Problem 1

* Current Stock Price 151.03
* Strike Price 165
* Current Date 03/13/2022
* Options Expiration Date 04/15/2022
* Risk Free Rate of 4.25%
* Continuously Compounding Coupon of 0.53%

Implement the closed form Greeks for GBSM. Implement a finite difference derivative calculation. Compare the values between the two methods for both a call and a put.
Implement the binomial tree valuation for American options with and without discrete dividends. Assume the stock above:

* Pays dividend on 4/11/2022 of $0.88

Calculate the value of the call and the put. Calculate the Greeks of each.
What is the sensitivity of the put and call to a change in the dividend amount?

Answer:
Through programming, we have obtained the Greeks calculated by closed form and finite difference using GBSM as below:

	Closed Form, GBSM		Finite Difference, GBSM	
	Call	Put	Call	Put
Delta	0.0830	-0.9166	0.0830	-0.9165
Gamma	0.0168	0.0168	0.0168	0.0168
Vega	6.9387	6.9387	6.9387	6.9387
Theta	-8.1265	-1.9409	-8.1263	-1.9408
Rho	1.1026	-13.7580	-0.0304	-1.2427
Carry Rho	1.1330	-12.5153	1.1330	-12.5153

As can be obtained from the table, Delta, Gamma, Vega, Theta, and Carry Rho for both of the methods are quite similar, but Rho calculated using closed form and finite difference are very different.

Then, I calculated the option values using binomial tree model with and without dividends, assuming $0.88 were paid as dividend on 4/11/2022.

	Without Discrete Dividends		With Discrete Dividends	
Option Type	Call	Put	Call	Put
Option Value	0.336	14.037	0.299	14.557

As can be obtained from the table, option values calculated without discrete dividends and with discrete dividends are quite similar.

Also, we have obtained the Greeks calculated by finite difference using binomial tree model, assuming with dividends as below:

	Finite Difference, Binomial Tree	
	Call	Put
Delta	0.0726	-0.9383
Gamma	-8.8818	1.3287
Vega	6.3194	5.6755
Theta	-7.4679	-0.4490

Rho	-0.0244	-1.1608
Carry Rho	0.9627	-11.3111

As can be obtained from the table, the Greeks calculated by finite difference using binomial tree model and those calculated by finite difference using GBSM are quite similar.

And sensitivity of put and call by finite difference to a change in the dividend amount are as below: for call it is -0.021, for put it is 0.941.

Problem 2

Using the options portfolios from Problem3 last week (named problem2.csv in this week's repo) and assuming:
* American Options
* Current Date 03/03/2023
* Current AAPL price is 165
* Risk Free Rate of 4.25%
* Dividend Payment of 1.00 on 3/15/2023

Using DailyPrices.csv. Fit a Normal distribution to AAPL returns – assume 0 mean return. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES.
Calculate VaR and ES using Delta-Normal.
Present all VaR and ES values a dollar loss, not percentages.
Compare these results to last week's results.

Answer:

I simulated 10-days-ahead AAPL returns using normal distribution and applied to the current AAPL price. The portfolio's values of mean, VaR, and ES through direct simulation are as follows:

Portfolio	Mean	VaR	ES
Call	1.074	6.001	6.230
CallSpread	-0.148	3.863	4.068
CoveredCall	-2.122	15.329	18.254
ProtectedPut	1.048	7.614	7.833
Put	2.263	4.257	4.547
PutSpread	0.755	2.584	2.790
Stock	-0.883	19.123	22.133
Straddle	3.337	0.004	0.005
SynLong	-1.189	20.195	23.372

The portfolio's values of mean, VaR, and ES through delta-normal are as follows:

Portfolio	Mean	VaR	ES
Call	1.567	-0.021	-0.005
CallSpread	0.104	1.088	2.003
CoveredCall	-1.609	5.839	7.287
ProtectedPut	1.681	0.018	0.021
Put	1.860	-0.023	-0.006
PutSpread	0.540	-0.006	0.071
Stock	0.000	0.000	0.000
Straddle	3.427	-0.047	-0.012
SynLong	-0.293	0.927	1.043

The portfolio's values of mean, VaR, and ES using AR(1) through direct simulation are as follows:

Portfolio	Mean	VaR	ES
Call	0.446017	6.016327	6.444773
CallSpread	-0.084402	3.509378	3.863553
CoveredCall	-0.118143	10.699119	13.954988

ProtectedPut	0.553295	7.766353	8.538461
Put	0.245108	5.133167	5.497061
PutSpread	0.171525	2.501650	2.732848
Stock	0.392643	14.833202	18.290305
Straddle	0.691126	1.594988	1.600536
SynLong	0.200909	14.994298	18.436803

As can be seen from the tables above, VaR and ES are the lowest through delta normal, which means this method bears the least risk.

Problem 3

Use the Fama French 3 factor return time series (F-F_Research_Data_Factors_daily.CSV) as well as the Carhart Momentum time series (F-F_Momentum_Factor_daily.CSV) to fit a 4 factor model to the following stocks.

AAPL	FB	UNH	MA
MSFT	NVDA	HD	PFE
AMZN	BRK-B	PG	XOM
TSLA	JPM	V	DIS
GOOGL	JNJ	BAC	CSCO

Fama stores values as percentages, you will need to divide by 100 (or multiply the stock returns by 100) to get like units.
Based on the past 10 years of factor returns, find the expected annual return of each stock.
Construct an annual covariance matrix for the 10 stocks.
Assume the risk-free rate is 0.0425. Find the super-efficient portfolio.

Answer:
Based on the past 10 years of factor returns, the annual returns of each stock are as follow:

Stock	Annual Return	Stock	Annual Return
AAPL	0.157	PG	0.082
META	0.018	XOM	0.522
UNH	0.254	TSLA	-0.033
MA	0.223	JPM	0.098
MSFT	0.156	V	0.241
NVDA	0.280	DIS	-0.155
HD	0.121	GOOGL	-0.017
PFE	0.077	JNJ	0.124
AMZN	-0.043	BAC	-0.112
BRK-B	0.130	CSCO	0.147

Sharpe ratio of super-efficient portfolio is 1.47 and the weights of super-efficient portfolio are as follows:

Stock	Weight	Stock	Weight
AAPL	0.00	PG	0.00
META	0.00	XOM	57.44
UNH	22.57	TSLA	0.00
MA	0.00	JPM	0.00
MSFT	0.00	V	12.93
NVDA	0.00	DIS	0.00
HD	0.00	GOOGL	0.00
PFE	0.00	JNJ	7.05
AMZN	0.00	BAC	0.00
BRK-B	0.00	CSCO	0.00