Documentation

Libraries Used

1. **pandas:** Handles Table data (Loading, Saving, Modifying tables)
2. **numpy:** Handles Matrix data (Define matrices and perform operations with them)
3. **matplotlib.pyplot:** Plot Graph (Scatter, Surface, curve, etc.)

Data Preprocessing

1. **Loading the dataset into a DataFrame:** Using the pandas library we load the given dataset into a pandas DataFrame.
2. **Normalizing the feature variable:** We normalize the feature variables by utilizing the formula: $X' = (X - \mu) / \sigma$ where μ represents the mean of the feature column, and σ represents the standard deviation of the feature column.
3. **Filling Null Values:** *–Only in Part B–* We predict the null values in cells using the mean of the existing values of the corresponding feature.
4. **Shuffle split the dataset into training and testing sets:** We shuffle the dataset and split the dataset into training and testing sets; using 80% data for training and 20% for testing.

Polynomial Regression

Polynomial Transformation and Dummy Ones

Using an input degree, we transform the input data by adding higher degree terms. We add a column of 1s at the start of the dataset to facilitate matrix operations.

Error Function

We use Mean squared error as our error metric and create a function for later use.

Batch Gradient Descent

We use matrix operations and the following formula to perform batch gradient descent:

```
for iteration in range(max_iterations):
  Y_pred = X @ W
  gradient = X.T @ (Y_pred – Y)
  W –= (learning_rate/n)*gradient
```

where W is our weight vector (includes Wo – bias term) and is initialized to 0. @ denotes matrix multiplication. X is the matrix of training points and features; while Y is the true target value of those points.

Stochastic Gradient Descent

We use matrix operations and the following formula to perform stochastic gradient descent:

```
for iteration in range(max_iterations):
  nextrow = random.randint(0, len(train)–1)
  x_i = X[nextrow]
  y_i = Y[nextrow]
  y_pred = x_i @ W
  gradient = x_i * (y_pred – y_i)
  W –= (learning_rate/n) * gradient
```

where W is our weight vector (includes Wo – bias term) and is initialized to 0. @ denotes matrix multiplication. nextrow is a randomly selected row from our dataset, containing a combination of features and their target value. X is the matrix of training points and features; while Y is the true target value of those points. x_i is the feature vector of nextrow and y_i is its target value.

Regularized Linear Regression

To implement the gradient descent algorithms; we must first differentiate both loss functions (to find the gradient) and then write them in matrix form (for faster operations). Differentiating the equation (w.r.t. W) we get:

SUM(tn – wT * Xn) + lambda * (0.5 * q) * SUM(|wj|^(q–1))

as a matrix equation:

Y – (W.T * X) + lambda * 0.5 * q * (W**(q–1)) (for q = 2 or 4)

Y – (W.T * X) + lambda * 0.5 * q * (abs(W)**(–0.5)) (for q = 0.5)

For q = 1:

SUM(tn – wT * Xn) + lambda * 0.5 * SUM(sign(wj))

as a matrix equation:

Y – (W.T * X) + lambda * 0.5 * (sign(W))

To modify our above fuctions to use these, we just need to change the gradient as follows:

Batch Gradient Descent

if q = 0.0: gradient = X.T @ (Y_pred – Y)

q = 0.5: gradient = X.T @ (Y_pred – Y) + (lmbda * 0.5 * q) * (np.abs(W) ** (–0.5))

q = 1.0: gradient = X.T @ (Y_pred – Y) + (lmbda * 0.5) * np.sign(W)

else: gradient = X.T @ (Y_pred – Y) + (lmbda * 0.5 * q) * (W ** (q–1))

Stochastic Gradient Descent

if q = 0.0: gradient = x_i * (y_pred – y_i)

q = 0.5: gradient = x_i * (y_pred – y_i) + (lmbda * 0.5 * q) * (np.abs(W) ** (–0.5))

