

1 a)  $a_{11} = 1$

b)  $a_{13} = 5$

c)  $a_{31} = 2$

d)  $\sum_{i=1}^3 = 1 + -2 + 1 = 0$

g)  $A^T \rightarrow \text{Transpose}$

$$A^T = \begin{bmatrix} -1 & 1 & 4 \\ 0 & 3 & 2 \\ 2 & -2 & 3 \end{bmatrix}$$

b)  $\begin{bmatrix} 1 & 2 & 5 \\ 3 & -4 & -2 \end{bmatrix} = C^T$

c)  $A+B = \begin{bmatrix} -1+6 & 0+8 & 7 \\ 5 & 1 & 5 \\ 7 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 8 & 7 \\ 5 & 1 & 5 \\ 7 & 3 & 5 \end{bmatrix}$

d) This cannot be done  $A$  is  $3 \times 3$ ;  $C$  is  $3 \times 2$ 

c)  $\begin{bmatrix} 5 & 5 & 7 \\ 8 & 1 & 3 \\ 7 & 5 & 5 \end{bmatrix} = (A+B)^T$

e)  $A^T+B^T = \begin{bmatrix} 6+1 & 4+1 & 3+4 \\ 0+8 & -2+3 & 2+1 \\ 2+5 & -2+7 & 2+3 \end{bmatrix}$

$$A^T+B^T = \begin{bmatrix} 5 & 5 & 7 \\ 8 & 1 & 3 \\ 7 & 5 & 5 \end{bmatrix}$$

g)  $\begin{bmatrix} 6 \cdot 2 & 4+8 & 3+5 \\ 8+4 & -2 \cdot 2 & 1+7 \\ 3+5 & 4+8 & 2 \cdot 2 \end{bmatrix} = B+B^T$

$$\begin{bmatrix} 12 & 12 & 8 \\ 12 & -4 & 8 \\ 8 & 12 & 4 \end{bmatrix} = B+B^T$$

b)

$C + C^t =$  incompatible  $C$  is  $3 \times 2$  while  $C^t$  is  $2 \times 3$

④ a) b)

$$a) \begin{bmatrix} -2 & 0 & 4 \\ 2 & 6 & -4 \\ 8 & 4 & 6 \end{bmatrix} = A + A$$

b) same as above

5) a de a) odd = 0 e = even)

$$\begin{bmatrix} \boxed{0} & \boxed{0} & \boxed{0} \\ \boxed{0} & \boxed{0} & \boxed{0} \\ \boxed{0} & \boxed{0} & \boxed{0} \\ -1 & 1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1+1 & 1+2 & 1+3 \\ 2+1 & 2+2 & 2+3 \\ 3+1 & 3+2 & 3+3 \\ 4+1 & 4+2 & 4+3 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \\ 5 & 6 & 7 \end{bmatrix}$$

$$a) A^t = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$$a) \begin{bmatrix} 3 & 2 & 5 \\ 2 & 5 & 4 \\ 5 & 4 & 7 \\ 4 & 5 & 6 \end{bmatrix}$$

 $a + B$

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$$(e) \begin{bmatrix} 3 & 2 & 5 & 4 \\ 2 & 5 & 4 & 5 \\ 5 & 4 & 7 & 6 \end{bmatrix}$$

(6) ?

$$(7) A) \begin{bmatrix} 3 \cdot 1 + 9 \cdot -2 & 3 \cdot -3 + 9 \cdot 6 \\ 1 \cdot 1 + 3 \cdot -2 & 1 \cdot -3 + 3 \cdot 6 \end{bmatrix} = AB$$

$$A) \begin{bmatrix} 3-18 & -9+54 \\ 1-6 & -3+18 \end{bmatrix} = \begin{bmatrix} -15 & 45 \\ -5 & 15 \end{bmatrix} = AB$$

(8)

$$BA = \begin{bmatrix} 1 \cdot 3 + -3 \cdot 1 & 1 \cdot 9 + -3 \cdot 3 \\ -2 \cdot 3 + 6 \cdot 1 & -2 \cdot 9 + 6 \cdot 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$(c) A^2 = \begin{bmatrix} 3 \cdot 3 + 9 \cdot 1 & 3 \cdot 9 + 9 \cdot 3 \\ 1 \cdot 3 + 3 \cdot 1 & 1 \cdot 9 + 3 \cdot 3 \end{bmatrix} = \begin{bmatrix} 18 & 54 \\ 6 & 18 \end{bmatrix}$$

$$(d) B^2 = \begin{bmatrix} 1 \cdot 1 + -3 \cdot -2 & 1 \cdot -3 + -3 \cdot 6 \\ -2 \cdot 1 + 6 \cdot -2 & -2 \cdot -3 + 6 \cdot 6 \end{bmatrix} = \begin{bmatrix} 7 & -21 \\ 14 & 42 \end{bmatrix}$$

