Problem 1

Assume you a call and a put option with the following:

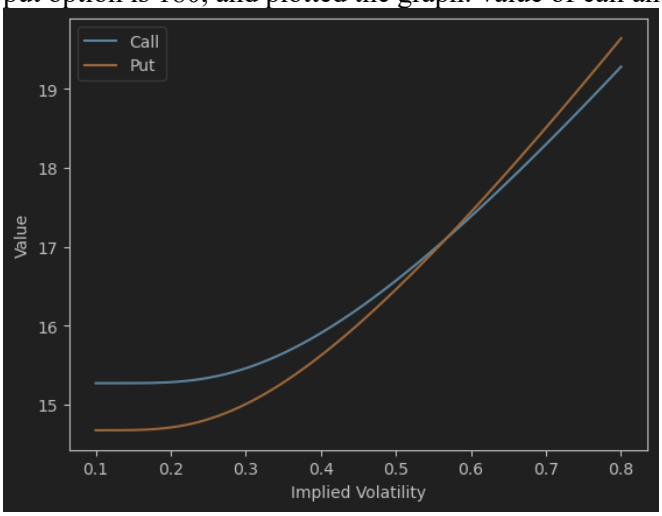
- Current Stock Price $165
- Current Date 03/03/2023
- Options Expiration Date 03/17/2023
- Risk Free Rate of 5.25%
- Continuously Compounding Coupon of 0.53%

Calculate the time to maturity using calendar days (not trading days). For a range of implied volatilities between 10% and 80%, plot the value of the call and the put. Discuss these graphs. How does the supply and demand affect the implied volatility?

Answer:

Firstly, I calculated the time to maturity. Since the current date is 03/03/2023, expiration date is 03/17/2023, and 365 days in 2023, the time to maturity is 0.0384.

Then I change the implied volatility between 10% and 80%, set strike price for call option is 150 and for put option is 180, and plotted the graph: value of call and put versus different implied volatility.



It can be seen that as implied volatility increase, the value of call and put option will increase as well. From the perspective of supply and demand, this is because an increase in implied volatility indicates an increase in the uncertainty or riskiness of the underlying asset. This, in turn, increases the potential payoff of the option and makes it more valuable to investors, who are willing to pay a higher price for the option. Additionally, an increase in implied volatility often leads to an increase in demand for options as investors seek to hedge against potential losses or speculate on potential gains. This increased demand for options can also drive up their value.

Also, when demand for an asset in the market exceeds its supply, the price increases and implied volatility also increases. This is because buyers are willing to pay higher prices to acquire the asset, which increases the market's expectations of future price fluctuations and leads to an increase in implied volatility. Conversely, when supply exceeds demand, the price drops and implied volatility decreases.

