ECE 271A HW #5 The Cheetah Problem (Continued)

In [174]:
```
import numpy as np
from scipy.io import loadmat
import matplotlib.pyplot as plt
from scipy.stats import multivariate_normal
%matplotlib inline
```

In [2]:
```
m = loadmat('../data/TrainingSamplesDCT_8_new.mat')
foreground,background = m['TrainsampleDCT_FG'],m['TrainsampleDCT_BG']
```

In [3]:
```
# define the zigzag transformation
zig_zag = np.array([[0,1,5,6,14,15,27,28],[2,4,7,13,16,26,29,42],[3,8,12
,17,25,30,41,43],
                   [9,11,18,24,31,40,44,53],[10,19,23,32,39,45,52,54],[2
0,22,33,38,46,51,55,60],
                   [21,34,37,47,50,56,59,61],[35,36,48,49,57,58,62,63]])
zz_flat = zig_zag.flatten()
def zig_zag_transform(a):
    result = np.zeros(64)
    for i in range(64):
        result[zz_flat[i]] = a[i]
    return result
```

In [4]:
```
# 2D DCT function
import scipy.fftpack
def dct2d(a):
    return scipy.fftpack.dct(scipy.fftpack.dct( a, axis=0, norm='ortho'
),axis=1,norm='ortho')
```

In [5]:
```
im = loadmat('../data/im_double.mat')
im_array = im['img']
print(im_array.shape)
```

```
(255, 270)
```

In [6]:
```
import imageio
# store the test data as a numpy array
im_test = imageio.imread('../data/cheetah_mask.bmp')
im_test_array = np.array(im_test)
# convert 255 to 1 for error calculation
im_test_array = im_test_array / 255
```

In [ ]:
```
def image_process(im_array):
    result = []
    for i in range(im_array.shape[0]-8):
        for j in range(im_array.shape[1]-8):
            row_start,row_end = i,i+8
            col_start,col_end = j,j+8
            block = im_array[row_start:row_end,col_start:col_end]
            block_dct = dct2d(block).flatten()
            block_dct= zig_zag_transform(block_dct)
            result.append(block_dct)
    result = np.array(result)
    return result
```

a) 5 foreground and 5 background mixture models with 8 components:

In [7]:
```
M,C,N_FG,N_BG = 5,8,foreground.shape[0],background.shape[0]
```

In [206]:
```
# random initialization
def rand_init(c,sample):
    pi = np.ones(c) * 1 / c
    mu = np.random.randn(c,64)
    cov = []
    for i in range(c):
        cov_temp = np.random.normal(5,0.3,size=64)
        cov.append(np.diag(cov_temp))
    cov = np.array(cov)
    return pi,mu,cov
```

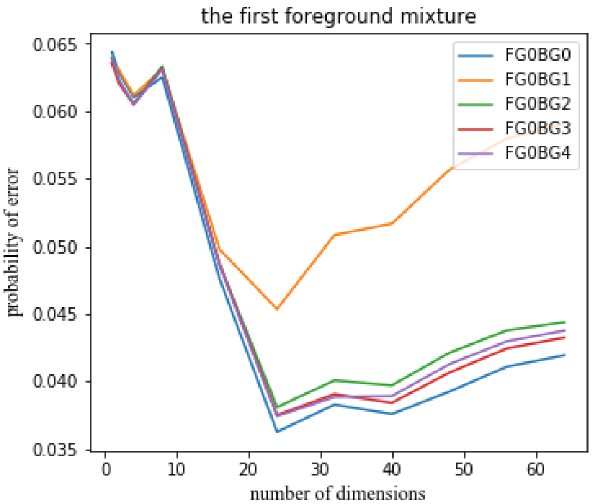
In [194]:
```
def EM(c,sample,max_iter):
    pi,mu,cov = rand_init(c,sample)
    for i in range(max_iter):
        # E-step
        H = []
        for j in range(c):
            H_temp = multivariate_normal.pdf(sample,mean=mu[j,:],cov=cov
[j,:,:]) * pi[j]
            H.append(H_temp)
        H = np.array(H).T
        H = H / np.sum(H,axis = 1)[:,np.newaxis]
        H_sum = np.sum(H,axis = 0)
        # M-step
        # update pi
        pi = 1 / sample.shape[0] * H_sum
        # update mean
        mu_update = []
        for j in range(c):
            mu_temp = np.sum(H[:,j][:,np.newaxis] * sample,axis = 0) / H
_sum[j]
            mu_update.append(mu_temp)
        # update covariance
        cov_update = []
        for j in range(c):
            x_temp = sample - mu[j,:]
            cov_temp = np.sum((x_temp ** 2) * H[:,j][:,np.newaxis],axis
= 0) / H_sum[j]
            # make sure cov is not too small
            cov_temp[cov_temp < 1e-6] = 1e-6
            cov_temp = np.diag(cov_temp)
            cov_update.append(cov_temp)
        cov = np.array(cov_update)
        mu = np.array(mu_update)
    return pi,mu,cov
```

