### Implementing Mitchell's Candidate Elimination Algorithm

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This report

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### 1 Introduction

Mitchell's candidate elimination algorithm (CEA) is an induction based system for learning a set of data and utilising this trained system for classifying new data.

The main concept in CEA is the version space, this is a set of hypotheses bounded by two sets, the general set (G) and the specific set (S). G contains the most general hypotheses that are consistent with the previously seen negative examples; while S contains the most specific hypotheses that cover all of the previously seen positive examples.

This generality and specificity introduce one important constraint on the hypothesis space of any problem that CEA is used to solve, it must be possible to say whether one hypothesis is more general or more specific than another and to be able to generate the set of more specific/general hypotheses from a single hypothesis.

Because of this constraint the hypothesis space can be viewed as a lattice, each hypothesis has a set of hypotheses that are one step more specific and a set of hypotheses that are one step more general. In the following implementation these are named descendant and ancestor hypotheses respectively.

To derive this version space from the training set a very simple algorithm can be used, this is very similar to the algorithm described in the handout with just a slight change to how hypotheses are generalised and specialised to better fit the data structure chosen, the overall result is identical:

```
1. Initialize set S to (null, null, ..., null), set G to (undefined, undefined, ..., undefined)
```

### 2. For each positive example d:

- a. Remove from G any hypothesis that fails to cover d
- b. While there is at least one hypothesis in S that fails to cover d
  - i. Remove all hypotheses s in S that fail to cover d
  - ii. Add all ancestors of each hypothesis s to S
  - iii. Remove from S any hypothesis that is more general than another in S

### 3. For each negative example d:

- a. Remove from S any hypothesis inconsistent with d
- b. While there is at least one hypothesis in G that is not consistent with d
  - i. Remove all hypotheses g in G that are not consistent with d
  - ii. Add all descendants of each hypothesis g to G
  - iii. Remove from G any hypothesis that is more specific than another in G

## **2** Results

### 2.1 Training

### 2.1.1 Using all examples

### **Recognising SOFT**

The starting version space is:

```
S: { astigmatic: null, tear_prod: null, age: null, prescription: null } G: { astigmatic: undefined, tear_prod: undefined, age: undefined, prescription: undefined }
```

The first example is a negative example and is covered by G. This causes G to be made more specific to ensure the case is no longer covered:

The next example is positive and is not covered by S. This causes S to be made more general to include the example and G to have all hypotheses that do not cover the new S to be removed:

The next example is negative and is consistent with G so the version space does not change:

The next example is negative and is inconsistent with G. This causes G to again be made more specific to not cover the example:

```
Example: {:astigmatic =>:yes, :tear_prod =>:normal, :age =>:young, :prescription =>:myope} ==> hard
Prior Classification: unknown
After Classification: negative
S: { astigmatic: no,
                               tear_prod: normal,
                                                       age: young,
                                                                         prescription: myope
                                                                                                     }
                                                      age: undefined, prescription: undefined
                            tear_prod: normal,
  { astigmatic: no,
  { astigmatic: undefined, tear_prod: normal,
                                                                        prescription: undefined
                                                      age: middle,
  { astigmatic: undefined, tear_prod: normal, { astigmatic: undefined, tear_prod: normal,
                                                                        prescription: undefined
                                                      age: old,
                                                      age: undefined, prescription: hyper
```

The next example is negative and consistent with G so nothing changes:

```
Example: {:astigmatic=>:no, :tear_prod=>:reduced, :age=>:young, :prescription=>:hyper} ==> none
Prior Classification: negative
After Classification: negative
S: { astigmatic: no,
                            tear_prod: normal,
                                                   age: young,
                                                                   prescription: myope
                                                                                             }
G:
                                                  age: undefined, prescription: undefined
  { astigmatic: no,
                          tear_prod: normal,
  { astigmatic: undefined, tear_prod: normal,
                                                  age: middle,
                                                                   prescription: undefined
  \{\ astigmatic:\ undefined\ ,\ tear\_prod:\ normal\ ,
                                                  age: old,
                                                                   prescription: undefined
  { astigmatic: undefined, tear_prod: normal,
                                                  age: undefined, prescription: hyper
   The next example is positive and not covered by S so S is made more general and the inconsistent
hypotheses in G are removed:
Example: {:astigmatic=>:no, :tear_prod=>:normal, :age=>:young, :prescription=>:hyper} ==> soft
Prior Classification: unknown
After Classification: positive
S: { astigmatic: no,
                            tear_prod: normal,
                                                   age: young,
                                                                   prescription: undefined }
G:
  { astigmatic: no,
                           tear_prod: normal,
                                                  age: undefined, prescription: undefined
  { astigmatic: undefined, tear_prod: normal,
                                                  age: undefined, prescription: hyper
```

The next example is negative and consistent with G so nothing changes:

The next example is negative and inconsistent with G so G is made more specific:

```
Example: {:astigmatic=>:yes, :tear_prod=>:normal, :age=>:young, :prescription=>:hyper} ==> hard
Prior Classification: unknown
After Classification: negative
                              tear_prod: normal,
S: \{ astigmatic: no, 
                                                                        prescription: undefined }
                                                      age: young,
G:
   astigmatic: no, tear_prod: normal, astigmatic: undefined, tear_prod: normal,
                                                     age: undefined, prescription: undefined
  { astigmatic: no,
                                                                       prescription: hyper
                                                     age: middle,
  { astigmatic: undefined, tear_prod: normal,
                                                     age: old,
                                                                       prescription: hyper
```

