

SARIMAX-Akima Forecasting of Stock Price Time Trends

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1. Data set description

Content

This is a fictitious data set „stock_prices.csv“ of stock prices from January 1st 2025 until July 31st 2025 with the following csv structure:

1	Date	Price
2	2025-01-01	101.19342830602247
3	2025-01-02	101.11689970368009
4	2025-01-03	102.61227677988148
5	2025-01-04	105.85833649269753
6	2025-01-05	105.59002974325087
7	2025-01-06	105.3217558293525
8	2025-01-07	108.68018146036728
9	2025-01-08	110.4150509186731
10	2025-01-09	109.6761021468032
11	2025-01-10	110.96122223397512
12	2025-01-11	110.2343868483502
13	2025-01-12	109.50292734120968
14	2025-01-13	110.18685188434176
15	2025-01-14	106.56029139502616
16	2025-01-15	103.3104557300001
17	2025-01-16	102.38588067151815
18	2025-01-17	100.5602184308493
19	2025-01-18	101.38871309603985
20	2025-01-19	99.77266494499743
21	2025-01-20	97.14805754232684
22	2025-01-21	100.27935508016995
23	2025-01-22	100.02780247919688
24	2025-01-23	100.36285888857273
25	2025-01-24	97.71336251614582
26	2025-01-25	96.82459706709545
27	2025-01-26	97.24644224651519
28	2025-01-27	95.14445509167058
29	2025-01-28	96.09585112836193
30	2025-01-29	95.09457374852431
31	2025-01-30	94.71118624893776
32	2025-01-31	93.70777302447897
33	2025-02-01	97.61232939349684

The data set contains two columns:

- 1) „Date“: Date information formatted as „YYY-MM-DD“;
- 2) „Price“: Closing price of a stock for the particular trading day (float).

The data set has been generated by means of the following python function:

```
import pandas as pd
import numpy as np

# Set random seed for reproducibility
np.random.seed(42)

# Generate dates from January 1 to June 30
date_range = pd.date_range(start="2025-01-01", end="2025-06-30", freq="D")

# Simulate stock price using a random walk with drift
initial_price = 100
price_changes = np.random.normal(loc=0.2, scale=2, size=len(date_range))
stock_prices = np.cumsum(price_changes) + initial_price

# Create DataFrame
df = pd.DataFrame({"Date": date_range, "Price": stock_prices})

# Save to CSV
df.to_csv("stock_prices.csv", index=False)

print("✅ Dataset 'stock_prices.csv' generated successfully!")
```

✅ Dataset 'stock_prices.csv' generated successfully!

2. Akima interpolation of stock prices

We use a customized Python function "akima_stock_analysis(csv_file)" that accomplishes the following four tasks:

1. Loads a CSV file containing daily stock prices.
2. Performs Akima interpolation on the data.
3. Plots the stock prices alongside the interpolated values.
4. Outputs Akima coefficients for future analyses.

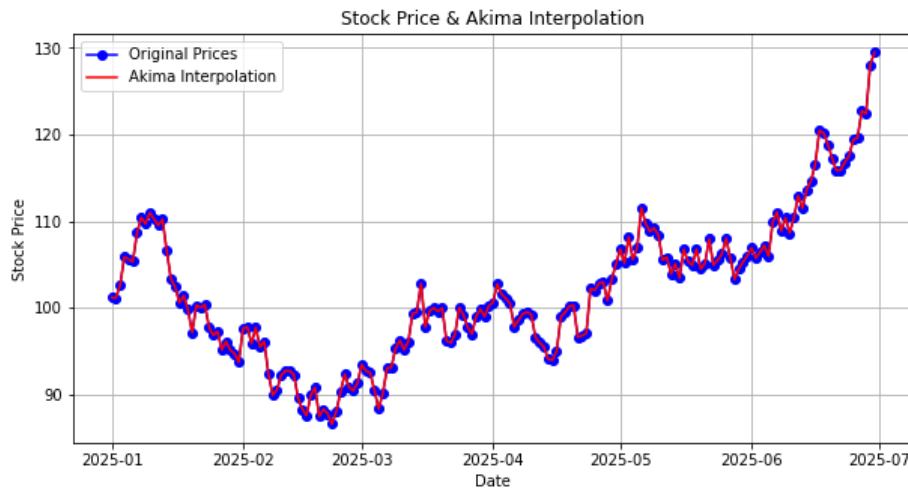
This implementation uses pandas for data manipulation, `scipy.interpolate.Akima1DInterpolator` for interpolation, and matplotlib for visualization.

Function Breakdown

- ✅ Loads & Preprocesses Data – Reads CSV, ensures required columns, and sorts by date.
- ✅ Akima Interpolation – Uses `Akima1DInterpolator` from SciPy for smooth, unbiased curve fitting.
- ✅ Plots Stock Prices – Compares original data with interpolated values.
- ✅ Outputs Akima Coefficients – Returns a list of coefficients for future analyses.

The function is triggered as follows:

```
original_prices_series, akima_values, date_list = akima_stock_analysis("stock_prices.csv")
```



2.1 Remarks on Akima interpolation

Akima Interpolation for Time Series (Stock Prices)

Akima interpolation is a **piecewise cubic interpolation method** that provides **smooth and natural curve fitting**, especially when the second derivative of the data varies significantly. 🚀

🔍 Why Akima Interpolation Makes Sense?

