Biostatistics Project

December 15, 2017

1 Final Exam

CS 555 Multivariate Data Analysis, Fall 2017 Due on December 15, 2017 100 total points

Use R to perform all necessary calculations. Attach your code and output. Give interpretation and discuss all relevant statistical measures.

2 Problem 1.

(25 points) We are interested in comparing four characteristics (sepal length, sepal width, petal length and petal width) of three flower species Setosa, Versicolor and Virginica. We want to analyze the dataset "iris" available in the package MVN. Perform all necessary data analysis steps and write a section summarizing the findings.

2.1 Load data and basic analysis

```
In [2]: # load data
        library(car)
        library(MVN)
        data("iris")
        head(iris)
```

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
5.1	3.5	1.4	0.2	setosa
4.9	3.0	1.4	0.2	setosa
4.7	3.2	1.3	0.2	setosa
4.6	3.1	1.5	0.2	setosa
5.0	3.6	1.4	0.2	setosa
5.4	3.9	1.7	0.4	setosa

```
In [3]: # understand the data
        dim(iris)
        table(iris$Species)
        summary(iris)
        str(iris)
```

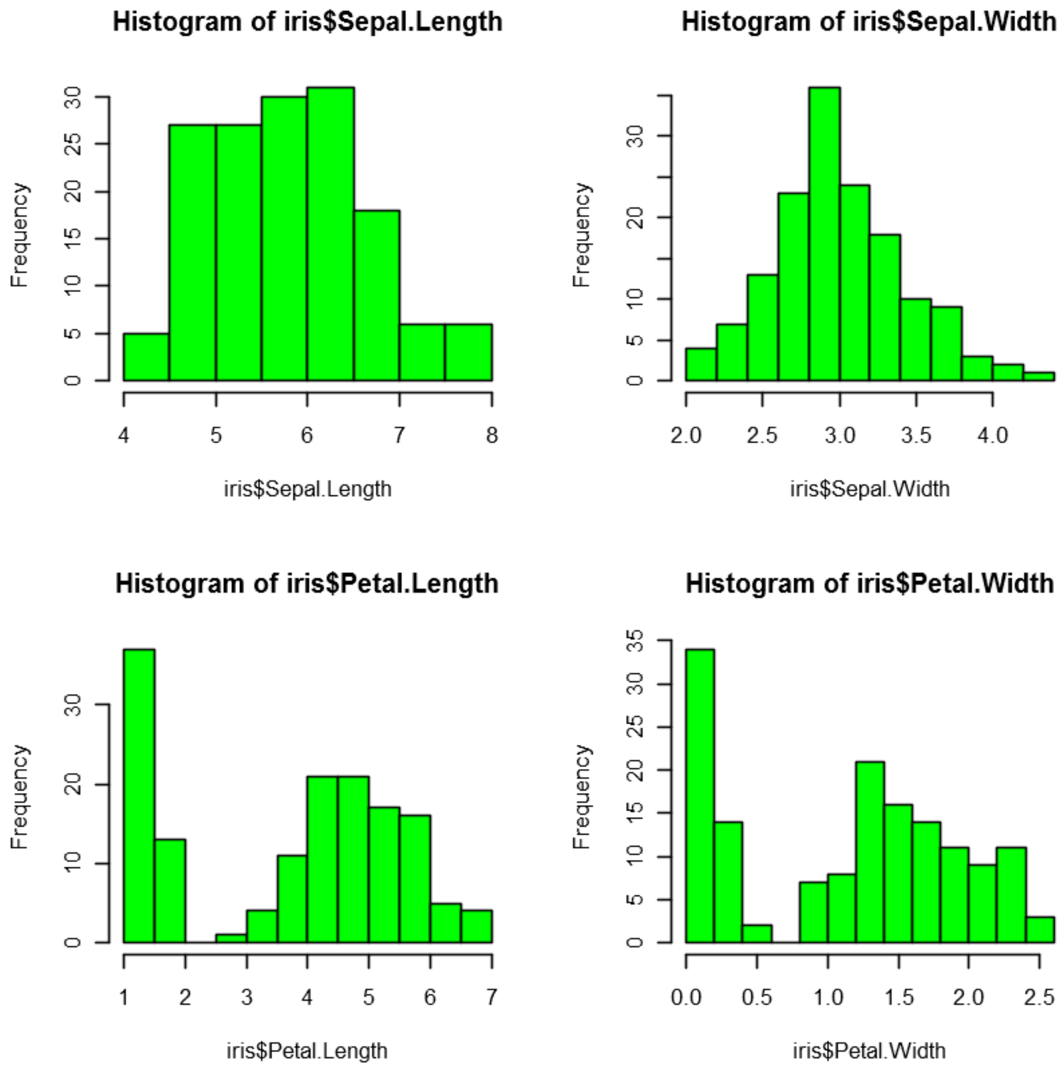
1. 150 2. 5

```
    setosa versicolor  virginica 
        50         50         50 
```

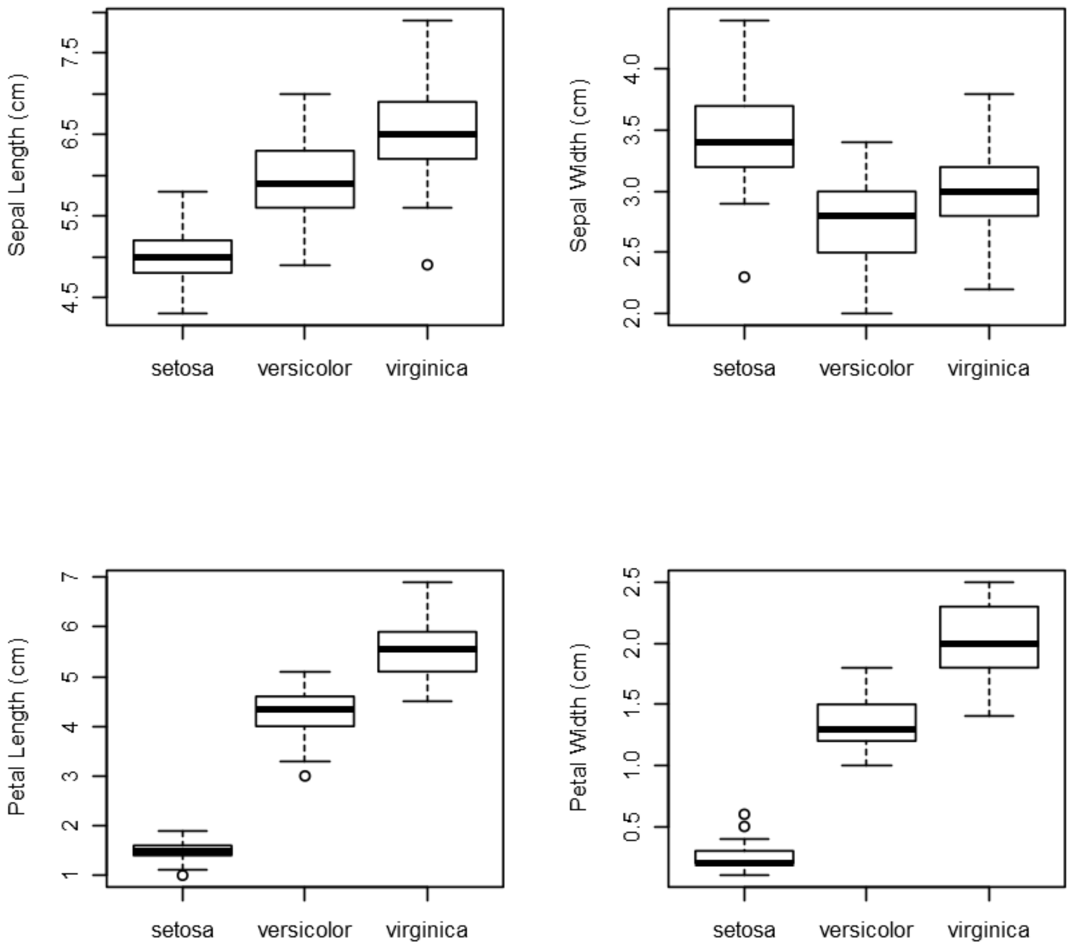
```
  Sepal.Length    Sepal.Width     Petal.Length    Petal.Width   
 Min.   :4.300   Min.   :2.000   Min.   :1.000   Min.   :0.100  
 1st Qu.:5.100   1st Qu.:2.800   1st Qu.:1.600   1st Qu.:0.300  
 Median :5.800   Median :3.000   Median :4.350   Median :1.300  
 Mean   :5.843   Mean   :3.057   Mean   :3.758   Mean   :1.199  
 3rd Qu.:6.400   3rd Qu.:3.300   3rd Qu.:5.100   3rd Qu.:1.800  
 Max.   :7.900   Max.   :4.400   Max.   :6.900   Max.   :2.500  
       Species  
 setosa    :50  
 versicolor:50  
 virginica :50  
```

```
'data.frame':	150 obs. of  5 variables:
 $ Sepal.Length: num  5.1 4.9 4.7 4.6 5 5.4 4.6 5 4.4 4.9 ...
 $ Sepal.Width : num  3.5 3 3.2 3.1 3.6 3.9 3.4 3.4 2.9 3.1 ...
 $ Petal.Length: num  1.4 1.4 1.3 1.5 1.4 1.7 1.4 1.5 1.4 1.5 ...
 $ Petal.Width : num  0.2 0.2 0.2 0.2 0.2 0.4 0.3 0.2 0.2 0.1 ...
 $ Species     : Factor w/ 3 levels "setosa","versicolor",..: 1 1 1 1 1 1 1 1 1 1 ...
```

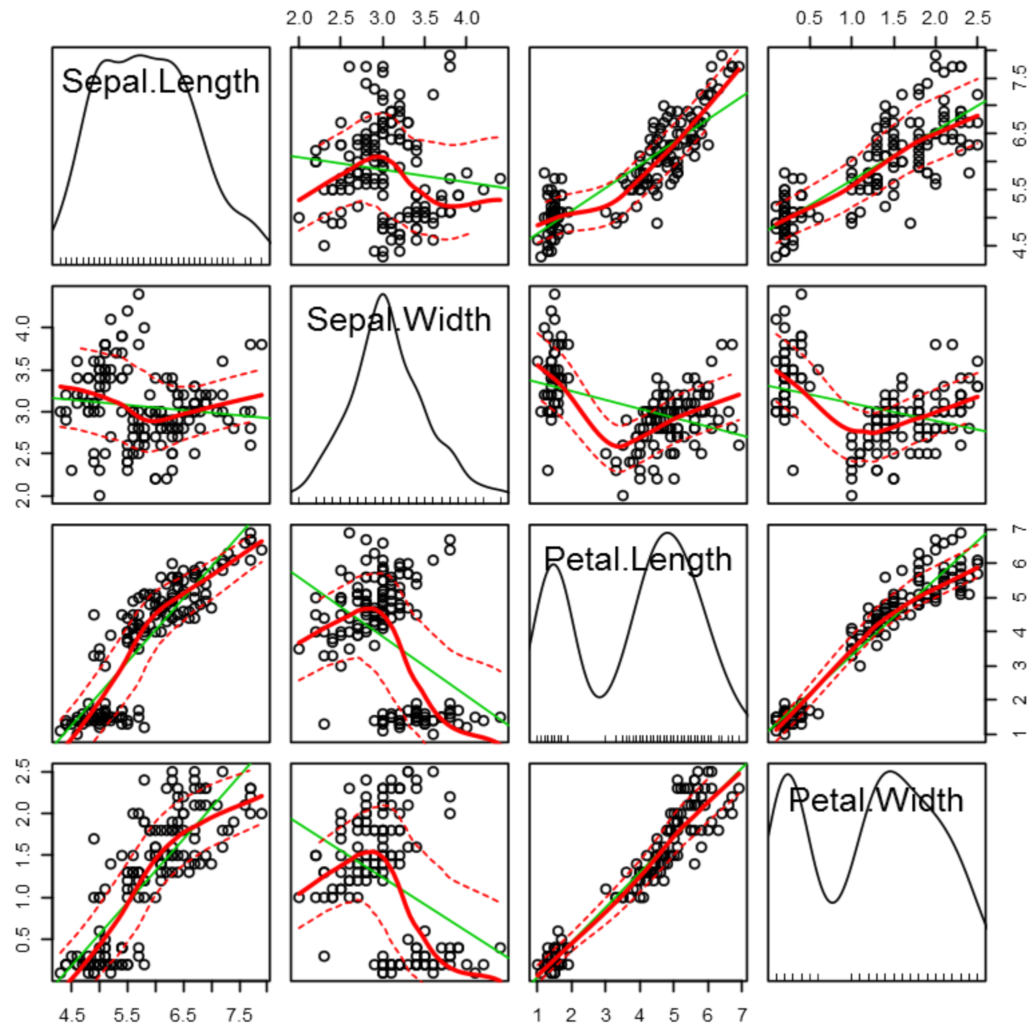
In [13]:
```
# histgram
par(mfrow=c(2,2))
hist(iris$Sepal.Length,col='green')
hist(iris$Sepal.Width,col='green')
hist(iris$Petal.Length,col='green')
hist(iris$Petal.Width,col='green')
```



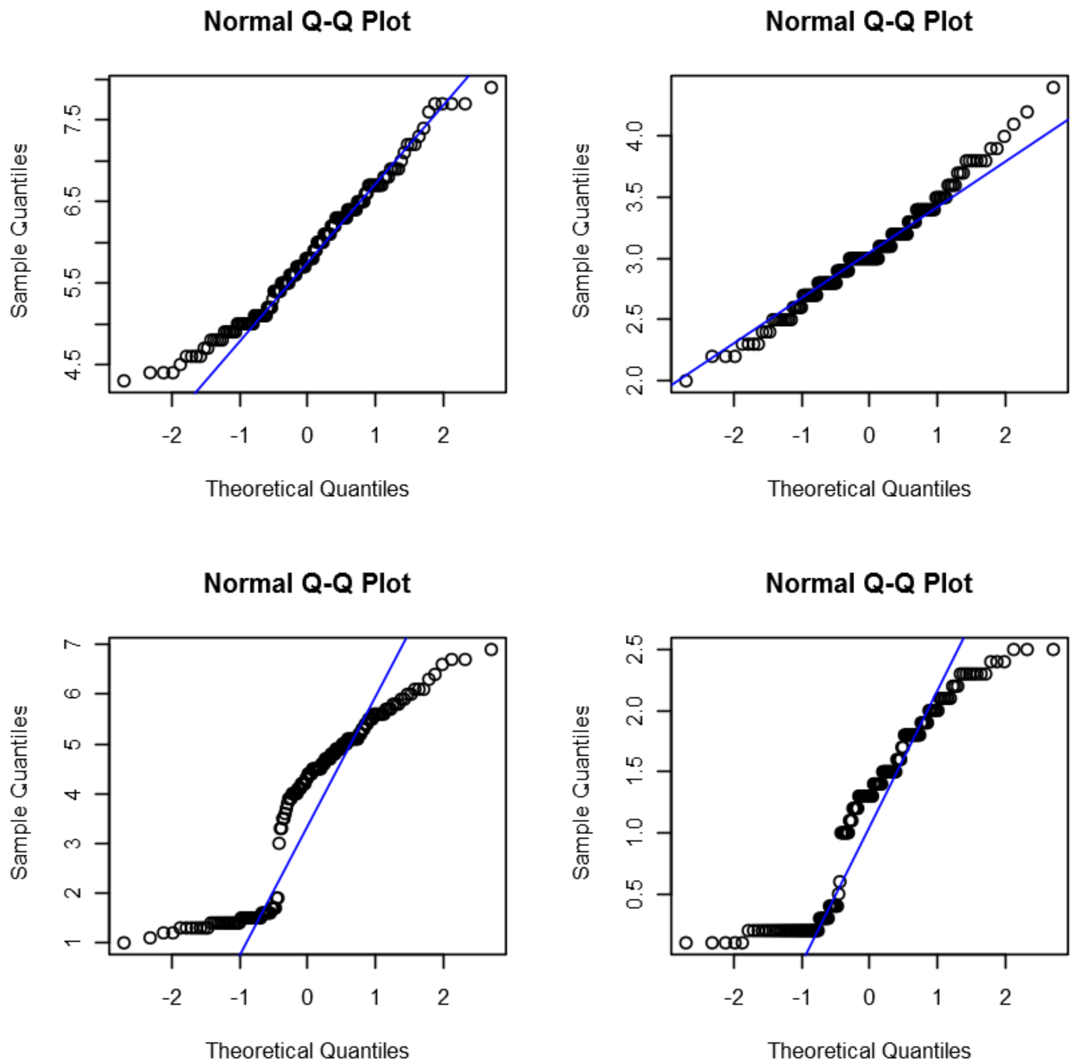

```
In [14]: # boxplot
         par(mfrow=c(2,2))
         boxplot(iris$Sepal.Length~iris$Species,ylab="Sepal Length (cm)")
         boxplot(iris$Sepal.Width~iris$Species,ylab="Sepal Width (cm)")
         boxplot(iris$Petal.Length~iris$Species,ylab="Petal Length (cm)")
         boxplot(iris$Petal.Width~iris$Species,ylab="Petal Width (cm)")
```



In [15]:
```
# scatter plot
par(mfrow=c(1,1))
scatterplotMatrix(~Sepal.Length + Sepal.Width + Petal.Length + Petal.Width, data=iris
```



In [34]:
```
# QQ Plot
par(mfrow=c(2,2))
qqnorm(iris$Sepal.Length);qqline(iris$Sepal.Length,col='blue')
qqnorm(iris$Sepal.Width);qqline(iris$Sepal.Width,col='blue')
qqnorm(iris$Petal.Length);qqline(iris$Petal.Length,col='blue')
qqnorm(iris$Petal.Width);qqline(iris$Petal.Width,col='blue')
```



In [18]:
```
# correlation
cor(iris[,c(1:4)])

# Petal Length is highly correlated with Sepal.Length
# Petal Width is highly correlated with Sepal.Length
# Petal.Width is highly correlated with Petal.Length
```

	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
Sepal.Length	1.0000000	-0.1175698	0.8717538	0.8179411
Sepal.Width	-0.1175698	1.0000000	-0.4284401	-0.3661259
Petal.Length	0.8717538	-0.4284401	1.0000000	0.9628654
Petal.Width	0.8179411	-0.3661259	0.9628654	1.0000000

2.2 Normality Test

```
In [5]: # check normality
        library(nortest)
        n <- dim(iris)[2]
        name <- names(iris)
        for (i in 1:(n-1)){
            print(name[i])
            print(ks.test(iris[,i],"pnorm"))
            print(shapiro.test(iris[,i]))
            print(pearson.test(iris[,i]))
            print(ad.test(iris[,i]))
            print(cvm.test(iris[,i]))
        }
        # Sepal.Length: Not Normal distributed, only Pearson chi-square test p-value=0.1352>0.
        # Sepal.Width: Not Normal distributed, only Shapiro-Wilk test p-value=0.1012>0.05
        # Petal.Length: Not Normal distributed, all p-value<0.05
        # Petal.Width: Not Normal distributed, all p-value<0.05
```

```
[1] "Sepal.Length"


Warning message in ks.test(iris[, i], "pnorm"):
"ties should not be present for the Kolmogorov-Smirnov test"


        One-sample Kolmogorov-Smirnov test

data:  iris[, i]
D = 0.99999, p-value < 2.2e-16
alternative hypothesis: two-sided


        Shapiro-Wilk normality test

data:  iris[, i]
W = 0.97609, p-value = 0.01018


        Pearson chi-square normality test

data:  iris[, i]
P = 17.4, p-value = 0.1352


        Anderson-Darling normality test

data:  iris[, i]
A = 0.8892, p-value = 0.02251
```

```
        Cramer-von Mises normality test

data:  iris[, i]
W = 0.1274, p-value = 0.04706

[1] "Sepal.Width"


Warning message in ks.test(iris[, i], "pnorm"):
"ties should not be present for the Kolmogorov-Smirnov test"


        One-sample Kolmogorov-Smirnov test

data:  iris[, i]
D = 0.97943, p-value < 2.2e-16
alternative hypothesis: two-sided


        Shapiro-Wilk normality test

data:  iris[, i]
W = 0.98492, p-value = 0.1012


        Pearson chi-square normality test

data:  iris[, i]
P = 46.2, p-value = 6.409e-06


        Anderson-Darling normality test

data:  iris[, i]
A = 0.90796, p-value = 0.02023


        Cramer-von Mises normality test

data:  iris[, i]
W = 0.18065, p-value = 0.009336

[1] "Petal.Length"


Warning message in ks.test(iris[, i], "pnorm"):
"ties should not be present for the Kolmogorov-Smirnov test"
```

```
        One-sample Kolmogorov-Smirnov test

data:  iris[, i]
D = 0.87653, p-value < 2.2e-16
alternative hypothesis: two-sided


        Shapiro-Wilk normality test

data:  iris[, i]
W = 0.87627, p-value = 7.412e-10


        Pearson chi-square normality test

data:  iris[, i]
P = 192.8, p-value < 2.2e-16


        Anderson-Darling normality test

data:  iris[, i]
A = 7.6785, p-value < 2.2e-16



Warning message in cvm.test(iris[, i]):
"p-value is smaller than 7.37e-10, cannot be computed more accurately"


        Cramer-von Mises normality test

data:  iris[, i]
W = 1.2223, p-value = 7.37e-10

[1] "Petal.Width"


Warning message in ks.test(iris[, i], "pnorm"):
"ties should not be present for the Kolmogorov-Smirnov test"


        One-sample Kolmogorov-Smirnov test

data:  iris[, i]
D = 0.54593, p-value < 2.2e-16
alternative hypothesis: two-sided
```

```
        Shapiro-Wilk normality test

data:  iris[, i]
W = 0.90183, p-value = 1.68e-08


        Pearson chi-square normality test

data:  iris[, i]
P = 155.6, p-value < 2.2e-16


        Anderson-Darling normality test

data:  iris[, i]
A = 5.1057, p-value = 1.125e-12


        Cramer-von Mises normality test

data:  iris[, i]
W = 0.72156, p-value = 4.338e-08
```

2.3 Equality of Variance test

```
In [57]: # check equality of variance
         # Not normal distributed
         lapply(iris[,c(1:4)],function(x) fligner.test(x~iris$Species))
         #Sepal.Length, Petal.Length, p-value<0.05, Petal.Width:  reject H0, variance are not
         #Sepal.Width: p-value>0.05, Not reject H0, variance are equal.

         # normal distributed
         # bartlett.test get the same results as fligner.test
         lapply(iris[,c(1:4)],function(x) bartlett.test(x~iris$Species))
```

```
$Sepal.Length

Fligner-Killeen test of homogeneity of variances

data:  x by iris$Species
Fligner-Killeen:med chi-squared = 11.618, df = 2, p-value = 0.003


$Sepal.Width
```

```
Fligner-Killeen test of homogeneity of variances

data:  x by iris$Species
Fligner-Killeen:med chi-squared = 0.9122, df = 2, p-value = 0.6338


$Petal.Length

Fligner-Killeen test of homogeneity of variances

data:  x by iris$Species
Fligner-Killeen:med chi-squared = 34.708, df = 2, p-value = 2.906e-08


$Petal.Width

Fligner-Killeen test of homogeneity of variances

data:  x by iris$Species
Fligner-Killeen:med chi-squared = 29.387, df = 2, p-value = 4.157e-07




$Sepal.Length

Bartlett test of homogeneity of variances

data:  x by iris$Species
Bartlett's K-squared = 16.006, df = 2, p-value = 0.0003345


$Sepal.Width

Bartlett test of homogeneity of variances

data:  x by iris$Species
Bartlett's K-squared = 2.0911, df = 2, p-value = 0.3515


$Petal.Length

Bartlett test of homogeneity of variances

data:  x by iris$Species
Bartlett's K-squared = 55.423, df = 2, p-value = 9.229e-13
```

```
$Petal.Width

Bartlett test of homogeneity of variances

data:  x by iris$Species
Bartlett's K-squared = 39.213, df = 2, p-value = 3.055e-09
```

From the Normality test and equality of variance test. The iris dataset are Not Normal distributed, and the variance is NOT equal. So I will use Kruskal-Wallis test to check whether there is difference in three different species. If there is difference, then use MANN-Whitney-Wilcoxon test to do pairwised test.