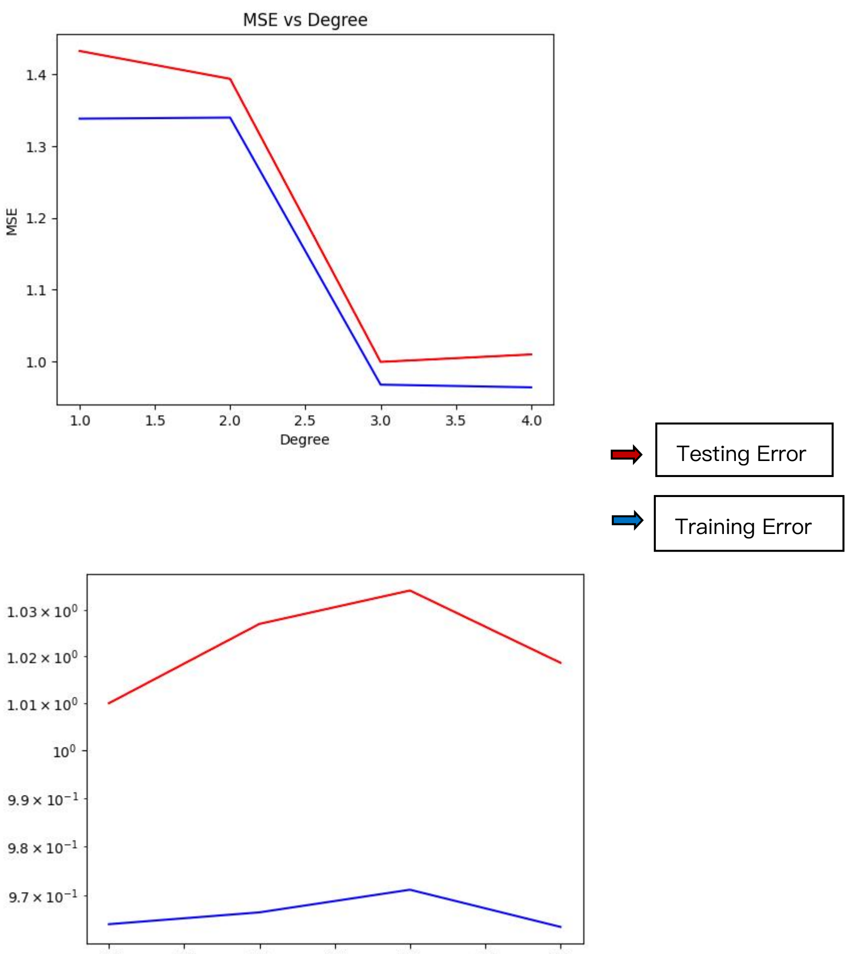
q = 1: gradient = x_i * (y_pred – y_i) + (lmbda * 0.5) * np.sign(W)

else: gradient = x_i * (y_pred – y_i) + (lmbda * 0.5 * q) * (W ** (q–1))

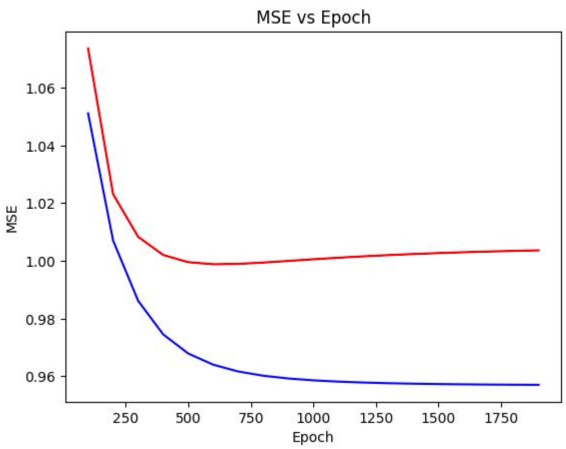
Graph Plotting

1-A

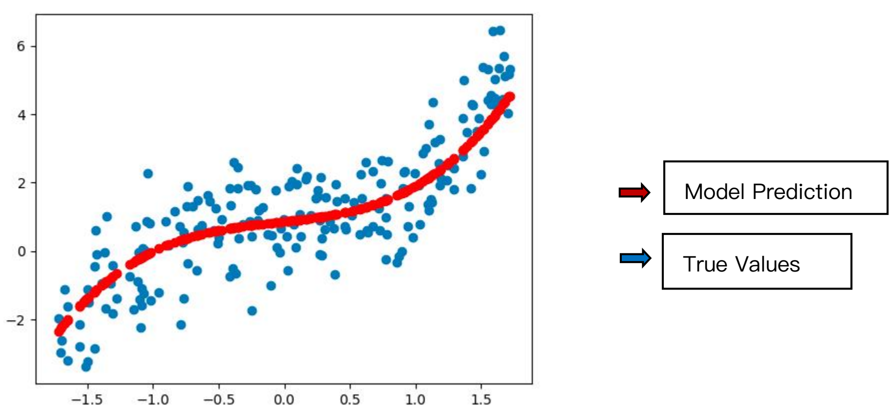
Training and Testing Error vs Degree of polynomial



