

The Art of Multiprocessor Programming  
Solutions to Exercises  
Chapter 9

July 14, 2009

**Exercise 100.** Describe how to modify each of the linked list algorithms if object hash codes are not guaranteed to be unique.

**Solution** The simplest solution is to keep entries with the same hash codes next to one another in the list. Replace every line of the form

```
while (curr.key < key) {
```

with

```
    while (curr.key < key || (curr.key == key && !curr.item.equals(item))) {
```

(Note that if items are naturally ordered, say by implementing the `Comparable` interface, then we wouldn't need to sort them by hash code.)

**Exercise 101.** Explain why the fine-grained locking algorithm is not subject to deadlock.

**Solution** We can describe the synchronization state of a lock-based data structure by a directed *waits-for* graph: each node represents a thread, and there is an edge from  $B$  to  $A$  if  $B$  is waiting for a lock held by  $A$ . There is a deadlock if and only if the waits-for graph has a cycle.

The fine-grained locking algorithm is not subject to deadlock because locks can be ordered by their position in the list. When a thread tries to acquire one lock, it holds only locks earlier in the list. If there were a cycle, then some thread would have to be holding a later lock and requesting an earlier one, which is impossible.

**Exercise 102.** Explain why the fine-grained list's `add()` method is linearizable.

**Solution** As usual, there are two cases to consider: successful and unsuccessful calls.

If the call is successful (returns *true*), then the linearization point occurs at Line ??, when it sets the `pred` node's `next` field to point to its newly-allocated node (while `pred` is locked).

If the call is unsuccessful, then the linearization point occurs at Line ?? or Line ?? when it locks a `curr` node in the list with the sought-after key.

**Exercise 103.** Explain why the optimistic and lazy locking algorithms are not subject to deadlock.

**Solution** In the both algorithm, method calls acquire locks in the order they appear in the list: earlier locks before later locks. (See discussion of waits-for graph above.)

```

1
2  public boolean contains(T item) {
3      Node pred = null, curr = null;
4      int key = item.hashCode();
5      head.lock();
6      try {
7          pred = head;
8          curr = pred.next;
9          curr.lock();
10         try {
11             while (curr.key < key) {
12                 pred.unlock();
13                 pred = curr;
14                 curr = curr.next;
15                 curr.lock();
16             }
17             return (curr.key == key);
18         } finally {
19             curr.unlock();
20         }
21     } finally {
22         pred.unlock();
23     }
24 }

```

Figure 1: The `FineList` class: `contains()` method

**Exercise 104.** Show a scenario in the optimistic algorithm where a thread is forever attempting to delete a node.

*Hint:* since we assume that all the individual node locks are starvation-free, the livelock is not on any individual lock, and a bad execution must repeatedly add and remove nodes from the list.

**Solution** The `remove()` method could starve if each call to `validate()` returns *false* because some other thread had deleted `pred` or `curr`, or inserted a node between them.

**Exercise 105.** Provide the code for the `contains()` method missing from the fine-grained algorithm. Explain why your implementation is correct.

**Solution** See Figure 1. This method follows the same hand-over-hand locking strategy as the other `FineList` methods, but it returns without modifying the list.

**Exercise 106.** Is the optimistic list implementation still correct if we switch the order in which `add()` locks the `pred` and `curr` entries?

**Solution** No, because it can deadlock with a `remove()` call that could lock the same nodes in the opposite order.

**Exercise 107.** Show that in the optimistic list algorithm, if `predA` is not *null*, then `tail` is reachable from `predA`, even if `predA` itself is not reachable.

**Solution** It is easier to show that once `tail` is reachable from any node, it remains reachable. Initially, `tail` is reachable from every node in the list. There are only two places the list is modified. Both `add()` and `remove()` update their `pred` nodes. If `tail` was reachable from a node without passing through `pred`, then it remains reachable after modifying `pred`. It remains to check that if `tail` was reachable from a node through `pred`, then it remains reachable after each modification.

- The `add()` method (Figure 2) prepares a new node by setting its `next` field to a node in the list Line 16, so `tail` is reachable from the new node. The new node is then spliced into the list by updating `pred`'s `next` field (Line 17). If `tail` was reachable from a node through `pred`, then it is still reachable.
- The `remove()` method (Figure 3) modifies the list at Line 38, it sets `pred.next` to `pred.next.next`, where by assumption `tail` is reachable through `pred.next`. It follows that if `tail` was reachable from a node through `pred`, then it is still reachable.

Each time the method assigns to `predA`, it assigns it a node reachable from

**Exercise 108.** Show that in the optimistic algorithm, the `add()` method needs to lock only `pred`.

**Solution** We examine the three possible ways a method call can overlap an `add()` call.

- Two concurrent `add()` calls: For two concurrent adds to cause any problems, they must be adding an entry at the same position. Since both threads must lock the predecessor, one thread will have to wait for the other. If one thread is trying to insert an element between entries *a* and *b* and another is trying to insert one between *b* and *c*, then there is no conflict since *b* is only modified by the thread inserting between *b* and *c*. If a thread is trying to insert after a value that is currently being added by another thread, then it must have validated it successfully in order to succeed, and so the insertion must have completed.

```

1  public boolean add(T item) {
2      int key = item.hashCode();
3      while (true) {
4          Node pred = head;
5          Node curr = pred.next;
6          while (curr.key <= key) {
7              pred = curr; curr = curr.next;
8          }
9          pred.lock(); curr.lock();
10         try {
11             if (validate(pred, curr)) {
12                 if (curr.key == key) {
13                     return false;
14                 } else {
15                     Node node = new Node(item);
16                     node.next = curr;
17                     pred.next = node;
18                     return true;
19                 }
20             }
21         } finally {
22             pred.unlock(); curr.unlock();
23         }
24     }
25 }