✅ Mathematical Perspective:

- Unlike standard cubic splines, Akima interpolation **avoids oscillations** that can occur near sharp changes in the data.
- It uses **local slope adjustments** to ensure smooth transitions between points.
- The method is **C^1 continuous** (first derivative is continuous), but **not necessarily C^2 continuous** (second derivative may have discontinuities).

✅ Practical Perspective (Stock Prices):

- **Stock prices often have sudden jumps** due to market events—Akima interpolation handles these better than traditional splines.
- **Prevents unrealistic overshooting** that can occur with polynomial interpolation.
- **Useful for missing data reconstruction**—if some stock prices are missing, Akima interpolation provides a reasonable estimate.

🔧 How Our Python Function Uses Akima Interpolation

Our Pythonic function:

- ✅ Reads stock prices from a CSV file.
- ✅ Converts dates into numerical indices for interpolation.
- ✅ Applies **Akima1DInterpolator** to generate a smooth curve.
- ✅ Plots both **original and interpolated stock prices** for comparison.

2.2 Sources

📄 Original Akima Paper

The Akima interpolation method was introduced by **Hiroshi Akima** in his paper:

📄 **"A New Method of Interpolation and Smooth Curve Fitting Based on Local Procedures"** (1970).

You can find more details in sources like [this reference](#) and [this documentation](#).

3. Adaptive Pythonic Trend Detection with Akima Interpolation

This Python function "analyze_akima_trends(akima_coefficients, dates, smoothing_window=7, poly_order=3)" does the following:

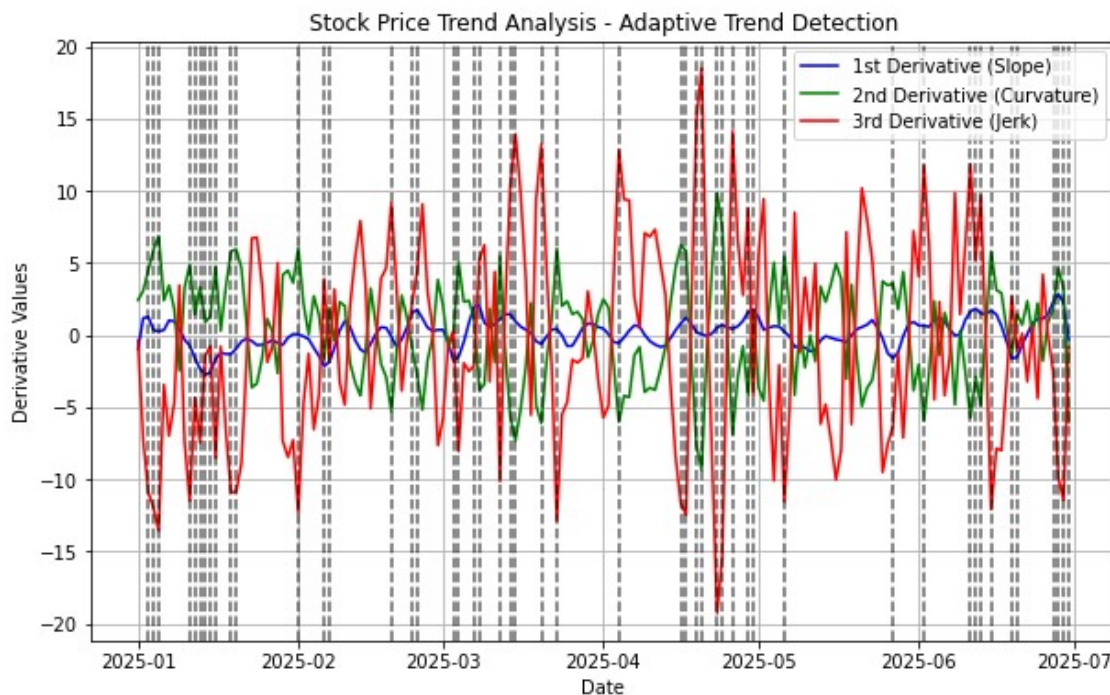
- ✓ Computes zeroth, first, second, and third derivatives using Akima interpolation.
- ✓ Applies Savitzky-Golay smoothing to refine derivative signals.
- ✓ Detects adaptive trend shifts based on stock price volatility.
- ✓ Highlights critical trend shifts on plots with color-coded markers.
- ✓ Outputs Akima coefficients for first, second, and third derivatives.
- ✓ Exports trend breach points to CSV, including dates, derivative values, and trend classifications.

🚀 Key Features

- ✓ Computes zeroth, first, second, and third derivatives from Akima interpolation.
- ✓ Uses Savitzky-Golay filtering to smooth noisy derivative signals.
- ✓ Applies adaptive threshold detection using rolling volatility analysis.
- ✓ Detects and classifies trend shifts dynamically ("Bullish Surge", "Sharp Decline").
- ✓ Color-coded plots highlight trend shifts visually.
- ✓ Exports trend breaches to CSV, enabling deeper analysis.