The next example is negative and consistent with G so the version space stays the same:

```
Example: {:astigmatic=>:no, :tear_prod=>:reduced, :age=>:middle, :prescription=>:myope} ==> none
Prior Classification: negative
After Classification: negative
S: { astigmatic: no,
                            tear_prod: normal,
                                                  age: young,
                                                                   prescription: undefined }
G:
  { astigmatic: no,
                           tear_prod: normal,
                                                  age: undefined, prescription: undefined
  { astigmatic: undefined, tear_prod: normal,
                                                  age: middle,
                                                                  prescription: hyper
  { astigmatic: undefined, tear_prod: normal,
                                                  age: old,
                                                                  prescription: hyper
```

The next example is positive and not covered by S so S is generalised and inconsistent hypotheses from G are removed. After this S and G only contain the same hypothesis so this version space has converged. Assuming the examples are consistent then neither S nor G will change from now on:

```
Example: {:astigmatic=>:no, :tear_prod=>:normal, :age=>:middle, :prescription=>:myope} ==> soft
Prior Classification: unknown
After Classification: positive
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
S: { astigmatic: no,
G: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
Example: {:astigmatic=>:yes, :tear_prod=>:reduced, :age=>:middle, :prescription=>:myope} ==> none
Prior Classification: negative
After Classification: negative
                            tear_prod: normal,
S: { astigmatic: no,
                                                  age: undefined, prescription: undefined
                                                  age: undefined, prescription: undefined
G: { astigmatic: no,
                            tear_prod: normal,
```

```
Example: {:astigmatic=>:yes, :tear_prod=>:normal, :age=>:middle, :prescription=>:myope} ==> hard
Prior Classification: negative
After Classification: negative
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
S: { astigmatic: no,
                                                  age: undefined, prescription: undefined }
G: { astigmatic: no,
                            tear_prod: normal.
Example: {:astigmatic=>:no, :tear_prod=>:reduced, :age=>:middle, :prescription=>:hyper} ==> none
Prior Classification: negative
After Classification: negative
                                                  age: undefined, prescription: undefined
S: { astigmatic: no,
                            tear_prod: normal,
G: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
Example: {:astigmatic=>:no, :tear_prod=>:normal, :age=>:middle, :prescription=>:hyper} ==> soft
Prior Classification: positive
After Classification: positive
S: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
G: { astigmatic: no,
Example: {:astigmatic=>:yes, :tear_prod=>:reduced, :age=>:middle, :prescription=>:hyper} ==> none
Prior Classification: negative
After Classification: negative
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined }
S: { astigmatic: no,
G: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
Example: {:astigmatic=>:yes, :tear_prod=>:normal, :age=>:middle, :prescription=>:hyper} ==> none
Prior Classification: negative
After Classification: negative
S: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
                            tear_prod: normal,
G: { astigmatic: no,
                                                  age: undefined, prescription: undefined
Example: {:astigmatic=>:no, :tear_prod=>:reduced, :age=>:old, :prescription=>:myope} ==> none
Prior Classification: negative
After Classification: negative
                                                  age: undefined, prescription: undefined
S: { astigmatic: no,
                            tear_prod: normal,
G: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
Example: {:astigmatic =>:yes, :tear_prod =>:reduced, :age =>:old, :prescription =>:myope} ==> none
Prior Classification: negative
After Classification: negative
S: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
                            tear_prod: normal,
G: { astigmatic: no,
                                                  age: undefined, prescription: undefined }
Example: {:astigmatic=>:yes, :tear_prod=>:normal, :age=>:old, :prescription=>:myope} ==> hard
Prior Classification: negative
After Classification: negative
S: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined }
G: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined }
Example: {:astigmatic=>:no, :tear_prod=>:reduced, :age=>:old, :prescription=>:hyper} ==> none
Prior Classification: negative
After Classification: negative
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
S: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined }
G: { astigmatic: no,
Example: {:astigmatic=>:no, :tear_prod=>:normal, :age=>:old, :prescription=>:hyper} ==> soft
Prior Classification: positive
After Classification: positive
                                                  age: undefined, prescription: undefined
S: { astigmatic: no,
                            tear_prod: normal,
G: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
```

```
Example: {:astigmatic=>:yes, :tear_prod=>:reduced, :age=>:old, :prescription=>:hyper} ==> none
Prior Classification: negative
After Classification: negative
S: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
G: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
Example: {:astigmatic=>:yes, :tear_prod=>:normal, :age=>:old, :prescription=>:hyper} ==> none
Prior Classification: negative
After Classification: negative
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
S: { astigmatic: no,
G: { astigmatic: no,
                            tear_prod: normal,
                                                  age: undefined, prescription: undefined
```

### **Recognising HARD**

The starting version spaces is:

```
S: { astigmatic: null, age: null, prescription: null, tear_prod: null } G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: undefined }
```

The first example is a negative example and covered by G, this causes G to be made more specific to no longer cover it:

```
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: unknown
After Classification: negative

S: { astigmatic: null, age: null, prescription: null, tear_prod: null }
G:
{ astigmatic: yes, age: undefined, prescription: undefined, tear_prod: undefined }
{ astigmatic: undefined, age: middle, prescription: undefined, tear_prod: undefined }
{ astigmatic: undefined, age: old, prescription: undefined, tear_prod: undefined }
{ astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: undefined }
{ astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: normal }
```

