ROB311

TP-6 Bayesian Network

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1 Bayesian Network Model

In this medical diagnosis problem, we design a Bayesian network model to aid in diagnosing tuberculosis, cancer, and bronchitis.

1.1 Variables

— **Asia** : Whether the patient has recently visited Asia.
— **Smoker** : Whether the patient is a smoker.
— **Tuberculosis** : Whether the patient has tuberculosis.
— **Cancer** : Whether the patient has lung cancer.
— **Bronchitis** : Whether the patient has bronchitis.
— **X-Ray** : Result of the X-ray test.
— **Stethoscope** : Result of the stethoscope test.

1.2 Bayesian Network Structure

— **Asia → Tuberculosis** : Visiting Asia increases the risk of tuberculosis.
— **Smoker → Cancer** : Smoking increases the risk of lung cancer.
— **Smoker → Bronchitis** : Smoking decreases the likelihood of bronchitis (since non-smokers are more likely to have bronchitis).
— **Tuberculosis, Cancer → X-Ray** : The X-ray can detect either tuberculosis or cancer.
— **Cancer, Bronchitis → Auscultation** : The auscultation(stethoscope) can detect either cancer or bronchitis.

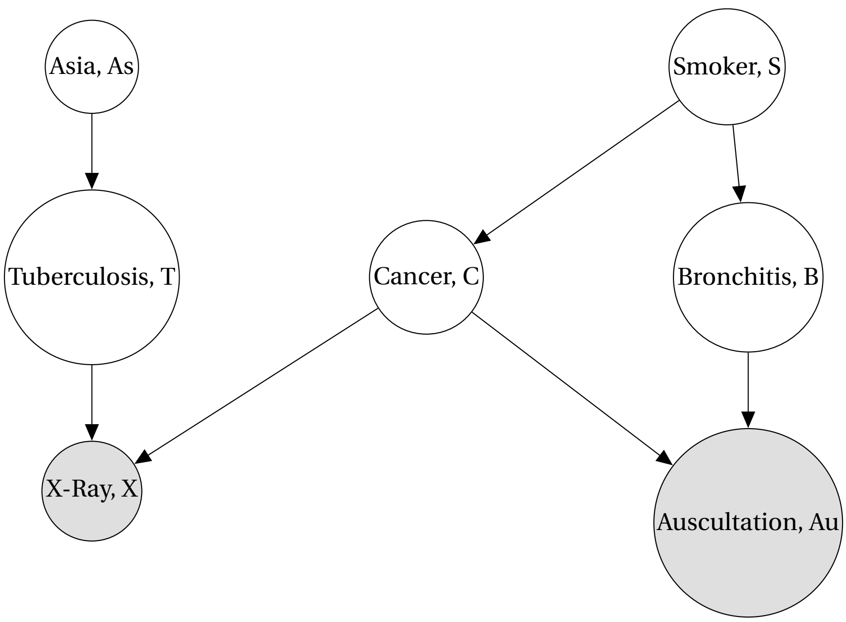

FIGURE 1 – Bayesian Network

1.3 Conditional Probabilities

The conditional probabilities of the problem are listed as follows.

1. $\mathbb{P}(As) = 0.1$

2. $\mathbb{P}(S) = 0.3$
3. $\mathbb{P}(T \mid As) = 0.1$
4. $\mathbb{P}(T \mid \neg As) = 0.01$
5. $\mathbb{P}(C \mid S) = 0.2$
6. $\mathbb{P}(C \mid \neg S) = 0.02$
7. $\mathbb{P}(B \mid S) = 0.6$
8. $\mathbb{P}(B \mid \neg S) = 0.8$
9. $\mathbb{P}(Au \mid B, \neg C) = 0.6$
10. $\mathbb{P}(Au \mid \neg B, C) = 0.6$
11. $\mathbb{P}(\neg Au \mid \neg B, \neg C) = 0.99$
12. $\mathbb{P}(X \mid T, \neg C) = 0.7$
13. $\mathbb{P}(X \mid \neg T, C) = 0.7$
14. $\mathbb{P}(\neg X \mid \neg T, \neg C) = 0.98$

2 Question 2

Given that the patient is neither smoking nor has recently visited Asia, we can draw inferences about the likelihood of each disease based on the Bayesian network structure and provided data :

2.1 Tuberculosis

Since the patient has not visited Asia, the probability of tuberculosis is low, specifically around 1%, as per the data from the hospital.

$$\mathbb{P}(T \mid \neg S, \neg As) = \mathbb{P}(T \mid \neg As) = 0.01$$

2.2 Cancer

For non-smokers, the probability of cancer is also significantly lower, around 2%.

$$\mathbb{P}(C \mid \neg S, \neg As) = \mathbb{P}(C \mid \neg S) = 0.02$$

2.3 Bronchitis

Non-smokers are more prone to bronchitis than smokers, with an 80% probability in cases where lung issues are present.

$$\mathbb{P}(B \mid \neg S, \neg As) = \mathbb{P}(B \mid \neg S) = 0.8$$

2.4 Conclusion

We can infer that the most likely diagnosis of the patient is bronchitis (80%).

