



**FIGURE 12.17** The inductive proof that a MERGER [8] network correctly merges two width-4 sequences  $x$  and  $x'$  that have the step property into a single width-8 sequence  $y$  that has the step property. The odd and even width-2 subsequences of  $x$  and  $x'$  all have the step property. Moreover, the difference in the number of tokens between the even sequence from one and the odd sequence from the other is at most 1 (in this example, 11 and 12 tokens, respectively). It follows from the induction hypothesis that the outputs  $z$  and  $z'$  of the two MERGER [4] networks have the step property, with at most 1 extra token in one of them. This extra token must fall on a specific numbered wire (wire 3 in this case) leading into the same balancer. In this figure, these tokens are darkened. They are passed to the southern-most balancer, and the extra token is pushed north, ensuring the final output has the step property.