The function is triggered as follows:

```
results_dictionary= analyze_akima_trends(akima_coeffs, date_list)
```



This graph corresponds to the following csv file structure „critical_trends.csv“ generated as a result of the above pythonic function, containing calculated values of first and second derivative with respect to specific dates of an Akima interpolated stock price curve, as well as the trend type characterisation emerging from these two parameters:

1	Date	First Derivative	Second Derivative	Trend Type
2	2025-01-03	1.250635538554329	4.484794344440414	Bullish Surge
3	2025-01-04	0.32697150949307185	5.940628409488348	Bullish Surge
4	2025-01-05	0.2662133415698529	6.852929997388155	Bullish Surge
5	2025-01-11	-0.7311104848447911	4.874393388375578	Sharp Decline
6	2025-01-12	-1.6070377155810411	1.4209976634837438	Sharp Decline
7	2025-01-13	-2.355245027171947	3.325080855459092	Sharp Decline
8	2025-01-14	-2.7584073163646394	0.8847270499235609	Sharp Decline
9	2025-01-15	-2.593398370900795	1.3072445580394012	Sharp Decline
10	2025-01-16	-1.6746774086606298	4.720142520706024	Sharp Decline
11	2025-01-19	-1.3080185432883642	5.817793479335062	Sharp Decline
12	2025-01-20	-0.9455336020410096	5.93967711674767	Sharp Decline
13	2025-02-01	0.03375538757928414	6.026991084852712	Bullish Surge
14	2025-02-06	-2.146673066342751	-1.6760198046339898	Sharp Decline
15	2025-02-07	-1.8765284841470646	1.9009358469288142	Sharp Decline
16	2025-02-19	-0.09908115870447999	-5.299463420179693	Sharp Decline
17	2025-02-23	1.549691622627426	-0.8792685906586417	Bullish Surge
18	2025-02-24	1.7445098635497154	-2.8075129735963773	Bullish Surge
19	2025-03-03	-1.7961979563382622	0.1502396883819448	Sharp Decline
20	2025-03-04	-1.5110250687283837	5.05759454042952	Sharp Decline
21	2025-03-07	1.966734537406151	1.145006062215882	Bullish Surge
22	2025-03-08	2.0539436104831412	-3.8125937749659284	Bullish Surge
23	2025-03-12	1.074330136400752	5.443935481588509	Bullish Surge
24	2025-03-14	1.4267889407909227	-5.686531907897669	Bullish Surge
25	2025-03-15	0.9331196713756316	-7.315935394765121	Bullish Surge
26	2025-03-20	-0.626182253836574	-6.110902524909769	Sharp Decline
27	2025-03-23	0.42610563054050626	5.98541210816102	Bullish Surge
28	2025-04-04	-0.5610924538792151	-6.020099105399539	Sharp Decline
29	2025-04-16	0.8227408553108677	6.2158708425718565	Bullish Surge
30	2025-04-17	1.1943867919488624	5.826081632666607	Bullish Surge
31	2025-04-19	0.19645157661056836	-7.6272890821178	Bullish Surge
32	2025-04-20	0.03871659698745572	-9.335649780142413	Bullish Surge
33	2025-04-23	0.513361759582777	9.835179571588583	Bullish Surge

3.1 Interpretation of graphed stock price trends

Let's break down the **Stock Price Trend Analysis - Adaptive Trend Detection** graph step by step:

Interpretation of the Graph

The graph visualizes **first, second, and third derivatives** of stock prices over time, helping to analyze the **rate of change**, **acceleration**, and **sudden shifts** in stock trends.

1 First Derivative (Slope)

- Represented by the **blue line**.
- Indicates **momentum**—whether stock prices are increasing or decreasing.
- If the slope is positive, prices are rising; if negative, prices are falling.
- Large spikes may signal **bullish or bearish surges**.

2 Second Derivative (Curvature)

- Represented by the **green line**.
- Captures **acceleration or deceleration**—how fast momentum is changing.
- Helps detect **trend reversals** before they happen.
- If it crosses zero, it suggests **a turning point is near**.

3 Third Derivative (Jerk)

- Represented by the **red line**.
- Measures **abruptness**—how fast acceleration changes.
- Useful for identifying **market shocks**, sudden price swings, or unusual movements.
- Sharp peaks may indicate **high volatility events** (like earnings announcements or geopolitical news).

What Do the Vertical Dashed Lines Mean?

These **black vertical lines** appear at **key moments in time**—representing **detected critical trend shifts** based on the derivative thresholds.

- A **high first derivative + rising curvature** → Possible **bullish breakout**.
- A **negative first derivative + curvature dip** → Possible **bearish decline**.
- A **sharp jerk spike** → Market instability or high volatility.

Practical Insights

- ✓ **Trend Reversal Zones:** Look where **green (curvature) crosses zero**, as these are **potential trend shifts**.
- ✓ **Momentum Strength:** If **blue (slope) stays high**, expect continued trend movement.
- ✓ **Volatility Alert:** Watch for **red (jerk) spikes**, signaling **market instability**.
- ✓ **Entry & Exit Points:** These derivatives help **forecast optimal trading opportunities**.