3 Question 3

The results of last question shows that bronchitis was initially considered the most probable diagnosis, with an 80% likelihood. To validate this diagnosis, the doctor chooses to perform auscultation using a stethoscope. This test has a 60% sensitivity for detecting bronchitis when it is indeed present. A positive outcome would strengthen the confidence in the bronchitis diagnosis, whereas a negative outcome would decrease the confidence.

Given that the auscultation test is negative($\neg Au$), we revise our probabilities to reflect the likelihood of each condition.

3.1 Probability of Negative Auscultation Result Given Different Conditions

As the auscultation test result is negative, we can conclude that

— **If the patient has only bronchitis** $(B, \neg C)$ **:** The probability of a negative auscultation result is calculated as :

$$\mathbb{P}(\neg Au \mid B, \neg C) = 1 - 0.6 = 0.4$$

— **If the patient has only lung cancer** $(C, \neg B)$ **:** Similarly, the probability of a negative auscultation result in this case is :

$$\mathbb{P}(\neg Au \mid C, \neg B) = 1 - 0.6 = 0.4$$

— **If the patient has both bronchitis and lung cancer** (B, C) **:** Since the auscultation detects either disease with a probability of 0.6 independently, the probability of a negative auscultation result when both diseases are present is the product of the individual probabilities :

$$\mathbb{P}(\neg Au \mid B, C) = 0.4 \times 0.4 = 0.16$$

— **If the patient has neither bronchitis nor lung cancer** $(\neg B, \neg C)$ **:** In the absence of both diseases, the probability of a negative auscultation result is very high :

$$\mathbb{P}(\neg Au \mid \neg B, \neg C) = 0.99$$

3.2 Calculate Total Probability of Negative Stethoscope Result

To update the probabilities, we use the total probability of a negative result, $\mathbb{P}(\neg Au)$:

$$\mathbb{P}(\neg Au) = \mathbb{P}(\neg Au \mid B, \neg C) \cdot \mathbb{P}(B, \neg C) + \mathbb{P}(\neg Au \mid C, \neg B) \cdot \mathbb{P}(C, \neg B)$$
$$+ \mathbb{P}(\neg Au \mid B, C) \cdot \mathbb{P}(B, C) + \mathbb{P}(\neg Au \mid \neg B, \neg C) \cdot \mathbb{P}(\neg B, \neg C)$$

As the patient is a non-smoker and has not visited Asia, we can use the result of last question $\mathbb{P}(B) = 0.8$, $\mathbb{P}(C) = 0.02$.

We can also infer that

— $\mathbb{P}(B, C) = \mathbb{P}(B) \cdot \mathbb{P}(C) = 0.016$

— $\mathbb{P}(B, \neg C) = \mathbb{P}(B) \cdot \mathbb{P}(\neg C) = 0.784$

— $\mathbb{P}(\neg B, C) = \mathbb{P}(\neg B) \cdot \mathbb{P}(C) = 0.004$

— $\mathbb{P}(\neg B, \neg C) = \mathbb{P}(\neg B) \cdot \mathbb{P}(\neg C) = 0.196$

Substituting in known probabilities, we get that

$$\mathbb{P}(\neg Au) = 0.512$$

3.3 Posterior Probability of Bronchitis and Cancer Given Negative Auscultation

Now, we use Bayes' theorem to compute the posterior probability of bronchitis and cancer given a negative auscultation test. We can calculate $\mathbb{P}(B \mid \neg Au)$ and $\mathbb{P}(C \mid \neg Au)$ by

$$\mathbb{P}(B \mid \neg Au) = \frac{\mathbb{P}(\neg Au \mid B) \cdot \mathbb{P}(B)}{\mathbb{P}(\neg Au)} = \frac{\mathbb{P}(\neg Au \mid B, C) \cdot \mathbb{P}(B, C) + \mathbb{P}(\neg Au \mid B, \neg C) \cdot \mathbb{P}(B, \neg C)}{\mathbb{P}(\neg Au)}$$

$$\mathbb{P}(C \mid \neg Au) = \frac{\mathbb{P}(\neg Au \mid C) \cdot \mathbb{P}(C)}{\mathbb{P}(\neg Au)} = \frac{\mathbb{P}(\neg Au \mid B, C) \cdot \mathbb{P}(B, C) + \mathbb{P}(\neg Au \mid \neg B, C) \cdot \mathbb{P}(\neg B, C)}{\mathbb{P}(\neg Au)}$$

Substituting in known probabilities, we get that

$$\mathbb{P}(B \mid \neg Au) = 0.618$$

$$\mathbb{P}(C \mid \neg Au) = 0.016$$

3.4 Conclusion

Given the negative result from the stethoscope test, the inferred diagnosis remains most likely **bronchitis** with an updated posterior probability of **61.8%**. The probability of **cancer** is also decreased to **1.6%**.

