Problem 1

Calculate and compare the expected value and standard deviation of price at time t (Pt), given each of the 3 types of price returns, assuming $rt \sim N(0, \sigma 2)$. Simulate each return equation using $rt \sim N(0, \sigma 2)$ and show the mean and standard deviation match your expectations.

Answer:

First, given 3 types of prices returns, I calculated the expected mean and standard deviation based on formula, assuming rt ~ (0, sigma), and there only being time t and t-1.

For Classic Brownian Motion:

$$P_t = P_{t-1} + r_t$$

Mean:

$$E(P_t) = P_{t-1}$$

Standard deviation:

$$std(P_t) = sigma$$

For Arithmetic Return System:

$$P_t = P_{t-1}(1 + r_t)$$

Mean:

$$E(P_t) = P_{t-1}$$

Standard deviation:

$$std(P_t) = P_{t-1} * sigma$$

For Geometric Brownian Motion:

$$P_t = P_{t-1} * e^{r_t}$$

Mean:

$$E(P_t) = P_{t-1} * e^{\frac{sigma^2}{2}}$$

Standard deviation:

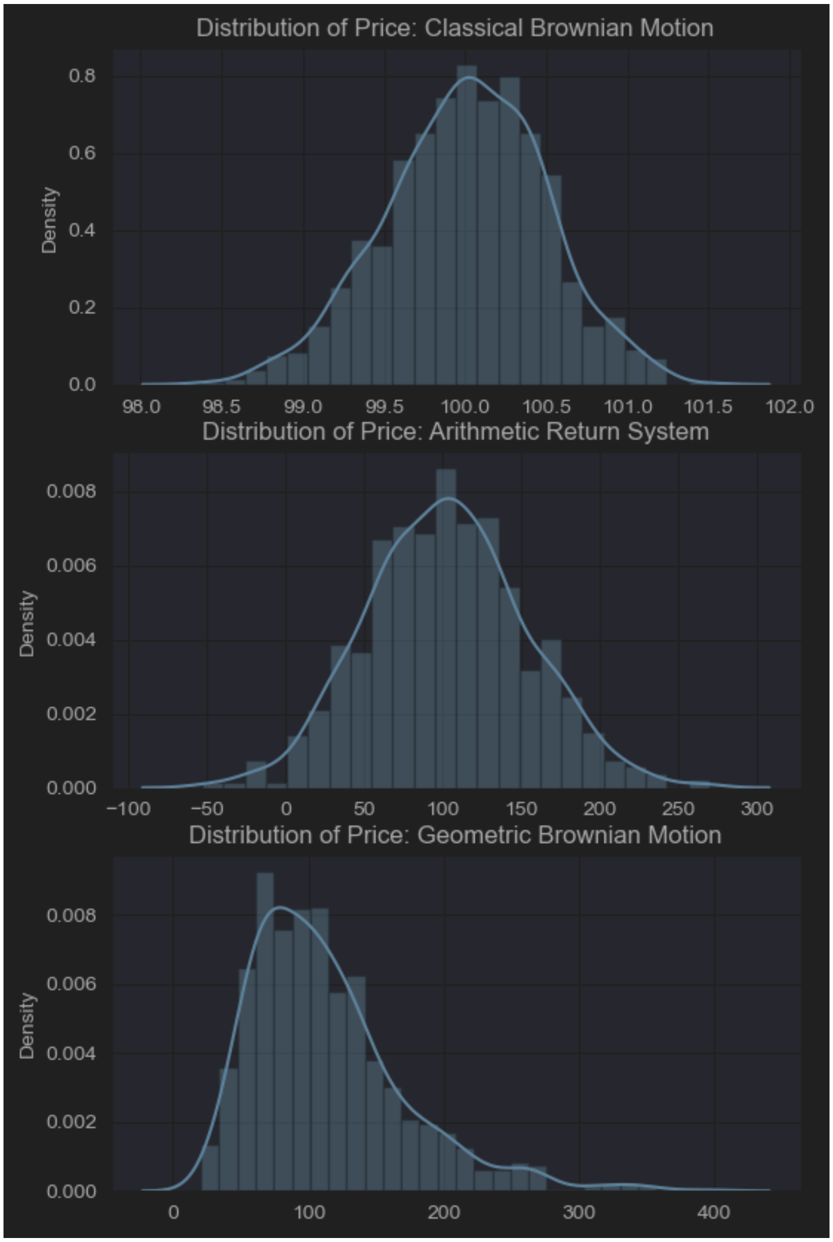
$$std(P_t) = P_{t-1}[(e^{\frac{sigma^2}{2}} - 1)e^{\frac{sigma^2}{2}}]^{\frac{1}{2}}$$

Then, I set Pt-1 = 1, std(rt) = 0.5, and simulated for 1000 times. The results of simulated parameters and expected parameters are below:

	Mean		Standard deviation	
	Expected Mean	Simulated Mean	Expected Std	Simulated Std
Classical Brownian	1.0000	1.0138	0.5000	0.5150
Arithmetic Return	1.0000	0.9901	0.5000	0.4796
Geometric Brownian	1.1331	1.1058	0.6039	0.6049

As can be obtained from the graph, the results of simulation are very similar to the expected values, as the differences are very small.