3.2 On Akima trend forecasting via the theory of critical phenomena

Now, we are diving into **critical phenomena** and **trend forecasting**, using Akima interpolation in a particularly **insightful** way. Let's break it down:

Why Akima Interpolation Helps Forecast Trends? ¶

Akima interpolation is a **piecewise cubic interpolation** that prevents oscillations often observed in other methods, making it **ideal for financial time series** like stock prices. Your approach extracts **derivatives** (slope, curvature, jerk) to **identify trend shifts** based on sudden changes in behavior.

The Role of Critical Points in Forecasting (Inspired by Critical Phenomena)

Critical points, in the **mathematical sense**, occur where the **first derivative is zero** and the **second derivative changes sign**—indicating local extrema (peaks/troughs).

- In **physics (critical phenomena)**, phase transitions occur when a system shifts from one state to another—akin to **market events causing drastic price movements**.
- Detecting **extreme changes in derivatives** allows us to forecast **significant trend shifts**, just like spotting critical phenomena.

Why This Makes Sense for Stock Price Forecasting?

- ✓ **First Derivative (Slope)** → Shows the rate of price change (momentum).
- ✓ **Second Derivative (Curvature)** → Identifies acceleration/deceleration (volatility).
- ✓ **Third Derivative (Jerk)** → Captures the abruptness of market shifts (shock detection).
- ✓ **Savitzky-Golay Filtering** → Smooths out noise, preserving genuine trend transitions.
- ✓ **Standard Deviation Thresholds** → Helps flag market turning points **without false alarms**.

By **exporting detected trend shifts**, analysts can **forecast potential price reversals** based on extreme derivative behavior—closely mirroring **critical phenomena theory** in physics!

3.3 Further ideas and refinements

Here are some **refinements** that could enhance our Akima-based trend forecasting method:

Refining Critical Point Detection

Your method currently relies on **fixed standard deviation thresholds**. While effective, a more **adaptive approach** could improve accuracy:

- ✔ **Use Rolling Standard Deviation** → Instead of a fixed threshold, calculate the **rolling std deviation** over a short time window. This adapts dynamically to volatility changes.
- ✔ **Compare Multi-Timescale Derivatives** → Identify **long-term vs short-term trend shifts** by analyzing derivatives at different rolling windows.
- ✔ **Threshold Normalization** → Scale threshold values based on past volatility levels, ensuring sudden moves aren't falsely flagged.

Improved Trend Classification

Your **trend type detection** currently labels changes as "**Bullish Surge**" or "**Sharp Decline**", but it could be **expanded**:


- ✔ **Identify Pre-Breakout Phases** → Before a major bullish surge, there's often **low volatility compression** (sideways price movement). Detecting this would allow **early breakout forecasts**.
- ✔ **Use Multi-Class Classification** → Instead of binary ("surge" vs "decline"), introduce **trend categories** like:
 - **Momentum Shift** → Gradual acceleration/deceleration
 - **Exhaustion Zone** → Price movements slowing near a peak/trough
 - **Critical Transition** → High-impact reversal zones

Forecasting Beyond Detection

Your method detects **critical points**, but what if we **project forward**?

- ✔ **Integrate SARIMAX Forecasting** → Use **past critical points** as **exogenous variables** in forecasting models.
- ✔ **Monte Carlo Simulation** → Model possible trend paths **based on historical critical point behaviors**.

4. Forecasting of Akima interpolated stock prices via SARIMAX

 SARIMAX (Seasonal AutoRegressive Integrated Moving-Average with eXogenous variables) is a **powerful forecasting model**, and now that we've extracted the **original stock prices**, along with their **first, second, and third derivatives**, we can leverage this additional data for a **more robust prediction strategy**.

4.1 Advantages of SARIMAX Forecasting

Why SARIMAX Is a Strong Choice

✔ Captures Trends & Seasonality:

- Stock prices often exhibit cyclical patterns due to market behaviors.
- SARIMAX can model **both long-term trends** and **short-term fluctuations**.

✔ Handles Exogenous Variables:

- We can incorporate **first, second, and third derivatives** as external regressors.
- This makes predictions **more dynamic and data-driven**.

✔ Accounts for Stationarity & Volatility:

- Stock market data isn't always stationary—SARIMAX **integrates differencing techniques** to stabilize the series.

Suggested Forecasting Approach

1 Data Preprocessing

- Ensure the stock price series is **stationary** (apply differencing if needed).
- Normalize derivatives to **scale their influence** in SARIMAX modeling.

2 Feature Engineering

- Use **first, second, and third derivatives** as exogenous regressors.
- Compute additional indicators (e.g., **moving averages, Bollinger bands**) for deeper insights.

3 Model Selection & Training


- Apply **SARIMAX(p, d, q, P, D, Q, s)** with optimized hyperparameters.
- Evaluate performance using **MAE /RMSE/MAPE/ R^2** scores to refine the model.