4 Question 4

The doctor orders an X-Ray test, and the result is positive. Since the X-Ray test can detect tuberculosis or cancer with a probability of 70%, a positive result increases the likelihood of these diseases. Given that the auscultation (stethoscope) test had already suggested a higher probability for bronchitis, this new X-Ray result shifts the diagnosis towards the possibility of either tuberculosis or cancer.

4.1 Probability of Positive X-ray Result Given Different Conditions

As the X-ray test result is positive, we can conclude that

- **If the patient has only tuberculosis** $(T, \neg C)$ **:** The probability of a positive X-ray result is calculated as :
$$\mathbb{P}(X \mid T, \neg C) = 0.7$$
- **If the patient has only lung cancer** $(C, \neg T)$ **:** The probability of a positive X-ray result is calculated as :
$$\mathbb{P}(X \mid C, \neg T) = 0.7$$
- **If the patient has both tuberculosis and lung cancer** (T, C) **:** The probability of a positive X-ray result is calculated as :
$$\mathbb{P}(X \mid T, C) = 0.7 \times 0.7 = 0.49$$
- **If the patient has neither tuberculosis nor lung cancer** $(\neg T, \neg C)$ **:** The probability of a positive X-ray result is calculated as :
$$\mathbb{P}(X \mid \neg T, \neg C) = 1 - 0.98 = 0.02$$

4.2 Calculate Total Probability of Positive X-ray Result

To update the probabilities, we use the total probability of a positive result, $\mathbb{P}(X)$:

$$\mathbb{P}(X) = \mathbb{P}(X \mid T, \neg C) \cdot \mathbb{P}(T, \neg C) + \mathbb{P}(X \mid C, \neg T) \cdot \mathbb{P}(C, \neg T)$$
$$+ \mathbb{P}(X \mid T, C) \cdot \mathbb{P}(T, C) + \mathbb{P}(X \mid \neg T, \neg C) \cdot \mathbb{P}(\neg T, \neg C)$$

Using the result of last questions, we get $\mathbb{P}(T) = 0.01$ (Question 2), $\mathbb{P}(C) = 0.016$ (Question 3).

We can also infer that
- $\mathbb{P}(T, C) = \mathbb{P}(T) \cdot \mathbb{P}(C) = 0.00016$
- $\mathbb{P}(T, \neg C) = \mathbb{P}(T) \cdot \mathbb{P}(\neg C) = 0.00984$
- $\mathbb{P}(\neg T, C) = \mathbb{P}(\neg T) \cdot \mathbb{P}(C) = 0.0158$
- $\mathbb{P}(\neg T, \neg C) = \mathbb{P}(\neg T) \cdot \mathbb{P}(\neg C) = 0.974$

Substituting in known probabilities, we get that

$$\mathbb{P}(X) = 0.0374$$

4.3 Posterior Probability of Tuberculosis and Cancer Given Positive X-ray Result

Now, we use Bayes' theorem to compute the posterior probability of tuberculosis and cancer given a positive X-ray test. We can calculate $\mathbb{P}(T \mid X)$ and $\mathbb{P}(C \mid X)$ by

$$\mathbb{P}(T \mid X) = \frac{\mathbb{P}(X \mid T) \cdot \mathbb{P}(T)}{\mathbb{P}(X)} = \frac{\mathbb{P}(X \mid T, C) \cdot \mathbb{P}(T, C) + \mathbb{P}(X \mid T, \neg C) \cdot \mathbb{P}(T, \neg C)}{\mathbb{P}(X)}$$
$$\mathbb{P}(C \mid X) = \frac{\mathbb{P}(X \mid C) \cdot \mathbb{P}(C)}{\mathbb{P}(X)} = \frac{\mathbb{P}(X \mid T, C) \cdot \mathbb{P}(T, C) + \mathbb{P}(X \mid \neg T, C) \cdot \mathbb{P}(\neg T, C)}{\mathbb{P}(X)}$$

Substituting in known probabilities, we get that

$$\mathbb{P}(T \mid X) = 0.187$$

$$\mathbb{P}(C \mid X) = 0.294$$

4.4 Conclusion

Given the positive result from the X-ray test, the inferred diagnosis remains most likely **bronchitis** with a probability of **61.8%**. Then is the **cancer**, which has a probability of **29.4%**. The probability of **tuberculosis** is the smallest, which is **18.7%**.

5 Question 5

Yes, the X-ray was needed.
- Before X-ray test :
  - **P(bronchitis)** = 61.8%
  - **P(cancer)** = 1.6%
  - **P(tuberculosis)** = 1%
- After X-ray test :

- **P(bronchitis)** = 61.8%
- **P(cancer)** = 29.4%
- **P(tuberculosis)** = 18.7%

The X-ray test provided critical information in this situation. Given the Bayesian network structure, a positive X-ray result pointed toward tuberculosis or cancer, diseases that were initially less likely. Thus, the X-ray was necessary to rule out or confirm these conditions, especially since tuberculosis and cancer cannot be ruled out solely by auscultation with a stethoscope. Therefore, the X-ray served as an essential diagnostic tool in refining the diagnosis. As the results above shows that the positive X-ray result significantly increases the probability of cancer and tuberculosis.