# Probability

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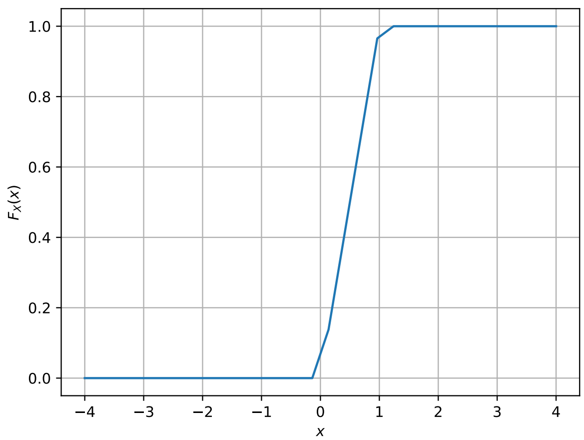


Fig. 1.2. The CDF of $U$

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## 1 Uniform Random Numbers

Let $U$ be a uniform random variable between 0 and 1.

1.1 Generate $10^6$ samples of $U$ using a C program and save into a file called uni.dat .
**Solution:** Download the following files and execute the C program.

```
codes/exrand.c ...... done
codes/coeffs.h ...... done
```

1.2 Load the uni.dat file into python and plot the empirical CDF of $U$ using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \tag{1.2.1}$$

**Solution:** The following code plots Fig. 1.2

```
codes/cdf_plot.py ...... done
```

1.3 Find a theoretical expression for $F_U(x)$.
**Solution:** Probability density function:

$$\begin{cases} \frac{1}{1-0} & \text{for } x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

Cumulative density function:

$$F_U(x) = f_U(x) = \int_{-\infty}^{\infty} f_U(x)(dx) \tag{1.3.1}$$
$$= \int_0^x f_U(x)(dx) \tag{1.3.2}$$
$$= \int_0^x \frac{1}{1-0}(dx) \tag{1.3.3}$$
$$= \frac{1}{1-0}[x]_0^x \tag{1.3.4}$$
$$= \frac{x-0}{1} \tag{1.3.5}$$

$$\begin{cases} 0 & \text{for } x < 0 \\ \frac{x-0}{1} & \text{for } x \in [0,1) \\ 1 & \text{for } x \geq 1 \end{cases}$$

1.4 The mean of $U$ is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{1.4.1}$$

**Solution:** mean of uniform= 0.500007
and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{1.4.2}$$

**Solution:** variance of uniform= 0.083301
Write a C program to find the mean and variance of $U$.
**Solution:**

```
codes/myexrand.c
codes/mycoeffs.h
```

1.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{1.5.1}$$

**Solution:** $E[U - E[U]]^2$
$= E[U^2 - 2UE[U] + E[U]^2]$
$= E[U^2] - E[U]^2$