School of Computing
FACULTY OF ENGINEERING

UNIVERSITY OF LEEDS

Signed Distance Fields as a Means of Improving the Speed of Rendering Three-Dimensional Fractals

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The candidate confirms that the following have been submitted.

Items	Format	Recipient(s) and Date
Project Report	Report	SSO (19/08/22)
Project Code	Zip File	SSO (19/08/22)

Type of project: Exploratory Software

The candidate confirms that the work submitted is their own and the appropriate credit has been given where reference has been made to the work of others.

I understand that failure to attribute material which is obtained from another source may be considered as plagiarism.

(Signature of Student) Nell Mills

Summary

<Concise statement of the problem you intended to solve and main achievements (no more than one A4 page)>

Acknowledgements

<The page should contain any acknowledgements to those who have assisted with your work. Where you have worked as part of a team, you should, where appropriate, reference to any contribution made by other to the project.>
Note that it is not acceptable to solicit assistance on ‘proof reading’ which is defined as the “the systematic checking and identification of errors in spelling, punctuation, grammar and sentence construction, formatting and layout in the test”;
see http://www.leeds.ac.uk/gat/documents/policy/Proof-reading-policy.pdf.

Contents

Chapter 1

Introduction

Chapter 2

Background

This project aims to improve the efficiency of real-time rendering of 3D fractals. This chapter will focus on background theory for fractals, signed distance functions and sphere tracing.

2.1 2D Fractals - The Mandelbrot Set

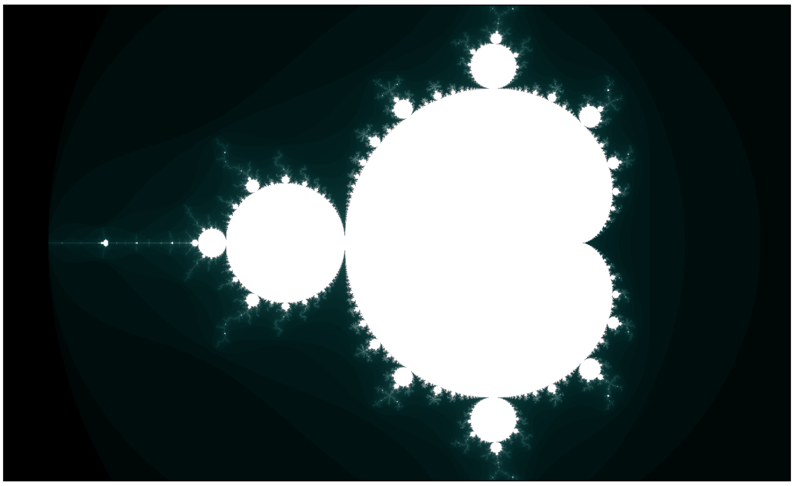

Figure 2.1: The Mandelbrot set. The white points in the centre are inside the set.

The Mandelbrot set is the set of two-dimensional points that satisfy a certain constraint on the following complex quadratic equation:

$$Z = Z^2 + C \tag{2.1}$$

where Z and C are complex numbers. The constraint on the points is that their orbit must be bounded. The value of Z is initialized to 0 and equation 2.1 is iterated over, each new value of Z being placed back in to the equation in the next iteration. If the length of the point Z does not exceed a threshold, then the point (represented by C) is in the Mandelbrot set [1].

Figure 2.1 shows a generated Mandelbrot set. The real part of the point C is represented by the x-axis, and the imaginary part by the y-axis. Equation 2.1 is iterated over a maximum of five hundred times, and the threshold value is two. The pixels are coloured according to how many iterations are achieved before the length of Z exceeds the threshold.