Training and Testing Error vs Epoch



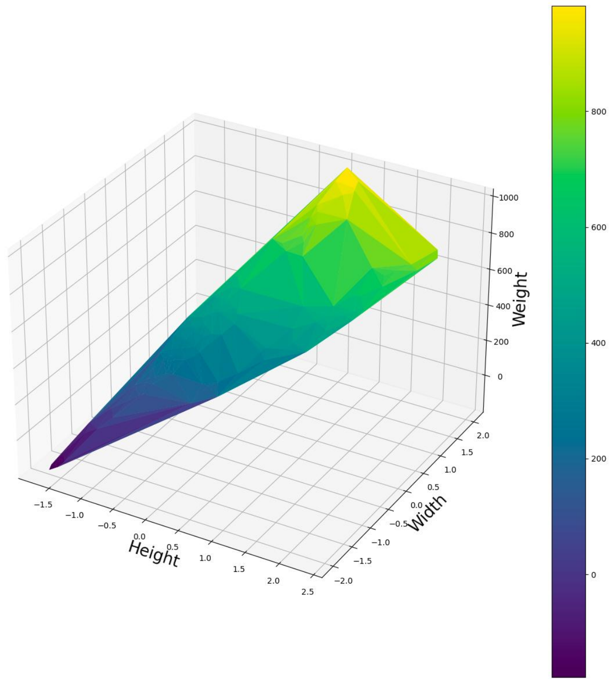
Plotting the best fit curve



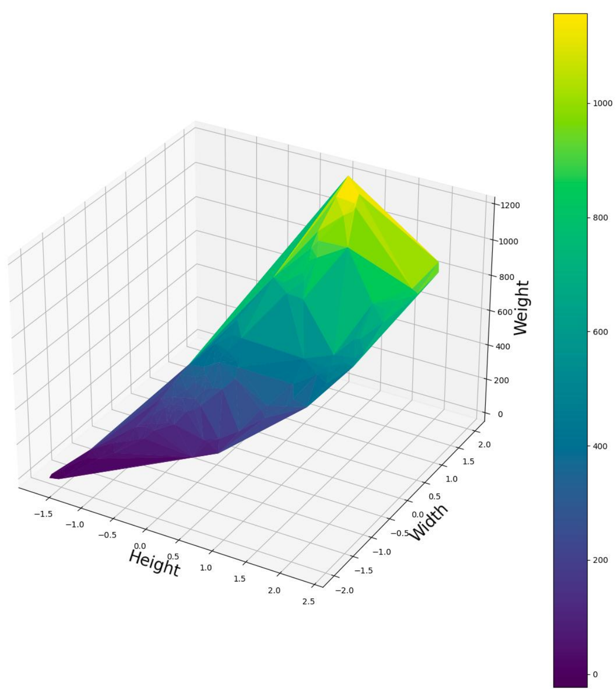
1-B

Polynomial Regression Surface plot

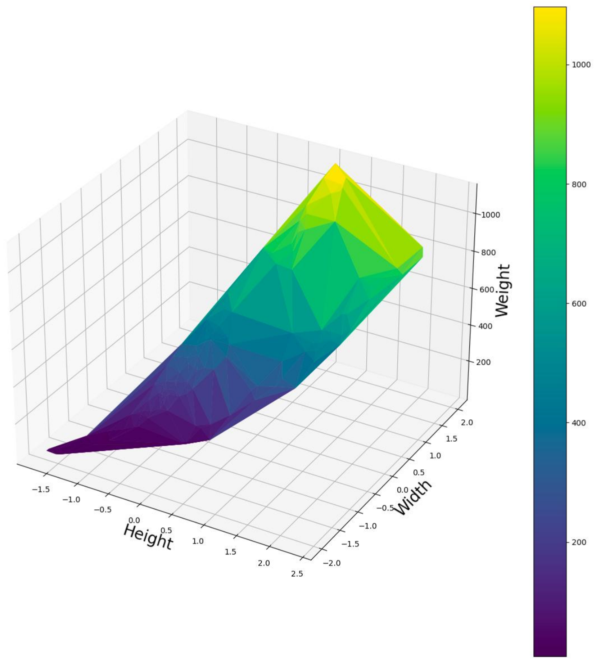
Degree 1 best fit



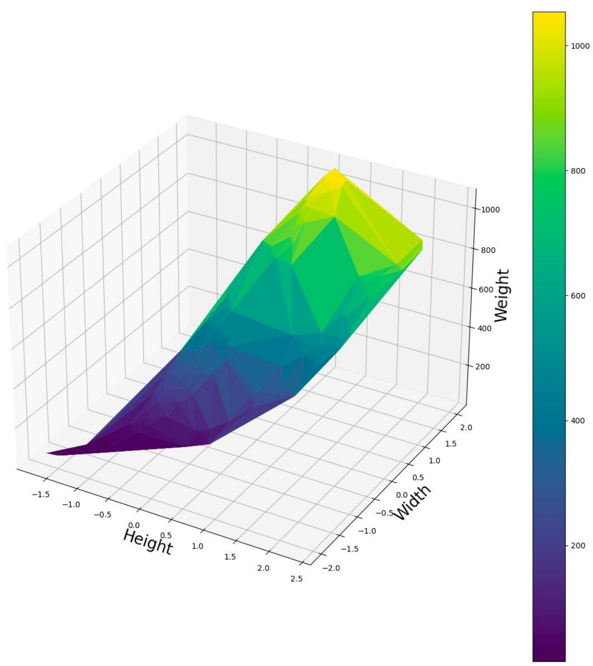
Degree 2 Best Fit



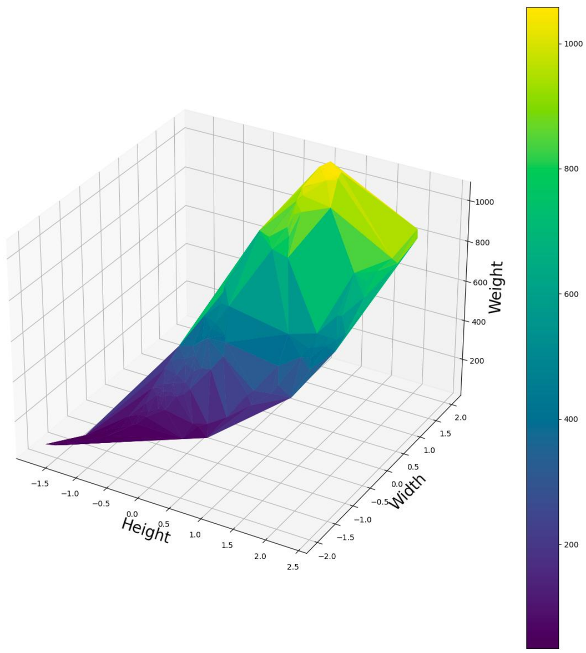
Degree 3 Best Fit



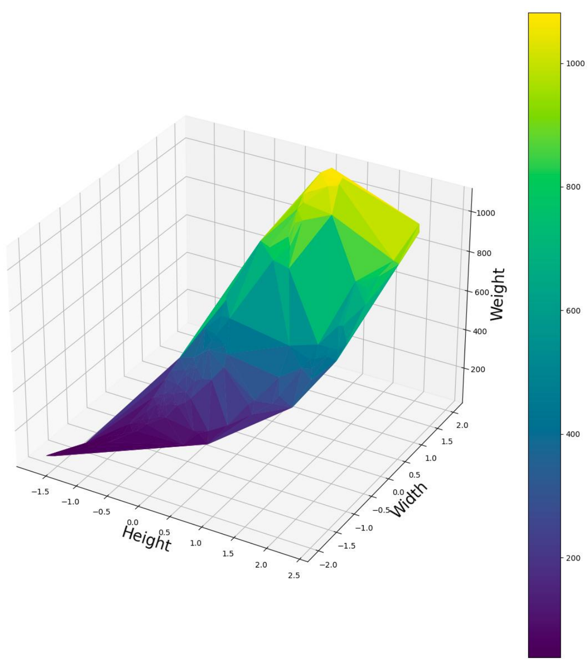
Degree 4 Best Fit



