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Investigation of positioning algorithm and method for increasing the linear measurement range for four-quadrant detector

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ABSTRACT

The paper studied the relationship between the light spot position and the output voltage in the case of laser spot which is in the four-quadrant (4Q) detector whose shape is circle or elliptic, energy distribution uniform or Gaussian. The study is based on add–subtract algorithm, diagonal algorithm and difference over sum ($\Delta/\sum$) algorithm. By using methods of polynomial fitting it can get a simple polynomial expression to enhance measuring speed and the simulation error is given. A method of using Binary Optics Elements (BOE) to shape the laser beam in the 4Q detector to square light spot with uniform intensity distribution is proposed to expand linear measurement range. The simulation result is verified for a circular spot with Gaussian energy distribution.

1. Introduction

4Q detector is a common used position-sensing detector. Due to its simple signal processing, lower inherent noise level, high sensitivity and high response speed advantages, 4Q detector has been widely used in areas such as laser beam position sensor, autocollimators, optical tweezers, ellipsometers, precise optical alignment, wavefront sensing, angle measurement and so on [1–5]. The spot characteristics [6–8] and the algorithm been used have a great influence on the sensitivity and the linear measurement range when use the 4Q detector for laser spot center orientation, so the appropriate algorithm should be chosen to fulfill the request for practical application. In the real-time case, the algorithm is difficult to get practical application because of the complicate expression. Thus it has to be simplified while control the error based on the actual engineering.

Add–subtract algorithm and diagonal algorithm [9] are most commonly used algorithms, but the linear range of them are limited. All the study at present stage does not overcome this shortage. In this paper, a method of using square light spot with uniform intensity of distribution is proposed and the linear measurement range is increased for linear range critical conditions to meet the linear range critical conditions. Based on the analysis of add–subtract algorithm, diagonal algorithm and $\Delta/\sum$ algorithm [10], the voltage-displacement curves of four spot models, circular spot with flat energy distribution, circular spot with Gaussian energy distribution, ellipse spot with flat energy distribution and ellipse spot with Gaussian energy distribution have been deduced. Through the comparison and analysis of graphs, the advantages and disadvantages of the three algorithms on sensitive and linear measurement range are obvious. Since the expression of the voltage-displacement curves are very complicated, using these expressions directly will slow detection speed of the 4Q detector laser spot detection system, so polynomial expression been used instead of the simulation results in order to improve detection speed.

2. 4Q detector work principle

4Q detector is an optical position sensor (OPS) that can measure a position of a light spot in two-dimensions on a sensor surface, it has four active areas which are positioned symmetrically around the center of the sensor and separated by two gap lines. The sensors have an isotropic sensor surface that has a raster-like structure that supplies continuous position data [10].

If the coordinate system is chosen as shown in Fig. 1(a), two coordinate systems coincide with each other, add–subtract algorithm is generally adopted. If as Fig. 1(b), two coordinate systems have a 45° included angle, diagonal algorithm and $\Delta/\sum$ algorithm could be adopted.

Incoming laser beam is focused or imaged on to the detector as a spot. Suppose the output voltages are u_1, u_2, u_3 and u_4. The equations used to describe the position of the centroid of the spot can be expressed as

$$E_x = K\frac{(u_1+u_4)-(u_2+u_3)}{u_1+u_2+u_3+u_4}, \qquad E_y = K\frac{(u_1+u_2)-(u_3+u_4)}{u_1+u_2+u_3+u_4} \tag{1}$$

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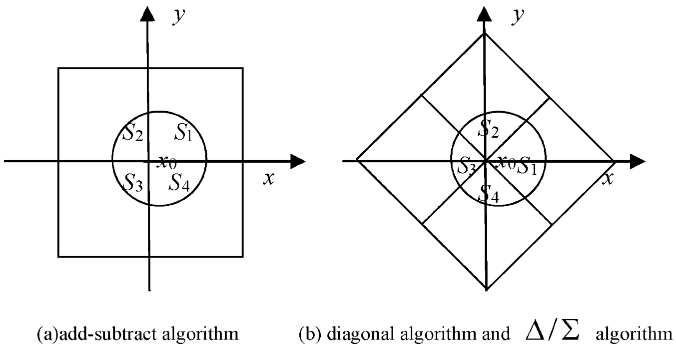

