Radbound University

Block cipher modes of use

Cryptography, Autumn 2021

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September 28, 2021

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Outline

Overview of symmetric cryptography

Modes for encryption

Stream encryption with block ciphers

Provable security of modes

Authentication with block ciphers

Overview of symmetric cryptography

Symmetric cryptography operations

- Basic operations
  - encryption
  - MAC computation
  - authenticated encryption (including sessions)
- Require a key shared between sender and receiver
- Auxiliary operations
  - cryptographic hashing
  - deterministic random bit generation (DRBG)
  - ...
- Not really symmetric crypto but often categorized as such
  - true random generators
  - secret sharing for key management

Need for secret keys in symmetric cryptography

- Symmetric stands for
  - same key for encryption and decryption
  - same key for MAC generation and verification
- Basic operations achieve following:
  - reduce problem of securing (big) data
  - to problem of securing (small) keys
- A secure solution requires secrecy of keys
  - key generation requires qualitative random generator
  - key transfer between entities requires other keys
  - modules performing crypto shall not leak keys
  - many potential weaknesses

Limit to security: exhaustive key search

- Exhaustive key search
  - given some plaintext and corresponding ciphertext ($M = 1$) ...
  - trying all different keys (N)
- Single-target attack: one particular k-bit key K
  - success prob. after N trials: $N2^{-k}$
  - expected effort $N \approx 2^k$
  - (implicit) security claim: this should be best attack
  - so a k-bit key limits security strength to k bits
- Multi-target attack:
  - attacker is happy if she finds one key out of n keys K_i
  - relevant in many cases
  - e.g., if keys K_i are on badges giving access to a building

Limit to security: multi-target exhaustive key search

- Multi-target attack setting example
  - attacker knows $Z_i = \mathsf{SC}_{K_i}(D = 1, \ell)$ for n keys K_i
- Attack:
  - guess K' and compute $Z' = \mathsf{SC}_{K'}(D = 1, \ell)$
  - until $Z' \in \{Z_i\}$: success
  - success probability per trial: $\geq n2^{-k}$
  - expected effort $N \approx 2^k/n$,
- Security erosion: 128-bit key offers much less than 128-bit strength
  - Security strength decreases to $k - log_2(n)$
- Can be prevented with globally unique diversifier: *global nonce*
  - e.g., key ID_i plus message counter Nr: $Z_i = \mathsf{SC}_{K_i}(ID_i \| Nr, \ell)$
  - or, random string R of sufficient length $Z_i = \mathsf{SC}_{K_i}(R, \ell)$

Security erosion

Security strength is smaller than key if multi-target attacks are possible

Modes for encryption

Block cipher modes for encryption

- DES can encipher 8-byte messages, AES 16-byte messages
  - what about longer and shorter messages?
  - two approaches: block encryption and stream encryption
- Block encryption modes
  - split the message in blocks
  - after padding last *incomplete* block if needed
  - apply permutation B_K to blocks in some way
- Stream encryption modes
  - build a stream cipher with a block cipher as updating function F or output function f

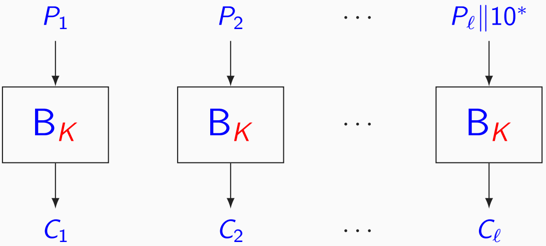
Block encryption modes

- Electronic Code Book (ECB) mode
  - *we consider only 16-byte messages*
  - longer messages are split in 16-byte blocks
  - shorter messages padded to 16 bytes
  - same for *last incomplete block*
- Cipher Block Chaining (CBC) mode
  - *ECB randomized with what's available*
  - requires also split in 16-byte blocks and padding
- Due to padding, cryptogram is longer than message