2.4 Kruskal-Wallis test

```
In [50]: # Kruskal-Wallis test
         kruskal.test(iris$Sepal.Length+iris$Sepal.Width+iris$Petal.Length+iris$Petal.Width~iri
         # There is difference in these three Species: setosa,versicolor,virginica.

         lapply(iris[,c(1:4)],function(x) kruskal.test(x~iris$Species)) #p-value<0.05, reject 
         # There is difference of Sepal Length in three Species: setosa,versicolor,virginica.
         # There is difference of Sepal Width in three Species: setosa,versicolor,virginica.
         # There is difference of Petal Length in three Species: setosa,versicolor,virginica.
         # There is difference of Petal Width in three Species: setosa,versicolor,virginica.
```

```
Kruskal-Wallis rank sum test

data:  iris$Sepal.Length + iris$Sepal.Width + iris$Petal.Length + iris$Petal.Width by iris$Spec
Kruskal-Wallis chi-squared = 124.17, df = 2, p-value < 2.2e-16
```

```
$Sepal.Length

Kruskal-Wallis rank sum test

data:  x by iris$Species
Kruskal-Wallis chi-squared = 96.937, df = 2, p-value < 2.2e-16


$Sepal.Width

Kruskal-Wallis rank sum test

data:  x by iris$Species
Kruskal-Wallis chi-squared = 63.571, df = 2, p-value = 1.569e-14
```

```
$Petal.Length

Kruskal-Wallis rank sum test

data:  x by iris$Species
Kruskal-Wallis chi-squared = 130.41, df = 2, p-value < 2.2e-16


$Petal.Width

Kruskal-Wallis rank sum test

data:  x by iris$Species
Kruskal-Wallis chi-squared = 131.19, df = 2, p-value < 2.2e-16
```

```
In [51]: # Even though the iris dataset is NOT Normal distributed, and the variance are not eq
         #I still want to check the result of ANOVA test. It gets the same results as  Kruskal

         # ANOVA test
         summary(aov(iris$Sepal.Length+iris$Sepal.Width+iris$Petal.Length+iris$Petal.Width~iris
         # There is difference in these three Species: setosa,versicolor,virginica.

         lapply(iris[,c(1:4)],function(x) summary(aov(x~iris$Species))) #p-value<0.05, reject
```

```
              Df Sum Sq Mean Sq F value Pr(>F)
iris$Species   2 1238.4   619.2   422.4 <2e-16 ***
Residuals    147  215.5     1.5
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


$Sepal.Length
              Df Sum Sq Mean Sq F value Pr(>F)
iris$Species   2  63.21  31.606   119.3 <2e-16 ***
Residuals    147  38.96   0.265
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

$Sepal.Width
              Df Sum Sq Mean Sq F value Pr(>F)
iris$Species   2  11.35   5.672   49.16 <2e-16 ***
Residuals    147  16.96   0.115
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

$Petal.Length
              Df Sum Sq Mean Sq F value Pr(>F)
iris$Species   2  437.1  218.55    1180 <2e-16 ***
Residuals    147   27.2    0.19
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

$Petal.Width
              Df Sum Sq Mean Sq F value Pr(>F)
iris$Species   2  80.41   40.21     960 <2e-16 ***
Residuals    147   6.16    0.04
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

2.5 pairwised test: MANN-Whitney-Wilcoxon test

In [72]:
```
# pairwised test: MANN-Whitney-Wilcoxon test
pairwise.wilcox.test(iris$Sepal.Length+iris$Sepal.Width+iris$Petal.Length+iris$Petal.W
# p-value < 0.05, reject H0.
# There are differences between setosa and versicolor, setosa and virginica, versicol

lapply(iris[,c(1:4)], function(x) pairwise.wilcox.test(x, iris$Species, paired=FALSE)]
# all of p-values < 0.05, reject H0.
```

```
Pairwise comparisons using Wilcoxon rank sum test

data:  iris$Sepal.Length + iris$Sepal.Width + iris$Petal.Length + iris$Petal.Width and iris$Spe

           setosa  versicolor
versicolor < 2e-16 -
virginica  < 2e-16 6.8e-14

P value adjustment method: holm


$Sepal.Length

Pairwise comparisons using Wilcoxon rank sum test

data:  x and iris$Species

           setosa  versicolor
versicolor 1.7e-13 -
```

```
virginica  < 2e-16 5.9e-07

P value adjustment method: holm

$Sepal.Width

Pairwise comparisons using Wilcoxon rank sum test

data:  x and iris$Species

           setosa  versicolor
versicolor 6.4e-13 -
virginica  1.4e-08 0.0046

P value adjustment method: holm

$Petal.Length

Pairwise comparisons using Wilcoxon rank sum test

data:  x and iris$Species

           setosa versicolor
versicolor <2e-16 -
virginica  <2e-16 <2e-16

P value adjustment method: holm

$Petal.Width

Pairwise comparisons using Wilcoxon rank sum test

data:  x and iris$Species

           setosa versicolor
versicolor <2e-16 -
virginica  <2e-16 <2e-16

P value adjustment method: holm
```

In [74]:
```
# Even though the iris dataset is NOT Normal distributed, and the variance are not eq
# I still want to use t-test to check the difference. It gets the same results as MAN
pairwise.t.test(iris$Sepal.Length+iris$Sepal.Width+iris$Petal.Length+iris$Petal.Width

lapply(iris[,c(1:4)], function(x) pairwise.t.test(x, iris$Species, paired=FALSE))
```

```
Pairwise comparisons using t tests with pooled SD

data:  iris$Sepal.Length + iris$Sepal.Width + iris$Petal.Length + iris$Petal.Width and iris$Spe

           setosa versicolor
versicolor <2e-16 -
virginica  <2e-16 <2e-16

P value adjustment method: holm


$Sepal.Length

Pairwise comparisons using t tests with pooled SD

data:  x and iris$Species

           setosa  versicolor
versicolor 1.8e-15 -
virginica  < 2e-16 2.8e-09

P value adjustment method: holm

$Sepal.Width

Pairwise comparisons using t tests with pooled SD

data:  x and iris$Species

           setosa  versicolor
versicolor < 2e-16 -
virginica  9.1e-10 0.0031

P value adjustment method: holm

$Petal.Length

Pairwise comparisons using t tests with pooled SD

data:  x and iris$Species

           setosa versicolor
versicolor <2e-16 -
virginica  <2e-16 <2e-16

P value adjustment method: holm

$Petal.Width
```

```
Pairwise comparisons using t tests with pooled SD

data:  x and iris$Species

           setosa versicolor
versicolor <2e-16 -
virginica  <2e-16 <2e-16

P value adjustment method: holm
```

2.6 Conclusion

From the above tests, we can tell that all the three species setosa, versicolor, and virginica are different with each other in all of their sepal length, sepal width, petal length and petal width.

3 Problem 2.

(25 points) We are interested in finding the important predictors of accumulated wealth at the time of retirement, assess their adjusted effect sizes (in direction and magnitude) and use the best linear regression model for interpretation and prediction. We want to analyze the Pension.txt dataset (available on Blackboard) that contains 194 observations on 17 variables: pyears - years of employment, prftshr - indicator for profit sharing company, choice - indicator for company giving a choice to contribute, female, married, age, educ - years of education, finc25, finc35, finc50, finc75, finc100, finc101- indicators for 25, 35, 50, 75, 100 and 101 levels of retirement contribution, wealth89 - wealth in thousands of dollars, race, stckin89 - percent of the portfolio in stock, irain89 - percent of the portfolio in IRA. Perform all necessary data analysis steps and write a section summarizing the findings.

3.1 Load data and basic analysis

```
In [148]: # load dataset
          pension_data <- read.table('Pension.txt',header=T)
          dim(pension_data)
          pension_data <- na.omit(pension_data)
          dim(pension_data)
          head(pension_data)
```

1. 194 2. 17
1. 191 2. 17

pyears	prftshr	choice	female	married	age	educ	finc25	finc35	finc50	finc75	finc100	finc
1	0	1	0	1	64	12	0	0	1	0	0	0
6	1	1	1	1	56	13	0	0	0	1	0	0
25	1	1	0	1	56	12	0	0	0	1	0	0
20	1	0	1	1	63	12	1	0	0	0	0	0
35	0	1	0	1	67	12	0	1	0	0	0	0
13	1	0	0	1	64	11	0	0	0	1	0	0

In [446]:
```
# 17 variables:
# prftshr - indicator for profit sharing company, 2 levels: 0 1
# choice - indicator for company giving a choice to contribute, 2 levels: 0 1
# female, 2 levels: 0 1
# married, 2 levels: 0 1
# finc25, finc35, finc50, finc75, finc100, finc101- indicators for 25, 35, 50, 75, 1
# race, 2 levels 0 1
# stckin89 - percent of the portfolio in stock, 2 levels 0 1
# irain89 - percent of the portfolio in IRA, 2 levels 0 1

# pyears - years of employment
# age
# educ - years of education
# wealth89 - wealth in thousands of dollars
str(pension_data)
summary(pension_data)
lapply(pension_data[, c(2:5,8:13,15:17)], function(x) unique(x))
```

```
'data.frame':	191 obs. of  17 variables:
 $ pyears  : int  1 6 25 20 35 13 2 10 26 5 ...
 $ prftshr : Factor w/ 2 levels "0","1": 1 2 2 2 1 2 1 2 1 1 ...
 $ choice  : Factor w/ 2 levels "0","1": 2 2 2 1 2 1 2 1 1 2 ...
 $ female  : Factor w/ 2 levels "0","1": 1 2 1 2 1 1 2 2 2 1 ...
 $ married : Factor w/ 2 levels "0","1": 2 2 2 2 2 2 2 2 2 2 ...
 $ age     : int  64 56 56 63 67 64 64 64 69 60 ...
 $ educ    : int  12 13 12 12 12 11 12 12 12 14 ...
 $ finc25  : Factor w/ 2 levels "0","1": 1 1 1 2 1 1 2 2 1 1 ...
 $ finc35  : Factor w/ 2 levels "0","1": 1 1 1 1 2 1 1 1 2 1 ...
 $ finc50  : Factor w/ 2 levels "0","1": 2 1 1 1 1 1 1 1 1 1 ...
 $ finc75  : Factor w/ 2 levels "0","1": 1 2 2 1 1 2 1 1 1 1 ...
 $ finc100 : Factor w/ 2 levels "0","1": 1 1 1 1 1 1 1 1 1 1 ...
 $ finc101 : Factor w/ 2 levels "0","1": 1 1 1 1 1 1 1 1 1 2 ...
 $ wealth89: num  77.9 154.9 154.9 232.5 179 ...
 $ race    : Factor w/ 2 levels "0","1": 1 1 1 1 1 1 1 1 1 1 ...
 $ stckin89: Factor w/ 2 levels "0","1": 2 2 2 2 1 2 1 1 1 2 ...
 $ irain89 : Factor w/ 2 levels "0","1": 2 2 2 2 2 1 2 2 1 2 ...
 - attr(*, "na.action")=Class 'omit'  Named int [1:3] 39 52 173
  .. ..- attr(*, "names")= chr [1:3] "39" "52" "173"
```

```
    pyears      prftshr choice  female  married      age               educ
```

```
 Min.   : 0.0   0:151   0: 74   0: 75   0: 47   Min.   :54.00   Min.   : 8.00
 1st Qu.: 4.0   1: 40   1:117   1:116   1:144   1st Qu.:57.00   1st Qu.:12.00
 Median : 9.0                                   Median :60.00   Median :12.00
 Mean   :11.3                                   Mean   :60.52   Mean   :13.53
 3rd Qu.:16.0                                   3rd Qu.:64.00   3rd Qu.:16.00
 Max.   :45.0                                   Max.   :73.00   Max.   :18.00
 finc25  finc35  finc50  finc75  finc100 finc101    wealth89      race
 0:151   0:157   0:146   0:165   0:165   0:181   Min.   :  -6.3   0:169
 1: 40   1: 34   1: 45   1: 26   1: 26   1: 10   1st Qu.:  65.8   1: 22
                                                 Median : 140.0
                                                 Mean   : 212.0
                                                 3rd Qu.: 253.4
                                                 Max.   :1485.0
 stckin89 irain89
 0:126    0:93
 1: 65    1:98
```

$prftshr 1. 0 2. 1

$choice 1. 1 2. 0

$female 1. 0 2. 1

$married 1. 1 2. 0

$finc25 1. 0 2. 1

$finc35 1. 0 2. 1

$finc50 1. 1 2. 0

$finc75 1. 0 2. 1

$finc100 1. 0 2. 1

$finc101 1. 0 2. 1

$race 1. 0 2. 1

$stckin89 1. 1 2. 0

$irain89 1. 1 2. 0

In [150]:
```
# change some variables to factors
pension_data[, c(2:5,8:13,15:17)] <- lapply(pension_data[, c(2:5,8:13,15:17)], funct
str(pension_data)
summary(pension_data)
```

```
'data.frame':	191 obs. of  17 variables:
 $ pyears  : int  1 6 25 20 35 13 2 10 26 5 ...
 $ prftshr : Factor w/ 2 levels "0","1": 1 2 2 2 1 2 1 2 1 1 ...
 $ choice  : Factor w/ 2 levels "0","1": 2 2 2 1 2 1 2 1 1 2 ...
 $ female  : Factor w/ 2 levels "0","1": 1 2 1 2 1 1 2 2 2 1 ...
 $ married : Factor w/ 2 levels "0","1": 2 2 2 2 2 2 2 2 2 2 ...
 $ age     : int  64 56 56 63 67 64 64 64 69 60 ...
 $ educ    : int  12 13 12 12 12 11 12 12 12 14 ...
 $ finc25  : Factor w/ 2 levels "0","1": 1 1 1 2 1 1 2 2 1 1 ...
 $ finc35  : Factor w/ 2 levels "0","1": 1 1 1 1 2 1 1 1 2 1 ...
 $ finc50  : Factor w/ 2 levels "0","1": 2 1 1 1 1 1 1 1 1 1 ...
 $ finc75  : Factor w/ 2 levels "0","1": 1 2 2 1 1 2 1 1 1 1 ...
 $ finc100 : Factor w/ 2 levels "0","1": 1 1 1 1 1 1 1 1 1 1 ...
 $ finc101 : Factor w/ 2 levels "0","1": 1 1 1 1 1 1 1 1 1 2 ...
 $ wealth89: num  77.9 154.9 154.9 232.5 179 ...
 $ race    : Factor w/ 2 levels "0","1": 1 1 1 1 1 1 1 1 1 1 ...
 $ stckin89: Factor w/ 2 levels "0","1": 2 2 2 2 1 2 1 1 1 2 ...
 $ irain89 : Factor w/ 2 levels "0","1": 2 2 2 2 2 1 2 2 1 2 ...
 - attr(*, "na.action")=Class 'omit'  Named int [1:3] 39 52 173
  .. ..- attr(*, "names")= chr [1:3] "39" "52" "173"
```

```
     pyears      prftshr choice  female  married      age             educ      
 Min.   : 0.0   0:151   0: 74   0: 75   0: 47   Min.   :54.00   Min.   : 8.00  
 1st Qu.: 4.0   1: 40   1:117   1:116   1:144   1st Qu.:57.00   1st Qu.:12.00  
 Median : 9.0                                   Median :60.00   Median :12.00  
 Mean   :11.3                                   Mean   :60.52   Mean   :13.53  
 3rd Qu.:16.0                                   3rd Qu.:64.00   3rd Qu.:16.00  
 Max.   :45.0                                   Max.   :73.00   Max.   :18.00  
 finc25  finc35  finc50  finc75  finc100 finc101    wealth89       race   
 0:151   0:157   0:146   0:165   0:165   0:181    Min.   :  -6.3   0:169  
 1: 40   1: 34   1: 45   1: 26   1: 26   1: 10    1st Qu.:  65.8   1: 22  
                                                  Median : 140.0          
                                                  Mean   : 212.0          
                                                  3rd Qu.: 253.4          
                                                  Max.   :1485.0          
 stckin89 irain89
 0:126    0:93   
 1: 65    1:98   
```

```
In [151]: # normalize variables: pyears, age, educ, wealth89
          #pension_data[,c(1,6,7,14)] <- lapply(pension_data[,c(1,6,7,14)],function(x) ((x-min
          #summary(pension_data)
```

3.2 Split the dataset into training and testing dataset

```
In [166]: # train dataset: 80%, test dataset: 20%
          (n <- dim(pension_data)[1])
          trainID <- sample(1:n,n*0.8)
          train <- pension_data[trainID,]
          test <- pension_data[-trainID,]
```

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3.3 Build Models

```
In [244]: library(MASS)
          lm.fit <- glm(wealth89~., data=train, family=gaussian)
          summary(lm.fit) # AIC: 2086.5
          # Best model 1: Smallest AIC value
          lm.best1 <- stepAIC(lm.fit, direction = "both", trace=F)
          summary(lm.best1)  # AIC: 2072.9
```

```
Call:
glm(formula = wealth89 ~ ., family = gaussian, data = train)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-323.12  -120.18   -37.47    67.09  1153.27

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -993.523    366.323  -2.712  0.00756 **
pyears        -1.068      2.207  -0.484  0.62918
prftshr1     -39.447     45.712  -0.863  0.38970
choice1      -35.341     39.596  -0.893  0.37369
female1       18.388     47.421   0.388  0.69881
married1      29.051     50.717   0.573  0.56773
age           14.572      5.123   2.845  0.00514 **
educ          14.698      7.746   1.897  0.05990 .
finc251        1.555     84.441   0.018  0.98534
finc351       42.812     87.199   0.491  0.62424
finc501       46.297     85.708   0.540  0.58997
finc751      148.742     97.188   1.530  0.12824
finc1001      95.619     95.286   1.003  0.31742
finc1011     250.303    106.848   2.343  0.02061 *
race1        -63.618     60.042  -1.060  0.29124
stckin891     95.298     42.546   2.240  0.02673 *
irain891      77.280     41.735   1.852  0.06625 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
(Dispersion parameter for gaussian family taken to be 47610.02)

    Null deviance: 9318957  on 151  degrees of freedom
Residual deviance: 6427352  on 135  degrees of freedom
AIC: 2086.5

Number of Fisher Scoring iterations: 2




Call:
glm(formula = wealth89 ~ age + educ + finc75 + finc101 + stckin89 +
    irain89, family = gaussian, data = train)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-333.23  -126.33   -41.41    60.88  1185.08

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -1007.733    282.800  -3.563 0.000496 ***
age            14.220      4.303   3.304 0.001200 **
educ           18.411      6.935   2.655 0.008826 **
finc751        97.291     55.670   1.748 0.082647 .
finc1011      222.799     75.823   2.938 0.003840 **
stckin891      98.490     40.350   2.441 0.015857 *
irain891       92.367     37.530   2.461 0.015021 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 46239.43)

    Null deviance: 9318957  on 151  degrees of freedom
Residual deviance: 6704717  on 145  degrees of freedom
AIC: 2072.9

Number of Fisher Scoring iterations: 2
```

In [248]:
```
# Best model: Smallest AIC value, add two way of interaction
lm.best2 <- stepAIC(lm.fit, ~.^2, direction = "both", trace=F)
summary(lm.best2)  # AIC: 2003.9
# race is not significant, and it isn't interactive with other variables. remove it.
```

```
Call:
```

```
glm(formula = wealth89 ~ pyears + choice + female + married +
    age + educ + finc50 + finc75 + finc100 + finc101 + race +
    stckin89 + irain89 + married:finc75 + age:finc101 + female:finc101 +
    female:irain89 + pyears:finc50 + age:stckin89 + educ:irain89 +
    educ:stckin89 + female:age + pyears:age + choice:finc50 +
    female:finc100 + age:finc100 + pyears:finc75 + choice:finc100 +
    choice:educ + finc100:irain89, family = gaussian, data = train)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-310.07   -65.77   -18.42    56.45   943.22

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)        -813.7271   611.4582  -1.331 0.185758
pyears               51.4753    26.2552   1.961 0.052227 .
choice1            -285.0433   158.4464  -1.799 0.074513 .
female1             937.7065   484.7226   1.935 0.055383 .
married1             54.3300    38.7141   1.403 0.163069
age                  13.1427     9.6119   1.367 0.174058
educ                  0.7784    10.8535   0.072 0.942943
finc501              31.2477    71.0050   0.440 0.660666
finc751            1109.2483   181.4240   6.114 1.23e-08 ***
finc1001          -2620.5919   823.3630  -3.183 0.001854 **
finc1011          -6149.0870   953.7124  -6.448 2.44e-09 ***
race1               -71.7749    45.2781  -1.585 0.115531
stckin891         -1489.1391   552.2316  -2.697 0.008004 **
irain891            603.1034   165.7642   3.638 0.000405 ***
married1:finc751  -1020.0467   176.5700  -5.777 6.03e-08 ***
age:finc1011         97.4925    15.0827   6.464 2.25e-09 ***
female1:finc1011    701.3971   150.2392   4.669 7.91e-06 ***
female1:irain891   -203.7372    59.4831  -3.425 0.000840 ***
pyears:finc501        6.3611     3.4606   1.838 0.068497 .
age:stckin891        18.0072     8.3033   2.169 0.032064 *
educ:irain891       -29.4550    11.5242  -2.556 0.011829 *
educ:stckin891       38.2214    12.4722   3.065 0.002689 **
female1:age         -14.4274     7.8070  -1.848 0.067042 .
pyears:age           -0.8397     0.4294  -1.955 0.052839 .
choice1:finc501    -158.4909    66.8751  -2.370 0.019372 *
female1:finc1001    193.2202    93.7932   2.060 0.041537 *
age:finc1001         46.8591    13.9634   3.356 0.001058 **
pyears:finc751       -7.3850     4.7362  -1.559 0.121549
choice1:finc1001   -328.4064   101.4060  -3.239 0.001551 **
choice1:educ         24.3864    12.0994   2.016 0.046067 *
finc1001:irain891  -158.3423    98.3715  -1.610 0.110083
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
(Dispersion parameter for gaussian family taken to be 25652.67)

    Null deviance: 9318957  on 151  degrees of freedom
Residual deviance: 3103974  on 121  degrees of freedom
AIC: 2003.9