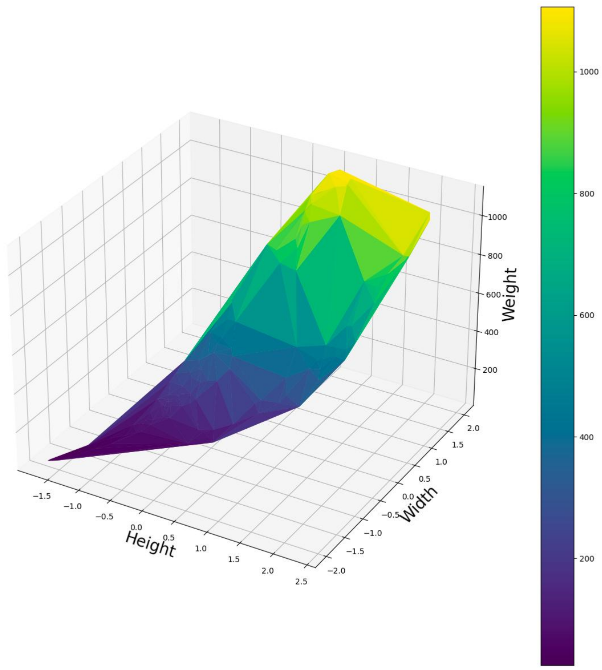
Degree 5 Best Fit



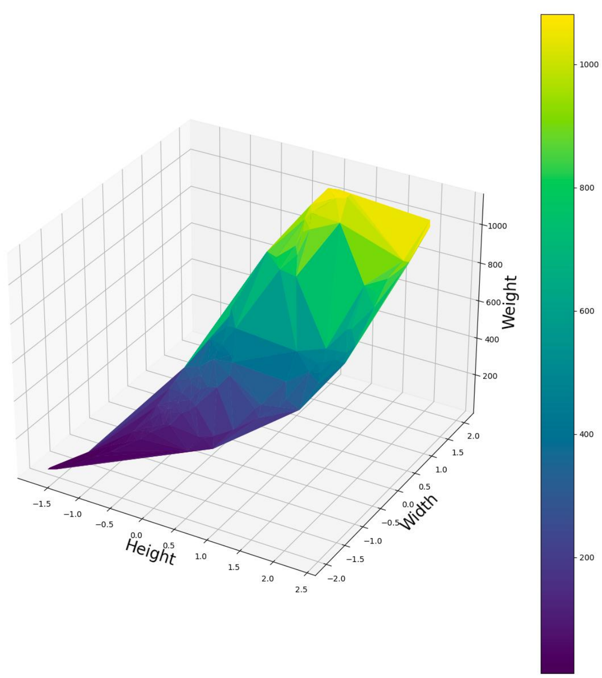
Degree 6 Best Fit



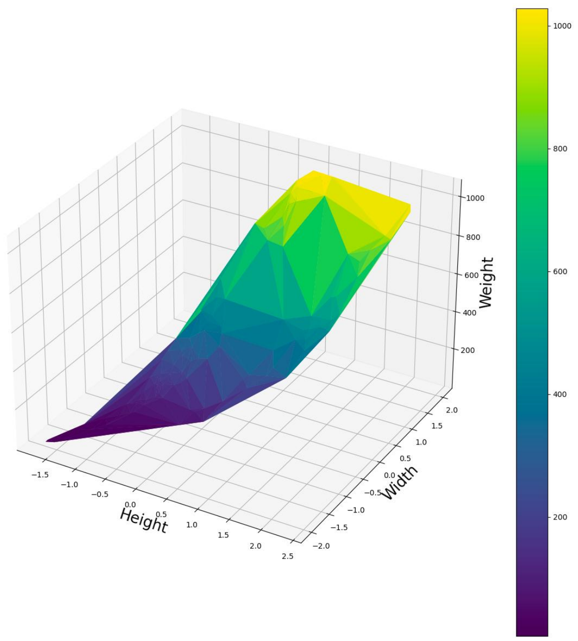
Degree 7 Best Fit



Degree 8 Best Fit

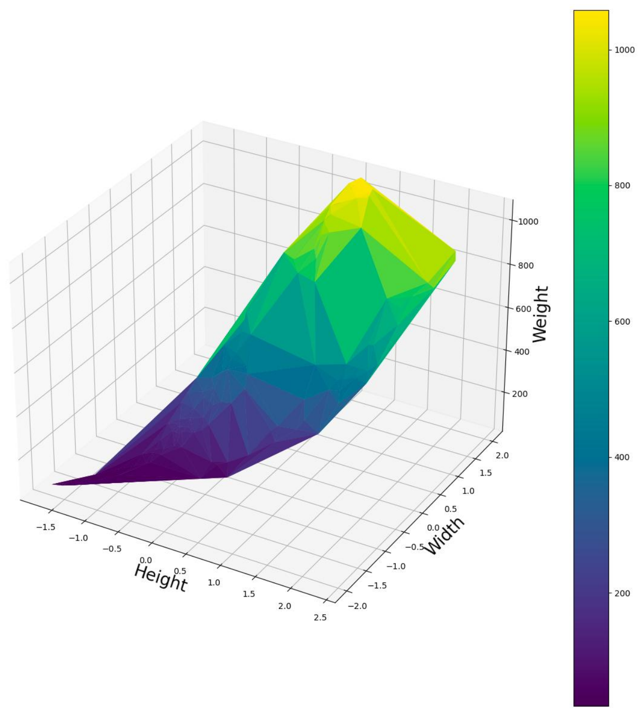


Degree 9 Best Fit

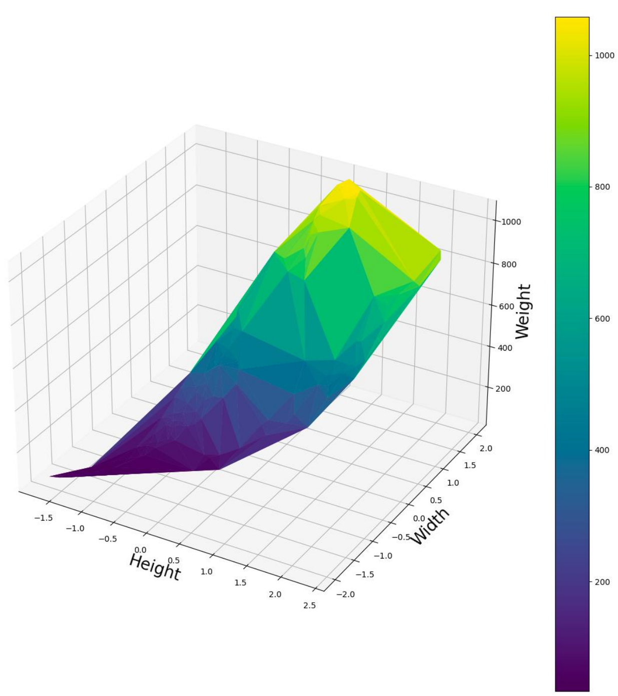


Optimal Regularized Linear Regression Model Surface Plot

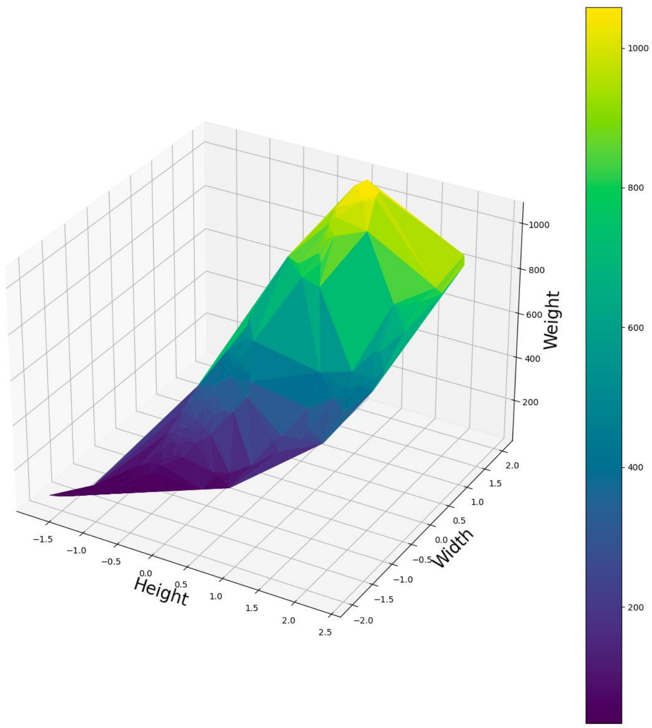
Q = 0.5



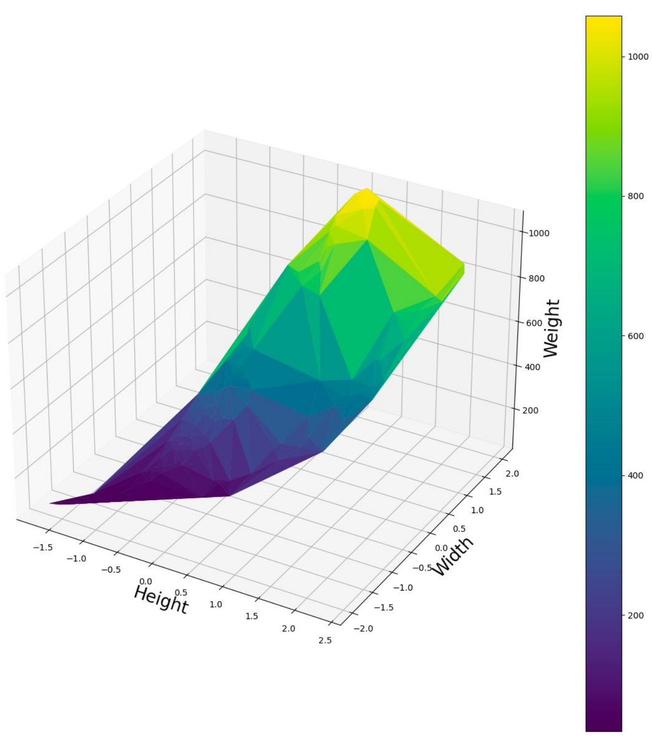
Q = 1



