School of Computing
FACULTY OF ENGINEERING

UNIVERSITY OF LEEDS

<Full title of Project>

<Full Name of Author>

Submitted in accordance with the requirements for the degree of
<Name of Degree>

<Session>

The candidate confirms that the following have been submitted.
<As an example>

Items	Format	Recipient(s) and Date
Beliverable 1, 2, 3	Report	SSO (DD/MM/YY)
Participant consent forms	Signed forms in envelop	SSO (DD/MM/YY)
Deliverable 4	Software codes or URL	Supervisor, Assessor (DD/MM/YY)
Deliverable 5	User manuals	Client, Supervisor (DD/MM/YY)

Type of project: ______________________________

The candidate confirms that the work submitted is their own and the appropriate credit has been given where reference has been made to the work of others.

I understand that failure to attribute material which is obtained from another source may be considered as plagiarism.

(Signature of Student) ______________________

Summary

<Concise statement of the problem you intended to solve and main achievements (no more than one A4 page)>

Acknowledgements

<The page should contain any acknowledgements to those who have assisted with your work. Where you have worked as part of a team, you should, where appropriate, reference to any contribution made by other to the project.>

Note that it is not acceptable to solicit assistance on ‘proof reading’ which is defined as the “the systematic checking and identification of errors in spelling, punctuation, grammar and sentence construction, formatting and layout in the test”;

see http://www.leeds.ac.uk/gat/documents/policy/Proof-reading-policy.pdf.

Contents

Chapter 1

Background

This project aims to improve the efficiency of real-time rendering of 3D fractals. This chapter will focus on background theory for fractals, signed distance functions and sphere tracing.

1.1 2D Fractals - The Mandelbrot Set

The Mandelbrot set is the set of two-dimensional points that satisfy a certain constraint on the following complex quadratic equation:

$$Z = Z^2 + C \tag{1.1}$$

where Z and C are complex numbers. The constraint on the points is that their orbit must be bounded. The value of Z is initialized to 0 and equation 1.1 is iterated over, each new value of Z being placed back in to the equation in the next iteration. If the length of the point Z does not exceed a threshold, then the point (represented by C) is in the Mandelbrot set [1].
Figure 1.1 shows a generated Mandelbrot set. In this case, the real part of the point C is represented by the x-axis, and the imaginary part by the y-axis. Equation 1.1 is iterated over a maximum of five hundred times, and the threshold value is two. The pixels are coloured according to how many iterations are achieved before the length of Z exceeds the threshold.
The image presented in figure 1.1 ranges from negative one to one on the y-axis (and is proportional in the x-axis), but the Mandelbrot set has infinite detail, so if one decreases the range of the axes, new patterns will emerge. Figure 1.1 shows two zoomed-in views of the edge of the original shape. New patterns can be seen, as well as repeated ones, and even new instances of the original shape.
Papers:

- Similarity between the Mandelbrot set and Julia sets [2].
- Evolutionary exploration of the mandelbrot set [3].
- The Mandelbrot set, the Farey tree, and the Fibonacci sequence [1].

1.2 3D Fractals - The Mandelbulb

Papers:

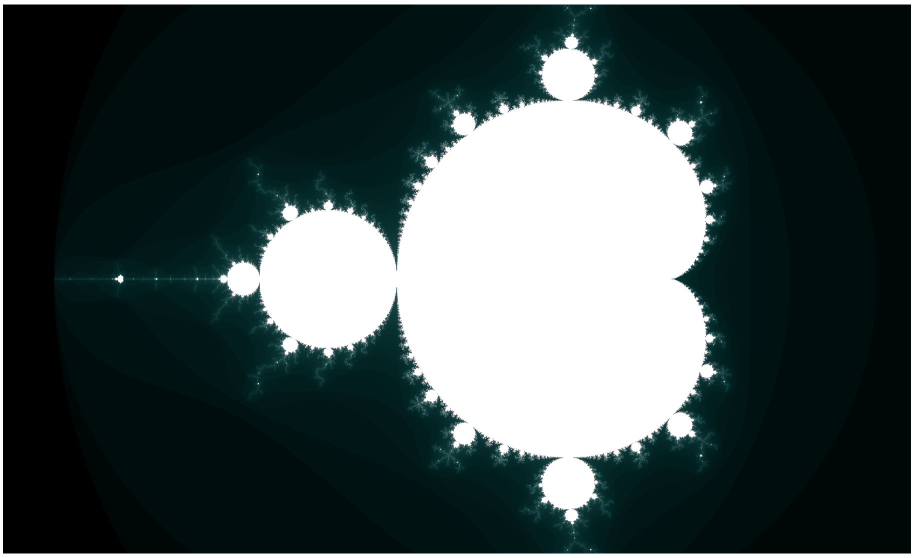

Figure 1.1: The Mandelbrot set. The white points in the centre are inside the set.