Number of Fisher Scoring iterations: 2
```

```
In [249]: lm.best2 <- glm(formula = wealth89 ~ pyears + choice + female + married + age + educ
              stckin89 + irain89 + married:finc75 + age:finc101 + female:finc101 + female:irai
              educ:stckin89 + female:age + pyears:age + choice:finc50 + female:finc100 + age:f
              choice:educ + finc100:irain89, family = gaussian, data = train)
          summary(lm.best2) # AIC: 2005
```

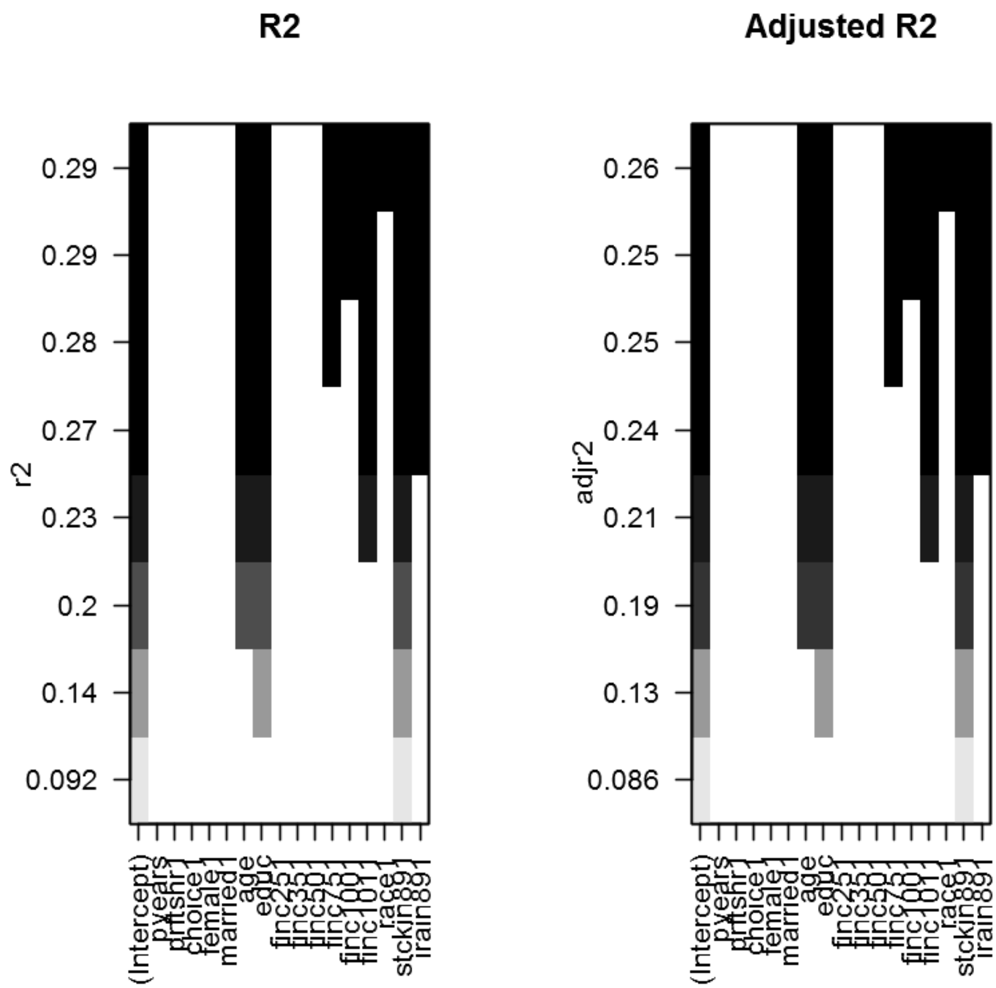
```
Call:
glm(formula = wealth89 ~ pyears + choice + female + married +
    age + educ + finc50 + finc75 + finc100 + finc101 + stckin89 +
    irain89 + married:finc75 + age:finc101 + female:finc101 +
    female:irain89 + pyears:finc50 + age:stckin89 + educ:irain89 +
    educ:stckin89 + female:age + pyears:age + choice:finc50 +
    female:finc100 + age:finc100 + pyears:finc75 + choice:finc100 +
    choice:educ + finc100:irain89, family = gaussian, data = train)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-309.29   -68.18   -15.90    64.20   943.01

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)        -843.4697   614.9481  -1.372 0.172702
pyears               49.7271    26.3942   1.884 0.061943 .
choice1            -294.4599   159.3137  -1.848 0.066980 .
female1             838.9570   483.6743   1.735 0.085346 .
married1             60.9021    38.7294   1.573 0.118423
age                  13.1785     9.6713   1.363 0.175507
educ                  1.4376    10.9126   0.132 0.895409
finc501              38.5126    71.2950   0.540 0.590053
finc751            1128.8213   182.1222   6.198 8.04e-09 ***
finc1001          -2600.8646   828.3577  -3.140 0.002121 **
finc1011          -6023.0753   956.2682  -6.299 4.94e-09 ***
stckin891         -1480.3054   555.6167  -2.664 0.008759 **
irain891            646.8110   164.4654   3.933 0.000140 ***
married1:finc751  -1029.4201   177.5618  -5.798 5.40e-08 ***
age:finc1011         95.4846    15.1223   6.314 4.58e-09 ***
female1:finc1011    709.4387   151.0817   4.696 7.02e-06 ***
```

```
female1:irain891   -202.9605    59.8488  -3.391 0.000938 ***
pyears:finc501        5.9580     3.4726   1.716 0.088752 .
age:stckin891        17.8365     8.3540   2.135 0.034753 *
educ:irain891       -32.4273    11.4409  -2.834 0.005376 **
educ:stckin891       38.5338    12.5477   3.071 0.002631 **
female1:age         -12.8637     7.7923  -1.651 0.101346
pyears:age           -0.8094     0.4316  -1.875 0.063152 .
choice1:finc501    -159.3892    67.2861  -2.369 0.019415 *
female1:finc1001    199.3274    94.2933   2.114 0.036559 *
age:finc1001         46.5178    14.0480   3.311 0.001222 **
pyears:finc751       -7.5582     4.7642  -1.586 0.115225
choice1:finc1001   -318.8968   101.8541  -3.131 0.002181 **
choice1:educ         25.3205    12.1597   2.082 0.039403 *
finc1001:irain891  -168.3136    98.7770  -1.704 0.090931 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 25970.78)

    Null deviance: 9318957  on 151  degrees of freedom
Residual deviance: 3168435  on 122  degrees of freedom
AIC: 2005

Number of Fisher Scoring iterations: 2
```

In [256]:
```r
# use regsubsets() to get the best model, which has the best R2 or ajusted-R2.
library(leaps)
lm.best3 <- regsubsets(wealth89~., data=train, nbest=1, method='exhaustive')
summary(lm.best3)
par(mfrow=c(1,2))
plot(lm.best3, scale='r2', main='R2')
plot(lm.best3, scale='adjr2', main='Adjusted R2')
```

```
Subset selection object
Call: regsubsets.formula(wealth89 ~ ., data = train, nbest = 1, method = "exhaustive")
16 Variables  (and intercept)
          Forced in Forced out
pyears        FALSE      FALSE
prftshr1      FALSE      FALSE
choice1       FALSE      FALSE
female1       FALSE      FALSE
married1      FALSE      FALSE
age           FALSE      FALSE
educ          FALSE      FALSE
finc251       FALSE      FALSE
finc351       FALSE      FALSE
```

```
finc501       FALSE      FALSE
finc751       FALSE      FALSE
finc1001      FALSE      FALSE
finc1011      FALSE      FALSE
race1         FALSE      FALSE
stckin891     FALSE      FALSE
irain891      FALSE      FALSE
1 subsets of each size up to 8
Selection Algorithm: exhaustive
         pyears prftshr1 choice1 female1 married1 age educ finc251 finc351
1  ( 1 ) " "    " "      " "     " "     " "      " " " "  " "     " "
2  ( 1 ) " "    " "      " "     " "     " "      " " "*"  " "     " "
3  ( 1 ) " "    " "      " "     " "     " "      "*" "*"  " "     " "
4  ( 1 ) " "    " "      " "     " "     " "      "*" "*"  " "     " "
5  ( 1 ) " "    " "      " "     " "     " "      "*" "*"  " "     " "
6  ( 1 ) " "    " "      " "     " "     " "      "*" "*"  " "     " "
7  ( 1 ) " "    " "      " "     " "     " "      "*" "*"  " "     " "
8  ( 1 ) " "    " "      " "     " "     " "      "*" "*"  " "     " "
         finc501 finc751 finc1001 finc1011 race1 stckin891 irain891
1  ( 1 ) " "     " "     " "      " "      " "   "*"       " "
2  ( 1 ) " "     " "     " "      " "      " "   "*"       " "
3  ( 1 ) " "     " "     " "      " "      " "   "*"       " "
4  ( 1 ) " "     " "     " "      "*"      " "   "*"       " "
5  ( 1 ) " "     " "     " "      "*"      " "   "*"       "*"
6  ( 1 ) " "     "*"     " "      "*"      " "   "*"       "*"
7  ( 1 ) " "     "*"     "*"      "*"      " "   "*"       "*"
8  ( 1 ) " "     "*"     "*"      "*"      "*"   "*"       "*"
```




```
In [257]: # from the plot, we can get the best model is
          lm.best3 <- lm(wealth89~age+educ+finc75+finc100+finc101+race+stckin89+irain89, data=
          summary(lm.best3) # Multiple R-squared:0.2945,Adjusted R-squared:0.255
          # The p-value of finc75, finc100 and race is greater than 0.05, not significant. But
          AIC(lm.best3)# AIC: 2073.93
```

```
Call:
lm(formula = wealth89 ~ age + educ + finc75 + finc100 + finc101 +
    race + stckin89 + irain89, data = train)

Residuals:
    Min      1Q  Median      3Q     Max
```

```
-330.88 -125.45  -37.86   66.91 1190.52

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -916.920    290.428  -3.157  0.00194 **
age           13.515      4.395   3.075  0.00252 **
educ          14.771      7.439   1.986  0.04898 *
finc751      106.187     57.824   1.836  0.06838 .
finc1001      69.640     55.626   1.252  0.21264
finc1011     231.057     77.810   2.970  0.00350 **
race1        -59.121     56.818  -1.041  0.29985
stckin891     91.264     40.486   2.254  0.02570 *
irain891      91.428     37.496   2.438  0.01598 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 214.4 on 143 degrees of freedom
Multiple R-squared:  0.2945,Adjusted R-squared:  0.255
F-statistic: 7.462 on 8 and 143 DF,  p-value: 2.749e-08
```

2073.93207827551

In [258]:
```
# model remove finc100 and race
lm.best3 <- lm(wealth89~age+educ+finc75+finc100+finc101+stckin89+irain89, data=train)
summary(lm.best3) # Multiple R-squared:0.2892,Adjusted R-squared:0.2546
# The p-value of race is greater than 0.05, not significant, remove it.
AIC(lm.best3) # AIC: 2073.08
```

```
Call:
lm(formula = wealth89 ~ age + educ + finc75 + finc100 + finc101 +
    stckin89 + irain89, data = train)

Residuals:
    Min      1Q  Median      3Q     Max
-336.11 -127.37  -38.84   68.11 1193.75

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -987.442    282.491  -3.495 0.000629 ***
age           14.483      4.297   3.371 0.000963 ***
educ          14.787      7.441   1.987 0.048793 *
finc751      115.297     57.174   2.017 0.045594 *
finc1001      73.405     55.524   1.322 0.188249
finc1011     242.389     77.066   3.145 0.002017 **
stckin891     95.134     40.327   2.359 0.019663 *
```

```
irain891      93.650     37.446   2.501 0.013506 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 214.5 on 144 degrees of freedom
Multiple R-squared:  0.2892,Adjusted R-squared:  0.2546
F-statistic: 8.368 on 7 and 144 DF,  p-value: 1.463e-08
```

2073.07860745362

3.4 Predict And Evaluate

In [259]:
```
# predict wealth for train dataset with lm.best1 from stepAIC
lm.pred1.train <- predict(lm.best1,newdata=train[,-14],type="response")
# Calculate R2 for test dataset with lm.best1
(lm.best1.R2.train <- 1-sum((train$wealth89-lm.pred1.train)^2)/sum((train$wealth89-m

# predict wealth for train dataset with lm.best2 from regsubsets
lm.pred2.train <- predict(lm.best2,newdata=train[,-14],type="response")
# Calculate R2 for test dataset with lm.best2
(lm.best2.R2.train <- 1-sum((train$wealth89-lm.pred2.train)^2)/sum((train$wealth89-m

# predict wealth for train dataset with lm.best3 from regsubsets
lm.pred3.train <- predict(lm.best3,newdata=train[,-14],type="response")
# Calculate R2 for test dataset with lm.best2
(lm.best3.R2.train <- 1-sum((train$wealth89-lm.pred3.train)^2)/sum((train$wealth89-m

# predict wealth for test dataset with lm.best3 from stepAIC
lm.pred1.test <- predict(lm.best1,newdata=test[,-14],type="response")
# Calculate R2 for test dataset with lm.best3
(lm.best1.R2.test <- abs(1-sum((test$wealth89-lm.pred1.test)^2)/sum((test$wealth89-m

# predict wealth for test dataset with lm.best2 from regsubsets
lm.pred2.test <- predict(lm.best2,newdata=test[,-14],type="response")
# Calculate R2 for test dataset with lm.best1
(lm.best2.R2 <- abs(1-sum((test$wealth89-lm.pred2.test)^2)/sum((test$wealth89-mean(t

# predict wealth for test dataset with lm.best3 from regsubsets
lm.pred3.test <- predict(lm.best3,newdata=test[,-14],type="response")
# Calculate R2 for test dataset with lm.best1
(lm.best3.R2 <- abs(1-sum((test$wealth89-lm.pred3.test)^2)/sum((test$wealth89-mean(t
```

0.28052923018587
0.660001103653225
0.289157110912427
0.147110695248097

0.460305366490338
0.211240853365952

3.5 Interpretation

From the results, we can see that: 1. The AIC value of lm.best2(2005) is the best, which the AIC value of lm.best1 is 2072.9 and lm.best3 is 2073.08 in train dataset; 2. The prediction of both train and test dataset, lm.best2 has the best R-square value(train: 0.66, test: 0.46), which for lm.best1 is (train: 0.28, test: 0.147), lm.best3 is (train: 0.289, test: 0.211).

In conclusion, lm.best2 is the best model. which is: wealth89 = -843.4697 + 49.7271*pyears - 294.4599*choice + 838.9570*female + 60.9021*married + 13.1785*age + 1.4376*educ + 38.5126*finc50 + 1128.8213*finc75 - 2600.864*finc100 - 6023.0753*finc101 - 1480.3054*stckin89 + 646.811*irain89 - 1029.4201*married:finc75 + 95.4846*age:finc101 + 709.4387*female:finc101 - 202.9605*female:irain89 + 5.9580*pyears:finc50 + 17.8365*age:stckin89 - 32.4273*educ:irain89 + 38.5338*educ:stckin89 - 12.8637*female:age - 0.8094*pyears:age - 159.3892*choice:finc50 + 199.3274*female:finc100 + 46.5178*age:finc100 - 7.5582*pyears:finc75 - 318.8968*choice:finc100 + 25.3205*choice:educ - 168.3136*finc100:irain89

Even though some of the single variables are not significant, it interact with other variables, and the interaction is significant. So I keep them.

From the results, we can tell that all of these variables have complicated effects to the wealth. For example, when the pyears increasing, it will affect the interaction with finc101, age, finc75 and affect wealth directly. Some of them are positive effect, some of them are negative effect.

Besides, the best model, lm.best2, has the highest accuracy, but it's not enough. they accuracy is still low. It may need get other variables, or not fit linear model, but logistic model or KNN, and so on.

4 Problem 3.

(25 points) We are interested in finding the important predictors of online customers booking a room at a hotel, assess their adjusted effect sizes (in direction and magnitude) and use the best logistic regression model for interpretation and prediction. We want to analyze the Travel.txt dataset (available on Blackboard) that contains 20,000 observations on 26 variables (description of all variables is presented in the Data_Dictionary_Travel file available on Blackboard). Perform all necessary data analysis steps and write a section summarizing the findings.

4.1 Load data and Basic Analysis

In [2]:
```
# load data
travel_data <- read.table('Travel.txt',header=T)
dim(travel_data)
# omit rows that has NA
travel_data <- na.omit(travel_data)
dim(travel_data)
head(travel_data)
names(travel_data)
str(travel_data)
summary(travel_data)
```

1. 20000 2. 25

1. 19993 2. 25

date_time	user_location_region	user_location_city	user_location_latitude	user_location_lo
2015-07-13 13:22:00	CA	STOCKTON	37.983835	-121.30217
2015-10-21 16:19:00	WA	KIRKLAND	47.70401	-122.190513
2015-08-04 08:15:00	BZ	BELIZE	NULL	NULL
2015-06-02 20:23:00	MAN	MANDALUYONG	NULL	NULL
2015-09-18 21:37:00	CO	DENVER	39.7478	-104.9406725
2015-12-02 01:07:00	CA	SAN DIEGO	32.760625	-117.168396

1. 'date_time' 2. 'user_location_region' 3. 'user_location_city' 4. 'user_location_latitude' 5. 'user_location_longitude' 6. 'orig_destination_distance' 7. 'user_id' 8. 'is_mobile' 9. 'is_package' 10. 'channel' 11. 'srch_ci' 12. 'srch_co' 13. 'srch_adults_cnt' 14. 'srch_children_cnt' 15. 'srch_rm_cnt' 16. 'srch_destination_id' 17. 'hotel_country' 18. 'is_booking' 19. 'hotel_id' 20. 'prop_is_branded' 21. 'prop_starrating' 22. 'distance_band' 23. 'hist_price_band' 24. 'popularity_band' 25. 'cnt'

```
'data.frame':        19993 obs. of  25 variables:
 $ date_time                : Factor w/ 19525 levels "2015-01-01 08:22:00",..: 10614 16391 1204
 $ user_location_region     : Factor w/ 367 levels "***","?","00",..: 107 348 105 206 112 107 
 $ user_location_city       : Factor w/ 4315 levels "?","AACHEN","ABANCAY",..: 3720 1938 289 22
 $ user_location_latitude   : Factor w/ 3088 levels "-14.809929","-15.791410500000001",..: 1172
 $ user_location_longitude  : Factor w/ 3090 levels "-100.453205",..: 521 598 3090 3090 48 278
 $ orig_destination_distance: Factor w/ 13865 levels "0.0056","0.0286",..: 1460 11683 13865 138
 $ user_id                  : int  1332946867 1826431285 -1744380080 -805781454 64349796 32077
 $ is_mobile                : int  0 0 0 0 0 0 0 0 1 0 ...
 $ is_package               : int  1 0 1 0 0 0 0 1 1 0 ...
 $ channel                  : int  262 541 541 541 541 541 541 541 262 541 ...
 $ srch_ci                  : Factor w/ 625 levels "2015-01-01","2015-01-02",..: 445 295 223 1
 $ srch_co                  : Factor w/ 634 levels "2015-01-03","2015-01-04",..: 448 299 223 1
 $ srch_adults_cnt          : int  2 2 2 2 2 2 2 2 2 6 ...
 $ srch_children_cnt        : int  2 2 0 1 2 0 1 0 1 0 ...
 $ srch_rm_cnt              : int  1 1 1 1 1 1 1 1 1 2 ...
 $ srch_destination_id      : int  5581115 5527578 5525315 5581053 5527578 5526679 5525315 557
 $ hotel_country            : Factor w/ 140 levels "ANGOLA","ANGUILLA",..: 80 135 135 124 135 
 $ is_booking               : int  0 0 0 0 0 0 0 0 0 0 ...
 $ hotel_id                 : int  24630615 182496 758476 145417930 254413465 391591 373983 17
 $ prop_is_branded          : int  0 1 1 0 0 1 1 1 0 1 ...
 $ prop_starrating          : int  4 4 4 4 2 4 3 5 3 5 ...
 $ distance_band            : Factor w/ 5 levels "C","F","M","VC",..: 1 1 1 4 3 1 3 3 4 3 ...
 $ hist_price_band          : Factor w/ 5 levels "H","L","M","VH",..: 3 4 3 3 2 3 3 3 2 1 ...
 $ popularity_band          : Factor w/ 5 levels "H","L","M","VH",..: 4 4 4 4 3 4 4 4 1 4 ...
 $ cnt                      : int  1 1 3 1 1 1 3 3 1 1 ...
 - attr(*, "na.action")=Class 'omit'  Named int [1:7] 116 3885 8877 11095 13029 17566 18759
  .. ..- attr(*, "names")= chr [1:7] "116" "3885" "8877" "11095" ...
```

```
               date_time     user_location_region   user_location_city
 2015-03-13 23:15:00:    3   CA     : 2924        NEW YORK   :  366
 2015-05-27 19:37:00:    3   NY     : 1236        LOS ANGELES:  266
```

```
2015-06-09 13:11:00:    3   TX     : 1163        TORONTO    :  232
2015-06-10 17:31:00:    3   FL     : 1063        HOUSTON    :  229
2015-07-07 20:11:00:    3   ON     :  938        CHICAGO    :  215
2015-07-25 11:51:00:    3   WA     :  591        CALGARY    :  179
(Other)            :19975   (Other):12078        (Other)    :18506
user_location_latitude       user_location_longitude orig_destination_distance
NULL      : 4242       NULL               : 4242      NULL     : 4242
40.75512 :  366        -73.98300900000001:  366      227.5021:   34
34.059768:  266        -118.312427       :  266      0.6328  :   17
43.667179:  232        -79.390203        :  232      0.1175  :   16
29.769607:  229        -95.42647         :  229      342.2687:   16
41.89042 :  215        -87.62904         :  215      196.1892:   15
(Other)  :14443        (Other)           :14443      (Other) :15653
   user_id             is_mobile        is_package        channel
Min.   :-2.147e+09   Min.   :0.0000   Min.   :0.0000   Min.   :231.0
1st Qu.:-1.031e+09   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:293.0
Median : 2.384e+07   Median :0.0000   Median :0.0000   Median :510.0
Mean   : 1.763e+07   Mean   :0.2233   Mean   :0.1949   Mean   :418.6
3rd Qu.: 1.086e+09   3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:541.0
Max.   : 2.147e+09   Max.   :1.0000   Max.   :1.0000   Max.   :541.0

      srch_ci           srch_co       srch_adults_cnt srch_children_cnt
2015-09-04:  134   2015-09-07:  152   Min.   :0.000   Min.   :0.0000
2015-07-03:  124   2015-07-05:  140   1st Qu.:2.000   1st Qu.:0.0000
2015-09-05:  121   2015-07-26:  124   Median :2.000   Median :0.0000
2015-08-14:  117   2015-08-09:  123   Mean   :2.056   Mean   :0.3105
2015-07-31:  114   2015-08-30:  122   3rd Qu.:2.000   3rd Qu.:0.0000
2015-08-07:  114   2016-01-02:  120   Max.   :9.000   Max.   :8.0000
(Other)   :19269   (Other)   :19212
 srch_rm_cnt    srch_destination_id                 hotel_country
Min.   :0.000   Min.   :      8152   UNITED STATES OF AMERICA:12009
1st Qu.:1.000   1st Qu.:   5527175   CANADA                  : 1141
Median :1.000   Median :   5627553   MEXICO                  : 1072
Mean   :1.077   Mean   :  67747386   ITALY                   :  537
3rd Qu.:1.000   3rd Qu.:187465121   UNITED KINGDOM          :  426
Max.   :8.000   Max.   :196871823   FRANCE                  :  376
                                     (Other)                 : 4432
  is_booking         hotel_id         prop_is_branded  prop_starrating
Min.   :0.00000   Min.   :      402   Min.   :0.0000   Min.   :0.000
1st Qu.:0.00000   1st Qu.:   725244   1st Qu.:0.0000   1st Qu.:3.000
Median :0.00000   Median : 21521873   Median :1.0000   Median :4.000
Mean   :0.08763   Mean   : 60268782   Mean   :0.6166   Mean   :3.528
3rd Qu.:0.00000   3rd Qu.: 76989646   3rd Qu.:1.0000   3rd Qu.:4.000
Max.   :1.00000   Max.   :410748015   Max.   :1.0000   Max.   :5.000

distance_band hist_price_band popularity_band      cnt
C :5129       H :4065         H :5972         Min.   : 1.000
F :2729       L :3870         L : 719         1st Qu.: 1.000
```

```
 M :7628       M :8076        M :5211        Median : 1.000
 VC:3155       VH:2108        VH:7969        Mean   : 1.421
 VF:1352       VL:1874        VL: 122        3rd Qu.: 1.000
                                             Max.   :38.000
```

