Matrix Calculations Assignment 1

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1

$2x + 5y = 7$
$-x + 4y = 3$

Cooefficient matrix

$$\begin{pmatrix} 2 & 5 \\ -1 & 4 \end{pmatrix}$$

Augmented matrix

$$\left(\begin{array}{cc|c} 2 & 5 & 7 \\ -1 & 4 & 3 \end{array}\right)$$

Row Echelon Form

$$\left(\begin{array}{cc|c} 2 & 5 & 7 \\ -1 & 4 & 3 \end{array}\right) R_2 := R_2 + \tfrac{1}{2}R_1$$

$$\left(\begin{array}{cc|c} 2 & 5 & 7 \\ 0 & \frac{13}{2} & \frac{13}{2} \end{array}\right)$$

This leads to:
$2x + 5y = 7$
$\frac{13}{2}y = \frac{13}{2}$
Which can easily be seen that $y = 1$, thus $x = 1$ because $5 + 2 = 7$

2

$x - 5y + 3z = 7$
$3x - 6z = -9$
$5x + y = -6$

Cooefficient matrix

$$\begin{pmatrix} 1 & -5 & 3 \\ 3 & 0 & -6 \\ 5 & 1 & 0 \end{pmatrix}$$

Augmented matrix

$$\left(\begin{array}{ccc|c} 1 & -5 & 3 & 7 \\ 3 & 0 & -6 & -9 \\ 5 & 1 & 0 & -6 \end{array}\right)$$

Row Echelon Form

$$\left(\begin{array}{ccc|c} 1 & -5 & 3 & 7 \\ 3 & 0 & -6 & -9 \\ 5 & 1 & 0 & -6 \end{array}\right) \; R_2 := R_2 - 3R_1$$

$$\left(\begin{array}{ccc|c} 1 & -5 & 3 & 7 \\ 0 & 15 & -15 & -30 \\ 5 & 1 & 0 & -6 \end{array}\right) \; R_3 := R_3 - 5R_1$$

$$\left(\begin{array}{ccc|c} 1 & -5 & 3 & 7 \\ 0 & 15 & -15 & -30 \\ 0 & 26 & -15 & -41 \end{array}\right) \; R_3 \Longleftrightarrow R_2$$

$$\left(\begin{array}{ccc|c} 1 & -5 & 3 & 7 \\ 0 & 26 & -15 & -41 \\ 0 & 15 & -15 & -30 \end{array}\right) \; R_3 := R_3 - \tfrac{15}{26} R_2$$

$$\left(\begin{array}{ccc|c} 1 & -5 & 3 & 7 \\ 0 & 26 & -15 & -41 \\ 0 & 0 & \frac{165}{26} & \frac{165}{26} \end{array}\right)$$

This results in:
$\frac{165}{26} z = \frac{165}{26} \implies z = 1$
$26y - 15z = -41 \implies y = -1$
$x + 5 + 3 = 7 \implies x = -1$
Which we can fill into every other formula and see that it is correct.

3

$x + 7y - 5z + 2t = 8$
$2x + 6y + 6z - 4t = -8$
$-x - 7y - z - 2t = 4$
$5x + 2y + 4z - 3t = -5$

Cooefficient matrix

$$\begin{pmatrix} 1 & 7 & -5 & 2 \\ 2 & 6 & 6 & -4 \\ -1 & -7 & -1 & -2 \\ 5 & 2 & 4 & -3 \end{pmatrix}$$

Augmented matrix

$$\left(\begin{array}{cccc|c} 1 & 7 & -5 & 2 & 8 \\ 2 & 6 & 6 & -4 & -8 \\ -1 & -7 & -1 & -2 & 4 \\ 5 & 2 & 4 & -3 & -5 \end{array}\right)$$

