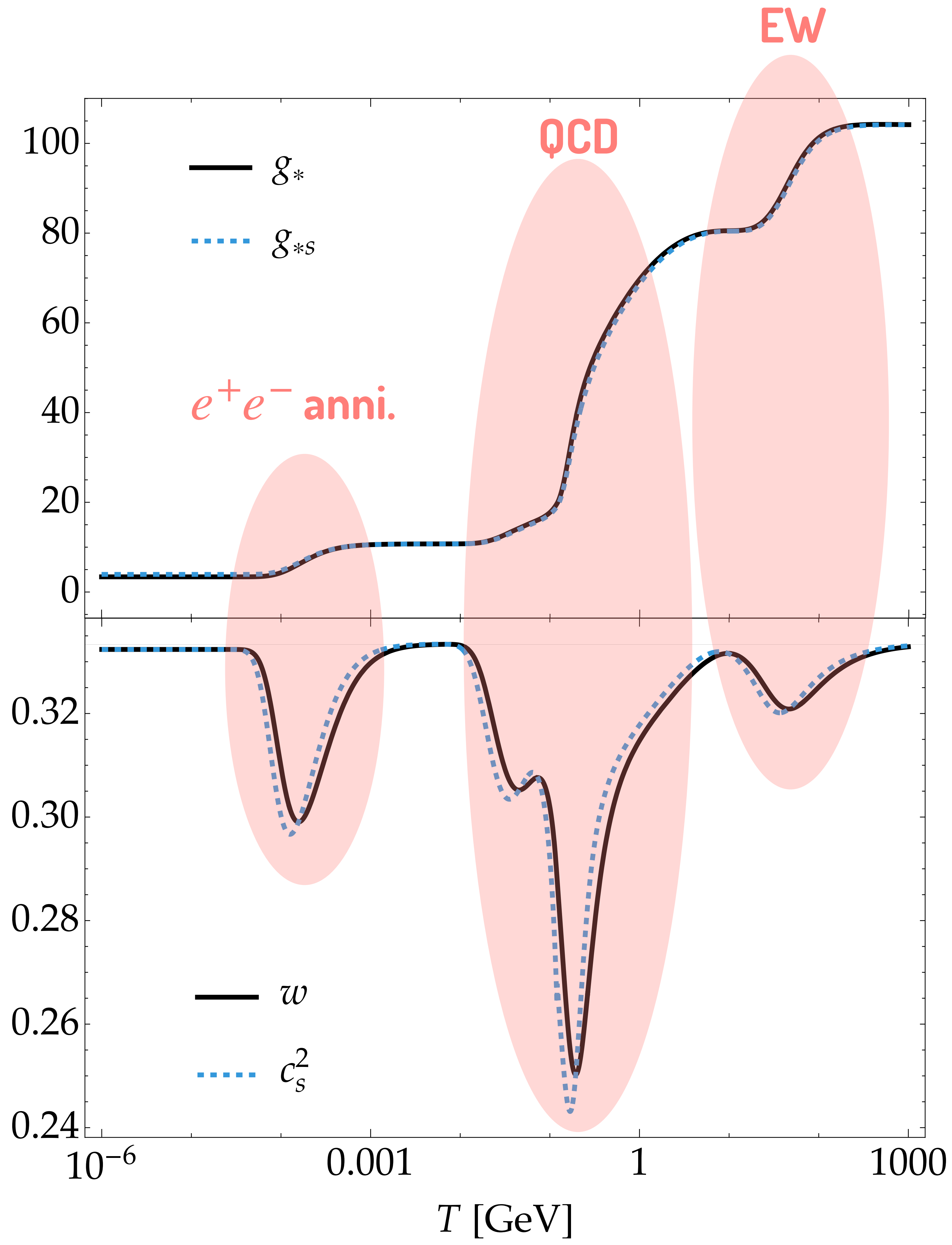


Carr, Kohri, Sendouda, Yokoyama '20

Positivist Perspective?



Carr, Clesse, García-Bellido, Hawkins, Kühnel '23



cf. Saikawa & Shirai '18

Evidence? 1

$$M < M_\odot$$

FAR [yr^{-1}]	$\ln\mathcal{L}$	UTC time	mass 1 [$M_\odot$]	mass 2 [$M_\odot$]	spin1z	spin2z	Network SNR	H1 SNR	L1 SNR
0.1674	8.457	2017-03-15 15:51:30	3.062	0.9281	0.08254	-0.09841	8.527	8.527	-
0.2193	8.2	2017-07-10 17:52:43	2.106	0.2759	0.08703	0.0753	8.157	-	8.157
0.4134	7.585	2017-04-01 01:43:34	4.897	0.7795	-0.05488	-0.04856	8.672	6.319	5.939
1.2148	6.589	2017-03-08 07:07:18	2.257	0.6997	-0.03655	-0.04473	8.535	6.321	5.736

Phukon+ '21

$m_1 = 0.62^{+0.46}_{-0.20} M_\odot$, $m_2 = 0.27^{+0.12}_{-0.10} M_\odot$ (Prunier+ '23)

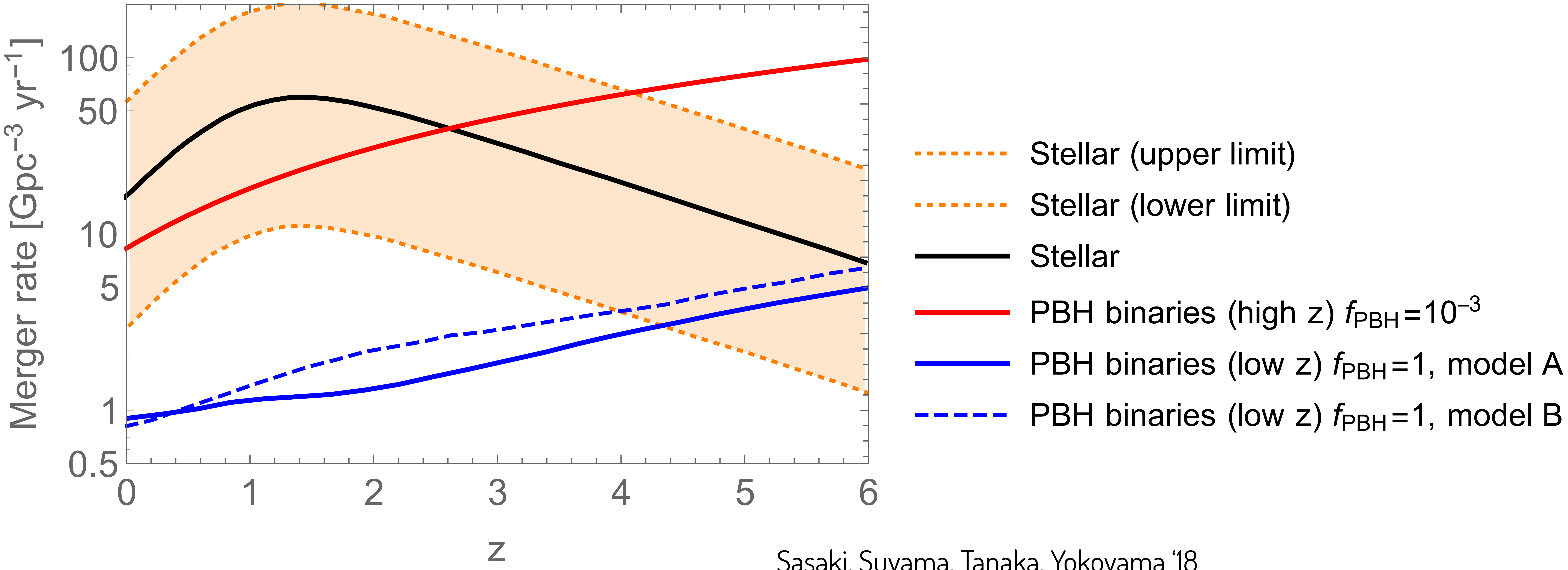
m_2 is even lighter than NS

FAR [yr^{-1}]	Pipeline	GPS time	m_1 [$M_\odot$]	m_2 [$M_\odot$]	χ_1	χ_2	H SNR	L SNR	V SNR	Network SNR
0.20	`GstLAL`	1267725971.02	0.78	0.23	0.57	0.02	6.31	6.28	-	8.90
1.37	`MBTA`	1259157749.53	0.40	0.24	0.10	-0.05	6.57	5.31	5.81	10.25
1.56	`GstLAL`	1264750045.02	1.52	0.37	0.49	0.10	6.74	6.10	-	9.10

LVK '22

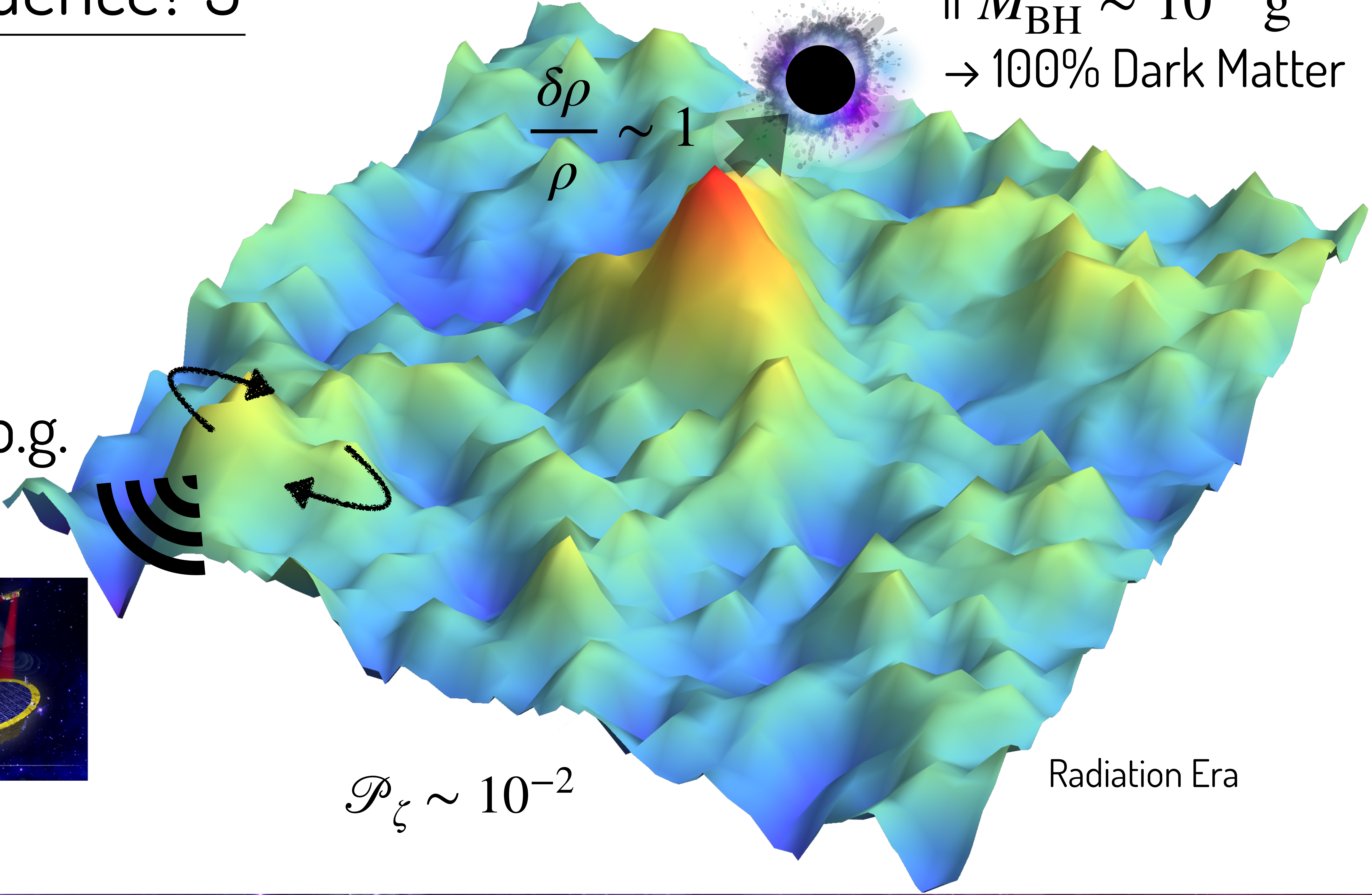
Evidence? 2

Redshift dependence

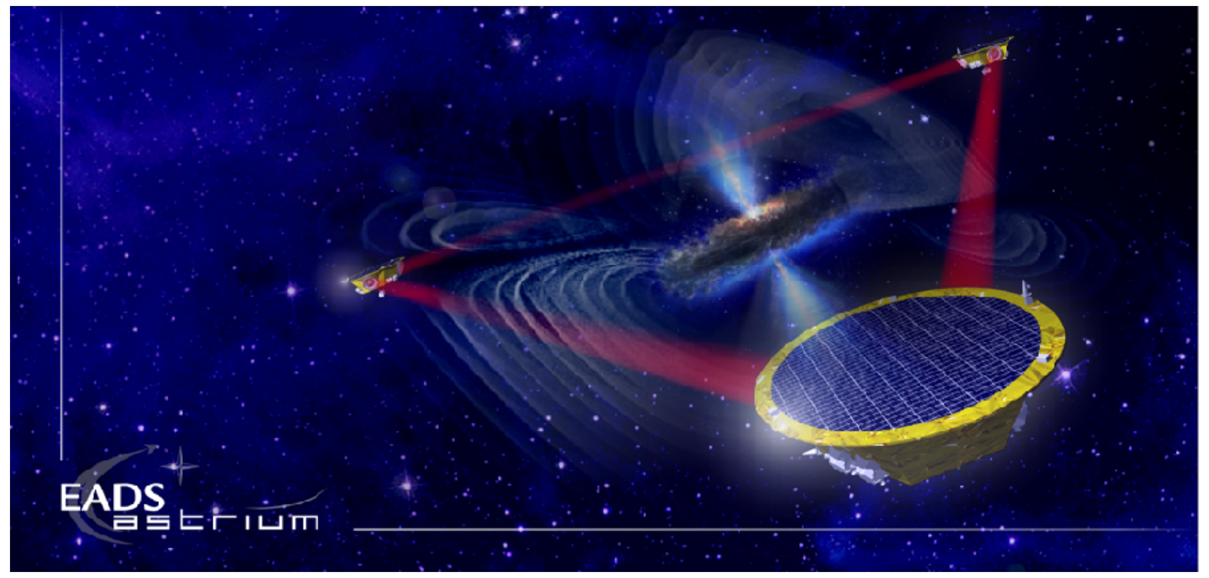


Sasaki, Suyama, Tanaka, Yokoyama '18

(indirect) Evidence? 3



induced GW b.g.



LISA

Triangle study

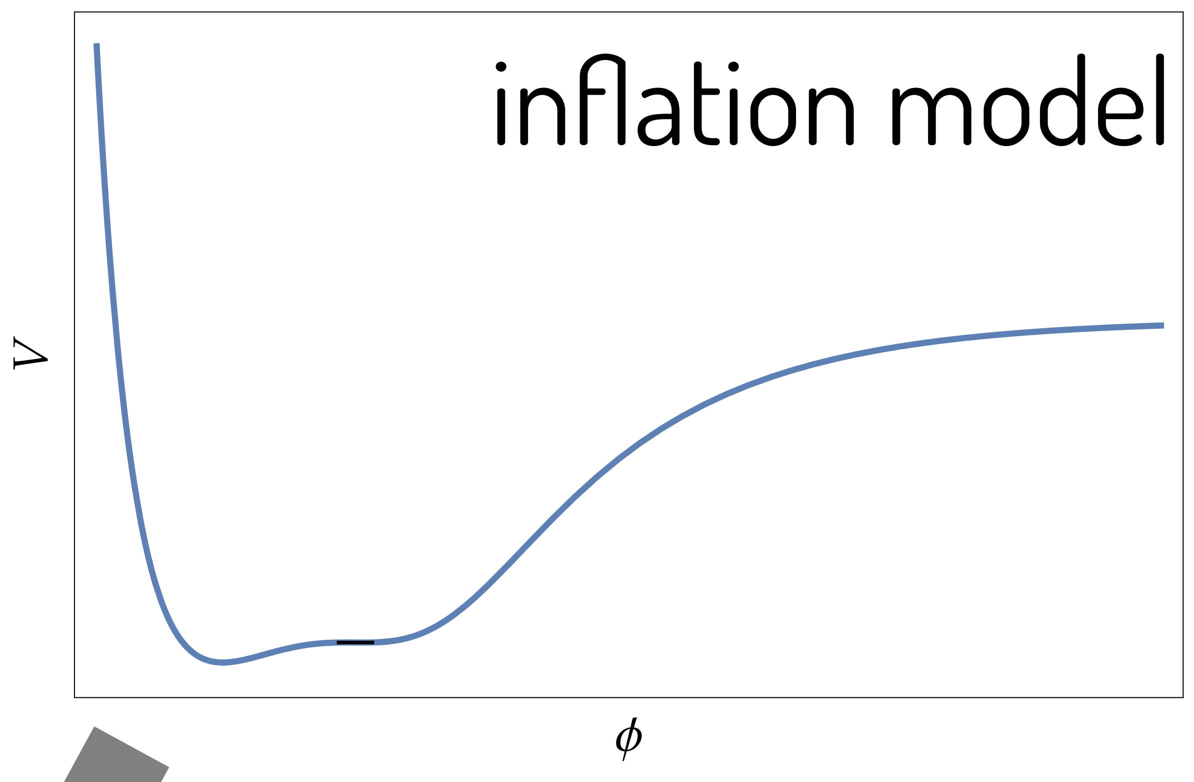
Universal Criterion
Atal+ '19
Escrivà, YT, Yokoyama, Yoo '22

$$\mathscr{C}(r) = \frac{2}{3}\left[1 - (1 + r\zeta'(r))^2\right]$$

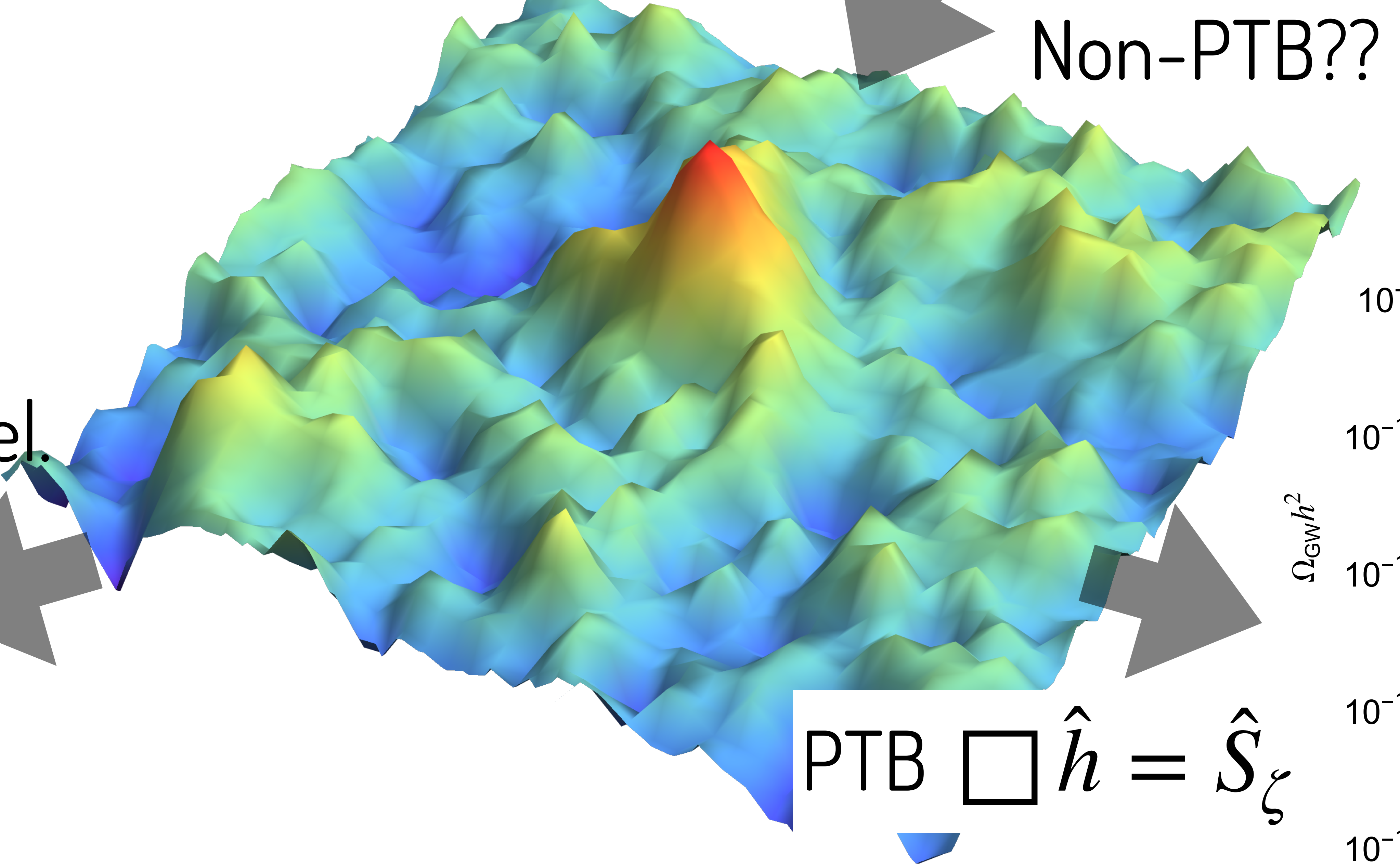
$$\bar{\mathscr{C}} = \frac{1}{V_{R_\mathrm{m}}}\int_0^{R_\mathrm{m}} 4\pi R^2 \mathscr{C} \mathrm{d}R > \bar{\mathscr{C}}_\mathrm{th} = \frac{2}{5}$$

Mass Formula Choptuik+ '93

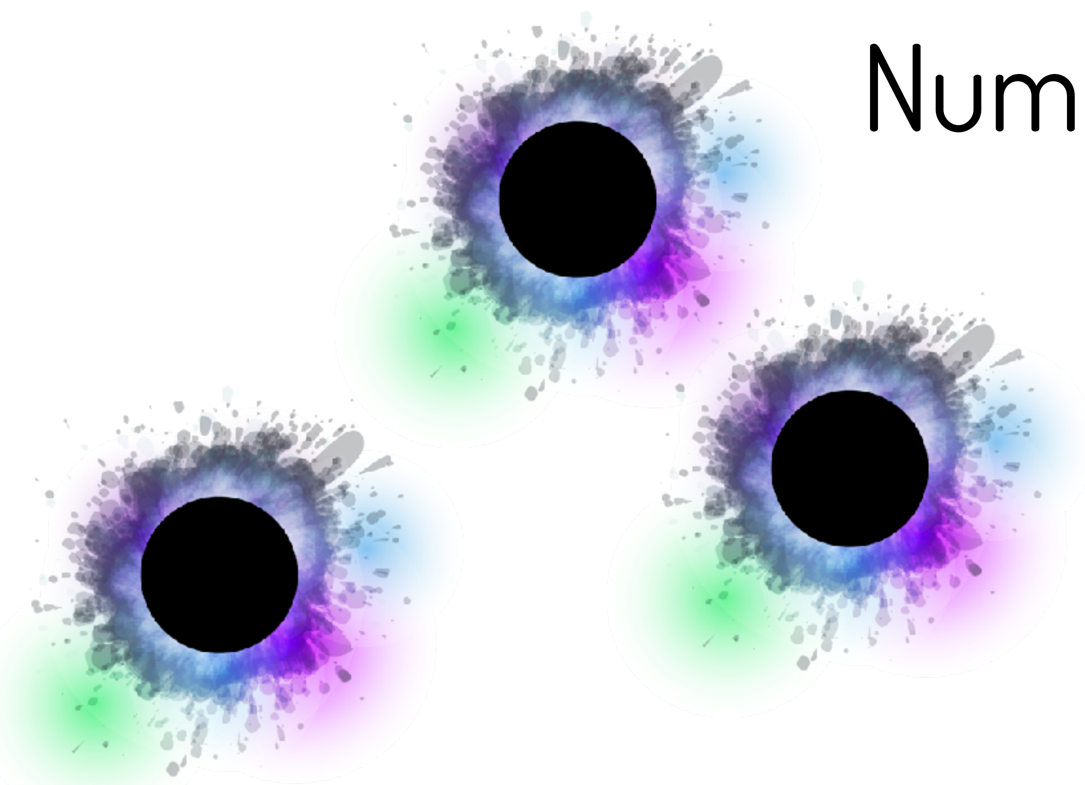
$$M \sim M_{R_\mathrm{m}}(\bar{\mathscr{C}} - \bar{\mathscr{C}}_\mathrm{th})^{0.36}$$



Non-PTB??



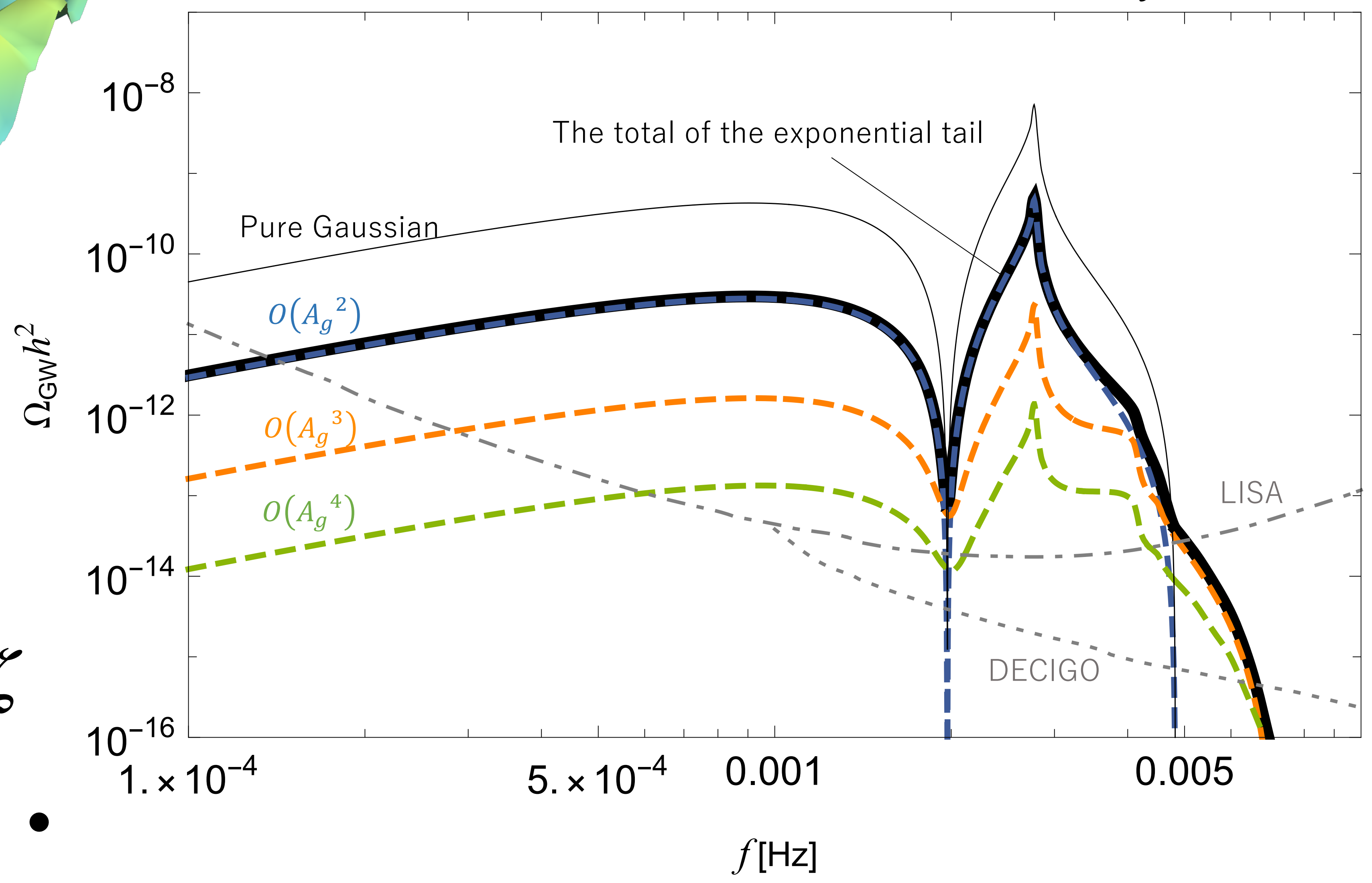
Num. Rel.



PBH abundance

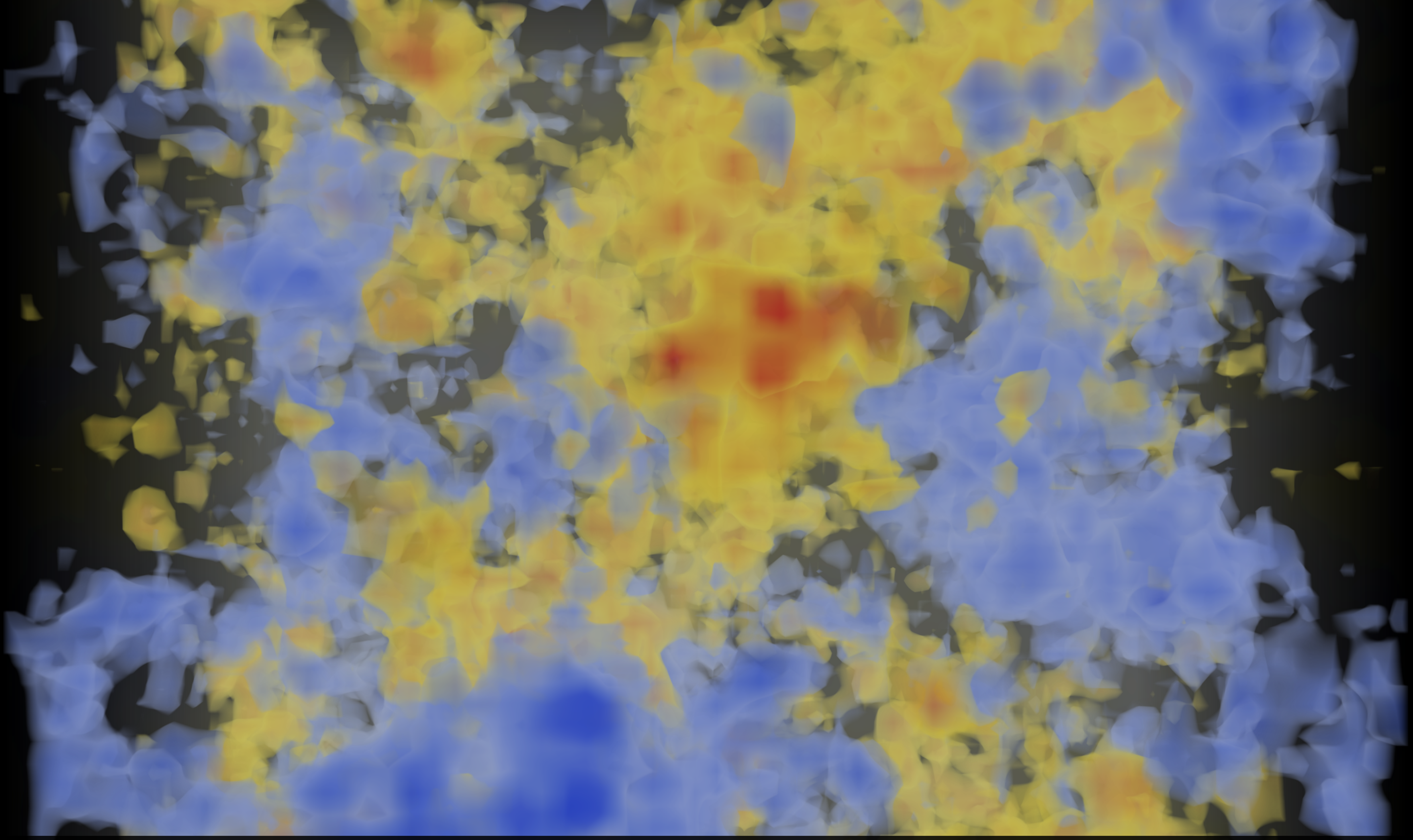
PTB $\Box \hat{h} = \hat{S}_\zeta$

Abe, Inui, YT, Yokoyama '22



induced GW b.g.

indirect evidence

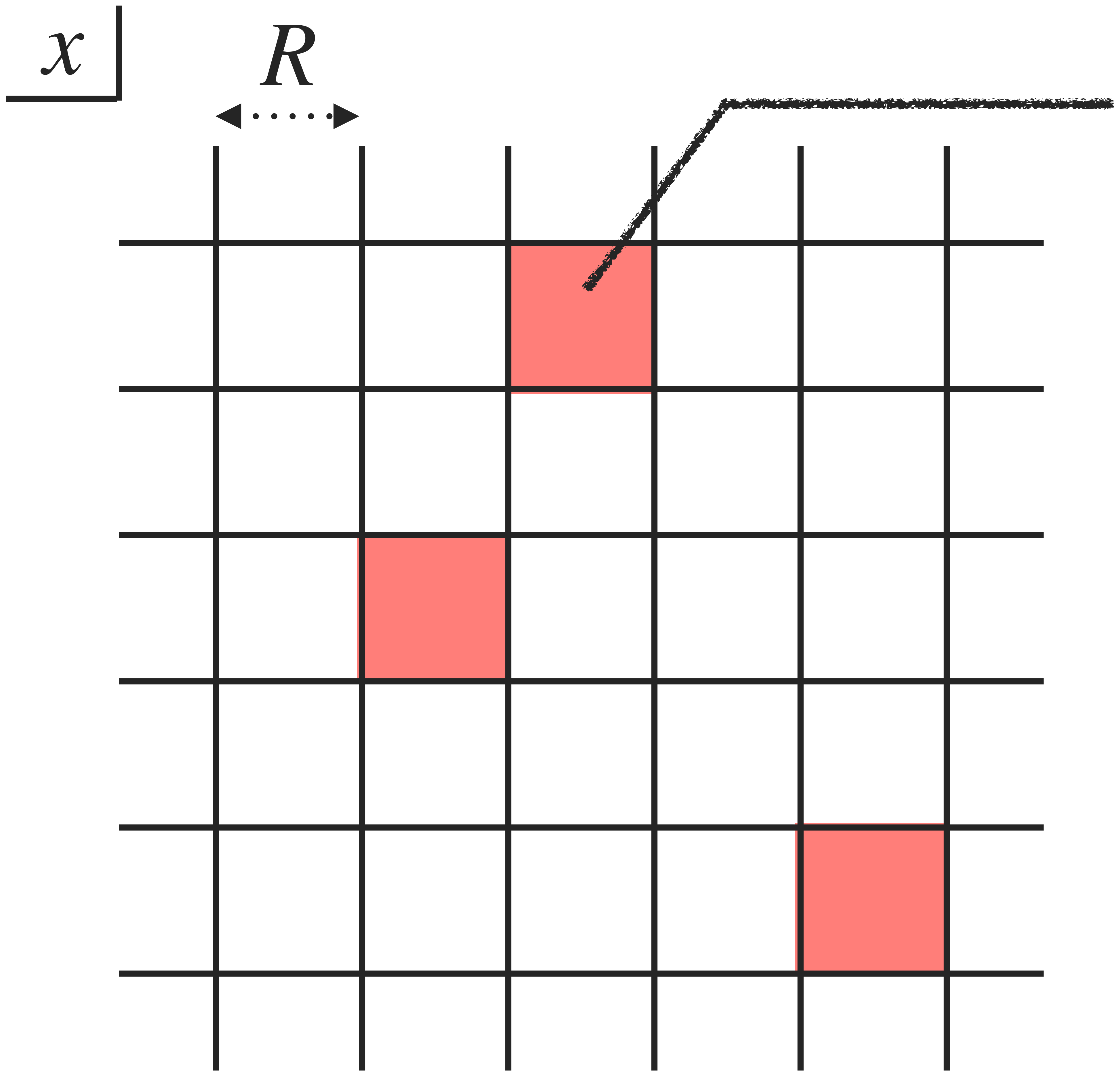


3. STOLAS

Carr's simplest way

Press—Schechter

$$\delta \sim -\frac{4}{9}\frac{\Delta}{(aH)^2}\zeta, \quad \sigma_R^2 = \left(\frac{4}{9}\right)^2 \int \mathrm{d}\ln k\,(kR)^2 \mathcal{P}_\zeta(k) W^2(kR)$$



$$\delta_R(\mathbf{x}) = \int \mathrm{d}^3y\, W_R(\mathbf{x}-\mathbf{y})\delta(\mathbf{y}) \gtrsim \frac{1}{3}\left(=\frac{p}{\rho}\right)$$

$\Rightarrow$ PBH!!