```
In [3]: # change some variables to factors
        travel_data[,c(8,9,10,18,20)] <- lapply(travel_data[,c(8,9,10,18,20)], function(x) as.i
        # change date_time to month
        library(lubridate)
        travel_data$month <- as.factor(month(as.Date(travel_data$date_time)))
        # change check-in and check-out day to the number of days stay in the hotel.
        # if the days greater than 30 days or less than 0 days, change them to 0.
        travel_data$days <- as.integer(as.Date(travel_data$srch_co)-as.Date(travel_data$srch_ci
        travel_data$days <- ifelse(0<travel_data$days & travel_data$days<30 ,travel_data$days,0
        # change orig_destination_distance NULL to 0.
        travel_data$orig_destination_distance <- as.numeric(levels(travel_data$orig_destinatio
        #travel_data$orig_destination_distance[is.na(travel_data$orig_destination_distance)] <
        # remove redundent and non-important variables
        travel_data <- travel_data[,-c(1,4,5,7,11,12,16,19)]
        travel_data <- na.omit(travel_data)

        dim(travel_data)
        str(travel_data)
        summary(travel_data)
```

```
Warning message:
"package 'lubridate' was built under R version 3.3.3"
Attaching package: 'lubridate'

The following object is masked from 'package:base':

    date

Warning message in eval(expr, envir, enclos):
"NAs introduced by coercion"
```

1. 19986 2. 19

```
'data.frame':        19986 obs. of  19 variables:
 $ user_location_region     : Factor w/ 367 levels "***","?","00",..: 107 348 105 206 112 107 
 $ user_location_city       : Factor w/ 4315 levels "?","AACHEN","ABANCAY",..: 3720 1938 289 2
 $ orig_destination_distance: num  1243 686 0 0 950 ...
 $ is_mobile                : Factor w/ 2 levels "0","1": 1 1 1 1 1 1 1 1 2 1 ...
 $ is_package               : Factor w/ 2 levels "0","1": 2 1 2 1 1 1 1 2 2 1 ...
 $ channel                  : Factor w/ 11 levels "231","262","293",..: 2 11 11 11 11 11 11 11
 $ srch_adults_cnt          : int  2 2 2 2 2 2 2 2 2 6 ...
 $ srch_children_cnt        : int  2 2 0 1 2 0 1 0 1 0 ...
```

```
$ srch_rm_cnt              : int  1 1 1 1 1 1 1 1 1 2 ...
$ hotel_country            : Factor w/ 140 levels "ANGOLA","ANGUILLA",..: 80 135 135 124 135
$ is_booking               : Factor w/ 2 levels "0","1": 1 1 1 1 1 1 1 1 1 1 ...
$ prop_is_branded          : Factor w/ 2 levels "0","1": 1 2 2 1 1 2 2 2 1 2 ...
$ prop_starrating          : int  4 4 4 4 2 4 3 5 3 5 ...
$ distance_band            : Factor w/ 5 levels "C","F","M","VC",..: 1 1 1 4 3 1 3 3 4 3 ...
$ hist_price_band          : Factor w/ 5 levels "H","L","M","VH",..: 3 4 3 3 2 3 3 3 2 1 ...
$ popularity_band          : Factor w/ 5 levels "H","L","M","VH",..: 4 4 4 4 3 4 4 4 1 4 ...
$ cnt                      : int  1 1 3 1 1 1 3 3 1 1 ...
$ month                    : Factor w/ 12 levels "1","2","3","4",..: 7 10 8 6 9 12 11 11 4 9
$ days                     : num  4 5 1 1 2 2 1 5 2 5 ...
- attr(*, "na.action")=Class 'omit'  Named int [1:7] 289 6822 7355 9678 12238 14162 17499
 .. ..- attr(*, "names")= chr [1:7] "290" "6824" "7357" "9681" ...
```

```
user_location_region   user_location_city orig_destination_distance is_mobile
CA     : 2921        NEW YORK   :  366    Min.   :    0.00          0:15527
NY     : 1236        LOS ANGELES:  266    1st Qu.:   26.58          1: 4459
TX     : 1163        TORONTO    :  232    Median :  422.30
FL     : 1063        HOUSTON    :  229    Mean   : 1330.88
ON     :  938        CHICAGO    :  215    3rd Qu.: 1767.37
WA     :  591        CALGARY    :  179    Max.   :11698.27
(Other):12074        (Other)    :18499
is_package    channel     srch_adults_cnt srch_children_cnt  srch_rm_cnt
0:16089    541    :7801   Min.   :0.000   Min.   :0.0000    Min.   :0.000
1: 3897    510    :3076   1st Qu.:2.000   1st Qu.:0.0000    1st Qu.:1.000
           231    :2612   Median :2.000   Median :0.0000    Median :1.000
           293    :2578   Mean   :2.056   Mean   :0.3106    Mean   :1.077
           262    :1796   3rd Qu.:2.000   3rd Qu.:0.0000    3rd Qu.:1.000
           324    :1276   Max.   :9.000   Max.   :8.0000    Max.   :8.000
           (Other): 847
                 hotel_country   is_booking prop_is_branded prop_starrating
UNITED STATES OF AMERICA:12003   0:18234    0: 7663         Min.   :0.000
CANADA                  : 1141   1: 1752    1:12323         1st Qu.:3.000
MEXICO                  : 1072                              Median :4.000
ITALY                   :  537                              Mean   :3.528
UNITED KINGDOM          :  426                              3rd Qu.:4.000
FRANCE                  :  376                              Max.   :5.000
(Other)                 : 4431
distance_band hist_price_band popularity_band      cnt              month
C :5129       H :4064         H :5970         Min.   : 1.000   7      :2148
F :2726       L :3869         L : 718         1st Qu.: 1.000   8      :2035
M :7626       M :8071         M :5208         Median : 1.000   6      :1974
VC:3155       VH:2108         VH:7968         Mean   : 1.421   5      :1750
VF:1350       VL:1874         VL: 122         3rd Qu.: 1.000   3      :1742
                                              Max.   :38.000   4      :1628
                                                               (Other):8709
     days
```

```
 Min.   : 0.000
 1st Qu.: 1.000
 Median : 2.000
 Mean   : 3.077
 3rd Qu.: 4.000
 Max.   :28.000
```

In [18]:
```
# basic analysis

# is_booking grouped by user_location_region,user_location_city
city<-aggregate(travel_data$is_booking, by=travel_data[,c(1,2)], FUN = length)
# is_booking grouped by channel
channel<-aggregate(travel_data$is_booking, by=list(travel_data$channel), FUN=length)
# is_booking grouped by is_mobile
mobile<-aggregate(travel_data$is_booking, by=list(travel_data$is_mobile), FUN=length)
# is_booking grouped by is_package
package<-aggregate(travel_data$is_booking, by=list(travel_data$is_package), FUN=lengt
# is_booking grouped by hotel_country
hcountry<-aggregate(travel_data$is_booking, by=list(travel_data$hotel_country), FUN=le
# is_booking grouped by prop_is_branded
brand<-aggregate(travel_data$is_booking, by=list(travel_data$prop_is_branded), FUN=le
# is_booking grouped by prop_starrating
stars<-aggregate(travel_data$is_booking, by=list(as.factor(travel_data$prop_starratin
# is_booking grouped by cnt
click<-aggregate(travel_data$is_booking, by=list(as.factor(travel_data$cnt)), FUN=len
# is_booking grouped by month
month<-aggregate(travel_data$is_booking, by=list(travel_data$month), FUN=length)
```

In [25]:
```
city[order(city$x,decreasing=T),]
```

	user_location_region	user_location_city	x
3095	NY	NEW YORK	366
2550	CA	LOS ANGELES	266
4469	ON	TORONTO	232
2002	TX	HOUSTON	229
835	IL	CHICAGO	215
645	AB	CALGARY	179
2797	FL	MIAMI	166
4029	WA	SEATTLE	156
3925	CA	SAN FRANCISCO	133
572	NY	BROOKLYN	130
4583	BC	VANCOUVER	128
3921	CA	SAN DIEGO	127
3928	CA	SAN JOSE	125
1297	AB	EDMONTON	124
1148	CO	DENVER	114
2916	QC	MONTREAL	113
1075	TX	DALLAS	107
2392	NV	LAS VEGAS	99
3473	AZ	PHOENIX	99
199	GA	ATLANTA	98
3468	PA	PHILADELPHIA	92
4685	DC	WASHINGTON	91
3301	FL	ORLANDO	87
3580	OR	PORTLAND	87
3325	ON	OTTAWA	78
1563	FL	FT LAUDERDALE	65
2997	BY	MUNICH	64
227	TX	AUSTIN	63
2849	MN	MINNEAPOLIS	63
395	BE	BERLIN	62
...	...	...	...
4922	MA	WRENTHAM	1
4923	NW	WUPPERTAL	1
4925	MN	WYKOFF	1
4927	PA	WYNNEWOOD	1
4930	CO	YAMPA	1
4931	SD	YANKTON	1
4932	M	YAVNE	1
4933	M	YEHUD	1
4934	NT	YELLOWKNIFE	1
4935	11	YEOKSAM-DONG	1
4936	11	YEONGEON-DONG	1
4938	NULL	YIGO	1
4939	16	YILDIRIM	1
4940	34	YILDIZ	1
4941	14	YOKOSUKA	1
4942	41	YONGIN	1
4945	ON	YORK	1
4947	YOR	YORK	1
4948	PA	YORK HAVEN	1
4953	NM	YOUNGSVILLE	1
4955	CA	YREKA	1

```
In [27]: channel[order(channel$x,decreasing=T),]
```

	Group.1	x
11	541	7801
10	510	3076
1	231	2612
3	293	2578
2	262	1796
4	324	1276
6	386	379
5	355	245
8	448	185
9	479	34
7	417	4

```
In [28]: mobile # more user not connect from mobile
```

Group.1	x
0	15527
1	4459

```
In [29]: package # more user not from package
```

Group.1	x
0	16089
1	3897

```
In [31]: hcountry[order(hcountry$x,decreasing=T),]
```

	Group.1	x
135	UNITED STATES OF AMERICA	12003
22	CANADA	1141
80	MEXICO	1072
62	ITALY	537
134	UNITED KINGDOM	426
42	FRANCE	376
114	SPAIN & CANARY ISLANDS	340
35	DOMINICAN REPUBLIC	275
45	GERMANY	223
126	THAILAND	193
64	JAPAN	175
133	UNITED ARAB EMIRATES	147
99	PUERTO RICO	138
47	GREECE	137
63	JAMAICA	136
129	TURKEY	125
88	NETHERLANDS	114
29	COSTA RICA	101
60	IRELAND	88
6	AUSTRALIA	78
26	CHINA	78
59	INDONESIA	78
96	PHILIPPINES	71
98	PORTUGAL	70
9	BAHAMAS	69
123	SWITZERLAND	66
55	HONG KONG	64
18	BRAZIL	63
27	COLOMBIA	62
113	SOUTH KOREA	56
...	...	...
17	BOSNIA AND HERZEGOVINA	2
38	ESTONIA	2
48	GRENADA	2
49	GUADELOUPE	2
53	HAITI	2
68	LAOS	2
71	LITHUANIA	2
74	MACEDONIA	2
78	MARTINIQUE	2
81	MOLDOVA	2
82	MONACO	2
103	RWANDA	2
121	STATE OF PALESTINE	2
139	ZAMBIA	2
1	ANGOLA	1
8	AZERBAIJAN	1
23	CAPE VERDE	1
44	GAMBIA	1
52	GUYANA	1
66	KAZAKHSTAN	1
83	MONTENEGRO	1

```
In [32]: brand # more book brand hotels
```

Group.1	x
0	7663
1	12323

```
In [34]: stars[order(stars$x,decreasing=T),] #4 and 3 stars hotel booked the most
```

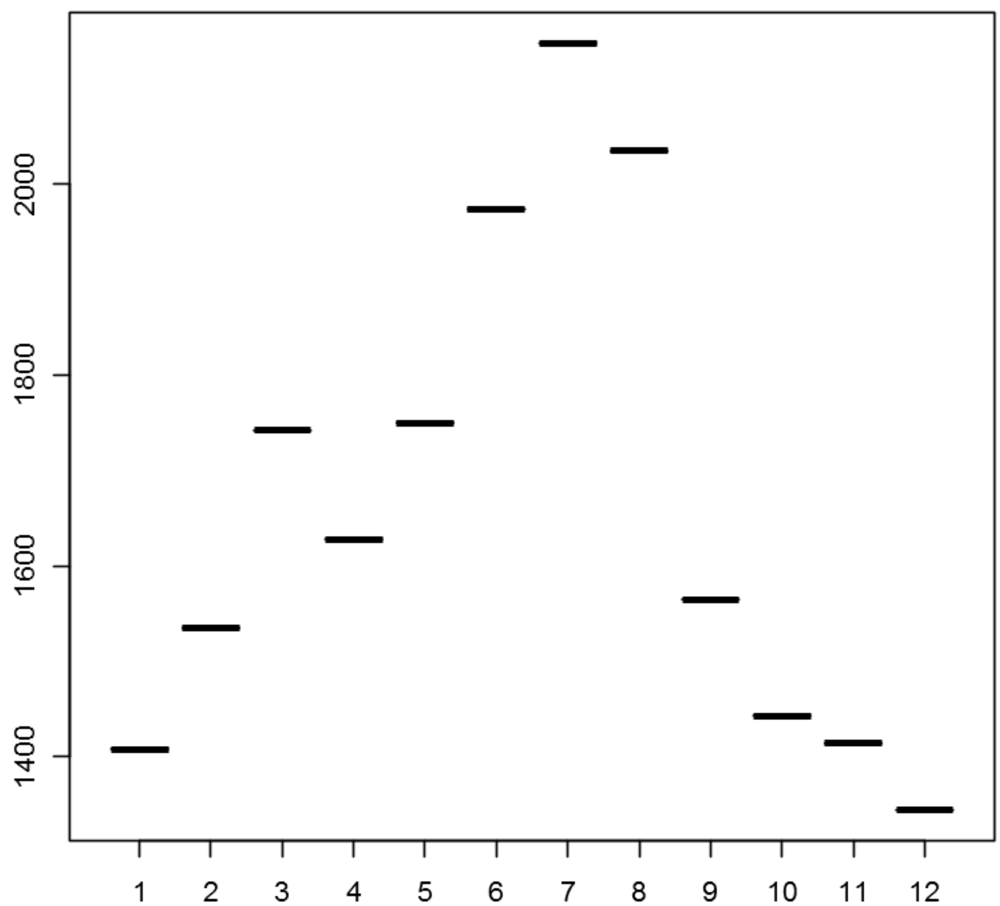
	Group.1	x
5	4	8222
4	3	6831
6	5	2623
3	2	1992
1	0	280
2	1	38

```
In [36]: click[order(click$x,decreasing=T),] # most of users click one or two times then booke
```

	Group.1	x
1	1	15717
2	2	2693
3	3	834
4	4	326
5	5	156
6	6	80
7	7	44
9	9	25
8	8	23
10	10	16
11	11	10
26	26	8
12	12	6
25	25	6
13	13	5
14	14	5
15	15	4
24	24	4
27	27	4
16	16	3
19	19	3
20	20	3
17	17	2
18	18	2
23	23	2
28	28	2
21	21	1
22	22	1
29	38	1

```
In [63]: month[order(month$x,decreasing=T),]
         plot(month$Group.1,month$x,type='l') # from may to august is the highest session book
```

	Group.1	x
7	7	2148
8	8	2035
6	6	1974
5	5	1750
3	3	1742
4	4	1628
9	9	1565
2	2	1535
10	10	1443
11	11	1414
1	1	1408
12	12	1344



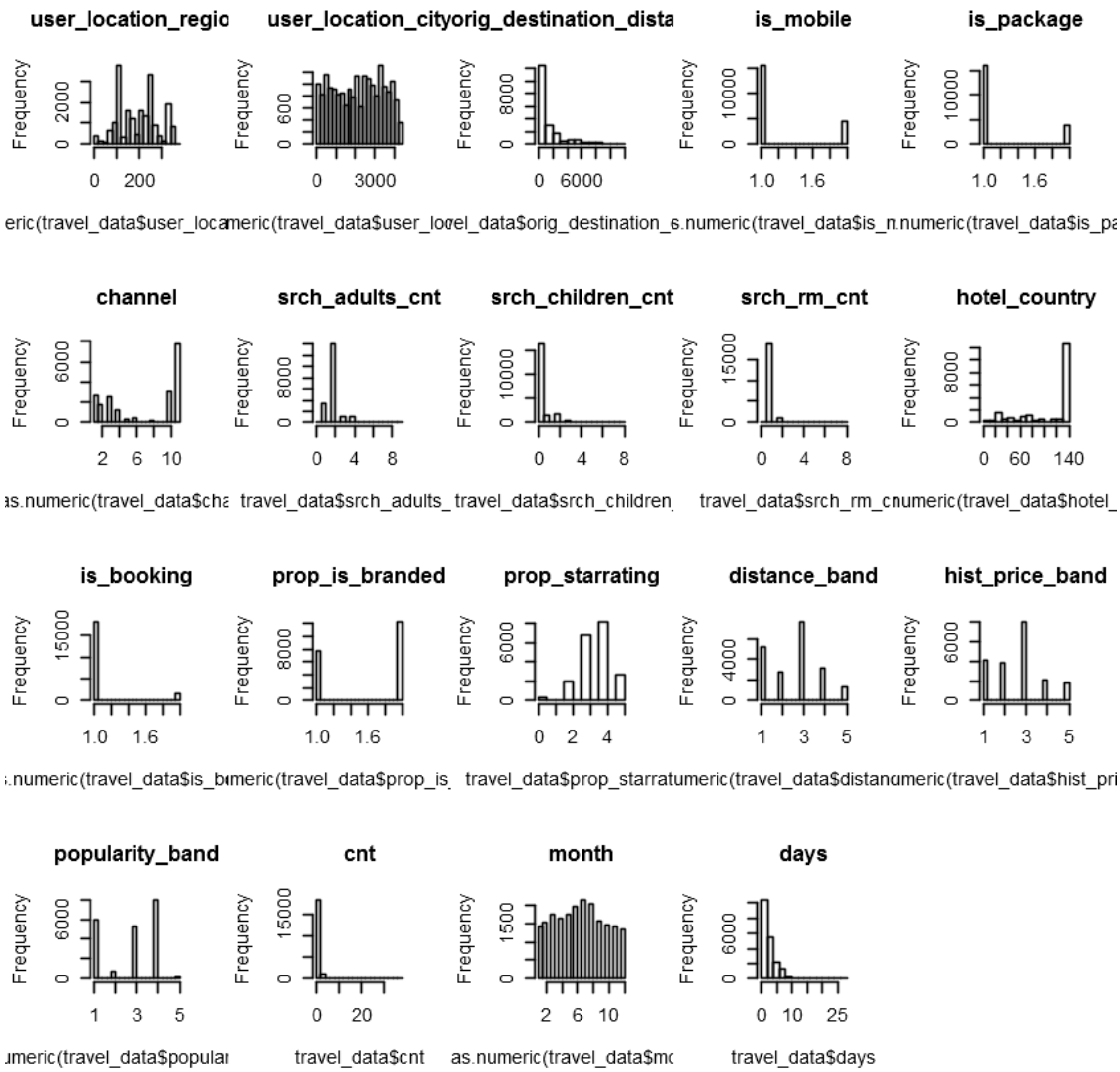
In [64]:
```
# change variables, user_location_region,user_location_city,hotel_country,distance_ba
travel_data$user_location_region <- as.factor(as.numeric(travel_data$user_location_reg
travel_data$user_location_city <- as.factor(as.numeric(travel_data$user_location_city)
travel_data$hotel_country <- as.factor(as.numeric(travel_data$hotel_country)-1)
travel_data$distance_band <- as.factor(as.numeric(travel_data$distance_band)-1)
travel_data$hist_price_band <- as.factor(as.numeric(travel_data$hist_price_band)-1)
travel_data$popularity_band <- as.factor(as.numeric(travel_data$popularity_band)-1)
summary(travel_data)
```

```
 user_location_region user_location_city orig_destination_distance is_mobile
 106    : 2921        2653   :  366      Min.   :    0.00          0:15527
 248    : 1236        2192   :  266      1st Qu.:   26.58          1: 4459
 327    : 1163        3887   :  232      Median :  422.30
 143    : 1063        1720   :  229      Mean   : 1330.88
 255    :  938        705    :  215      3rd Qu.: 1767.37
 347    :  591        538    :  179      Max.   :11698.27
 (Other):12074        (Other):18499
 is_package    channel     srch_adults_cnt srch_children_cnt  srch_rm_cnt
 0:16089    541    :7801   Min.   :0.000   Min.   :0.0000    Min.   :0.000
 1: 3897    510    :3076   1st Qu.:2.000   1st Qu.:0.0000    1st Qu.:1.000
            231    :2612   Median :2.000   Median :0.0000    Median :1.000
            293    :2578   Mean   :2.056   Mean   :0.3106    Mean   :1.077
            262    :1796   3rd Qu.:2.000   3rd Qu.:0.0000    3rd Qu.:1.000
            324    :1276   Max.   :9.000   Max.   :8.0000    Max.   :8.000
            (Other): 847
 hotel_country   is_booking prop_is_branded prop_starrating distance_band
 134    :12003   0:18234    0: 7663         Min.   :0.000   0:5129
 21     : 1141   1: 1752    1:12323         1st Qu.:3.000   1:2726
 79     : 1072                              Median :4.000   2:7626
 61     :  537                              Mean   :3.528   3:3155
 133    :  426                              3rd Qu.:4.000   4:1350
 41     :  376                              Max.   :5.000
 (Other): 4431
 hist_price_band popularity_band      cnt              month
 0:4064          0:5970          Min.   : 1.000   7      :2148
 1:3869          1: 718          1st Qu.: 1.000   8      :2035
 2:8071          2:5208          Median : 1.000   6      :1974
 3:2108          3:7968          Mean   : 1.421   5      :1750
 4:1874          4: 122          3rd Qu.: 1.000   3      :1742
                                 Max.   :38.000   4      :1628
                                                  (Other):8709
      days
 Min.   : 0.000
 1st Qu.: 1.000
 Median : 2.000
```

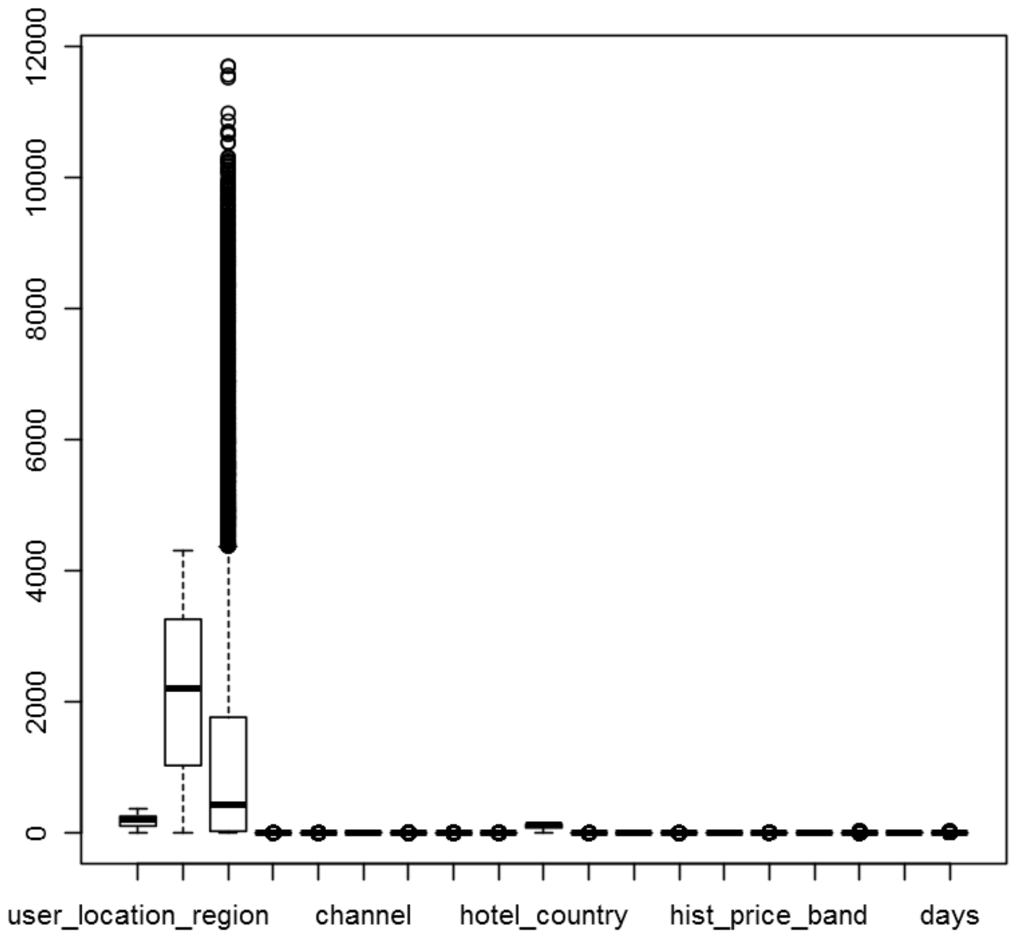
```
 Mean   : 3.077
 3rd Qu.: 4.000
 Max.   :28.000
```

