

LIMITS, CONTINUITY

EE24BTECH11046 - NENAVATH VASU *

I. MCQs WITH ONE CORRECT ANSWER

- 1) For a real number y , let $[y]$ denote the greatest integer less than or equal to y . Then the function $f(x) = \frac{\tan \pi[x-\pi]}{1+[x]^2}$ is
(1981 - 2 Marks)
 - a) discontinuous at some x
 - b) continuous at all x , but the derivative $f'(x)$ does not exist for some x
 - c) $f'(x)$ exists for all x , but the second derivative $f''(x)$ does not exist for some x
 - d) $f'(x)$ exists for all x
- 2) There exists a function $f(x)$, satisfying $f(0) = 1, f'(0) = -1, f(x) > 0$ for all x , and
(1982 - 2 Marks)
 - a) $f'(x) > 0$ for all x
 - b) $-1 < f''(x) < 0$ for all x
 - c) $-2 \leq f''(x) \leq -1$ for all x
 - d) $f''(x) < -2$ for all x
- 3) If $G(x) = -\sqrt{25-x^2}$ then $\lim_{x \rightarrow 1} \frac{G(x)-G(1)}{x-1}$ has the value
(1983 - 1 Mark)
 - a) $\frac{1}{24}$
 - b) $\frac{1}{5}$
 - c) $-\sqrt{24}$
 - d) none of these
- 4) If $f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2$, then the value of $\lim_{x \rightarrow a} \frac{g(x)f(a)-g(a)f(x)}{x-a}$ is
(1983 - 1 Mark)
 - a) -5
 - b) $\frac{1}{5}$
 - c) 5
 - d) none of these
- 5) The function $f(x) = \frac{\ln(1+ax)-\ln(1-bx)}{x}$ is not defined at $x=0$. The value which should be assigned to f at $x=0$ so that it is continuous at $x=0$, is
(1983 - 1 Mark)
 - a) $a-b$
 - b) $a+b$
 - c) $\ln a - \ln b$
 - d) none of these
- 6) $\lim_{n \rightarrow \infty} \left(\frac{1}{1-n^2} + \frac{2}{1-n^2} + \dots + \frac{n}{1-n^2} \right)$ is equal to
(1984 - 2 Marks)
 - a) 0
 - b) $-\frac{1}{2}$
 - c) $\frac{1}{2}$
 - d) none of these

7) If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x , then $\lim_{x \rightarrow 0} f(x)$ equals (1985 - 2 Marks)

- a) 1
- b) 0
- c) -1
- d) none of these

8) Let $f : R \rightarrow R$ be a differentiable function and $f(1) = 4$. Then the value of

$$\lim_{x \rightarrow 1} \int_4^{f(x)} \frac{2t}{x-1} dt \quad (1)$$

is

(1990 - 2 Marks)

- a) $8f'(1)$
- b) $4f'(1)$
- c) $2f'(1)$
- d) $f''(1)$

9) Let $[.]$ denote the greatest integer function and $f(x) = [\tan^2 x]$, then (1993 - 1 Mark)

- a) $\lim_{x \rightarrow 0}$ does not exist
- b) $f(x)$ is continuous at $x=0$
- c) $f(x)$ is not differentiable at $x=0$
- d) $f'(0) = 1$

10) The function $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, is discontinuous at (1995S)

- a) All x
- b) All integer points
- c) No x
- d) x which is not an integer

11) $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}}$ equals (1997 - 2 Marks)

- a) $1 + \sqrt{5}$
- b) $-1 + \sqrt{5}$
- c) $-1 + \sqrt{2}$
- d) $1 + \sqrt{2}$

12) The function $f(x) = [x]^2 - [x]^2$ (where $[y]$ is the greatest integer less than or equal to y), is discontinuous at (1992 - 2 Marks)

- a) all integers
- b) all integers except 0 and 1
- c) all integers except 0
- d) all integers except 1

13) The function $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos(|x|)$ is NOT differentiable at (1999 - 2 Marks)

- a) -1
- b) 0
- c) 1
- d) 2

14) $\lim_{x \rightarrow 0} \frac{x \tan(2x) - 2x \tan(x)}{(1 - \cos(2x))^2}$ is (1999 - 2 Marks)

- a) 2

b) -2

c) $\frac{1}{2}$

d) $\frac{-1}{2}$

15) For $x \in \mathbb{R}$, $\lim_{x \rightarrow \infty} \left(\frac{x-3}{x+2} \right)^x =$

(2000S)

a) e

b) e^{-1}

c) e^{-5}

d) e^5