The next example is a negative example and covered by G as well, this causes G to be made more specific again so it no longer covers it:

```
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:myope, :tear_prod=>:normal} ==> soft
Prior Classification: unknown
After Classification: negative
                              age: null,
S: { astigmatic: null,
                                               prescription: null,
                                                                         tear_prod: null
                                                                                                 }
G:
                            age: undefined, prescription: undefined, tear_prod: undefined
  { astigmatic: yes,
  { astigmatic: yes, age: undefine { astigmatic: undefined, age: middle,
                                              prescription: undefined, tear_prod: undefined
  { astigmatic: undefined, age: old,
                                              prescription: undefined, tear_prod: undefined
  { astigmatic: undefined, age: undefined, prescription: hyper,
                                                                        tear_prod: undefined
```

Again the example is negative and covered by G so G is made more specific:

```
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: unknown

After Classification: negative

S: { astigmatic: null, age: null, prescription: null, tear_prod: null }
G:
{ astigmatic: undefined, age: middle, prescription: undefined, tear_prod: undefined }
{ astigmatic: undefined, age: old, prescription: undefined, tear_prod: undefined }
{ astigmatic: undefined, age: undefined, prescription: hyper, tear_prod: undefined }
{ astigmatic: yes, age: undefined, prescription: undefined, tear_prod: normal }
```

Finally we get a positive example, this isn't covered by S so S is generalised to cover it. Then any hypotheses in G that are inconsistent with S are removed:

```
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:myope, :tear_prod=>:normal} ==> hard
Prior Classification: unknown
After Classification: positive

S: { astigmatic: yes, age: young, prescription: myope, tear_prod: normal }
G: { astigmatic: yes, age: undefined, prescription: undefined, tear_prod: normal }
```

### Nothing changes as these are negative examples consistent with G:

```
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: negative
After Classification: negative
S: { astigmatic: yes,
                            age: young,
                                             prescription: myope,
                                                                      tear_prod: normal
                            age: undefined, prescription: undefined, tear_prod: normal
G: { astigmatic: yes,
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:hyper, :tear_prod=>:normal} ==> soft
Prior Classification: negative
After Classification: negative
S: { astigmatic: yes,
                                             prescription: myope,
                            age: young,
                                                                      tear_prod: normal
                            age: undefined, prescription: undefined, tear_prod: normal
G: { astigmatic: yes,
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: negative
After Classification: negative
S: { astigmatic: yes,
                            age: young,
                                             prescription: myope,
                                                                      tear_prod: normal
G: { astigmatic: yes,
                            age: undefined, prescription: undefined, tear_prod: normal
   A new positive example causes S to be made more general again:
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:hyper, :tear_prod=>:normal} ==> hard
Prior Classification: unknown
After Classification: positive
S: \{ astigmatic: yes, 
                                             prescription: undefined, tear_prod: normal
                            age: young,
                            age: undefined, prescription: undefined, tear_prod: normal
G: { astigmatic: yes,
   And some more negative examples consistent with G:
Example: {:astigmatic=>:no, :age=>:middle, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: negative
After Classification: negative
S: { astigmatic: yes,
                            age: young,
                                             prescription: undefined, tear_prod: normal
                            age: undefined, prescription: undefined, tear_prod: normal
G: { astigmatic: yes,
Example: {:astigmatic=>:no, :age=>:middle, :prescription=>:myope, :tear_prod=>:normal} ==> soft
Prior Classification: negative
After Classification: negative
S: { astigmatic: yes,
                            age: young,
                                             prescription: undefined, tear_prod: normal
                            age: undefined, prescription: undefined, tear_prod: normal
G: { astigmatic: yes,
Example: {:astigmatic=>:yes, :age=>:middle, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: negative
After Classification: negative
S: { astigmatic: yes,
                             age: young,
                                             prescription: undefined, tear_prod: normal
                            age: undefined, prescription: undefined, tear_prod: normal
G: { astigmatic: yes,
   A new positive example. S is again generalised and is now converged with G. If the dataset is
consistent then nothing will change from here on:
Example: {:astigmatic=>:yes, :age=>:middle, :prescription=>:myope, :tear_prod=>:normal} ==> hard
Prior Classification: unknown
After Classification: positive
                            age: undefined, prescription: undefined, tear_prod: normal
S: { astigmatic: yes,
                            age: undefined, prescription: undefined, tear_prod: normal
G: { astigmatic: yes,
Example: {:astigmatic =>:no, :age =>:middle, :prescription =>:hyper, :tear_prod =>:reduced} ==> none
Prior Classification: negative
After Classification: negative
                            age: \ undefined \ , \ prescription: \ undefined \ , \ tear\_prod: \ normal
S: { astigmatic: yes,
G: { astigmatic: yes,
                            age: undefined, prescription: undefined, tear_prod: normal
Example: {:astigmatic=>:no, :age=>:middle, :prescription=>:hyper, :tear_prod=>:normal} ==> soft
Prior Classification: negative
```

```
After Classification: negative
```

```
S: { astigmatic: yes, age: undefined, prescription: undefined, tear_prod: normal } G: { astigmatic: yes, age: undefined, prescription: undefined, tear_prod: normal } Example: {:astigmatic=>:yes, :age=>:middle, :prescription=>:hyper, :tear_prod=>:reduced} ==> none Prior Classification: negative

After Classification: negative

S: { astigmatic: yes, age: undefined, prescription: undefined, tear_prod: normal } G: { astigmatic: yes, age: undefined, prescription: undefined, tear_prod: normal }
```

