**Problem 1**

\* Current Stock Price 151.03

\* Strike Price 165

\* Current Date 03/13/2022

\* Options Expiration Date 04/15/2022

\* Risk Free Rate of 4.25%

\* Continuously Compounding Coupon of 0.53%

Implement the closed form Greeks for GBSM. Implement a finite difference derivative calculation.

Compare the values between the two methods for both a call and a put.

Implement the binomial tree valuation for American options with and without discrete dividends. Assume the stock above:

\* Pays dividend on 4/11/2022 of $0.88

Calculate the value of the call and the put. Calculate the Greeks of each.

What is the sensitivity of the put and call to a change in the dividend amount?

**Answer:**

Through programming, we have obtained the Greeks calculated by closed form and finite difference using GBSM as below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Closed Form, GBSM | | Finite Difference, GBSM | |
| Call | Put | Call | Put |
| Delta | 0.0830 | -0.9166 | 0.0830 | -0.9165 |
| Gamma | 0.0168 | 0.0168 | 0.0168 | 0.0168 |
| Vega | 6.9387 | 6.9387 | 6.9387 | 6.9387 |
| Theta | -8.1265 | -1.9409 | -8.1263 | -1.9408 |
| Rho | 1.1026 | -13.7580 | -0.0304 | -1.2427 |
| Carry Rho | 1.1330 | -12.5153 | 1.1330 | -12.5153 |

As can be obtained from the table, Delta, Gamma, Vega, Theta, and Carry Rho for both of the methods are quite similar, but Rho calculated using closed form and finite difference are very different.

Then, I calculated the option values using binomial tree model with and without dividends, assuming $0.88 were paid as dividend on 4/11/2022.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Without Discrete Dividends | | With Discrete Dividends | |
| Option Type | Call | Put | Call | Put |
| Option Value | 0.336 | 14.037 | 0.299 | 14.557 |

As can be obtained from the table, option values calculated without discrete dividends and with discrete dividends are quite similar.

Also, we have obtained the Greeks calculated by finite difference using binomial tree model, assuming with dividends as below:

|  |  |  |
| --- | --- | --- |
|  | Finite Difference, Binomial Tree | |
|  | Call | Put |
| Delta | 0.0726 | -0.9383 |
| Gamma | -8.8818 | 1.3287 |
| Vega | 6.3194 | 5.6755 |
| Theta | -7.4679 | -0.4490 |
| Rho | -0.0244 | -1.1608 |
| Carry Rho | 0.9627 | -11.3111 |

As can be obtained from the table, the Greeks calculated by finite difference using binomial tree model and those calculated by finite difference using GBSM are quite similar.

And sensitivity of put and call by finite difference to a change in the dividend amount are as below: for call it is -0.021, for put it is 0.941.

**Problem 2**

Using the options portfolios from Problem3 last week (named problem2.csv in this week’s repo) and assuming:

\* American Options

\* Current Date 03/03/2023

\* Current AAPL price is 165

\* Risk Free Rate of 4.25%

\* Dividend Payment of 1.00 on 3/15/2023

Using DailyPrices.csv. Fit a Normal distribution to AAPL returns – assume 0 mean return. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES.

Calculate VaR and ES using Delta-Normal.

Present all VaR and ES values a dollar loss, not percentages.

Compare these results to last week’s results.

**Answer:**

I simulated 10-days-ahead AAPL returns using normal distribution and applied to the current AAPL price. The portfolio’s values of mean, VaR, and ES through direct simulation are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Portfolio | Mean | VaR | ES |
| Call | 1.074 | 6.001 | 6.230 |
| CallSpread | -0.148 | 3.863 | 4.068 |
| CoveredCall | -2.122 | 15.329 | 18.254 |
| ProtectedPut | 1.048 | 7.614 | 7.833 |
| Put | 2.263 | 4.257 | 4.547 |
| PutSpread | 0.755 | 2.584 | 2.790 |
| Stock | -0.883 | 19.123 | 22.133 |
| Straddle | 3.337 | 0.004 | 0.005 |
| SynLong | -1.189 | 20.195 | 23.372 |