Fig. 1. Schematic diagram for comparison of different algorithms. (a) Add–subtract algorithm, (b) diagonal algorithm and $\Delta/\sum$ algorithm.

where E_x and E_x are position signals in x and y directions after processing with add–subtract algorithm [9], K the slope factor. Used diagonal algorithm and $\Delta/\sum$ algorithm, E_x and E_y are calculated using the following equations respectively [9,10]:

$$E_x = K\frac{u_1 - u_3}{u_1 + u_2 + u_3 + u_4}, \qquad E_y = K\frac{u_2 - u_4}{u_1 + u_2 + u_3 + u_4} \tag{2}$$

$$E_x = K\frac{u_1 - u_3}{u_1 + u_3}, \qquad E_y = K\frac{u_2 - u_4}{u_2 + u_4} \tag{3}$$

within the liner range, E_x and E_y meet the above positive correlation. Slope factor K is a constant depends on the diameter and energy distribution of the spot.

3. Algorithm simulation and comparison

Because the x and y direction have the same analytical method, the paper only simulation analysis of the algorithm on the x direction. The relationship between spot size and the dimensions of the detector must be considered when using a 4Q detector. Suppose the radius of the light spot and 4Q detector are r and R respectively. If r is smaller than $R/2$, there will represent a dead zone, where beam tracking will lost completely. Conversely, if r is larger than $R/2$, the spot energy of the spot is not fully used and the sensitivity of the detector is reduced. While tracking range increases with spot size positional resolution is inversely proportional to it. In general, in order to meet both of them, the most reasonable value is $r = 0.5R$ [6,11,12].

The 4Q detector used in the experiment has a square photosensitive surface. Suppose the side-length of square surface is D. In the simulation analysis below, $r = 0.25D$.

3.1. Circular spot with uniform energy distribution

Circular spot with uniform energy distribution is a simple and ideal model. Suppose the radius of the spot is 1. The diagram illustrating the relationship between E_x and x_0 (the real position of the spot) is shown in Fig. 2 while using (1)–(3).

3.2. Circular spot with Gaussian energy distribution

The model of circular spot with Gaussian energy distribution has huge difference with the practical application. The energy distribution of the general laser's output spot is Gaussian. Suppose the radius of the spot is 1, then the energy distribution is:

$$I = I_0 \exp\left[-2(x - x_0)^2 - (y - y_0)^2\right] \tag{4}$$

where (x_0, y_0) denotes the center position of the spot, I the energy distribution on the position of point (x, y) on the transverse section

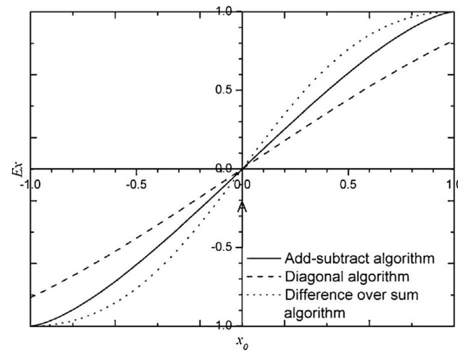

Fig. 2. Circular spot with flat energy distribution.

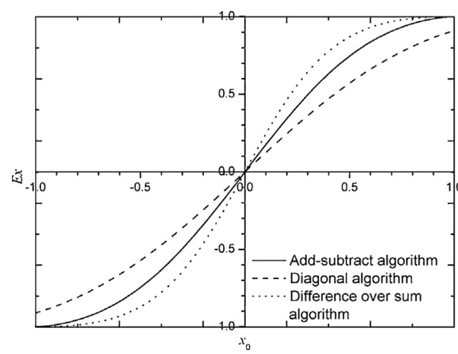

Fig. 3. Circular spot with Gaussian energy distribution.

of the laser beam, I_0 maximum of the energy on the transverse section. The result is as shown in Fig. 3 while using (1), (2) and (4).

