LIMITS,CONTINUITY

EE24BTECH11046 - NENAVATH VASU *

I. MCQs with One Correct Answer

2) For a real number y, let $[y]$ denote the greatest integer less thsn or equal to y.Then the function $f(x) = \frac{\tan \pi [x - \pi]}{1 + [x]^2}$ is

(1981 - 2 Marks)

a) discontinuous at some x

b) continuous at all x, but the derivative $f'(x)$ does not exist for some x

c) $f'(x)$ exists for all x, but the second derivative $f''(x)$ does not exist for some x

d) $f'(x)$ exists for all x

3) There exists a function $f(x)$,satisfying $f(0) = 1, f'(0) = -1, f(x) > 0$ for all x, and

(1982 - 2 Marks)

a) $f'(x) > 0$ for all x

b) $-1 < f''(x) < 0$ for all x

c) $-2 \leq f''(x) \leq -1$ for all

d) $f''(x) < -2$ for all x

4) If $G(x) = -\sqrt{25 - x^2}$ then $\lim_{x \to 1} \frac{G(x) - G(1)}{x - 1}$ has the value

(1983 - 1 Mark)

a) $\frac{1}{24}$

b) $\frac{1}{5}$

c) $-\sqrt{24}$

d) none of these

5) If $f(a) = 2, f'(a) = 1, g(a) = -1, g'(a)2$, then the value of $\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$ is

(1983 - 1 Mark)

a) -5

b) $\frac{1}{5}$

c) 5

d) none of these

6) The function $f(x) = \frac{\ln(1 + ax) - \ln(1 - bx)}{x}$ is not defined at $x=0$.The value which should be assigned to f at $x = 0$ so that it is continuous at $x=0$, is

(1983 - 1 Mark)

a) a-b

b) a+b

c) $\ln a - \ln b$

d) none of these

7) $\lim_{n \to \infty} \left(\frac{1}{1 - n^2} + \frac{2}{1 - n^2} + ... + \frac{n}{1 - n^2} \right)$ is equal to

(1984 - 2 Marks)

a) 0

b) $-\frac{1}{2}$

c) $\frac{1}{2}$

d) none of these

8) If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, [x] \neq 0 \\ = 0, [x] = 0 \end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x, then $\lim_{x \to 0}$ equals

(1985 - 2 Marks)

a) 1

b) 0

c) -1

d) none of these

9) Let $f : R \to R$ be a differentiable function and $f(1) = 4$.Then the value of $\lim_{x \to 1}$

$$\int_4^{f(x)} \frac{2t}{x - 1} dt$$

is

(1990 - 2 Marks)

a) $8f'(1)$

b) $4f'(1)$

c) $2f'(1)$

d) $f''(1)$

10) Let $[.]$ denote the greatest integer function and $f(x) = [\tan^2 x]$, then

(1993 - 1 Mark)

a) $\lim_{x \to 0}$ does not exist

b) $f(x)$ is continuous at $x=0$

c) $f(x)$ is not differentiable at $x=0$

d) $f'(0) = 1$

11) The function $f(x) = [x] \cos \left(\frac{2x - 1}{2} \right) \pi$,denotes[.] denotes the greatest integer function, is discontinuous at

(1995S)

a) All x

b) All integer points

c) No x

d) x which is not an integer

12) $\lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$ equals (1997 - 2 Marks)

a) $1 + \sqrt{5}$

b) $-1 + \sqrt{5}$

c) $-1 + \sqrt{2}$

d) $1 + \sqrt{2}$

13) The function $f(x) = [x]^2 - [x^2]$ (where $[y]$ is the greatest integer less than or equal to y), is discontinuous at   (1992 - 2 Marks)

   a) all integers

   b) all integers except 0 and 1

   c) all integers except 0

   d) all integers except 1

14) The function $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos(|x|)$ is NOT differentiable at

                          (1999 - 2 Marks)

   a) -1

   b) 0

   c) 1

   d) 2

15) $\lim_{x \to 0} \frac{x \tan(2x) - 2x \tan(x)}{(1 - \cos(2x))^2}$ is   (1999 - 2 Marks)

   a) 2

   b) -2

   c) $\frac{1}{2}$

   d) $\frac{-1}{2}$

16) For $x \in R, \lim_{x \to \infty} \left(\frac{x-3}{x+2}\right)^x =$ (2000S)

   a) e

   b) e^{-1}

   c) e^{-5}

   d) e^5