(8) (A)  $(A+B)^2$ 

$$A+B = \begin{bmatrix} 4 & 6 \\ -1 & 9 \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} 4 \cdot 4 + 6 \cdot -1 & 4 \cdot 6 + 6 \cdot 9 \\ -1 \cdot 4 + 9 \cdot -1 & -1 \cdot 6 + 9 \cdot 9 \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} 10 & 78 \\ -13 & 75 \end{bmatrix}$$

B) a)

$$A^2 = \begin{bmatrix} 18 & 54 \\ 6 & 18 \end{bmatrix} \quad B^2 = \begin{bmatrix} 7 & -21 \\ -14 & 42 \end{bmatrix} \quad AB = \begin{bmatrix} -15 & 45 \\ -5 & 15 \end{bmatrix}$$

$$A^2 + B^2 + 2AB = \begin{bmatrix} 18+7-15 \cdot 2 & 54-21+45 \cdot 2 \\ 6-14-5 \cdot 2 & 18+42+15 \cdot 2 \end{bmatrix} = \begin{bmatrix} -5 & 123 \\ -18 & 90 \end{bmatrix}$$

b) No

13) ABC  $c(aA + bB) = caA + cbB = (ca)A + (cb)B$

a) are equal?

b)  $-aA = (-a)A = a(-A)$

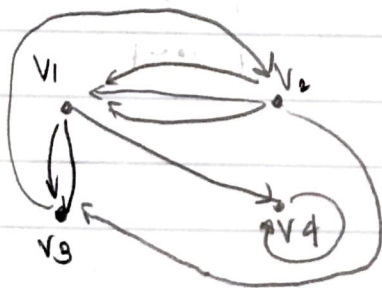
c)  $(aA)^T = aA^T$

15) a) 4

(6) A)

	$V_1$	$V_2$	$V_3$	$V_4$
$V_1$	2	1	1	0
$V_2$	1	1	0	0
$V_3$	1	0	0	1
$V_4$	0	0	1	0

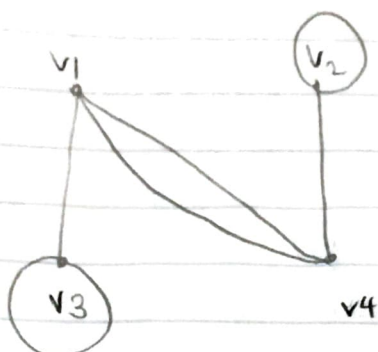
17) a)





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(8) b)



section 11.3

(1) (a) 
$$\begin{bmatrix} 1 \cdot 2 + 2 \cdot -1 + 4 \cdot -2 & 1 \cdot 2 + 2 \cdot 0 + 4 \cdot 3 \\ 3 \cdot 2 + 0 \cdot -1 + 2 \cdot -2 & 3 \cdot 1 + 0 \cdot 2 \cdot 3 \end{bmatrix}$$

a) 
$$\begin{bmatrix} -8 & 13 \\ 2 & 9 \end{bmatrix}$$

(b) 
$$\begin{aligned} BA &= \begin{bmatrix} 2 & -1 \\ -1 & 0 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 2+3 & 4+0 & 8+2 \\ -1+0 & -2+0 & -4+0 \\ -2+3 \cdot 3 & -4+0 & -8+6 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 4 & 10 \\ -1 & -2 & -4 \\ 7 & -4 & -2 \end{bmatrix} \end{aligned}$$

(c) 
$$= \begin{bmatrix} -8 & 13 \\ 2 & 9 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 2 \end{bmatrix}$$
  
(AB)

$$ABA = \begin{bmatrix} -8+13 \cdot 3 & -16+0 & -8 \cdot 4+13 \cdot 2 \\ 2 \cdot 1+9 \cdot 3 & 2 \cdot 2+9 \cdot 0 & 2 \cdot 4+9 \cdot 2 \end{bmatrix} = \begin{bmatrix} 31 & -16 & -6 \\ 29 & 4 & 26 \end{bmatrix}$$

(d)

$$A+B^T = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -1 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$A+B^T = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 0 & 5 \end{bmatrix}$$

$$(e) \quad 3A^T - 2B = 3 \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ 4 & 2 \end{bmatrix} - 2 \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ -2 & 3 \end{bmatrix}$$

$$3A^T - 2B = \begin{bmatrix} 3 & 9 \\ 6 & 0 \\ 12 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ -2 & 0 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} -1 & 7 \\ 8 & 0 \\ 16 & 0 \end{bmatrix}$$

$$(f) \quad (AB)^2 \rightarrow AB = \begin{bmatrix} -8 & 13 \\ 2 & 9 \end{bmatrix}$$

$$AB^2 = \begin{bmatrix} -8 \cdot -8 + 13 \cdot 2 & -8 \cdot 13 + 13 \cdot 9 \\ 2 \cdot -8 + 9 \cdot 2 & 2 \cdot 13 + 9 \cdot 9 \end{bmatrix}$$

$$AB^2 = \begin{bmatrix} 90 & 13 \\ 2 & 107 \end{bmatrix}$$

3) A

A<sup>2</sup> does not exist

B)

$$B^2 = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 6 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

c)

