CBGT model equations

1. STN

$$
\begin{aligned}
C_{\mathrm{m}}\dot{\boldsymbol{v}} &= -I_{\mathrm{L}} - I_{\mathrm{K}} - I_{\mathrm{Na}} - I_{\mathrm{T}} - I_{\mathrm{Ca}} - I_{\mathrm{AHP}} - I_{\mathrm{G\to S}} \\
I_{\mathrm{L}} &= g_{\mathrm{L}}(\boldsymbol{v} - v_{\mathrm{L}}) \\
I_{\mathrm{K}} &= g_{\mathrm{K}}\boldsymbol{n}^4(\boldsymbol{v} - v_{\mathrm{K}}) \\
I_{\mathrm{Na}} &= g_{\mathrm{Na}} m_\infty^3 \boldsymbol{h}(\boldsymbol{v} - v_{\mathrm{Na}}) \\
I_{\mathrm{T}} &= g_{\mathrm{T}} a_\infty^3 b_\infty^2(\boldsymbol{v} - v_{\mathrm{Ca}}) \\
I_{\mathrm{Ca}} &= g_{\mathrm{Ca}} s_\infty^2(\boldsymbol{v} - v_{\mathrm{Ca}}) \\
b_\infty &= \left(1 + \exp\left(\frac{\boldsymbol{r} - \theta_b}{\sigma_b}\right)^{-1}\right) - \left(1 + \exp\left(-\frac{\theta_b}{\sigma_b}\right)\right)^{-1} \\
x_\infty &= \left(1 + \exp\left(-\frac{\boldsymbol{v} - \theta_x}{\sigma_x}\right)\right)^{-1} && , x \in \{n, m, h, a, r, s\} \\
&\begin{cases} \dot{\boldsymbol{x}} = \Phi_x \dfrac{x_\infty - x}{\tau_x} \\ \tau_x = \tau_x^0 + \tau_x^1\left(1 + \exp\left(-\dfrac{v - \theta_x^\tau}{\sigma_x^\tau}\right)\right) \end{cases} && , x \in \{n, h, r\} \\
I_{\mathrm{AHP}} &= g_{\mathrm{AHP}}(v - v_{\mathrm{K}})\frac{[\mathrm{Ca}]}{[\mathrm{Ca}] + k_1} \\
\dot{[\mathrm{Ca}]} &= \varepsilon(-I_{\mathrm{Ca}} - I_{\mathrm{T}} - k_{\mathrm{Ca}}[\mathrm{Ca}]) \\
\dot{s} &= \alpha H_\infty(v - \theta_g)(1 - s) - \beta s \\
H_\infty(v) &= \left(1 + \exp\left(-\frac{v - \theta_g^H}{\sigma_g^H}\right)\right)^{-1} \\
I_{\mathrm{G\to S}} &= g_{\mathrm{G\to S}}(\boldsymbol{v} - v_{\mathrm{G\to S}}) \sum_{j \in \mathrm{GPe}} w_j \boldsymbol{s_j}
\end{aligned}
$$

2. GPe

Same as STN except

$$
\begin{aligned}
C_{\mathrm{m}}\dot{\boldsymbol{v}} &= -I_{\mathrm{L}} - I_{\mathrm{K}} - I_{\mathrm{Na}} - I_{\mathrm{T}} - I_{\mathrm{Ca}} - I_{\mathrm{AHP}} - I_{\mathrm{S\to G}} - I_{\mathrm{G\to G}} + I_{\mathrm{app}} \\
I_{\mathrm{T}} &= g_{\mathrm{T}} a_\infty^3 \boldsymbol{r}(\boldsymbol{v} - v_{\mathrm{Ca}}) \\
\dot{\boldsymbol{r}} &= \Phi_r \frac{r_\infty - \boldsymbol{v}}{\tau_r}
\end{aligned}
$$

3. STDP

$$
\begin{aligned}
\dot{p}_{\mathrm{pre}} &= -\frac{p_{\mathrm{pre}}}{\tau_{\mathrm{pre}}} + \delta(t - t_{\mathrm{spike}}) \\
\dot{p}_{\mathrm{post}} &= -\frac{p_{\mathrm{post}}}{\tau_{\mathrm{post}}} + \delta(t - t_{\mathrm{spike}}) \\
\delta(t) &= \begin{cases} 0 & \text{if } x \neq 0 \\ \infty & \text{if } x = 0 \end{cases} \qquad , \int_{-\infty}^{\infty} \delta(t)dt = 1
\end{aligned}
$$

For a weight from neuron $i \in I$ to neuron $j \in J$

$$\Delta w_{i,j} = \begin{cases} A^{J}_{\text{pre}}\ \boldsymbol{p}^{\boldsymbol{i}}_{\mathbf{pre}} & \text{if } t = t^{i}_{\text{spike}} \\ -A^{J}_{\text{post}}\ \boldsymbol{p}^{\boldsymbol{j}}_{\mathbf{post}} & \text{if } t = t^{i}_{\text{spike}} \\ 0 & \text{otherwise} \end{cases}$$

Or in matrix form

$$\Delta W = \eta\left(A^{J}_{\text{pre}} \left(\left(\boldsymbol{p}^{\boldsymbol{I}}_{\mathbf{pre}}\right)^{\top} \cdot \mathbb{1}_{\{\boldsymbol{p}^{\boldsymbol{J}}_{\mathbf{post}} = 1\}} \right) - A^{J}_{\text{post}} \left(\left(\boldsymbol{p}^{\boldsymbol{J}}_{\mathbf{post}}\right)^{\top} \cdot \mathbb{1}_{\{\boldsymbol{p}^{\boldsymbol{I}}_{\mathbf{post}} = 1\}} \right) \right) \odot C_{I \to J}$$

Legend

$x \to$ Parameters
$\boldsymbol{x} \to$ State Variables