Q = 2



Q = 4



Comparative Analysis

1-A

We observe that a polynomial model of degree 3, with learning rate of 0.01; trained with batch gradient descent algorithm for 600 iterations gives us the best result of 0.9988638271467665 testing error and 0.9640454936180258 training error out of all polynomial models.

1-B

<u>Tabulation of MSE vs Degree</u>

Q	Degree	Batch Test Error	Batch Train Error	Stochastic Test Error	Stochastic Train Error
0	1	54929.7961	20953.2505	51559.6331	22073.3716
	2	48393.6692	13907.1528	44622.4110	15285.5285
	3	46574.5700	12922.6224	42194.1122	14242.0322
	4	45299.5922	12405.5280	40691.8961	13862.1792
	5	45460.6792	12233.4381	40220.3483	14803.3771
	6	46192.7053	12176.7766	43016.1331	18953.8453
	7	46834.5874	12166.2085	81165.9661	39570.2850
	8	47042.6682	12160.4672	2.5522019e+118	1.3328255e+118
	9	46850.2169	12150.4923	inf	inf

Q	Degree	Batch Test Error	Batch Train Error	Stochastic Test Error	Stochastic Train Error
0.5	1	54929.7961	20953.2505	51559.6331	22073.3716
	2	48393.6692	13907.1528	44622.4110	15285.5285
	3	46574.5700	12922.6224	42194.1122	14242.0322
	4	45299.5922	12405.5280	40691.8961	13862.1792
	5	45460.6792	12233.4381	40220.3483	14803.3771
	6	46192.7053	12176.7766	43016.133	18953.8453
	7	46834.5874	12166.2085	81165.9661	39570.2850
	8	47042.6682	12160.4672	2.55220e+118	1.332825e+118
	9	46850.2169	12150.49231	inf	inf