But wait, the dataset is inconsistent. This is a negative example and was covered by G so G had to be made more specific. However this meant that the only hypothesis in S had to be removed to make S consistent with G. Because it was S that got wiped out our version space is now only usable for determining negative examples, there will be no false positives but there can be false negatives:

```
Example: {:astigmatic=>:yes, :age=>:middle, :prescription=>:hyper, :tear_prod=>:normal} ==> none
Prior Classification: positive
After Classification: negative
S:
G:
  { astigmatic: yes,
                            age: young,
                                            prescription: undefined, tear_prod: normal
  { astigmatic: yes,
                            age: old,
                                            prescription: undefined, tear_prod: normal
                            age: undefined, prescription: myope,
  { astigmatic: yes,
                                                                      tear_prod: normal
   And we're back to consistent negative examples:
Example: {:astigmatic=>:no, :age=>:old, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: negative
After Classification: negative
S:
G:
  { astigmatic: yes,
                           age: young,
                                            prescription: undefined, tear_prod: normal
  { astigmatic: yes,
                                            prescription: undefined, tear_prod: normal
                            age: old,
  { astigmatic: yes,
                            age: undefined, prescription: myope,
                                                                      tear prod: normal
Example: {:astigmatic=>:yes, :age=>:old, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: negative
After Classification: negative
S:
\mathbf{G}
```

Until we get to a positive one, notice that even after using this to train the classification is still unknown because S is empty:

age: undefined, prescription: myope,

prescription: undefined, tear\_prod: normal

prescription: undefined, tear\_prod: normal

tear prod: normal

```
Example: {:astigmatic=>:yes, :age=>:old, :prescription=>:myope, :tear_prod=>:normal} ==> hard
Prior Classification: unknown
After Classification: unknown

S:
G:
{ astigmatic: yes, age: old, prescription: undefined, tear_prod: normal }
{ astigmatic: yes, age: undefined, prescription: myope, tear_prod: normal }
```

### Three more consistent negative examples:

age: young,

age: old,

astigmatic: yes,

astigmatic: yes,

{ astigmatic: yes,

```
Example: {:astigmatic=>:no, :age=>:old, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: negative

After Classification: negative

S:
G:
{ astigmatic: yes, age: old, prescription: undefined, tear_prod: normal }
{ astigmatic: yes, age: undefined, prescription: myope, tear_prod: normal }
```

```
Prior Classification: negative
After Classification: negative
S:
G:
                            age: old, prescription: undefined, tear_prod: normal age: undefined, prescription: myope, tear_prod: normal
  { astigmatic: yes,
  { astigmatic: yes,
Example: {:astigmatic=>:yes, :age=>:old, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: negative
After Classification: negative
\mathbf{G}
  { astigmatic: yes,
                             age: old,
                                              prescription: undefined, tear_prod: normal
                             age: undefined, prescription: myope,
  { astigmatic: yes,
                                                                        tear_prod: normal
    And another inconsistent negative example to make G more specific and probably increase the num-
ber of false negatives:
Example: {:astigmatic=>:yes, :age=>:old, :prescription=>:hyper, :tear_prod=>:normal} ==> none
Prior Classification: unknown
After Classification: negative
G: { astigmatic: yes,
                            age: undefined, prescription: myope,
                                                                        tear_prod: normal
Recognising NONE
The starting version space is:
S: { astigmatic: null, age: null, prescription: null, tear_prod: null } G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: undefined }
    The first example is positive and not covered by S so S is made more general to cover it:
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: unknown
After Classification: positive
                             age: young,
                                              prescription: myope,
                                                                          tear_prod: reduced
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: undefined
    The next example is negative and inconsistent with G so G is made more specific to not cover it:
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:myope, :tear_prod=>:normal} ==> soft
Prior Classification: unknown
After Classification: negative
S: { astigmatic: no,
                             age: young,
                                               prescription: myope,
                                                                         tear_prod: reduced
                                                                                                 }
G:
  { astigmatic: yes,
                            age: undefined, prescription: undefined, tear_prod: undefined
  { astigmatic: undefined, age: middle, prescription: undefined, tear_prod: undefined
  { astigmatic: undefined, age: old,
                                              prescription: undefined , tear_prod: undefined
  { astigmatic: undefined, age: undefined, prescription: hyper,
                                                                        tear_prod: undefined
  { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
    Another positive example not covered by S causes S to be made more general and inconsistent
hypotheses from G to be removed:
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: unknown
After Classification: positive
S: { astigmatic: undefined, age: young,
                                             prescription: myope,
                                                                         tear_prod: reduced
                                                                                                 }
G:
                           age: undefined, prescription: undefined, tear_prod: undefined
  { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
```

