Weekly Assignment 2:
Breadth-First Search

September 2022

1. Apply the BFS algorithm to the directed graph displayed in Figure 1 using the node with label 2 as source.

   Specify:

   - The content of the queue, after initialization and after each iteration of the loop,
   - For any discovered node, its predecessor and the iteration when it was discovered

   You may assume that the adjacent vertices of a node are visited in the order of their sequence number.

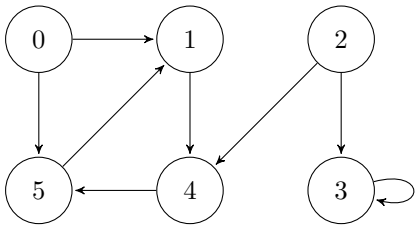

Figure 1: Directed graph

2. Stacks and Queues are data structures which implement Dynamic Sets and are fundamental building blocks for many algorithms. They implement `Insertion` and `Deletion` through the stack operations: `Pop`, `Push`, and the queue operations: `Enqueue`, `Dequeue`.

   Show how to implement a stack using two queues. Analyse the running time of the queue operations.

3. Most graph algorithms that take an adjacency-matrix representation as input require time $\mathcal{O}(|V|^2)$, but there are some exceptions.

   a) Show how to determine whether a directed graph $G = (V, E)$ contains a *universal sink*. A universal sink is a vertex with in-degree $n - 1$ (i.e. $n - 1$ incoming edges) and out-degree 0, in time $\mathcal{O}(|V|)$ given an adjacency matrix for G.

   b) Show how to determine whether a directed graph $G = (V, E)$ contains a universal sink in time $\mathcal{O}(|V| + |E|)$, given an adjacency list representation for G. Argue why an $\mathcal{O}(|V|)$-algorithm does not exist.

4. Consider a directed graph G and a vertex $s \in V$. Let C be the set of cycles in G that visit vertex s. Give an efficient algorithm that returns `true` and a cycle from C in case C is nonempty, and `false` otherwise. Explain why your algorithm is correct.

5. Consider a group of n human-looking entities of whom some are androids and some are human beings but we don't know who is a human and who is an android. Some of these have a (mutual) inter-species friendship. Assume we have a complete list of the r pairs of inter-species friends.

   Give an $\mathcal{O}(r+n)$ time algorithm that determines whether it is possible to partition the group of entities into humans and the rest into androids in such a way that inter-species friendships is only between humans and androids. If a species designation exists your algorithms should produce one. Discuss the time complexity and correctness of your algorithm.