* Abundance : $\displaystyle \frac{\rho_{\mathrm{PBH}}}{\rho_{\mathrm{tot}}} = \int_{1/3}^{\infty} \frac{1}{\sqrt{2\pi\sigma_R^2}} \mathrm{e}^{-\delta_R^2/2\sigma_R^2}$
* Mass : $\displaystyle M_{\mathrm{PBH}} \sim M_H\Big|_{R=H^{-1}} = \frac{4\pi}{3}\rho R^3\Big|_{R=H^{-1}}$

Compaction & Peak th.

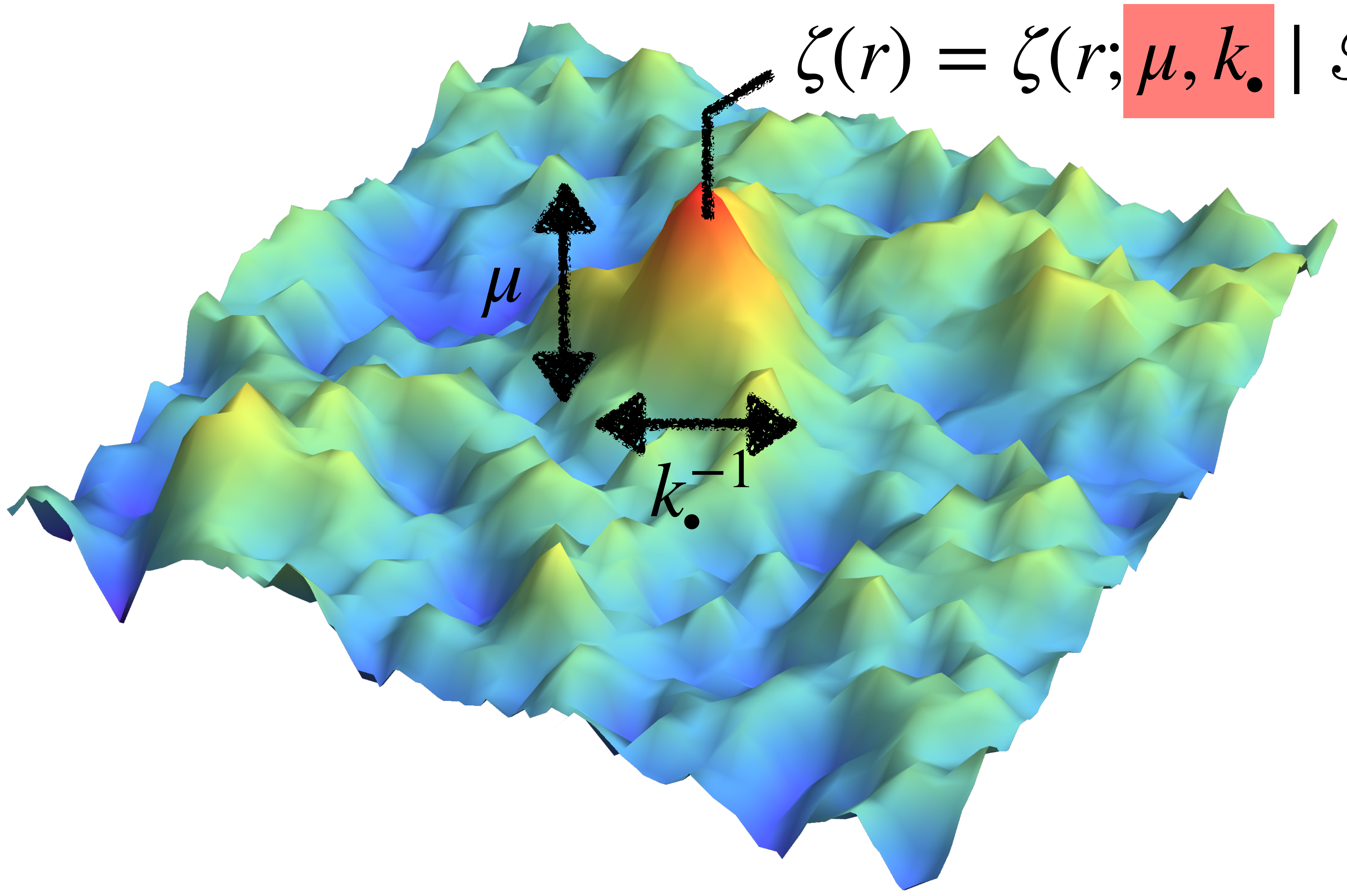
Yoo, Harada, Garriga, Kohri '18

$$\delta = -\frac{8}{9}\frac{1}{a^2H^2}e^{-5\zeta/2}\Delta e^{\zeta/2}$$

if ζ is Gaussian (Bardeen, Bond, Kaiser, Szalay '86)

comb. Gaussian

$$\zeta(r) = \zeta(r; \boxed{\mu, k_\bullet} \mid \mathscr{P}_\zeta)$$



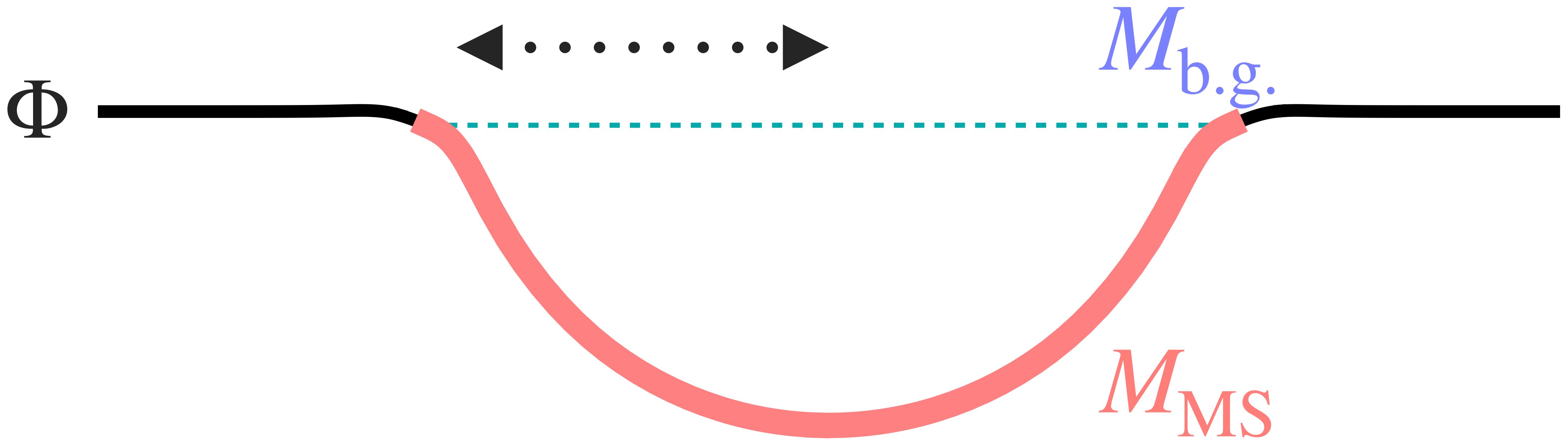
Compaction Function Shibata & Sasaki '99

$$\mathscr{C} = 2G\frac{M_{\mathrm{MS}} - M_{\mathrm{b.g.}}}{R}$$

$$= \left.\frac{1}{V(R)}\int_0^R \delta \times 4\pi R^2 \mathrm{d}R\right|_{R=H^{-1}}$$

$$= \frac{2}{3}\left[1 - (1 + r\zeta')^2\right]$$

$$R = ae^{\zeta}r$$



Compaction & Peak th.

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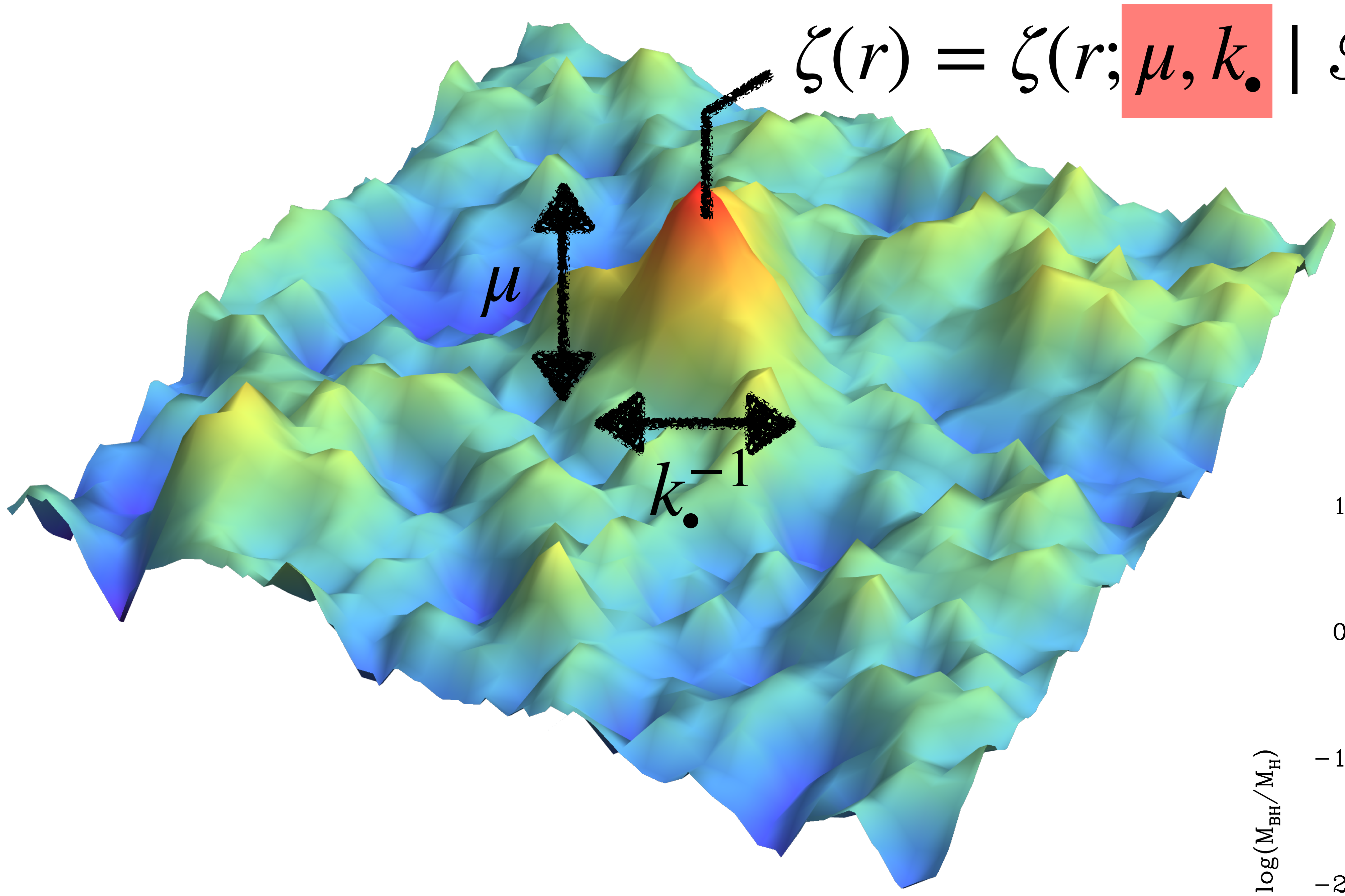


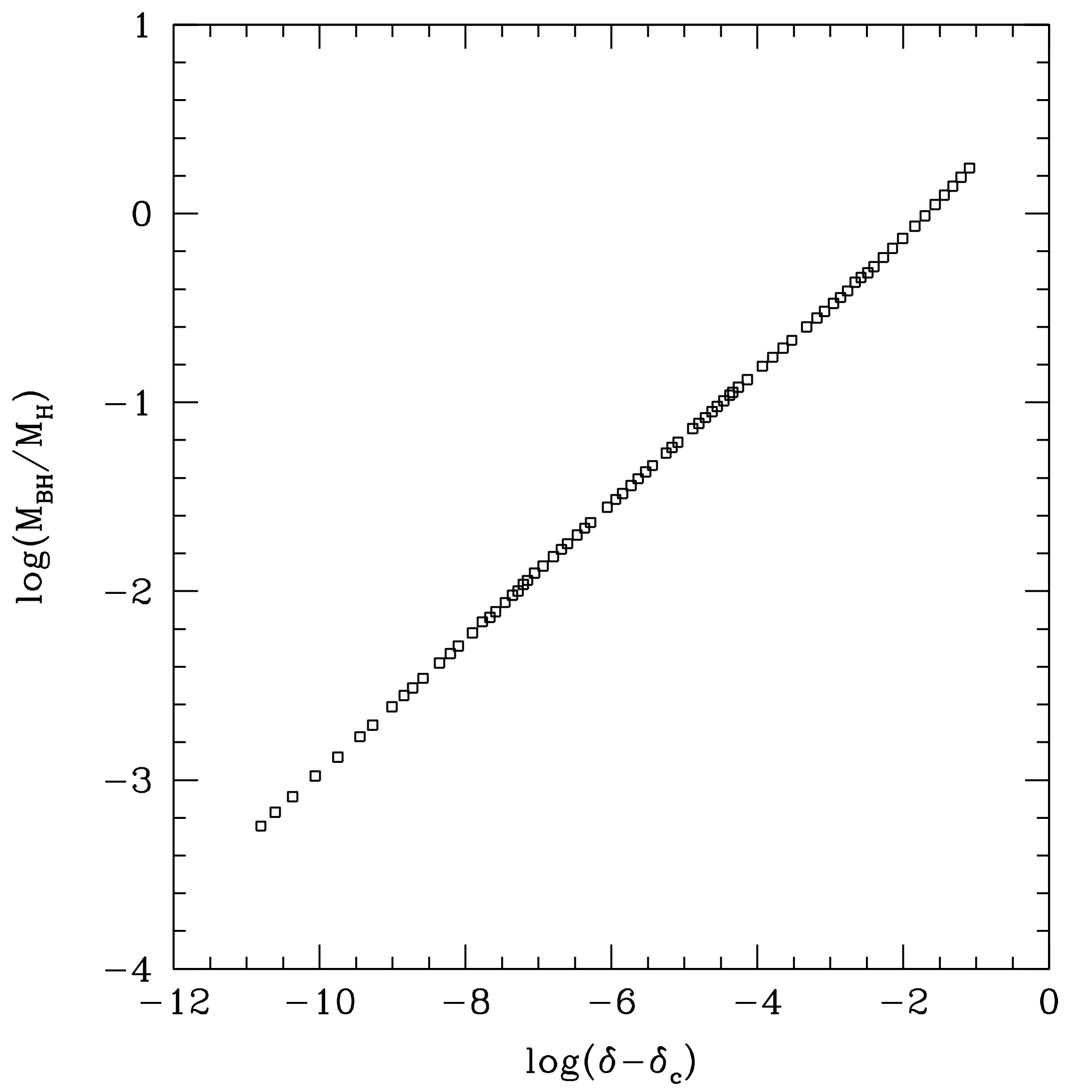
$$\delta = -\frac{8}{9}\frac{1}{a^2H^2}\mathrm{e}^{-5\zeta/2}\Delta\mathrm{e}^{\zeta/2}$$

Compaction Function Shibata & Sasaki '99

$$\mathscr{C} = \frac{2}{3}\left[1 - (1 + r\zeta')^2\right]$$

Universal Criterion Atal, Cid, Escriva, Garriga, '19

$$\bar{\mathscr{C}} = \frac{1}{V(R)}\int_0^R \mathscr{C} \times 4\pi R^2 > \bar{\mathscr{C}}_{\mathrm{th}} \simeq \frac{2}{5}$$



Mass Musco, Miller, Polnarev '08

$$M_{\mathrm{PBH}} \simeq \left(\mu - \mu_{\mathrm{th}}(k_\bullet, \cdots)\right)^{0.36} M_H\Big|_{R=H^{-1}}$$

Stochastic Approach

Starobinsky '86

= EFT of superH fields

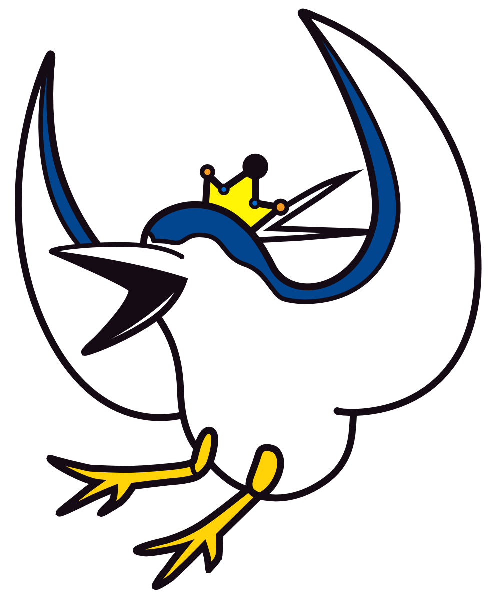
= Local FLRW + Correlated Brownian motion

$$\approx \frac{H(N,\mathbf{x})}{2\pi}$$

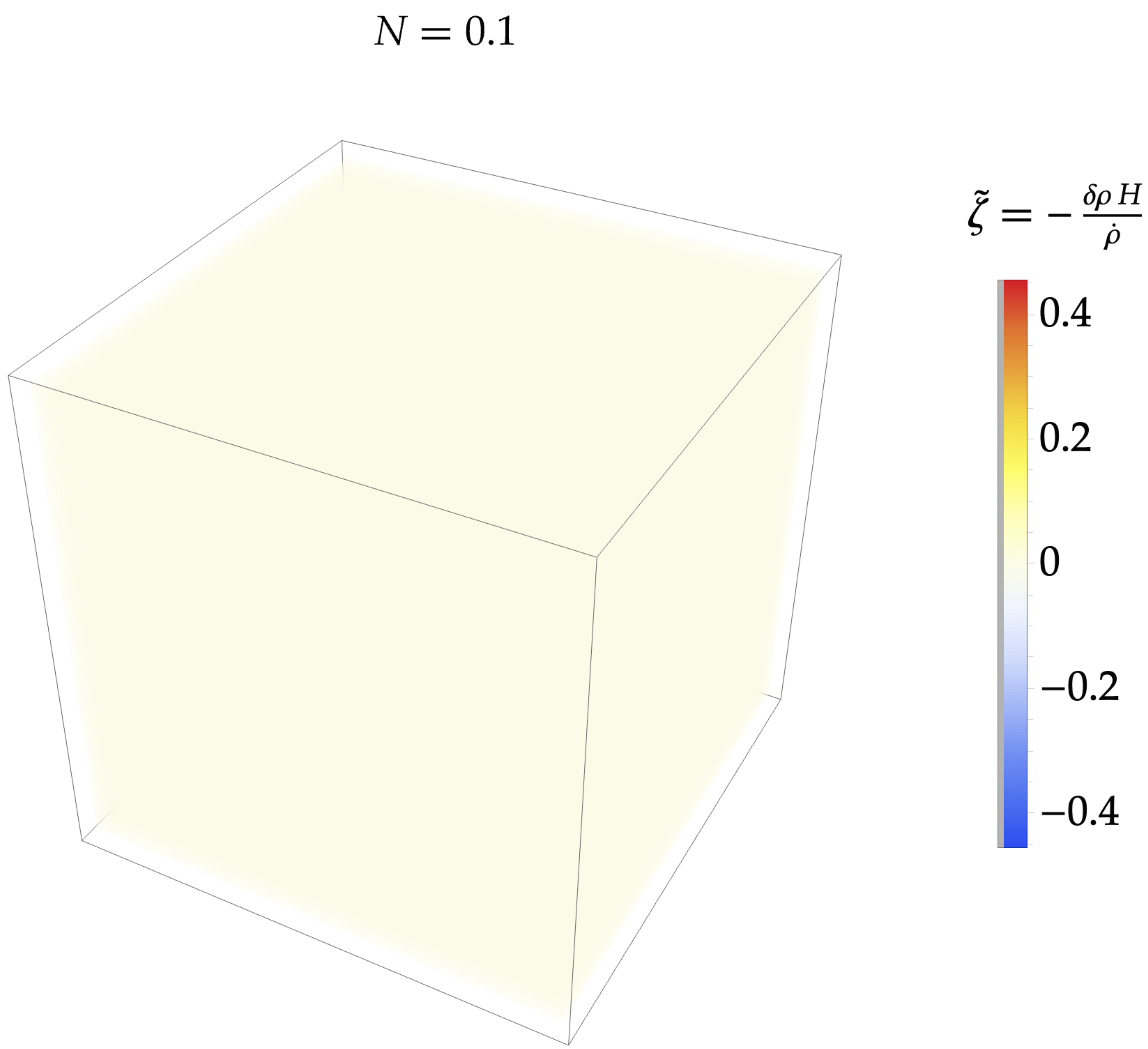
$$\begin{cases} \mathrm{d}\phi(N,\mathbf{x}) &= \dfrac{\pi(N,\mathbf{x})}{H(N,\mathbf{x})}\mathrm{d}N + \boxed{\sqrt{\mathscr{P}_\phi(N,\mathbf{x})}}\,\mathrm{d}W(N,\mathbf{x}), \\ \mathrm{d}\pi(N,\mathbf{x}) &= \left(-3\pi(N,\mathbf{x}) - \dfrac{V'(\phi(N,\mathbf{x}))}{H(N,\mathbf{x})}\right)\mathrm{d}N, \\ 3M_{\mathrm{Pl}}^2 H^2(N,\mathbf{x}) &= \dfrac{1}{2}\pi^2(N,\mathbf{x}) + V(\phi(N,\mathbf{x})), \\ \mathrm{d}W(N,\mathbf{x})\mathrm{d}W(N',\mathbf{y}) &= \dfrac{\sin k_\sigma(N)\,|\,\mathbf{x}-\mathbf{y}\,|}{k_\sigma(N)\,|\,\mathbf{x}-\mathbf{y}\,|}\delta_{NN'}\mathrm{d}N \end{cases}$$