The portfolio’s values of mean, VaR, and ES through delta-normal are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Portfolio | Mean | VaR | ES |
| Call | 1.567 | -0.021 | -0.005 |
| CallSpread | 0.104 | 1.088 | 2.003 |
| CoveredCall | -1.609 | 5.839 | 7.287 |
| ProtectedPut | 1.681 | 0.018 | 0.021 |
| Put | 1.860 | -0.023 | -0.006 |
| PutSpread | 0.540 | -0.006 | 0.071 |
| Stock | 0.000 | 0.000 | 0.000 |
| Straddle | 3.427 | -0.047 | -0.012 |
| SynLong | -0.293 | 0.927 | 1.043 |

The portfolio’s values of mean, VaR, and ES using AR(1) through direct simulation are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Portfolio | Mean | VaR | ES |
| Call | 0.446017 | 6.016327 | 6.444773 |
| CallSpread | -0.084402 | 3.509378 | 3.863553 |
| CoveredCall | -0.118143 | 10.699119 | 13.954988 |
| ProtectedPut | 0.553295 | 7.766353 | 8.538461 |
| Put | 0.245108 | 5.133167 | 5.497061 |
| PutSpread | 0.171525 | 2.501650 | 2.732848 |
| Stock | 0.392643 | 14.833202 | 18.290305 |
| Straddle | 0.691126 | 1.594988 | 1.600536 |
| SynLong | 0.200909 | 14.994298 | 18.436803 |

As can be seen from the tables above, VaR and ES are the lowest through delta normal, which means this method bears the least risk.

**Problem 3**

Use the Fama French 3 factor return time series (F-F\_Research\_Data\_Factors\_daily.CSV) as well as the Carhart Momentum time series (F-F\_Momentum\_Factor\_daily.CSV) to fit a 4 factor model to the following stocks.

|  |  |  |  |
| --- | --- | --- | --- |
| AAPL | FB | UNH | MA |
| MSFT | NVDA | HD | PFE |
| AMZN | BRK-B | PG | XOM |
| TSLA | JPM | V | DIS |
| GOOGL | JNJ | BAC | CSCO |

Fama stores values as percentages, you will need to divide by 100 (or multiply the stock returns by 100) to get like units.

Based on the past 10 years of factor returns, find the expected annual return of each stock.

Construct an annual covariance matrix for the 10 stocks.

Assume the risk-free rate is 0.0425. Find the super-efficient portfolio.

**Answer:**

Based on the past 10 years of factor returns, the annual returns of each stock are as follow:

|  |  |  |  |
| --- | --- | --- | --- |
| Stock | Annual Return | Stock | Annual Return |
| AAPL | 0.157 | PG | 0.082 |
| META | 0.018 | XOM | 0.522 |
| UNH | 0.254 | TSLA | -0.033 |
| MA | 0.223 | JPM | 0.098 |
| MSFT | 0.156 | V | 0.241 |
| NVDA | 0.280 | DIS | -0.155 |
| HD | 0.121 | GOOGL | -0.017 |
| PFE | 0.077 | JNJ | 0.124 |
| AMZN | -0.043 | BAC | -0.112 |
| BRK-B | 0.130 | CSCO | 0.147 |

Sharpe ratio of super-efficient portfolio is 1.47 and the weights of super-efficient portfolio are as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Stock | Weight | Stock | Weight |
| AAPL | 0.00 | PG | 0.00 |
| META | 0.00 | XOM | 57.44 |
| UNH | 22.57 | TSLA | 0.00 |
| MA | 0.00 | JPM | 0.00 |
| MSFT | 0.00 | V | 12.93 |
| NVDA | 0.00 | DIS | 0.00 |
| HD | 0.00 | GOOGL | 0.00 |
| PFE | 0.00 | JNJ | 7.05 |
| AMZN | 0.00 | BAC | 0.00 |
| BRK-B | 0.00 | CSCO | 0.00 |