Assignment 1 - Probability

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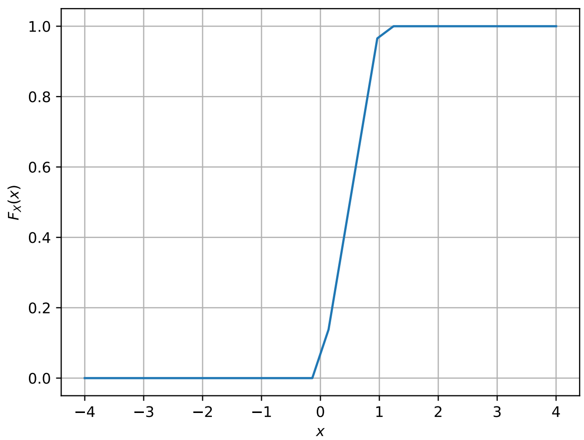

Fig. 1.2. The CDF of U

Contents

1 Uniform Random Numbers

Let U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Download the following files and execute the C program.

```
Code/myexrand.c
Code/mycoeffs.h
```

1.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \tag{1.2.1}$$

Solution: The following code plots Fig. 1.2

```
Code/cdf_plot.py
```

1.3 Find a theoretical expression for $F_U(x)$.
Solution: Probability density function:

$$\begin{cases} \frac{1}{1-0} & \text{for } x \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$

Cumulative density function:

$$\begin{aligned} F_U(x) &= \int_{-\infty}^{\infty} f_U(x)(dx) \\ &= \int_{0}^{x} f_U(x)(dx) \\ &= \int_{0}^{x} \frac{1}{1-0}(dx) \\ &= \frac{1}{1-0}[x]_0^x \\ &= \frac{x-0}{1} \end{aligned}$$

$$\begin{cases} 0 & \text{for } x < 0 \\ \frac{x-0}{1} & \text{for } x \in [0,1) \\ 1 & \text{for } x \geq 1 \end{cases}$$

1.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{1.4.1}$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{1.4.2}$$

Write a C program to find the mean and variance of U.
Solution:

```
Code/myexrand.c
Code/mycoeffs.h
```

- Mean of uniform distribution = 0.500007.
- Variance of uniform distribution = 0.083301.

1.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{1.5.1}$$

Solution:

$$= E[U - E[U]]^2$$
$$= E[U^2 - 2UE[U] + E[U]^2]$$
$$= E[U^2] - E[U]^2$$

$$Given, E[U]^k = \int_{-\infty}^{\infty} x^k dF_U(x)$$

$For\ E[U],\ k = 1\ and\ limits\ are\ from\ 0 - 1.$

$$E[U] = \int_0^1 x dF_U(x)$$
$$= \int_0^1 x dF_U(x)$$

$dF_U(x)$ for uniformly distributed function is $\frac{1}{b-a}$ where b=1 and a=0.

$$= \frac{1}{1-0} \int_0^1 x.dx$$
$$= \frac{1}{1-0} * [\frac{x^2}{2}]_0^1$$
$$= \frac{1}{2} = 0.5.$$
$$E[U^2] = \frac{1}{1-0} \int_0^1 x^2.dx$$
$$= \frac{1}{1-0} * [\frac{x^3}{3}]_0^1$$
$$= \frac{1}{3} = 0.3333.$$
$$= E[U^2] - E[U]^2$$
$$= 0.3333 - 0.5^2$$
$$= 0.08333.$$