# 202unsold-bootstrap

B-MAT-400

# Previously: random experiments

- Random experiment: procedure with a set of outcomes
- $\Omega$: set of outcomes (sample space)
- $\omega$: a specific outcome
- $P(\omega)$: probability of the outcome
- Event: subset of sample space
- $P(A) = \sum_{\omega \in A} P(\omega)$ (outcomes are mutually exclusives)

# Discrete random variables

- Random variable: values that depends on the outcome of an experiment
  - Example: sum of 2 dice rolls
- Defined as function $X: \Omega \to E$
  - In the previous example, $E = \{2,3,4,5,6,7,8,9,10,11,12\}$
- If $E$ is countable, $X$ is a discrete random variable
  - The sum of 2 dice rolls is a discrete random variable

# Probability distribution (law)

- Function that maps every value of a random variable to its probability

$$p(x) = P(X = x), x \in E$$

- Example: sum of 2 dice roll$(X = x)$

$$p(2) = p(12) = \frac{1}{36} \qquad p(3) = p(11) = \frac{2}{36}$$

$$p(4) = p(10) = \frac{3}{36} \qquad p(5) = p(9) = \frac{4}{36}$$

$$p(6) = p(8) = \frac{5}{36} \qquad p(7) = \frac{6}{36}$$

- Probability of rolling less than 5:

$$P(X \leq 5) = \sum_{x \leq 5} p(x) = p(2) + p(3) + p(4) + p(5) = \frac{10}{36}$$

# Joint probability distribution (joint law)

- Probability distribution for multiple random variables
$$p(x, y) = P\big((X = x) \cap (Y = y)\big), x \in E, y \in F$$
- Example: sum ($X$) and product ($Y$) of 2 dice rolls:

$p(5,6) = \frac{2}{36}$ $\quad$ $p(6,9) = \frac{1}{36}$ $\quad$ $p(7,8) = 0$

- Tip: For two discrete random variables, the distribution can be nicely presented in a two dimensional array

# Marginal distribution (marginal law)

- Deducing the probability distribution of a random variable from a joint distribution

$$p_X(x) = P(X = x) = \sum_{y \in F} P\big((X = x) \cap (Y = y)\big)$$
$$= \sum_{y \in F} p(x, y)$$

- Example: rolling 2 dice and getting 6 as a product

$$p_Y(6) = \sum_{x=2}^{12} p(x, 6) = p(5,6) + p(7,6) = \frac{4}{36}$$

# Expected value

- The expected value of random variable is the weighted average of all possible values, with the probability as the weights

$$E[X] = \sum_{x \in E} x.p(x)$$

- Example: expected value of the sum of 2 dice rolls:

$$E[X] = 2\frac{1}{36} + 3\frac{2}{36} + 4\frac{3}{36} + \cdots + 11\frac{2}{36} + 12\frac{1}{36} = 7$$

# Variance

- Measures how far from the expected value a random variable can deviate
- It is the expected value of the squared deviation of $X$ from its expected value:

$$
\begin{aligned}
Var(X) &= E[(X - E[X])^2] \\
&= \sum_{x \in E} (x - E[X])^2 . p(x) \\
&= \left( \sum_{x \in E} x^2 . p(x) \right) - E[X]^2
\end{aligned}
$$

# 202unsold

- Compute the probability distribution of three random variables, their expected values and variance
  - $X$: price of a jacket (\$10, \$20, \$30, \$40, \$50)
  - $Y$: price of trousers (\$10, \$20, \$30, \$40, \$50)
  - $Z$: price of a suit (jacket and trousers)
- Given only the following joint probability distribution

$$p(x, y) = \frac{(a - x)(b - y)}{(5a - 150)(5b - 150)}$$

# Exercises

- Implement a function that takes a, b, x, y as parameters and returns the value of $p(x, y)$
- Implement a function that takes a, b, x as parameters and returns the value of $p_X(x)$
- Implement a function that takes a, b, z as parameters and returns the value of $P(Z = z)$
  - Tip: $Z = X + Y$
- Implement a function that takes an array of values and probabilities and returns the expected value and the variance