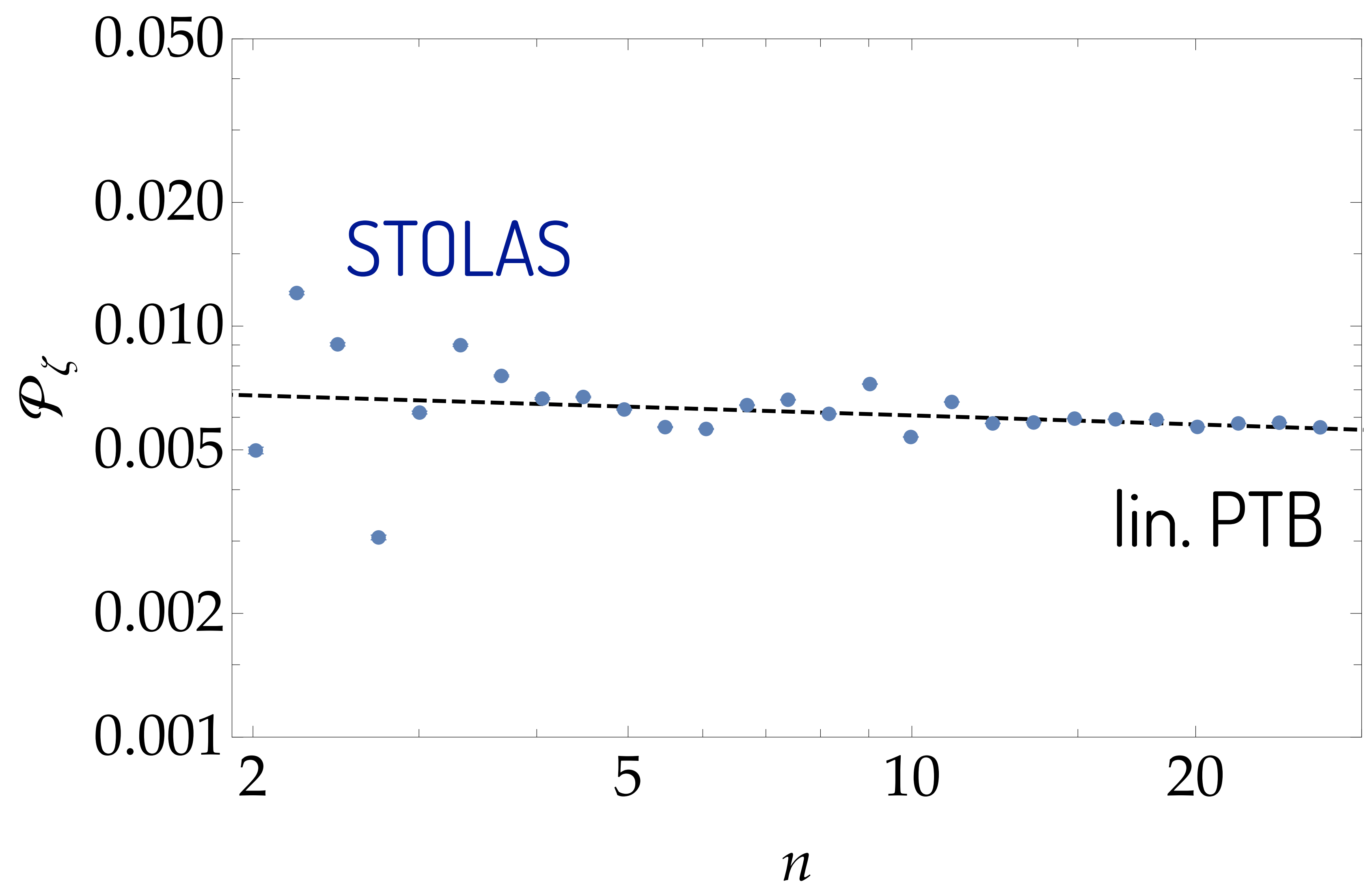
STOLAS

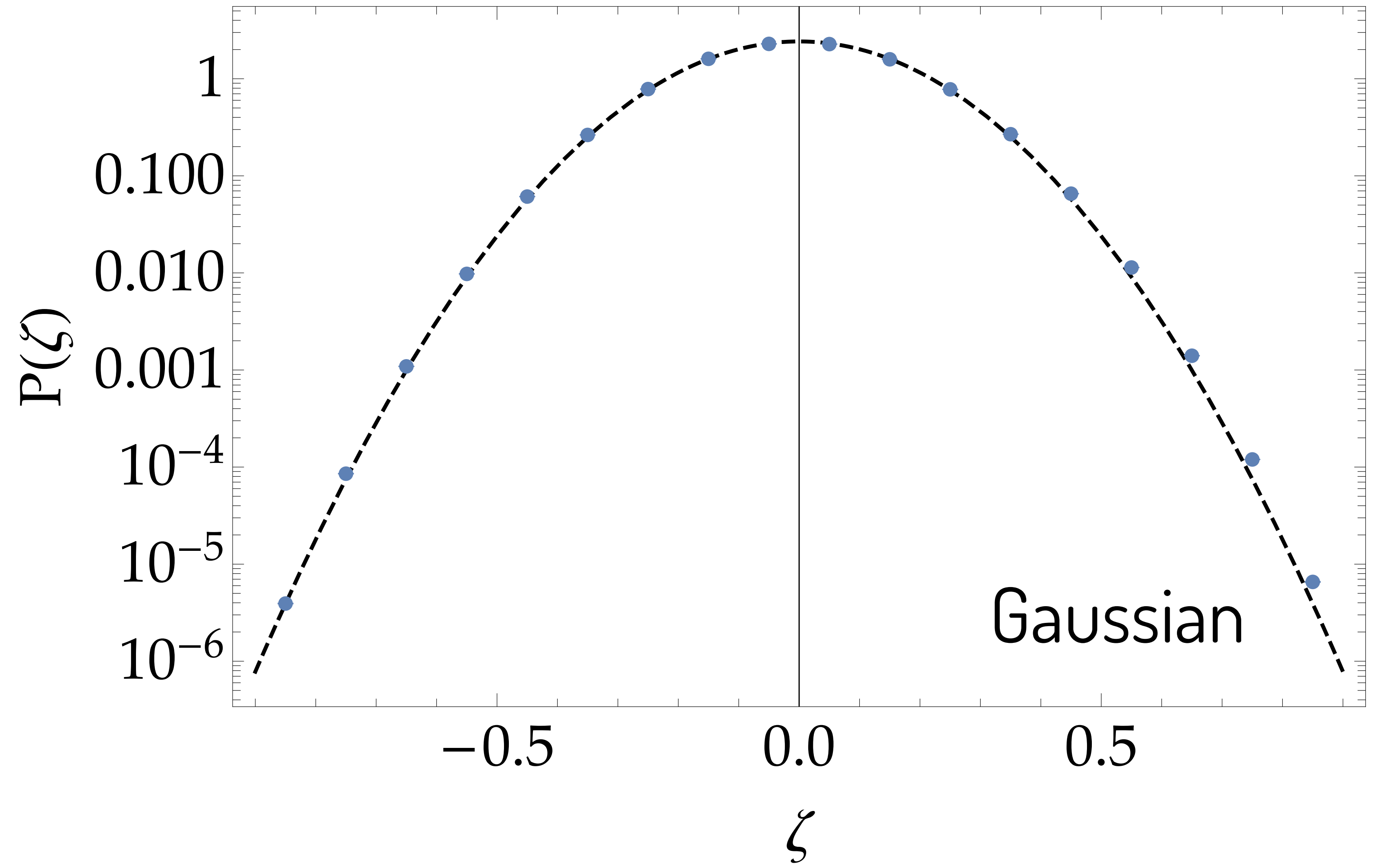
Mizuguchi, Murata, YT '24



Ex. 1: Chaotic $V = \frac{1}{2}m^2\phi^2$



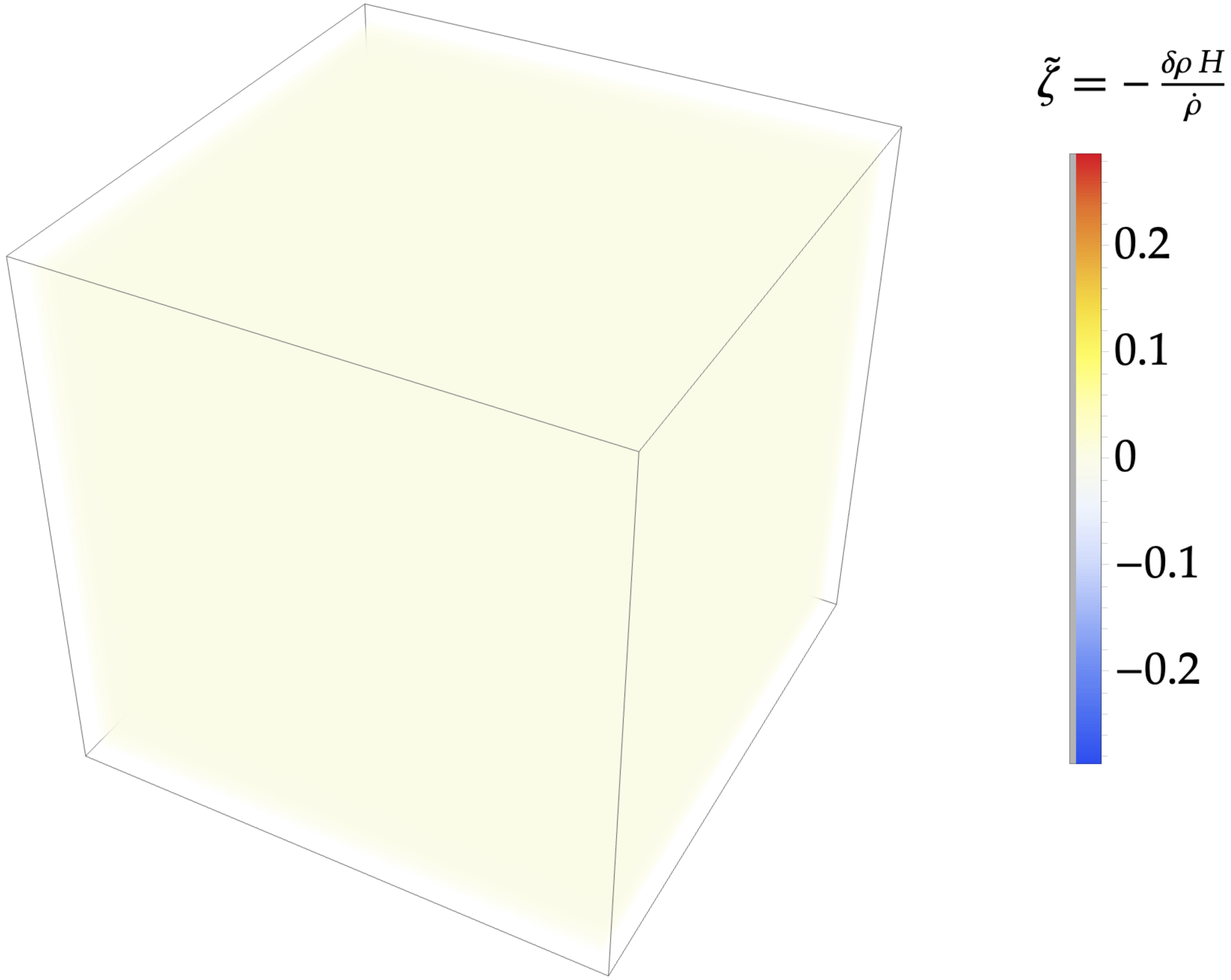




STOLAS

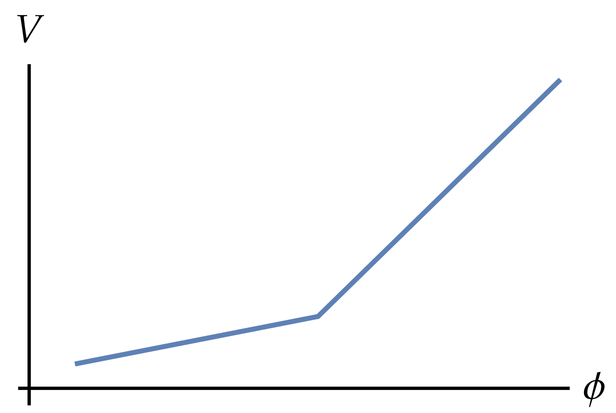
Mizuguchi, Murata, YT '24

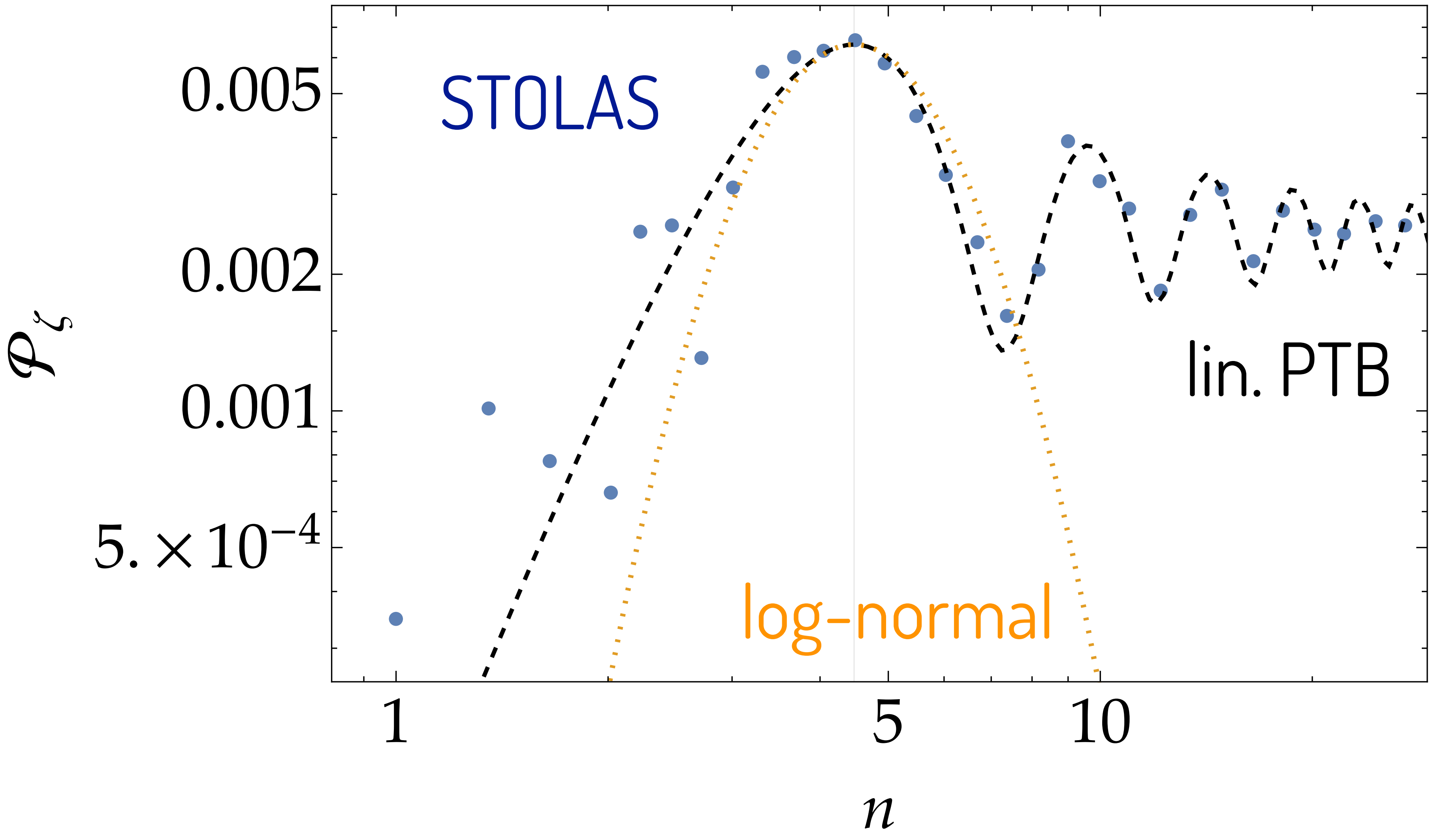
$N = 0.1$

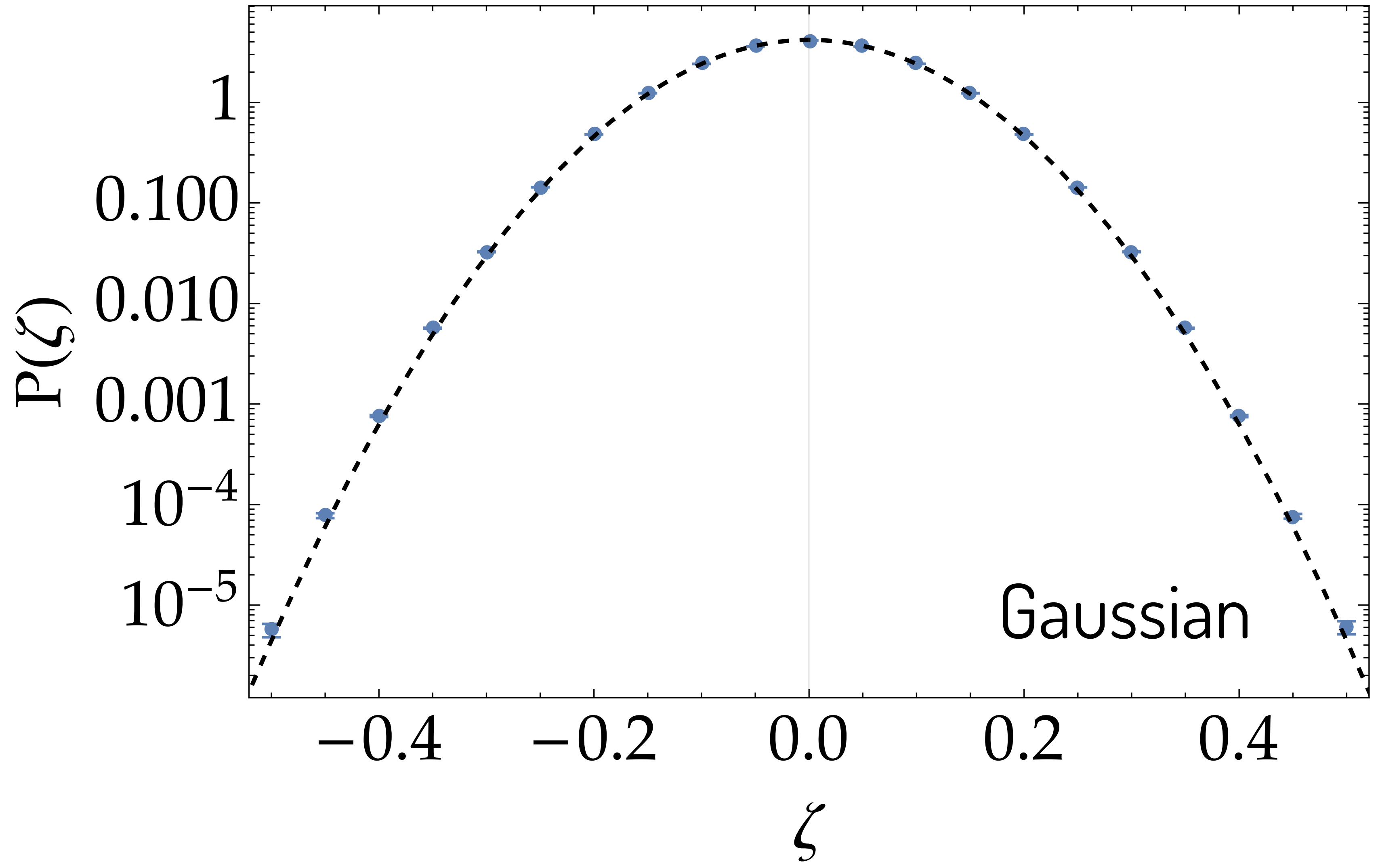


Ex. 2: Starobinsky's linear

Starobinsky '92







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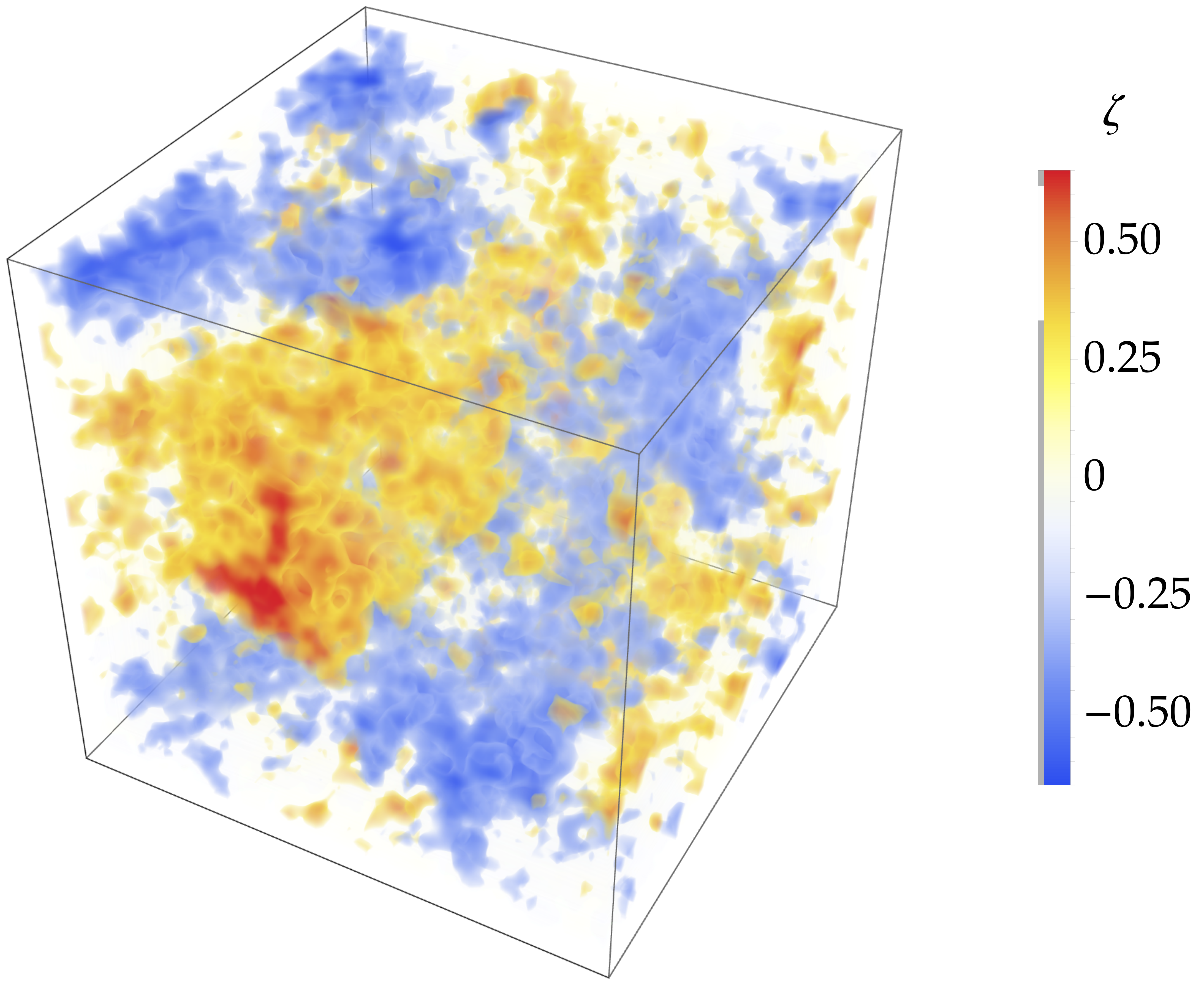
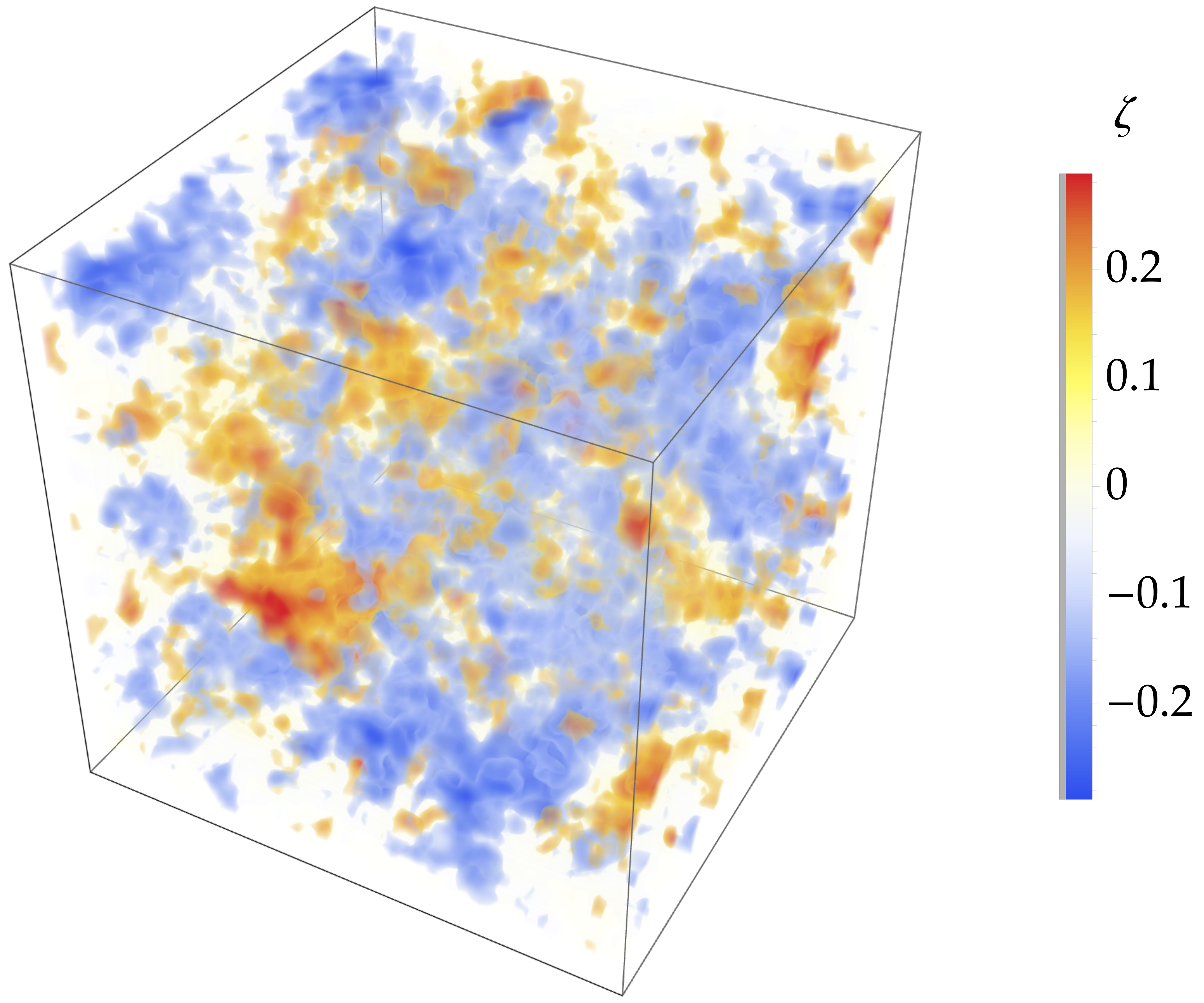


Ex. 2 : Starobinsky's linear

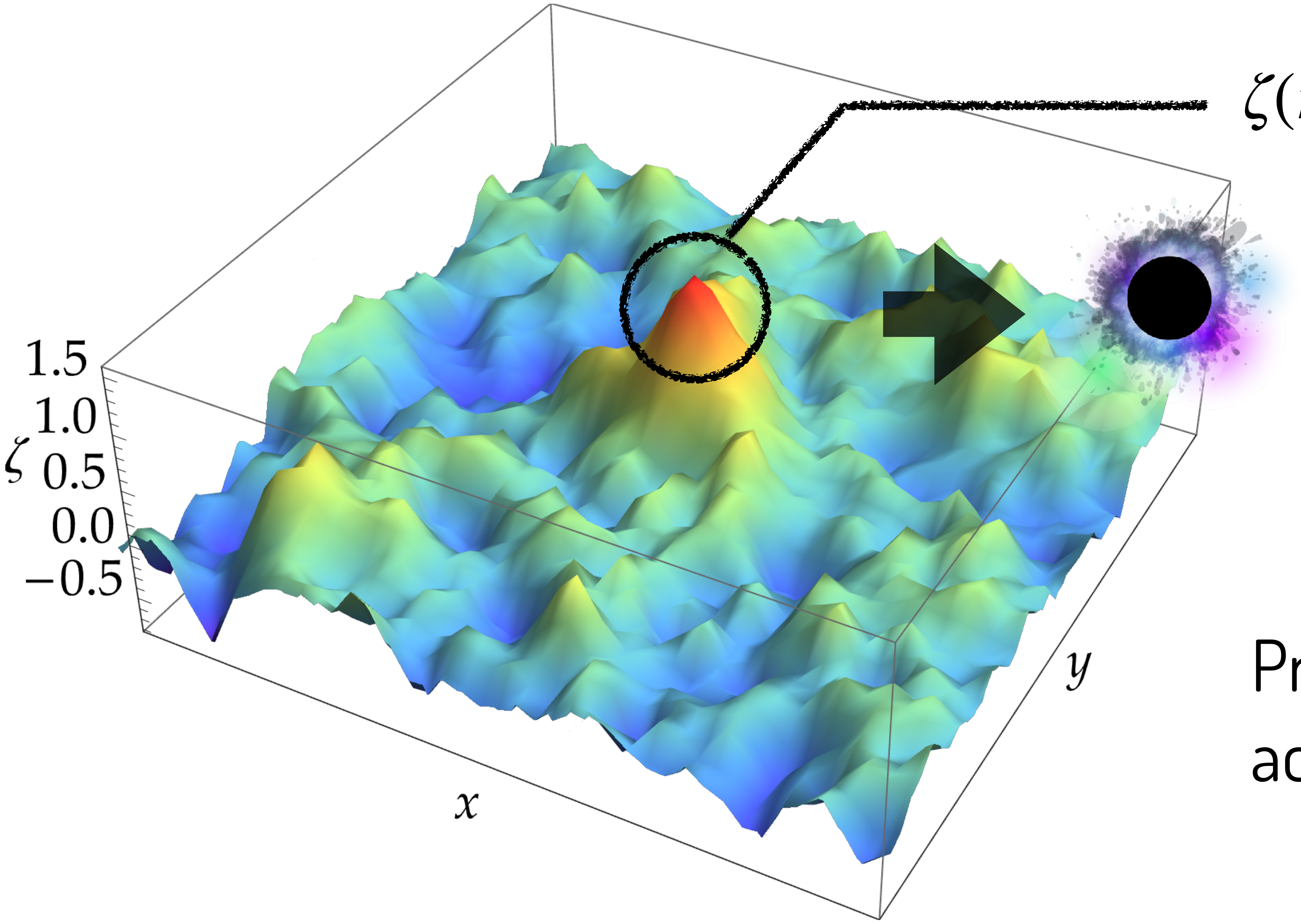


Importance Sampling

see, e.g., Jackson+ '22

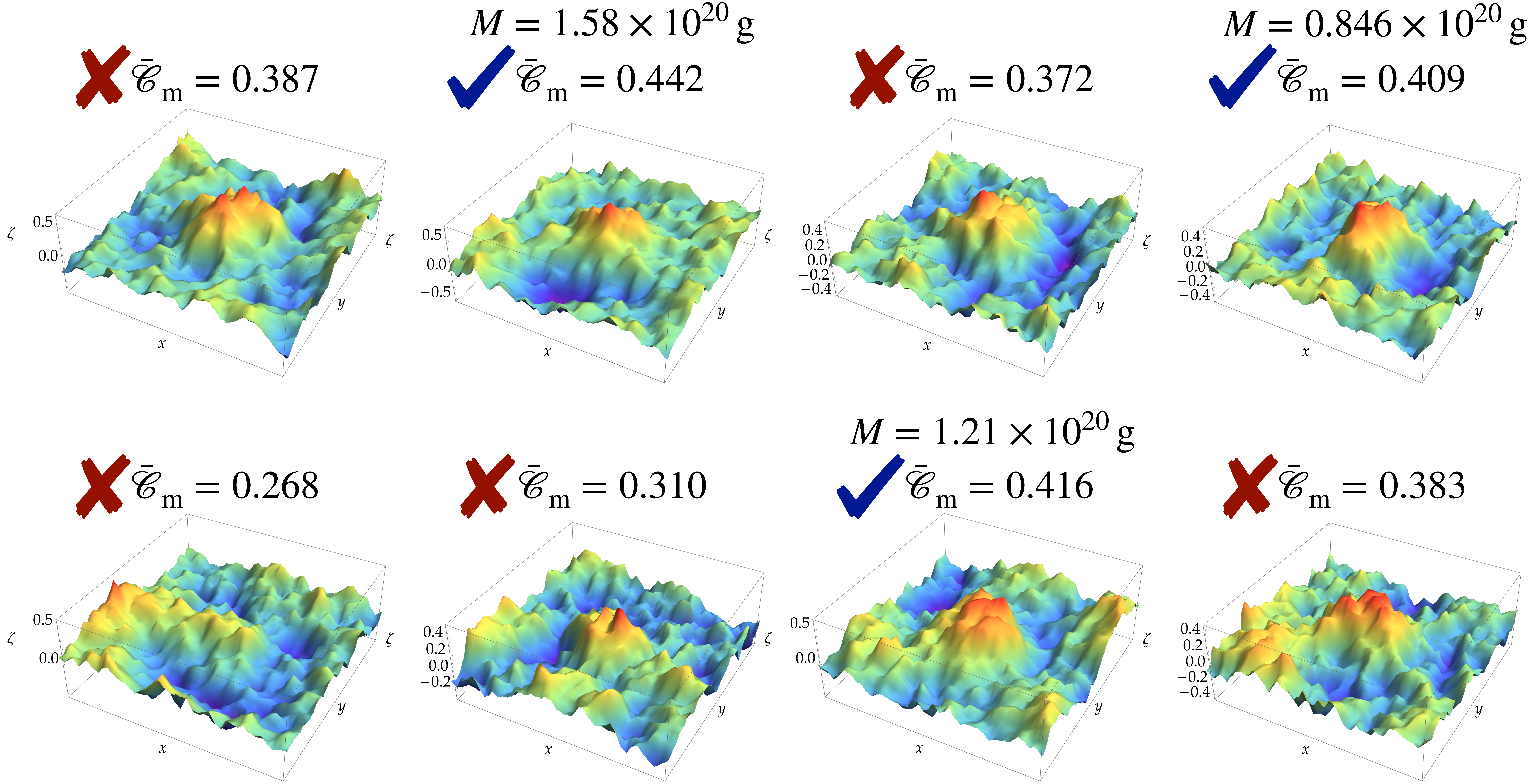
Ex. 2 : Starobinsky's linear

Intentionally large noise



$$\zeta(r) \to \mathscr{C}(r) \to \bar{\mathscr{C}}_{\mathrm{m}} = 0.56 > \bar{\mathscr{C}}_{\mathrm{th}} = \frac{2}{5}$$

Probability is re-weighted
according to the probability of large noise!!

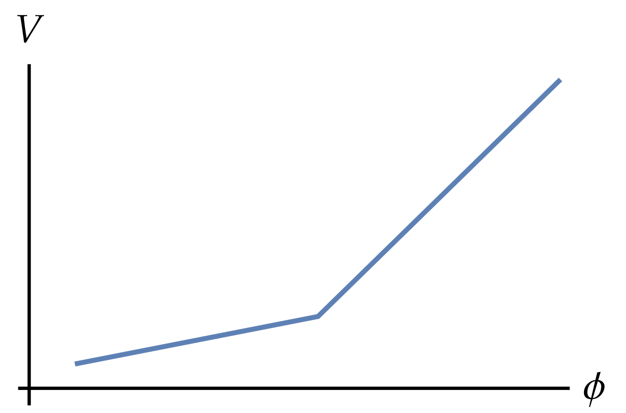

$M = 1.58 \times 10^{20}$ g
$M = 0.846 \times 10^{20}$ g
$\bar{\mathscr{C}}_\mathrm{m} = 0.387$
$\bar{\mathscr{C}}_\mathrm{m} = 0.442$
$\bar{\mathscr{C}}_\mathrm{m} = 0.372$
$\bar{\mathscr{C}}_\mathrm{m} = 0.409$
$M = 1.21 \times 10^{20}$ g
$\bar{\mathscr{C}}_\mathrm{m} = 0.268$
$\bar{\mathscr{C}}_\mathrm{m} = 0.310$
$\bar{\mathscr{C}}_\mathrm{m} = 0.416$
$\bar{\mathscr{C}}_\mathrm{m} = 0.383$
ζ
x
y

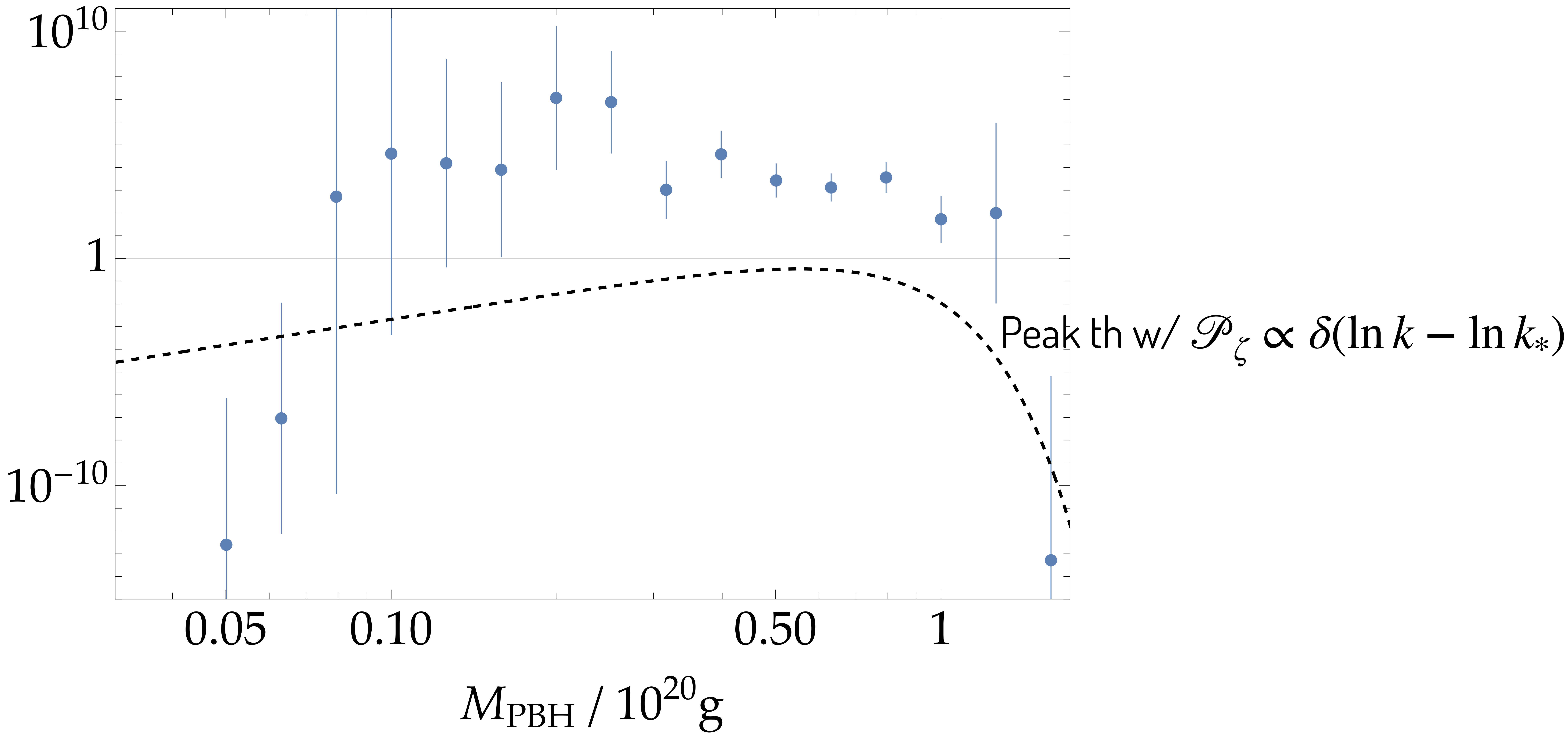
Importance Sampling

Mizuguchi, Murata, YT '24

Ex. 2: Starobinsky's linear

Starobinsky '92





4. Other topics

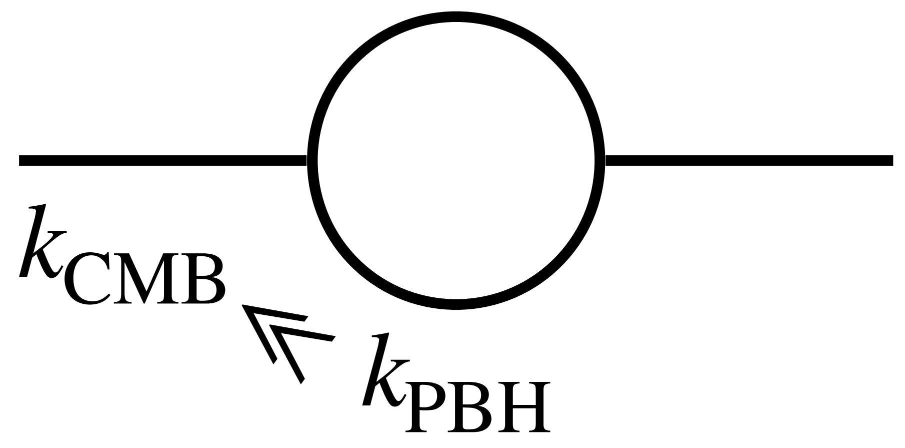
PBH ruled out?

2211.03395

Ruling Out Primordial Black Hole Formation From Single-Field Inflation

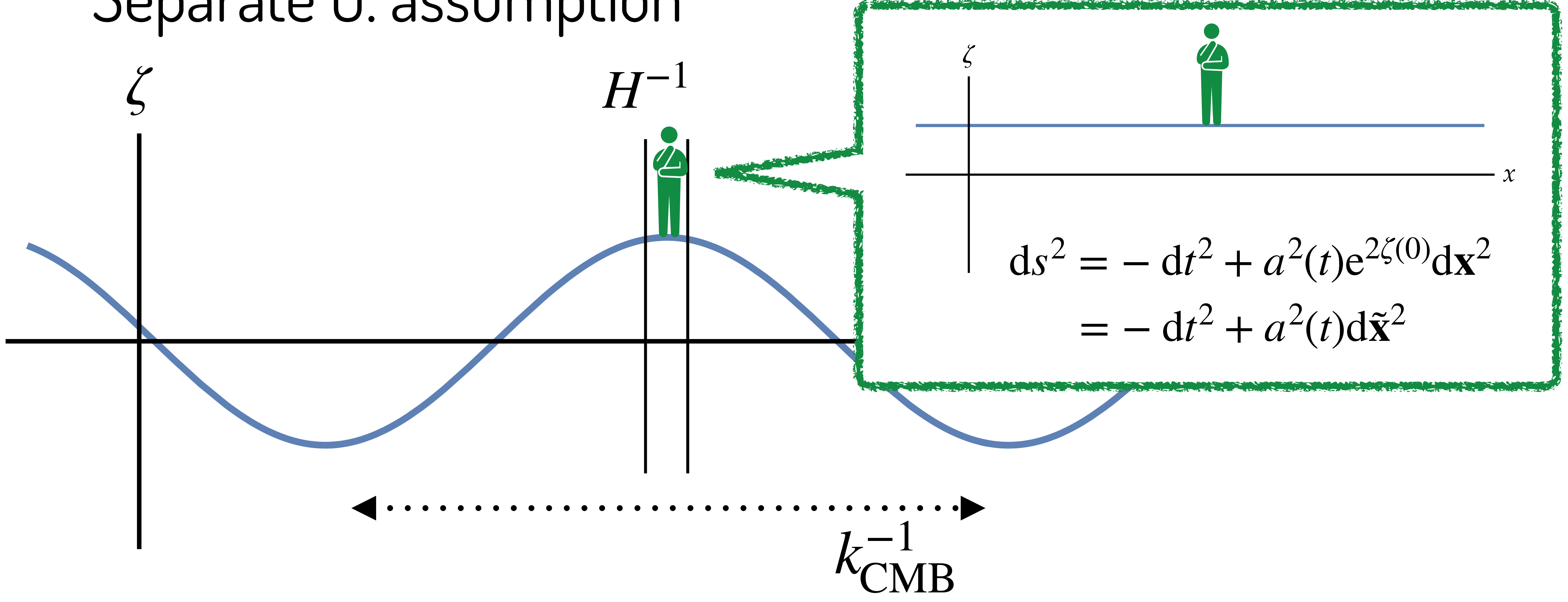
Jason Kristiano[1,2,*] and Jun'ichi Yokoyama[1,2,3,4,†]

$S^{(3)}[\zeta]$



$$\mathcal{P}_\zeta^{\rm tree}(k_{\rm CMB}) \sim \Delta\mathcal{P}_\zeta^{1-{\rm loop}}(k_{\rm CMB})$$

Separate U. assumption



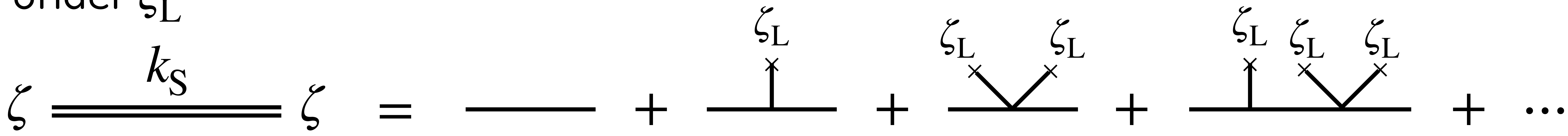
- ζ as NG boson of asymp. dilatation (e.g. Assassi, Baumann, Green '12)
- (classically) soft ζ is conserved (Lyth, Malik, Sasaki '05)
- Maldacena's consistency relation ('03)

$$S^{(3)}[\zeta] \Rightarrow \langle \zeta_{k_L}\zeta_{k_S}\zeta_{k_S}\rangle \propto \mathcal{P}_\zeta(k_L)\frac{\mathrm{d}\mathcal{P}_\zeta(k_S)}{\mathrm{d}\ln k_S}$$

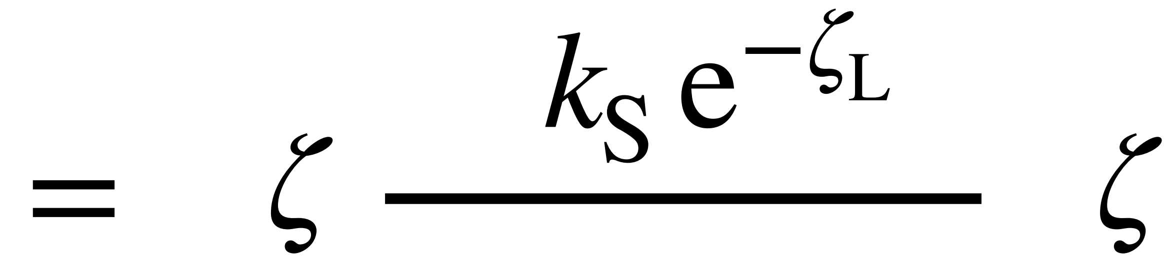
merely scale-redefinition

Soft th. on loops

prop. under ζ_L



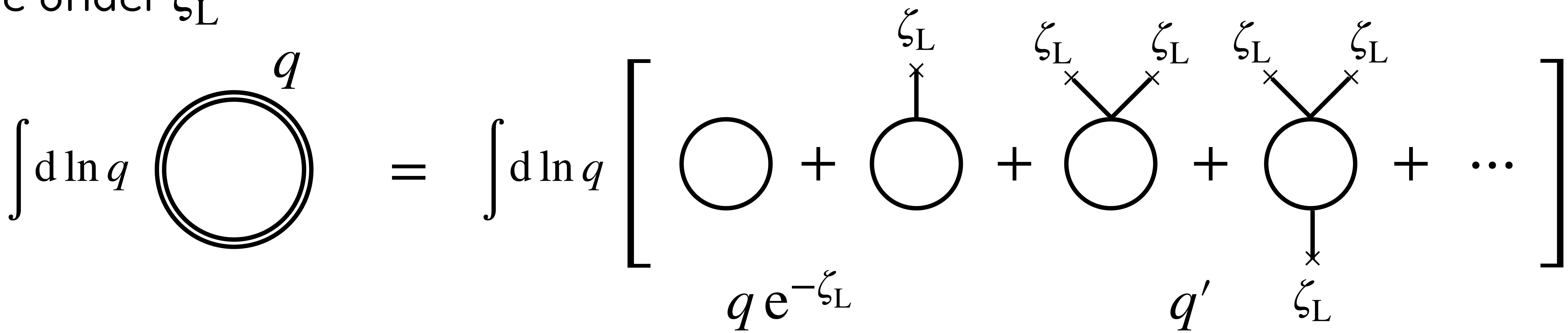
Maldacena's CR



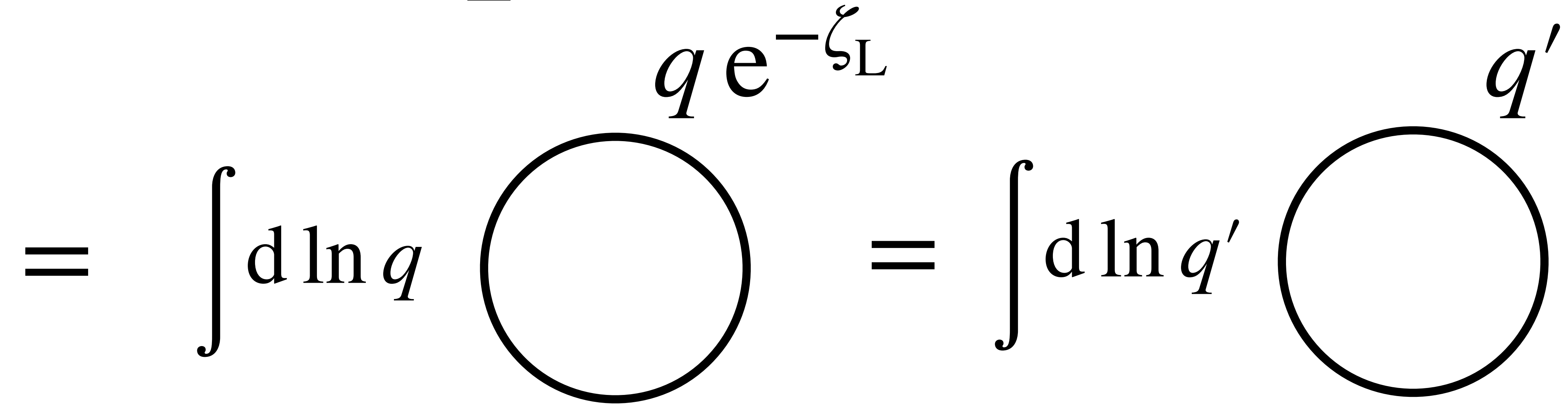
prop. w/o ζ_L

bubble under ζ_L

all 1-loop corrections on $n$pt. func.