Intermezzo: padding

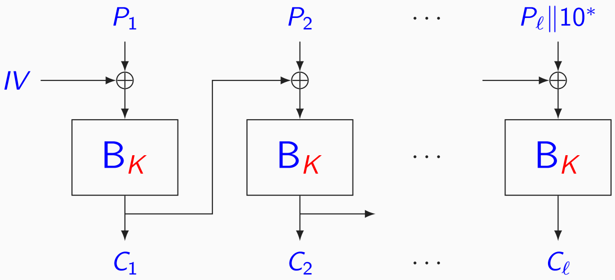
- Simplest padding: append zeroes
  - up to length multiple of block length (e.g., 16 bytes)
  - shortest possible padding
  - as such not usable for our purposes because it is not injective
- Decryption of cryptogram gives *padded* message
- Recovering message requires removing padding
  - send along message or padding length with cryptogram, or
  - impose padding is injective (or reversible)
- Simplest reversible padding: a single 1 and then zeroes
  - extends message in all cases
  - turns 16-byte message into 32-byte string
- Padding with exotic requirements
  - random-length padding: to hide message length
  - random padding: to add entropy
- Badly designed padding is often source of security problems

Electronic CodeBook Mode (ECB)



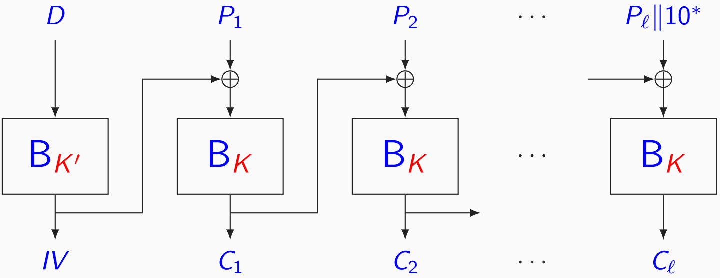
- Advantages
  - simple
  - parallelizable
- Limitation: equal plaintext blocks $\rightarrow$ equal ciphertext blocks:
  - likely to happen in low-entropy messages
  - problem in padded last block, that can be a single byte

Cipher Block Chaining mode (CBC)



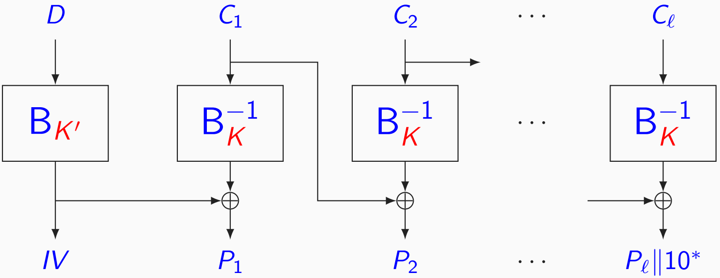
- *ECB with plaintext block randomized by previous ciphertext block*
- First plaintext block randomized with **random** Initial Value (IV)
- Solves leakage in ECB (partially):
  - equal plaintext blocks do not lead to equal ciphertext blocks
  - requires randomly generating and transferring IV

Cipher Block Chaining mode (cont'd)



- Replacing IV randomness by D nonce requirement: $IV = B_{K'}(D)$
  - with different key K' to avoid chosen-plaintext attacks
- CBC properties
  - encryption strictly serial
  - IV or diversifier D must be managed and transferred

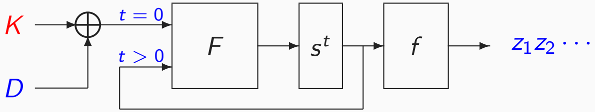
Cipher Block Chaining decryption



- Decryption can be done in parallel
- Bottom line
  - *we still need a nonce despite doing block encryption*
  - but ok, nonce re-use leaks less information

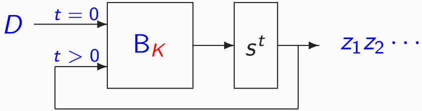
Stream encryption with block ciphers

Stream encryption with a block cipher



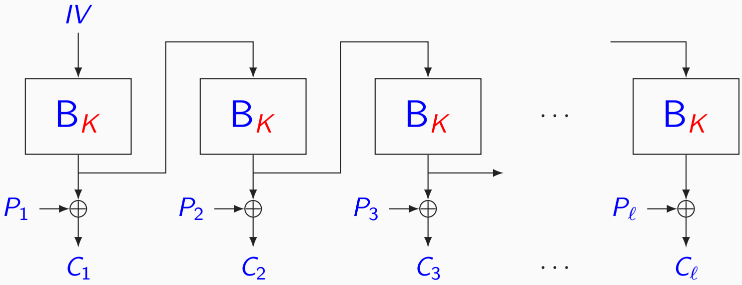
- Remember structure of iterative stream ciphers:
  - state update function $s^t = F(s^{t-1})$
  - output function $z_t = f(s^t)$
- Stream encryption modes of a block cipher:
  - use a block cipher for F or f