3.3. Ellipse spot with uniform energy distribution

The shape of the spot is always ellipse but not standard circle in the practical application, suppose the length of the major semi-axis and the minor semi-axis are 1.25 and 1and the energy distribution is uniform. The result is as shown in Fig. 4 while using (1)–(3).

3.4. Ellipse spot with Gaussian Energy distribution

In actual condition, because of the influence and constriction of many factors, such as beam-splitting system, reflecting system and vibration, the spot is actually a elliptical Gaussian beam [13]. The function of its energy distribution is as follows:

$$I = I0\frac{2}{\pi\omega_a\omega_b}\exp\left[-\frac{2(x - x_0)^2}{{\omega_a}^2} - \frac{2(y - y_0)^2}{{\omega_b}^2}\right] \tag{5}$$

where ω_a, ω_b denotes the length of the major semi-axis and the minor semi-axis, (x_0, y_0) the center position of the spot, I the energy

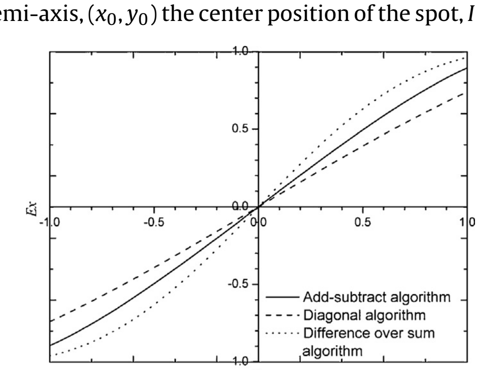

Fig. 4. Ellipse spot with flat energy distribution.

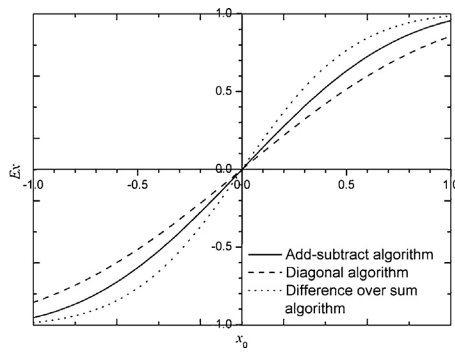

Fig. 5. Ellipse spot with Gaussian energy distribution.

distribution on the position of point (x, y) on the transverse section of the laser beam, I_0 maximum of the energy on the transverse section, the power in each of the four quadrants is double integrals of intensity. Suppose $I_0 = 1$, $\omega_a = 1.25$, $\omega_b = 1$. The result is as shown in Fig. 5 while using (2)–(4).

3.5. Square spot with uniform energy distribution

The diagonal algorithm is presented in order to improve the weakness of small linear measurement range of the add–subtract algorithm but it is still limited. Theoretically, when the spot only moves along the axial, if use add–subtract algorithm, the linear range is $2r$, if use diagonal algorithm, the linear range is $2\sqrt{2}r$ [1]. In the large linear range case, using BOE (Binary Optics Elements) converts a circular laser beam with Gaussian energy distribution into a spot with square and uniform spot intensity profile can be considered [14]. As shown in Fig. 6, the linear range can improve to $4r$. Consequently, it is found that use square spot with uniform energy distribution can have the largest linear range.

From Fig. 6, it can get the conclusion that $\Delta/\sum$ algorithm has the smallest linear measurement range and highest sensitivity than add–subtract algorithm and diagonal algorithm at same beam size. If use diagonal algorithm, the linearity of the position signal curve gets better, but the sensitivity is decreased. Add–subtract algorithm not stand out in any one area. For the same add–subtract algorithm, if we use the square spot with flat energy distribution, the linear range improved a lot.