Maldacena's CR

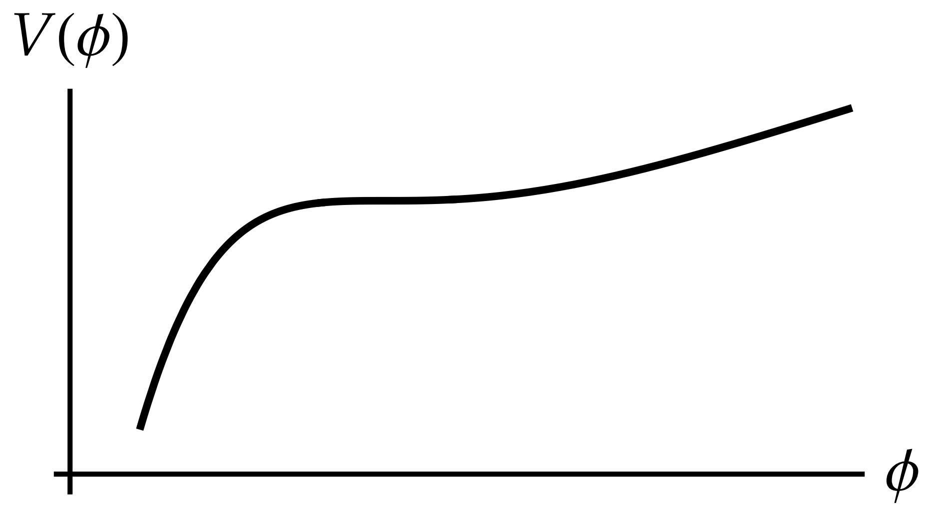


In bubble-vanishing QFT, all 1-loop corrections vanish?

cf. Pimentel, Senatore, Zaldarriaga '12

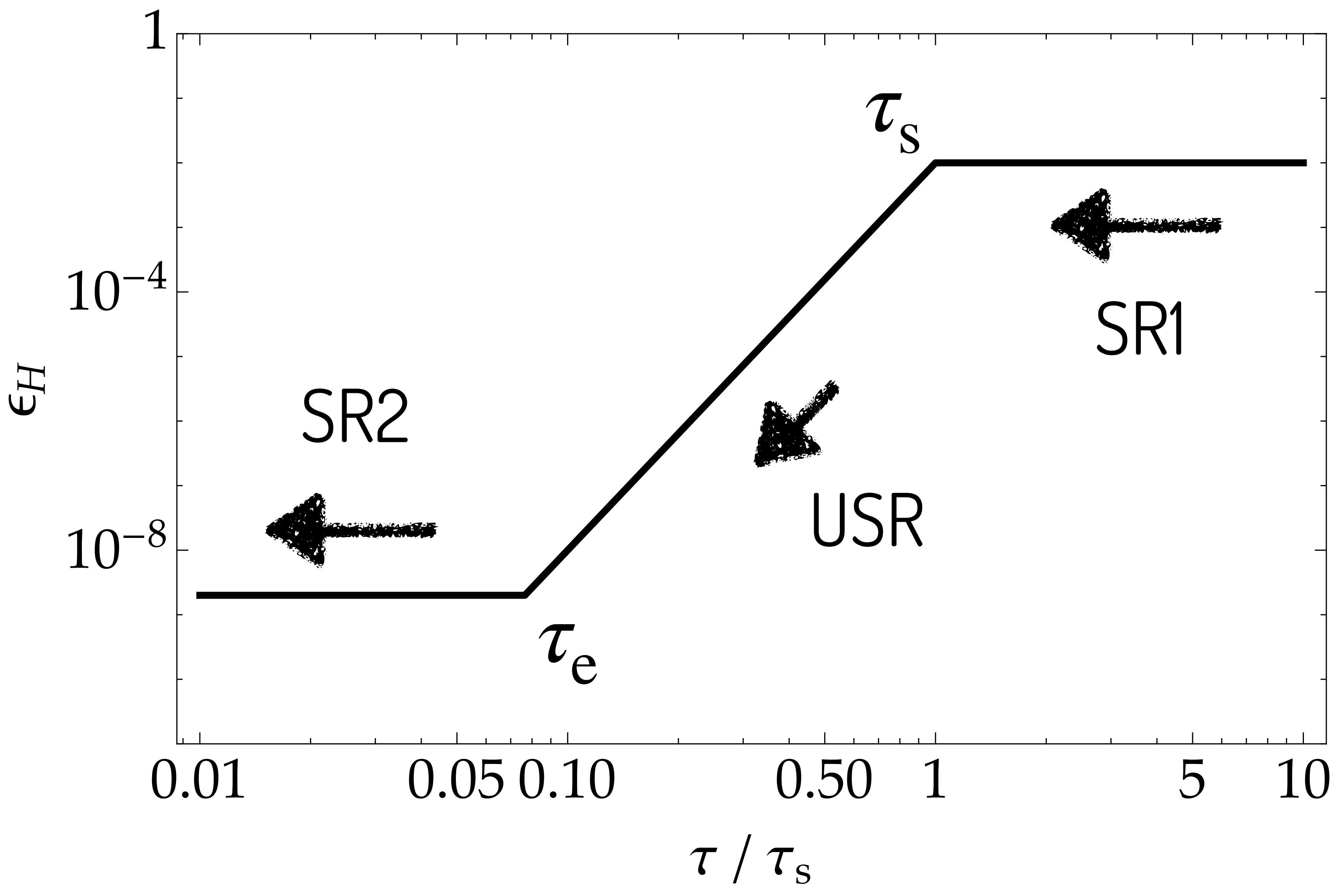
Transient USR

Kristiano & Yokoyama '22

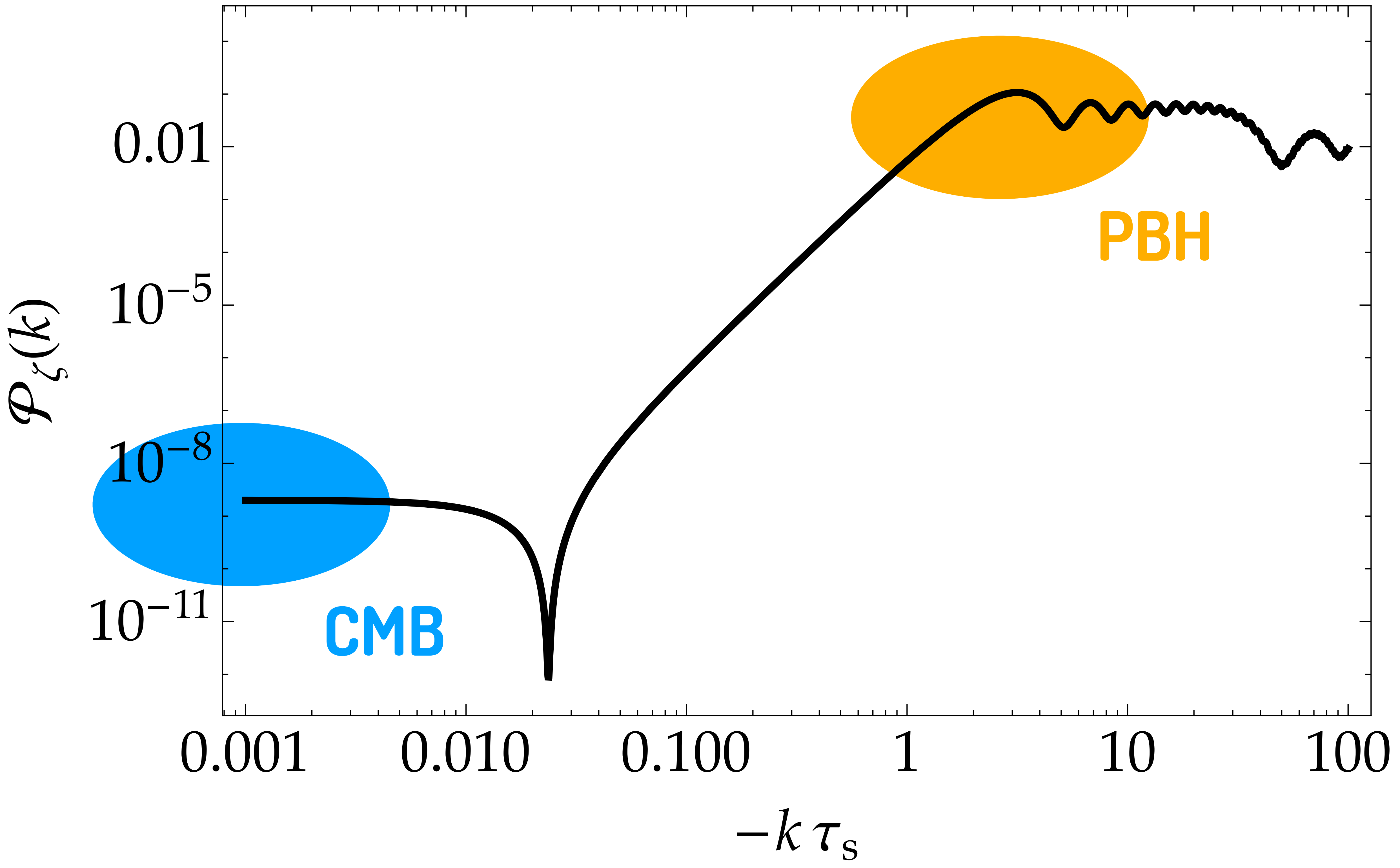


$$\eta = \frac{\dot{\epsilon}}{\epsilon H} = \begin{cases} 0 & \text{for } \tau \leq \tau_s, \\ -6 & \text{for } \tau_s \leq \tau < \tau_e, \\ 0 & \text{for } \tau_e \leq \tau\,. \end{cases}$$

- $\zeta_k(\tau) = C_k\sqrt{-k\tau}H^{(1)}_{3/2}(-k\tau) + D_k\sqrt{-k\tau}H^{(2)}_{3/2}(-k\tau)$ @ each stage
- $\zeta(\tau-0) = \zeta(\tau+0),\ \zeta'(\tau-0) = \zeta'(\tau+0)$ @ τ_s, τ_e



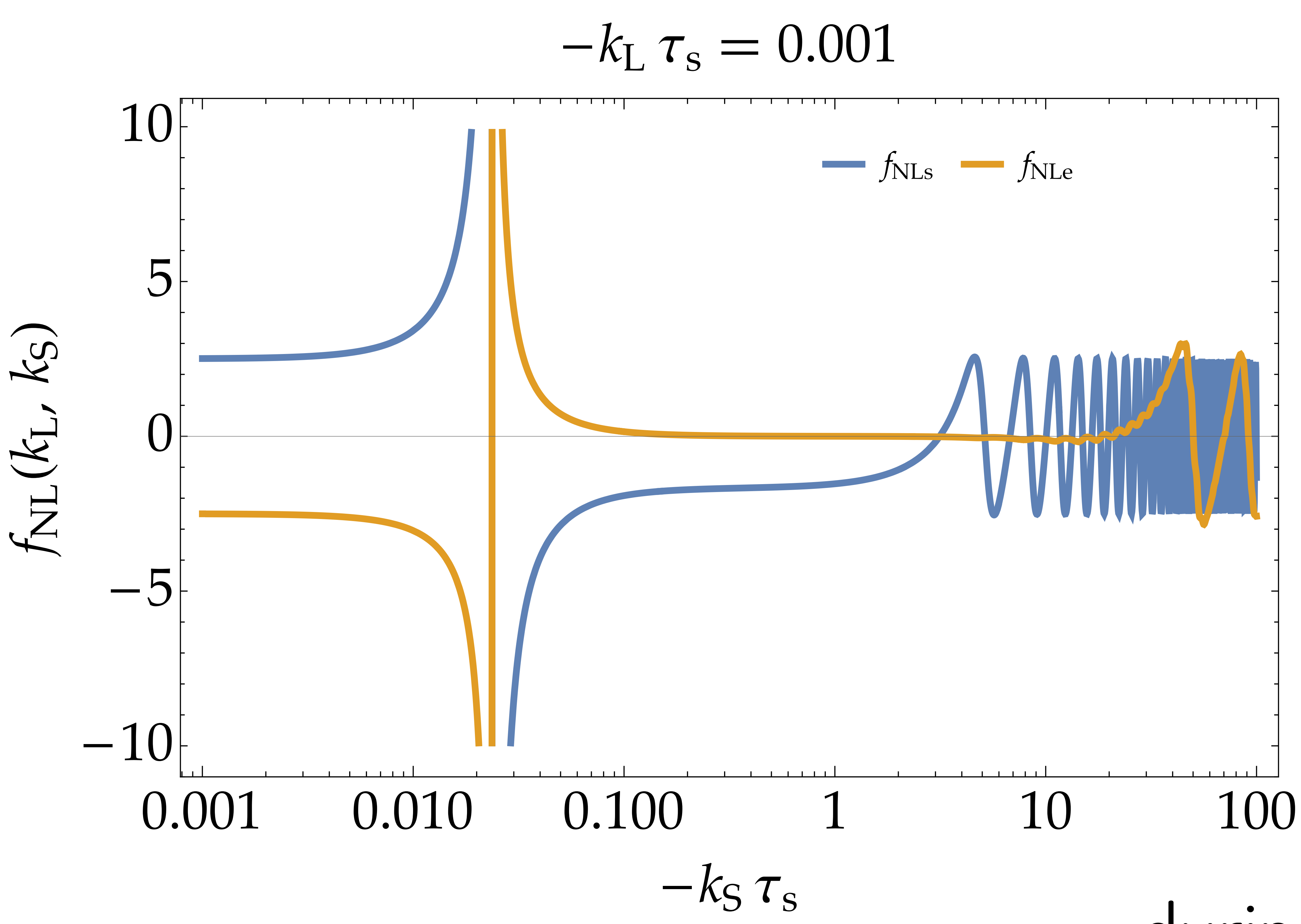
(see Appendix of Motohashi & YT '23 for V reconstruction)

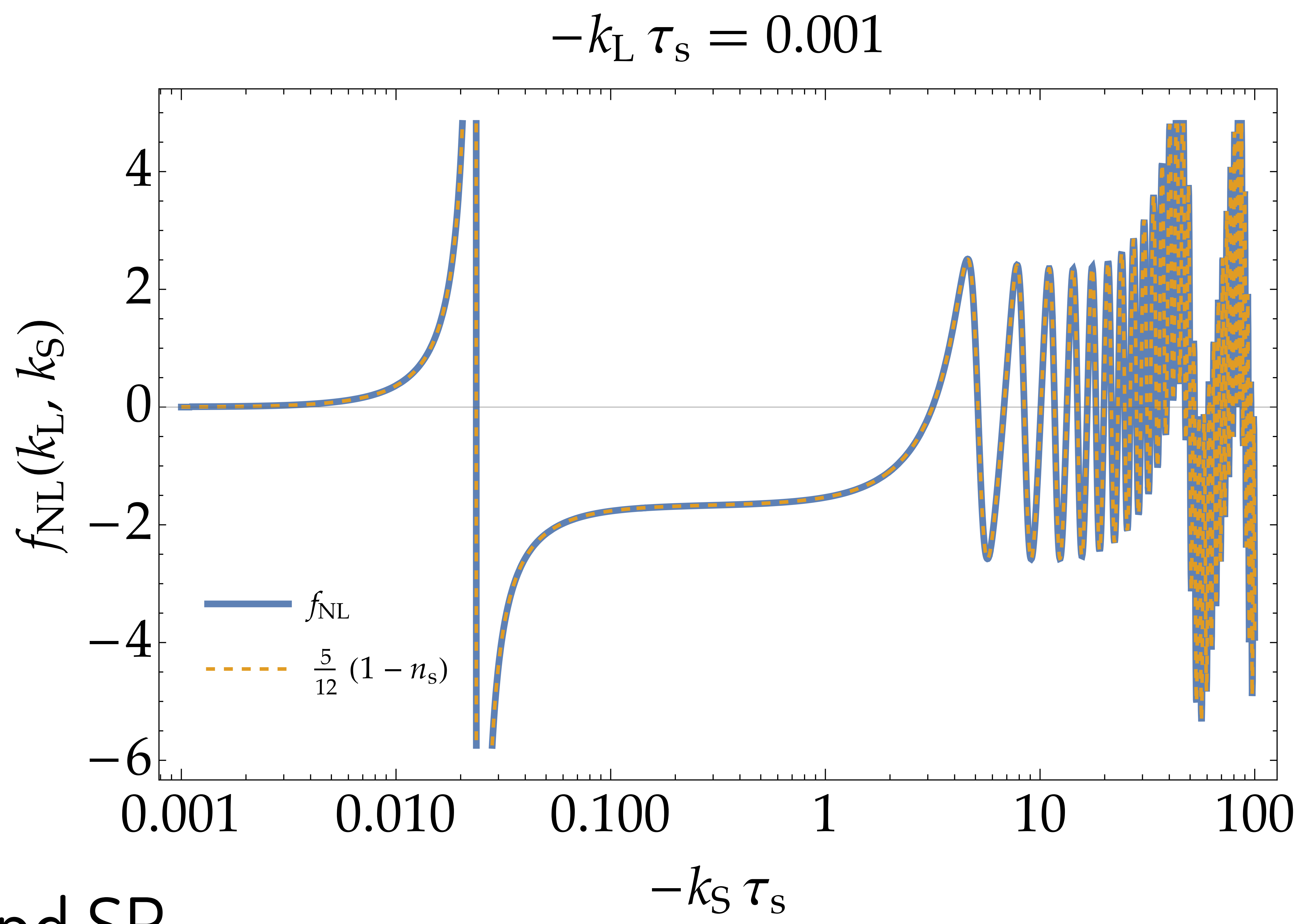


Squeezed Bispectrum

Motohashi & YT '23

$$S^{(3)} \supset \int \mathrm{d}\tau \mathrm{d}^3\mathbf{x} \left[\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' \right] \Rightarrow \langle \zeta_{\mathbf{k}_\mathrm{L}} \zeta_{\mathbf{k}_\mathrm{S}} \zeta_{\mathbf{k}_\mathrm{S}} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{K}) \frac{12}{5} f_\mathrm{NL}(k_\mathrm{L}, k_\mathrm{S}) P_\zeta(k_\mathrm{L}) P_\zeta(k_\mathrm{S})$$





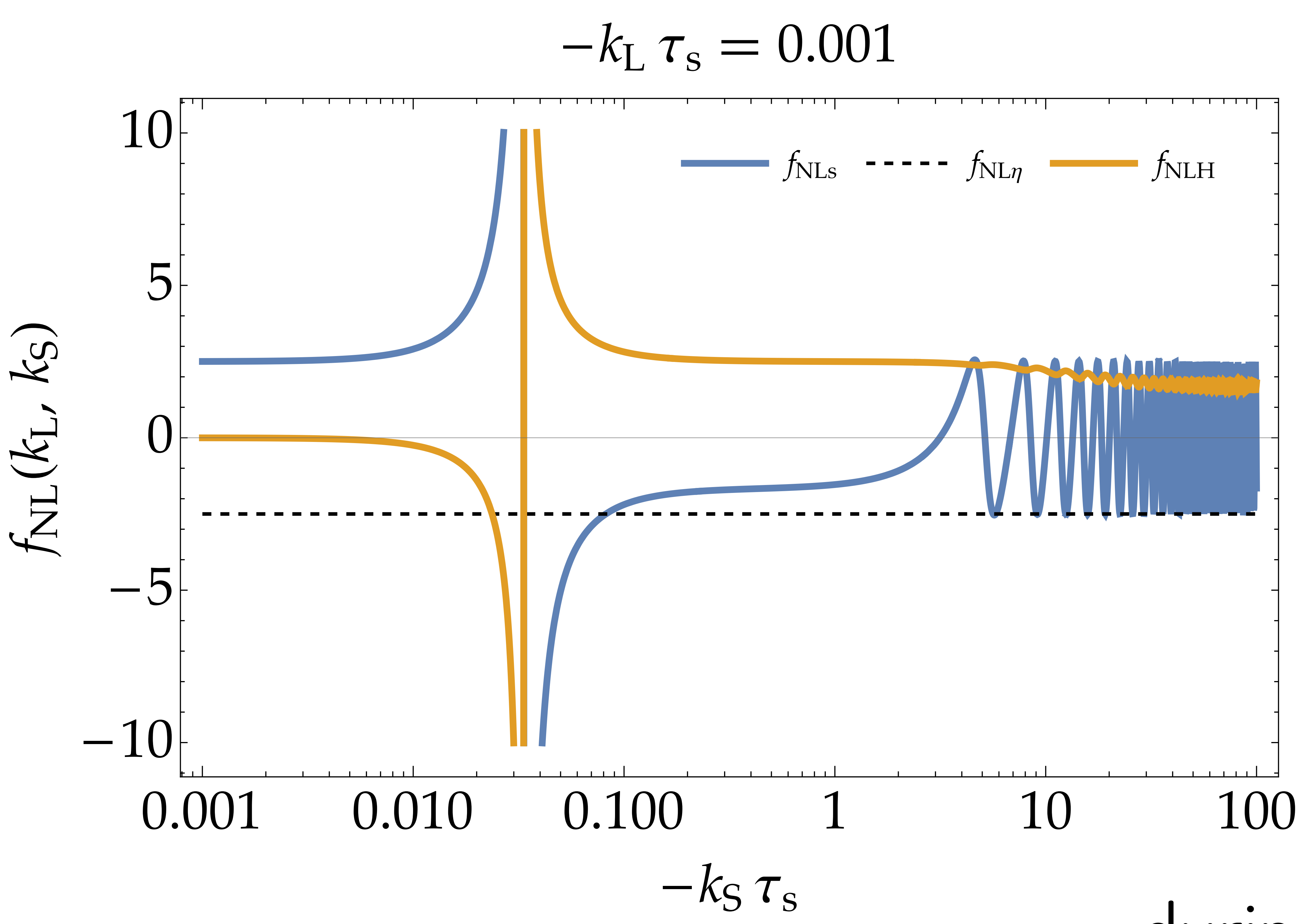
during 2nd SR

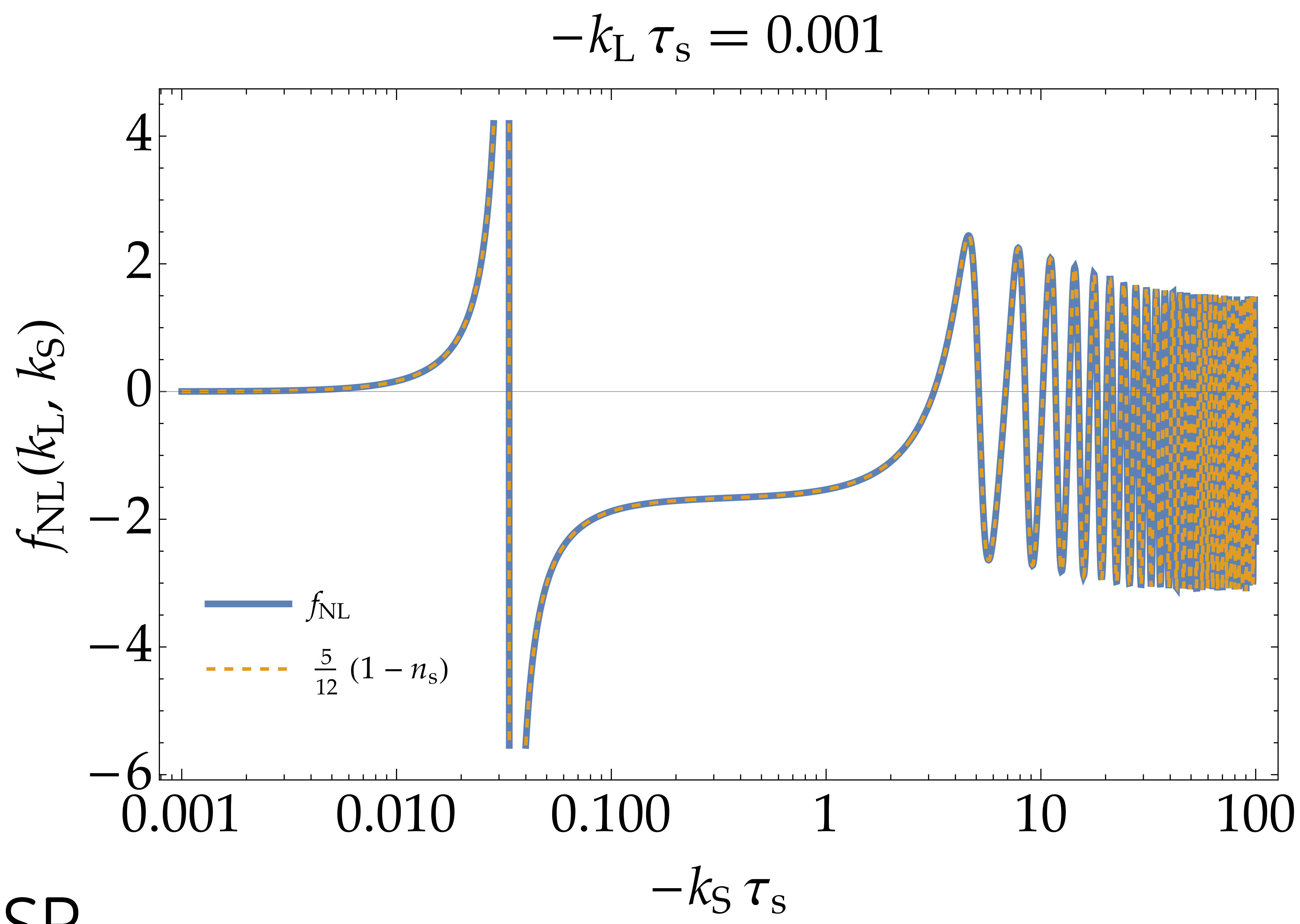
Squeezed Bispectrum

Motohashi & YT '23

cf. Arroja & Tanaka '11

$$S^{(3)} \supset \int d\tau d^3\mathbf{x} \left[\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' - \boxed{\frac{d}{d\tau}\left(\frac{a^2\epsilon}{2}\eta\zeta^2\zeta' + \frac{a\epsilon}{H}\zeta\zeta'^2\right)} \right] \Rightarrow \langle \zeta_{\mathbf{k}_L}\zeta_{\mathbf{k}_S}\zeta_{\mathbf{k}_S}\rangle = (2\pi)^3\delta^{(3)}(\mathbf{K})\frac{12}{5}f_{NL}(k_L,k_S)P_\zeta(k_L)P_\zeta(k_S)$$





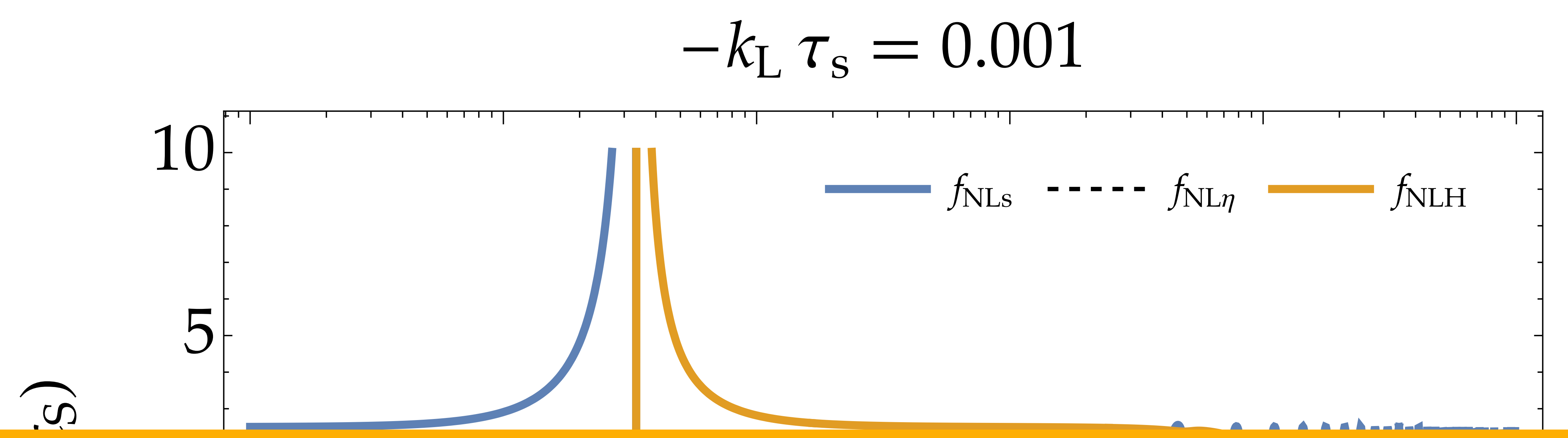
during USR

Squeezed Bispectrum

Motohashi & YT '23

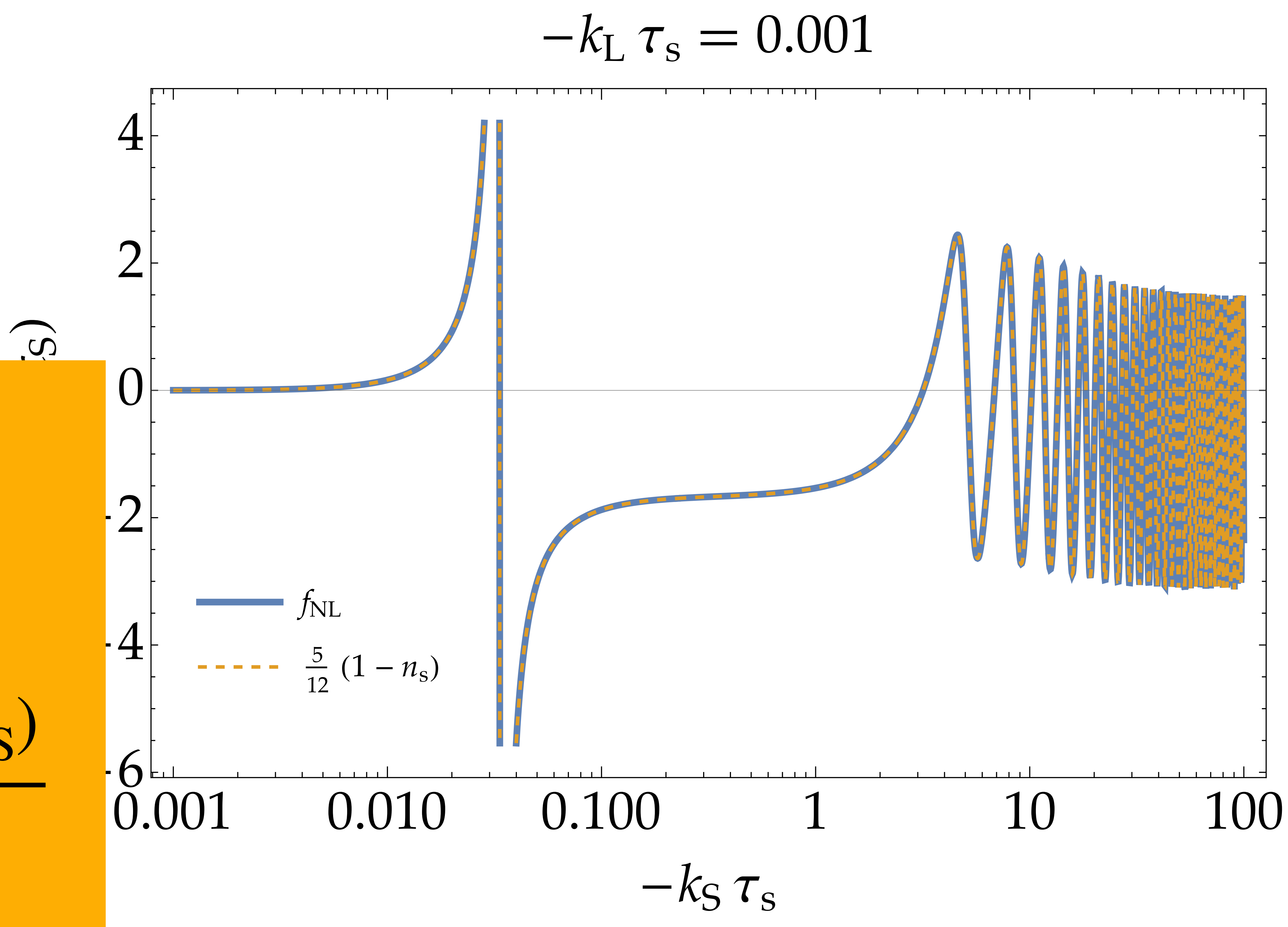
cf. Arroja & Tanaka '11

$$S^{(3)} \supset \int \mathrm{d}\tau \mathrm{d}^3\mathbf{x} \left[\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' - \boxed{\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{a^2\epsilon}{2}\eta\zeta^2\zeta' + \frac{a\epsilon}{H}\zeta\zeta'^2 \right)} \right] \Rightarrow \langle \zeta_{\mathbf{k}_\mathrm{L}}\zeta_{\mathbf{k}_\mathrm{S}}\zeta_{\mathbf{k}_\mathrm{S}} \rangle = (2\pi)^3\delta^{(3)}(\mathbf{K})\frac{12}{5}f_\mathrm{NL}(k_\mathrm{L},k_\mathrm{S})P_\zeta(k_\mathrm{L})P_\zeta(k_\mathrm{S})$$



