

Application of Union Find in Undirected Graph Algorithm Problems

<https://github.com/Neo-Hao/union-find>

Union Find / Disjoint Set

Union-find is a data structure that keeps track of a set of elements partitioned into a number of disjoint sets. It typically has three operations:

- ***makeSet***: Create a set using the only one element that is given
- ***findSet***: Determine which set a particular element is in. *findSet* typically returns an item from this set that serves as the "representative"; By comparing the returned values of two *findSet* operations on two elements, one can determine whether the two elements are in the same set.
- ***union***: Join two sets into a single set.

Time Complexity:

- Naïve Approach: $O(n)$ for per *findSet* and *union* operation.
- Optimized approach: Amortized time $O(\alpha(n))$ per operation, where $\alpha(n)$ is less than 5 for all remotely practical values of n .

Naïve Approach:

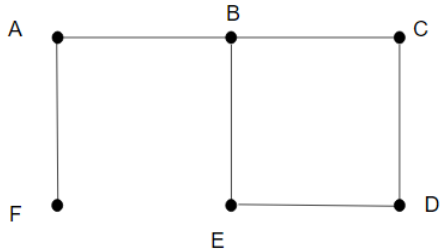
<pre> makeSet(v): n = Node(v) n.parent = n findSet(n): if n.parent == n: return n return findSet(n.parent) union(n1, n2): i = findSet(n1) j = findSet(n2) if i == j: return else: j.parent = i </pre>	<pre> class Node(object): def __init__(self, v): self.v = v self.parent = None class Union(object): def __init__(self): self.table = {} # makeSet method def makeSet(self, v): # input: vertex node = Node(v) node.parent = node self.table[v] = node # union method def union(self, v1, v2): # input: vertex1, vertex2 i = self.findSet(v1) j = self.findSet(v2) if i == j: return j.parent = i # findSet method def findSet(self, v): # input vertex n = self.table[v] return self.findSetHelper(n) def findSetHelper(self, n): # input node if n.parent == n: return n return self.findSetHelper(n.parent) </pre>
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Optimized Approach:

<pre> makeSet(v): n = Node(v) n.parent = n union(n1, n2): i = findSet(n1) j = findSet(n2) if i == j: return if i.rank > j.rank: j.parent = i elif i.rank < j.rank: i.parent = j else: j.parent = i i.rank += 1 findSet(n): if n.parent != n: n.parent = findSet(n.parent) return n.parent </pre>	<pre> class Node(object): def __init__(self, v): self.v = v self.parent = None self.rank = 0 class Union(object): def __init__(self): self.table = {} # makeSet method def makeSet(self, val): node = Node(val) node.parent = node self.table[val] = node # union method def union(self, v1, v2): i = self.findSet(v1) j = self.findSet(v2) if i == j: return if i.rank > j.rank: j.parent = i elif i.rank < j.rank: i.parent = j else: j.parent = i i.rank += 1 # findSet method def findSet(self, v): node = self.table[v] return self.findSetHelper(node) def findSetHelper(self, n): # representative itself if n == n.parent: return n n.parent = self.findSetHelper(n.parent) return n.parent </pre>
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Cycle Detection

Cycle detection refers to the algorithmic problem of finding a cycle in a sequence of iterated function values.



Implementations:

```
cycleDetect(G):
    foreach v ∈ G.V:
        MAKE-SET(v)
    foreach (u, v) in G.E:
        if FIND-SET(u) ≠ FIND-SET(v):
            UNION(u, v)
        else:
            return True
    return False
```

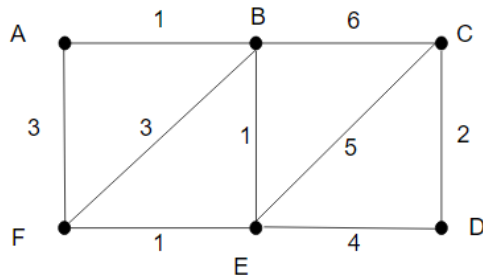
```
def cycleDetection(edges):
    union = Union()
    # step 1: makeSet
    for u, v in edges:
        union.makeSet(u)
        union.makeSet(v)
    # step 2: traverse the edges
    for u, v in edges:
        if union.findSet(u) ==
           union.findSet(v):
            return True
        else:
            union.union(u, v)
    return False
```