A negative example inconsistent with G makes G more specific:

```
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:myope, :tear_prod=>:normal} ==> hard
Prior Classification: unknown
After Classification: negative
S: { astigmatic: undefined, age: young,
                                                                              prescription: myope,
                                                                                                                          tear_prod: reduced
                                                                                                                                                                   }
   { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
   { astigmatic: yes,
                                              age: middle,
                                                                             prescription: undefined, tear_prod: undefined prescription: undefined, tear_prod: undefined
   { astigmatic: yes,
                                                age: old,
   { astigmatic: yes,
                                                age: undefined, prescription: hyper,
                                                                                                                         tear_prod: undefined
       A positive example not covered by S so S is made more general and inconsistent hypotheses from G
are removed:
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: unknown
After Classification: positive
S: { astigmatic: undefined, age: young, prescription: undefined, tear_prod: reduced G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
      Some negative examples consistent with G and positive examples covered by S so nothing changes:
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:hyper, :tear_prod=>:normal} ==> soft
Prior Classification: negative
After Classification: negative
S: { astigmatic: undefined, age: young, prescription: undefined, tear_prod: reduced G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: positive
After Classification: positive
S: \{ \ astigmatic: \ undefined \ , \ age: \ young \ , \\ G: \{ \ astigmatic: \ undefined \ , \ age: \ undefined \ , \ prescription: \ undefined \ , \ tear\_prod: \ reduced \ , \ tear\_prod: \ , \ reduced \ , \ reduced
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:hyper, :tear_prod=>:normal} ==> hard
Prior Classification: negative
After Classification: negative
S: { astigmatic: undefined, age: young, prescription: undefined, tear_prod: reduced G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
      A positive example not covered by S so S is made more general. S and G are now converged:
Example: {:astigmatic=>:no, :age=>:middle, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: unknown
After Classification: positive
S: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
      Some more negative/consistent and positive/covered examples:
Example: {:astigmatic=>:no, :age=>:middle, :prescription=>:myope, :tear_prod=>:normal} ==> soft
Prior Classification: negative
After Classification: negative
S: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
Example: {:astigmatic=>:yes, :age=>:middle, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: positive
After Classification: positive
S: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
Example: {:astigmatic=>:yes, :age=>:middle, :prescription=>:myope, :tear_prod=>:normal} ==> hard
Prior Classification: negative
After Classification: negative
```

S: { astigmatic: undefined, age: undefined, prescription: undefined, tear\_prod: reduced

```
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
Example: {:astigmatic=>:no, :age=>:middle, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: positive
After Classification: positive
S: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
Example: {:astigmatic=>:no, :age=>:middle, :prescription=>:hyper, :tear_prod=>:normal} ==> soft
Prior Classification: negative
After Classification: negative
S{:}\ \{\ astigmatic:\ undefined\ ,\ age:\ undefined\ ,\ prescription:\ undefined\ ,\ tear\_prod:\ reduced
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
Example: {:astigmatic=>:yes, :age=>:middle, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: positive
After Classification: positive
S: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
                                                                                             }
Example: {:astigmatic=>:yes, :age=>:middle, :prescription=>:hyper, :tear_prod=>:normal} ==> none
```

And we hit an inconsistency, a positive examples that is not covered by G or S. To fix this S is made more general and G is blanked out as it was inconsistent with the new S:

```
Prior Classification: negative

After Classification: positive

S: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: undefined } G:

Example: {:astigmatic=>:no, :age=>:old, :prescription=>:myope, :tear_prod=>:reduced} ==> none

Prior Classification: positive

After Classification: positive

S: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: undefined } G:

Example: {:astigmatic=>:yes, :age=>:old, :prescription=>:myope, :tear_prod=>:reduced} ==> none

Prior Classification: positive

After Classification: positive

S: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: undefined } G:
```

And another inconsistency, a negative example that is now covered by S. This causes the offending hypothesis to be removed from S making this version space useless. With how the classification is set up this will now simply return negative for all examples:

```
Example: {:astigmatic=>:yes, :age=>:old, :prescription=>:myope, :tear_prod=>:normal} ==> hard
Prior Classification: positive
After Classification: negative
S:
G:
Example: {:astigmatic=>:no, :age=>:old, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: negative
After Classification: negative
S:
G:
Example: {:astigmatic=>:no, :age=>:old, :prescription=>:hyper, :tear_prod=>:normal} ==> soft
Prior Classification: negative
After Classification: negative
S:
G:
Example: {:astigmatic=>:yes, :age=>:old, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
```

```
Prior Classification: negative
After Classification: negative
G:
Example: {:astigmatic=>:yes, :age=>:old, :prescription=>:hyper, :tear_prod=>:normal} ==> none
Prior Classification: negative
After Classification: negative
S:
G
```

### 2.1.2 Using just 10 examples

### Recognising SOFT with just 10 examples

```
The starting version space:
```

```
S: { astigmatic: null, age: null, prescription: null, tear_prod: null }
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: undefined }
                                                                                                             tear_prod: null
```