```
In [65]: names(travel_data)
         par(mfrow=c(4,5))
         hist(as.numeric(travel_data$user_location_region),main='user_location_region')
         hist(as.numeric(travel_data$user_location_city),main='user_location_city')
         hist(travel_data$orig_destination_distance,main='orig_destination_distance')
         hist(as.numeric(travel_data$is_mobile),main='is_mobile')
         hist(as.numeric(travel_data$is_package),main='is_package')
         hist(as.numeric(travel_data$channel),main='channel')
         hist(travel_data$srch_adults_cnt,main='srch_adults_cnt')
         hist(travel_data$srch_children_cnt,main='srch_children_cnt')
         hist(travel_data$srch_rm_cnt,main='srch_rm_cnt')
         hist(as.numeric(travel_data$hotel_country),main='hotel_country')
         hist(as.numeric(travel_data$is_booking),main='is_booking')
         hist(as.numeric(travel_data$prop_is_branded),main='prop_is_branded')
         hist(travel_data$prop_starrating,main='prop_starrating')
         hist(as.numeric(travel_data$distance_band),main='distance_band')
         hist(as.numeric(travel_data$hist_price_band),main='hist_price_band')
         hist(as.numeric(travel_data$popularity_band),main='popularity_band')
         hist(travel_data$cnt,main='cnt')
         hist(as.numeric(travel_data$month),main='month')
         hist(travel_data$days,main='days')
```

1. 'user_location_region' 2. 'user_location_city' 3. 'orig_destination_distance' 4. 'is_mobile' 5. 'is_package' 6. 'channel' 7. 'srch_adults_cnt' 8. 'srch_children_cnt' 9. 'srch_rm_cnt' 10. 'hotel_country' 11. 'is_booking' 12. 'prop_is_branded' 13. 'prop_starrating' 14. 'distance_band' 15. 'hist_price_band' 16. 'popularity_band' 17. 'cnt' 18. 'month' 19. 'days'



In [66]:
```
# boxplot
par(mfrow=c(1,1))
boxplot(travel_data)
```



4.2 Split the dataset into training and testing dataset

```
In [67]: # train dataset: 80%, test dataset: 20%
         set.seed(6)
         (n <- dim(travel_data)[1])
         trainID <- sample(1:n,n*0.8)
         train <- travel_data[trainID,]
         test <- travel_data[-trainID,]
         dim(train)
         dim(test)
```

19986
1. 15988 2. 19
1. 3998 2. 19

4.3 Build Models

```
In [169]: # use stepAIC find the logit.best1 model
          #I runed about 2 hours, it still not get the results and my laptop shutdown automati
          # So remove user_location_region and user_location_city
          logit.fit1=glm(is_booking~.,family=binomial,data=train[,-c(1,2)])
          summary(logit.fit1) # AIC: 8637.5
```

```
Call:
glm(formula = is_booking ~ ., family = binomial, data = train[,
    -c(1, 2)])

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.2697  -0.5164  -0.3523  -0.1161   3.3599

Coefficients:
                            Estimate Std. Error z value Pr(>|z|)
(Intercept)               -1.317e+01  3.956e+03  -0.003 0.997344
orig_destination_distance  5.083e-06  2.067e-05   0.246 0.805770
is_mobile1                -1.869e-01  7.357e-02  -2.541 0.011056 *
is_package1               -6.379e-01  1.083e-01  -5.892 3.81e-09 ***
channel262                -5.339e-02  1.240e-01  -0.431 0.666759
channel293                -3.680e-01  1.188e-01  -3.097 0.001954 **
channel324                 1.091e-01  1.345e-01   0.811 0.417313
channel355                 5.488e-01  2.214e-01   2.479 0.013174 *
channel386                 2.533e-01  2.207e-01   1.148 0.251157
channel417                -1.588e+01  2.282e+03  -0.007 0.994450
channel448                -1.244e+00  4.675e-01  -2.661 0.007782 **
channel479                 7.344e-01  5.214e-01   1.408 0.159028
channel510                -1.584e-02  1.226e-01  -0.129 0.897209
channel541                -3.787e-02  9.460e-02  -0.400 0.688893
srch_adults_cnt           -9.428e-02  3.998e-02  -2.359 0.018348 *
srch_children_cnt         -7.429e-02  4.252e-02  -1.747 0.080615 .
srch_rm_cnt                5.540e-02  8.488e-02   0.653 0.513949
hotel_country1             1.818e+01  3.956e+03   0.005 0.996333
hotel_country2             7.200e-01  4.100e+03   0.000 0.999860
hotel_country3             1.596e+01  3.956e+03   0.004 0.996781
hotel_country4             1.516e+01  3.956e+03   0.004 0.996942
hotel_country5             1.499e+01  3.956e+03   0.004 0.996977
hotel_country6             1.500e+01  3.956e+03   0.004 0.996975
hotel_country7            -5.777e-01  5.595e+03   0.000 0.999918
hotel_country8             1.382e+01  3.956e+03   0.003 0.997213
hotel_country9             6.170e-01  4.236e+03   0.000 0.999884
hotel_country10            5.142e-01  4.022e+03   0.000 0.999898
hotel_country11            1.459e+01  3.956e+03   0.004 0.997058
hotel_country12            1.512e+01  3.956e+03   0.004 0.996951
hotel_country13            1.551e+01  3.956e+03   0.004 0.996872
```

hotel_country14	1.707e+01	3.956e+03	0.004	0.996557
hotel_country15	-5.051e-02	4.845e+03	0.000	0.999992
hotel_country16	3.531e+01	5.595e+03	0.006	0.994965
hotel_country17	1.546e+01	3.956e+03	0.004	0.996882
hotel_country18	1.668e+01	3.956e+03	0.004	0.996636
hotel_country19	4.079e-01	4.517e+03	0.000	0.999928
hotel_country20	1.466e+01	3.956e+03	0.004	0.997044
hotel_country21	1.497e+01	3.956e+03	0.004	0.996981
hotel_country22	3.905e+01	5.595e+03	0.007	0.994431
hotel_country23	3.724e-01	4.092e+03	0.000	0.999927
hotel_country24	2.882e-01	4.030e+03	0.000	0.999943
hotel_country25	2.805e-02	3.983e+03	0.000	0.999994
hotel_country26	1.516e+01	3.956e+03	0.004	0.996943
hotel_country27	1.756e+01	3.956e+03	0.004	0.996459
hotel_country28	1.524e+01	3.956e+03	0.004	0.996926
hotel_country29	1.580e+01	3.956e+03	0.004	0.996813
hotel_country30	5.560e-01	4.148e+03	0.000	0.999893
hotel_country31	6.200e-01	4.211e+03	0.000	0.999883
hotel_country32	1.521e+01	3.956e+03	0.004	0.996932
hotel_country33	1.575e+01	3.956e+03	0.004	0.996824
hotel_country34	1.431e+01	3.956e+03	0.004	0.997114
hotel_country35	-2.702e-02	4.315e+03	0.000	0.999995
hotel_country36	1.569e+01	3.956e+03	0.004	0.996835
hotel_country37	7.385e-01	4.720e+03	0.000	0.999875
hotel_country38	1.736e+01	3.956e+03	0.004	0.996500
hotel_country39	1.578e+01	3.956e+03	0.004	0.996817
hotel_country40	-2.415e-01	4.137e+03	0.000	0.999953
hotel_country41	1.470e+01	3.956e+03	0.004	0.997034
hotel_country42	4.034e-01	4.075e+03	0.000	0.999921
hotel_country43	4.731e+00	5.595e+03	0.001	0.999325
hotel_country44	1.452e+01	3.956e+03	0.004	0.997072
hotel_country45	3.048e-01	4.515e+03	0.000	0.999946
hotel_country46	1.482e+01	3.956e+03	0.004	0.997011
hotel_country48	5.596e-01	4.838e+03	0.000	0.999908
hotel_country49	1.537e+00	4.551e+03	0.000	0.999731
hotel_country50	1.500e+01	3.956e+03	0.004	0.996974
hotel_country52	3.466e+00	5.595e+03	0.001	0.999506
hotel_country53	-9.789e-04	4.207e+03	0.000	1.000000
hotel_country54	1.480e+01	3.956e+03	0.004	0.997015
hotel_country55	1.420e+01	3.956e+03	0.004	0.997136
hotel_country56	1.529e+01	3.956e+03	0.004	0.996916
hotel_country57	1.380e+01	3.956e+03	0.003	0.997217
hotel_country58	1.545e+01	3.956e+03	0.004	0.996885
hotel_country59	1.454e+01	3.956e+03	0.004	0.997068
hotel_country60	-3.270e-01	4.019e+03	0.000	0.999935
hotel_country61	1.459e+01	3.956e+03	0.004	0.997058
hotel_country62	1.396e+01	3.956e+03	0.004	0.997185
hotel_country63	1.476e+01	3.956e+03	0.004	0.997023

hotel_country64	1.705e+01	3.956e+03	0.004	0.996561
hotel_country65	5.821e+00	5.595e+03	0.001	0.999170
hotel_country66	3.642e-01	4.255e+03	0.000	0.999932
hotel_country67	2.069e+00	5.595e+03	0.000	0.999705
hotel_country68	5.561e-01	4.366e+03	0.000	0.999898
hotel_country69	7.842e+00	5.595e+03	0.001	0.998882
hotel_country70	-1.931e-01	4.832e+03	0.000	0.999968
hotel_country71	-1.339e+00	5.595e+03	0.000	0.999809
hotel_country72	1.711e+01	3.956e+03	0.004	0.996550
hotel_country73	4.842e-02	4.836e+03	0.000	0.999992
hotel_country74	1.446e+01	3.956e+03	0.004	0.997084
hotel_country75	6.788e-01	4.003e+03	0.000	0.999865
hotel_country76	1.648e+01	3.956e+03	0.004	0.996675
hotel_country77	1.865e+00	4.615e+03	0.000	0.999678
hotel_country78	2.021e-01	4.178e+03	0.000	0.999961
hotel_country79	1.480e+01	3.956e+03	0.004	0.997016
hotel_country80	1.849e+01	3.956e+03	0.005	0.996271
hotel_country81	7.631e-01	5.595e+03	0.000	0.999891
hotel_country82	1.744e+00	5.595e+03	0.000	0.999751
hotel_country83	1.699e+00	4.051e+03	0.000	0.999665
hotel_country84	-5.531e-01	4.546e+03	0.000	0.999903
hotel_country85	4.885e+00	5.595e+03	0.001	0.999303
hotel_country86	1.694e+01	3.956e+03	0.004	0.996583
hotel_country87	1.479e+01	3.956e+03	0.004	0.997016
hotel_country88	1.525e+01	3.956e+03	0.004	0.996925
hotel_country89	1.542e+01	3.956e+03	0.004	0.996891
hotel_country90	1.499e+00	5.595e+03	0.000	0.999786
hotel_country91	1.492e+01	3.956e+03	0.004	0.996991
hotel_country92	8.844e-01	4.458e+03	0.000	0.999842
hotel_country93	1.469e+01	3.956e+03	0.004	0.997036
hotel_country94	1.517e+01	3.956e+03	0.004	0.996941
hotel_country95	1.362e+01	3.956e+03	0.003	0.997253
hotel_country96	1.514e+01	3.956e+03	0.004	0.996946
hotel_country97	1.575e+01	3.956e+03	0.004	0.996824
hotel_country98	1.443e+01	3.956e+03	0.004	0.997090
hotel_country99	2.046e-01	4.203e+03	0.000	0.999961
hotel_country100	-6.795e-02	4.388e+03	0.000	0.999988
hotel_country101	1.608e+01	3.956e+03	0.004	0.996757
hotel_country102	-1.830e-01	4.760e+03	0.000	0.999969
hotel_country103	1.582e+01	3.956e+03	0.004	0.996810
hotel_country104	7.537e+00	5.595e+03	0.001	0.998925
hotel_country105	9.656e+00	5.595e+03	0.002	0.998623
hotel_country106	2.834e-02	4.833e+03	0.000	0.999995
hotel_country107	1.423e+01	3.956e+03	0.004	0.997131
hotel_country108	9.334e-01	4.128e+03	0.000	0.999820
hotel_country109	-1.042e+00	5.595e+03	0.000	0.999851
hotel_country110	1.168e-01	5.595e+03	0.000	0.999983
hotel_country111	1.566e+01	3.956e+03	0.004	0.996841

```
hotel_country112           1.489e+01  3.956e+03   0.004 0.996998
hotel_country113           1.470e+01  3.956e+03   0.004 0.997035
hotel_country114          -3.721e-01  4.415e+03   0.000 0.999933
hotel_country116           7.163e-01  4.388e+03   0.000 0.999870
hotel_country117           9.963e-01  4.025e+03   0.000 0.999802
hotel_country118           5.714e-01  4.159e+03   0.000 0.999890
hotel_country119           2.866e+00  4.844e+03   0.001 0.999528
hotel_country120           4.716e-01  4.754e+03   0.000 0.999921
hotel_country121           1.489e+01  3.956e+03   0.004 0.996997
hotel_country122           1.440e+01  3.956e+03   0.004 0.997095
hotel_country123           1.535e+01  3.956e+03   0.004 0.996903
hotel_country124           1.176e+00  4.206e+03   0.000 0.999777
hotel_country125           1.504e+01  3.956e+03   0.004 0.996967
hotel_country126           1.198e-01  4.560e+03   0.000 0.999979
hotel_country127           3.152e-01  4.843e+03   0.000 0.999948
hotel_country128           1.498e+01  3.956e+03   0.004 0.996979
hotel_country129           7.257e-01  4.025e+03   0.000 0.999856
hotel_country130           1.460e+01  3.956e+03   0.004 0.997055
hotel_country131           5.444e-01  4.188e+03   0.000 0.999896
hotel_country132           1.555e+01  3.956e+03   0.004 0.996863
hotel_country133           1.521e+01  3.956e+03   0.004 0.996933
hotel_country134           1.509e+01  3.956e+03   0.004 0.996957
hotel_country135           4.651e-01  4.290e+03   0.000 0.999913
hotel_country136           4.834e-01  5.595e+03   0.000 0.999931
hotel_country137           1.484e+01  3.956e+03   0.004 0.997008
hotel_country138          -1.665e-01  4.844e+03   0.000 0.999973
prop_is_branded1           2.705e-01  6.728e-02   4.021 5.81e-05 ***
prop_starrating           -2.047e-01  4.087e-02  -5.010 5.44e-07 ***
distance_band1            -1.566e-01  1.032e-01  -1.518 0.129117
distance_band2             5.682e-02  7.435e-02   0.764 0.444770
distance_band3            -1.141e-01  9.400e-02  -1.213 0.224961
distance_band4             1.450e-01  1.238e-01   1.171 0.241475
hist_price_band1           1.237e-01  9.956e-02   1.242 0.214152
hist_price_band2           1.689e-02  8.373e-02   0.202 0.840123
hist_price_band3           1.710e-01  1.146e-01   1.493 0.135421
hist_price_band4          -2.398e-01  1.319e-01  -1.817 0.069142 .
popularity_band1          -8.256e-01  2.090e-01  -3.950 7.81e-05 ***
popularity_band2          -1.212e-01  8.101e-02  -1.496 0.134546
popularity_band3           2.775e-01  7.137e-02   3.889 0.000101 ***
popularity_band4          -7.086e-01  4.718e-01  -1.502 0.133146
cnt                       -2.709e+00  2.232e-01 -12.139  < 2e-16 ***
month2                     2.967e-02  1.578e-01   0.188 0.850892
month3                     1.446e-02  1.501e-01   0.096 0.923264
month4                    -6.771e-02  1.536e-01  -0.441 0.659379
month5                     8.374e-02  1.607e-01   0.521 0.602384
month6                     7.190e-02  1.606e-01   0.448 0.654326
month7                    -2.108e-01  1.650e-01  -1.277 0.201443
month8                    -1.345e-02  1.614e-01  -0.083 0.933597
```

```
month9                      1.217e-01  1.668e-01   0.730 0.465492
month10                     1.785e-01  1.689e-01   1.057 0.290540
month11                     7.827e-02  1.744e-01   0.449 0.653504
month12                     2.314e-01  1.721e-01   1.345 0.178604
days                       -1.808e-01  1.959e-02  -9.230  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 9483.3  on 15987  degrees of freedom
Residual deviance: 8279.5  on 15809  degrees of freedom
AIC: 8637.5

Number of Fisher Scoring iterations: 16
```

In [168]:
```
# use stepAIC to get the best model
library(MASS)
logit.best1=stepAIC(logit.fit1,direction="both", trace = FALSE,data=train[,-c(1,2)])
summary(logit.best1) # AIC: 8507.1
# variables:srch_children_cnt is not significant, remove it.
```

```
Warning message:
"package 'MASS' was built under R version 3.3.3"Warning message:
"glm.fit: fitted probabilities numerically 0 or 1 occurred"Warning message:
"glm.fit: fitted probabilities numerically 0 or 1 occurred"Warning message:
"glm.fit: fitted probabilities numerically 0 or 1 occurred"Warning message:
"glm.fit: fitted probabilities numerically 0 or 1 occurred"Warning message:
"glm.fit: fitted probabilities numerically 0 or 1 occurred"Warning message:
"glm.fit: fitted probabilities numerically 0 or 1 occurred"Warning message:
"glm.fit: fitted probabilities numerically 0 or 1 occurred"Warning message:
"glm.fit: fitted probabilities numerically 0 or 1 occurred"Warning message:
"glm.fit: fitted probabilities numerically 0 or 1 occurred"Warning message:
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"glm.fit: fitted probabilities numerically 0 or 1 occurred"Warning message:
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Call:
glm(formula = is_booking ~ is_mobile + is_package + channel +
    srch_adults_cnt + srch_children_cnt + prop_is_branded + prop_starrating +
    distance_band + hist_price_band + popularity_band + cnt +
    days, family = binomial, data = train[, -c(1, 2)])

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.0124  -0.5192  -0.3660  -0.1389   3.5189

Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
(Intercept)         1.807502   0.297697   6.072 1.27e-09 ***
is_mobile1         -0.176136   0.072336  -2.435 0.014893 *
is_package1        -0.700497   0.105022  -6.670 2.56e-11 ***
channel262         -0.052573   0.123091  -0.427 0.669303
channel293         -0.381748   0.117692  -3.244 0.001180 **
channel324          0.074915   0.132592   0.565 0.572069
channel355          0.561647   0.218070   2.576 0.010009 *
channel386          0.176829   0.217327   0.814 0.415842
channel417        -10.758173 187.377734  -0.057 0.954215
channel448         -1.238959   0.466169  -2.658 0.007867 **
channel479          0.788797   0.518156   1.522 0.127930
channel510         -0.076177   0.106833  -0.713 0.475813
channel541         -0.017783   0.088897  -0.200 0.841452
srch_adults_cnt    -0.081639   0.036589  -2.231 0.025665 *
srch_children_cnt  -0.068989   0.042111  -1.638 0.101371
prop_is_branded1    0.305120   0.063121   4.834 1.34e-06 ***
prop_starrating    -0.187794   0.037203  -5.048 4.47e-07 ***
distance_band1     -0.181276   0.101803  -1.781 0.074967 .
distance_band2      0.057788   0.073548   0.786 0.432027
distance_band3     -0.103343   0.093326  -1.107 0.268151
```

```
distance_band4      0.158380   0.121498   1.304 0.192386
hist_price_band1    0.121853   0.096986   1.256 0.208972
hist_price_band2    0.001405   0.082080   0.017 0.986343
hist_price_band3    0.115093   0.113105   1.018 0.308877
hist_price_band4   -0.210420   0.128388  -1.639 0.101227
popularity_band1   -0.795602   0.208055  -3.824 0.000131 ***
popularity_band2   -0.092947   0.079296  -1.172 0.241137
popularity_band3    0.275299   0.070514   3.904 9.45e-05 ***
popularity_band4   -0.723237   0.469936  -1.539 0.123801
cnt                -2.640927   0.216090 -12.221  < 2e-16 ***
days               -0.191243   0.018783 -10.181  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 9483.3  on 15987  degrees of freedom
Residual deviance: 8445.1  on 15957  degrees of freedom
AIC: 8507.1

Number of Fisher Scoring iterations: 11
```

```
In [170]: # remove srch_children_cnt
          logit.best1 <- glm(formula = is_booking ~ is_mobile + is_package + channel +
              srch_adults_cnt  + prop_is_branded + prop_starrating +
              distance_band + hist_price_band + popularity_band + cnt +
              days, family = binomial, data = train[, -c(1, 2)])
          summary(logit.best1) # AIC: 8507.9
```

```
Warning message:
"glm.fit: fitted probabilities numerically 0 or 1 occurred"


Call:
glm(formula = is_booking ~ is_mobile + is_package + channel +
    srch_adults_cnt + prop_is_branded + prop_starrating + distance_band +
    hist_price_band + popularity_band + cnt + days, family = binomial,
    data = train[, -c(1, 2)])

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.0273  -0.5192  -0.3663  -0.1396   3.5259

Coefficients:
                   Estimate Std. Error z value Pr(>|z|)
(Intercept)        1.804052   0.297813   6.058 1.38e-09 ***
```

```
is_mobile1        -0.175075   0.072318  -2.421  0.01548 *
is_package1       -0.697306   0.105028  -6.639 3.15e-11 ***
channel262        -0.051581   0.123065  -0.419  0.67512
channel293        -0.380355   0.117678  -3.232  0.00123 **
channel324         0.081007   0.132512   0.611  0.54099
channel355         0.557752   0.218065   2.558  0.01054 *
channel386         0.181859   0.217270   0.837  0.40258
channel417       -10.735697 187.379383  -0.057  0.95431
channel448        -1.228628   0.466082  -2.636  0.00839 **
channel479         0.805183   0.517946   1.555  0.12005
channel510        -0.075253   0.106810  -0.705  0.48109
channel541        -0.018351   0.088880  -0.206  0.83643
srch_adults_cnt   -0.088386   0.036429  -2.426  0.01525 *
prop_is_branded1   0.299662   0.063032   4.754 1.99e-06 ***
prop_starrating   -0.185572   0.037237  -4.984 6.24e-07 ***
distance_band1    -0.184832   0.101769  -1.816  0.06934 .
distance_band2     0.054592   0.073510   0.743  0.45769
distance_band3    -0.103447   0.093322  -1.109  0.26765
distance_band4     0.152857   0.121444   1.259  0.20815
hist_price_band1   0.124505   0.096965   1.284  0.19913
hist_price_band2   0.002048   0.082066   0.025  0.98009
hist_price_band3   0.113960   0.113099   1.008  0.31364
hist_price_band4  -0.204585   0.128326  -1.594  0.11088
popularity_band1  -0.795968   0.208006  -3.827  0.00013 ***
popularity_band2  -0.092916   0.079283  -1.172  0.24122
popularity_band3   0.274264   0.070496   3.890  0.00010 ***
popularity_band4  -0.717964   0.469819  -1.528  0.12647
cnt               -2.643898   0.216083 -12.236  < 2e-16 ***
days              -0.192581   0.018787 -10.251  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 9483.3  on 15987  degrees of freedom
Residual deviance: 8447.9  on 15958  degrees of freedom
AIC: 8507.9