4. Experimental results

The experimental system mainly includes a 650 nm semiconductor laser, displacement platform, optical system, 4Q detector, signal processing circuit and upper monitor, as shown in Fig. 7. System settings were as follows: spot diameter is 5 mm, 4Q detector with 6 mm×6 mm active area. Then the best spot diameter is $d = D/2 = 3$ mm. Fix the laser on the displacement platform first, then use an optical attenuator to reduce the power level of the spot to meet the maximum received power (5 mW) of the 4Q detector, finally, use the Galilean beam expander to compress the spot size. The experiment data is shown in Fig. 8, dot stands for add–subtract algorithm, triangle diagonal algorithm and five-pointed star $\Delta/\sum$ algorithm.

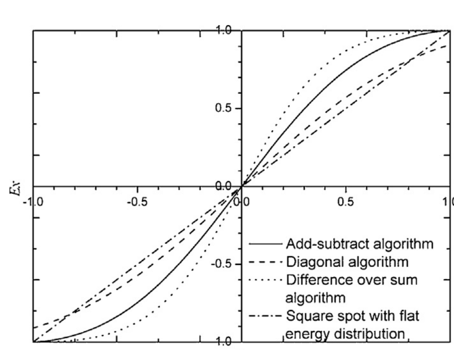

Fig. 6. Comparison of the curves of square light spot with uniform intensity of distribution using add–subtract algorithm with curves in Fig. 3.

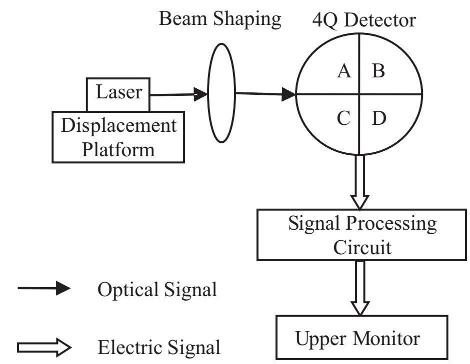

Fig. 7. Model of experimental system.

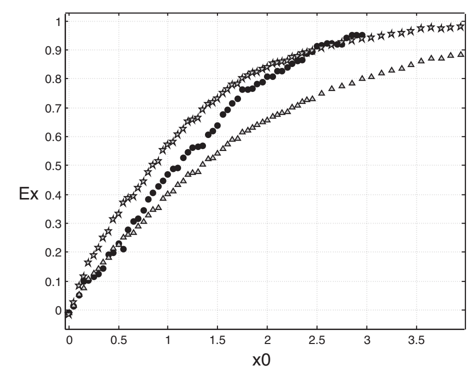

Fig. 8. Real observation of E_x curves.

The experimental results demonstrate that add–subtract algorithm has the smallest linear range, when the laser spot displacement is bigger than its diameter, the measurement data is invalid. The angle contained by the coordinate system of the 4Q detector and the measurement system is 45° in add–subtract algorithm and diagonal algorithm, so the measurement range is wider than spot diameter. The data within linear range is always be used in actual application. The data exceed the linear range for reference only due to the low sensitivity.

The experimental data were fitted by polynomial using MATLAB and the relationship between x_0 and E_x has been obtained. Different high degree fitting curves can be chosen to meet different demand. This paper only use add–subtract algorithm as an example to realize data fitting. Table 1 lists the coefficients of third to ninth degree polynomial fitting. Table 2 lists the sum of squares for error (SSE), R-square, adjusted R-square and root-mean-square error (RMSE), these parameters may serve as the basis for choosing fitting curves. Figs. 9 and 10 are two example of them.

$$E_x = A_{10} \times x_0^9 + A_9 \times x_0^8 + A_8 \times x_0^7 + A_7 \times x_0^6 + A_6 \times x_0^5 + A_5 \times x_0^4 + A_4 \times x_0^3 + A_3 \times x_0^2 + A_2 \times x_0 + A_1 \quad (6)$$

In data processing, suppose the central position of the spot is where E_x takes the minimum value. But if the minimum value is not 0, the fitting functions are different from the actual. In order to reduce the error, the above result can be optimized. Ninth degree polynomial curve, for instance, when $E_x = 0$, $x = -0.1276157380$ can

Table 1
The coefficients of third to ninth degree polynomial fitting.