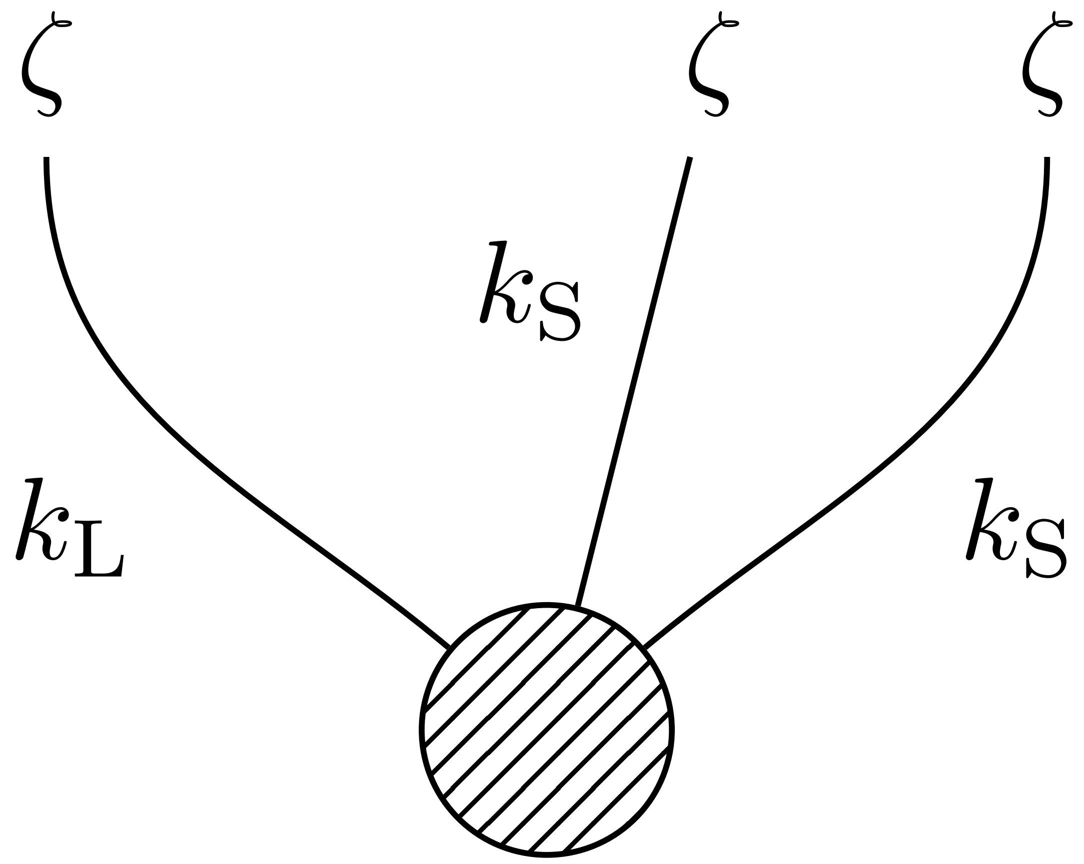
- Boundary terms are relevant!
- Maldacena's CR holds at any time!

$$f_\mathrm{NL}(k_\mathrm{L},k_\mathrm{S}) = \frac{5}{12}\left(1 - n_\mathrm{s}(k_\mathrm{S})\right) = -\frac{5}{12}\frac{\mathrm{d}\ln\mathscr{P}_\zeta(k_\mathrm{S})}{\mathrm{d}\ln k_\mathrm{S}}$$

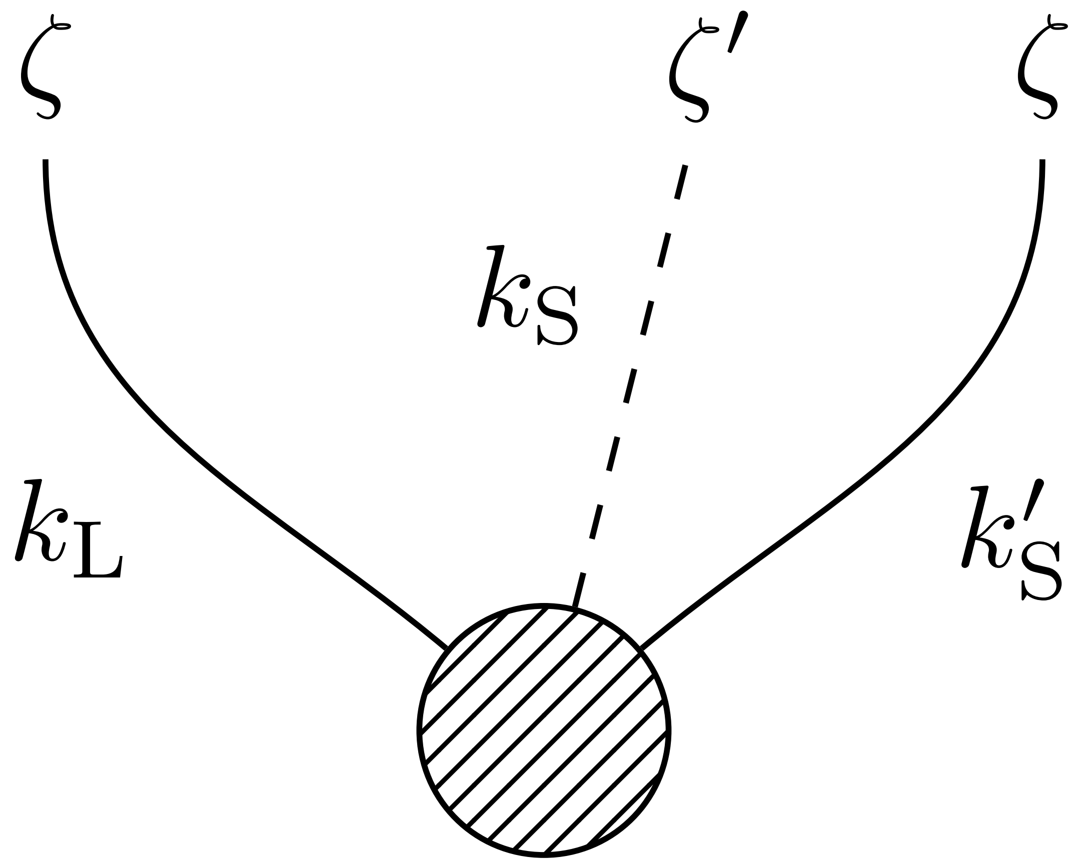


CR in diagrams

YT, Terada, Tokuda '23



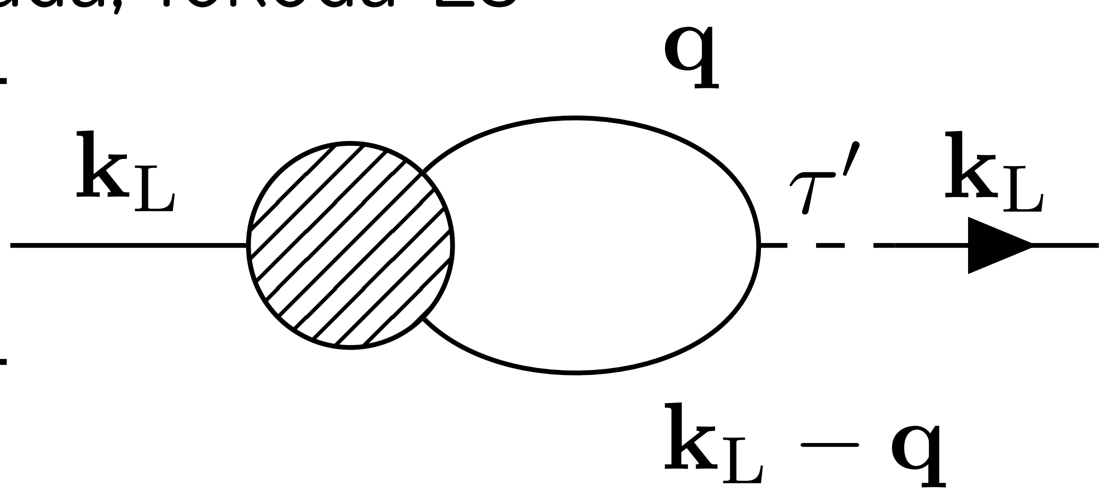
$$= -\frac{2\pi^2}{k_{\mathrm{S}}^3} P_\zeta(k_{\mathrm{L}}) \frac{\mathrm{d}\mathcal{P}_\zeta(k_{\mathrm{S}})}{\mathrm{d}\ln k_{\mathrm{S}}}$$

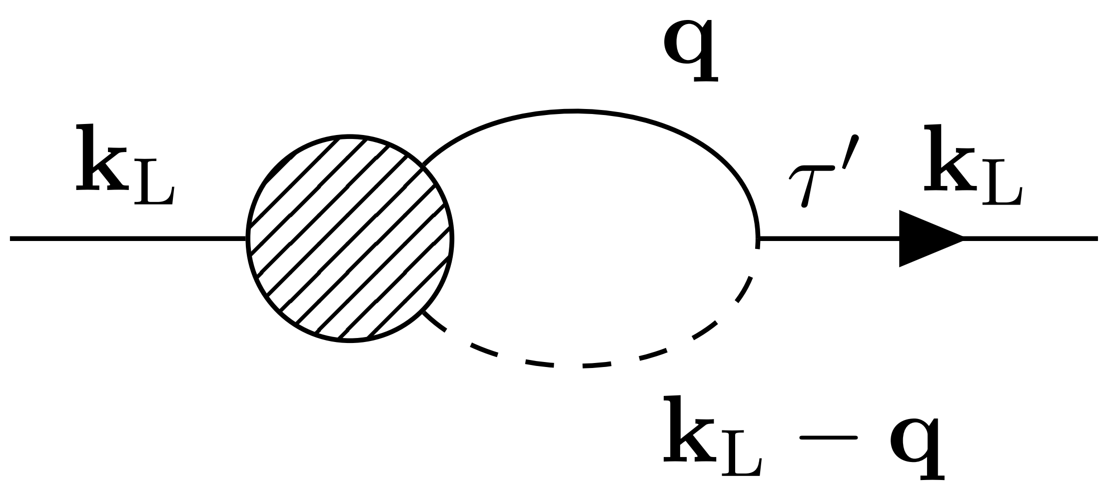


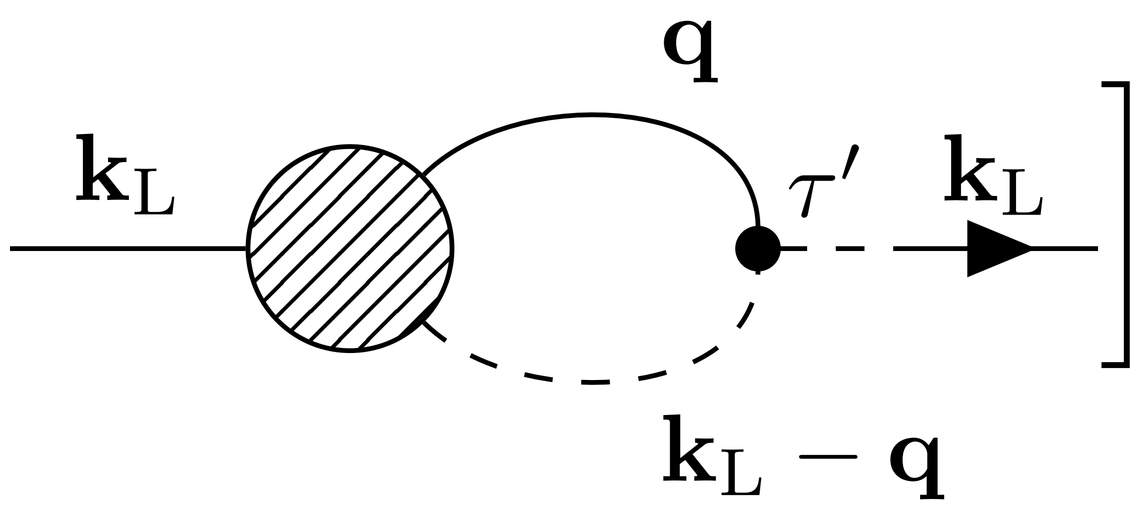
$$+ \ (k_{\mathrm{S}} \leftrightarrow k'_{\mathrm{S}}) = -\frac{2\pi^2}{k_{\mathrm{S}}^3} P_\zeta(k_{\mathrm{L}}) \frac{\mathrm{d}(\partial_\tau \mathcal{P}_\zeta(k_{\mathrm{L}}))}{\mathrm{d}\ln k_{\mathrm{L}}}$$

One-loop Cancellation

YT, Terada, Tokuda '23

$$P_\zeta^{(\mathrm{soft})}(k_\mathrm{L}) = \int^\tau \mathrm{d}\tau' \int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} \Bigg[$$ 

$+$ 

$+$ 

$\Bigg]$

$$\propto P_\zeta(k_\mathrm{L}) \int \mathrm{d}\ln q \frac{\mathrm{d}\mathscr{P}_\zeta(q)}{\mathrm{d}\ln q} \qquad \propto P_\zeta(k_\mathrm{L}) \int \mathrm{d}\ln q \frac{\mathrm{d}(\partial_\tau \mathscr{P}_\zeta(q))}{\mathrm{d}\ln q} \qquad \propto P_\zeta(k_\mathrm{L}) \int \mathrm{d}\ln q \frac{\mathrm{d}(\partial_\tau \mathscr{P}_\zeta(q))}{\mathrm{d}\ln q}$$

→ 0 by $i\varepsilon$ (i.e. $\tau \to (1+i\varepsilon)\tau$)

$$= P_\zeta(k_\mathrm{L})\mathscr{P}_\zeta(q)\Big|_{q\to k_\mathrm{L}}^{q\to\infty} \qquad = P_\zeta(k_\mathrm{L})(\partial_\tau \mathscr{P}_\zeta(q))\Big|_{q\to k_\mathrm{L}}^{q\to\infty} \qquad = P_\zeta(k_\mathrm{L})(\partial_\tau \mathscr{P}_\zeta(q))\Big|_{q\to k_\mathrm{L}}^{q\to\infty}$$

$$\sim \mathscr{O}(10^{-9}) \qquad \to 0 \qquad \to 0$$

cf. Pimentel, Senatore, Zaldarriaga '12

$$\sim \mathscr{O}(10^{-9}) \times P_\zeta(k_\mathrm{L}) \ll P_\zeta(k_\mathrm{L})$$ ✔

Conserved ζ @ tree ⇔ Maldacena's CR ⇔ Cancellation of One-loop

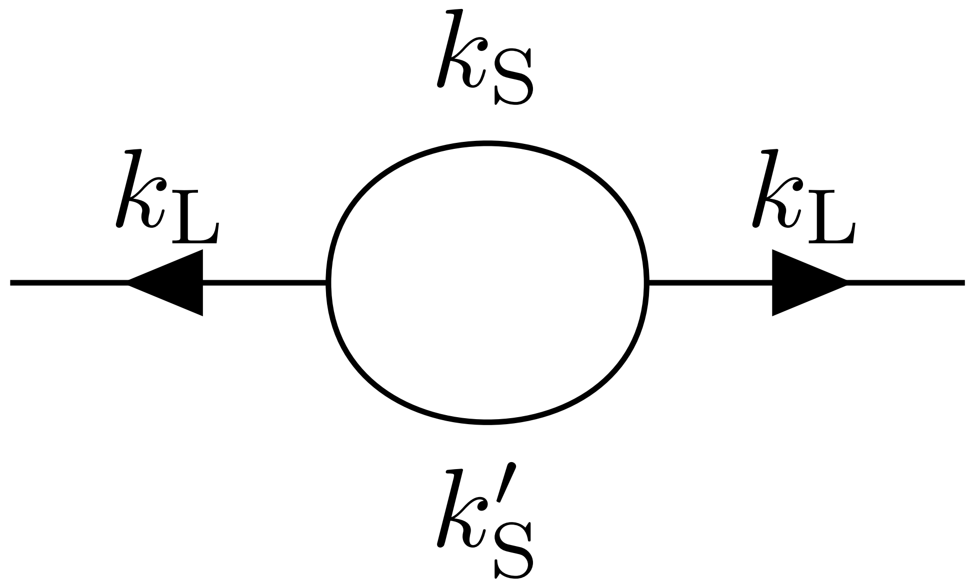
cf. Ward—Takahashi by asymptotic dilatation

Comments

YT, Terada, Tokuda '23

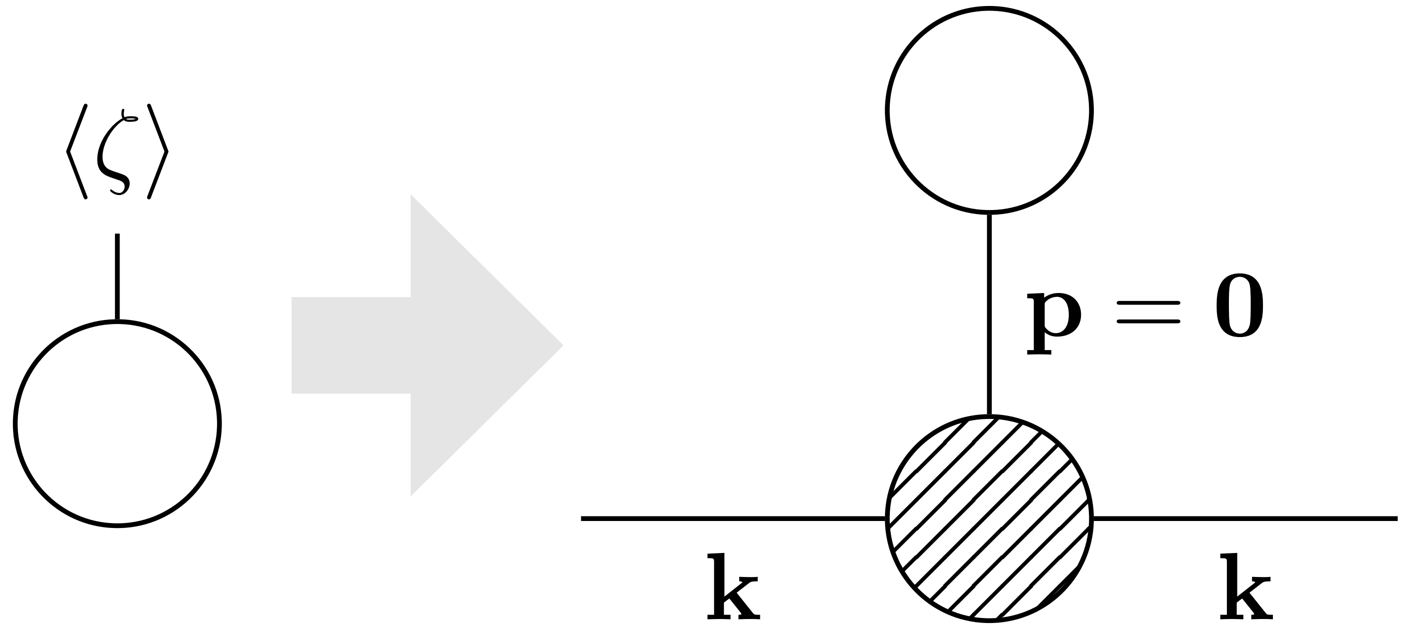
- induced scalar (cf. induced GW)

$$P_\zeta^{(\mathrm{induced})}(k_\mathrm{L}) \propto$$



$$\propto \left(\frac{k_\mathrm{L}}{k_\mathrm{S}}\right)^3$$

- tadpole



$$= -\langle\zeta\rangle \frac{\mathrm{d}\mathcal{P}_\zeta(k)}{\mathrm{d}\ln k}$$

$$P_\zeta^{\mathrm{ren}}(k) := P_\zeta(k e^{-\langle\zeta\rangle})$$

cf. stochastic-δN

$$\langle\zeta\rangle = \langle\mathcal{N}\rangle - N_{\mathrm{cl}}$$

- tensor (Ota, Sasaki, Wang '22x2)

not total derivative...

(2311.11053; no mass correction in dS?)

$$B_{h_\lambda\zeta\zeta}(k_\mathrm{L},k_\mathrm{S},k_\mathrm{S}) = -\frac{1}{2}P_{h_\lambda}(k_\mathrm{L})P_\zeta(k_\mathrm{S})e^\lambda_{ij}(\hat{\mathbf{k}}_\mathrm{L})\hat{k}^i_\mathrm{S}\hat{k}^j_\mathrm{S}\frac{\mathrm{d}\ln P_\zeta(k_\mathrm{S})}{\mathrm{d}\ln k_\mathrm{S}}$$

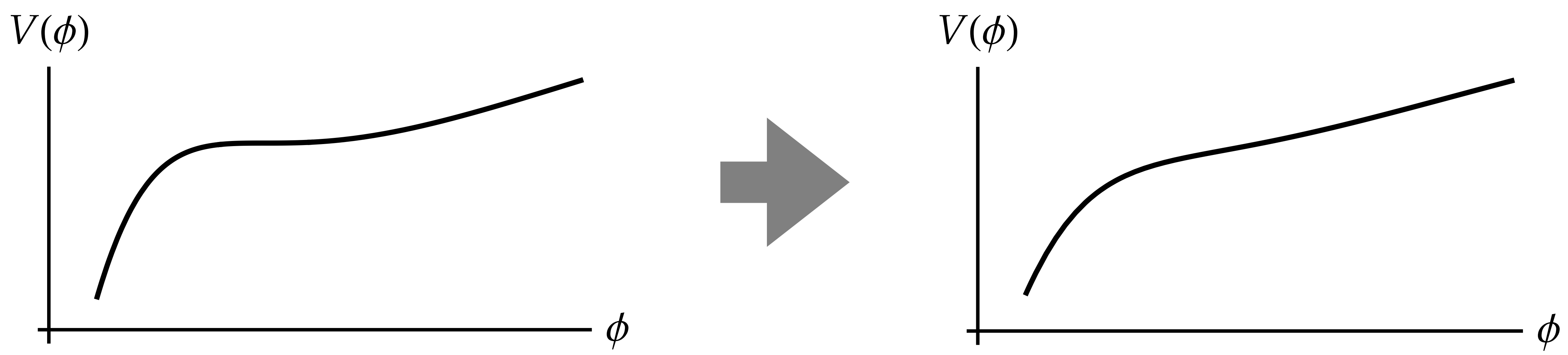
$$P_h^{(\mathrm{soft})}(k_\mathrm{L}) \propto P_h(k_\mathrm{L})\int \mathrm{d}\ln q\, q^5 \frac{\mathrm{d}P_\zeta(q)}{\mathrm{d}\ln q}$$

Comments

- other works

  e.g. Firouzjahi 2311.0408

  $S_{\rm EFT}[\delta\phi]$: large corrections on coupling consts. are possible
  but it simultaneously changes B.G. and relation $\delta\phi \leftrightarrow \zeta$



$V(\phi)$ may change but ζ-conservation should NOT depend on the details of $V(\phi)$

Riotto, Choundhury+, Fumagalli, Tasinato, Maity+, Mulryne+, etc., etc., ...

- symmetry?

  e.g. Soft dS Effective Theory by Cohen & Green '20

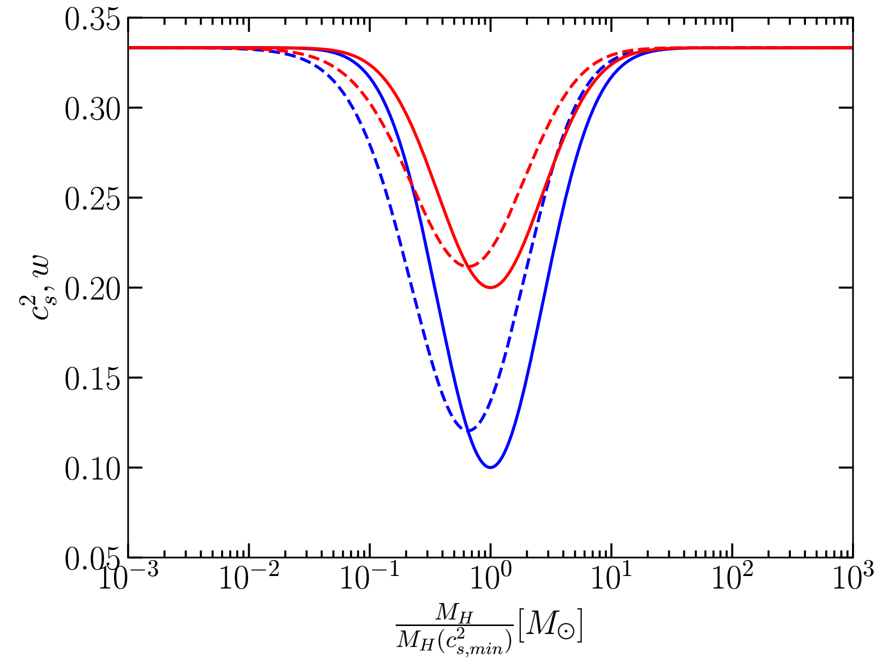
  "conservation of 'const.'-modes of ζ and h is preserved @ all orders"

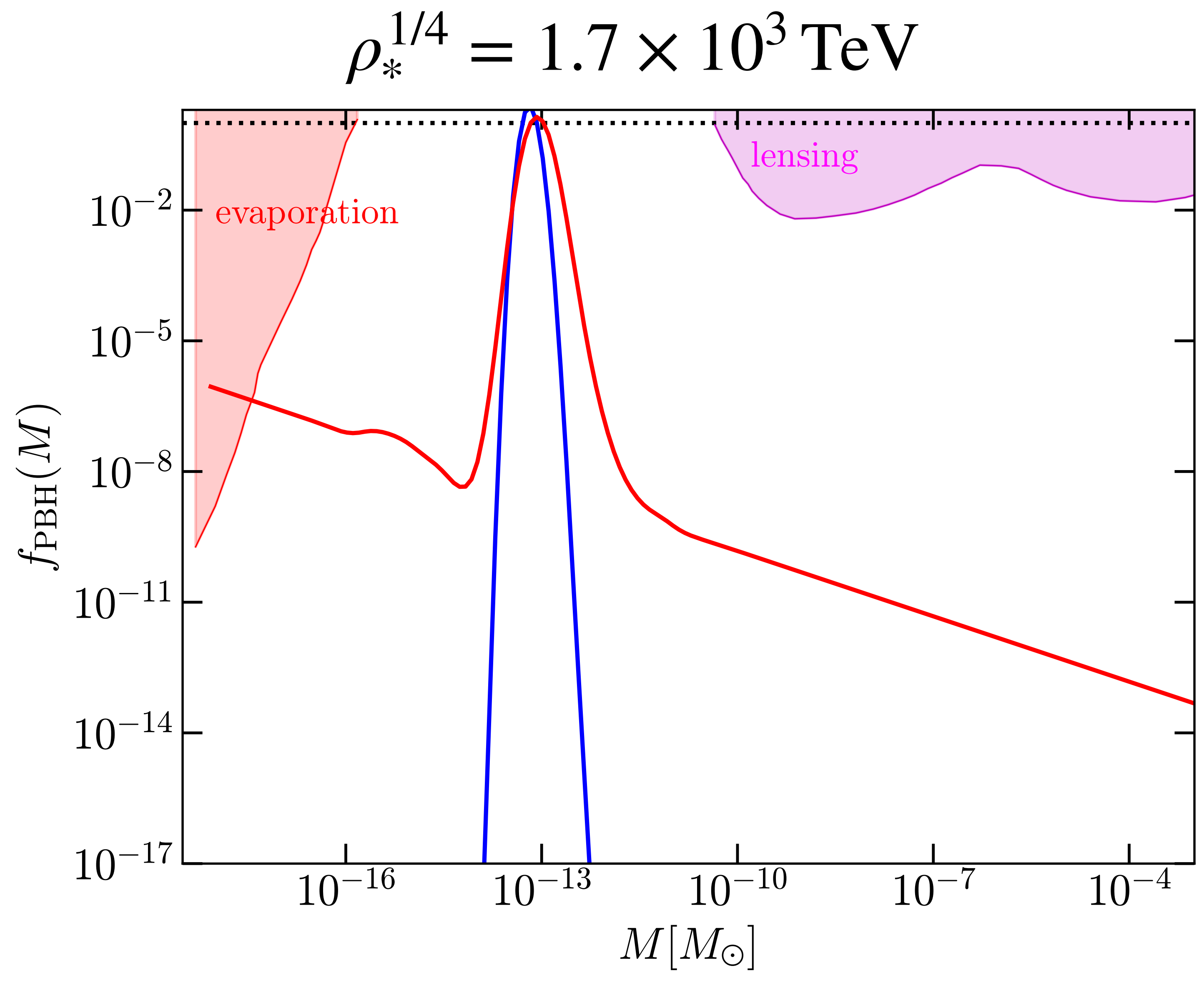
BSM crossover

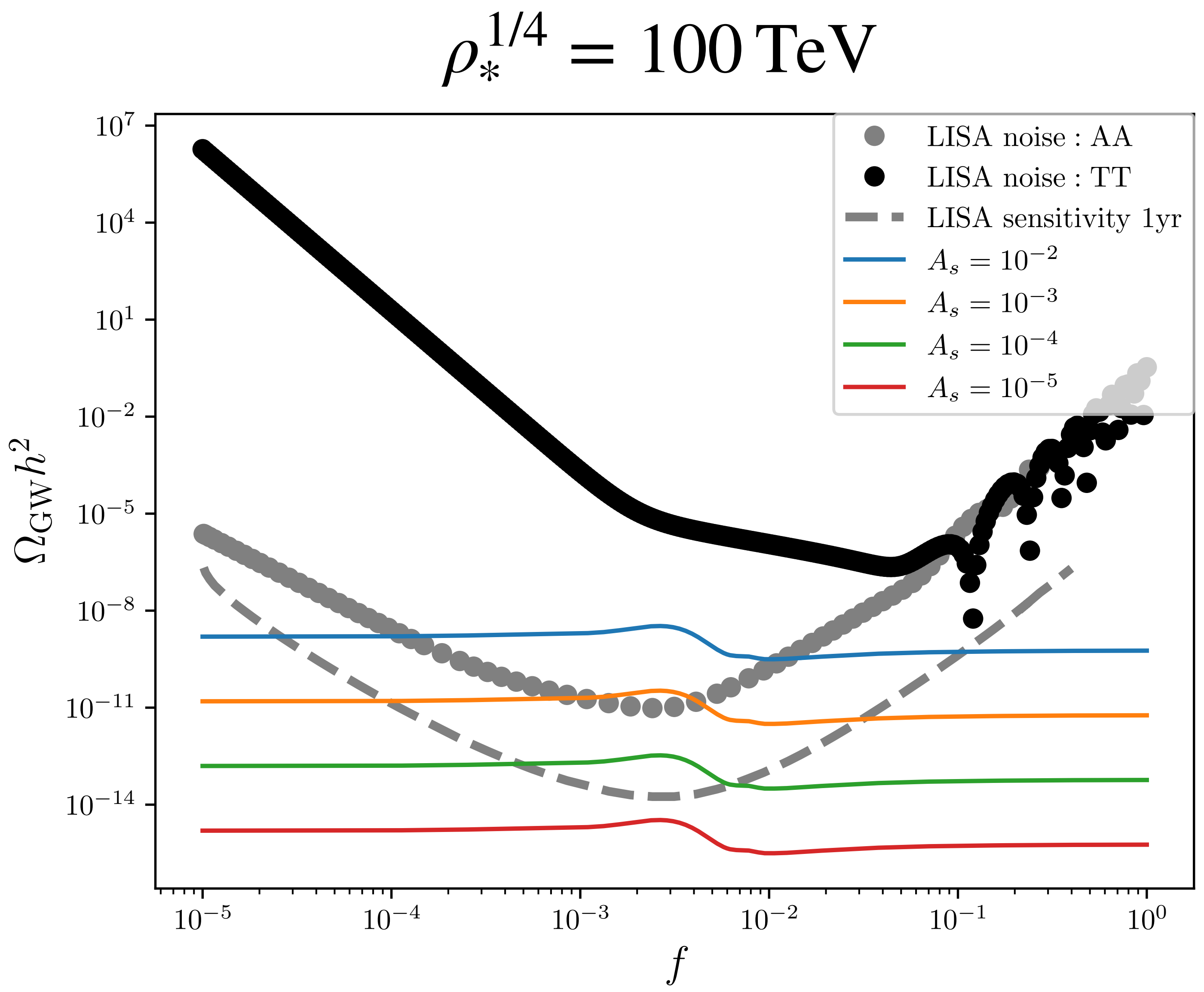
Escrivà, YT, Yoo '24, +Inui '24

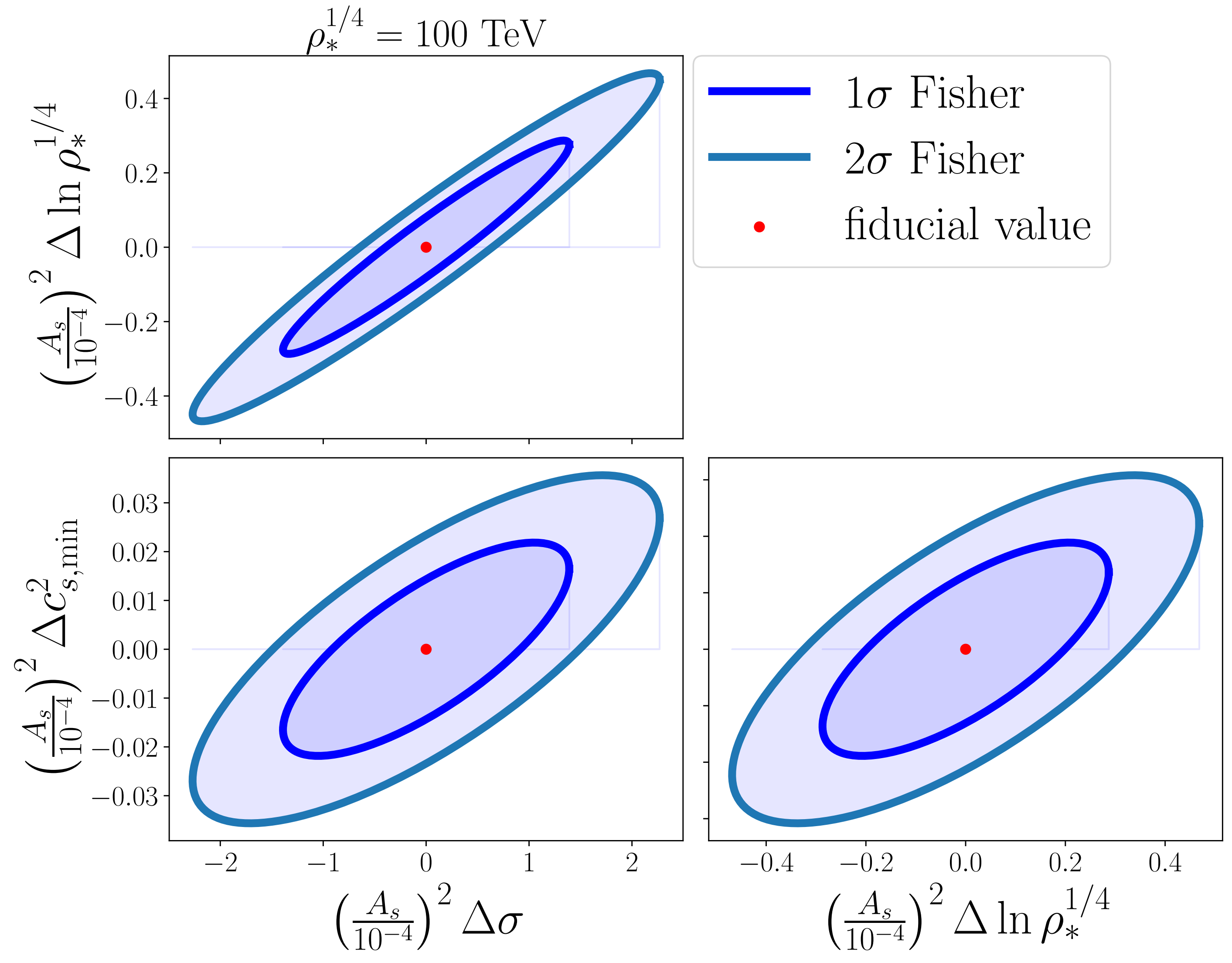
Plasma pressure reduces during smooth CO → PBH production enhanced!

$$\mathscr{P}_\zeta(k) = A_\mathrm{s} \quad + \quad \Delta c_\mathrm{s}^2 \propto \exp\left(-\frac{(\ln\rho - \ln\rho_*)^2}{2\sigma^2}\right)$$









Summary

- Evidences:

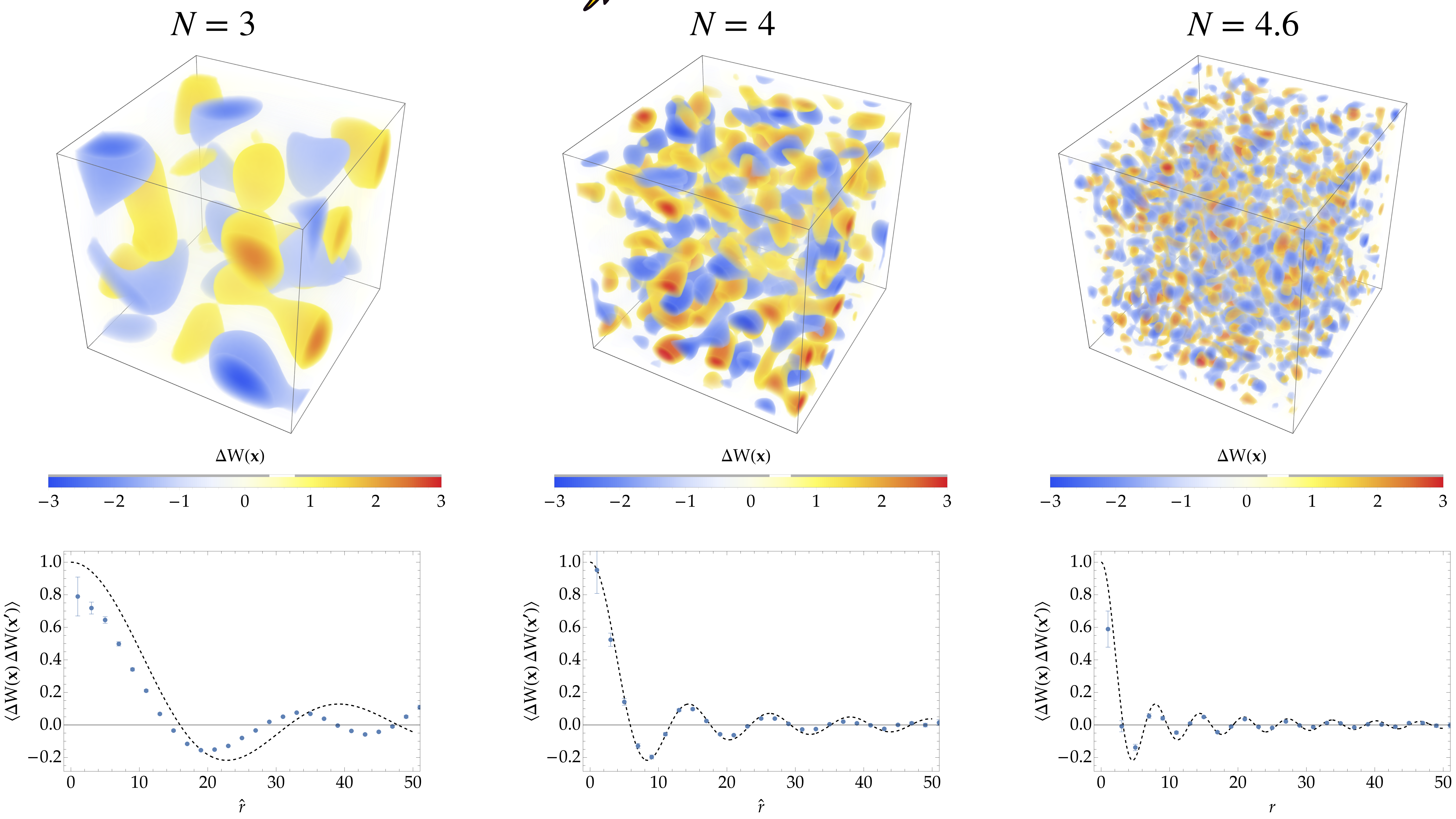
  1) subsolar BHs   2) merger GWs @ high-z 3) SIGWs

- STOLAS can directly sample the PBH abundance
- Loop correction on soft ζ is prohibited
- $\mathcal{O}(100\text{–}10^3)\,\mathrm{TeV}$ BSM crossover?

