Weekly Assignment 4 :
Dijkstra

September 2022

1. Apply Dijkstra's algorithm on the following graph, starting at the node labelled 0.

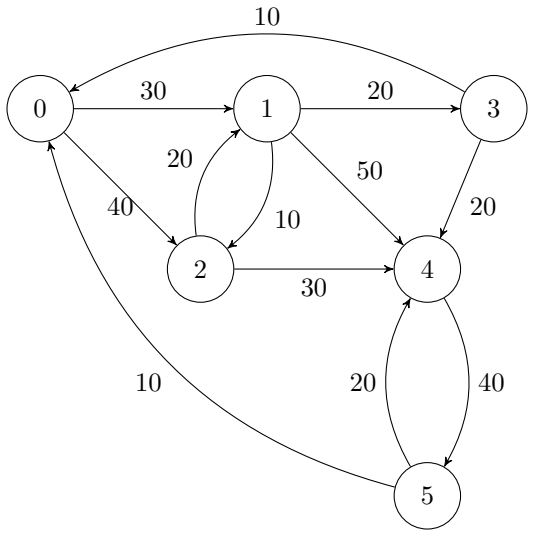

Figure 1: Graph of Exercise 1.

2. You managed to buy a second-hand electric car which can drive for L kilometers without recharging. Unfortunately, L is not that big... You are given a set of cities, along with a set of roads between them, in the form of an undirected graph $G = (V, E)$. Each stretch of road $e \in E$ connects two cities, and you know its length in kilometers, $w(e)$. You want to get from city s to city t. You may recharge your car in cities, but not in between cities. Therefore, you can only take a route if every one of its edges e has length $w(e) \leq L$.
   (a) Given the limited range of your car, show how to determine in linear time whether there is a feasible route from s to t.
   (b) You are now planning to buy a new electric car, and you want to know the minimum range that is needed to travel from s to t. Give an $\mathcal{O}((|V|+|E|)\log|V|)$ algorithm to determine this, and explain why your algorithm is correct.

3. Professor Balthazaar suggests the following algorithm for finding the shortest path from node s to node t in a directed graph with some negative edges: add a large constant to each edge weight so that all the weights become positive, then run Dijkstra's algorithm starting at node s, and return the shortest path found to node t.

   Is this a valid method? Either prove that it works correctly, or give a counterexample.

4. (a) From elements 1, 4, 7 and 9, draw all possible **max**-heaps.

   (b) A *d-ary heap* is like a binary heap, but (with one possible exception) non-leaf nodes have d children instead of 2 children.

   Over an array, implement the two functions

   - D-Ary-Parent(i), which retrieves the index of the parent of the i:th element,
   - D-Ary-Child(i, k), which retrieves the k:th child of the i:th element.

   (**HINT**: Draw an example 3-ary heap from the elements: 18, 15, 14, 10, 12, 9.)

   (c) What is the height of a d-ary heap of n elements in terms of n and d?

5. **NB:** In this exercise you are asked to implement operations associated with a d-ary **max**-heap.

   (a) Give an efficient implementation of EXTRACT-MAX in a d-ary max-heap. Analyze its running time in terms of d and n.

   (b) Give an efficient implementation of INSERT in a d-ary max-heap. Analyze its running time in terms of d and n.

   (c) Give an efficient algorithm of INCREASE-KEY(A, i, k), which flags an error if $k < A[i]$, but otherwise sets $A[i] = k$ and then updates the d-ary max-heap structure appropriately. Analyze its running time in terms of d and n.