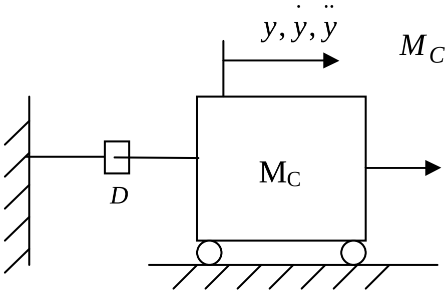
ENEL430 Assignment (Due Monday 12th September by 5 pm)

System:

A cart is put into motion by a force, *F(t)*, that is commanded by a voltage, *V*.



$$M_C \ddot{y} = F(t)$$

$$F(t) = \bar{\beta} V - \bar{C} \dot{y}$$

where $\bar{\beta}$ is the proportionality constant relating input voltage to translational force provided by a DC motor and gears, and $\bar{C}$ is the net damping of the system, which includes the motor back EMF and mechanical friction. The overall system is written in the form:

$$\ddot{y} = \beta V - C \dot{y}, \quad \beta = \frac{\bar{\beta}}{M_C}, \quad C = \frac{\bar{C}}{M_C} \tag{1}$$

The goal is to do a system identification on this cart system to find the voltage constant β and the damping constant C. To allow an effective frequency response a proportional controller is considered. Setting $V = K_p(r - y)$ in Equation (1) yields:

$$\ddot{y} + C\dot{y} + \beta K_p y = \beta K_p r(t), \quad K_p = 60, \quad r(t) = 0.06\sin(\omega t) \tag{2}$$

The constants in Equation (2) are chosen to ensure a sufficiently high voltage (around 5-10 V) so that the effects of static friction and other non-linearities are minimized in the physical system. A frequency response experiment is performed for the following frequencies:

$$\omega = 2\pi f, \quad f = 0.6, 0.8, 1.0, \ldots, 2.8 \tag{3}$$

And the output displacement $y(t)$ (metres) is synchronized with the reference input $r(t)$ in Equation (2) so that time can be assumed to start at $t = 0$ for any given frequency. Several cycles of the data are saved for each frequency in .mat files. For the purpose of an analytical solution, Equation (2) is written in the form:

$$\ddot{y} + C\dot{y} + Ky = \bar{K}\sin(\omega t), \quad K = \beta K_p, \quad \bar{K} = 0.06\beta K_p \tag{4}$$

The analytical solution for Equation (4) is the model y_{model} of the system and is defined:

$$y_{model} = A_1\cos(\omega t) + A_2\sin(\omega t), \quad A_1 = -\frac{\bar{K}C\omega}{C^2\omega^2 + \omega^4 - 2\omega^2 K + K^2}, \quad A_2 = -\frac{\bar{K}(\omega^2 - K)}{C^2\omega^2 + \omega^4 - 2\omega^2 K + K^2} \tag{5}$$

The parameters A_1 and A_2 in Equation (5) can be rearranged (as in notes) to yield:

$$C = -\frac{A_1 \bar{K}}{\omega(A_1^2 + A_2^2)}, \quad K = \frac{\bar{K} A_2}{A_1^2 + A_2^2} + \omega^2 \tag{6}$$

Matlab assignment:

1 Given the inputs C, β, ω and t_{end}, create code for computing the analytical solution of Equation (5). Use the time interval $t = [0, t_{end}]$ and evaluate y at 1 kHz.

2 Compute the transfer function $G = \frac{Y}{R}$ of Equation (2), and using the command lsim, check your solution in part 1 for C=5, beta=1, Kp=60 and $f = 1.8$, $t_{end} = 3$. Explain any differences that occur.

3 (a) Given general input data y_data sampled at 1kHz, create a function for identifying the numerical values of the parameters C and β in Equation (4).

(b) Check the answer of (a) using simulated "measured data" from question 1 based on the inputs in question 2, with 0, 5, 10, 15, 20, …, 40% normally distributed noise, but choose $t_{end} = 1$. Are the identifications of C and K significantly affected by noise? In each case do 100 simulations, and report the 90% confidence interval (CI) of C and β and the median in a table. Also present the results in a graph (with the amount of noise on x axis) with errorbars as in notes. Explain the results.

4 (a) Consider the measured cart response, with f=1.8 Hz and Kp=60. By using the function developed and tested in simulation in question 3, calculate the values of C and β in Equation (2) that "best fit" the data (using linear least squares). The resulting values will correspond to a system identification of the cart dynamics.

(b) Check how good the resulting model of (a) is at predicting the cart response over all the other frequency responses. Compare by first computing the absolute error relative to the mean absolute displacement for each frequency to give 12 error vectors: error1, error2, …, error12. Hence, do an errorbar plot over all the frequencies. Finally, define a total error metric as:

$$error = [error1, error2, ..., error12]$$

Report the median and 90% CI for this error vector. Also do a histogram to see the shape of the distribution. Explain the results.

5 **Extra credit:**

(a) Compute the bode plot (||G|| versus ω and arg(G) versus ω) of the measured cart response. [Hint: fit waveforms of the form of Equation (5), to get "average" magnitude and phase for each set of data]. Using the identified parameters C and β of question 4 (a) (with f=1.8 Hz), compute the model predicted bode plot. Use relative error with respect to mean magnitude and phase as a metric for comparing the measured versus model bode plots.

(b) Identify the parameters C and β for all measured frequency responses of the cart system. This data corresponds to values of ω given in Equation (3). Plot the identified C and β versus ω, and compute the relative error in the magnitude and phase versus ω. Explain the results. What is the best ω to use? Suggest some reasons why it performs the best? What's the worst ω and why?