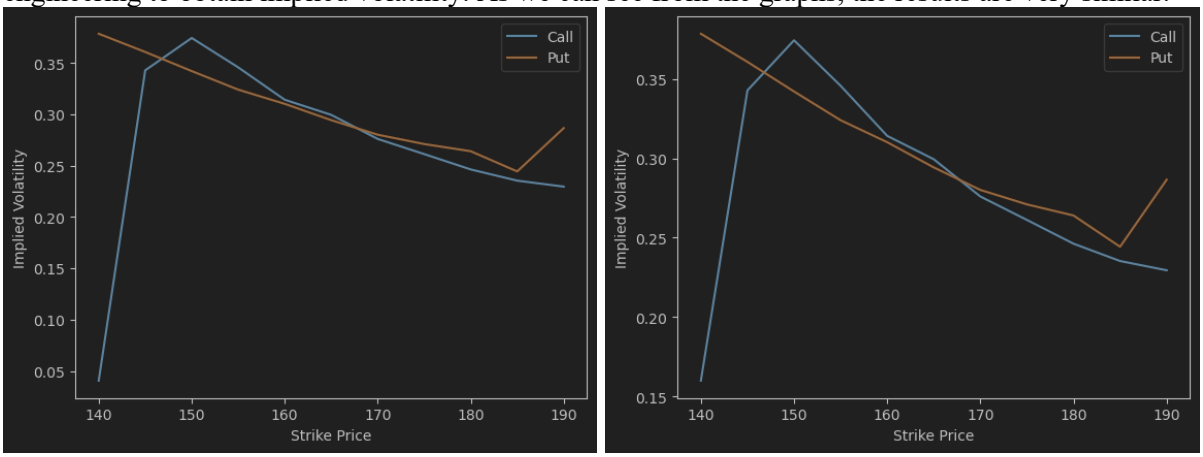
Problem 2

Use the options found in AAPL_Options.csv:
- Current AAPL price is 170.15
- Current Date: 10/30/2023
- Risk Free Rate: 5.25%
- Dividend Rate: 0.57%.

Calculate the implied volatility for each option. Plot the implied volatility vs the strike price for Puts and Calls. Discuss the shape of these graphs. What market dynamics could make these graphs? There are bonus points available on this question based on your discussion. Take some time to research if needed.

Answer:

I used the method that is like the root finder in Julia to find implied volatilities and plotted the implied volatility vs the strike price for Puts and Calls. The left-side graph is obtained using fsolve from scipy package, while the right-side graph is obtained using brentq, also from scipy package, both using reversed engineering to obtain implied volatility. As we can see from the graphs, the results are very similar.



It can be demonstrated from the graphs that the implied volatility for call options rises first before it falls as the strike price increases, while the implied volatility for put options falls first before it rises as the strike price increases.

The pattern we are describing in the implied volatility across different strike prices is known as the "volatility smile" for call options and a "volatility smirk" for put options. These phenomena occur due to various market dynamics and trader expectations, such as market psychology and risk aversion, leverage effect, supply and demand dynamics, stochastic volatility models, and impact of asymmetric information.

Market Psychology and Risk Aversion: Investors are generally more concerned about market crashes or rapid declines than they are about equivalent upward movements. This risk aversion can lead to higher premiums (and thus higher IV) for put options, particularly for those that are OTM, as they serve as insurance against market drops.

Leverage Effect: When a company's stock price falls, its leverage increases if it has debt, making the stock riskier. This increased risk can cause the IV of put options to increase, as the downside risk is now greater.

Supply and Demand Dynamics: The supply and demand for options at different strike prices can also affect IV. If many traders are buying OTM options for speculative reasons or as a hedge, the increased demand can drive up the IV.

Stochastic Volatility Models: Advanced models of option pricing consider that volatility is not constant and can change unpredictably. This stochastic nature of volatility can lead to the observed patterns in IV across strike prices.

Impact of Asymmetric Information: Some investors in the market may have more or more accurate information than other investors. This information can lead to changes in the demand or supply of certain

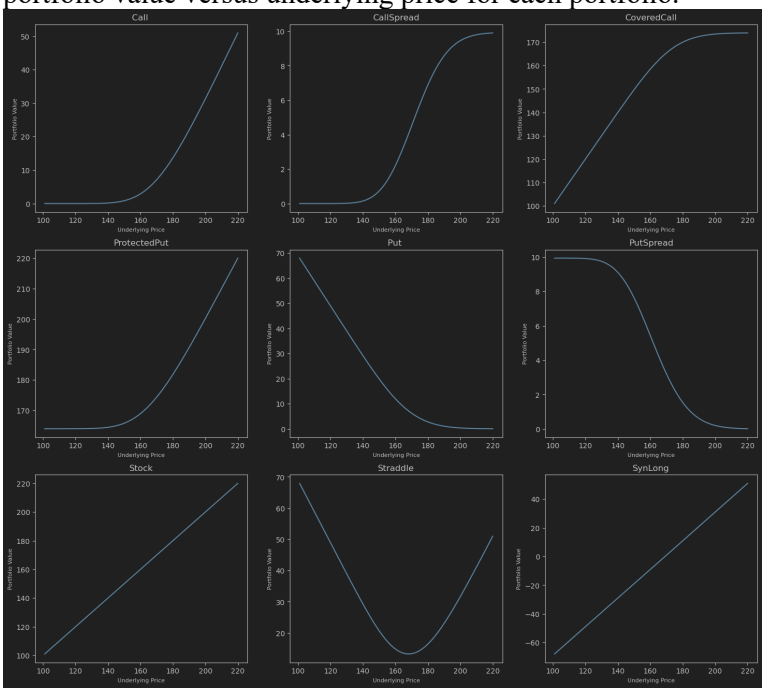
options in the market, thereby affecting the implied volatility of these options. Especially in times of high market risk, investors are more likely to be concerned about uncertain information, increasing the demand for lower strike put options and resulting in higher implied volatility for these options and the phenomenon of volatility smile.

