# LIMITS,CONTINUITY

EE24BTECH11046 - NENAVATH VASU *

## I. MCQs with One Correct Answer

1) For a real number $y$, let $[y]$ denote the greatest integer less than or equal to $y$. Then the function $f(x) = \frac{\tan \pi[x-\pi]}{1+[x]^2}$ is

(1981 - 2 Marks)

a) discontinuous at some $x$
b) continuous at all $x$, but the derivative $f'(x)$ does not exist for some $x$
c) $f'(x)$ exists for all $x$, but the second derivative $f''(x)$ does not exist for some $x$
d) $f'(x)$ exists for all $x$

2) There exists a function $f(x)$,satisfying $f(0) = 1, f'(0) = -1, f(x) > 0$ for all $x$, and

(1982 - 2 Marks)

a) $f'(x) > 0$ for all $x$
b) $-1 < f''(x) < 0$ for all $x$
c) $-2 \leq f''(x) \leq -1$ for all
d) $f''(x) < -2$ for all $x$

3) If $G(x) = -\sqrt{25 - x^2}$ then $\lim_{x\to 1} \frac{G(x)-G(x)}{x-1}$ has the value (1983 - 1 Mark)

a) $\frac{1}{24}$
b) $\frac{1}{5}$
c) - $\sqrt{24}$
d) none of these

4) If $f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2$, then the value of $\lim_{x\to a} \frac{g(x)f(a)-g(a)f(x)}{x-a}$ is

(1983 - 1 Mark)

a) -5
b) $\frac{1}{5}$
c) 5
d) none of these

5) The function $f(x) = \frac{\ln(1+ax)-\ln(1-bx)}{x}$ is not defined at $x$=0.The value which should be assigned to $f$ at $x = 0$ so that it is continuous at $x$=0, is

(1983 - 1 Mark)

a) a-b
b) a+b
c) $\ln a - \ln b$
d) none of these

6) $\lim_{n\to\infty} \left(\frac{1}{1-n^2} + \frac{2}{1-n^2} + ... + \frac{n}{1-n^2}\right)$ is equal to

(1984 - 2 Marks)

a) 0
b) -$\frac{1}{2}$
c) $\frac{1}{2}$
d) none of these

7) If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$ where $[x]$ denotes the greatest integer less than or equal to $x$, then $\lim_{x\to 0} f(x)$ equals (1985 - 2 Marks)

a) 1
b) 0
c) -1
d) none of these

8) Let $f : R \to R$ be a differentiable function and $f(1) = 4$.Then the value of

$$\lim_{x \to 1} \int_4^{f(x)} \frac{2t}{x-1}\, dt \tag{1}$$

is (1990 - 2 Marks)

a) $8f'(1)$
b) $4f'(1)$
c) $2f'(1)$
d) $f''(1)$

9) Let [.] denote the greatest integer function and $f(x) = \left[\tan^2 x\right]$, then (1993 - 1 Mark)

a) $\lim_{x\to 0}$ does not exist
b) $f(x)$ is continuous at $x$=0
c) $f(x)$ is not differentiable at $x$=0
d) $f'(0) = 1$

10) The function $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$ , where $[x]$ denotes the greatest integer function, is discontinuous at (1995S)

a) All $x$
b) All integer points
c) No $x$
d) $x$ which is not an integer

11) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}}$ equals (1997 - 2 Marks)

a) 1+ $\sqrt{5}$
b) -1+ $\sqrt{5}$
c) -1+ $\sqrt{2}$
d) 1+ $\sqrt{2}$

12) The function $f(x) = [x]^2$-$\left[x^2\right]$ , where $[y]$ is the greatest integer less than or equal to $y$, is discontinuous at (1992 - 2 Marks)

a) all integers
b) all integers except 0 and 1
c) all integers except 0
d) all integers except 1

13) The function $f(x) = \left(x^2 - 1\right)\left|x^2 - 3x + 2\right| + \cos(|x|)$ is NOT differentiable at (1999 - 2 Marks)

a) -1
b) 0
c) 1
d) 2

14) $\lim_{x\to 0} \frac{x\tan(2x)-2x\tan(x)}{(1-\cos(2x))^2}$ is (1999 - 2 Marks)

a) 2
b) -2
c) $\frac{1}{2}$
d) $\frac{-1}{2}$

15) For $x \in R, \lim_{x\to\infty} \left(\frac{x-3}{x+2}\right)^x$ = (2000S)

a) $e$
b) $e^{-1}$
c) $e^{-5}$
d) $e^5$