LIMITS,CONTINUITY

EE24BTECH11046 - NENAVATH VASU *

I. MCQs with One Correct Answer

1) For a real number y, let $[y]$ denote the greatest integer less than or equal to y.Then the function$f(x) = \frac{\tan\pi[x-\pi]}{1+[x]^2}$ is

(1981 - 2 Marks)

a) discontinuous at some x
b) continuous at all x, but the derivative $f'(x)$ does not exist for some x
c) $f'(x)$ exists for all x, but the second derivative $f''(x)$ does not exist for some x
d) $f'(x)$ exists for all x

2) There exists a function $f(x)$,satisfying $f(0) = 1, f'(0) = -1, f(x) > 0$ for all x, and

(1982 - 2 Marks)

a) $f'(x) > 0$ for all x
b) $-1 < f''(x) < 0$ for all x
c) $-2 \leq f''(x) \leq -1$ for all
d) $f''(x) < -2$ for all x

3) If $G(x) = -\sqrt{25-x^2}$ then $\lim_{x\to 1} \frac{G(x)-G(1)}{x-1}$ has the value (1983 - 1 Mark)

a) $\frac{1}{24}$
b) $\frac{1}{5}$
c) - $\sqrt{24}$
d) none of these

4) If $f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2$, then the value of $\lim_{x\to a} \frac{g(x)f(a)-g(a)f(x)}{x-a}$ is

(1983 - 1 Mark)

a) -5
b) $\frac{1}{5}$
c) 5
d) none of these

5) The function $f(x) = \frac{\ln(1+ax)-\ln(1-bx)}{x}$ is not defined at x=0.The value which should be assigned to f at $x = 0$ so that it is continuous at x=0, is

(1983 - 1 Mark)

a) a-b
b) a+b
c) $\ln a - \ln b$
d) none of these

6) $\lim_{n\to\infty} \left(\frac{1}{1-n^2} + \frac{2}{1-n^2} + ... + \frac{n}{1-n^2}\right)$ is equal to

(1984 - 2 Marks)

a) 0
b) -$\frac{1}{2}$
c) $\frac{1}{2}$
d) none of these

7) If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, [x] \neq 0 \\ 0, [x] = 0 \end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x, then $\lim_{x\to 0} f(x)$ equals (1985 - 2 Marks)
   a) 1
   b) 0
   c) -1
   d) none of these

8) Let $f : R \to R$ be a differentiable function and $f(1) = 4$.Then the value of

$$\lim_{x\to 1} \int_4^{f(x)} \frac{2t}{x-1}\, dt \tag{1}$$

is (1990 - 2 Marks)
   a) $8f'(1)$
   b) $4f'(1)$
   c) $2f'(1)$
   d) f"(1)

9) Let [.] denote the greatest integer function and $f(x) = [\tan^2 x]$, then (1993 - 1 Mark)
   a) $\lim_{x\to 0}$ does not exist
   b) $f(x)$ is continuous at x=0
   c) $f(x)$ is not differentiable at x=0
   d) $f'(0) = 1$

10) The function $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$,denotes[.] denotes the greatest integer function, is discontinuous at (1995S)
   a) All x
   b) All integer points
   c) No x
   d) x which is not an integer

11) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}}$ equals (1997 - 2 Marks)
   a) 1+ $\sqrt{5}$
   b) -1+ $\sqrt{5}$
   c) -1+ $\sqrt{2}$
   d) 1+ $\sqrt{2}$

12) The function $f(x) = [x]^2$-$[x^2]$ (where $[y]$ is the greatest integer less than or equal to y), is discontinuous at (1992 - 2 Marks)
   a) all integers
   b) all integers except 0 and 1
   c) all integers except 0
   d) all integers except 1

13) The function $f(x) = (x^2-1)\left|x^2-3x+2\right| + \cos(|x|)$ is NOT differentiable at (1999 - 2 Marks)
   a) -1
   b) 0
   c) 1
   d) 2

14) $\lim_{x\to 0} \frac{x\tan(2x)-2x\tan(x)}{(1-\cos(2x))^2}$ is (1999 - 2 Marks)
   a) 2
   b) -2

c) $\frac{1}{2}$
d) $\frac{-1}{2}$

15) For $x \in R, \lim_{x\to\infty}(\frac{x-3}{x+2})^x =$ (2000S)

a) e
b) e^{-1}
c) e^{-5}
d) e^5