Output FeedBack mode (OFB)



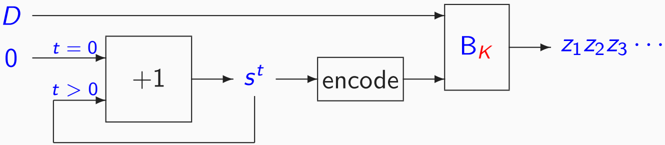
- $F = \mathsf{B}_K$, so $s_t \leftarrow \mathsf{B}_K(s_{t-1})$
- f is identity: $z_t \leftarrow s^t$
- Initialization: storage of K and $s_0 \leftarrow D$ (often called IV)
- Properties:
  - strictly serial
  - cycle lengths not known in advance
  - no need for B_K^{-1} (valid for all stream encryption)

OFB encryption presented in the classical way



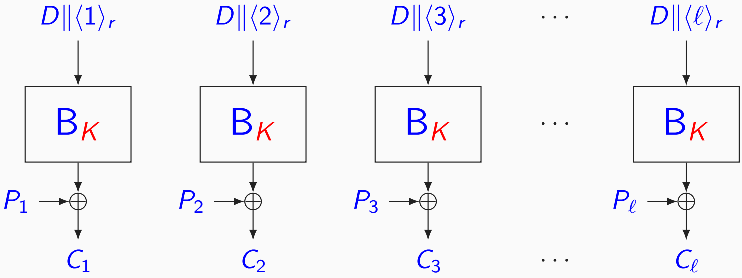
Note: the diversifier is often denoted as IV

Counter mode

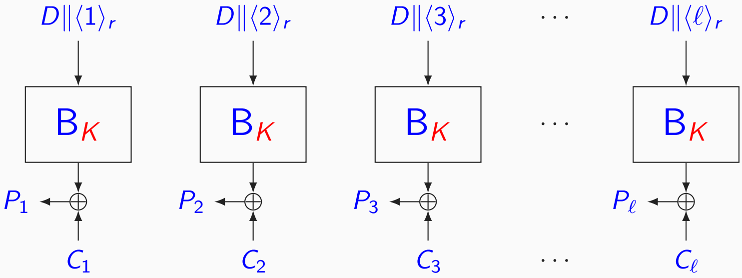


- F: interpret s^t as integer and add 1: $s^t = s^{t-1} + 1$
- $f = \mathsf{B}_K$, so $z_t = \mathsf{B}_K(D \|$ encoding of $s^t)$
- Initialization: storage of K and $s_0 \leftarrow 0$
- Properties:
  - fully parallelizable
  - number of blocks $\ell = |Z|$ is at most $2^{b-|D|}$
  - no risk of short cycles

Counter mode encryption presented in the usual way



Counter mode decryption presented in the usual way



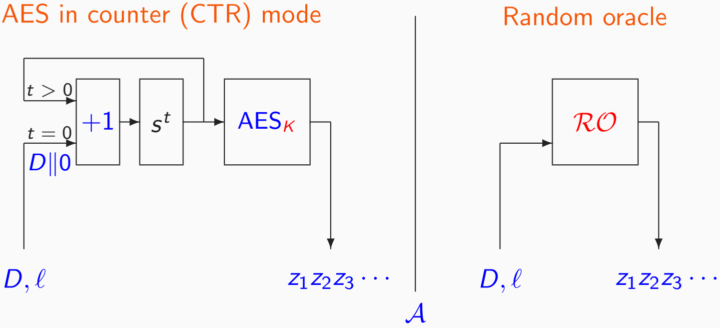
Encryption modes: overview

	ECB	CBC	OFB	CTR
parallel encryption	✓	—	—	✓
parallel decryption	✓	✓	—	✓
inverse free	—	—	✓	✓
absence of message expansion	—	—	✓	✓
tolerant to bit flips in $C \rightarrow P$	—	—	✓	✓
graceful degradation if nonce violation	n/a	✓	—	—

Provable security of modes

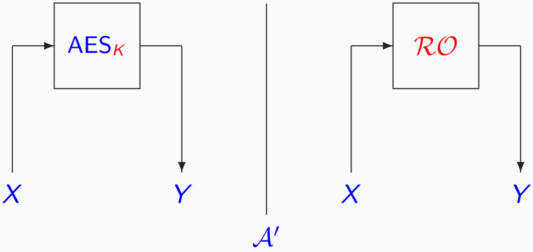
Provable security of a counter mode scheme