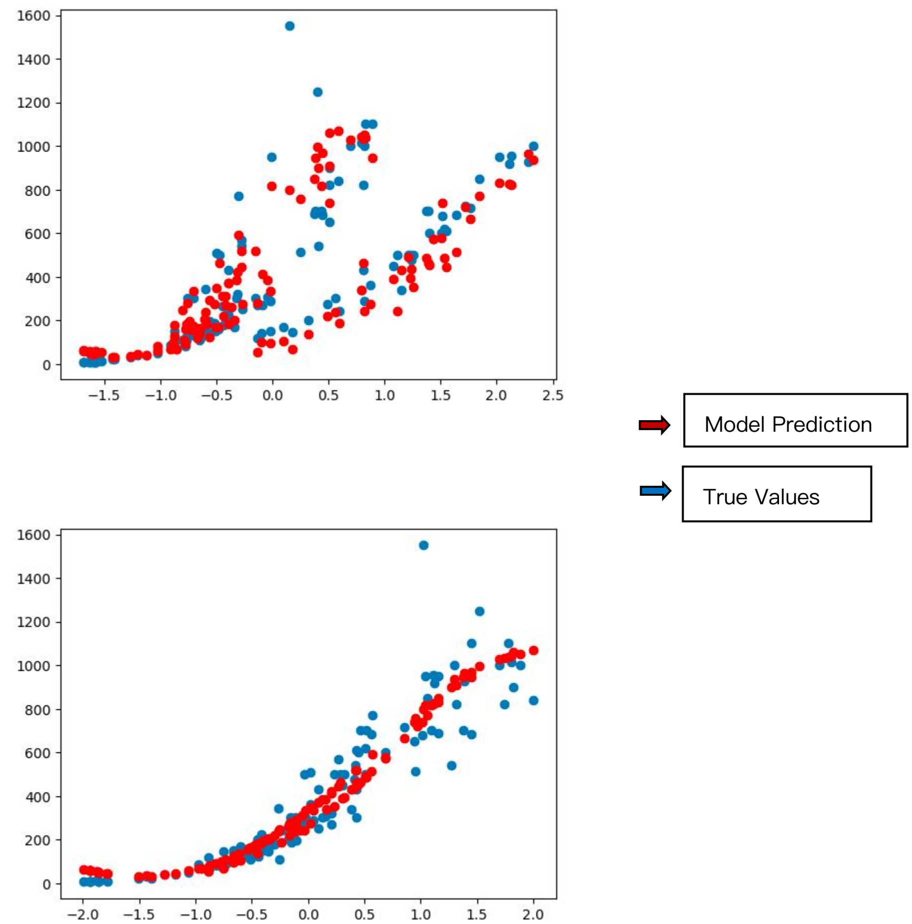
Q	Degree	Batch Test Error	Batch Train Error	Stochastic Test Error	Stochastic Train Error
1	1	54929.7961	20953.2505	51559.6331	22073.3716
	2	48393.6692	13907.1528	44622.4110	15285.5285
	3	46574.5700	12922.6224	42194.1122	14242.0322
	4	45299.5922	12405.5280	40691.8961	13862.1792
	5	45460.6792	12233.4381	40220.3483	14803.3771
	6	46192.7053	12176.7766	43016.1331	18953.8453
	7	46834.5874	12166.2085	81165.9661	39570.2850
	8	47042.6682	12160.4672	2.552201e+118	1.332825e+118
	9	46850.2169	12150.4923	inf	inf

Q	Degree	Batch Test Error	Batch Train Error	Stochastic Test Error	Stochastic Train Error
2	1	54929.7961	20953.2505	51559.6331	22073.3716
	2	48393.6692	13907.1528	44622.4110	15285.5285
	3	46574.5700	12922.6224	42194.1122	14242.0322
	4	45299.5922	12405.5280	40691.8961	13862.1792
	5	45460.6792	12233.4381	40220.3483	14803.3771
	6	46192.7053	12176.7766	43016.1331	18953.8453
	7	46834.5874	12166.2085	81165.9661	39570.2850
	8	47042.6682	12160.4672	2.552201e+118	1.332825e+118
	9	46850.2169	12150.4923	inf	inf

Q	Degree	Batch Test Error	Batch Train Error	Stochastic Test Error	Stochastic Train Error
4	1	54929.7961	20953.2505	51559.6331	22073.3716
	2	48393.6692	13907.1528	44622.4110	15285.5285
	3	46574.5700	12922.6224	42194.1122	14242.0322
	4	45299.5922	12405.5280	40691.8961	13862.1792
	5	45460.6792	12233.4381	40220.3483	14803.3771
	6	46192.7053	12176.7766	43016.1331	18953.8453
	7	46834.5874	12166.2085	81165.9661	39570.2850
	8	47042.6682	12160.4672	inf	inf
	9	inf	inf	inf	inf

The best plot

We observe that a regularized polynomial model of degree 5, q 2 and lambda 10^(−20), with learning rate of 7; trained with stochastic gradient descent algorithm for 50000 iterations gives us the best result of 40220.3483 testing error and 14803.3771 training error out of all models.



Team Members

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