Number of Fisher Scoring iterations: 11
```

In [68]:
```
# Lasso regression
library(glmnet)
# create dummy variables
train.xmat <- model.matrix(is_booking~., data=train)[,-1]
test.xmat <- model.matrix(is_booking~., data=test)[,-1]
```

```
lasso.fit <- cv.glmnet(train.xmat, train$is_booking, alpha=1,family="binomial")
(lambda <- lasso.fit$lambda.min)  # 0.00472111648596817
```

```
Warning message:
"package 'glmnet' was built under R version 3.3.3"Loading required package: Matrix
Warning message:
"package 'Matrix' was built under R version 3.3.3"Loading required package: foreach
Warning message:
"package 'foreach' was built under R version 3.3.3"Loaded glmnet 2.0-13
```

0.00472111648596817

```
In [89]: # coeffecient
         ce <- coef(lasso.fit, s = "lambda.min")[which(coef(lasso.fit, s = "lambda.min") != 0)]
         # features that selected.
         fe <- colnames(train.xmat)[which(coef(lasso.fit, s = "lambda.min") != 0)]
         (n <- length(fe))
         # the fomula of this model
         fomula <- paste('log(pr(is_booking=1)/(1-pr(is_booking=1)))','=',ce[1],'*',fe[1])
         for (i in 2:n){
             fomula <- paste(fomula,'+',ce[i],'*',fe[i])
         }
         fomula
```

300

'log(pr(is_booking=1)/(1-pr(is_booking=1))) = -0.840175372380394 * user_location_region1 + 1.70064665164346 * user_location_region22 + 1.75037430767309 * user_location_region50 + 1.34757146927526 * user_location_region52 + 1.92621638805609 * user_location_region57 + 2.1966114724833 * user_location_region96 + 1.64498572322229 * user_location_region97 + 1.47093185065033 * user_location_region132 + 1.66825471520191 * user_location_region136 + 1.65327994128566 * user_location_region170 + 3.30869745225522 * user_location_region203 + 1.48393368588763 * user_location_region227 + 0.0776302603633484 * user_location_region230 + 0.379842165348271 * user_location_region262 + 2.3547967267212 * user_location_region266 + 1.72465244977372 * user_location_region279 + 0.183637427426377 * user_location_region282 + 2.36255454869789 * user_location_region286 + 1.61669538815595 * user_location_region296 + 1.73346642783372 * user_location_region323 + 1.71242619221405 * user_location_region345 + 2.66866636854355 * user_location_region357 + 1.72830370889992 * user_location_city23 + 1.91328689457381 * user_location_city25 + 1.82170701871056 * user_location_city43 + 1.34578747648795 * user_location_city62 + 0.171065000451806 * user_location_city145 + 0.769946669660274 * user_location_city154 + 1.46179218712341 * user_location_city161 + 1.91255710647529 * user_location_city188 + 1.57340288072404 * user_location_city190 + 2.66744055985309 * user_location_city202 + 0.625201658293635 * user_location_city210 + 1.20527540484896 * user_location_city225 + 0.00815313008617242 * user_location_city233 + 1.26156000929005 * user_location_city255 + 2.21353619712526 * user_location_city281 + 1.58240429249761 * user_location_city292 + 1.30318576331268 * user_location_city308 + 1.48776649780589 * user_location_city326 + 0.221991050010289 * user_location_city370 + 0.00897283018513941 * user_location_city396 + 2.08388660865866 * user_location_city399

+ 1.50810302908296 * user_location_city426 + 0.673150030556413 * user_location_city461 + 0.116011183583241 * user_location_city480 + 1.41594090800985 * user_location_city488 + 1.70405084239088 * user_location_city508 + 2.2956656986328 * user_location_city522 + 1.34608110011558 * user_location_city556 + 1.423711359596 * user_location_city559 + 1.42371135871114 * user_location_city573 + 0.20720038179701 * user_location_city595 + 0.618241346360835 * user_location_city607 + 0.0411031171108332 * user_location_city618 + 1.68226468836783 * user_location_city621 + 0.202959456053917 * user_location_city674 + 0.759334581797492 * user_location_city701 + 1.39265613284325 * user_location_city717 + 1.30446902568766 * user_location_city745 + 1.80742677118418 * user_location_city766 + 1.28519822850426 * user_location_city780 + 1.89229105223772 * user_location_city781 + 1.41594046593867 * user_location_city803 + 0.00764552989074245 * user_location_city807 + 1.56053485931506 * user_location_city811 + 1.51959680530411 * user_location_city819 + 1.37958746251617 * user_location_city823 + 2.1441767229971 * user_location_city824 + 1.35086935651256 * user_location_city860 + 0.0710943297740069 * user_location_city885 + 0.0466781230604732 * user_location_city903 + 1.43681538411199 * user_location_city907 + 0.130698050124748 * user_location_city910 + 0.271604556939314 * user_location_city911 + 1.94684193332267 * user_location_city915 + 1.93310636708276 * user_location_city952 + 1.58761010304687 * user_location_city965 + 1.20620315023935 * user_location_city975 + 1.10073170629572 * user_location_city977 + 1.6031510117803 * user_location_city980 + 1.69097798917911 * user_location_city988 + 1.42810151061056 * user_location_city1013 + 0.0173664772915361 * user_location_city1032 + 1.97783520565801 * user_location_city1060 + 1.69531316730153 * user_location_city1080 + 1.28899959933342 * user_location_city1086 + 0.0167581868446094 * user_location_city1101 + 1.5721576153883 * user_location_city1145 + 1.64260145085358 * user_location_city1154 + 0.705731557702907 * user_location_city1163 + 1.15517055215293 * user_location_city1170 + 1.54288955764277 * user_location_city1173 + 1.56438252651794 * user_location_city1180 + 0.412690526502291 * user_location_city1245 + 1.73717444923818 * user_location_city1257 + 2.38126002372614 * user_location_city1261 + 2.3787338292697 * user_location_city1291 + 1.45142075135272 * user_location_city1328 + 1.75502832813195 * user_location_city1335 + 2.70859467559473 * user_location_city1340 + 1.63818113374493 * user_location_city1360 + 1.61444253481964 * user_location_city1385 + 1.65524642533126 * user_location_city1412 + 1.64488800378399 * user_location_city1430 + 1.48776117301537 * user_location_city1433 + 1.32444037806607 * user_location_city1440 + 1.35478954587678 * user_location_city1442 + 1.30453579653865 * user_location_city1449 + 2.81148304322146 * user_location_city1478 + 3.06542496623121 * user_location_city1524 + 1.41593552700032 * user_location_city1525 + 0.141838688657448 * user_location_city1551 + 0.0180337853924606 * user_location_city1556 + 2.39285315223249 * user_location_city1564 + 1.45553683117395 * user_location_city1575 + 3.37109136045686 * user_location_city1594 + 1.42370603411847 * user_location_city1596 + 1.70754792902532 * user_location_city1597 + 1.74382248139009 * user_location_city1604 + 1.99578799236085 * user_location_city1607 + 1.87968702402652 * user_location_city1618 + 1.8484835534846 * user_location_city1626 + 0.195452375133235 * user_location_city1640 + 1.55038735070886 * user_location_city1698 + 0.0223783732009064 * user_location_city1720 + 2.83241933485316 * user_location_city1730 + 1.44099476551541 * user_location_city1771 + 1.67675235689419 * user_location_city1774 + 2.36442573933774 * user_location_city1800 + 0.0187045158538939 * user_location_city1804 + 1.06823492552214 * user_location_city1810 + 2.13849975397308 * user_location_city1819 + 0.685539856971392 * user_location_city1860 + 1.63841214275929 * user_location_city1874 + 1.75253611695693 * user_location_city1881 + 1.2933853385294 * user_location_city1894 + 1.29676516573578 * user_location_city1895 + 1.58760510419444 * user_location_city1908 +

1.76409180290395 * user_location_city1950 + 2.63653535284903 * user_location_city1956 + 1.52787846172861 * user_location_city1962 + 0.0223255307280383 * user_location_city1968 + 1.99064707662391 * user_location_city1979 + 2.18853880939357 * user_location_city1980 + 0.768887557596195 * user_location_city1984 + 2.01749304129641 * user_location_city1992 + 0.0140053569477208 * user_location_city2014 + 1.41593540858525 * user_location_city2034 + 1.35965079573736 * user_location_city2039 + 1.73614041442272 * user_location_city2044 + 1.51959174180391 * user_location_city2072 + 0.300559715124701 * user_location_city2078 + 1.43241959305018 * user_location_city2102 + 0.00347860498983713 * user_location_city2103 + 2.76174063262805 * user_location_city2114 + 2.25665304908358 * user_location_city2119 + 0.641235304492185 * user_location_city2121 + 0.535850561451285 * user_location_city2142 + 0.645058462797759 * user_location_city2145 + 1.71688377443262 * user_location_city2146 + 0.0280810470530736 * user_location_city2159 + 1.32859581144242 * user_location_city2180 + 0.0295914509071797 * user_location_city2205 + 1.44028055705431 * user_location_city2217 + 1.65771368540736 * user_location_city2219 + 2.49726189945697 * user_location_city2232 + 1.77160273732166 * user_location_city2241 + 1.59009697116719 * user_location_city2264 + 1.54906452921943 * user_location_city2300 + 1.36521074381114 * user_location_city2339 + 1.71282882770461 * user_location_city2344 + 1.36082007590793 * user_location_city2349 + 0.316885555520117 * user_location_city2372 + 1.57215246844488 * user_location_city2374 + -0.100530683153615 * user_location_city2405 + 2.23144801119273 * user_location_city2410 + 0.026486576985917 * user_location_city2417 + 1.34775152906851 * user_location_city2422 + 1.4402805430411 * user_location_city2460 + 0.677163400896585 * user_location_city2465 + 0.0356138983968096 * user_location_city2475 + 1.4867497564737 * user_location_city2513 + 1.74725545171385 * user_location_city2519 + 2.01954811849554 * user_location_city2531 + 1.50149434487202 * user_location_city2538 + 1.5919935394011 * user_location_city2577 + 1.61398477058887 * user_location_city2578 + 1.65771160686041 * user_location_city2601 + 0.641168791645382 * user_location_city2605 + 1.71912856848315 * user_location_city2607 + 1.26932268654331 * user_location_city2652 + 0.0323713575015137 * user_location_city2692 + 1.58714734022335 * user_location_city2703 + 0.428308556786427 * user_location_city2740 + 2.10560576684296 * user_location_city2750 + 1.59649526982019 * user_location_city2757 + 1.58769097934811 * user_location_city2759 + 0.00383717182970205 * user_location_city2799 + 1.44182943160601 * user_location_city2808 + 1.56793277815932 * user_location_city2833 + 1.24345450476768 * user_location_city2835 + 1.45553404302367 * user_location_city2880 + 0.137906917624647 * user_location_city2900 + 1.33997846365577 * user_location_city2909 + 1.53475305721225 * user_location_city2934 + 1.60540920670948 * user_location_city2936 + 0.0303729715924401 * user_location_city2965 + 1.58347371591598 * user_location_city2967 + 1.96942591317618 * user_location_city3001 + 1.57331940760417 * user_location_city3026 + 1.44348401842475 * user_location_city3038 + 0.502143637809217 * user_location_city3058 + 1.59365558190658 * user_location_city3069 + 1.51958862908655 * user_location_city3097 + 2.40590668375856 * user_location_city3114 + 1.41593227937248 * user_location_city3116 + 1.63840897727578 * user_location_city3132 + 1.53497881124773 * user_location_city3143 + 1.90738642508415 * user_location_city3166 + 0.136534080085431 * user_location_city3179 + 1.70499387252118 * user_location_city3244 + 1.39626266878445 * user_location_city3261 + 0.633666017607251 * user_location_city3269 + 0.291362620342793 * user_location_city3272 + 0.816417504131623 * user_location_city3284 + 1.32262598063512 * user_location_city3295 + 1.42370129699512 * user_location_city3301 + 0.98419094292816 * user_location_city3323 + 0.033826970645109 * user_location_city3339 + 1.44054848086501 * user_location_city3347 + 0.136698147263468 * user_location_city3349 + 0.607483822279127 * user_location_city3417 + 0.589289651161736 * user_location_city3423 + 1.8570528105965 * user_location_city3444

+ 1.59536984809037 * user_location_city3465 + 1.96752427660863 * user_location_city3477 + 1.67311340995418 * user_location_city3521 + 1.30115005436785 * user_location_city3523 + 0.0345032723031656 * user_location_city3527 + 1.903469155243 * user_location_city3565 + 1.47092109786354 * user_location_city3571 + 1.53497619488896 * user_location_city3592 + 1.46842906682882 * user_location_city3618 + 1.90106766605376 * user_location_city3631 + 1.53497618909107 * user_location_city3640 + 1.36520513604405 * user_location_city3658 + 1.47521094304858 * user_location_city3675 + 1.44195573618911 * user_location_city3680 + 2.42565685590999 * user_location_city3706 + 1.91813862366892 * user_location_city3721 + 1.61091072984783 * user_location_city3735 + 1.4603150668246 * user_location_city3767 + 0.420769294985221 * user_location_city3778 + 1.47092078669489 * user_location_city3795 + 2.56014162544353 * user_location_city3810 + 1.52603593853385 * user_location_city3826 + 0.0407796478970972 * user_location_city3835 + 1.6139808775342 * user_location_city3836 + 1.30114970925876 * user_location_city3868 + 1.42369980888356 * user_location_city3917 + 2.47712664823362 * user_location_city3945 + 1.70129168082216 * user_location_city3970 + 1.44520582988759 * user_location_city3978 + 0.0376645629145996 * user_location_city4001 + 1.69750535019174 * user_location_city4003 + 0.865496003488552 * user_location_city4048 + 1.40497338048667 * user_location_city4057 + 2.37745671716887 * user_location_city4081 + 1.57308983045378 * user_location_city4088 + 1.99809357128654 * user_location_city4097 + 1.37635508185944 * user_location_city4116 + 0.742650182802601 * user_location_city4149 + 1.56437605167919 * user_location_city4151 + 1.63723653095967 * user_location_city4183 + 1.62428689756246 * user_location_city4231 + 0.544013496634376 * user_location_city4285 + 1.4205142708612 * user_location_city4290 + 1.44027445616191 * user_location_city4295 + 2.55541530964901 * user_location_city4313 + -0.0318264177362581 * is_package1 + -0.448261666576075 * channel262 + -0.171683756628001 * channel324 + 0.140754753599276 * channel386 + -0.120724797287833 * channel479 + -0.0077713399951557 * srch_children_cnt + 0.570918745126533 * hotel_country2 + 2.3720265789286 * hotel_country17 + 0.46042139651852 * hotel_country23 + 1.22444921059706 * hotel_country39 + 0.110133301416886 * hotel_country135 + 0.135671015399629 * prop_starrating + -0.0640694520679812 * distance_band1 + -0.016576192445178 * distance_band2 + 0.027438314403372 * hist_price_band2 + -0.272422482034485 * popularity_band2 + 0.126954456011052 * popularity_band4 + -0.865817642323192 * month2 + -0.00907385931340032 * month8 + -0.148443004226004 * NA'

4.4 Predict and Evaluate

