problem 1:

①

$$\left.\begin{aligned} f_x(a,b)&=0 \\ f_y(a,b)&=0 \end{aligned}\right\} \nabla f(a,b) = \langle 0,0\rangle$$

ⓐ $D_u f(a,b) = \nabla f(a,b) \cdot u$

$$= \langle 0,0\rangle \cdot \langle u_1, u_2\rangle = 0$$

ⓑ show $D_u{}^2 f = D_u(D_u f) = f_{xx} u_1{}^2 + 2 f_{xy} u_1 u_2 + f_{yy} u_2{}^2$

$D_u f = \langle f_x, f_y\rangle \cdot \langle u_1, u_2\rangle = f_x u_1 + f_y u_2$

$D_u{}^2 f = (f_x u_1 + f_y u_2)^2 = f_{xx} u_1{}^2 + 2 f_{xy} u_1 u_2 + f_{yy} u_2{}^2$

②

$D = f_{xx} f_{yy} - f^2_{xy}$ show $D_u{}^2 f(a,b) > 0$ if $f_{xx}(a,b) > 0$ & $D(a,b) > 0$

$D_u{}^2 f = f_{xx} u_1{}^2 + 2 f_{xy} u_1 u_2 + f_{yy} u_2{}^2$

$f_{xx}\left(u_1{}^2 + \frac{2f_{xy}}{f_{xx}} u_1 u_2 + \frac{f_{yy}}{f_{xx}} u_2{}^2\right)$

$f_{xx}\left(\left(u_1 + \frac{f_{xy}}{f_{xx}} u_2\right)^2 + \left(-\frac{(f_{xy})^2}{f_{xx}{}^2} u_2{}^2 + \frac{f_{yy}}{f_{xx}} u_2{}^2\right)\right)$

$f_{xx}\left(\left(u_1 + \frac{f_{xy}}{f_{xx}} u_2\right)^2 + \left(\frac{f_{yy} f_{xx} - (f_{xy})^2}{(f_{xx})^2}\right) u_2{}^2\right)$

$\geq$ than 0 because it is squared

$$\left(\frac{f_{yy}f_{xx} - (f_{xy})^2}{(f_{xx})^2} \right) u_2^2 \Big)$$

>0 we know that $f_{yy}f_{xx} - (f_{xy})^2 = D$ and $D(a,b) > 0$

$f(xx)^2$ is squared so ≥ 0

we know it is just >0 not ≥ 0 because for $\frac{f_{yy}f_{xx} - f(xy)^2}{(f_{xx})^2} u_2^2$, u_2 has to $=0$, then for $(u_1 + \frac{f_{xy}}{f_{xx}} u_2)^2 = 0$ $f_{xx}(u_1)^2 = 0$, u_1 must also be 0, if $\vec{u}$ is a unit vector, u_1 & u_2 cannot possibly both be 0.

$\therefore$ given $f_{xx}(a,b) > 0$ and $D(a,b) > 0$ $D_{u^2} f(a,b) > 0$