```

Figure 2: The OptimisticList class: the add().

```

26 public boolean remove(T item) {
27     int key = item.hashCode();
28     while (true) {
29         Node pred = head;
30         Node curr = pred.next;
31         while (curr.key < key) {
32             pred = curr; curr = curr.next;
33         }
34         pred.lock(); curr.lock();
35         try {
36             if (validate(pred, curr)) {
37                 if (curr.key == key) {
38                     pred.next = curr.next;
39                     return true;
40                 } else {
41                     return false;
42                 }
43             }
44         } finally {
45             pred.unlock(); curr.unlock();
46         }
47     }
48 }

```

Figure 3: The OptimisticList class: the remove() method.

- `remove()` call overlapping with `mAnadd` call: First, let's see what happens when the node marked as `curr` in the `add()` method is being deleted. Then one method would lock the other out since both try to acquire the same `pred` lock. If the node marked as `pred` is the one being deleted, then we are saved again by the locks since the node is being locked as the `pred` in `add()` and as the `currEntry` in `remove()`.
- `contains()` overlaps `add()`: The `contains()` method does not cause any problems since it simply goes through the list and checks if it finds an element in the list. By not locking `curr` in `add()`, `contains()` is left completely as it was.

**Exercise 109.** In the optimistic algorithm, the `contains()` method locks two entries before deciding whether a key is present. Suppose, instead, it locks no entries, returning *true* if it observes the value, and *false* otherwise.

Either explain why this alternative is linearizable, or give a counterexample showing it is not.

**Solution** It is linearizable. Suppose the `contains(x)` call returns *true*. Then there must have been some instant during that call when *x*'s node was reachable from `head`. Linearize the call at any such moment.

Suppose the `contains(x)` call returns *false*. Then there must have been some instant during that call when *x*'s node was not reachable from `head`. Linearize the call at any such moment.

**Exercise 110.** Would the lazy algorithm still work if we marked a node as removed simply by setting its next field to *null*? Why or why not? What about the lock-free algorithm?

**Solution** The lazy algorithm would not work. The problem is that by removing the node by simply setting its next field to null we lose information - specifically, the rest of the list! This could be modified to work if we make a couple of changes. First, we must save a pointer to the rest of the list after setting the removed node's next field to null. We also need to have a special sentinel node to mark the end of the list instead of using the null pointer. Note that this "fix" is pointless since we aren't really setting the next field to null, we are just copying it to another location.

Not only would the `LockFreeList` algorithm not work if we remove a node by setting its next field to null, our "solution" above would also not work because of the `compareAndSet()` operation. We would need to lock the tail of the list to remedy this problem, but then the list would no longer be lock-free.

**Exercise 111.** In the lazy algorithm, can `predA` ever be unreachable? Justify your answer.

**Solution** Yes, there is nothing to prevent one thread from unlinking `predA` while another thread is scanning through the list.

**Exercise 112.** Your new employee claims that the lazy list's validation method (Fig. ??) can be simplified by dropping the check that `pred.next` is equal to `curr`. After all, the code always sets `pred` to the old value of `curr`, and before `pred.next` can be changed, the new value of `curr` must be marked, causing the validation to fail. Explain the error in this reasoning.

**Solution** The problem is that another thread could insert a new node between `pred` and `curr`. An `add()` or `remove()` call using the simplified validation would remove all the nodes between `pred` and `curr`.

**Exercise 113.** Can you modify the lazy algorithm's `remove()` so it locks only one node?

**Solution** It is not necessary to lock `curr`.

Both `add()` and `remove()` lock `pred` before accessing `curr`. Once `pred` is locked, by either method, then no other `add()` or `remove()` can access that node. The `contains()` method is lock-free, and works whether or not `curr` is locked.

**Exercise 114.** In the lock-free algorithm, argue the benefits and drawbacks of having the `contains()` method help in the cleanup of logically removed entries.

**Solution** The principal disadvantage is that the `contains()` method is usually the most frequently called, and calling `compareAndSet()` would slow it down considerably.

The principal advantage is that doing so might speed up the other methods, which would have to do fewer physical deletions.

**Exercise 115.** In the lock-free algorithm, if an `add()` method call fails because `pred` does not point to `curr`, but `pred` is not marked, do we need to traverse the list again from `head` in order to attempt to complete the call?

**Solution** Since any unmarked reachable node is reachable, it would be correct to resume the traversal from `pred`.

**Exercise 116.** Would the `contains()` method of the lazy and lock-free algorithms still be correct if logically removed entries were not guaranteed to be sorted?

**Solution** No, because the `contains()` method might stop searching prematurely if it found a logically deleted node with a higher key.

**Exercise 117.** The `add()` method of the lock-free algorithm never finds a marked node with the same key. Can one modify the algorithm so that it will simply insert its new added object into the existing marked node with same key if such a node exists in the list, thus saving the need to insert a new node?

**Solution** No, because the node might be physically removed while the new value is being installed.

**Exercise 118.** Explain why this cannot happen in the `LockFreeList` algorithm. A node with item  $x$  is logically but not yet physically removed by some thread, then the same item  $x$  is added into the list by another thread, and finally a `contains()` call by a third thread traverses the list, finding the logically removed node, and returning *false*, even though the linearization order of the `remove()` and `add()` implies that  $x$  is in the set.

**Solution** The thread adding  $x$  the second time would physically remove the marked node containing  $x$ .