The examples are exactly the same as the first 10 in Recognising SOFT:

```
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: unknown
After Classification: negative
S: { astigmatic: null,
                                 age: null,
                                                     prescription: null,
                                                                                   tear_prod: null
                                                                                                              }
G:
  { astigmatic: yes,
                               age: undefined, prescription: undefined, tear_prod: undefined
  { astigmatic: undefined, age: middle, prescription: undefined, tear_prod: undefined
  { astigmatic: undefined, age: old,
                                                    prescription: undefined, tear_prod: undefined
  { astigmatic: undefined, age: undefined, prescription: hyper,
                                                                                tear_prod: undefined
  { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: normal
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:myope, :tear_prod=>:normal} ==> soft
Prior Classification: unknown
After Classification: positive
S: \{ \ astigmatic: \ no, \qquad age: \ young, \qquad prescription: \ myope, \qquad tear\_prod: \ normal \\ G: \{ \ astigmatic: \ undefined, \ age: \ undefined, \ prescription: \ undefined, \ tear\_prod: \ normal \} \}
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: negative
After Classification: negative
S: { astigmatic: no, age: young, prescription: myope, tear_prod: normal G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: normal
                                                                                   tear_prod: normal
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:myope, :tear_prod=>:normal} ==> hard
Prior Classification: unknown
After Classification: negative
S: { astigmatic: no,
                                 age: young,
                                                     prescription: myope,
                                                                                  tear_prod: normal
                                                                                                              }
G:
  { astigmatic: no, age: undefined, prescription: undefined, tear_prod: normal astigmatic: undefined, age: middle, prescription: undefined, tear_prod: normal
                                                   prescription: undefined, tear_prod: normal
  { astigmatic: undefined, age: old,
                                                    prescription: undefined, tear_prod: normal
  { astigmatic: undefined, age: undefined, prescription: hyper,
                                                                                 tear_prod: normal
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: negative
After Classification: negative
S: { astigmatic: no,
                                 age: young,
                                                    prescription: myope,
                                                                                   tear_prod: normal
                                                                                                              }
G:
  { astigmatic: no, age: undefined, prescription: undefined, tear_prod: normal astigmatic: undefined, age: middle, prescription: undefined, tear_prod: normal astigmatic: undefined age: old prescription: undefined tear_prod: normal
  { astigmatic: undefined, age: old,
                                                   prescription: undefined, tear_prod: normal
  { astigmatic: undefined, age: undefined, prescription: hyper,
```

tear\_prod: normal

```
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:hyper, :tear_prod=>:normal} ==> soft
Prior Classification: unknown
After Classification: positive
                                            prescription: undefined, tear_prod: normal
S: { astigmatic: no,
                          age: young,
                                                                                            }
G:
                          age: undefined, prescription: undefined, tear_prod: normal
  { astigmatic: undefined, age: undefined, prescription: hyper,
                                                                    tear_prod: normal
Example: {:astigmatic =>:yes, :age =>:young, :prescription =>:hyper, :tear_prod =>:reduced} ==> none
Prior Classification: negative
After Classification: negative
                                            prescription: undefined, tear_prod: normal
S: { astigmatic: no,
                            age: young,
                                                                                            }
G:
  { astigmatic: no.
                          age: undefined, prescription: undefined, tear_prod: normal
  { astigmatic: undefined, age: undefined, prescription: hyper,
                                                                    tear_prod: normal
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:hyper, :tear_prod=>:normal} ==> hard
Prior Classification: unknown
After Classification: negative
S: { astigmatic: no,
                            age: young,
                                            prescription: undefined, tear_prod: normal
G:
  { astigmatic: no,
                         age: undefined, prescription: undefined, tear_prod: normal
  { astigmatic: undefined, age: middle,
                                           prescription: hyper,
                                                                     tear_prod: normal
  { astigmatic: undefined, age: old,
                                           prescription: hyper,
                                                                     tear_prod: normal
Example: {:astigmatic=>:no, :age=>:middle, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: negative
After Classification: negative
S: \{ astigmatic: no, 
                                            prescription: undefined, tear_prod: normal
                            age: young,
                                                                                            }
G:
                          age: undefined, prescription: undefined, tear_prod: normal
  { astigmatic: no.
  { astigmatic: undefined, age: middle, prescription: hyper,
                                                                    tear_prod: normal
  { astigmatic: undefined, age: old,
                                           prescription: hyper,
                                                                     tear_prod: normal
Example: {:astigmatic=>:no, :age=>:middle, :prescription=>:myope, :tear_prod=>:normal} ==> soft
Prior Classification: unknown
After Classification: positive
                            age: \ undefined \ , \ prescription: \ undefined \ , \ tear\_prod: \ normal
S: { astigmatic: no,
                            age: undefined, prescription: undefined, tear_prod: normal
G: { astigmatic: no,
```

This version space has ended up converged so any example that is covered by S/G will be classified as positive and any that is inconsistent will be negative.