And below is the plotted distribution of the simulated Pt using three methods:


Distribution of Price: Classical Brownian Motion
Density
0.0
0.2
0.4
0.6
0.8
98.0
98.5
99.0
99.5
100.0
100.5
101.0
101.5
102.0
Distribution of Price: Arithmetic Return System
Density
0.000
0.002
0.004
0.006
0.008
−100
−50
0
50
100
150
200
250
300
Distribution of Price: Geometric Brownian Motion
Density
0.000
0.002
0.004
0.006
0.008
0
100
200
300
400

Problem 2

Implement a function similar to the "return_calculate()" in this week's code. Allow the user to specify the method of return calculation. Use DailyPrices.csv. Calculate the arithmetic returns for all prices. Remove the mean from the series so that the mean(META)=0

Calculate VaR

1. Using a normal distribution.
2. Using a normal distribution with an Exponentially Weighted variance ($\lambda = 0.94$)
3. Using a MLE fitted T distribution.
4. Using a fitted AR(1) model.
5. Using a Historic Simulation.

Compare the 5 values.

Answer:

First, I created a function as was instructed in Julia "return_calculate()" to calculate the returns of all stocks in DailyPrice.csv, taking all prices, method designated, and "Date" column, returning a data frame having stock name, date and return.

Then, I extracted the return data of META and removed its mean from every return. VaR was calculated based on centered return using 5 methods:

1. Using a normal distribution.
2. Using a normal distribution with an Exponentially Weighted variance ($\lambda = 0.94$)
3. Using a MLE fitted T distribution.
4. Using a fitted AR(1) model.
5. Using a Historic Simulation.

Below are VaR in percentage:

Method	VaR in Percentage
Normal Distribution	5.2412%
Norma Distribution with EW Variance	3.0177%
MLE fitted T- Distribution	4.5809%
AR(1)	5.7664%
Historical Simulation	3.9484%

Below are VaR in dollar amount:

Method	VaR in Dollar Amount
Normal Distribution	14.8418
Norma Distribution with EW Variance	8.1918
MLE fitted T- Distribution	12.8671
AR(1)	16.4126
Historical Simulation	10.9754

As can be obtained from the tables, VaR calculated by exponentially weighted variance is the lowest, while that of AR(1) is the highest. VaR calculated by MLE fitted T – distribution and calculated by historical distribution are quite similar, while that of normal distribution and AR(1) are quite similar. VaR calculated by normal distribution with EW variance is much lower than the rest VaRs, this may be because there are relative small changes in prices (more stable than average) in most recent time, of which that method attaches more significance to, or maybe just because less information was obtained in that method due to w_lambda.

Problem 3

Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.
This file contains the stock holdings of 3 portfolios. You own each of these portfolios. Using an exponentially weighted covariance with lambda = 0.94, calculate the VaR of each portfolio as well as your total VaR (VaR of the total holdings). Express VaR as a $.
Discuss your methods and your results. Choose a different model for returns and calculate VaR again. Why did you choose that model? How did the model change affect the results?

Answer:

First, I used delta normal VaR with exponentially weighted covariance lambda = 0.94, this method assumes payoffs are linear and returns are distributed multivariate normal. Alpha was set to be 0.05, and the last price was considered to be the latest price to calculate asset value or simulation (if simulation is required).

Below are the results for delta normal VaR:

	VaR in Dollar Amount
Portfolio A	15426.968
Portfolio B	8082.572
Portfolio C	18163.292
Portfolio Total	38941.376

However, it is more often that first, assumption of normality of returns does not hold, as during irrational market periods like financial crisis returns can be very extreme; and second, the linear assumption of portfolio value with returns does not hold, though in this particular set we are just using stocks and not options or bonds, such assumption poses many restrictions on our further applications. These situations undermine the validity of delta normal VaR, thus other methods should be used.

Thus, we decided to opt for historical distribution simulation, in which we used real historical data to obtain VaR. And since we don't consider certain historical period is much more important than others, we did not add weights to historical values.

Below are the results for historical simulated VaR:

	VaR in Dollar Amount
Portfolio A	16525.829
Portfolio B	10757.91
Portfolio C	21981.393
Portfolio Total	47618.778

As can be obtained from the tables, the results of historical VaRs are higher than those of delta normal VaR using exponentially weighted covariance. This may be due to the fact that more data is obtained in historical methods, which makes it more precise, or EW covariance attaches more importance to more recent data and thus some features of earlier data are omitted in delta normal with EW covariance.