Appendices

STOLAS

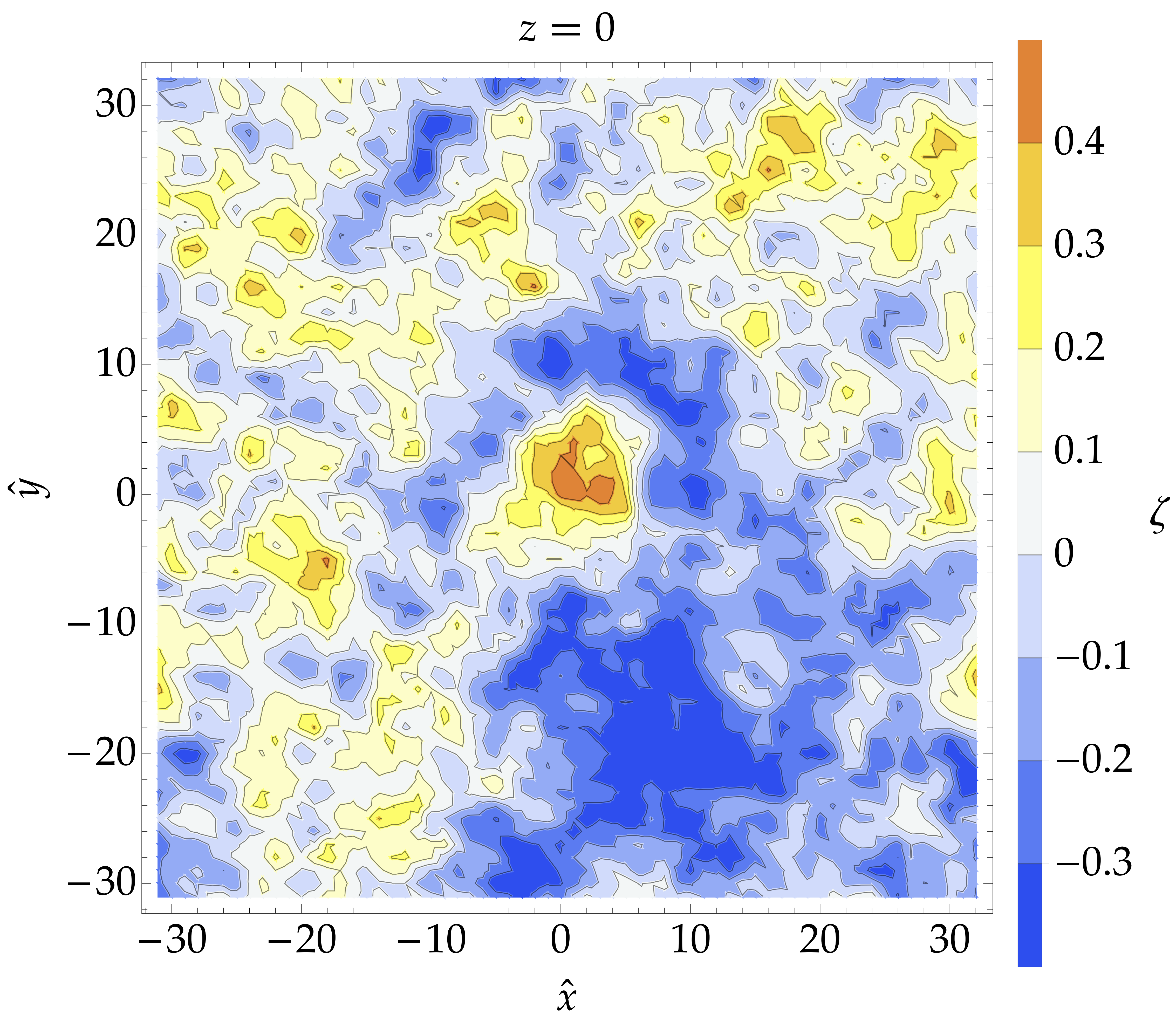
Mizuguchi, Murata, YT '24

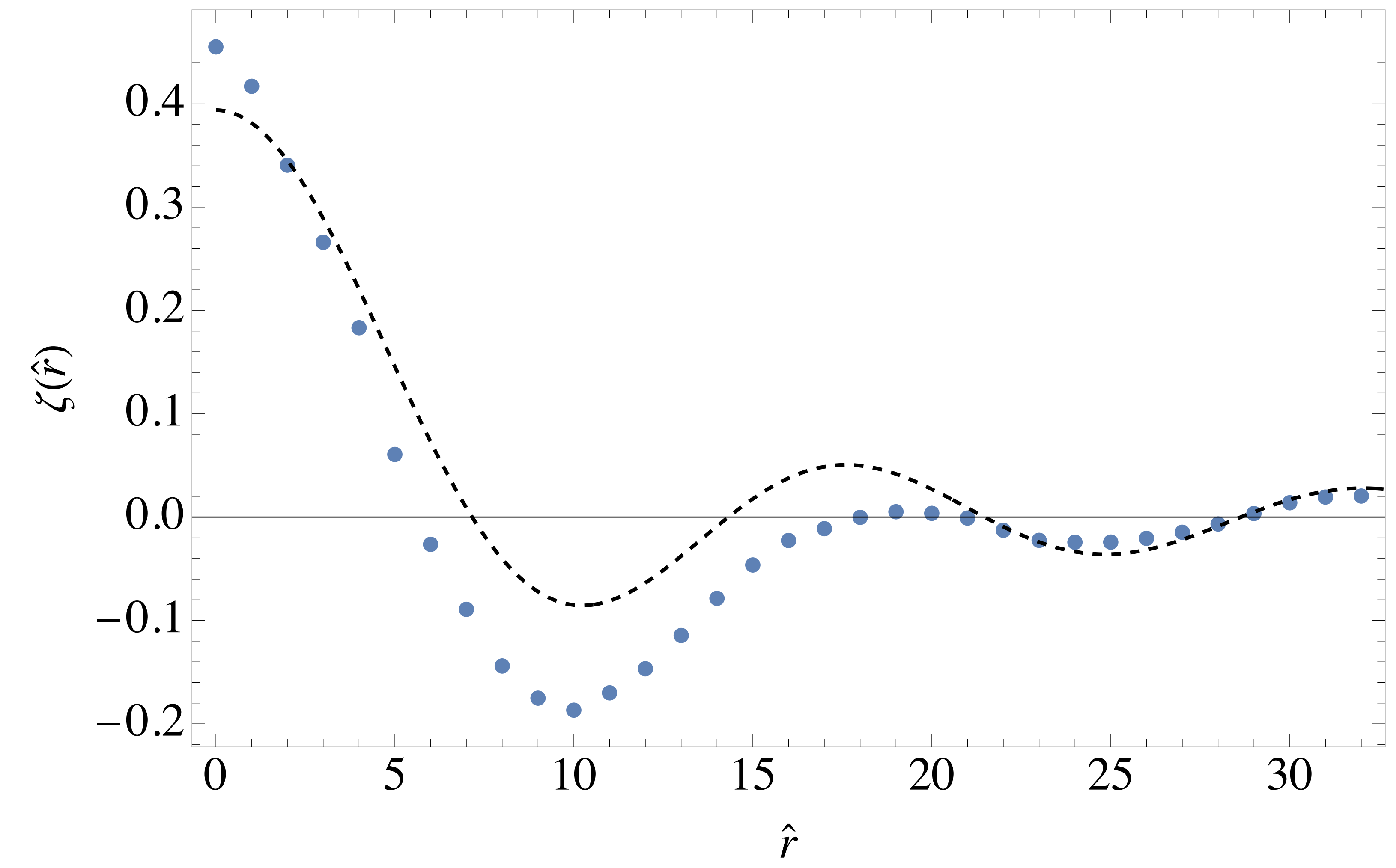


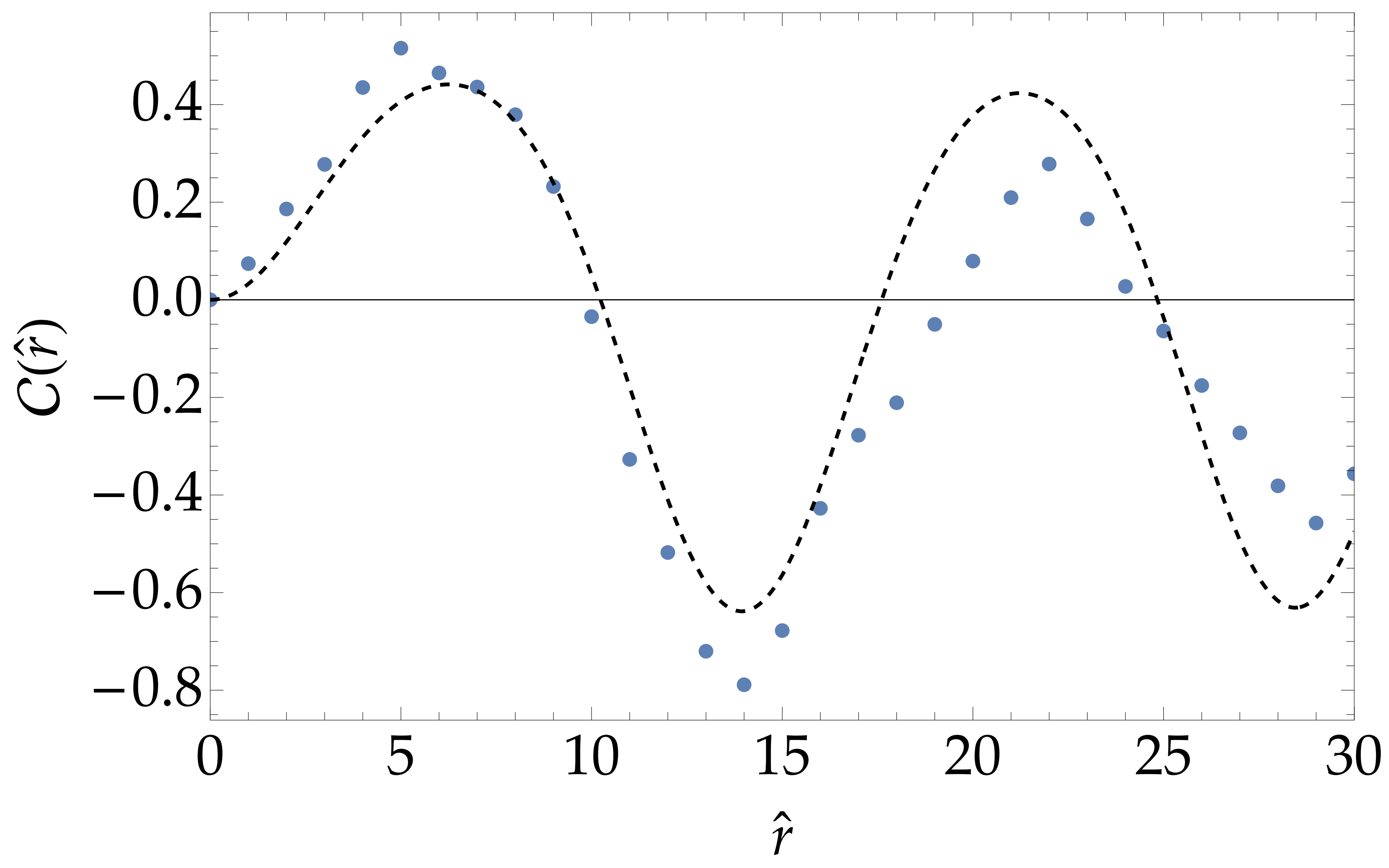
STOLAS

Mizuguchi, Murata, YT '24

Ex. 1: Chaotic $V = \frac{1}{2}m^2\phi^2$





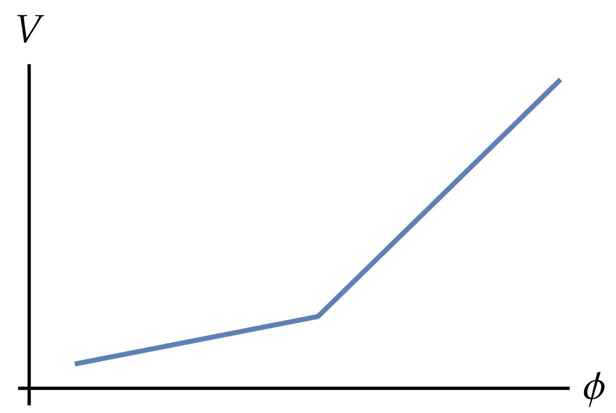


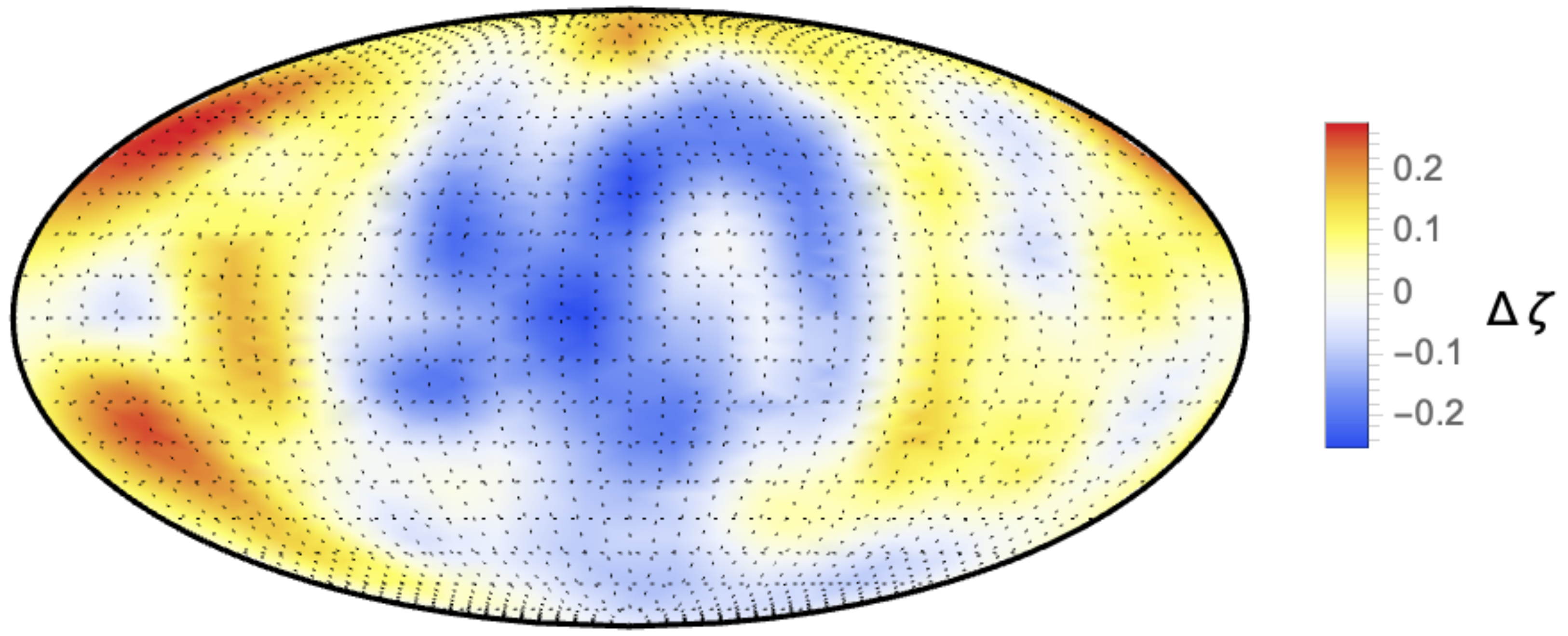
STOLAS

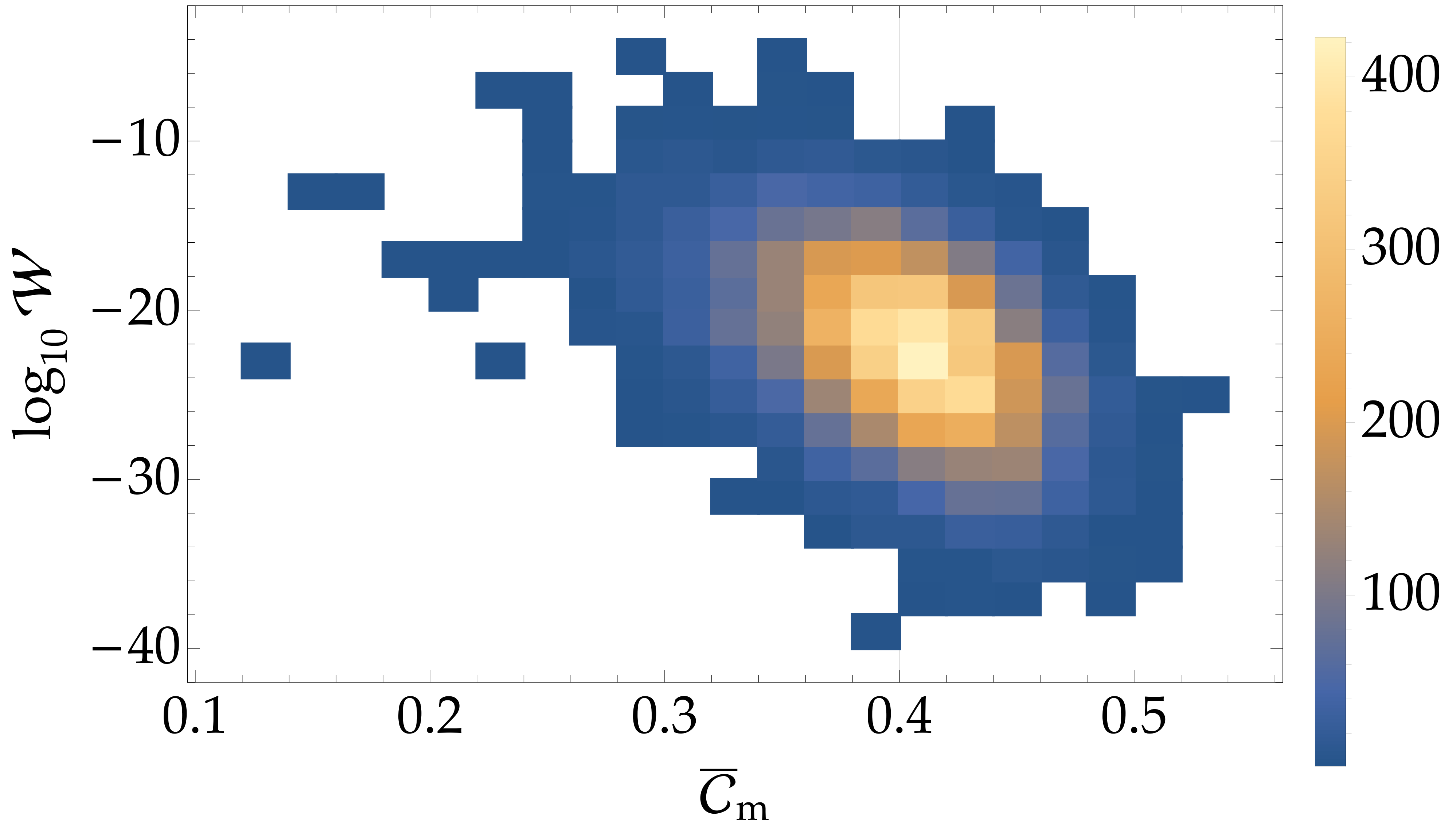
Mizuguchi, Murata, YT '24

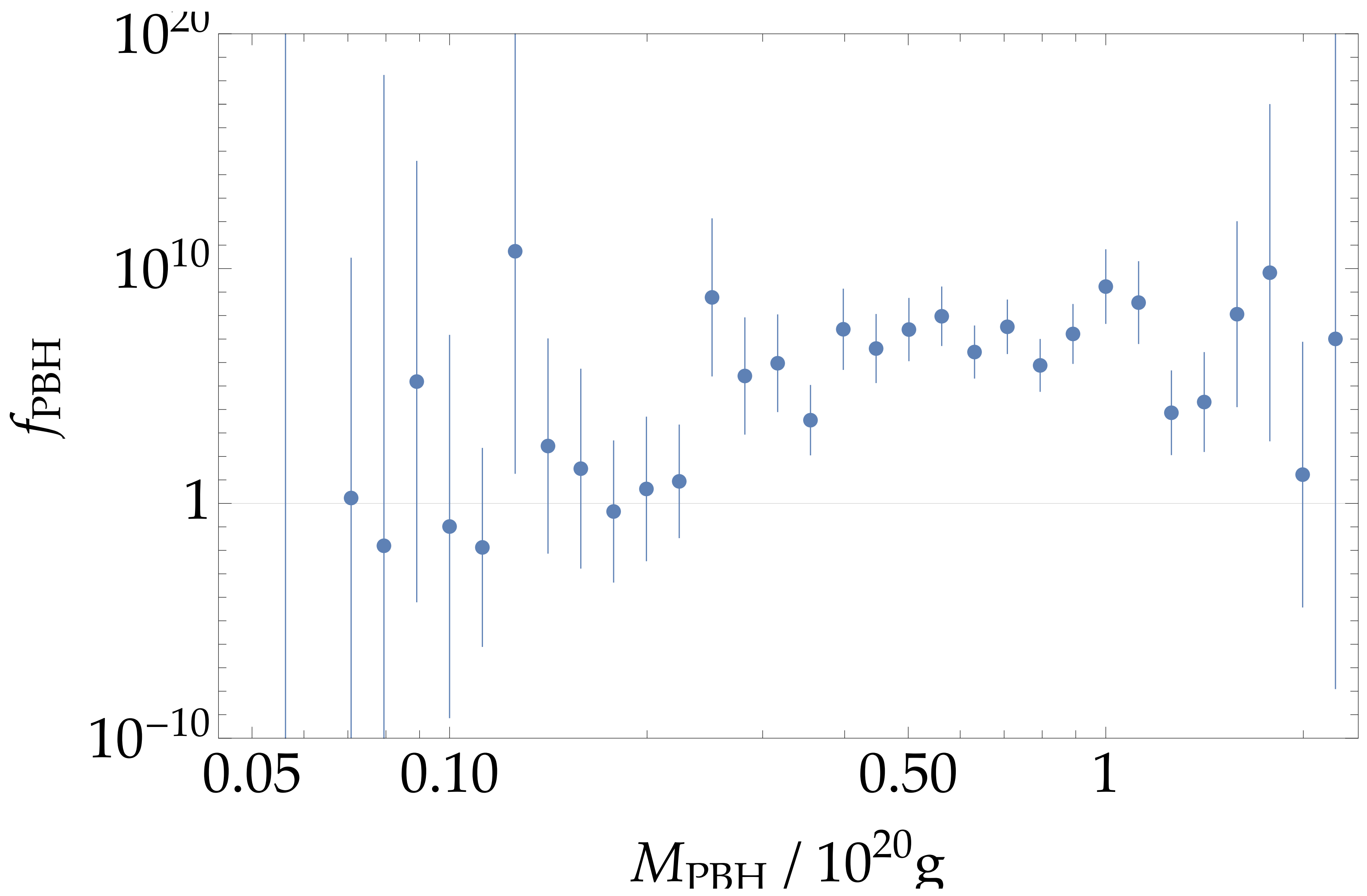
Ex. 2: Starobinsky's linear

Starobinsky '92







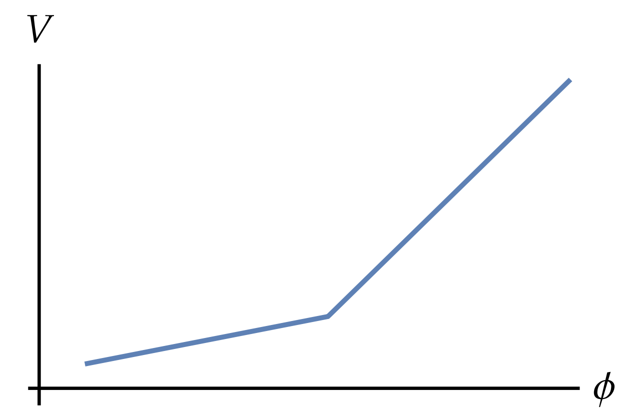


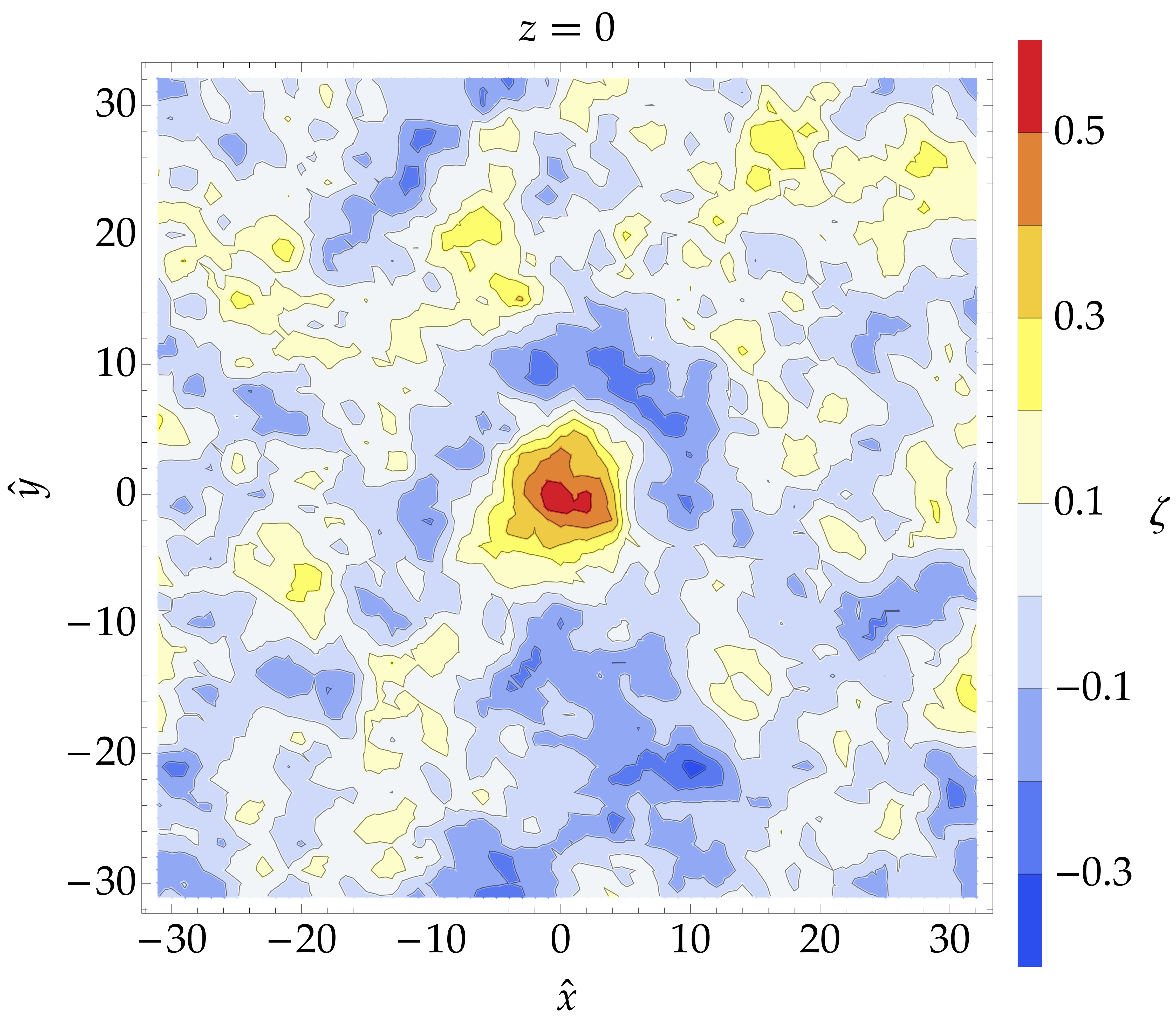
STOLAS

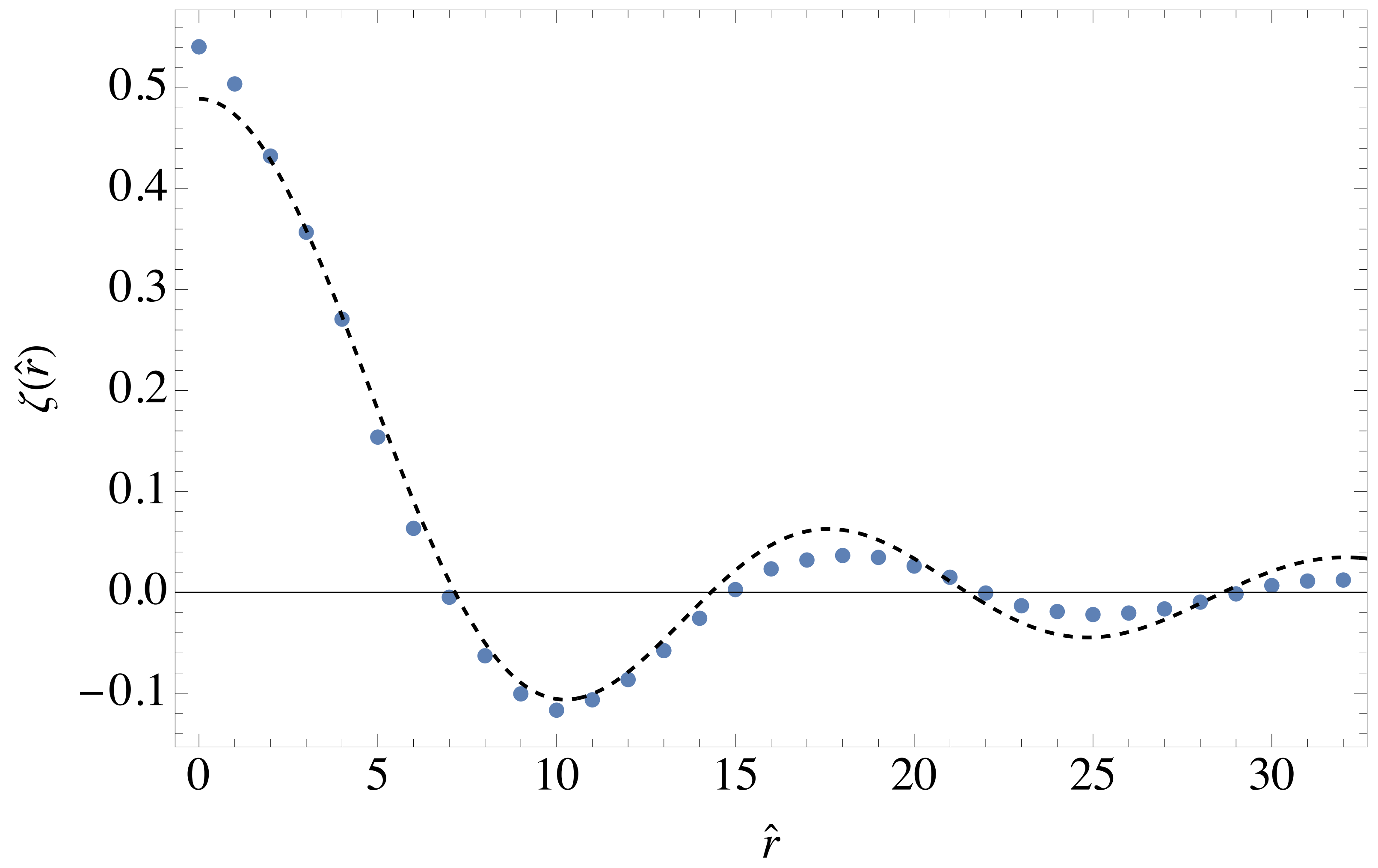
Mizuguchi, Murata, YT '24

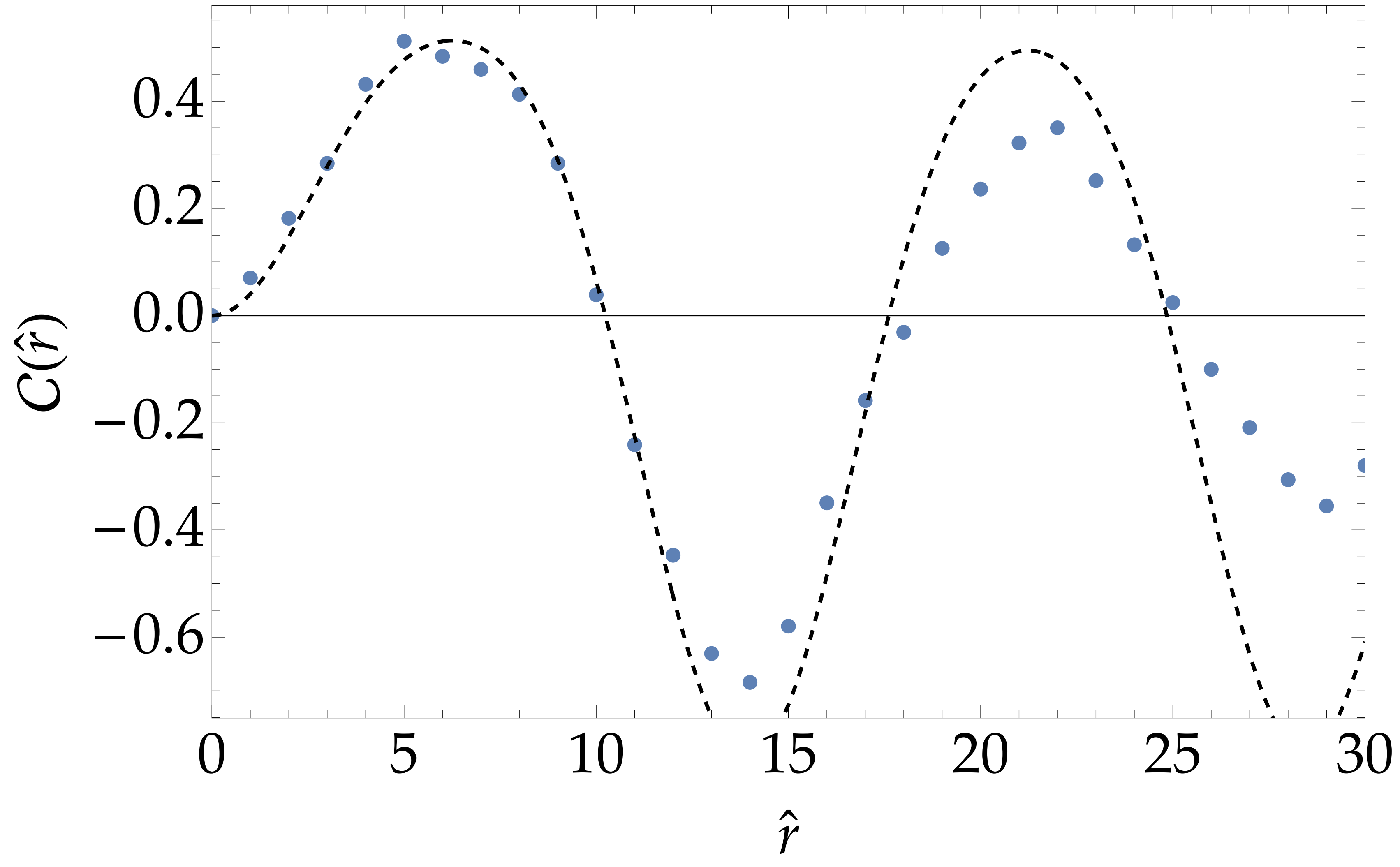
Ex. 2: Starobinsky's linear

Starobinsky '92







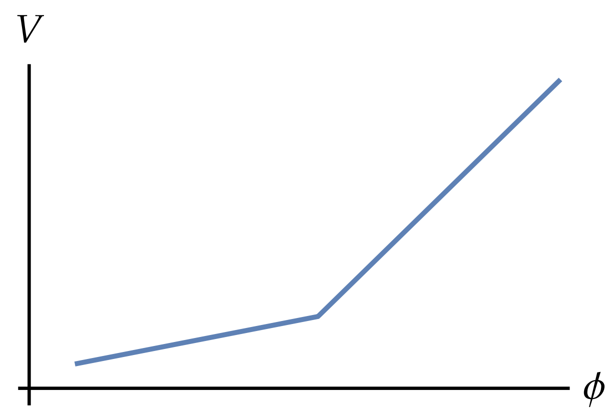


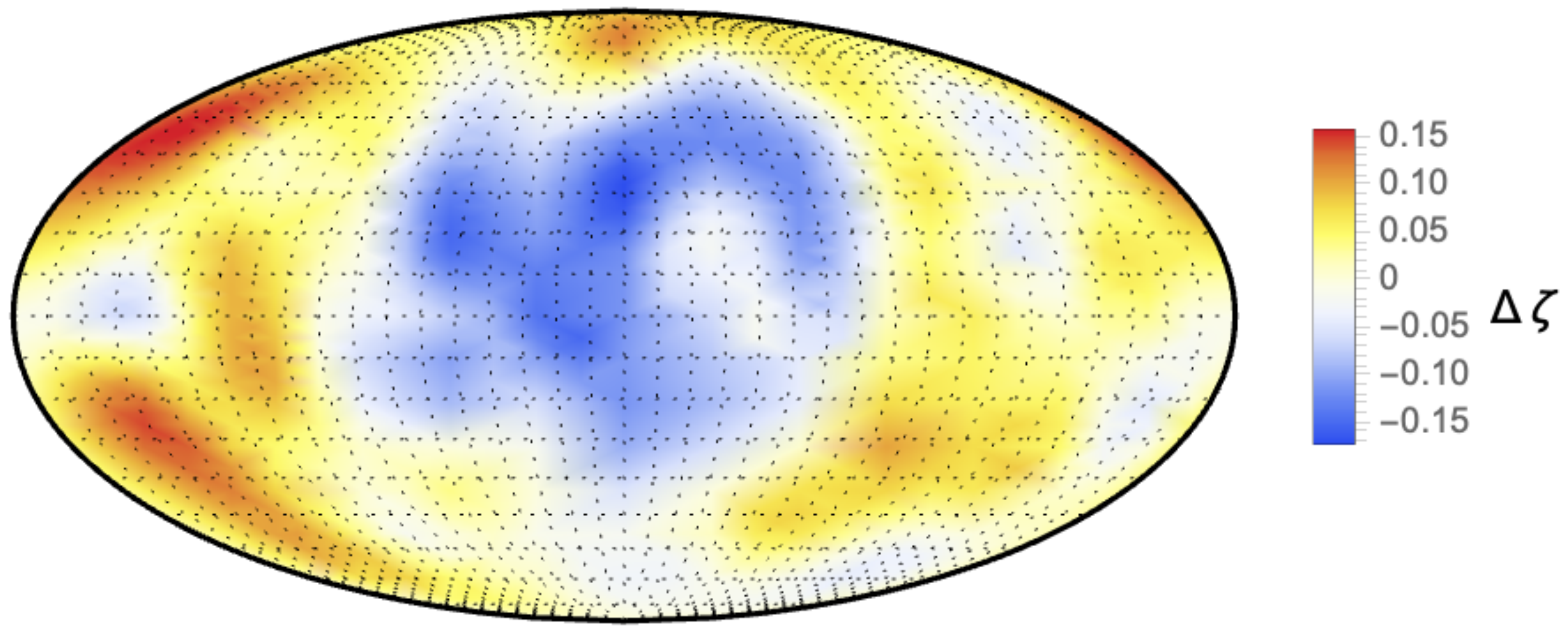
STOLAS

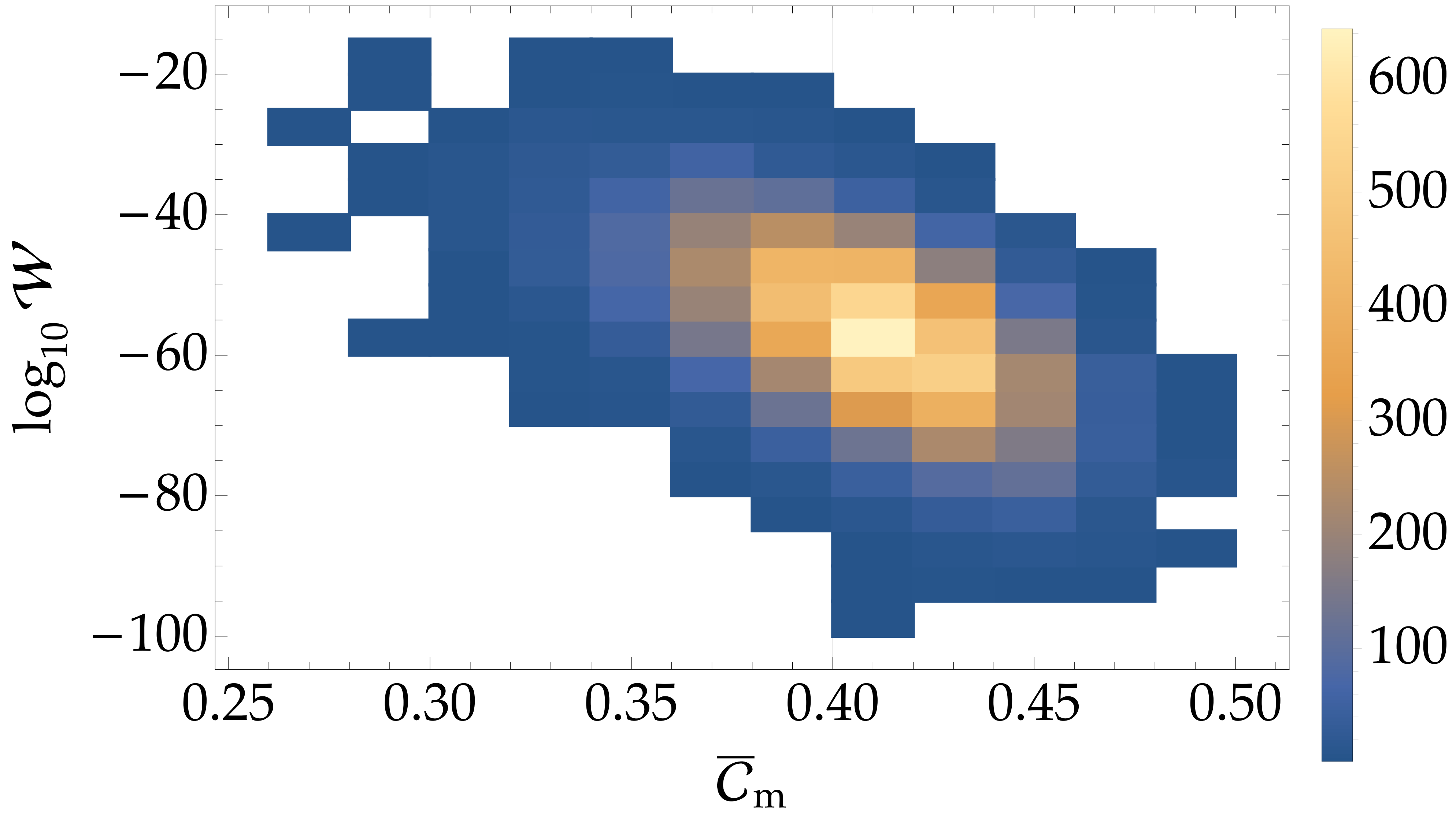
Mizuguchi, Murata, YT '24

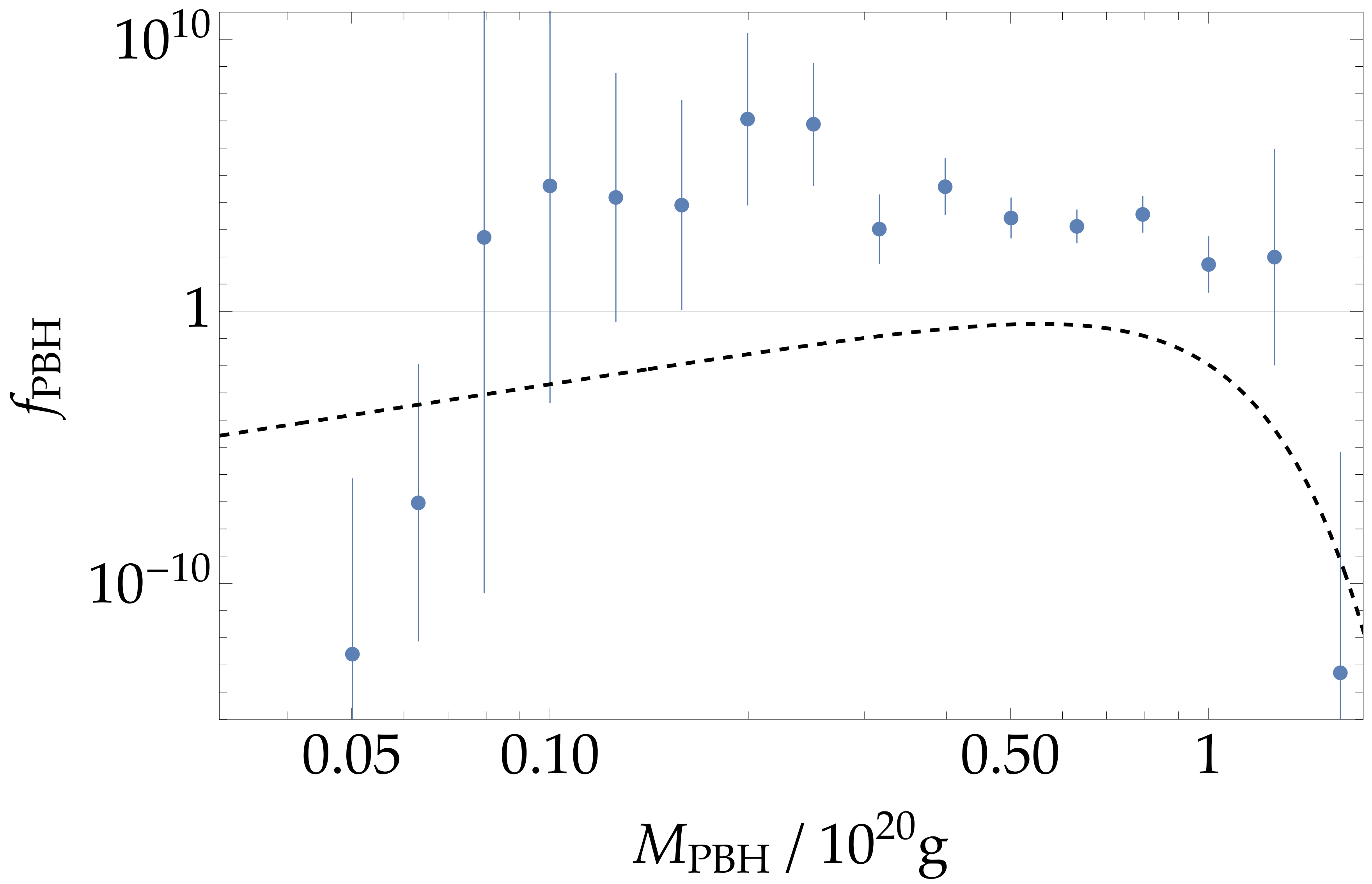
Ex. 2: Starobinsky's linear

Starobinsky '92









Field Redefinition?

Kristiano & Yokoyama '22

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$ & comoving gauge $(\delta\phi = 0)$

$$S^{(3)}[\zeta] = \int \mathrm{d}^4x \,(\text{ tedious terms.})$$

Arroja & Tanaka '11

$$= \int \mathrm{d}^4x \left[\mathcal{O}(\epsilon^2) + \frac{a^3\epsilon}{2}\dot{\eta}\zeta^2\dot{\zeta} + 2f(\zeta)\frac{\delta L}{\delta\zeta}\bigg|_1 + \frac{\mathrm{d}}{\mathrm{d}t}\left((\zeta^3\text{-terms}) - \frac{a^3\epsilon}{2}\eta\zeta^2\dot{\zeta} - \frac{a^3\epsilon}{H}\zeta\dot{\zeta}^2 + \mathcal{O}(\epsilon^2)\right)\right]$$

$$\frac{\delta L}{\delta\zeta}\bigg|_1 = 0 : \text{tree EoM}, \quad f(\zeta) = \frac{\eta}{4}\zeta^2 + \frac{1}{H}\zeta\dot{\zeta} + \cdots$$

field redef. : $\zeta = \tilde{\zeta} + f(\tilde{\zeta})$ Maldacena '02

$$S^{(3)}[\tilde{\zeta}] = \int \mathrm{d}^4x\left[\mathcal{O}(\epsilon^2) + \boxed{\frac{a^3\epsilon}{2}\dot{\eta}\tilde{\zeta}^2\dot{\tilde{\zeta}}}\right]$$

unique relevant vertex $\rightarrow H_{\rm int}$

Field Redefinition?

Kristiano & Yokoyama '22

Weinberg '05

$$\langle \hat{\tilde{\zeta}}_{\mathbf{k}_\mathrm{L}}(\tau_\mathrm{e})\hat{\tilde{\zeta}}_{\mathbf{k}'_\mathrm{L}}(\tau_\mathrm{e})\rangle_{(1)} = \left\langle \left[\bar{T}\exp\left(i\int_{-\infty}^{\tau_\mathrm{e}}\hat{H}_\mathrm{int}(\tau)\mathrm{d}\tau\right)\right]\hat{\tilde{\zeta}}_{\mathrm{I},\mathbf{k}_\mathrm{L}}(\tau_\mathrm{e})\hat{\tilde{\zeta}}_{\mathrm{I},\mathbf{k}'_\mathrm{L}}(\tau_\mathrm{e})\left[T\exp\left(-i\int_{-\infty}^{\tau_\mathrm{e}}\hat{H}_\mathrm{int}(\tau)\mathrm{d}\tau\right)\right]\right\rangle_{(1)}$$

$$= i^2\int^{\tau_\mathrm{e}}\mathrm{d}\tau_1\int^{\tau_1}\mathrm{d}\tau_2\langle[\hat{H}_\mathrm{int}(\tau_1),[\hat{H}_\mathrm{int}(\tau_2),\hat{\tilde{\zeta}}_{\mathrm{I},\mathbf{k}_\mathrm{L}}(\tau_\mathrm{e})\hat{\tilde{\zeta}}_{\mathrm{I},\mathbf{k}'_\mathrm{L}}(\tau_\mathrm{e})]]\rangle$$

$$\sim \eta^2 P_\zeta(k_\mathrm{L})\int_{k_\mathrm{s}}^{k_\mathrm{e}}\mathrm{d}\ln k\,\mathscr{P}_\zeta(k) \sim P_\zeta(k_\mathrm{L})$$

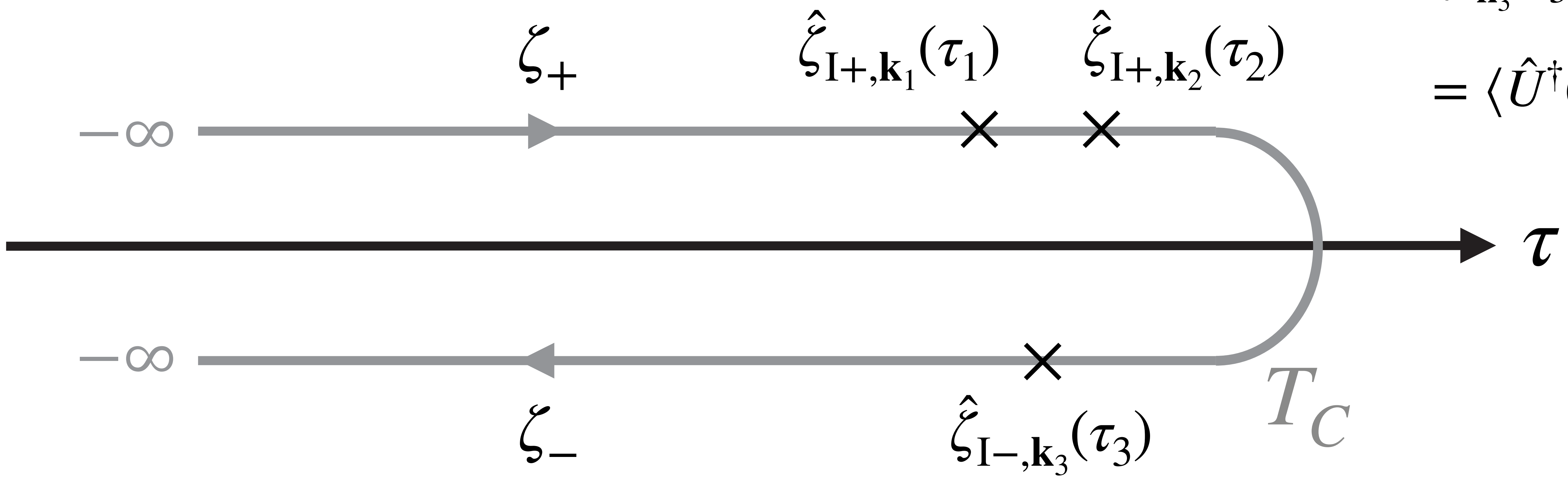
PTB approach breaks down even on CMB scale?

Rmk:

- $\tilde{\zeta}(\tau) \to \zeta(\tau)$ @ SR2
- $S^{(4)}[\tilde{\zeta}]$?
- non-lin. field-redef. works @ one-loop?
- $\cdots$

Closed Time Path

Schwinger '65, Keldysh '65



Example