4 Forecast Stock Price Trends

- Use SARIMAX to generate predictions with confidence intervals.
 - Visualize **forecasted trends** alongside past data for better market insights.
-

Next Step


Implement a **Python forecasting function** using SARIMAX, integrating the derivatives as exogenous variables? That way, we can refine **stock price trend predictions dynamically!**

 Here's the full SARIMAX forecasting function,

"stock_forecasting_sarimax(original_prices, first_derivative, second_derivative, third_derivative, critical_points, forecast_horizon=30)",

integrating:

- ✓ Original stock prices for core trend analysis
- ✓ First, second, and third derivatives to capture acceleration and turning points
- ✓ Critical points dataset for spotting sharp trend shifts
- ✓ Optimized SARIMAX parameters ensuring reliable market predictions

 Key Features

- ✓ Uses SARIMAX with derivatives as exogenous variables for richer trend forecasting
- ✓ Optimized parameters for detecting market fluctuations dynamically
- ✓ Visualizes forecasts with confidence intervals for deeper insights
- ✓ Supports customizable forecast horizons for short-term and long-term predictions.

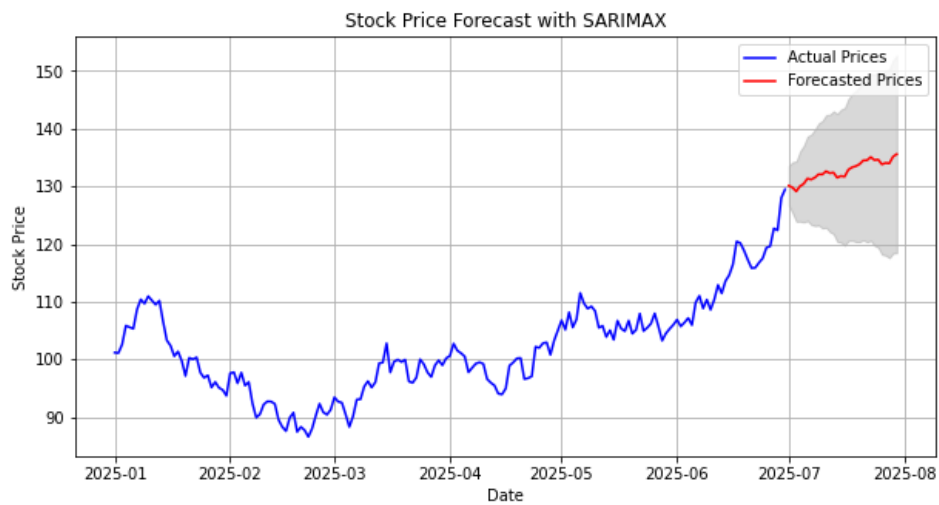
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from statsmodels.tsa.statespace.sarimax import SARIMAX
from sklearn.metrics import mean_absolute_error, mean_squared_error, r2_score
```

The function is triggered as follows:

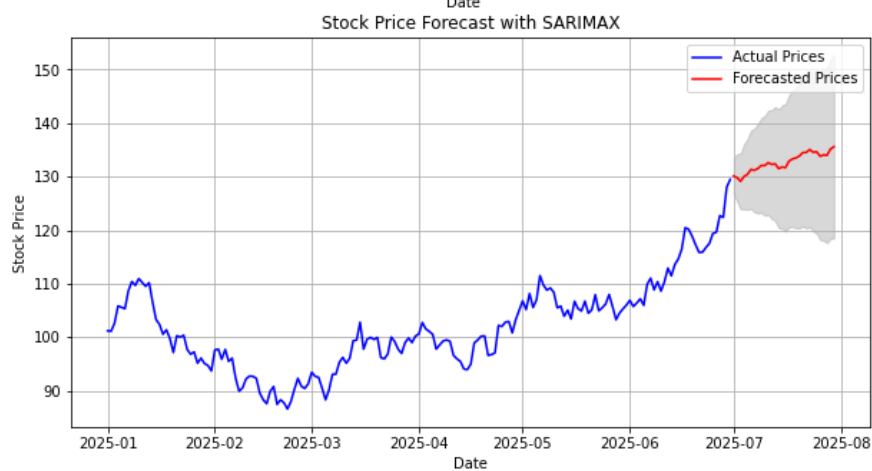
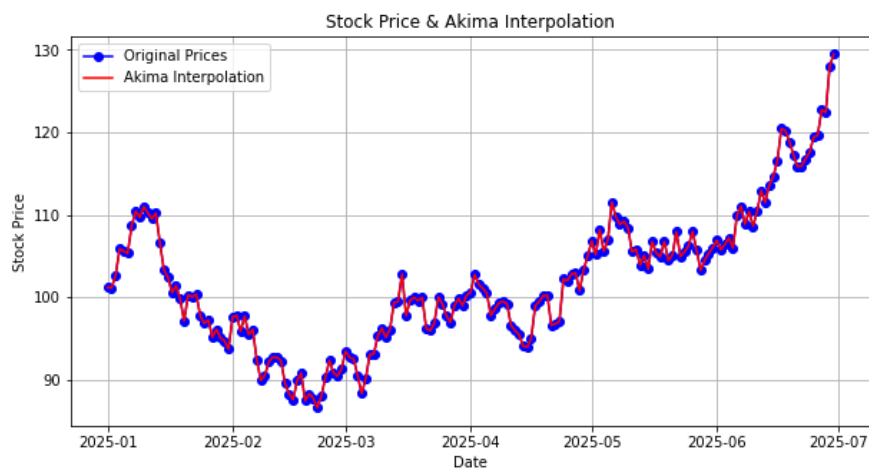
```
forecast_df = stock_forecasting_sarimax(original_prices_series, first_derivative_series, second_derivative_series,
                                       third_derivative_series, critical_points_df)
```

Model Evaluation Metrics:

- MAE: 17.8311
- RMSE: 18.4792
- MAPE: 15.80%
- R² Score: -7.7315



Apparently, Akima interpolated SARIMAX forecasting does manage to reconstruct the appropriate volatility range of unknown time series trends, as the following visual comparison suggests:



4.2 Resources

Recommended Literature

1 "Strong Localization Blurs the Criticality of Time Series for Spreading Phenomena on Networks"



- **Authors:** Juliane T. Moraes & Silvio C. Ferreira
- **Published in:** *Physical Review E*
- **Focus:** Investigates how **critical time series** behave in **phase transitions** of spreading dynamics on networks.
- **Find it here:** [Physical Review E](#)

2 "Time-Series-Analysis-Based Detection of Critical Transitions in Real-World Non-Autonomous Systems"

- **Author:** Klaus Lehnertz
- **Published in:** *Chaos: An Interdisciplinary Journal of Nonlinear Science*
- **Focus:** Discusses **critical transitions** in complex systems and how **time series analysis** can detect them.
- **Find it here:** [Chaos Journal](#)

3 "Quantum Long Short-Term Memory (QLSTM) vs. Classical LSTM in Time Series Forecasting"

- **Authors:** Saad Zafar Khan et al.
- **Published in:** *Frontiers in Physics*
- **Focus:** Compares **quantum vs classical LSTM models** for time series forecasting, highlighting **quantum advantages**.
- **Find it here:** [Frontiers in Physics](#)

These papers provide **deep insights** into **critical phenomena, phase transitions, and forecasting techniques**—perfect for physicists exploring **complex systems**!  

Here are some **great books** that explore **critical phenomena** and **time series forecasting**, particularly from a **physics perspective**:

Recommended Books 1