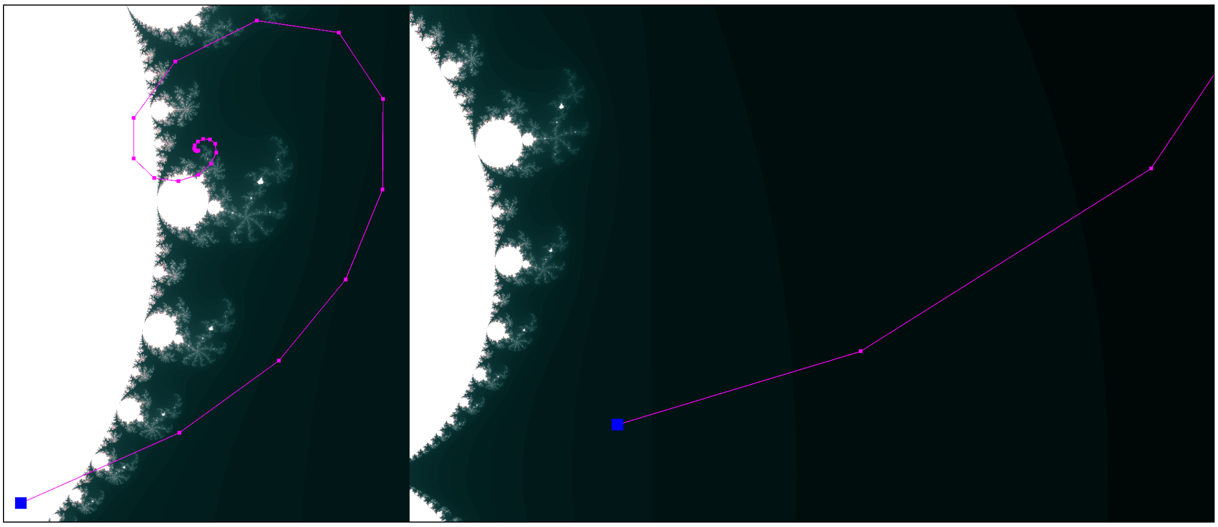

Figure 2.2: Visualization of the first twenty five iterations of equation 2.1 on the initial points [0.3, 0.05] (left) and [0.5, 0.04] (right). The initial points are shown in blue.

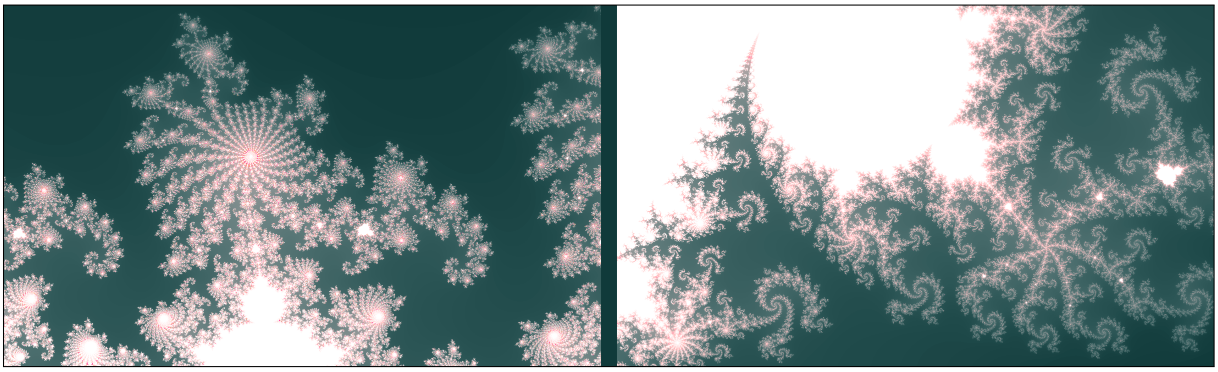

Figure 2.3: Two different views of the Mandelbrot set, zoomed in.

Figure 2.2 illustrates the first twenty five iterations on two different points. For the first point, the iterations converge in a spiral shape and the length of Z never exceeds the threshold of two, therefore the point is in the Mandelbrot set and is coloured white. For the second point, the iterations diverge and exceed the threshold of two within a few iterations, so this point is not in the Mandelbrot set and is coloured dark.

Figure 2.3 shows two zoomed-in views at the edge of the original shape. New patterns can be seen, as well as repeated ones, and even new instances of the original shape. This is because the Mandelbrot set has infinite detail, so if one decreases the range of the axes, new patterns will emerge [2].

This project makes use of three-dimensional fractal rendering, so the next section will look at the challenge of bringing this infinite level of detail to three dimensions.

2.2 3D Fractals - The Mandelbulb

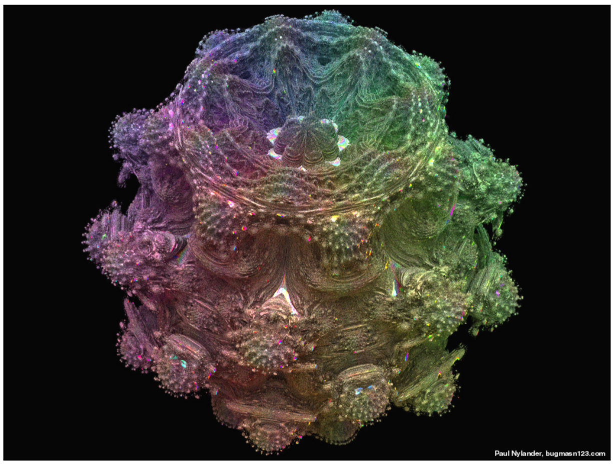

Figure 2.4: First look at the Mandelbulb, from Paul Nylander's website [3].

Since the Mandelbrot set is in two dimensions, and since complex numbers have only two components (real and imaginary), obtaining a true three-dimensional Mandelbrot set is a challenging task, and a mathematically rigorous three-dimensional Mandelbrot has not yet been found [4].

One attempt that seems to have come close was originated by Rudy Rucker in 1987. Rucker thought that expressing the three-dimensional points in spherical coordinates would allow the manipulation of the points in a similar way to the complex-number operations performed on the points in the two-dimensional Mandelbrot set [5].

Rucker did not have the computational power to accomplish a rendering of this idea, so it was put aside for twenty years, until Daniel White independently published a formula in 2007, which took the approach proposed by Rucker. White decided to approach the problem by considering the geometrical consequences of multiplying numbers in the complex plane, which amounts to rotating them [4].