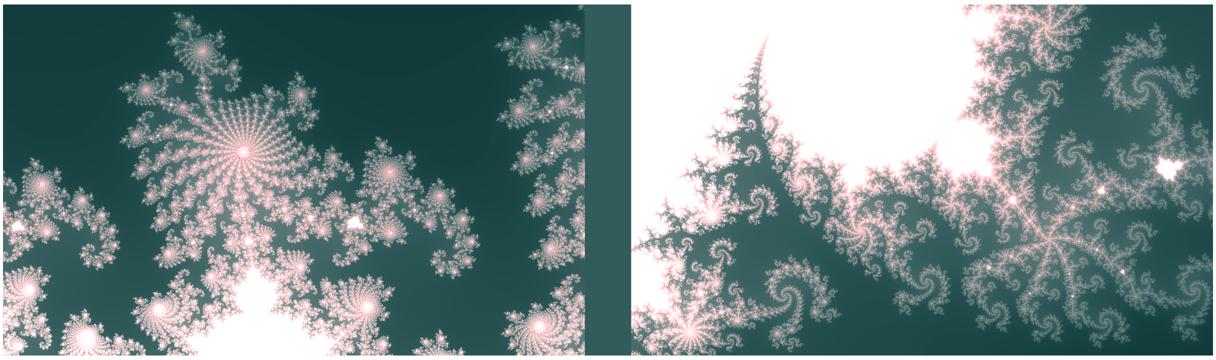

Figure 1.2: The Mandelbrot set

- The mandelbulb: first 'true'3D image of famous fractal [4].

[4]
- A few ways to represent mandelbrot in 3D.
- 1. Slices of 2D mandelbrot.
- 2. Quaternion mandelbrot with fourth dimension = 0.
- 3. Informal extension to 3D with polar coordinates.
- Explain each. I will be going with option 3.

1.3 Signed Distance Functions and Sphere Tracing

Papers:

- Ray Tracing Gems II [5].

- Signed distance function is an estimate of how close the point is to the surface of the fractal.
- If it's negative, the point is inside the surface.
- Demonstrate derivation from Mandelbulb equation.
- Distance estimators can be used for raymarching - known as sphere tracing.
- Diagram of sphere tracing.

1.4 Signed Distance Fields

Papers:

- Signed distance fields: A natural representation for both mapping and planning [6].

- Stored values from signed distance function, stored as 2D texture or 3D grid.
- Saves having to recompute SDF all the time.
- Fractal scenes with high view distance could benefit from some pre-calculated values to give them a head start - show room of pillars and remark on frame rate (no SDF).
- Remark on memory costs.
- Briefly talk about storage methods - octrees.

Chapter 2

Theory

2.1 Project aim

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2.2 Code Performance Theory

Chapter 3

Implementation

3.1 Project Structure and Overview

3.2 Rendering 3D Fractals

3.3 Signed Distance Field

3.4 Second Attempt: 2D Texture SDF

3.4.1 Single-pixel Sampling

3.4.2 Sampling a small area

3.5 Possibly Additional - Cone Marching

3.6 Performance Measurement

3.6.1 Data Types Collected

3.6.2 Representative Views

3.6.3 Animation

Chapter 4

Results

4.1 Static SDF

- Wife bad - was slow and low quality compared to original
- 100+ fractals?
- Faster at even lower resolutions but bad quality

4.2 Representative Views

4.2.1 Front View

Default view, Mandelbulb taking up most of screen. Single Mandelbulb.

4.2.2 Angled Top-Down View

Chapter 5

Conclusion

5.0.1 Static SDF

Wife bad.

Use Cases

Maybe wife not so bad with 100+ fractals?

5.0.2 Texture SDF

Single-pixel Sampling

Multi-pixel Sampling

Use Cases

References

[1] R. L. Devaney, "The mandelbrot set, the farey tree, and the fibonacci sequence," *The American Mathematical Monthly*, vol. 106, no. 4, pp. 289–302, 1999.

[2] T. Lei, "Similarity between the mandelbrot set and julia sets," *Communications in mathematical physics*, vol. 134, no. 3, pp. 587–617, 1990.

[3] D. Ashlock, "Evolutionary exploration of the mandelbrot set," in *2006 IEEE International Conference on Evolutionary Computation*, pp. 2079–2086, IEEE, 2006.

[4] J. Aron, "The mandelbulb: first 'true'3d image of famous fractal," *New Scientist*, vol. 204, no. 3736, pp. 54–55, 2009.

[5] A. Marrs, P. Shirley, and I. Wald, *Ray Tracing Gems II: Next Generation Real-Time Rendering with DXR, Vulkan, and OptiX*. Springer Nature, 2021.

[6] H. Oleynikova, A. Millane, Z. Taylor, E. Galceran, J. Nieto, and R. Siegwart, "Signed distance fields: A natural representation for both mapping and planning," in *RSS 2016 workshop: geometry and beyond-representations, physics, and scene understanding for robotics*, University of Michigan, 2016.

Appendices

Appendix A

External Material

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Appendix B

Ethical Issues Addressed