### Recognising HARD with just 10 examples

```
The starting version space:
```

```
S: { astigmatic: null, age: null, prescription: null, tear_prod: null G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: undefined
```

```
The examples are exactly the same as the first 10 in Recognising HARD:
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: unknown
After Classification: negative
                               age: null,
                                                 prescription: null,
S: { astigmatic: null,
                                                                             tear_prod: null
                                                                                                      }
G:
  { astigmatic: yes, age: undefined, prescription: undefined, tear_prod: undefined { astigmatic: undefined, age: middle, prescription: undefined, tear_prod: undefined
  { astigmatic: undefined, age: old,
                                                prescription: undefined, tear_prod: undefined
  \{\ astigmatic:\ undefined\ ,\ age:\ undefined\ ,\ prescription:\ hyper\ ,
                                                                           tear_prod: undefined
  { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: normal
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:myope, :tear_prod=>:normal} ==> soft
Prior Classification: unknown
After Classification: negative
```

```
age: null,
                                                     prescription: null,
S: { astigmatic: null,
                                                                                   tear_prod: null
G:
  { astigmatic: yes, age: undefined, prescription: undefined, tear_prod: undefined astigmatic: undefined, age: middle, prescription: undefined, tear_prod: undefined astigmatic: undefined, age: old, prescription: undefined, tear_prod: undefined
  { astigmatic: undefined, age: undefined, prescription: hyper,
                                                                                   tear_prod: undefined
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: unknown
After Classification: negative
S: { astigmatic: null,
                                  age: null,
                                                     prescription: null,
                                                                                   tear_prod: null
                                                                                                               }
G:
  { astigmatic: undefined, age: middle,
                                                    prescription: undefined, tear_prod: undefined
  { astigmatic: undefined, age: old, prescription: undefined, tear_prod: undefined { astigmatic: undefined, age: undefined, prescription: hyper, tear_prod: undefined
  { astigmatic: yes,
                             age: undefined, prescription: undefined, tear_prod: normal
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:myope, :tear_prod=>:normal} ==> hard
Prior Classification: unknown
After Classification: positive
                                  age: young, prescription: myope, tear_prod: normal age: undefined, prescription: undefined, tear_prod: normal
S: { astigmatic: yes,
G: { astigmatic: yes,
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: negative
After Classification: negative
                                  age: young, prescription: myope, tear_prod: normal age: undefined, prescription: undefined, tear_prod: normal
S: { astigmatic: yes,
G: { astigmatic: yes,
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:hyper, :tear_prod=>:normal} ==> soft
Prior Classification: negative
After Classification: negative
                                  age: young, prescription: myope, tear_prod: normal age: undefined, prescription: undefined, tear_prod: normal
S: { astigmatic: yes,
G: { astigmatic: yes,
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: negative
After Classification: negative
S: { astigmatic: yes,
                                                     prescription: myope,
                                                                                    tear_prod: normal
                                  age: young,
                                  age: undefined, prescription: undefined, tear_prod: normal
G: { astigmatic: yes,
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:hyper, :tear_prod=>:normal} ==> hard
Prior Classification: unknown
After Classification: positive
                                  age: young, prescription: undefined, tear_prod: normal age: undefined, prescription: undefined, tear_prod: normal
S: { astigmatic: yes,
G: { astigmatic: yes,
Example: {:astigmatic =>:no, :age =>:middle, :prescription =>:myope, :tear_prod =>:reduced} ==> none
Prior Classification: negative
After Classification: negative
                                  age: young, prescription: undefined, tear_prod: normal age: undefined, prescription: undefined, tear_prod: normal
S: { astigmatic: yes,
G: { astigmatic: yes,
Example: {:astigmatic=>:no, :age=>:middle, :prescription=>:myope, :tear_prod=>:normal} ==> soft
Prior Classification: negative
After Classification: negative
                                  age: young, prescription: undefined, tear_prod: normal age: undefined, prescription: undefined, tear_prod: normal
S: { astigmatic: yes,
G: { astigmatic: yes,
```

The final version space is not converged, this means that there are three possible outputs from classification:

positive	If the example is covered by <i>S</i> it is definitely positive.	
unknown	If the example is consistent with $G$ but is not covered by $S$ then it is unknown,	
	in this case this is only 4 examples: the ones with astigmatic: yes, age: old or	
	middle, prescription: either and tear_prod: normal.	
	This case is assumed to be a weak positive.	
negative	If the example is inconsistent with $G$ then it is negative.	

### Recognising NONE with just 10 examples

 $S: \{ astigmatic: undefined, age: young, \}$ 

```
The starting version space:
S: { astigmatic: null,
                               age: null,
                                                 prescription: null,
                                                                           tear_prod: null
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: undefined
    The examples are exactly the same as the first 10 in Recognising NONE:
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: unknown
After Classification: positive
                               age: young,
                                                 prescription: myope,
S: { astigmatic: no,
                                                                              tear_prod: reduced
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: undefined
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:myope, :tear_prod=>:normal} ==> soft
Prior Classification: unknown
After Classification: negative
S: { astigmatic: no,
                                                 prescription: myope,
                                                                              tear prod: reduced
                                                                                                       }
                               age: young,
G:
                             age: undefined, prescription: undefined, tear_prod: undefined
  { astigmatic: yes,
  { astigmatic: undefined, age: middle, prescription: undefined, tear_prod: undefined
  { astigmatic: undefined, age: old, prescription: undefined, tear_prod: undefined { astigmatic: undefined, age: undefined, prescription: hyper, tear_prod: undefined { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: unknown
After Classification: positive
S: { astigmatic: undefined, age: young,
                                                 prescription: myope,
                                                                              tear_prod: reduced
  { astigmatic: yes, age: undefined, prescription: undefined, tear_prod: undefined { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:myope, :tear_prod=>:normal} ==> hard
Prior Classification: unknown
After Classification: negative
S: { astigmatic: undefined, age: young,
                                                 prescription: myope,
                                                                              tear_prod: reduced
                                                                                                       }
G:
  { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
  { astigmatic: yes,
                              age: middle,
                                                prescription: undefined, tear_prod: undefined
  { astigmatic: yes,
                                                prescription: undefined , tear_prod: undefined
                              age: old,
  { astigmatic: yes,
                              age: undefined, prescription: hyper,
                                                                            tear_prod: undefined
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: unknown
After Classification: positive
S: \{ astigmatic: undefined, age: young, \\ G: \{ astigmatic: undefined, age: undefined, prescription: undefined, tear\_prod: reduced \} \}
Example: {:astigmatic=>:no, :age=>:young, :prescription=>:hyper, :tear_prod=>:normal} ==> soft
Prior Classification: negative
After Classification: negative
```

G: { astigmatic: undefined, age: undefined, prescription: undefined, tear\_prod: reduced

prescription: undefined, tear\_prod: reduced

```
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:hyper, :tear_prod=>:reduced} ==> none
Prior Classification: positive
After Classification: positive
S: { astigmatic: undefined, age: young, prescription: undefined, tear_prod: reduced G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
                                                                                                  }
Example: {:astigmatic=>:yes, :age=>:young, :prescription=>:hyper, :tear_prod=>:normal} ==> hard
Prior Classification: negative
After Classification: negative
S: { astigmatic: undefined, age: young,
                                               prescription: undefined, tear_prod: reduced
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
Example: {:astigmatic=>:no, :age=>:middle, :prescription=>:myope, :tear_prod=>:reduced} ==> none
Prior Classification: unknown
After Classification: positive
S: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
Example: {:astigmatic=>:no, :age=>:middle, :prescription=>:myope, :tear_prod=>:normal} ==> soft
Prior Classification: negative
After Classification: negative
S: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced
```

This version space has ended up converged so any example that is covered by S/G will be classified as positive and any that is inconsistent will be negative.

