CBGT model equations

Poster

$$C_{\mathrm{m}}\dot{\boldsymbol{v}} = -I_{\mathrm{L}} - I_{\mathrm{K}} - I_{\mathrm{Na}} - I_{\mathrm{T}} - I_{\mathrm{Ca}} - I_{\mathrm{AHP}} - I_{\mathrm{GPe}\to\mathrm{STN}} + \eta_{\mathrm{M2}}$$

$$I_{\mathrm{GPe}\to\mathrm{STN}} = g_{\mathrm{GPe}\,\to\,\mathrm{STN}}(\boldsymbol{v} - v_{\mathrm{GPe}\,\to\,\mathrm{STN}})\sum_{s_j\in\mathrm{GPe}}\boldsymbol{w_j s_j}$$

$$\dot{s} = \alpha H_\infty\left(v-\theta_g\right)(1-s) - \beta s, \quad H_\infty(v) = \left(1+\exp\left(-\left(\left(v-\theta_g^H\right)/\sigma_g^H\right)\right)\right)^{-1}$$

$$\tau_{\mathrm{Ca}}[\dot{\mathbf{Ca}}]_{\mathbf{syn}} = -[\mathbf{Ca}]_{\mathbf{syn}} + \mathrm{Ca}_{\mathrm{pre}}\delta\left(t - t_{\mathrm{spike}}^{\mathrm{pre}}\right) + \mathrm{Ca}_{\mathrm{post}}\delta\left(t - t_{\mathrm{spike}}^{\mathrm{post}}\right)$$

$$\dot{\boldsymbol{w}} = \begin{cases} 0 & , [\mathbf{Ca}]_{\mathbf{syn}} < \theta_D \\ \eta_D(F_D - \boldsymbol{w}) & , \theta_D < [\mathbf{Ca}]_{\mathbf{syn}} < \theta_P \\ \eta_P(F_P - \boldsymbol{w}) & , \theta_P < [\mathbf{Ca}]_{\mathbf{syn}} \end{cases}$$

1. STN

$$C_{\mathrm{m}}\dot{\boldsymbol{v}} = -I_{\mathrm{L}} - I_{\mathrm{K}} - I_{\mathrm{Na}} - I_{\mathrm{T}} - I_{\mathrm{Ca}} - I_{\mathrm{AHP}} - I_{\mathrm{G}\to\mathrm{S}}$$

$$I_{\mathrm{L}} = g_{\mathrm{L}}(\boldsymbol{v} - v_{\mathrm{L}})$$

$$I_{\mathrm{K}} = g_{\mathrm{K}}\boldsymbol{n}^4(\boldsymbol{v} - v_{\mathrm{K}})$$

$$I_{\mathrm{Na}} = g_{\mathrm{Na}}m_\infty^3\boldsymbol{h}(\boldsymbol{v} - v_{\mathrm{Na}})$$

$$I_{\mathrm{T}} = g_{\mathrm{T}}a_\infty^3 b_\infty^2(\boldsymbol{v} - v_{\mathrm{Ca}})$$

$$I_{\mathrm{Ca}} = g_{\mathrm{Ca}}s_\infty^2(\boldsymbol{v} - v_{\mathrm{Ca}})$$

$$b_\infty = \left(1+\exp\left(\frac{\boldsymbol{r}-\theta_b}{\sigma_b}\right)^{-1}\right) - \left(1+\exp\left(-\frac{\theta_b}{\sigma_b}\right)\right)^{-1}$$

$$x_\infty = \left(1+\exp\left(-\frac{\boldsymbol{v}-\theta_x}{\sigma_x}\right)\right)^{-1} \qquad , x \in \{n, m, h, a, r, s\}$$

$$\begin{cases} \dot{\boldsymbol{x}} = \Phi_x\dfrac{x_\infty - x}{\tau_x} \\ \tau_x = \tau_x^0 + \tau_x^1\left(1+\exp\left(-\dfrac{v-\theta_x^\tau}{\sigma_x^\tau}\right)\right) \end{cases} \qquad , x \in \{n, h, r\}$$