(counter mode depicted slightly differently for compactness)
(calls to internals symbolizing computational complexity omitted)



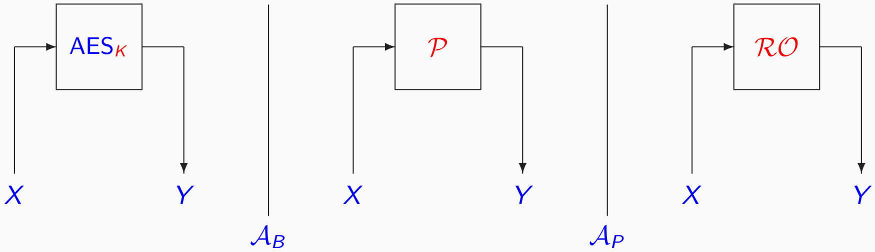
- Security of concrete scheme
  - advantage in distinguishing real and ideal world
  - denoted as $\mathrm{Adv}_{\mathcal{A}}(\mathsf{CTR}_{\mathsf{AES}_K}, \mathcal{RO})$
- Hard to analyze as such ...
  - we break this into simpler problems with some techniques
  - this set of techniques form the discipline of *provable security*

Provable security: simulation



- We replace $\mathcal{A}$ by an adversary $\mathcal{A}'$ that has more power
- $\mathcal{A}$ can be *simulated* by $\mathcal{A}'$
  - response to any query sent by $\mathcal{A}$ can be obtained by $\mathcal{A}'$
  - being asked to distinguish, $\mathcal{A}'$ can just ask $\mathcal{A}$
  - ... as she could do it herself
  - advantage of $\mathcal{A}'$ cannot be smaller than that of $\mathcal{A}$:

$$\mathrm{Adv}_{\mathcal{A}}(\mathsf{CTR}_{\mathsf{AES}_K}, \mathcal{RO}) \leq \mathrm{Adv}_{\mathcal{A}'}(\mathsf{AES}_K, \mathcal{RO})$$



- We add a step in between, here a random permutation $\mathcal{P}$
  - Adversary $\mathcal{A}_B$ distinguishes between AES_K and $\mathcal{P}$
  - Adversary $\mathcal{A}_P$ distinguishes between $\mathcal{P}$ and $\mathcal{RO}$
- Triangle inequality:

$$\mathrm{Adv}_{\mathcal{A}'}(\mathsf{AES}_K, \mathcal{RO}) \leq \mathrm{Adv}_{\mathcal{A}_B}(\mathsf{AES}_K, \mathcal{P}) + \mathrm{Adv}_{\mathcal{A}_P}(\mathcal{P}, \mathcal{RO})$$

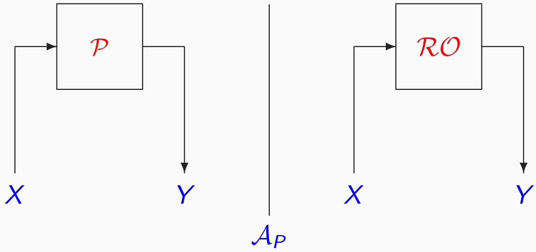
Separation of primitive cryptanalysis and mode security proofs

$$\mathrm{Adv}_{\mathcal{A}'}(\mathsf{AES}_K, \mathcal{RO}) \leq \mathrm{Adv}_{\mathcal{A}_B}(\mathsf{AES}_K, \mathcal{P}) + \mathrm{Adv}_{\mathcal{A}_P}(\mathcal{P}, \mathcal{RO})$$

The advantage has two components:

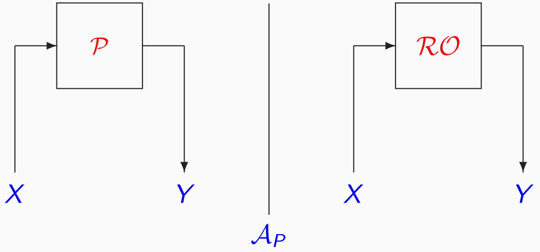
- Advantage of the primitive
  - here: PRP-security of AES
  - domain of *cryptanalysis*
  - cannot be proven, only assumed, claimed and challenged
- Advantage of the mode assuming ideal component
  - here: (CTR mode of a) random permutation $\mathcal{P}$
  - domain of *provable security*
  - bounds can be proven using probability theory

