Model Predictive Control for Spacecraft Rendezvous and Docking

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Abstract

Spacecraft rendezvous and docking (RVD) demands precise 6-DOF control of translational and rotational dynamics under stringent accuracy requirements. Traditional controllers such as PD or LQR perform adequately in simplified scenarios but degrade in coupled dynamics and when constraints must be explicitly enforced. This work implements and evaluates a Model Predictive Control (MPC) framework for autonomous spacecraft RVD using a nonlinear 6-DOF plant model with coupled dynamics. A baseline LQR controller is included for benchmarking. Results show MPC achieves stable convergence in both coupled and uncoupled cases, with positional errors under 0.05 m and attitude errors within 0.024 rad RMS. These findings highlight the advantages of MPC in handling constraints and complex dynamics.

1 Introduction

High-precision RVD requires accurate trajectory and attitude tracking in six degrees of freedom. Control must remain stable under coupling between translational and rotational states while meeting strict precision demands. While traditional controllers such as LQR offer elegant solutions for linearized systems, they lack the ability to enforce constraints or optimize over a prediction horizon. Model Predictive Control (MPC) addresses these limitations, making it a strong candidate for spacecraft guidance and docking.

2 Methodology

The project follows a bottom-up modelling and simulation approach:

- **Nonlinear Plant Modelling**: The chaser spacecraft is represented by a nonlinear, coupled 6-DOF model. Translational motion is expressed in Cartesian coordinates, and rotational motion uses Euler angles. Inputs include forces and torques with thruster offsets, capturing real-world translation–rotation coupling.

$$\dot{x} = f(x,u),\ x \in R^{12},\ u \in R^{6}$$

where $x=[p,v,\phi,\theta,\psi,\omega]^T$ *represents position, velocity, attitude, and angular rates.*

- **Linearization**: At each time-step, the nonlinear plant is linearized using numerical Jacobians to obtain discrete-time matrices. This allows local dynamics to be captured while preserving coupling effects.

$$A = \partial x/\partial f|_{(x0,u0)}\ ,\quad B = \partial u/\partial f|_{(x0,u0)}$$

- **Control Design**:
    - **MPC**: A receding-horizon controller implemented in Python with *cvxpy and OSQP* minimizes quadratic state and input costs. Weighting matrices Q and R are tuned via Bryson's rule to balance translation, rotation, and angular rates.

      The receding-horizon optimization problem is:

      $$\min_{\{u(0),\ldots u(N-1)\}} \sum_{k=0}^{N-1} (x_k - x_{ref})^T Q (x_k - x_{ref}) + u_k^T R u_k$$

      Subject to:

      $$x_{k+1} = Ax_k + Bu_k$$

    - **LQR Benchmark**: A standard Linear Quadratic Regulator is implemented for comparison against MPC performance.

      The optimal gain matrix is:

