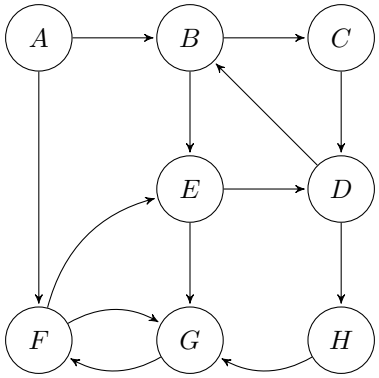
Weekly Assignment 3:
Depth-First Search

September 2022

1. Perform depth-first search on the following graph. Whenever there is a choice of vertices, pick the one that is alphabetically first. Classify each edge as a tree edge, forward edge, back edge, or cross edge, and give the discovery and finishing times of each vertex.



2. Prof. Marcos claims that the algorithm for strongly connected components can be simplified. Instead of using the transpose graph in the second DFS one can just use the original graph, but the vertices are visited in order of *increasing* finishing times. Will this simpler algorithm always give correct results? Either prove that it works correctly, or give a counterexample.

3. For each of the following statements, determine whether they are true or false. Explain your choice. (Your explanation is worth more than your choice of true or false.)
   (a) If a depth-first search on a directed graph $G = (V, E)$ produces exactly one back edge, then it is possible to choose an edge $e \in E$ such that the graph $G' = (V, E - \{e\})$ is acyclic.
   (b) If a directed graph G is cyclic but can be made acyclic by removing a single edge, then a depth-first search in G will encounter exactly one back edge.

4. The DFS algorithm classifies the edges of a digraph into four groups. Explain the classification and how the algorithm classifies edges.

5. Give an efficient algorithm which takes as input a directed graph $G = (V, E)$, and determines whether or not there is a vertex $s \in V$ from which all other vertices are reachable.