```
In [191]: logit.pred1.test <- predict(logit.best1,type='response',newdata=test[,-c(1,2,11)])
          min(logit.pred1.test)
          max(logit.pred1.test)
          unique(logit.pred1.test)
```

2.22044604925031e-16
0.403230212677895
1. 0.15371960423574 2. 0.196080876419912 3. 0.0587912600381845 4. 0.00125421510926031 5. 0.132423608432973 6. 0.0396861816807097 7. 0.0160537099137261 8. 0.0847520267639034 9. 0.0123355944721956 10. 0.0159211931161573 11. 0.048252591387093 12. 0.0524276569216523 13. 0.000978789203763779 14. 0.00353980290572955 15. 0.0922842074125851 16. 0.0100557268283705 17. 0.169473676177605 18. 0.184092604160856 19. 0.168159676901507 20. 0.215992760965985 21. 0.172603927891027 22. 0.132991969035463 23. 0.155312932566763 24. 0.000914429853907556 25. 0.121011833859889 26. 0.222547281985495 27. 0.0752964681038247

28. 0.209991489152388 29. 0.174190531281142 30. 0.0806490871639141 31. 0.171718482703863 32. 0.0972782012000671 33. 0.000585149792140442 34. 0.0933351670952805 35. 0.0307946419270644 36. 0.24602356793587 37. 0.167934865309499 38. 0.10581035802915 39. 0.00447868578787114 40. 0.0993502063858279 41. 0.113590164992191 42. 0.0813711026139276 43. 0.115975921696321 44. 0.00110292615075012 45. 0.0486543398493723 46. 0.0187029513896591 47. 0.0928758667997458 48. 0.168967517629418 49. 0.233269750531505 50. 0.0364007490517683 51. 0.080577681977002 52. 0.0854762450495025 53. 0.0845452409389507 54. 0.0860063595414643 55. 0.17884076406466 56. 0.0797517924713882 57. 0.113392775481006 58. 0.00933253231009517 59. 0.0532216702113453 60. 0.131425320548771 61. 0.0931619620401907 62. 0.0012107043792003 63. 0.00735543580904943 64. 0.000746142708521499 65. 0.0677873677508406 66. 0.141108409597164 67. 0.090492385236277 68. 0.170397210365865 69. 0.0399995702813921 70. 0.119235227846469 71. 0.169427326139594 72. 0.153173257616347 73. 0.10121019908798 74. 0.122693370940037 75. 0.0111569476371399 76. 0.0608284545859356 77. 0.00466334005690663 78. 0.0907877068515882 79. 0.0688095570008723 80. 0.194102632080644 81. 0.131799889599798 82. 0.0286470889609362 83. 0.00742202118598025 84. 0.0382044027109237 85. 0.14698478474695 86. 0.104422191258428 87. 0.101205815239725 88. 0.0380734271138187 89. 0.0584244627902995 90. 0.170807390636845 91. 0.0871202274866631 92. 0.0112132457596484 93. 0.131689238835481 94. 0.205101111285541 95. 0.0119893665744137 96. 0.0832486188146488 97. 0.214172369601674 98. 0.007297380728874 99. 0.151038486154062 100. 0.0682122211024601 101. 0.0415641722590144 102. 0.0635772705739719 103. 0.0466977851313856 104. 0.171477123331847 105. 5.54016911726286e-06 106. 0.0650871881255389 107. 0.221336970182166 108. 0.257320549851923 109. 0.00984437583652095 110. 0.0212535864841841 111. 0.0360764738462893 112. 0.00316459408819306 113. 0.00787364575925141 114. 0.0597456346766667 115. 0.108385921595767 116. 0.0545406575537666 117. 0.00813181437443863 118. 0.169271217518385 119. 0.0175420642916249 120. 0.174265945998708 121. 0.0559856636664471 122. 0.15778075892631 123. 0.138595764042007 124. 0.0445022333979972 125. 0.0540904172409825 126. 0.021981594997313 127. 0.014152489985729 128. 0.00980021976518355 129. 0.0651825456606913 130. 0.0014090684682003 131. 0.00331594166351338 132. 0.0779030439844852 133. 0.214724852089376 134. 0.111504646162052 135. 0.0956228750435141 136. 0.0927228993750739 137. 0.110192609831289 138. 0.0919752464407558 139. 0.00315210176352689 140. 0.0791245623169048 141. 0.207293385792985 142. 0.00661203435166702 143. 0.133983238213508 144. 0.0729847124737043 145. 0.198909762685845 146. 0.107320380975232 147. 0.128555093826857 148. 0.230503368267161 149. 0.209009949459428 150. 0.013072731246722 151. 0.187858417697366 152. 0.210665474853066 153. 0.0239484327596495 154. 0.0581063732626024 155. 0.1108267141831 156. 0.181936399664246 157. 0.084219924324744 158. 0.124722814098704 159. 0.0265107969666664 160. 0.0804260513419043 161. 0.00384011345816582 162. 0.0897087247977339 163. 0.0996413329102617 164. 0.232135868319775 165. 0.122647296074489 166. 0.0836895581597679 167. 6.96001112225765e-05 168. 0.0509400427077214 169. 0.047674162584635 170. 0.0750704178494485 171. 0.0740603359059777 172. 0.136772903460507 173. 0.129524201034465 174. 0.0349970777947528 175. 0.274958143621156 176. 0.0337975286926561 177. 0.143505492523669 178. 0.00572540709346485 179. 0.0331470720905459 180. 0.23748524112536 181. 0.145497388135788 182. 0.179332207828645 183. 0.0881742561864182 184. 3.38419428598802e-05 185. 0.084108883030288 186. 0.206651750261052 187. 0.0124106523426121 188. 0.0406540962763935 189. 0.012875487374671 190. 0.0037576270127154 191. 0.0284807088693756 192. 0.0230650064127305 193. 0.0235052620231478 194. 0.0785951292335289 195. 0.116368691726379 196. 0.00501362912129866 197. 0.116696443509907 198. 0.200469082960803 199. 0.146098944949433 200. 0.0451181257503151 201. 0.160784520521889 202. 0.096423390446227 203. 0.118911661458776 204. 0.000652366863043845

205. 0.0241685291133283 206. 0.100092919688574 207. 0.0158045645333918 208. 0.0841543245026593
209. 0.127249331910682 210. 0.220817512298232 211. 0.223477713870624 212. 0.103152358943157
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3506. 0.0311661538973198 3507. 0.121728401735662 3508. 0.0838225406426926 3509. 7.04363912358041e-05
3510. 0.0787151370268179 3511. 0.135855810951232 3512. 0.111395100788069 3513. 0.0375430636451105
3514. 0.0214451714135428 3515. 0.0849755235825456 3516. 0.19810135570553 3517. 0.214058587722386
3518. 0.0499201418086119 3519. 0.121795303186554 3520. 0.0479119848506843 3521. 0.00984847332139165
3522. 0.0154024930323064 3523. 0.189183494299134 3524. 0.16409229595037 3525. 0.0548505303990657
3526. 0.00472760359270932 3527. 0.0144589627919433 3528. 0.095377875153543 3529. 0.107130098931859
3530. 0.00646757632442479 3531. 0.0385506575052127 3532. 0.101989522387843 3533. 0.123398670513184
3534. 0.0445723604191613 3535. 0.00708380333656784 3536. 0.0199517854145515 3537. 0.0473299899339162
3538. 0.110305493682775 3539. 0.110983544841272 3540. 0.0428812896963631 3541. 0.0969331538426617
3542. 0.0693984871942778 3543. 0.112815414135567 3544. 0.00438944329029737 3545. 0.0518586265174763
3546. 0.000370370822394855 3547. 0.000291004098277332 3548. 1.64636976105961e-06 3549. 0.07200239806091
3550. 0.0795022012569347 3551. 0.110659127857515 3552. 0.00036906753478554 3553. 0.0590682764958098
3554. 0.151070097371163 3555. 0.0764438649610778 3556. 0.177204096431278 3557. 0.000754429002101991
3558. 0.101759524878005 3559. 0.128195221256176 3560. 0.0653044420789327 3561. 0.024851616577239
3562. 0.00407971718019351 3563. 0.221537608834213 3564. 0.216095381575936 3565. 0.129673370502063
3566. 0.0311701125743756 3567. 0.0806355555696028 3568. 0.0395165313844915 3569. 0.207222891942127
3570. 0.0011811664036589 3571. 0.20260070338569 3572. 0.0119399075073749 3573. 2.21884937929732e-08
3574. 0.146714025143062 3575. 0.0484304295550643 3576. 0.000215927096214122 3577. 2.64762634448856e-05
3578. 0.129133134684042 3579. 0.0742753721393637 3580. 0.0983916413541819 3581. 0.00353301239145605
3582. 0.217058699439566 3583. 0.0472917020341308 3584. 0.1031157405074 3585. 0.244998217119302
3586. 0.094765359693496 3587. 0.0554390510280213 3588. 0.0955127046077512 3589. 3.37457507593244e-06
3590. 0.128920703638444 3591. 0.123466807775555 3592. 0.0133386860716123 3593. 0.0955958553995933
3594. 0.0776445387975979 3595. 0.114704003331366 3596. 0.0268391148705145 3597. 0.188126572374704
3598. 0.0991315362476562 3599. 5.50543724082611e-05 3600. 0.089120004945207 3601. 0.176953778117839
3602. 0.1340387034496 3603. 0.072641137138791 3604. 4.78418780448209e-05 3605. 0.0247553734022383

3606. 0.0547809085931345 3607. 0.0830033478597822 3608. 0.0581760508222719 3609. 0.0395244799188041 3610. 0.0983507427539001 3611. 0.0471833241217635 3612. 0.0857006778109695 3613. 0.0782580705324895 3614. 0.154288259965349 3615. 0.124898754434107 3616. 0.0562734467688876 3617. 1.25898383914651e-05 3618. 0.129026316893499 3619. 0.00622612228738438 3620. 0.154950367967547 3621. 0.102545353939842 3622. 0.0060653279412865 3623. 0.028344804449464 3624. 0.144489839289378 3625. 0.145636509486855 3626. 0.182183747393188 3627. 0.137461867563431 3628. 0.123037137930629 3629. 0.186241489509814 3630. 0.0103371094335435 3631. 0.0920040083599195 3632. 0.00801259827220425 3633. 0.0793321889304545 3634. 0.0429467578176204 3635. 0.032005589457977 3636. 0.135078554968512 3637. 0.00804155270586953 3638. 0.0695767903018929 3639. 0.0868902947310243 3640. 0.00833268081627328 3641. 0.201345523839111 3642. 0.00329757050572958 3643. 0.106235940809332 3644. 0.0578960543293345 3645. 0.178810919861844 3646. 0.109472708046718 3647. 0.111239802297677 3648. 0.0847437523148552 3649. 0.0881626561047546 3650. 0.0489093942237208 3651. 0.0555954849735504 3652. 0.125082224234951 3653. 0.217348101648833 3654. 3.77453843488917e-05 3655. 0.000434063029777052 3656. 0.0442051544275936 3657. 0.0111308593206375 3658. 0.050252710906739 3659. 0.17407651110159 3660. 0.00352629796725853 3661. 0.0691648552447488 3662. 0.0160344785405365 3663. 0.00632142503595582 3664. 0.00275189760388385 3665. 0.0118565066405449 3666. 0.120291886731497 3667. 0.0826096105272194 3668. 0.0597561641933947 3669. 0.0886511436189296 3670. 0.106636270169144 3671. 0.117523266197185 3672. 0.0854908988815136 3673. 0.119839252330057 3674. 0.13469253348233 3675. 0.146349861823319 3676. 0.129040629330503 3677. 0.0380723351480771 3678. 0.197126354390823 3679. 0.130531510322541 3680. 0.141573891414284 3681. 2.47072900643319e-06 3682. 5.05801986221827e-12 3683. 0.160874751230055 3684. 0.0130278052809087 3685. 0.156424519168853 3686. 0.0431706021385955 3687. 0.167725668156745 3688. 0.0329634277484425 3689. 0.0900625974774133 3690. 0.0472199799154547 3691. 0.106269354509136 3692. 0.0305399412689747 3693. 0.108076483952743 3694. 0.15057948116061 3695. 0.000400592828420038 3696. 0.262784213891133 3697. 0.0601342212530846 3698. 0.0789937675910654 3699. 0.00712999731904766 3700. 0.084931621211061 3701. 0.0798952913383591 3702. 9.85906478237219e-11 3703. 0.000390622151641622 3704. 0.000275961882423262 3705. 0.101681145508036 3706. 0.049337056828393 3707. 7.19353157874883e-12 3708. 0.00541230867531682 3709. 0.0429879641102183 3710. 0.116111973781688 3711. 0.104075126429935 3712. 0.0619168540896346 3713. 0.132613416239165 3714. 0.0110375266636061 3715. 0.146253021244301 3716. 0.123576467812992 3717. 0.0998821634795586 3718. 0.0587230470014263 3719. 0.0927243761791674 3720. 0.151224575341267 3721. 0.204304654413075 3722. 0.045720933041452 3723. 0.039150306221659 3724. 0.2002060834664 3725. 0.000743782085070824 3726. 0.121469493855542 3727. 0.224094907519568 3728. 0.157712851923218 3729. 0.0333291058153329 3730. 0.110120000834067 3731. 0.096985544790372 3732. 0.15962271314357 3733. 0.0106662813967996 3734. 0.113553660117426 3735. 0.025084694039649 3736. 0.03363891114251 3737. 0.193803338166788 3738. 0.00156726414778945 3739. 0.170444581382865 3740. 0.102370313544366 3741. 0.0955641357601637 3742. 0.110946007564115 3743. 0.147978386427318 3744. 0.139563154178447

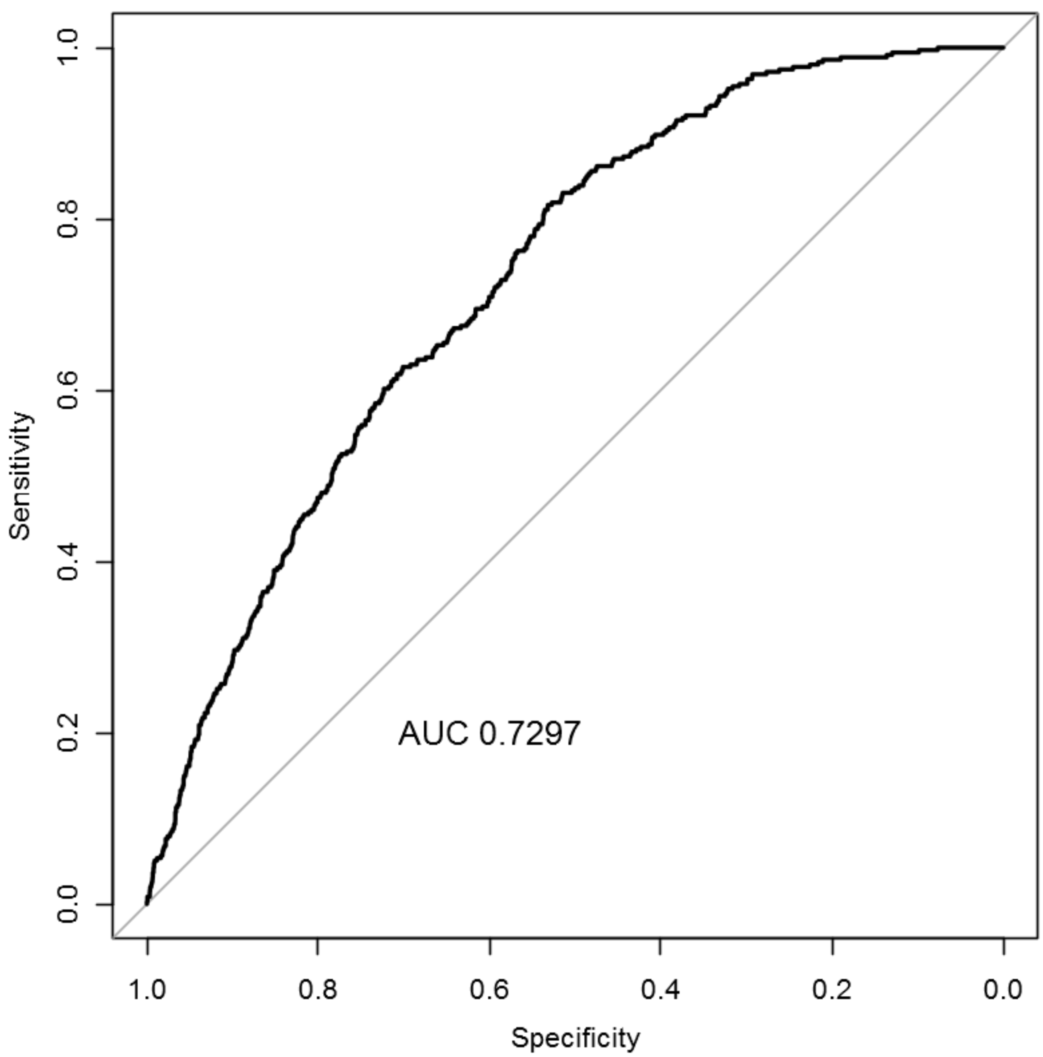
In [202]:
```
# logit.best1 from stepAIC
logit.pred1.test <- predict(logit.best1,type='response',newdata=test[,-c(1,2,11)])
library(pROC)
# ROC
(roc1 <- roc(test$is_booking,logit.pred1.test,plot=TRUE))
par(cex=1.2)
text(0.6,0.2,"AUC 0.7297")
# Confusion Table and Accuracy
logit.pred1.test <- ifelse(logit.pred1.test > 0.35, 1, 0) #Best thereshold: 0.35
(confTable1 <- table(logit.pred1.test, test$is_booking))
(acc1 <- sum(diag(confTable1))/sum(confTable1)) #Accuracy: 0.911705852926463
```

```
Call:
roc.default(response = test$is_booking, predictor = logit.pred1.test,     plot = TRUE)

Data: logit.pred1.test in 3644 controls (test$is_booking 0) < 354 cases (test$is_booking 1).
Area under the curve: 0.7297
```

```
logit.pred1.test    0    1
               0 3644  353
               1    0    1
```

0.911705852926463



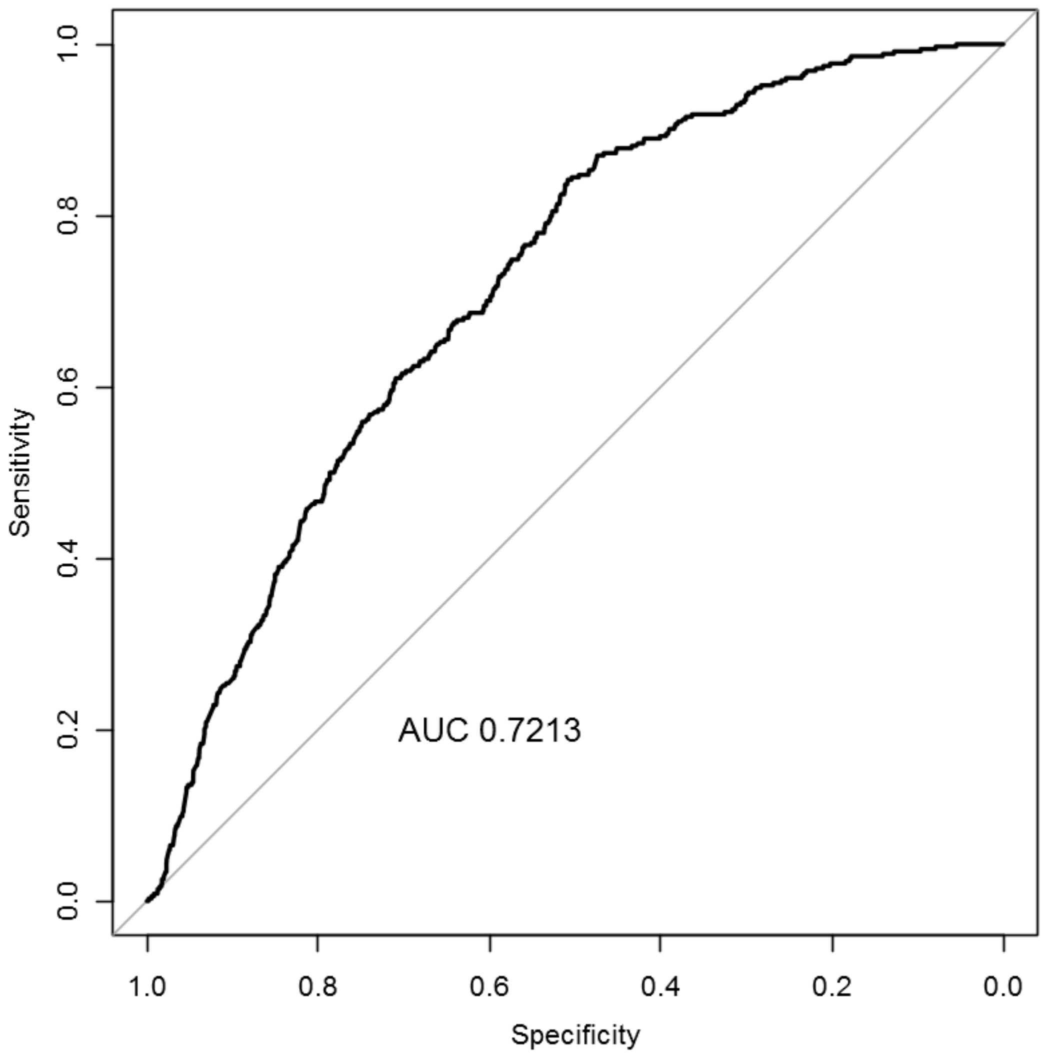
```
In [136]: library(pROC)
          lasso.pred <- predict(lasso.fit, s=lambda, newx=test.xmat)
          lasso.pred <- as.numeric(lasso.pred)
          # ROC
          (lasso.roc <- roc(test$is_booking,lasso.pred,plot=TRUE)) # AUC: 0.7213
          par(cex=1.2)
          text(0.6,0.2,"AUC 0.7213")
          # Confusion Table and Accuracy
          lasso.pred <- ifelse(lasso.pred > 0.3, 1, 0) #Best thereshold: 0.3
          (lasso.confTable <- table(lasso.pred, test$is_booking))
          (lasso.acc <- sum(diag(lasso.confTable))/sum(lasso.confTable)) #Accuracy: 0.91120560
```

```
Call:
roc.default(response = test$is_booking, predictor = lasso.pred,     plot = TRUE)

Data: lasso.pred in 3644 controls (test$is_booking 0) < 354 cases (test$is_booking 1).
Area under the curve: 0.7213
```

```
lasso.pred    0    1
         0 3643  354
         1    1    0
```

0.911205602801401



4.5 Interpretation

From the results, we can teel that logit.best1 is better than lasso.fit. the AUC logit.best1 vs lasso.fit is 0.7297 vs 0.7213 and the accuracy of logit.best1 is better than lasso.fit, which is 0.9117 vs 0.9112. So model logit.best1 is better than lasso.fit.

The best model is: log(pr(is_booking=1)/(1-pr(is_booking=1))) = 1.804052 - 0.175075*is_mobile1 - 0.697306*is_package1 - 0.051581*channel262 - 0.380355*channel293 + 0.081007*channel324 + 0.557752*channel355 + 0.181859*channel386 - 10.735697*channel417 - 1.228628*channel448 + 0.805183*channel479 - 0.075253*channel510 - 0.018351*channel541 - 0.088386*srch_adults_cnt + 0.299662*prop_is_branded1 - 0.185572*prop_starrating - 0.184832*distance_band1 + 0.054592*distance_band2 - 0.103447*distance_band3 + 0.152857*distance_band4 + 0.124505*hist_price_band1 + 0.002048*hist_price_band2 + 0.113960*hist_price_band3 - 0.204585*hist_price_band4 -

*0.795968*popularity_band1 - 0.092916*popularity_band2 + 0.274264*popularity_band3 - 0.717964*popularity_band4 - 2.643898*cnt - 0.192581*days

From the model, we can tell that: 1. is_mobile,is_package,srch_adults_cnt,prop_starrating, cnt, living days have negative effect to log of odds ratio of is_booking. 2. varibles: channel,distance_band,hist_price_band,popularity_band have both positive and negative effect to the log of odds ratio of is_booking.

5 Problem 4.

(25 points) We are interested in finding the important predictors of number of non-spinal bone fractures in women with low bone densities, assess their adjusted effect sizes (in direction and magnitude) and use the best Poisson regression model for interpretation and prediction. We want to analyze the FITglm2.txt dataset (available on Blackboard, use the command read.delim("…\FITglm2.txt",sep="")) that contains 6459 observations on 18 variables: alloc - id, ra_age –age in years, frx – indicator for spinal fractures, nosp – indicator for non-spinal fractures, numnosp – number of non-spinal fractures (outcome variable), trt01 – indicator for treatment, p3_weigh – weight over 100 pounds, htotbmd – bone mass density 1, nbmd – bone mass density 2, trialyrs – duration of follow-up, riskcat4 – risk category, tneck – bone density at the neck, bmd25 – indicator for osteoporosis (based on tneck values), hplac – indicator for high placebo dose, htrt – indicator for high dose treatment, lplac – indicator for low placebo dose, ltrt- indicator for lose dose treatment, rtgroup – risk group for falling. Perform all necessary data analysis steps and write a section summarizing the findings.

5.1 Load data and Basic Analysis

In [30]:
```
# load data
fitglm2_data <- read.table("FITglm2.txt",sep="\t",header=T)
dim(fitglm2_data)
# omit rows which has NA
fitglm2_data <- na.omit(fitglm2_data)
dim(fitglm2_data)
head(fitglm2_data)
```

1. 6459 2. 18

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alloc	ra_age	frx	nosp	numnosp	trt01	p3_weigh	htotbmd	nbmd	trialyrs	riskcat4
1	69	1	0	0	0	66.6	0.517	0.464	3.022587	1: HIGH RISK
2	76	0	0	0	1	69.0	0.583	0.518	3.003422	1: HIGH RISK
3	66	0	0	0	1	61.6	0.709	0.602	2.869268	0: LOW RISK
4	72	0	0	0	0	57.6	0.738	0.653	3.236140	0: LOW RISK
5	58	0	0	0	1	75.2	0.690	0.633	2.992471	0: LOW RISK
6	74	0	0	0	1	50.2	0.480	0.485	2.970568	0: LOW RISK

In [31]:
```
str(fitglm2_data)
summary(fitglm2_data)
```

```
'data.frame':        6366 obs. of  18 variables:
 $ alloc   : int  1 2 3 4 5 6 7 8 9 10 ...
```

```
$ ra_age  : int  69 76 66 72 58 74 66 63 62 69 ...
$ frx     : int  1 0 0 0 0 0 0 0 0 0 ...
$ nosp    : int  0 0 0 0 0 0 0 0 0 0 ...
$ numnosp : int  0 0 0 0 0 0 0 0 0 0 ...
$ trt01   : int  0 1 1 0 1 1 0 1 0 0 ...
$ p3_weigh: num  66.6 69 61.6 57.6 75.2 50.2 86.6 54 74 64.8 ...
$ htotbmd : num  0.517 0.583 0.709 0.738 0.69 0.48 0.816 0.538 0.726 0.514 ...
$ nbmd    : num  0.464 0.518 0.602 0.653 0.633 0.485 0.658 0.431 0.584 0.47 ...
$ trialyrs: num  3.02 3 2.87 3.24 2.99 ...
$ riskcat4: Factor w/ 3 levels "","0: LOW RISK",..: 3 3 2 2 2 2 2 2 2 2 ...
$ tneck   : num  -3.28 -2.83 -2.13 -1.71 -1.88 ...
$ bmd25   : int  1 1 0 0 0 1 0 1 0 1 ...
$ hplac   : int  1 0 0 0 0 0 0 0 0 0 ...
$ htrt    : int  0 1 0 0 0 0 0 0 0 0 ...
$ lplac   : int  0 0 0 1 0 0 1 0 1 1 ...
$ ltrt    : int  0 0 1 0 1 1 0 1 0 0 ...
$ rtgroup : Factor w/ 5 levels "","1:HIGH FALL RISK, PLACEBO GROUP",..: 2 3 5 4 5 5 4 5 4 4 .
- attr(*, "na.action")=Class 'omit'  Named int [1:93] 211 283 482 556 732 750 879 949 952 105
 .. ..- attr(*, "names")= chr [1:93] "211" "283" "482" "556" ...


    alloc          ra_age           frx              nosp
Min.   :   1   Min.   :54.00   Min.   :0.0000   Min.   :0.000
1st Qu.:2062   1st Qu.:64.00   1st Qu.:0.0000   1st Qu.:0.000
Median :3800   Median :68.00   Median :0.0000   Median :0.000
Mean   :3784   Mean   :68.12   Mean   :0.1407   Mean   :0.128
3rd Qu.:5583   3rd Qu.:73.00   3rd Qu.:0.0000   3rd Qu.:0.000
Max.   :7230   Max.   :81.00   Max.   :1.0000   Max.   :1.000
   numnosp          trt01           p3_weigh         htotbmd
Min.   :0.0000   Min.   :0.0000   Min.   : 36.30   Min.   :0.3700
1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.: 56.90   1st Qu.:0.6350
Median :0.0000   Median :1.0000   Median : 63.10   Median :0.6980
Mean   :0.1528   Mean   :0.5009   Mean   : 64.54   Mean   :0.6925
3rd Qu.:0.0000   3rd Qu.:1.0000   3rd Qu.: 70.80   3rd Qu.:0.7540
Max.   :4.0000   Max.   :1.0000   Max.   :124.60   Max.   :0.9860
     nbmd           trialyrs               riskcat4        tneck
Min.   :0.3370   Min.   :0.005476              :   0   Min.   :-4.342
1st Qu.:0.5420   1st Qu.:3.014374   0: LOW RISK :5239   1st Qu.:-2.633
Median :0.5900   Median :4.027379   1: HIGH RISK:1127   Median :-2.233
Mean   :0.5842   Mean   :3.795054                       Mean   :-2.282
3rd Qu.:0.6350   3rd Qu.:4.484600                       3rd Qu.:-1.858
Max.   :0.7830   Max.   :4.821355                       Max.   :-0.625
    bmd25            hplac              htrt              lplac
Min.   :0.0000   Min.   :0.00000   Min.   :0.00000   Min.   :0.0000
1st Qu.:0.0000   1st Qu.:0.00000   1st Qu.:0.00000   1st Qu.:0.0000
Median :0.0000   Median :0.00000   Median :0.00000   Median :0.0000
Mean   :0.3227   Mean   :0.08891   Mean   :0.08812   Mean   :0.4101
3rd Qu.:1.0000   3rd Qu.:0.00000   3rd Qu.:0.00000   3rd Qu.:1.0000
```