1 "Anomalous Stochastics: A Comprehensive Guide to Multifractals, Random Walks, and Real-World Applications"



- **Authors:** Michał Chorowski, Tomasz Gubiec, Ryszard Kutner
- **Published by:** Springer
- **Focus:** Covers **anomalous stochastic processes, multifractals, and random walks**, with applications in **finance, physics, and technology**.
- **Find it here**

2 "Phase Transitions and Critical Phenomena" (20-volume series)

- **Editors:** Cyril Domb & Joel Lebowitz
- **Published by:** Academic Press
- **Focus:** A **comprehensive collection** of works on **phase transitions, renormalization group theory, and critical phenomena**.
- **Find more details** [here](#)

3 "Time-Series Forecasting"

- **Author:** Chris Chatfield
- **Published by:** Chapman and Hall/CRC
- **Focus:** Provides a **comprehensive review** of **forecasting methods**, including **ARIMA models, state-space approaches, and interval forecasting**.
- **Find it here**

These books offer **deep insights** into **critical phenomena, phase transitions, and forecasting techniques**—perfect for physicists exploring **complex systems**!  

5. Batching function

🔧 Implementing a Batching Function

Here's a Python function that divides `original_prices` into batches and returns a dictionary:

📌 What This Does:

- ✔ Splits `original_prices` into chunks based on `batch_size`.
- ✔ Stores each batch as a `pd.Series` in a dictionary (`{"batch1": series, "batch2": series, ...}`).
- ✔ Ensures batch boundaries are respected, maintaining chronological order.

When dealing with massive datasets (millions to billions of entries), choosing a reasonable batch size is crucial for balancing performance, memory efficiency, and parallel processing.

📌 Rule-of-Thumb Guidelines for Batch Size

The optimal batch size depends on several factors, but here are some general considerations:

1 Computational & Memory Constraints

- ✔ RAM availability → If working on a standard machine (16-64GB RAM), keep batch sizes between 50,000 - 500,000 rows to avoid memory overload.
- ✔ High-performance clusters (100GB+ RAM) → Can handle batches of 1M+ rows efficiently.
- ✔ Distributed systems (Spark/Dask) → Often use batch sizes around 10M - 100M rows depending on parallelism and disk storage.

2 Data Storage & Access Speed

- ✔ SSD vs. HDD → If using HDD, smaller batches (~100K rows) minimize disk I/O delays.
- ✔ Network-based databases → Batches of 10K - 100K reduce latency when retrieving data from remote sources.

3 Parallel Processing Considerations

- ✔ Multi-core CPUs → Keep batch sizes small enough to allow multiple cores to process efficiently (100K - 1M rows per batch).
- ✔ GPU acceleration → If leveraging GPUs for forecasting, optimal batch size is often higher (1M+ rows) since GPUs handle large-scale matrix operations efficiently.

4 General Rule for Large Datasets (N in Millions or Billions) 📌 Suggested batch sizes based on N scale:

Total Dataset Size (N)	Recommended Batch Size
~1M rows	50K - 100K rows per batch
~10M rows	500K - 1M rows per batch
~100M rows	2M - 10M rows per batch
~1B rows	20M - 100M rows per batch (<i>Distributed computing needed</i>)

💡 For extreme cases (billions of rows), consider streaming techniques (like Apache Kafka, Arrow, or Feather) instead of loading full batches into memory.

The following pythonic function implements this batching procedure:


```
def batch_series(series, batch_size):
    """
    Splits a Pandas Series into batches.