Provable security of counter mode as such



- Difference in behaviour between $\mathcal{P}$ and $\mathcal{RO}$
  - $\mathcal{P}$ returns uniformly random responses, with restriction that they don't collide
  - $\mathcal{RO}$ returns uniformly random responses
- This implies that $\mathcal{A}_P$ can distinguish $\mathcal{P}$ from $\mathcal{RO}$ if and only if
  - she is speaking to $\mathcal{RO}$ AND
  - $\mathcal{RO}$ returns colliding outputs

Provable security of counter mode as such (cont'd)



- After queries, $\mathcal{A}_P$ returns 1 if there was a collision and 0 otherwise

  $\mathrm{Adv}_{\mathcal{A}_P}(\mathcal{P}, \mathcal{RO}) = |\Pr(\mathcal{A}_P = 1 \mid \mathcal{RO}) - \Pr(\mathcal{A}_P = 1 \mid \mathcal{P})| = \Pr(\text{coll.} \mid \mathcal{RO})$

- We have

$$\Pr(\text{coll.} \mid \mathcal{RO}) \leq \binom{M}{2} 2^{-128} \leq M^2 2^{-129}$$

- Advantage gets close to 1 when $M \approx 2^{64}$: the *birthday bound*

Authentication with block ciphers

Message authentication code (MAC) functions

- MAC: cryptographic checksum
  - input: key K and arbitrary-length message m
  - output: tag (aka MAC) T with some length ℓ
- Applications:
  - message authentication: append tag to message
  - entity authentication: compute tag over challenge

We can formally write: $T \leftarrow \mathrm{MAC}_K(m)$

Two types of MAC function (online) queries:

- Generation: give m and get $T \leftarrow \mathrm{MAC}_K(m)$
- Verification: give (m, T) and get 1 if $T = \mathrm{MAC}_K(m)$ and else 0

Security goal

MAC forgery

Generating a couple (m, T) such that tag verification returns 1 without knowing K and without querying tag generation with m

- Security goal of a MAC function: forgery should be hard. How hard?
- Ideal MAC function:
  - tags fully unpredictable when keyed with unknown K
  - ...except that same message returns same tag
  - like a random oracle with ℓ-bit output!
- Success probability of forgery after M attempts for $\mathcal{RO}$: $M2^{-\ell}$
- Try (m, T) with same m and different T until we hit the right tag

So we want our keyed MAC function to be like a random oracle

Pseudorandom function (PRF) security of a MAC function

MAC() is PRF-secure if $\mathrm{MAC}_K(m)$ is hard to distinguish from $\mathcal{RO}$

Note: same security concept as for a stream cipher

Implications of PRF-security bound of a MAC function

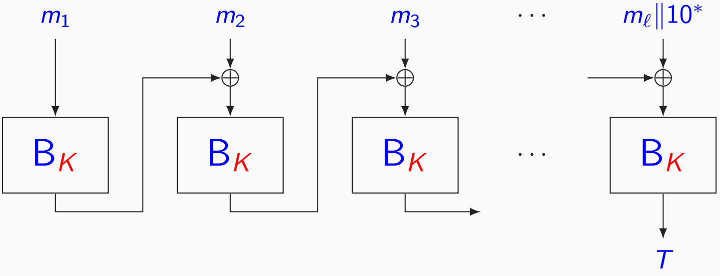
PRF-advantage of a MAC function

$\mathrm{Adv}_{\mathcal{A}}(\mathsf{MAC}_K, \mathcal{RO}) = |\Pr(\mathcal{A} = 1 \mid \mathsf{MAC}_K) - \Pr(\mathcal{A} = 1 \mid \mathcal{RO})|$

A (claimed) advantage $\mathrm{Adv}_{\mathcal{A}}(\mathsf{MAC}_K, \mathcal{RO}) \leq \epsilon(M, N)$ says something about the success probability of forgery

