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Controlling a Rocket

ENEL430 Assignment Two

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1 Introduction

In order to correctly design controllers for complex systems such as a rocket an accurate model must be created. One major method for creating this model is known as System Identification. System Identification is about utilising statistical methods to build mathematical models of dynamical systems from measured data.

The system used in this report consisted of a vertical wind tunnel containing a 1.4m long rocket suspended by a string. This allowed the rocket to freely rotate with negligible extraneous damping while wind flowed past at up to 110 km/hr.

The form of the model in relation to the roll rate ($\dot{\theta}$) was found to be:

$$\ddot{\theta} = -a_{\text{final}}\dot{\theta} + b_{\text{final}}u_{\text{f}} + f_0 \tag{1}$$

With the measured fin angle (u_{f}) obeying a non-linear function (F) in terms of the commanded fin angle (u_{cmd}).

$$u_{\text{f}} = F\left(u_{\text{cmd}}\right) \tag{2}$$

$$u_{\text{cmd}} = k_p\left(R_\theta - \theta\right) + k_d\left(-\dot{\theta}\right) \tag{3}$$

R_θ is the set point for the controller.

This can be converted into a more detailed model:

$$\ddot{\theta} = -a_{\text{final}}\dot{\theta} + b_{\text{final}}y_1 + f_0 \tag{4}$$

$$\dot{u}_{\text{cmd}} = -k_p\dot{\theta} - k_d\ddot{\theta} \tag{5}$$

$$\dot{y}_1 = y_2 \tag{6}$$

$$\dot{y}_2 = -C_1y_2 - K_1y_1 + C_2\left(-k_p\dot{\theta} - k_d\ddot{\theta}\right) + K_2\left(k_p\left(R_\theta - \theta\right) - k_dr\right) \tag{7}$$

With this model three controllers were designed. The first was designed with a relatively high k_p gain and low k_d gain in order to induce oscillations. The second was aimed at an optimal response and the third was another attempt at an optimal response with higher gains.

2 Results and Discussion

The three controllers were designed using a provided model with $C_1 = 6.3965$, $C_2 = 6.3141$, $K_1 = 1.2494$, $K_2 = 1.7439$ and $\beta = 2.0193$. The values found were:

Controller	k_p	k_d
One	1.00	0.20
Two	0.37	0.48
Three	0.80	0.60

Figure 1 shows the response of the first controller. The model appears to have much less damping than the measured data. This is likely partly from saturation of the fin angle. The controller on the rocket limits the fin angle to ± 20 degrees, viewing the output of the fin angle it can be seen that these limits are being hit a lot while this controller was running.

Figure 2 shows the response of the second controller, this is much better than the first. In fact the error in the model is less than 2% compared to over 20% for the first model.

Figure 3 shows the response of the third controller. This is not as good as the second controller – at around 6% error – but it is still much better than the first one. The rise time on this controller was quite a bit better than the second controller however, although not as much better as the models were predicting.

The fact that all models had faster responses than the measured data indicates that the real rocket had higher damping than the models were using. An attempt at finding better a_{final} and b_{final} values was performed using a brute force system based off the first controllers response. This resulted in values of $a_{\text{final}} = 0.89$ and $b_{\text{final}} = 2.2$ with the associated model responses shown in red on the figures. The errors in the three model responses with these values was 6%, 11% and 3%.

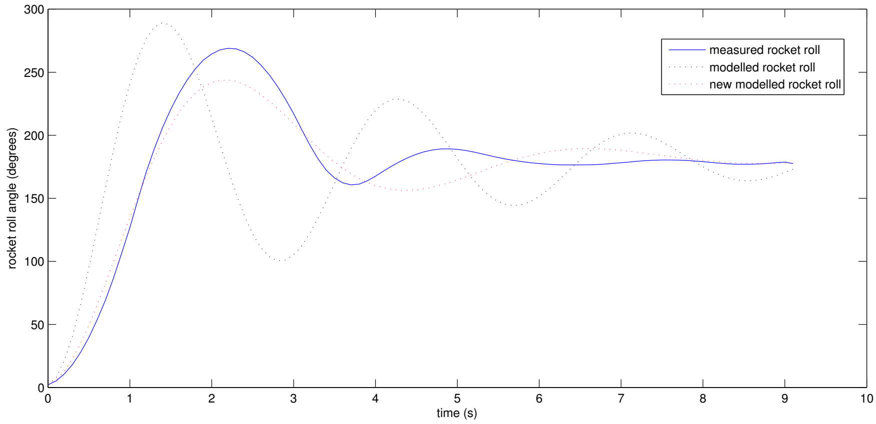

Figure 1: Response of first controller

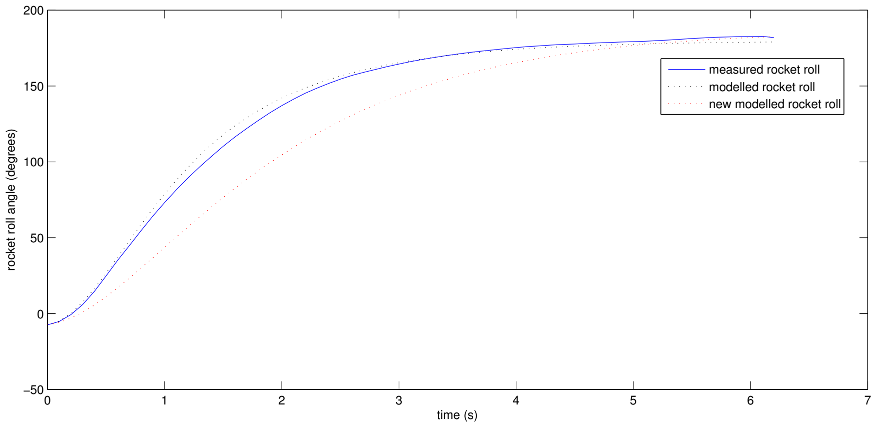

Figure 2: Response of second controller

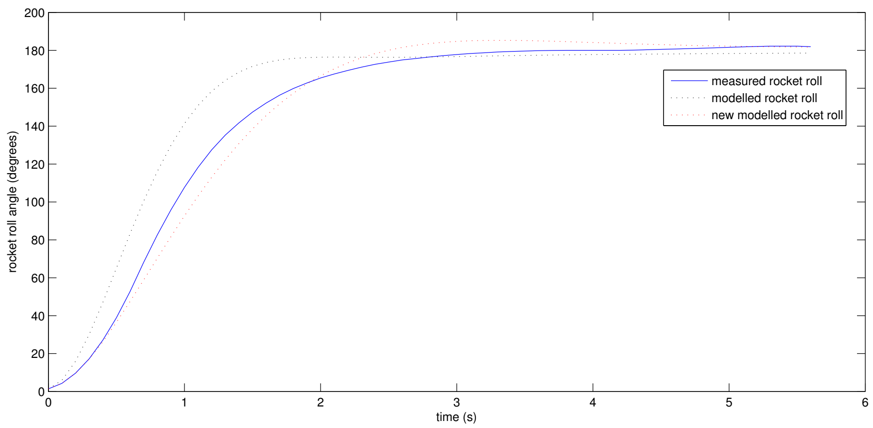

Figure 3: Response of third controller