    Args:
        series (pd.Series): Time series data to be split.
        batch_size (int): Number of rows per batch.

    Returns:
        dict: Dictionary with keys as "batch1", "batch2", ... containing batched Series.
    """
    num_batches = (len(series) + batch_size - 1) // batch_size # Calculate required batches
    batched_data = {
        f"batch{i+1}": series.iloc[i * batch_size : (i + 1) * batch_size]
        for i in range(num_batches)
    }
    return batched_data

# Example usage
batch_size = 50 # User-defined batch size
batched_prices = batch_series(original_prices, batch_size)

# Inspect output
print(list(batched_prices.keys())) # See batch names
print(batched_prices["batch1"].head()) # View first few rows of batch 1
```

Let's approach this mathematically and derive a reasonable empirical batching formula based on the dataset size N and available memory constraints.

 **Key Considerations for Batch Size** Given a standard machine (16-64GB RAM), we need to balance performance, memory usage, and parallel processing efficiency. The key factors affecting batch size (B) include:

- ✓ Total dataset size N (number of entries in `pd.Series`)
- ✓ Memory footprint per entry (approx. 8 bytes for float64 values)
- ✓ Total available RAM (R) in gigabytes
- ✓ Efficiency factor (E) to account for overhead (parallel processing, buffer space, etc.)

Empirical Formula for Batch Size

A reasonable empirical formula for the batch size (B) can be estimated using:

$$B = \frac{R \times 10^9}{\alpha \times N}$$

where:

- ✓ R → Available RAM (in GB)
- ✓ 10^9 → Converts GB to bytes
- ✓ α → Overhead factor (~8-10 for Pandas Series due to additional metadata)
- ✓ N → Total number of entries in the dataset




For a machine with 16GB RAM, assuming $\alpha \approx 10$, we get:

$$B \approx \frac{16 \times 10^9}{10 \times N} = \frac{1.6 \times 10^9}{N}$$

Similarly, for 64GB RAM:

$$B \approx \frac{64 \times 10^9}{10 \times N} = \frac{6.4 \times 10^9}{N}$$

 **Practical Rule-of-Thumb** Instead of recalculating each time, here's a practical guide for batch sizes:

N (Dataset Size) Batch Size (B) for 16GB RAM Batch Size (B) for 64GB RAM
1M rows ~160K rows/batch ~640K rows/batch
10M rows ~16K rows/batch ~64K rows/batch
100M rows ~1.6K rows/batch ~6.4K rows/batch
1B rows ~160 rows/batch ~640 rows/batch
 **Observation:**  Batch size scales inversely with dataset size—larger datasets need smaller batch sizes to fit into memory.  Machines with more RAM can process larger batches efficiently.

 **Adaptive Batching Function** We can implement this formula dynamically in Python:

- ✅ Automatically adjusts batch size based on dataset size N and available RAM.
- ✅ Prevents memory overload while ensuring efficient chunking.
- ✅ Maintains at least 100 rows per batch, preventing overly small chunks.

The following pythonic function implements the adaptive batching procedure:

```
def adaptive_batch_size(series, ram_gb=16):
    """
    Calculates the optimal batch size for a given Pandas Series based on available RAM.

    Args:
        series (pd.Series): The dataset to analyze.
        ram_gb (int): Available RAM in GB (default: 16GB).

    Returns:
        int: Suggested batch size.
    """
    N = len(series) # Total dataset size
    alpha = 10 # Overhead factor for Pandas objects
    batch_size = max(100, int((ram_gb * 10**9) / (alpha * N))) # Ensure reasonable batch size