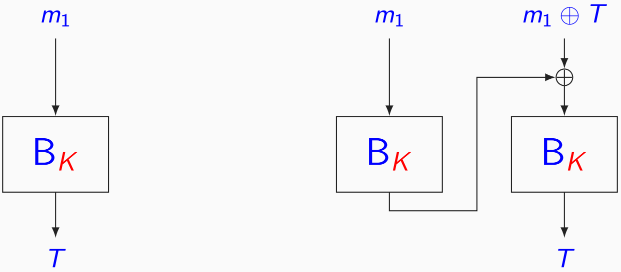
- Recipe for distinguishing adversary $\mathcal{A}$ based on forging ability:
  (1) Spend resources N and M on trying to generate forgery
  (2) If it works, return 1, else return 0
- $\Pr(\mathcal{A} = 1 \mid \mathcal{RO}) = \Pr(\text{forgery success for } \mathcal{RO}) \leq M2^{-\ell}$
- $\Pr(\mathcal{A} = 1 \mid \mathsf{MAC}_K) = \Pr(\text{forgery success for } \mathsf{MAC}_K)$
- Due to the claim: $\Pr(\text{forgery success for } \mathsf{MAC}_K) \leq M2^{-\ell} + \epsilon(M, N)$

Cipher Block Chaining MAC mode (CBC-MAC)



- Observation: in CBC ciphertext block C_i depends on m_0 to m_i
- Idea:
  - apply CBC encryption with zero IV to (padded) message
  - take tag equal to last ciphertext block
  - throw away other blocks (essential for security)
- This is the basis for most block-cipher based MAC functions

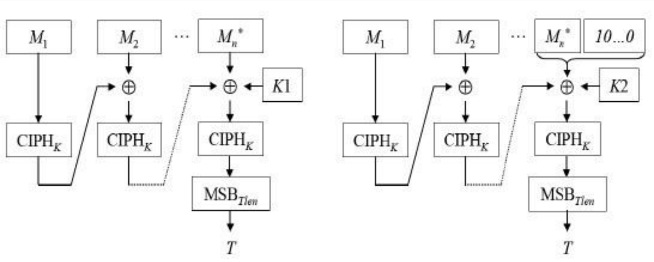
CBC-MAC weakness: length extension



- Distinguishing from random oracle $\mathcal{RO}$ in two queries:
  - query m_1 returns $T = B_K(m_1)$
  - query $m_1 \| m_2$ with $m_2 = m_1 \oplus T$ returns

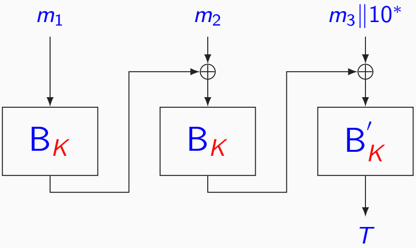
$$B_K(m_2 \oplus B_K(m_1)) = B_K(m_1 \oplus T \oplus B_K(m_1)) = B_K(m_1) = T$$

- A random oracle will give two completely unrelated tags
- Note: attack ignores padding, but this can be dealt with
- Truncating the tag T helps (somewhat) against this attack



- Trick: avoid length-extension problem by *doing something different at the end*: finalization
- Here: addition of a *subkey* before last application of B_K
- Advantage in distinguishing this from $\mathcal{RO}$ assuming random $\mathcal{P}$
  - birthday bound $M^2 2^{-(b+1)}$ due to *inner collisions*
  - see next slide

Security of CBC-MAC based modes: inner collisions



- Consider CBC-MAC with *finalization* B'_K, e.g., C-MAC
- Distinguishing this from a $\mathcal{RO}$:
  - query for many 3-block inputs $m^{(i)}$ of the form $m_1 m_2 m_3$
  - m_1 and m_2 different in each query, m_3 always the same
- Collision for $i \neq j$ at input of B'_K gives colliding tags
  - probability $\approx M^2 2^{-(b+1)}$ with M number of queries
  - detect *internal collision* by tag collision plus some check queries
  - then $\forall m'$: $m^{(i)}\|m'$ gives same tag as $m^{(j)}\|m'$
- $\mathcal{RO}$ has no internal collisions

Summary

Summary

- Block ciphers are versatile:
  - block encryption modes: e.g., ECB and CBC
  - stream encryption modes: e.g., OFB and counter
  - MAC computation modes: e.g., CBC-MAC and C-MAC
- Inverse permutation only used in block encryption modes
- Security analysis of cryptographic schemes splits in two parts
  - primitives must be cryptanalyzed, no security proofs
  - modes can be proven secure with probability theory
- Most modes only secure up to *birthday bound*
  - processing $2^{b/2}$ blocks with same key will show non-ideal behaviour