White's formula produced images that looked promising, but they didn't have the level of detail that was expected from a true three-dimensional equivalent of the Mandelbrot set. A mathematician, Paul Nylander, raised White's formula to a higher power (eight), which would be equivalent to increasing the number of rotations of the point. The resulting image is shown in figure 2.4. The shape maintains excellent detail, even at high levels of magnification [4].

The new shape, known as the Mandelbulb, has roughly the same formula as the Mandelbrot set (equation 2.1):

$$Z = Z^k + C \tag{2.2}$$

where Z is raised to an arbitrary power like so:

$$Z^k = r^k(sin[k\theta]cos[k\phi], sin[k\theta]sin[k\phi], cos[k\theta]). \tag{2.3}$$

The variable r is the norm of Z (|Z|), θ is equal to arctan(Z_y/Z_x) and ϕ is equal to $|(Z_x, Z_y)|/Z_z$. The spherical coordinates of the point Z/|Z| are represented by θ and ϕ. Equations 2.2 and 2.3 are sourced from Chapter 33 of the book Ray Tracing Gems II [6].

Papers:

- The mandelbulb: first 'true'3D image of famous fractal [4].
- In Search of a Beautiful 3D Mandelbrot Set [5].
- .
- A glimpse of the Mandelbulb with memory [7].
- On the Algebraic Foundation of the Mandelbulb [8].

- A few ways to represent mandelbrot in 3D.
- 1. Slices of 2D mandelbrot.
- 2. Quaternion mandelbrot with fourth dimension = 0.
- 3. Informal extension to 3D with polar coordinates.
- Explain each. I will be going with option 3.

2.3 Signed Distance Functions and Sphere Tracing

Papers:

- Ray Tracing Gems II [6].

- Signed distance function is an estimate of how close the point is to the surface of the fractal.
- If it's negative, the point is inside the surface.
- Demonstrate derivation from Mandelbulb equation.
- Distance estimators can be used for raymarching - known as sphere tracing.
- Diagram of sphere tracing.

2.4 Signed Distance Fields

Papers:

- Signed distance fields: A natural representation for both mapping and planning [9].

- Stored values from signed distance function, stored as 2D texture or 3D grid.
- Saves having to recompute SDF all the time.
- Fractal scenes with high view distance could benefit from some pre-calculated values to give them a head start - show room of pillars and remark on frame rate (no SDF).
- Remark on memory costs.
- Briefly talk about storage methods - octrees.

Chapter 3

Implementation

3.1 Project Structure and Overview

3.2 Rendering 3D Fractals

3.3 Signed Distance Field

3.4 Second Attempt: 2D Texture SDF

3.4.1 Single-pixel Sampling

3.4.2 Sampling a small area

3.5 Possibly Additional - Cone Marching

3.6 Performance Measurement

3.6.1 Data Types Collected

3.6.2 Representative Views

3.6.3 Animation

Chapter 4

Results

4.1 Static SDF

- Wife bad - was slow and low quality compared to original
- 100+ fractals?
- Faster at even lower resolutions but bad quality

4.2 Representative Views

4.2.1 Front View

Default view, Mandelbulb taking up most of screen. Single Mandelbulb.

4.2.2 Angled Top-Down View

Chapter 5

Conclusion

5.0.1 Static SDF

Wife bad.

Use Cases

Maybe wife not so bad with 100+ fractals?

5.0.2 Texture SDF

Single-pixel Sampling

Multi-pixel Sampling

Use Cases

References

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[3] P. Nylander, "Hypercomplex fractals." `http://www.bugman123.com/Hypercomplex`, 2009. accessed: 31/07/22.

[4] J. Aron, "The mandelbulb: first 'true'3d image of famous fractal," *New Scientist*, vol. 204, no. 3736, pp. 54–55, 2009.

[5] R. Rucker, "In search of a beautiful 3d mandelbrot set." `https://www.rudyrucker.com/blog/notebooks/rucker_mandelbulb_ver7_sept24_2009.pdf`, 2009. accessed: 31/07/22.

[6] A. Marrs, P. Shirley, and I. Wald, *Ray Tracing Gems II: Next Generation Real-Time Rendering with DXR, Vulkan, and OptiX*. Springer Nature, 2021.

[7] R. Alonso-Sanz, "A glimpse of the mandelbulb with memory," *Complex Systems*, vol. 25, no. 2, pp. 109–126, 2016.

[8] V. Boily and D. Rochon, "On the algebraic foundation of the mandelbulb," *arXiv preprint arXiv:2206.06332*, 2022.

[9] H. Oleynikova, A. Millane, Z. Taylor, E. Galceran, J. Nieto, and R. Siegwart, "Signed distance fields: A natural representation for both mapping and planning," in *RSS 2016 workshop: geometry and beyond-representations, physics, and scene understanding for robotics*, University of Michigan, 2016.

Appendices

Appendix A

External Material

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Appendix B

Ethical Issues Addressed