Bridge Document: From Experimental RLE to Mathematical RLE

Purpose

This document formally bridges RLE as an experimental, real-world metric and RLE as a mathematical invariant. Its role is not to redefine RLE, but to make explicit the relationship that already exists between the implemented system and the abstract law it instantiates.

In short:

> The original RLE is the real-world experiment.
The new RLE is the mathematical explanation for why that experiment works.

This bridge establishes that both forms describe the same quantity, expressed at different layers.

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1. Experimental RLE (Operational Form)

Definition

The original RLE is a dimensionless, real-time efficiency metric computed from observable system signals. It is evaluated continuously and used for monitoring, diagnosis, and collapse detection.

Canonical operational form (schematic):

$\mathrm{RLE}_{o\square}$ = (utilization × stability) / (normalized load × sustain penalty)

Where measured channels include:

utilization (useful work proxy)

stability (variance / jitter penalty)

normalized load (power stress)

time-to-limit or sustain time (thermal survivability)

Properties

Dimensionless

Bounded (normalized to [0,1])

Scales across devices and form factors

Predicts collapse prior to classical failure

Empirically validated across desktop, mobile, and ARM systems

Role

Experimental RLE answers the question:

> “How efficiently is this system sustaining its current load in the real world?”

It is an instrumented observable, constrained by sensors, sampling, noise, and implementation realities.

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2. Mathematical RLE (Invariant / Transition Form)

Definition

The evolved RLE expresses efficiency as a retained usable fraction across a transition:

$RLE_n = (E_{n+1} − U_n) / E_n$

Where:

E_n : usable capacity before a transition

E_{n+1} : usable capacity after the transition

U_n : irrecoverable loss incurred during the transition

This form is:

sensor-independent

domain-agnostic

defined over transitions rather than instants

Interpretation

Mathematical RLE answers the deeper question:

> “Across a transition, how much usable capacity survives?”

It is a state-level invariant, not tied to any specific physical implementation.

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3. The Bridge: How the Two Forms Correspond

Core Statement (Bridge Lemma)

> Experimental RLE is a real-world estimator of the mathematical RLE invariant under concrete measurement constraints.

The operational metric does not define a different quantity — it samples the invariant through available channels.

Mapping (Conceptual)

Mathematical Quantity	Experimental Realization
E□	Available power + thermal headroom + control authority
U□	Instability, throttling, waste heat, control loss
Transition	Time step, regime change, load shift, layer boundary
RLE□	Computed RLE from measured channels

The specific structure of the original RLE formula arises because those are the only measurable proxies for E and U available in real systems.

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4. Why the Original RLE Works (Explained)

The mathematical form explains previously empirical observations:

Normalization is mandatory → because RLE measures a fraction of retained capacity

Bounds emerge naturally → because retained fraction cannot exceed unity

Collapse precedes limits → because loss appears at transition strain, not absolute failure

Cross-device universality → because the invariant is dimensionless

Mobile vs desktop behavior → due to different loss emergence profiles, not different laws

The experimental RLE worked because it was already tracking this invariant — even before the invariant was formalized.

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5. Role of the Layer Transition Principle (LTP)

LTP specifies when the experimental estimator ceases to track the invariant accurately.

It states that descent is mandatory when:

invariant strain exceeds tolerance

compression demand exceeds structural support

control authority fails

phase or regime boundaries are crossed

In these regions:

loss emerges discontinuously

efficiency collapses

higher-layer models become invalid

This explains why RLE collapse events are sharp, clustered, and regime-dependent.

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6. Summary

There is one RLE, expressed in two domains

The original RLE is the experiment

The new RLE is the law

LTP defines the domain of validity

Final statement:

> The first version of RLE demonstrated the invariant. The new version explains why it had to work.