AB does not exist

$$a) \quad BA = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & -4 & 3 & 1 \\ 2 & 0 & 1 & -2 \\ -1 & 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -2 & -3 \\ 6 & -3 & 7 & -3 \\ 3 & -1 & -1 & -2 \end{bmatrix}$$

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④  $B = 3 \times 3$

$A^T = 4 \times 3$ , thus,  $BA^T$  does not exist  
 $4 \times 3 \times 3 \times 3$

⑤  $A^T B = \begin{bmatrix} 3 & 2 & -1 \\ -4 & 0 & 1 \\ 3 & 1 & 2 \\ 1 & -2 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix} =$

$\begin{bmatrix} 3 \cdot -1 + 2 \cdot 1 - 1 \cdot 0 & 3 \cdot 1 + 2 \cdot 2 - 1 \cdot (-1) & 3 \cdot 0 + 2 \cdot 1 - 1 \cdot (-1) \\ (-1) & (6) & (3) \\ -4 \cdot -1 + 0 \cdot 1 + 1 \cdot 0 & -4 \cdot 1 + 0 \cdot 2 + 1 \cdot 1 & -4 \cdot 0 + 0 \cdot 1 + 1 \cdot (-1) \\ 3 \cdot -1 + 1 \cdot 1 + 2 \cdot 0 & 3 \cdot 1 + 1 \cdot 2 + 2 \cdot (-1) & 3 \cdot 0 + 1 \cdot 1 + 2 \cdot (-1) \\ 1 \cdot -1 - 2 \cdot 1 + 0 & 1 \cdot 1 - 2 \cdot 2 + 0 & 1 \cdot 0 - 2 \cdot 1 + 0 \end{bmatrix} = \begin{bmatrix} -1 & 6 & 3 \\ 4 & -3 & -1 \\ -2 & 7 & -1 \\ -3 & -3 & -2 \end{bmatrix}$

⑥

$B^T \times A^T$

does not exist

$3 \times 3 \times 4 \times 3$   
x

⑦ (a)

$M^2$ ,  $V_1$  to itself is  $\boxed{2}$  b)  $\boxed{9}$

(c)  $\boxed{12}$

d) 0

⑧

a) ?

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①

a) Same  $\rightarrow$  equivalence relations,  $(T)$ ,  $(R)$ ,  $(S)$ ,

b) Not reflexive, not transitive, No equivalence, is symmetric, Not transitive

c) Not  $R$ ,  $S$ ,  $T$ , Not equivalence relationd)  $R$ , is  $S$ , Not  $T$ , Not equivalence relatione) is  $R$ , is  $S$ , Not  $T$ , Not equivalence relationf) is  $R$ , is  $S$ , is  $T$ , is an Equivalence relation

②

a)  $L$  and  $M$  are parallel a) No

b) No equivalence class

f)  $d(p) = \{q \in S : q \text{ is the son of } p\}$ 

c) No

d) No

⑦

a)  $(R)(S)(T)(e)$ b)  $m^2 = n^2$  $m^2 - n^2$ 

$$(m+n)(m-n) = 0$$

$$m = \pm n \rightarrow \boxed{2}$$

⑨

a)  $o(n) = |n|$ 

$$d(0) = \{0\}$$

$$d(n) = \{n, -n\}$$

$$\{0\}, \{n, -n\}, n \in \mathbb{Z}$$

b)  $h(n) = 1 + (-1)^n$ 

$$n = 2p, p \in \mathbb{Z}$$

$$d(n=2p) = \{2\}$$

$$d(n=2p+1) = \{0\}$$

$$\left\{ \begin{array}{l} 2, \text{ when } n=2p, p \in \mathbb{Z} \\ 0, \text{ when } n=2p+1, p \in \mathbb{Z} \end{array} \right.$$



10) a) R

$$m+l = h+k$$

$$m+l = h+k$$

$$m+l = h+k$$

$$k+n = l+m \text{ thus is } (S)$$

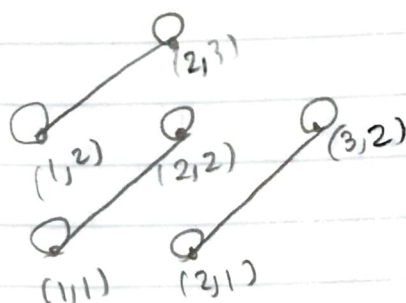
$m+l = h+k$  | is transitive  
 $k+n = l+m$  | is an equivalence relation

$$(m+l) + (k+n) = (h+k) + (l+m)$$

$$(m+n) + (l+k) = (h+p) + (l+k)$$

$$m+n = h+p$$

$$(1,1), (1,2), (2,2), (2,1)$$



1)