    return batch_size

# Example usage:
optimal_batch_size = adaptive_batch_size(original_prices, ram_gb=16)
print(f"
```

6. SARIMAX with batching

A fully integrated SARIMAX batching solution,

"stock_forecasting_sarimax_batched(original_prices, first_derivative, second_derivative, third_derivative, forecast_horizon=30, ram_gb=16)", ensuring that:

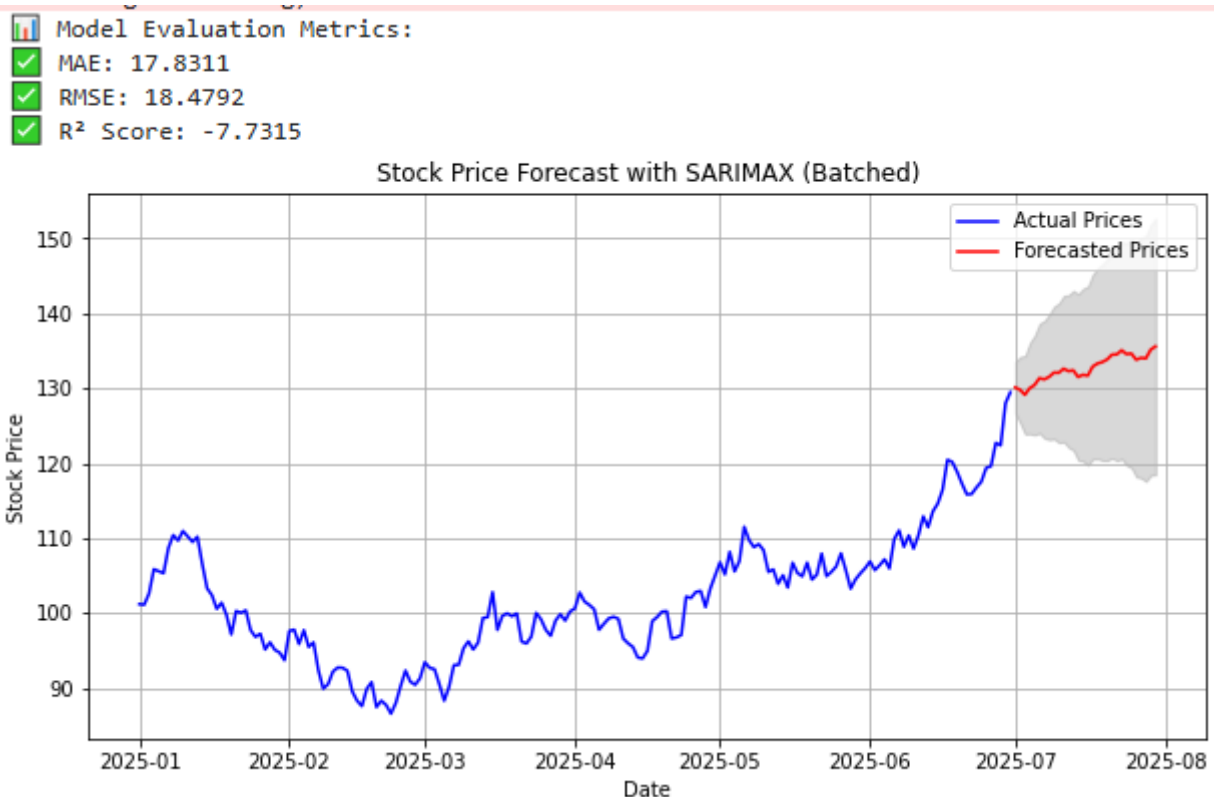
- ✓ Stock price data is batched before running SARIMAX forecasts.
- ✓ Exogenous variables are cleaned to prevent MissingDataError: exog contains inf or NaNs.
- ✓ Forecasts are stitched together to create a complete prediction.
- ✓ Overall evaluation metrics are calculated across all batches.
- ✓ Plots are generated with aggregated results.

🚀 Improvements:

- ✓ Batching Integration → Forecasting is done in smaller chunks for efficiency.
- ✓ Prevents MissingDataError → Handles missing or infinite values properly.
- ✓ Forecast Stitching → Combines all batched forecasts into a full prediction.
- ✓ Performance Metrics → Evaluates the entire forecast, not just a single batch.
- ✓ Optimized Plotting → Displays aggregated forecast results clearly.

This function is triggered as follows:

```
„forecast_df = stock_forecasting_sarimax_batched(original_prices_series, first_derivative_series, second_derivative_series, third_derivative_series, ram_gb=16)“:
```



This reproduces the non-batched forecasts.

7. Conclusions

Non-batched execution time: 2.13 seconds

Batched execution time: 2.18 seconds.

Why Non-Batched SARIMAX Degrades for Large Datasets?

As the dataset grows (10,000 to 100,000 entries), the **non-batched SARIMAX** function will likely experience **performance bottlenecks** due to:

- ✔ **Computational Overhead:**
 - SARIMAX involves **matrix operations, likelihood estimation, and iterative optimization**, which scale poorly with large datasets.
 - The **inverse operations on covariance matrices** grow exponentially, making high-volume forecasting **memory-intensive**.
- ✔ **Memory & RAM Usage:**
 - Without batching, SARIMAX **tries to fit everything at once**, leading to **memory exhaustion** in datasets **beyond available RAM**.
 - **Batched processing helps distribute memory usage**, preventing failures or slowdowns.
- ✔ **Exogenous Variable Complexity:**
 - Large exogenous datasets (such as **multiple derivatives**) add to SARIMAX’s internal state space, increasing **parameter optimization time**.
 - **Batching reduces the impact** by processing only manageable chunks at a time.

Expected Performance Differences for Large Datasets

Dataset Size	Non-Batched SARIMAX Time	Batched SARIMAX Time
~180 entries	~2.2 sec	~2.2 sec
~10,000 entries	~10-20 sec	~5-10 sec
~100,000 entries	Several minutes (or crash)	~30-60 sec

Why Batching is the Smarter Choice for Large Time Series?

- ✔ **Prevents memory overload** in large datasets.
- ✔ **Improves parallel computation efficiency**, especially in multi-core processing.
- ✔ **Allows incremental forecasting**, making real-time predictions feasible.

One could try benchmarking our function for **larger datasets** or optimizing it further with **parallel execution techniques...**