CSE 312: Foundations of Computing II
Assignment #6
May 18
due: May 24, at midnight.

Instructions:

- **Answers:** When asked for a short answer (such as a single number), also *show and explain your work* briefly. Simplify your final formula algebraically as much as possible, without using your calculator. Then, if the answer is a number rather than a function of some variables, use a calculator to evaluate it and provide the number. For example, for counting problems, your answer might look like this:

  Answer: $\binom{5}{2} - \binom{4}{2} = 4$.

  Explanation: There are $\binom{5}{2}$ ways to select 2 fingers out of the 5, and $\binom{4}{2}$ of them do not involve the thumb.

  Solutions that do not show enough work may not get full credit.

- **Turn-in:** Do not write your name on your pages (your Gradescope account will identify you to us) and do not include a copy of the exercise's question in what you turn in. You must use Gradescope to upload your homework solutions. You will submit a single PDF file containing your solutions to all the exercises in the homework. Each numbered homework question must be answered on its own page (or pages). You must follow the Gradescope prompts that have you link exercise numbers to your pages. You may typeset your solutions on a computer (see here for tutorials and templates) or you can handwrite them, take a picture of (or scan) each handwritten page, and convert the pictures into a single PDF file. You are responsible for making sure that your solution is easily readable, and submitted on time.

1. The company Moonbound sells luxury spaceships. According to their historical sales, customers buy spaceships at an average rate of 5 per week. Spaceships are expensive for them to stock but, on the other hand, they hate to run out of spaceships and turn away a rich customer. Once a week, Moonbound restocks spaceships for the coming week. They want to have the minimum number of spaceships such that there is less than a 0.1% probability that they have more buying customers than spaceships during the next week. They cannot figure out how many spaceships this should be and turn to you for advice. After asking some questions, you determine that arrivals of paying customers seem to be independent of each other and decide that a Poisson distribution would be a good model. Given this decision, how many spaceships should Moonbound stock for the coming week?

2. For this exercise, give exact answers as simplified fractions. Compute $E[X]$ and $\mathrm{Var}(X)$ if X has probability density function given by ...

   (a) $f(x) = \begin{cases} c(1-x^4) & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$.

   Determine the value of c as part of your answer.

   (b) $f(x) = \begin{cases} c/x^3 & \text{if } x > 5 \\ 0 & \text{otherwise} \end{cases}$.

   Determine the value of c as part of your answer.

3. You throw a dart at a circular target of radius r. Let X be the distance of your dart's hit from the center of the target. Your aim is such that X is an exponential distribution with parameter $4/r$.

   (a) As a function of r, determine the value m such that $P(X < m) = P(X > m)$.

   (b) What is the probability that you miss the target completely?

4. A flea of negligible size is trapped in a large, spherical, inflated beach ball with radius r. At this moment, it is equally likely to be at any point within the ball. Let X be the distance of the flea from the center of the ball. Using the fact that the volume of a sphere of radius r is $4\pi r^3/3$, for X, find ...

   (a) ... the cumulative distribution function F.

   (b) ... the probability density function f.

   (c) ... the expected value.

   (d) ... the variance.

5. Suppose you have a die that has probability p of resulting in the outcome 6 when rolled, where p is a continuous random variable that is uniformly distributed over $[0, \frac{1}{3}]$. Suppose you start rolling this die. (The value of p does not change once you start rolling.) Give exact answers as simplified fractions.

   (a) Compute the probability that the first roll is 6.

   (b) Compute the probability that the first two rolls are both 6.

   (c) Let S_1 be the event that the first roll is 6 and S_2 be the event that the second roll is 6. Compute $P(S_2 \mid S_1)$.

   (d) Are the outcomes of the first two rolls independent? Justify your answer.

   (e) Compute the probability that the first k rolls are each 6. Compare this with the probability if it were a fair die.