Time Complexity: $O(V)$

Kruskal's algorithm

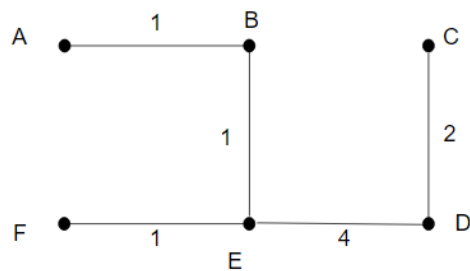
Weighted Graph:

A weighted graph refers to an edge-weighted graph, where edges have weights or values.



Minimum Spanning Tree:

A minimum spanning tree is a subset of the edges of a connected, edge-weighted graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight.



Implementation:

```

KRUSKAL(G):
    result = ∅
    foreach v ∈ G.V:
        MAKE-SET(v)
    foreach (u, v) in G.E ordered by
        increasing order of weight(u, v):
        if FIND-SET(u) ≠ FIND-SET(v):
            result = result ∪ {(u, v)}
            UNION(u, v)
    return result

```

```

def kruskal(weightedEdges):
    union = Union()
    result = []
    # step 1
    for u, v in weightedEdges:
        union.makeSet(v)
    # step 2
    weightedEdges.sort()
    # step 3
    for w, u, v in weightedEdges:
        if union.findSet(u) !=
           union.findSet(v):
            result.append((u, v))
            union.union(u, v)

    return result

```

Time Complexity: $O(E \log E)$

Q1. Number of Connected Components in an Undirected Graph

Given n nodes labeled from 0 to $n - 1$ and a list of undirected edges (each edge is a pair of nodes), write a function to find the number of connected components in an undirected graph.

Example 1:

```

    0          3
    |          |
    1 --- 2    4
    
```

Given $n = 5$ and $\text{edges} = [[0, 1], [1, 2], [3, 4]]$, return 2.

Example 2:

```

    0          4
    |          |
    1 --- 2 --- 3
    
```

Given $n = 5$ and $\text{edges} = [[0, 1], [1, 2], [2, 3], [3, 4]]$, return 1.

Requirement:

```

def countComponents(self, n, edges):
    """
    :type n: int
    :type edges: List[List[int]]
    :rtype: int
    """
    
```

Q2. Number of Islands

A 2d grid map of m rows and n columns is initially filled with water. We may perform an *addLand* operation which turns the water at position (row, col) into a land. Given a list of positions to operate, count the number of islands after each *addLand* operation. An island is surrounded by water and is formed by connecting adjacent lands horizontally or vertically. You may assume all four edges of the grid are all surrounded by water.

Example:

Given $m = 3$, $n = 3$, positions = $[[0,0], [0,1], [1,2], [2,1]]$.

Initially, the 2d grid grid is filled with water. (Assume 0 represents water and 1 represents land).

```
0 0 0
0 0 0
0 0 0
```

Operation #1: addLand(0, 0) turns the water at grid[0][0] into a land.

```
1 0 0
0 0 0   Number of islands = 1
0 0 0
```

Operation #2: addLand(0, 1) turns the water at grid[0][1] into a land.

```
1 1 0
0 0 0   Number of islands = 1
0 0 0
```

Operation #3: addLand(1, 2) turns the water at grid[1][2] into a land.

```
1 1 0
0 0 1   Number of islands = 2
0 0 0
```

We return the result as an array: $[1, 1, 2]$.

Requirement:

```
def numIslands2(self, m, n, positions):
    """
    :type m: int
    :type n: int
    :type positions: List[List[int]]
    :rtype: List[int]
    """
```

Q3. Graph Valid Tree

Given n nodes labeled from 0 to $n - 1$ and a list of undirected edges (each edge is a pair of nodes), write a function to check whether these edges make up a valid tree.

For example:

Given $n = 5$ and edges = $[[0, 1], [0, 2], [0, 3], [1, 4]]$, return true.

Given $n = 5$ and edges = $[[0, 1], [1, 2], [2, 3], [1, 3], [1, 4]]$, return false.

Requirement:

```
def validTree(self, n, edges):  
    """  
        :type n: int  
        :type edges: List[List[int]]  
        :rtype: bool  
    """
```