```
 Max.   :1.0000   Max.   :1.00000   Max.   :1.00000   Max.   :1.0000
     ltrt                                        rtgroup
 Min.   :0.0000                                      :   0
 1st Qu.:0.0000   1:HIGH FALL RISK, PLACEBO GROUP  : 566
 Median :0.0000   2:HIGH FALL RISK, TREATMENT GROUP: 561
 Mean   :0.4128   3:LOW FALL RISK, PLACEBO GROUP   :2611
 3rd Qu.:1.0000   4:LOW FALL RISK, TREATMENT GROUP :2628
 Max.   :1.0000
```

In [38]:
```
# change rg_age into age
fitglm2_data$age <- (117-fitglm2_data$ra_age)
# change indicator variables to factor
fitglm2_data[,c(3,4,6,13,14,15,16,17)]<-lapply(fitglm2_data[,c(3,4,6,13,14,15,16,17)]
# change riskcat4 and rtgroup to numeric factors.
fitglm2_data$riskcat4 <- as.factor(as.numeric(fitglm2_data$riskcat4)-2)
fitglm2_data$rtgroup <- as.factor(as.numeric(fitglm2_data$rtgroup)-2)
# remove alloc and ra_age
fitglm2_data <- fitglm2_data[,-c(1,2)]
dim(fitglm2_data)
summary(fitglm2_data)
```

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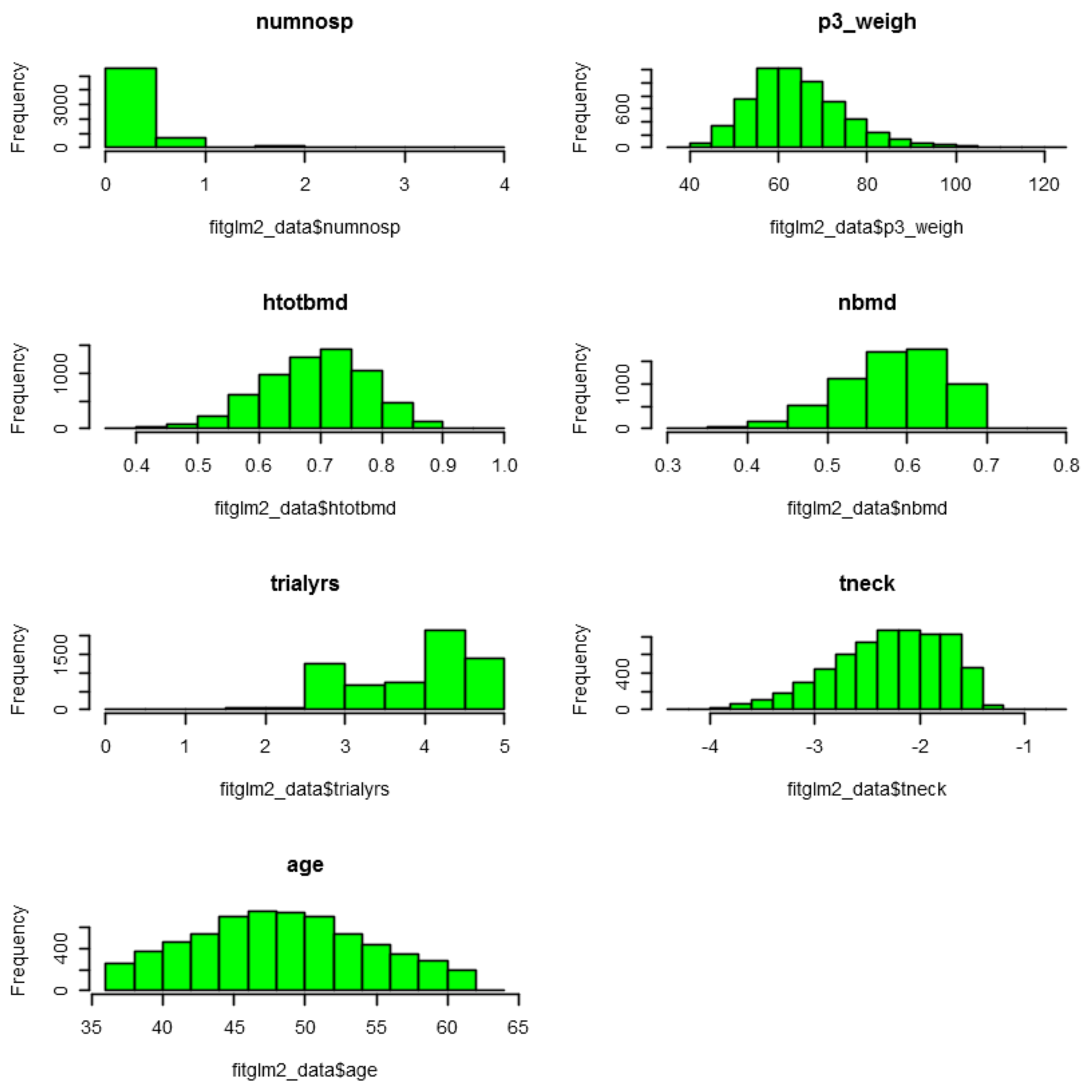
```
 frx      nosp        numnosp        trt01       p3_weigh         htotbmd
 0:5470   0:5551   Min.   :0.0000   0:3177   Min.   : 36.30   Min.   :0.3700
 1: 896   1: 815   1st Qu.:0.0000   1:3189   1st Qu.: 56.90   1st Qu.:0.6350
                   Median :0.0000            Median : 63.10   Median :0.6980
                   Mean   :0.1528            Mean   : 64.54   Mean   :0.6925
                   3rd Qu.:0.0000            3rd Qu.: 70.80   3rd Qu.:0.7540
                   Max.   :4.0000            Max.   :124.60   Max.   :0.9860
      nbmd           trialyrs          riskcat4     tneck          bmd25
 Min.   :0.3370   Min.   :0.005476   0:5239   Min.   :-4.342   0:4312
 1st Qu.:0.5420   1st Qu.:3.014374   1:1127   1st Qu.:-2.633   1:2054
 Median :0.5900   Median :4.027379            Median :-2.233
 Mean   :0.5842   Mean   :3.795054            Mean   :-2.282
 3rd Qu.:0.6350   3rd Qu.:4.484600            3rd Qu.:-1.858
 Max.   :0.7830   Max.   :4.821355            Max.   :-0.625
 hplac    htrt     lplac    ltrt     rtgroup       age
 0:5800   0:5805   0:3755   0:3738   0: 566   Min.   :36.00
 1: 566   1: 561   1:2611   1:2628   1: 561   1st Qu.:44.00
                                     2:2611   Median :49.00
                                     3:2628   Mean   :48.88
                                              3rd Qu.:53.00
                                              Max.   :63.00
```

In [41]:
```
par(mfrow=c(4,2))
hist(fitglm2_data$numnosp,col='green',main='numnosp')
```

```
hist(fitglm2_data$p3_weigh,col='green',main='p3_weigh')
hist(fitglm2_data$htotbmd,col='green',main='htotbmd')
hist(fitglm2_data$nbmd,col='green',main='nbmd')
hist(fitglm2_data$trialyrs,col='green',main='trialyrs')
hist(fitglm2_data$tneck,col='green',main='tneck')
hist(fitglm2_data$age,col='green',main='age')
```



5.2 Split the dataset into training and testing dataset

In [43]:
```
# train dataset: 80%, test dataset: 20%
(n <- dim(fitglm2_data)[1])
trainID <- sample(1:n,n*0.8)
train <- fitglm2_data[trainID,]
```

```
test <- fitglm2_data[-trainID,]
dim(train)
dim(test)
```

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5.3 Build Models

In [111]:
```
poreg.fit1 <- lm(log(numnosp+1)~., data=train)
# use stepAIC to get the best poisson regression model: poreg.best1.
library(MASS)
poreg.best1 <- stepAIC(poreg.fit1,data=train,trace=F)
summary(poreg.best1) #Multiple R-squared:0.9481,Adjusted R-squared:0.9481
# p3_weigh is not significant, remove it
```

```
Call:
lm(formula = log(numnosp + 1) ~ nosp + p3_weigh + bmd25 + hplac,
    data = train)

Residuals:
     Min       1Q   Median       3Q      Max
-0.08201 -0.00254  0.00024  0.00220  0.84796

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.022e-02  5.386e-03  -1.898   0.0578 .
nosp1        7.625e-01  2.513e-03 303.483   <2e-16 ***
p3_weigh     1.273e-04  7.938e-05   1.604   0.1088
bmd251       4.468e-03  1.868e-03   2.392   0.0168 *
hplac1       6.826e-03  2.921e-03   2.337   0.0195 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05974 on 5087 degrees of freedom
Multiple R-squared:  0.9481,Adjusted R-squared:  0.9481
F-statistic: 2.323e+04 on 4 and 5087 DF,  p-value: < 2.2e-16
```

In [117]:
```
# remove p3_weigh from model
poreg.best1 <- lm(formula = log(numnosp + 1) ~ nosp+bmd25+hplac, data = train)
summary(poreg.best1) # Multiple R-squared: 0.9481,Adjusted R-squared:0.948
```

```
Call:
lm(formula = log(numnosp + 1) ~ nosp + bmd25 + hplac, data = train)
```

```
Residuals:
     Min       1Q   Median       3Q      Max 
-0.07846 -0.00184  0.00176  0.00176  0.84852 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.001763   0.001087  -1.622   0.1048    
nosp1        0.762680   0.002511 303.701   <2e-16 ***
bmd251       0.003600   0.001788   2.013   0.0441 *  
hplac1       0.007088   0.002917   2.430   0.0151 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05975 on 5088 degrees of freedom
Multiple R-squared:  0.9481,Adjusted R-squared:  0.948 
F-statistic: 3.096e+04 on 3 and 5088 DF,  p-value: < 2.2e-16
```

```
In [116]: # add two way interaction
          poreg.best2 <- stepAIC(poreg.fit1, ~.^2, data=train,trace=F)
          summary(poreg.best2) #Multiple R-squared:0.949,Adjusted R-squared:0.9489
```

```
Call:
lm(formula = log(numnosp + 1) ~ nosp + trt01 + p3_weigh + bmd25 + 
    hplac + nosp:hplac + nosp:bmd25 + nosp:p3_weigh + trt01:p3_weigh + 
    p3_weigh:bmd25 + p3_weigh:hplac, data = train)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.15106 -0.00191 -0.00040  0.00082  0.85543 

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)    
(Intercept)      0.0127413  0.0090299   1.411   0.1583    
nosp1            0.6786481  0.0165184  41.084  < 2e-16 ***
trt011          -0.0185642  0.0105840  -1.754   0.0795 .  
p3_weigh        -0.0001963  0.0001365  -1.438   0.1506    
bmd251          -0.0221942  0.0112788  -1.968   0.0491 *  
hplac1           0.0249963  0.0170851   1.463   0.1435    
nosp1:hplac1     0.0455534  0.0081987   5.556 2.90e-08 ***
nosp1:bmd251     0.0292358  0.0052909   5.526 3.45e-08 ***
nosp1:p3_weigh   0.0010332  0.0002407   4.293 1.79e-05 ***
trt011:p3_weigh  0.0002907  0.0001622   1.792   0.0732 .  
p3_weigh:bmd251  0.0003605  0.0001807   1.995   0.0461 *  
p3_weigh:hplac1 -0.0003689  0.0002542  -1.451   0.1468    
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05926 on 5080 degrees of freedom
Multiple R-squared:  0.949,Adjusted R-squared:  0.9489
F-statistic:  8594 on 11 and 5080 DF,  p-value: < 2.2e-16
```

```
In [115]: # use regsubsets() to get the best model, which has the best R2 or ajusted-R2.
          library(leaps)
          poreg.best3 <- regsubsets(log(numnosp + 1)~., data=train, nbest=1, method='exhaustive
          summary(poreg.best3)
          plot(poreg.best3, scale='adjr2', main='Adjusted R2')
```

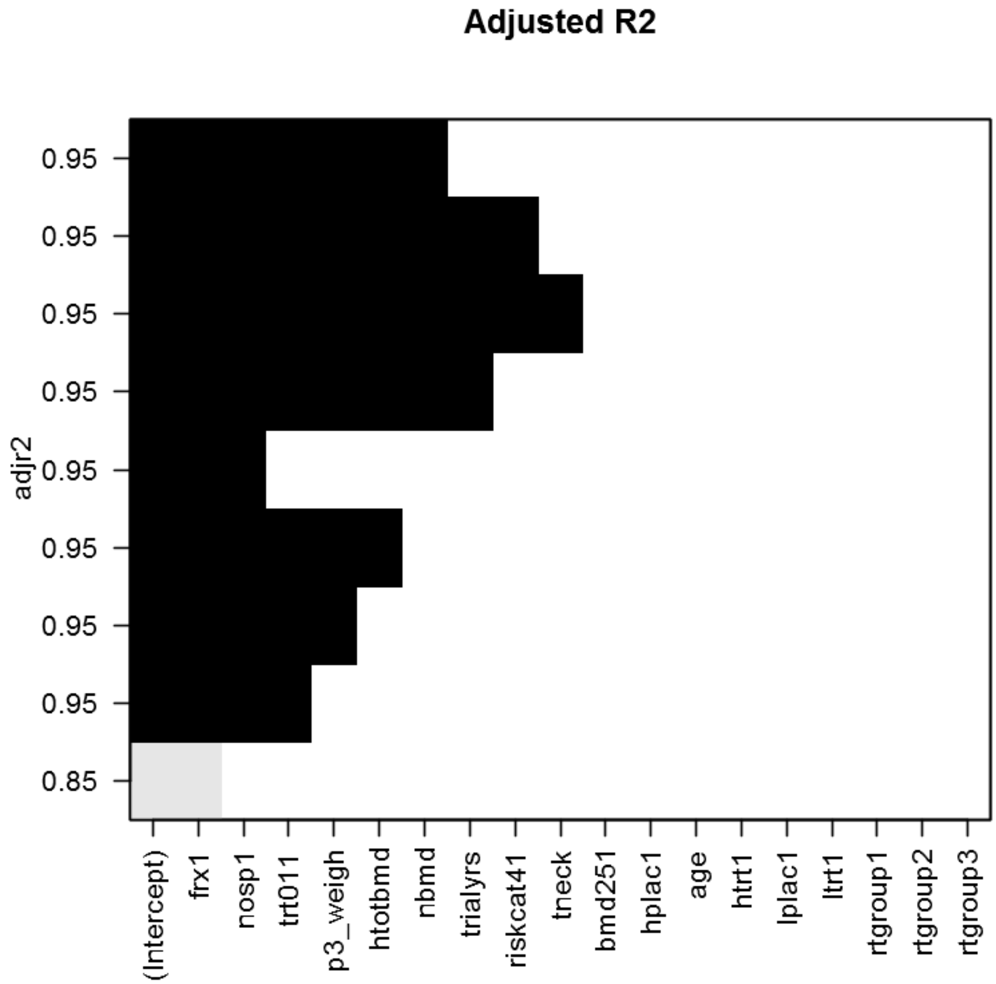
```
Warning message in leaps.setup(x, y, wt = wt, nbest = nbest, nvmax = nvmax, force.in = force.in
"6  linear dependencies found"

Reordering variables and trying again:


Warning message in leaps.exhaustive(a, really.big):
"XHAUST returned error code -999"

Subset selection object
Call: regsubsets.formula(log(numnosp + 1) ~ ., data = train, nbest = 1,
    method = "exhaustive")
18 Variables  (and intercept)
          Forced in Forced out
frx1          FALSE      FALSE
nosp1         FALSE      FALSE
trt011        FALSE      FALSE
p3_weigh      FALSE      FALSE
htotbmd       FALSE      FALSE
nbmd          FALSE      FALSE
trialyrs      FALSE      FALSE
riskcat41     FALSE      FALSE
tneck         FALSE      FALSE
bmd251        FALSE      FALSE
hplac1        FALSE      FALSE
age           FALSE      FALSE
htrt1         FALSE      FALSE
lplac1        FALSE      FALSE
ltrt1         FALSE      FALSE
rtgroup1      FALSE      FALSE
rtgroup2      FALSE      FALSE
rtgroup3      FALSE      FALSE
1 subsets of each size up to 9
Selection Algorithm: exhaustive
```

```
         frx1 nosp1 trt011 p3_weigh htotbmd nbmd trialyrs riskcat41 tneck
1  ( 1 ) "*"  " "   " "    " "      " "     " "  " "      " "       " "
2  ( 1 ) "*"  "*"   " "    " "      " "     " "  " "      " "       " "
3  ( 1 ) "*"  "*"   "*"    " "      " "     " "  " "      " "       " "
4  ( 1 ) "*"  "*"   "*"    "*"      " "     " "  " "      " "       " "
5  ( 1 ) "*"  "*"   "*"    "*"      "*"     " "  " "      " "       " "
6  ( 1 ) "*"  "*"   "*"    "*"      "*"     "*"  " "      " "       " "
7  ( 1 ) "*"  "*"   "*"    "*"      "*"     "*"  "*"      " "       " "
8  ( 1 ) "*"  "*"   "*"    "*"      "*"     "*"  "*"      "*"       " "
9  ( 1 ) "*"  "*"   "*"    "*"      "*"     "*"  "*"      "*"       "*"
         bmd251 hplac1 htrt1 lplac1 ltrt1 rtgroup1 rtgroup2 rtgroup3 age
1  ( 1 ) " "    " "    " "   " "    " "   " "      " "      " "      " "
2  ( 1 ) " "    " "    " "   " "    " "   " "      " "      " "      " "
3  ( 1 ) " "    " "    " "   " "    " "   " "      " "      " "      " "
4  ( 1 ) " "    " "    " "   " "    " "   " "      " "      " "      " "
5  ( 1 ) " "    " "    " "   " "    " "   " "      " "      " "      " "
6  ( 1 ) " "    " "    " "   " "    " "   " "      " "      " "      " "
7  ( 1 ) " "    " "    " "   " "    " "   " "      " "      " "      " "
8  ( 1 ) " "    " "    " "   " "    " "   " "      " "      " "      " "
9  ( 1 ) " "    " "    " "   " "    " "   " "      " "      " "      " "
```



In [137]:
```
# from the plot, we get the poreg.best3
poreg.best3 <- lm(log(numnosp+1)~frx+nosp+trt01+p3_weigh+htotbmd+nbmd, data=train)
summary(poreg.best3)
#Multiple R-squared:0.948,Adjusted R-squared: 0.948
# variables frx, trt01, p3_weigh, htotbmd are not significant, remove them
```

```
Call:
lm(formula = log(numnosp + 1) ~ frx + nosp + trt01 + p3_weigh +
    htotbmd + nbmd, data = train)

Residuals:
     Min       1Q   Median       3Q      Max
```

```
-0.07643 -0.00219 -0.00026  0.00146  0.84822

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  8.864e-03  8.206e-03   1.080   0.2801
frx1        -1.098e-03  7.505e-03  -0.146   0.8837
nosp1        7.638e-01  7.788e-03  98.063   <2e-16 ***
trt011      -1.250e-03  1.677e-03  -0.745   0.4561
p3_weigh     1.309e-04  8.277e-05   1.582   0.1137
htotbmd      1.142e-02  1.548e-02   0.738   0.4607
nbmd        -4.199e-02  2.072e-02  -2.027   0.0427 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05978 on 5085 degrees of freedom
Multiple R-squared:  0.948,Adjusted R-squared:  0.948
F-statistic: 1.546e+04 on 6 and 5085 DF,  p-value: < 2.2e-16
```

```
In [140]: poreg.best3 <- lm(formula = log(numnosp + 1) ~  nosp + nbmd, data = train)
          summary(poreg.best3)
          #Multiple R-squared:0.948,Adjusted R-squared: 0.948
          # nbmd is not significant, remove it
```

```
Call:
lm(formula = log(numnosp + 1) ~ nosp + nbmd, data = train)

Residuals:
     Min       1Q   Median       3Q      Max
-0.07433 -0.00144 -0.00006  0.00106  0.84859

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.012851   0.007749   1.658   0.0973 .
nosp1        0.762830   0.002514 303.425   <2e-16 ***
nbmd        -0.021941   0.013141  -1.670   0.0950 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05978 on 5089 degrees of freedom
Multiple R-squared:  0.948,Adjusted R-squared:  0.948
F-statistic: 4.638e+04 on 2 and 5089 DF,  p-value: < 2.2e-16
```

```
In [141]: poreg.best3 <- lm(formula = log(numnosp + 1) ~  nosp, data = train)
```

```
summary(poreg.best3)
#Multiple R-squared:0.948,Adjusted R-squared: 0.948
```

```
Call:
lm(formula = log(numnosp + 1) ~ nosp, data = train)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.07002  0.00000  0.00000  0.00000  0.84627 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1.979e-15  8.975e-04     0.0        1    
nosp1       7.632e-01  2.506e-03   304.5   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0598 on 5090 degrees of freedom
Multiple R-squared:  0.948,Adjusted R-squared:  0.948 
F-statistic: 9.273e+04 on 1 and 5090 DF,  p-value: < 2.2e-16
```

5.4 Predict and Evaluate

In [142]:
```
# evaluate models
pchisq(deviance(poreg.best1),df.residual(poreg.best1),lower.tail=FALSE) # p-value=1>
pchisq(deviance(poreg.best2),df.residual(poreg.best2),lower.tail=FALSE) # p-value=1>
pchisq(deviance(poreg.best3),df.residual(poreg.best3),lower.tail=FALSE) # p-value=1>


# predict and evaluate
# poreg.best1
# train
poreg.pred1.train <- predict(poreg.best1,type='response',data=train[,-3])
# unscale the predict value
poreg.pred1.train <- round(exp(poreg.pred1.train))-1
(confTable <- table(train$numnosp,poreg.pred1.train))
(acc1.train <- sum(diag(confTable))/sum(confTable)) # train accuracy: 0.97977

# test
poreg.pred1.test <- predict(poreg.best1,type='response',newdata=test[,-3])
# unscale the predict value
poreg.pred1.test <- round(exp(poreg.pred1.test))-1
(confTable <- table(test$numnosp,poreg.pred1.test))
(acc1.test <- sum(diag(confTable))/sum(confTable)) # test accuracy: 0.97252
```

```r
# poreg.best2
# train
poreg.pred2.train <- predict(poreg.best2,type='response',data=train[,-3])
# unscale the predict value
poreg.pred2.train <- round(exp(poreg.pred2.train))-1
(confTable <- table(train$numnosp,poreg.pred2.train))
(acc2.train <- sum(diag(confTable))/sum(confTable)) # train accuracy: 0.97977

# test
poreg.pred2.test <- predict(poreg.best2,type='response',newdata=test[,-3])
# unscale the predict value
poreg.pred2.test <- round(exp(poreg.pred2.test))-1
(confTable <- table(test$numnosp,poreg.pred2.test))
(acc2.test <- sum(diag(confTable))/sum(confTable)) # test accuracy: 0.97252


# poreg.best3
# train
poreg.pred3.train <- predict(poreg.best3,type='response',data=train[,-3])
# unscale the predict value
poreg.pred3.train <- round(exp(poreg.pred3.train))-1
(confTable <- table(train$numnosp,poreg.pred3.train))
(acc3.train <- sum(diag(confTable))/sum(confTable)) # train accuracy: 0.97977

# test
poreg.pred3.test <- predict(poreg.best3,type='response',newdata=test[,-3])
# unscale the predict value
poreg.pred3.test <- round(exp(poreg.pred3.test))-1
(confTable <- table(test$numnosp,poreg.pred3.test))
(acc3.test <- sum(diag(confTable))/sum(confTable)) # test accuracy: 0.97252
```

1
1
1

```
 poreg.pred1.train
     0    1
0 4439    0
1    0  550
2    0   90
3    0   12
4    0    1
```

0.979772191673213

```
 poreg.pred1.test
     0    1
```

```
0 1112    0
1    0  127
2    0   30
3    0    4
4    0    1
```

0.972527472527473

```
 poreg.pred2.train
     0    1
0 4439    0
1    0  550
2    0   90
3    0   12
4    0    1
```

0.979772191673213

```
 poreg.pred2.test
     0    1
0 1112    0
1    0  127
2    0   30
3    0    4
4    0    1
```

0.972527472527473

```
 poreg.pred3.train
     0    1
0 4439    0
1    0  550
2    0   90
3    0   12
4    0    1
```

0.979772191673213

```
 poreg.pred3.test
     0    1
0 1112    0
1    0  127
2    0   30
3    0    4
4    0    1
```

0.972527472527473

5.5 Interpretation

From the results, we can see all of these three best models get the same R-squared, 0.948, which is very good. After predict, they get the same train and test accuracy, 0.97977 and 0.9725, very high. As they get the same accuracy, I would choose poreg.best3 as the best model. Because it is the simplest.

The best modesl is: log(numnosp + 1) = 1.979e-15 + 0.7632*nosp

From the equation, if the bone is non-spinal bone, the number of non-spinal bone fractures in women with low bone densities will be 1, otherwise 0.

```
In [146]: exp(1.979e-15 + 0.7632)-1
```

1.14512966406385