Problem 3

Use the portfolios found in problem3.csv:
● Current AAPL price is 170.15
● Current Date: 10/30/2023
● Risk Free Rate: 5.25%
● Dividend Rate: 0.57%.
For each of the portfolios, graph the portfolio value over a range of underlying values. Plot the portfolio values and discuss the shapes. Bonus points available for tying these graphs to other topics discussed in the lecture.
Using DailyPrices.csv. Calculate the log returns of AAPL. Demean the series so there is 0 mean. Fit an AR(1) model to AAPL returns. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES. Discuss.
Hints:
● you will need to calculate the implied volatility - might not be the same as #2
● you need to take into account the change in dates for option valuations. You are simulating forward in time and options valuations are a function of time
● Calculate the PL from the current portfolio value using Current Date

Answer:

Since current AAPL price is 170.15, I chose the range of 100 – 220 for underlying prices and plotted portfolio value versus underlying price for each portfolio.



Call: The value of a call option rises slowly at first, then linearly when the underlying asset's price rises.
CallSpread: The value of a call spread rises slowly at first, then linearly, then slowly again as the underlying asset's price rises.
CoveredCall: The value of a covered call option rises linearly at first, then slowly, as the underlying asset's price rises.
Put: The value of a put option declines linearly at first, then gradually as the underlying asset's price rises.
ProtectedPut: The value of a protected put option rises slowly at first, then linearly when the underlying asset's price rises.

PutSpread: The value of a put spread declines gradually at first, then linearly, and then gradually again as the underlying asset's price rises.
Stock: The value of a stock is proportional to the underlying asset's price.
Straddle: The value of a straddle option initially declines, then increases when the underlying asset price rises.
Synlong: The value of a synthetic long position is positively correlated with the underlying asset's price.
The graph below is the simulation of portfolio features after simulating returns 10 days ahead:

Portfolio	Mean	VaR	ES
Call	0.446017	6.016327	6.444773
CallSpread	-0.084402	3.509378	3.863553
CoveredCall	-0.118143	10.699119	13.954988
ProtectedPut	0.553295	7.766353	8.538461
Put	0.245108	5.133167	5.497061
PutSpread	0.171525	2.501650	2.732848
Stock	0.392643	14.833202	18.290305
Straddle	0.691126	1.594988	1.600536
SynLong	0.200909	14.994298	18.436803

Call and Put are simple options with moderate risks.
CallSpread and PutSpread are options strategies that add a limit to ordinary Call and Put options, minimizing risk while losing possible gains.
CoveredCall and ProtectedPut strategies use a combination of stocks and options and follow similar patterns to Call and Put options. The risks are comparable to those of stocks and options.
SynLong and Stock: SynLong follows the same simple pattern as Stock, but with double returns. Both are fraught with danger.
Straddle: A particular portfolio that can profit as long as the stock price moves, independent of direction. In comparison to the other portfolios, the risk is low.