$$I_{\mathrm{AHP}} = g_{\mathrm{AHP}}(v - v_{\mathrm{K}})\frac{[\mathrm{Ca}]}{[\mathrm{Ca}] + k_1}$$

$$[\dot{\mathrm{Ca}}] = \varepsilon(-I_{\mathrm{Ca}} - I_{\mathrm{T}} - k_{\mathrm{Ca}}[\mathrm{Ca}])$$

$$\dot{s} = \alpha H_\infty\left(v-\theta_g\right)(1-s) - \beta s$$

$$H_\infty(v) = \left(1+\exp\left(-\frac{v-\theta_g^H}{\sigma_g^H}\right)\right)^{-1}$$

$$I_{\mathrm{G}\to\mathrm{S}} = g_{\mathrm{G}\to\mathrm{S}}(\boldsymbol{v} - v_{\mathrm{G}\to\mathrm{S}})\sum_{s_j\in\mathrm{GPe}} w_j\boldsymbol{s_j}$$

2. GPe

Same as STN except

$$
\begin{aligned}
C_\mathrm{m}\dot{\boldsymbol{v}} &= -I_\mathrm{L} - I_\mathrm{K} - I_\mathrm{Na} - I_\mathrm{T} - I_\mathrm{Ca} - I_\mathrm{AHP} - I_{\mathrm{S}\to\mathrm{G}} - I_{\mathrm{G}\to\mathrm{G}} + I_\mathrm{app} \\
I_\mathrm{T} &= g_\mathrm{T} a_\infty^3 \boldsymbol{r}(\boldsymbol{v} - v_\mathrm{Ca}) \\
\dot{\boldsymbol{r}} &= \Phi_r \frac{r_\infty - \boldsymbol{v}}{\tau_r}
\end{aligned}
$$

3. STDP

$$
\dot{p}_\mathrm{pre} = -\frac{p_\mathrm{pre}}{\tau_\mathrm{pre}} + \delta\left(t - t_\mathrm{spike}\right)
$$

$$
\dot{p}_\mathrm{post} = -\frac{p_\mathrm{post}}{\tau_\mathrm{post}} + \delta\left(t - t_\mathrm{spike}\right)
$$

$$
\text{Where} \quad \delta(t) = \begin{cases} 0 & \text{if } x \neq 0 \\ \infty & \text{if } x = 0 \end{cases} \text{, s.t.} \int_{-\infty}^{\infty} \delta(t)dt = 1
$$

For a weight from neuron $i \in I$ to neuron $j \in J$

$$\Delta w_{i,j} = \begin{cases} A^J_{\text{pre}}\, \boldsymbol{p}^{\boldsymbol{i}}_{\textbf{pre}} & \text{if } t = t^i_{\text{spike}} \\ -A^J_{\text{post}}\, \boldsymbol{p}^{\boldsymbol{j}}_{\textbf{post}} & \text{if } t = t^i_{\text{spike}} \\ 0 & \text{otherwise} \end{cases}$$

Or in matrix form

$$\Delta W = \eta\left(A^J_{\text{pre}} \left(\left(\boldsymbol{p}^{\boldsymbol{I}}_{\textbf{pre}}\right)^\top \cdot \mathbb{1}_{\{\boldsymbol{p}^{\boldsymbol{J}}_{\textbf{post}} = 1\}} \right) - A^J_{\text{post}} \left(\left(\boldsymbol{p}^{\boldsymbol{J}}_{\textbf{post}}\right)^\top \cdot \mathbb{1}_{\{\boldsymbol{p}^{\boldsymbol{I}}_{\textbf{post}} = 1\}} \right) \right) \odot C_{I \to J}$$

Legend

$x \rightarrow$ Parameters

$\boldsymbol{x} \rightarrow$ State Variables