$$\langle \hat\zeta_{\mathbf{k}_3}(\tau_3)\hat\zeta_{\mathbf{k}_2}(\tau_2)\hat\zeta_{\mathbf{k}_1}(\tau_1)\rangle = \left\langle T_C\, \hat\zeta_{\mathrm{I}-,\mathbf{k}_3}(\tau_3)\hat\zeta_{\mathrm{I}+,\mathbf{k}_2}(\tau_2)\hat\zeta_{\mathrm{I}+,\mathbf{k}_1}(\tau_1)\mathrm{e}^{-i\int \hat H_{\mathrm{int}}\mathrm{d}\tau}\right\rangle$$

$$= \langle \hat U^\dagger(\tau_3,-\infty)\hat\zeta_{\mathrm{I},\mathbf{k}_3}(\tau_3)\hat U^\dagger(\tau_2,\tau_3)\hat\zeta_{\mathrm{I},\mathbf{k}_2}(\tau_2)\hat U(\tau_2,\tau_1)\hat\zeta_{\mathrm{I},\mathbf{k}_1}(\tau_1)\hat U(\tau_1,-\infty)\rangle$$

Propagator

$$G_{ab}(x,x') = \langle T_C\, \hat\zeta_{\mathrm{I}a}(x)\hat\zeta_{\mathrm{I}b}(x')\rangle$$

$$= \begin{cases} \Theta(\tau-\tau')\langle\hat\zeta_{\mathrm{I}}(x)\hat\zeta_{\mathrm{I}}(x')\rangle + \Theta(\tau'-\tau)\langle\hat\zeta_{\mathrm{I}}(x')\hat\zeta_{\mathrm{I}}(x)\rangle, & (a,b)=(+,+) \\ \langle\hat\zeta_{\mathrm{I}}(x')\hat\zeta_{\mathrm{I}}(x)\rangle, & (a,b)=(+,-) \\ \langle\hat\zeta_{\mathrm{I}}(x)\hat\zeta_{\mathrm{I}}(x')\rangle, & (a,b)=(-,+) \\ \Theta(\tau'-\tau)\langle\hat\zeta_{\mathrm{I}}(x)\hat\zeta_{\mathrm{I}}(x')\rangle + \Theta(\tau-\tau')\langle\hat\zeta_{\mathrm{I}}(x')\hat\zeta_{\mathrm{I}}(x)\rangle, & (a,b)=(-,-) \end{cases}$$

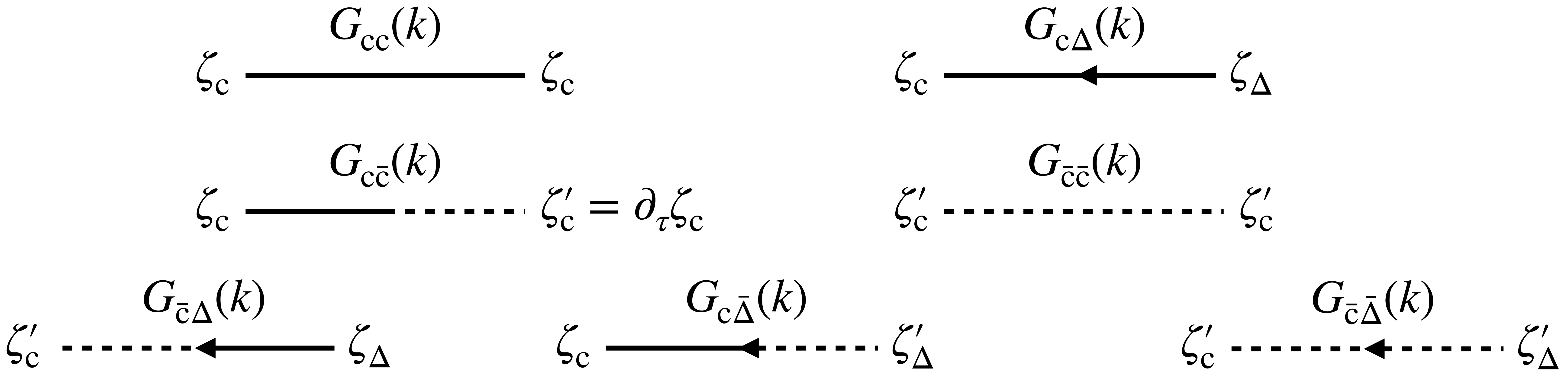
Closed Time Path

Schwinger '65, Keldysh '65

Shcwinger—Keldysh basis $\quad \zeta_{\rm c} = \dfrac{\zeta_+ + \zeta_-}{2}, \quad \zeta_\Delta = \zeta_+ - \zeta_-$

$$G_{\alpha\beta}(x,x') = \begin{cases} \frac{1}{2}\langle\{\hat{\zeta}_{\rm I}(x), \hat{\zeta}_{\rm I}(x')\}\rangle, & (\alpha,\beta) = ({\rm c},{\rm c}) \\ \Theta(\tau-\tau')\langle[\hat{\zeta}_{\rm I}(x), \hat{\zeta}_{\rm I}(x')]\rangle, & (\alpha,\beta) = ({\rm c},\Delta) \\ \Theta(\tau'-\tau)\langle[\hat{\zeta}_{\rm I}(x'), \hat{\zeta}_{\rm I}(x)]\rangle, & (\alpha,\beta) = (\Delta,{\rm c}) \\ 0, & (\alpha,\beta) = (\Delta,\Delta) \end{cases}$$

$$G_{\alpha\beta}(\tau,\tau';k) = \begin{cases} \Re\zeta_k(\tau)\zeta_k^*(\tau'), & (\alpha,\beta) = ({\rm c},{\rm c}) \\ 2i\Theta(\tau-\tau')\Im\zeta_k(\tau)\zeta_k^*(\tau'), & (\alpha,\beta) = ({\rm c},\Delta) \\ 2i\Theta(\tau'-\tau)\Im\zeta_k(\tau')\zeta_k^*(\tau), & (\alpha,\beta) = (\Delta,{\rm c}) \\ 0, & (\alpha,\beta) = (\Delta,\Delta) \end{cases}$$



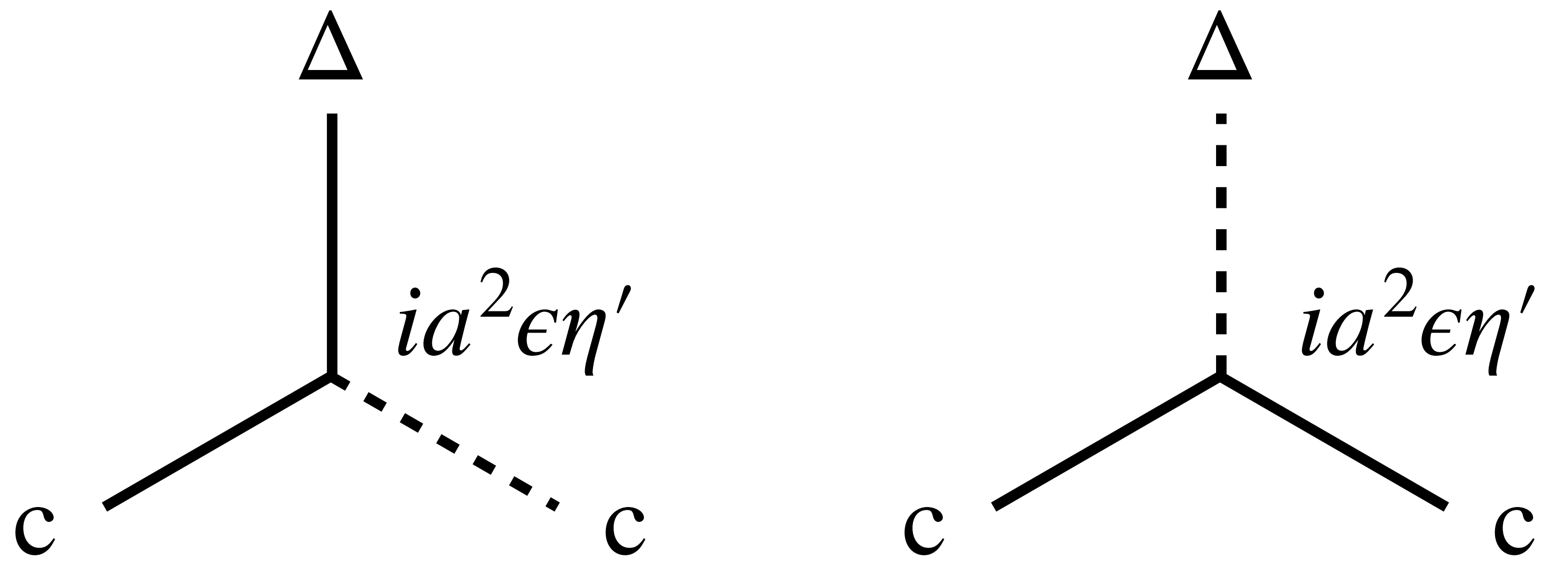
Boundary Vertices

Motohashi & YT '23

$$S^{(3)} \supset \int \mathrm{d}\tau \mathrm{d}^3\mathbf{x} \left[\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' - \frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{a^2\epsilon}{2}\eta\zeta^2\zeta' + \frac{a\epsilon}{H}\zeta\zeta'^2 \right) \right]$$

$$\mathscr{L}^{(3)}_{\rm bulk} = \frac{a^2\epsilon}{2}\eta'(\zeta_+^2\zeta_+' - \zeta_-^2\zeta_-')$$

$$= a^2\epsilon\eta' \left(\zeta_{\rm c}\zeta_\Delta\zeta_{\rm c}' + \frac{1}{2}\zeta_{\rm c}^2\zeta_\Delta' \right) + \mathscr{O}(\zeta_\Delta^3)$$



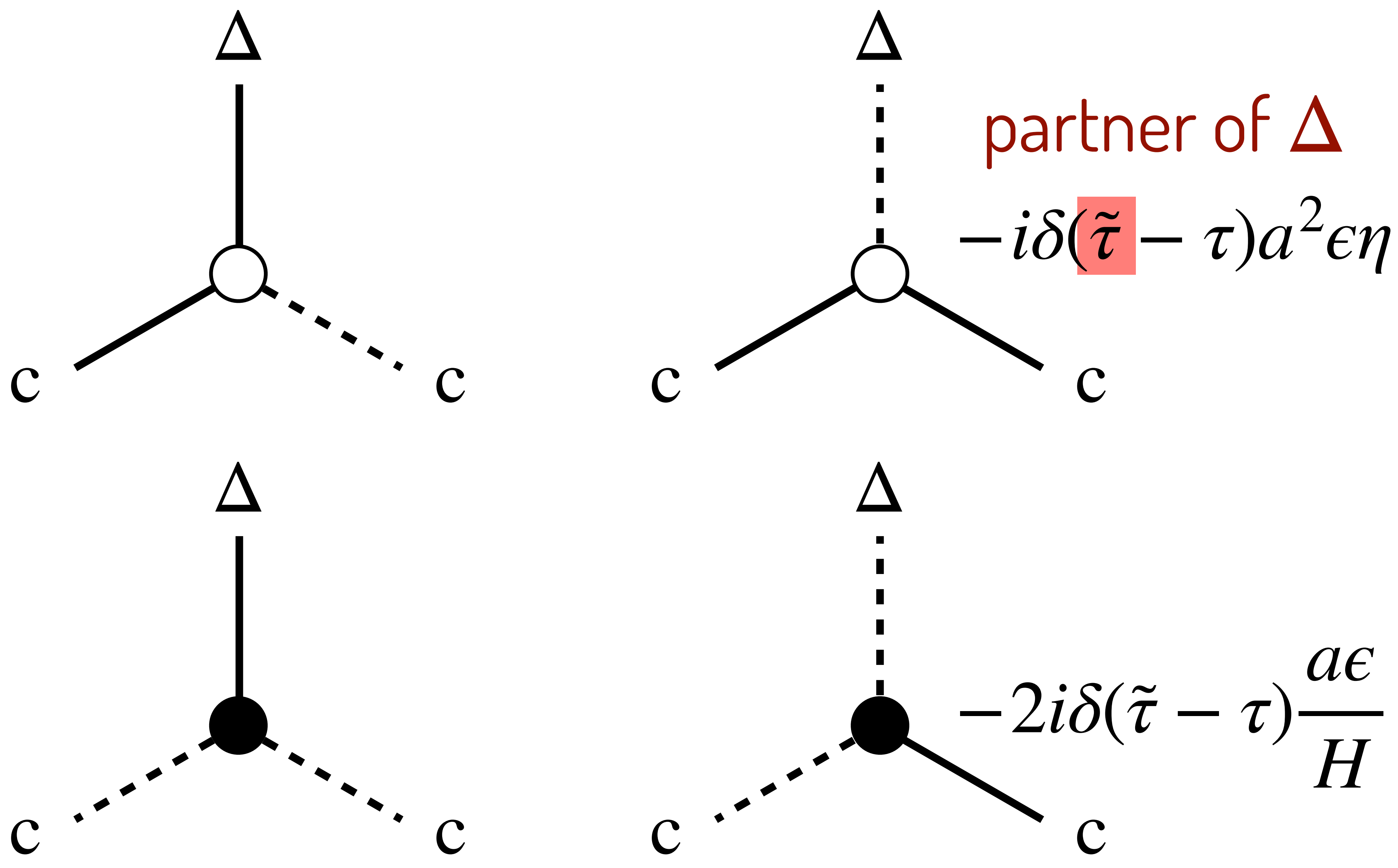
Boundary Vertices $\qquad \mathscr{L}^{(3)}_{\rm B} = \frac{\mathrm{d}}{\mathrm{d}\tau}\mathscr{B}$

$$\left\langle T_C\, \hat{\mathscr{O}}(\tau_1, \tau_2, \cdots)\frac{\mathrm{d}}{\mathrm{d}\tau}\hat{\mathscr{B}}(\tau) \right\rangle$$

$$= \cancel{\frac{\mathrm{d}}{\mathrm{d}\tau}\langle T_C\, \hat{\mathscr{O}}(\tau_1, \tau_2, \cdots)\hat{\mathscr{B}}(\tau)\rangle}$$

$$+\delta(\tau_1 - \tau)G_{{\rm c}\Delta}(\tau_1, \tau)G_{\rm cc}(\tau_2, \tau)\cdots + \cdots$$

$\frac{\mathrm{d}}{\mathrm{d}\tau}\Theta(\tau_1 - \tau)$ in $G_{{\rm c}\Delta}(\tau_1, \tau)$