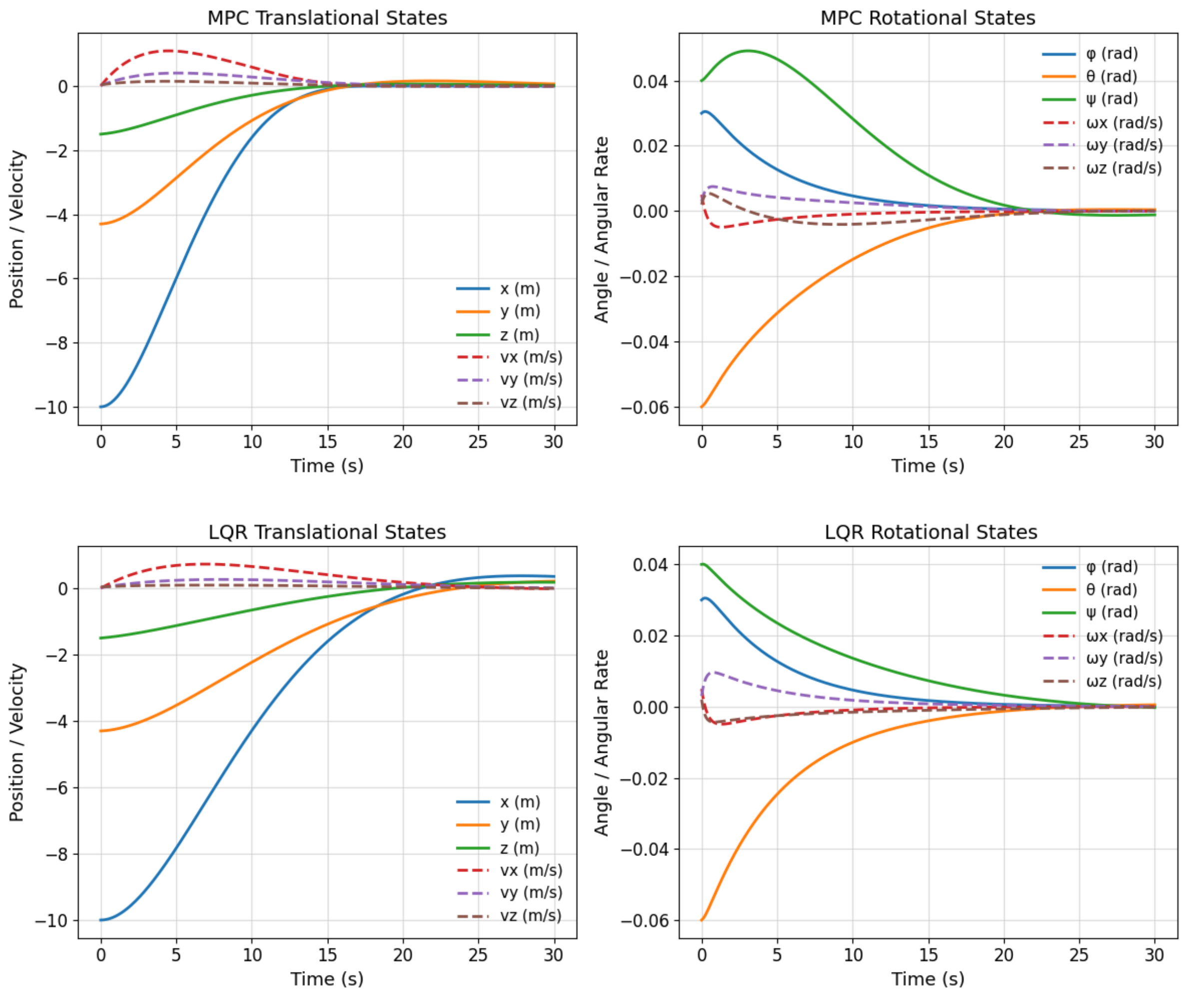
      $$K = R^{-1} B^T P$$

      where P solves the discrete Riccati equation.

- **Simulation Framework**: The plant, linearized model, and controllers are integrated into a closed-loop simulation. At each step, the linear model is updated with the current state, MPC computes the optimal input, and the nonlinear plant is propagated. Monte Carlo runs are used to test robustness across uncertain initial states and actuator limits.

3 Results

- **MPCvsLQR**: MPC consistently demonstrates tighter error bounds than LQR, particularly under coupled dynamics.
- **Convergence**: Positional errors remain below **0.05 m**; rotational errors remain within safe limits, with RMS pitch/yaw error ≈ **0.024 rad**.
- **Stability**: Steady-state rotational error is negligible (~0.002 rad).
- **Visualization**: Trajectory plots and convergence graphs confirm MPC achieves smoother and more precise performance than LQR benchmarks across uncoupled and coupled cases.



MPC vs LQR comparison for translations and rotations

In the translational states, both controllers drive the position and velocity to zero, but MPC exhibits faster initial response with slightly higher overshoot, particularly in the *x and y* directions, settling around 15 seconds, whereas LQR demonstrates a slower convergence. Similarly, in the rotational states, MPC achieves quicker damping of angular deviations and rates, reaching near-zero values slightly earlier than LQR.

4 Conclusion

This study demonstrates that MPC provides superior performance to traditional controllers such as LQR in spacecraft RVD, particularly in handling 6-DOF coupled dynamics and enforcing state constraints. The framework provides a foundation for more advanced control formulations. Future work will extend this framework to incorporate delay-tolerant strategies to address communication and actuation latencies.