Row Echelon Form

$$\left(\begin{array}{cccc|c} 1 & 7 & -5 & 2 & 8 \\ 2 & 6 & 6 & -4 & -8 \\ -1 & -7 & -1 & -2 & 4 \\ 5 & 2 & 4 & -3 & -5 \end{array}\right) \; R_2 := R_2 - 2R_1$$

$$\left(\begin{array}{cccc|c} 1 & 7 & -5 & 2 & 8 \\ 0 & -8 & 16 & -8 & -24 \\ -1 & -7 & -1 & -2 & 4 \\ 5 & 2 & 4 & -3 & -5 \end{array}\right) R_3 := R_3 + R_1$$
$$\left(\begin{array}{cccc|c} 1 & 7 & -5 & 2 & 8 \\ 0 & -8 & 16 & -8 & -24 \\ 0 & 0 & -6 & 0 & 12 \\ 5 & 2 & 4 & -3 & -5 \end{array}\right) R_4 := R_4 - 5R_1$$
$$\left(\begin{array}{cccc|c} 1 & 7 & -5 & 2 & 8 \\ 0 & -8 & 16 & -8 & -24 \\ 0 & 0 & -6 & 0 & 12 \\ 0 & -33 & 29 & -13 & -45 \end{array}\right) R_4 := R_4 - \tfrac{33}{8}R_2$$
$$\left(\begin{array}{cccc|c} 1 & 7 & -5 & 2 & 8 \\ 0 & -8 & 16 & -8 & -24 \\ 0 & 0 & -6 & 0 & 12 \\ 0 & 0 & -37 & 20 & 54 \end{array}\right) R_4 := R_4 - \tfrac{37}{6}R_3$$
$$\left(\begin{array}{cccc|c} 1 & 7 & -5 & 2 & 8 \\ 0 & -8 & 16 & -8 & -24 \\ 0 & 0 & -6 & 0 & 12 \\ 0 & 0 & 0 & 20 & -20 \end{array}\right)$$
This results in:
$20t = -20 \implies t = -1$
$-6z = 12 \implies z = -2$
$-y + 16z - 8t = -24 \implies -y - 32 + 8 = -24 \implies y = 0$
$x + 7y - 5z + 2t = 8 \implies x + 0 + 10 - 2 = 8 \implies x = 0$

Which we can check by filling in $5x + 2y + 4z - 3t = -5 \implies 0 + 0 - 8 + 3 = -5$

4

$3x - 9y = -12$
$2x - 5y + Az = 7$
$Ay + z = A$

Cooefficient matrix

$$\begin{pmatrix} 3 & -9 & 0 \\ 2 & -5 & A \\ 0 & A & 1 \end{pmatrix}$$

Augmented matrix

$$\left(\begin{array}{ccc|c} 3 & -9 & 0 & -12 \\ 2 & -5 & A & 7 \\ 0 & A & 1 & A \end{array}\right)$$

Row Echelon Form

$$\left(\begin{array}{ccc|c} 3 & -9 & 0 & -12 \\ 2 & -5 & A & 7 \\ 0 & A & 1 & A \end{array}\right) R_2 \longleftrightarrow R_3$$
$$\left(\begin{array}{ccc|c} 3 & -9 & 0 & -12 \\ 0 & A & 1 & A \\ 2 & -5 & A & 7 \end{array}\right) R_3 := R_3 - \tfrac{2}{3}R_1$$

$$\left(\begin{array}{ccc|c} 3 & -9 & 0 & -12 \\ 0 & A & 1 & A \\ 0 & 1 & A & -1 \end{array}\right)$$

If $A = 0$ then we can say that $y = -1$ and $z = 0$, which means that $3x - 9y = -12 \implies 3x + 9 = -12 \implies x = -7$. We can then test this with $2x - 5y + Az = 7 \implies -14 + 5 = 7$, which is false.
So $A = 0$ is unsolvable.

We know by $Ay + z = A$ that either $y = 1, z = 0$ or that y is a fraction and z complements that fraction, so we have to look for an A that makes that happen.