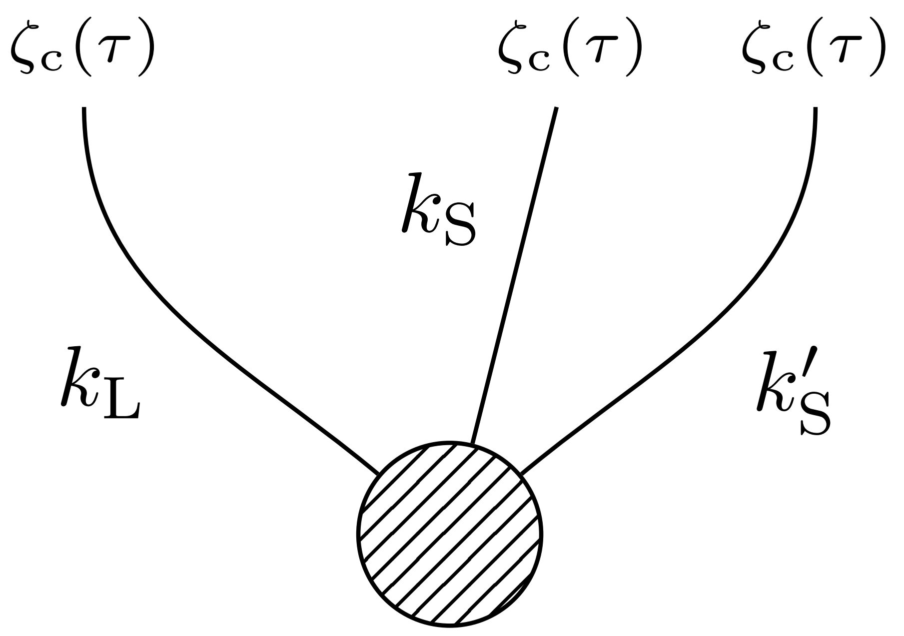


Bispectrum

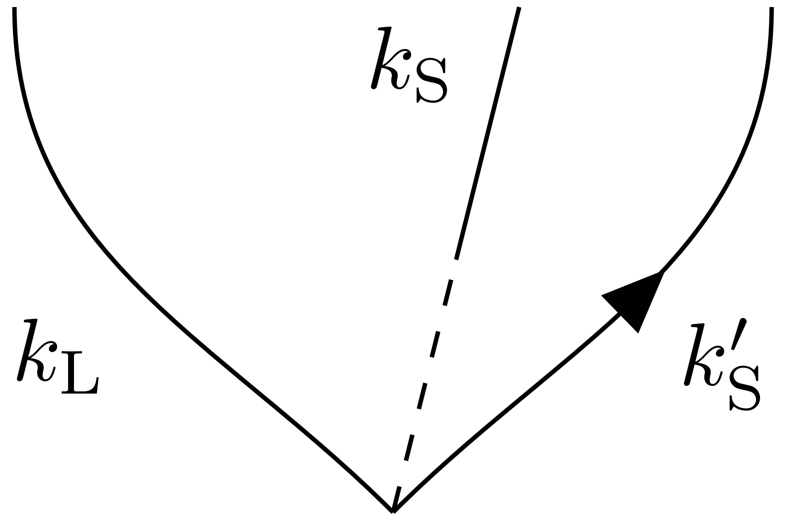
Motohashi & YT '23

during SR2 $\tau \gg \tau_{\rm e}$

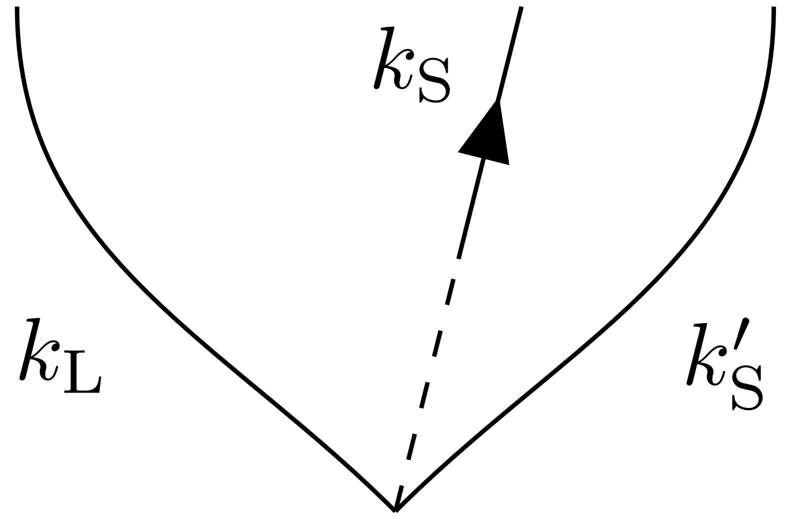
$B_{\zeta\zeta\zeta}(k_{\rm L}, k_{\rm S}, k_{\rm S}'; \tau) =$



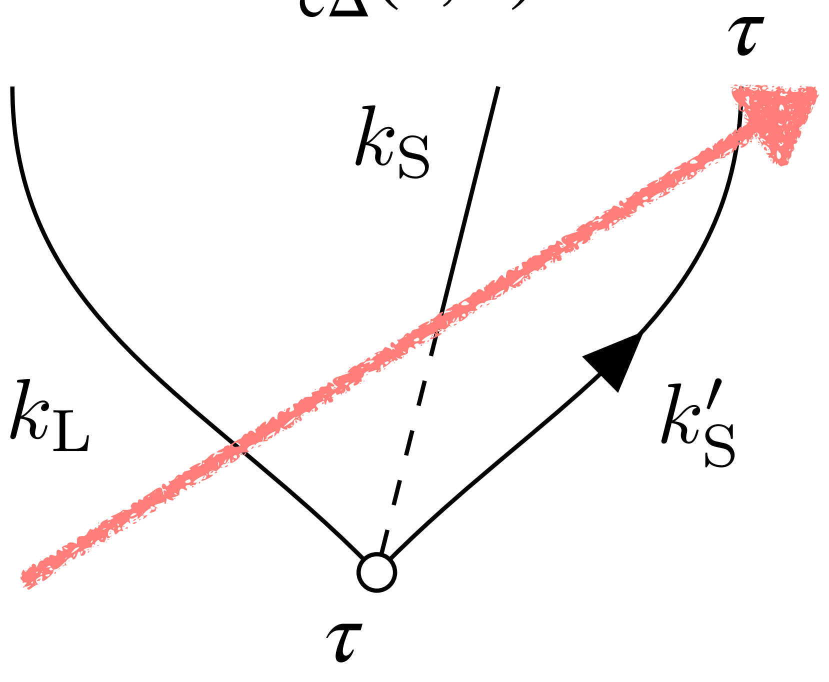
$=$ $\sim P_\zeta(k_{\rm L}) \propto \frac{1}{k_{\rm L}^3}$



$+$



$+$ $\propto G_{{\rm c}\Delta}(\tau, \tau) = 0$



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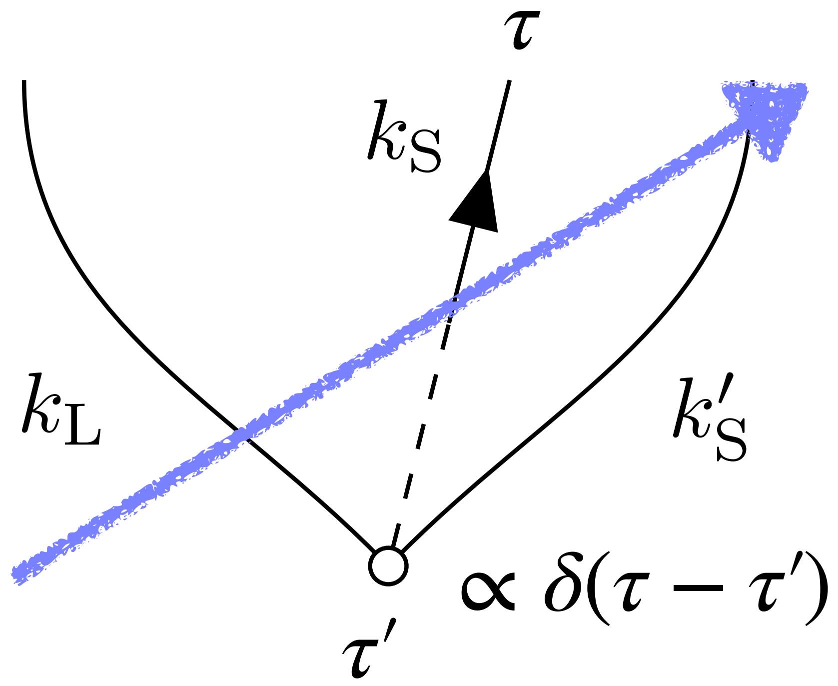

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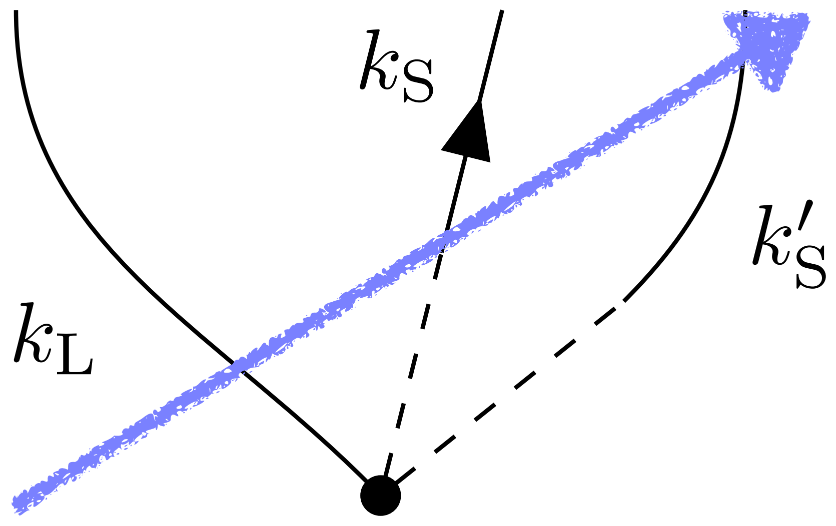

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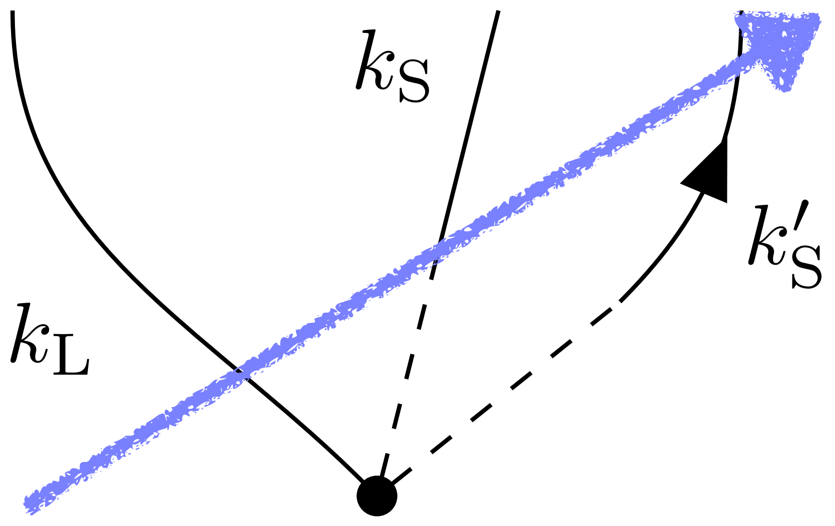
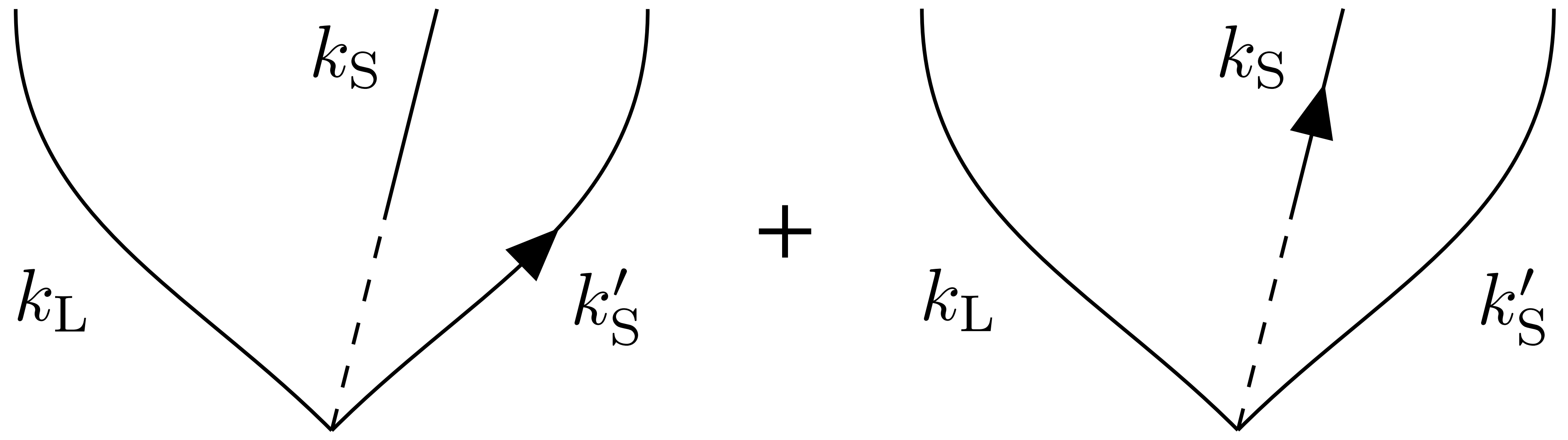
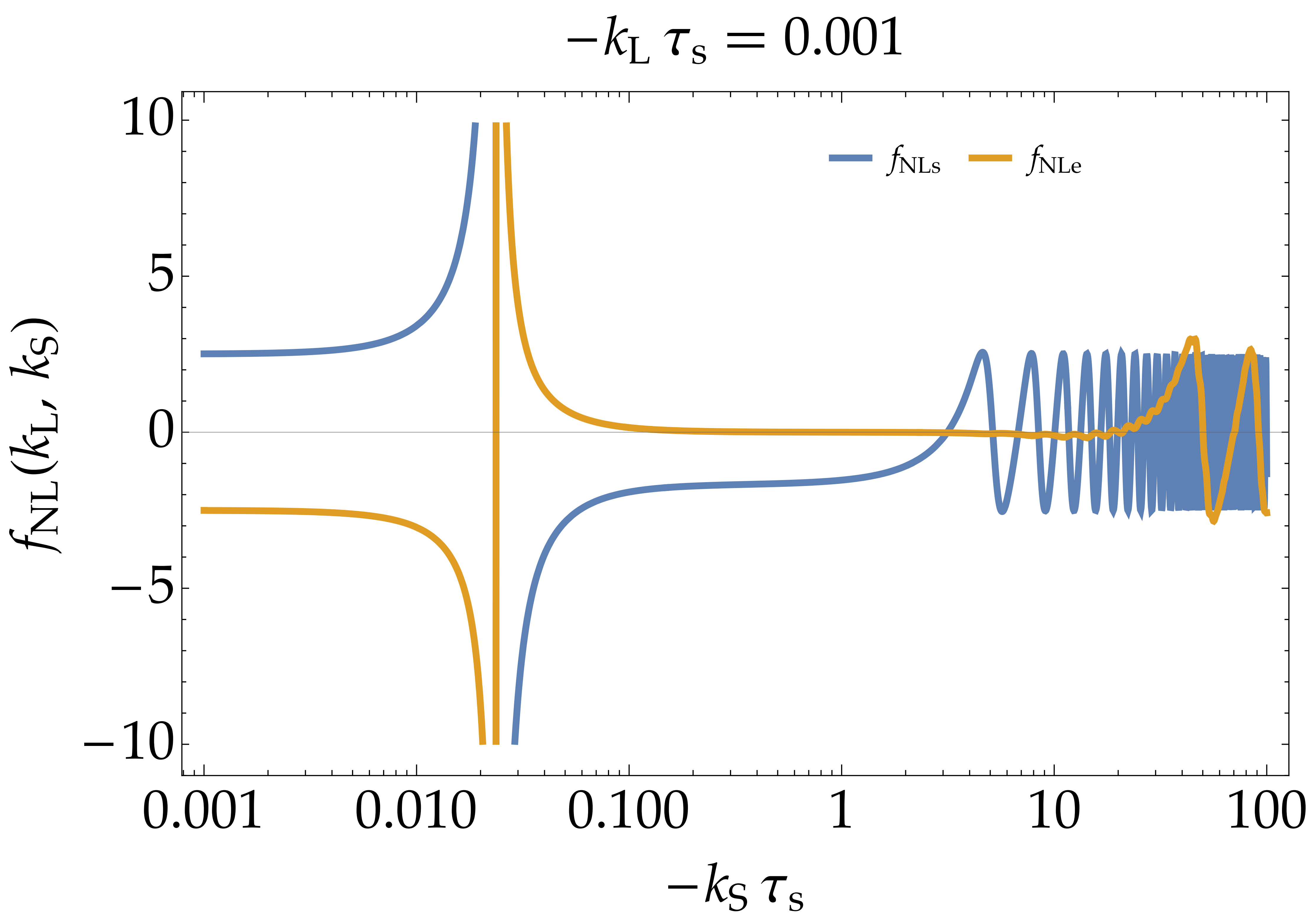


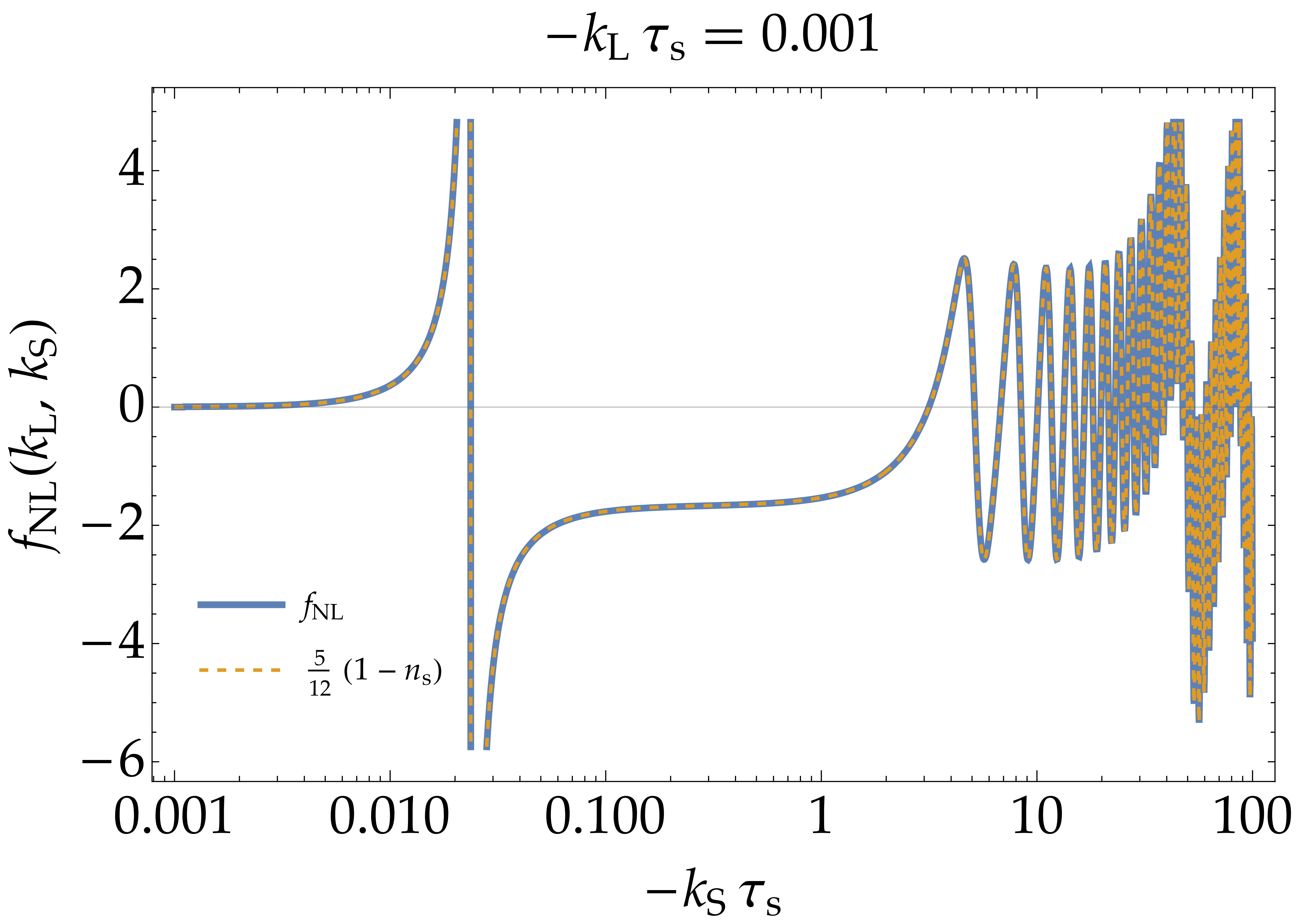
Bispectrum

Motohashi & YT '23

during SR2 $\tau \gg \tau_e$



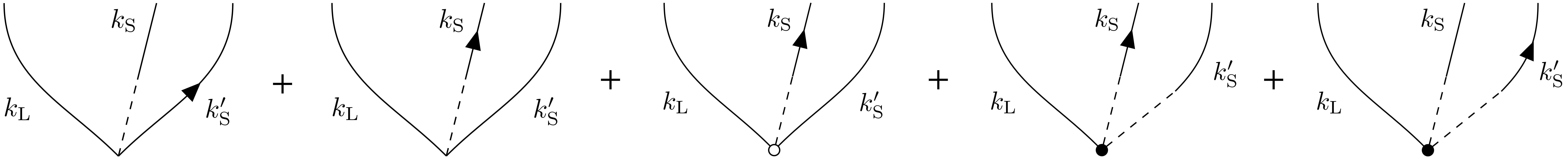


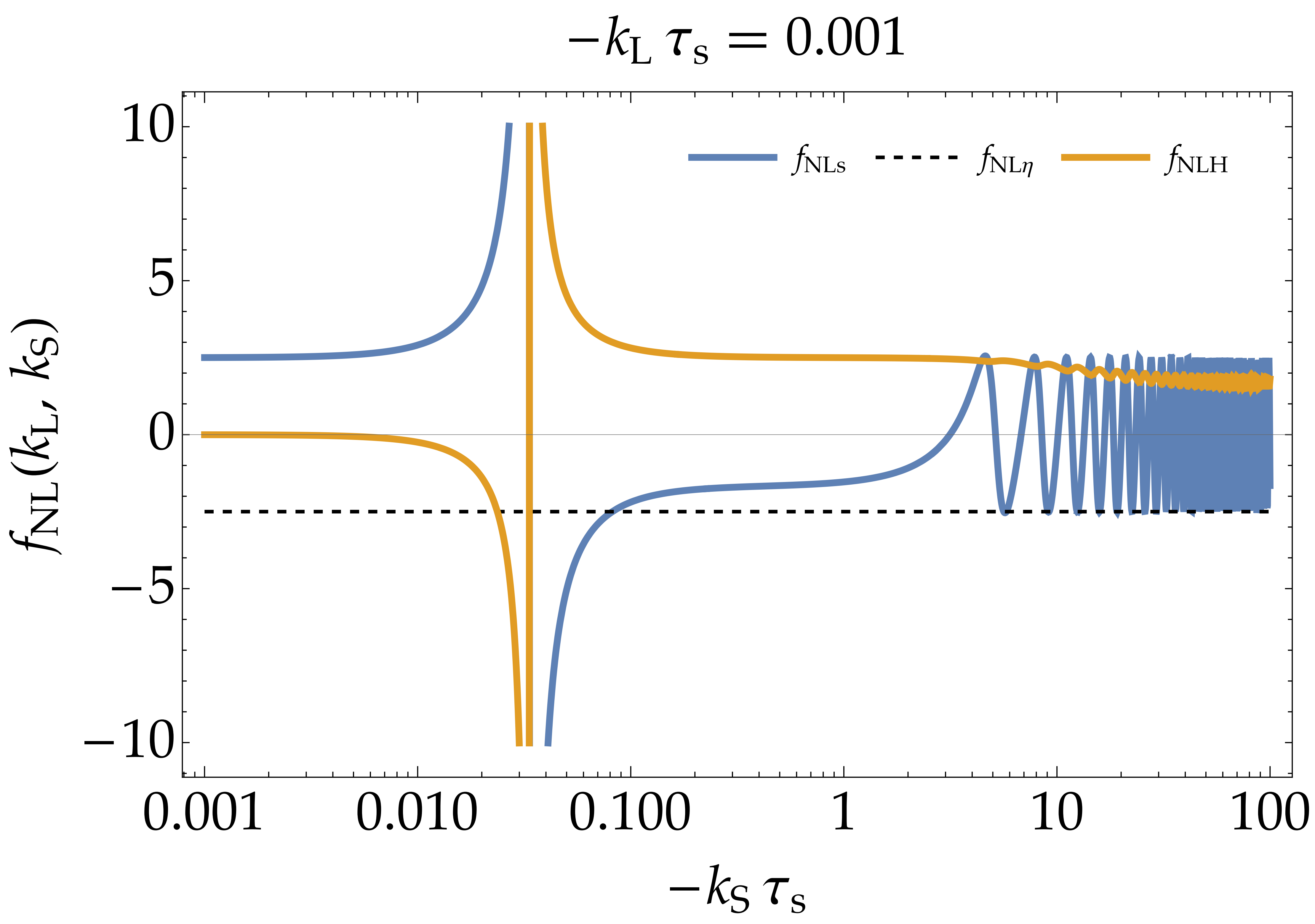


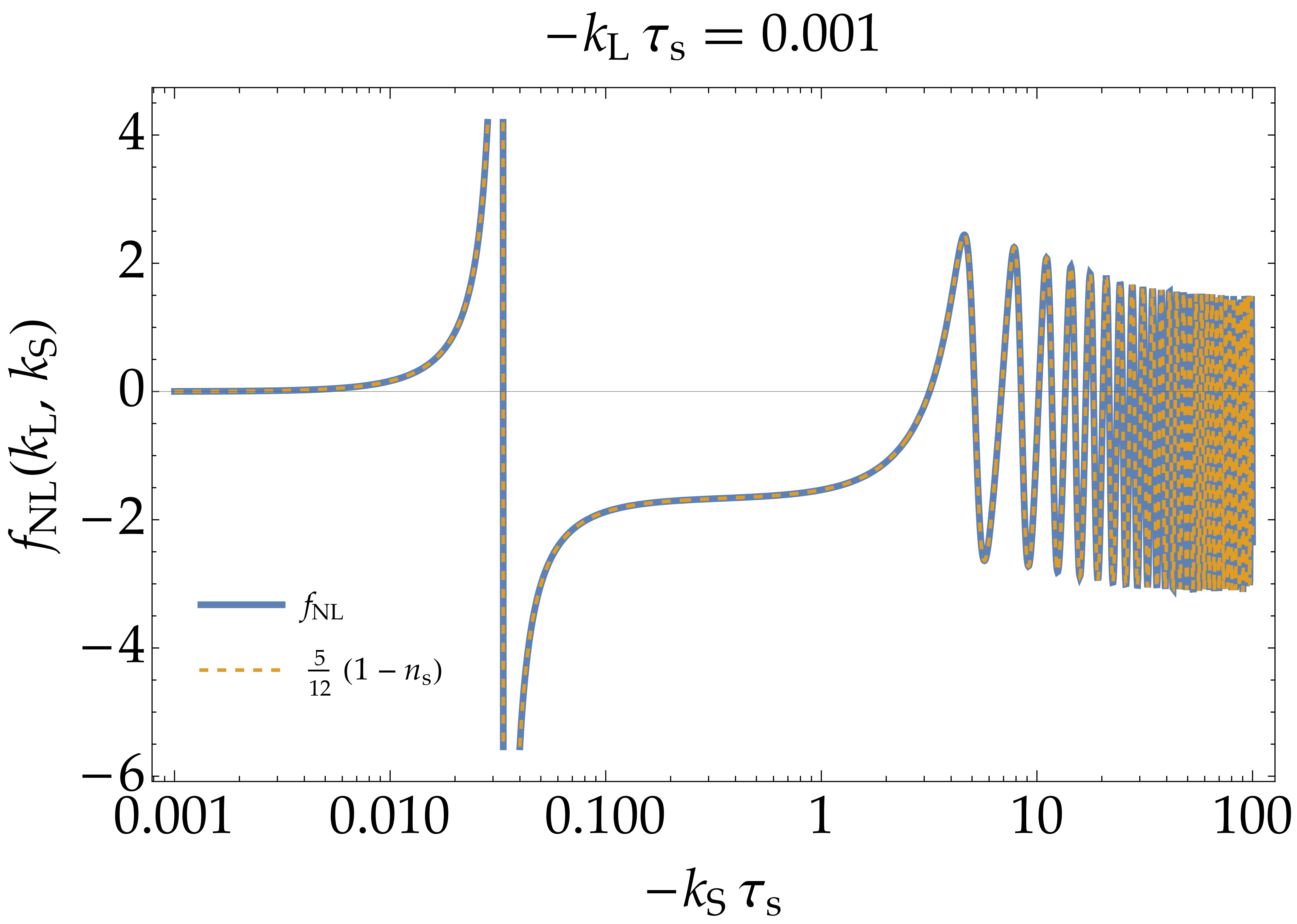
Bispectrum

Motohashi & YT '23

during USR $\tau_s < \tau < \tau_e$



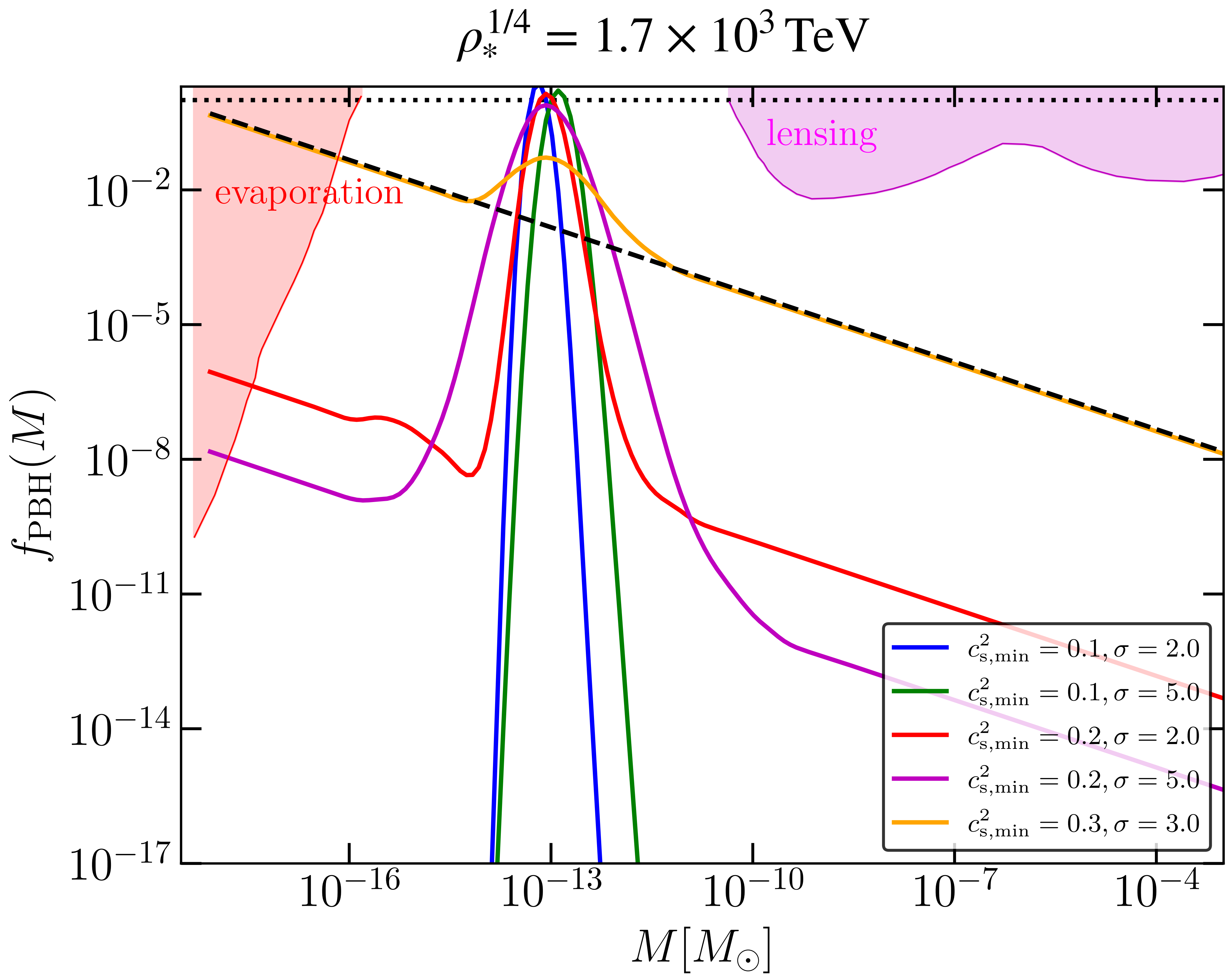


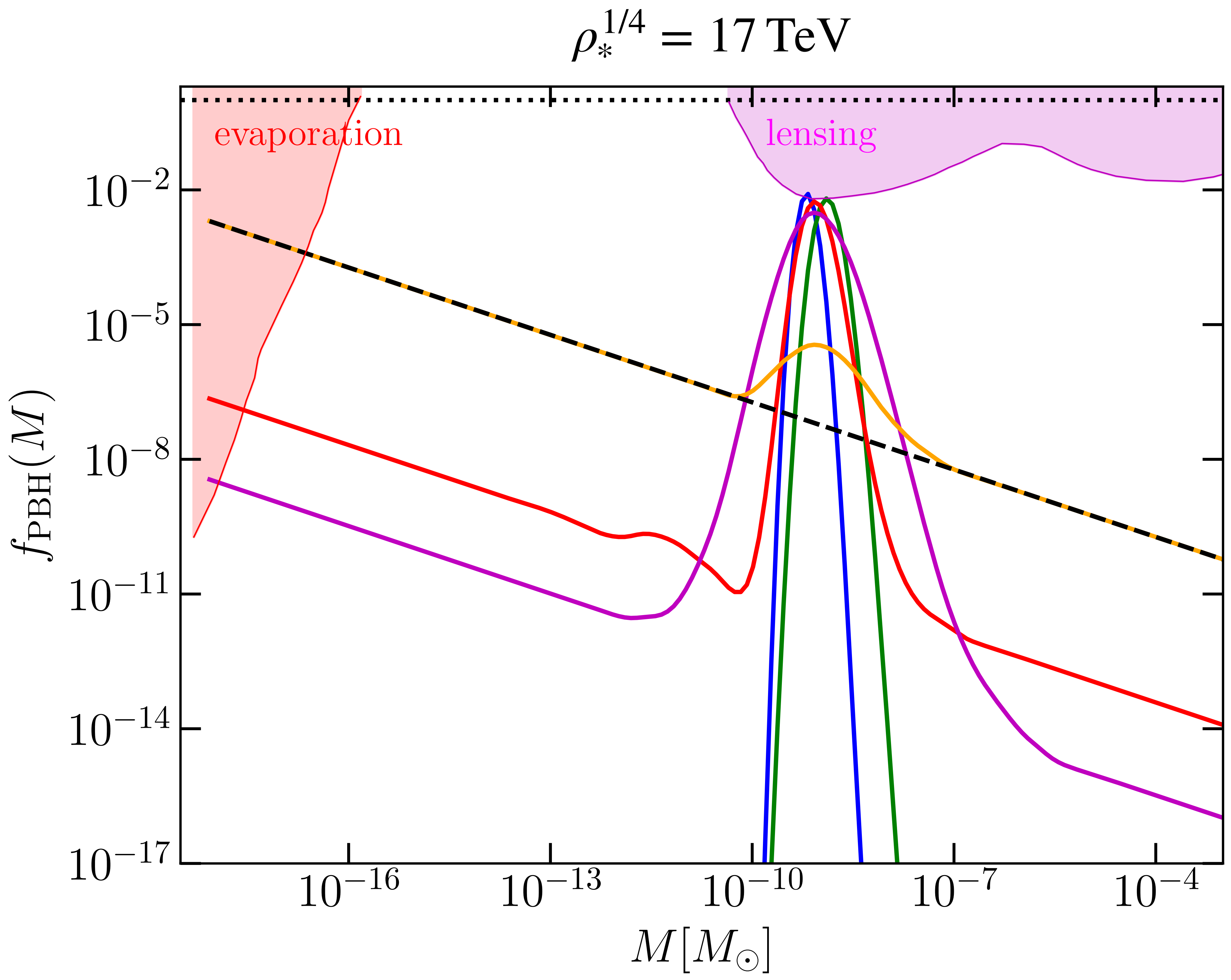


BSM crossover

Escrivà, YT, Yoo '24

$c^2_{s,\mathrm{min}}$	σ	$\mathcal{A}^{\mathbf{A}}/10^{-3}$	$\mathcal{A}^{\mathbf{B}}/10^{-3}$
0.1	2.0	1.972	1.920
0.1	5.0	1.539	1.504
0.2	2.0	3.393	3.320
0.2	5.0	3.156	3.086
0.3	3.0	4.268	3.840





BSM crossover

Inui, Escrivà, YT, Yoo '24