	3	4	5	6	7	8	9
A_1	0.05992	0.04512	0.06185	0.05893	0.06849	0.06797	0.07484
A_2	0.4905	0.5007	0.5411	0.5453	0.566	0.5676	0.5807
A_3	0.007392	0.006071	−0.0148	−0.009682	−0.0311	−0.02957	−0.05469
A_4	−0.018	−0.01992	−0.03703	−0.03915	−0.05545	−0.05688	−0.07335
A_5		−0.00131	0.001207	−0.000122	0.006239	0.005491	0.01906
A_6			0.001331	0.001521	0.004641	0.00494	0.01069
A_7				8.506e−005	−0.0003578	−0.0002416	−0.002459
A_8					−0.0001708	−0.000188	−0.0009389
A_9						−5.658e−006	0.0001041
A_{10}							3.29e−005

Table 2
Assessment parameters.

	SSE	R-square	Adjusted R-square	RMSE
3	0.1912	0.9962	0.9961	0.04247
4	0.1609	0.9968	0.9967	0.03914
5	0.05429	0.9989	0.9989	0.02285
6	0.05278	0.9989	0.9989	0.02264
7	0.03223	0.9994	0.9993	0.01778
8	0.03216	0.9994	0.9993	0.01784
9	0.02376	0.9995	0.9995	0.01542

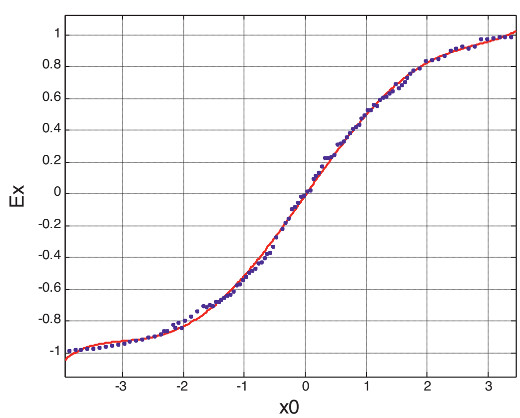

Fig. 9. Fifth degree polynomial curve.

be found, the ninth degree polynomial curve after optimizing is shown as below:

$$E_x = (3.29e-005) \times x_0^9 + (6.556e-005) \times x_0^8 - 0.001027 \times x_0^7 - 0.001562 \times x_0^6 + 0.01227 \times x_0^5 + 0.01156 \times x_0^4 + 0.01156 \times x_0^3 - 0.02439 \times x_0^2 + 0.591 \times x_0 - 0.001406 \quad (7)$$

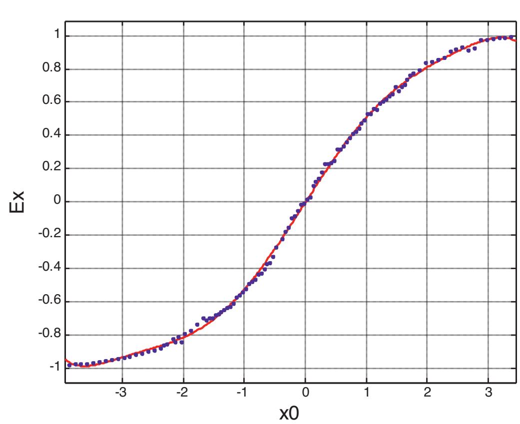

Fig. 10. Seventh degree polynomial curve.

5. Conclusions

In this paper, the voltage-displacement curves of add–subtract algorithm, diagonal algorithm and $\Delta/\sum$ algorithm in four different kinds of spot model are described and the advantages and disadvantages of every algorithm been compared. Based on the above theory analysis, it comes to the conclusion: for circular spot with uniform energy distribution, circular spot with Gaussian energy distribution, ellipse spot with uniform energy distribution and ellipse spot with Gaussian energy distribution, the linear measurement range of diagonal algorithm is better than that of add–subtract algorithm and $\Delta/\sum$ algorithm has the best sensitivity. This conclusion can also be reached on the basis of the experimental data. Fig. 6 showed that use square light spot with uniform intensity of distribution the linear measurement range can improve to 4r, it is an efficient method to improve the linear measurement range of 4Q detector.

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