### 2.2 Classifying

### 2.2.1 The Version Spaces

Two are converged (soft, none) while hard isn't:

```
For case: soft, Version space is:

S: { astigmatic: no, age: undefined, prescription: undefined, tear_prod: normal } 
G: { astigmatic: no, age: undefined, prescription: undefined, tear_prod: normal } 
====

For case: none, Version space is:

S: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced } 
G: { astigmatic: undefined, age: undefined, prescription: undefined, tear_prod: reduced } 
====

For case: hard, Version space is:

S: { astigmatic: yes, age: young, prescription: undefined, tear_prod: normal } 
G: { astigmatic: yes, age: undefined, prescription: undefined, tear_prod: normal } 
}
```

### 2.2.2 The Classifications

The first example is correctly classified as none as that is the only version space that returned positive:

```
For Example: {:astigmatic=>:yes, :age=>:middle, :prescription=>:myope, :tear_prod=>:reduced} => none

soft (conv) classifies as: negative
none (conv) classifies as: positive
hard (unco) classifies as: negative
Class: none
```

The second example is classified as don't know because only the unconverged version space returned positive:

```
For Example: {:astigmatic=>:yes, :age=>:middle, :prescription=>:myope, :tear_prod=>:normal} => hard soft (conv) classifies as: negative none (conv) classifies as: negative hard (unco) classifies as: unknown Class: don't know
```

The third example is correctly classified as only the none version space returned positive:

```
For Example: {:astigmatic=>:no, :age=>:middle, :prescription=>:hyper, :tear_prod=>:reduced}
=> none
soft (conv) classifies as: negative
none (conv) classifies as: positive
hard (unco) classifies as: negative
Class: none
```

The fourth example is correctly classified as only the soft version space returned positive:

The fifth example is correctly classified as only the none version space returned positive:

```
For Example: {:astigmatic=>:yes, :age=>:middle, :prescription=>:hyper, :tear_prod=>:reduced} => none

soft (conv) classifies as: negative
none (conv) classifies as: positive
hard (unco) classifies as: negative
Class: none
```

The sixth example is classified as don't know as only the hard version space returned positive (unknown is a weak positive):

```
For Example: {:astigmatic=>:yes, :age=>:middle, :prescription=>:hyper, :tear_prod=>:normal} => none

soft (conv) classifies as: negative
none (conv) classifies as: negative
hard (unco) classifies as: unknown
Class: don't know
```

The seventh example is correctly classified as only the none version space returned positive:

```
For Example: {:astigmatic=>:no, :age=>:old, :prescription=>:myope, :tear_prod=>:reduced}
=> none
soft (conv) classifies as: negative
none (conv) classifies as: positive
hard (unco) classifies as: negative
Class: none
```

The eighth example is correctly classified as only the none version space returned positive:

```
For Example: {:astigmatic=>:yes, :age=>:old, :prescription=>:myope, :tear_prod=>:reduced} => none
soft (conv) classifies as: negative
none (conv) classifies as: positive
hard (unco) classifies as: negative
Class: none
```

The ninth example is classified as don't know as only the non converged version space classified it:

```
For Example: {:astigmatic=>:yes, :age=>:old, :prescription=>:myope, :tear_prod=>:normal} => hard

soft (conv) classifies as: negative
none (conv) classifies as: negative
hard (unco) classifies as: unknown

Class: don't know
```

The tenth example is classified correctly since only the none version space returned positive:

```
For Example: {:astigmatic=>:no, :age=>:old, :prescription=>:hyper, :tear_prod=>:reduced} => none

soft (conv) classifies as: negative
none (conv) classifies as: positive
hard (unco) classifies as: negative
Class: none
```

The eleventh example is classified correctly as only the soft version space returned positive:

```
For Example: {:astigmatic=>:no, :age=>:old, :prescription=>:hyper, :tear_prod=>:normal} => soft (conv) classifies as: positive
```

```
none (conv) classifies as: negative
hard (unco) classifies as: negative
Class: soft
```

The twelfth example is classified correctly as only the none version space returned positive:

```
For Example: {:astigmatic=>:yes, :age=>:old, :prescription=>:hyper, :tear_prod=>:reduced}
=> none

soft (conv) classifies as: negative
none (conv) classifies as: positive
hard (unco) classifies as: negative
Class: none
```

The thirteenth example is classified as don't know as only the hard version space returned positive:

```
For Example: {:astigmatic=>:yes, :age=>:old, :prescription=>:hyper, :tear_prod=>:normal} => none

soft (conv) classifies as: negative
none (conv) classifies as: negative
hard (unco) classifies as: unknown
Class: don't know
```

# 3 Conclusion