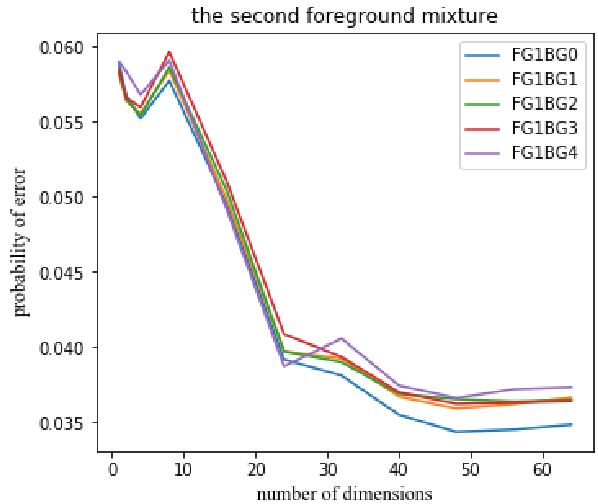
In [195]:
```
def Gaussian_mixture_BDR(pi_FG,mu_FG,cov_FG,pi_BG,mu_BG,cov_BG,FG,BG,im_
array,dim):
    c = mu_FG.shape[0]
    im_blocks = image_process(im_array)[:,:dim]
    FG_prob,BG_prob,A = np.zeros(247*262),np.zeros(247*262),np.zeros(247
*262)
    for k in range(c):
        FG_prob += multivariate_normal.pdf(im_blocks,mean = mu_FG[k],cov
= cov_FG[k]) * pi_FG[k]
    for k in range(c):
        BG_prob += multivariate_normal.pdf(im_blocks,mean = mu_BG[k],cov
= cov_BG[k]) * pi_BG[k]
    A = FG_prob - BG_prob
    A = np.where(A>0,1,0)
    A_matrix = np.reshape(A,(247,262))
    A_matrix = np.lib.pad(A_matrix,(4,4),'constant',constant_values = 0)
    return A_matrix
```

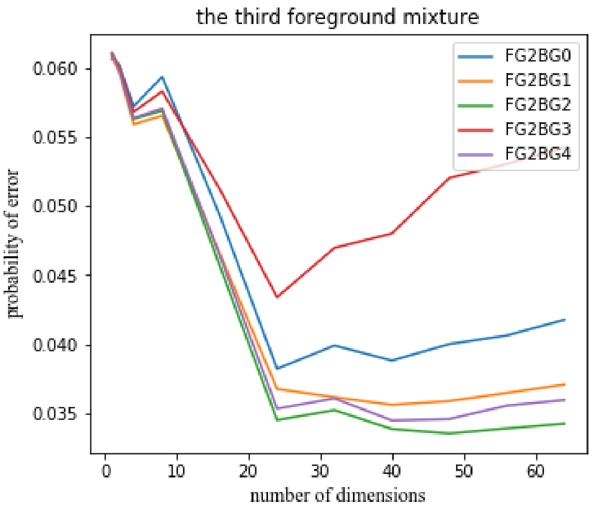
In [196]:
```
def prob_error(A,B):
    return np.sum(np.absolute(A-B)) / (255*270)
```

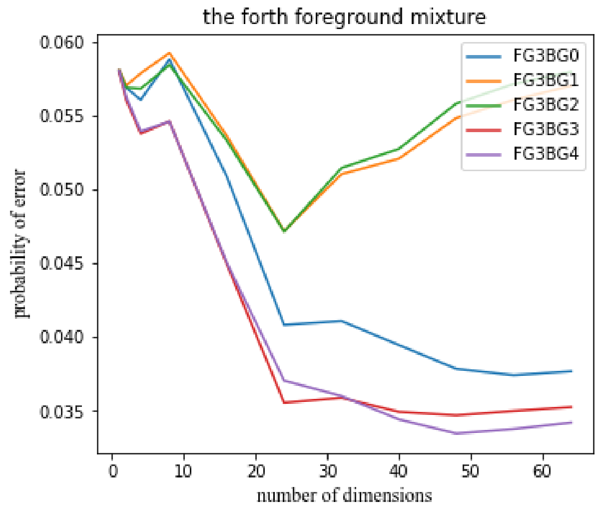
In [212]:
```
error_dic0 = {}
dim = np.array([1,2,4,8,16,24,32,40,48,56,64])
for idx_FG in range(M):
    print('FG '+ str(idx_FG))
    pi_FG,mu_FG,cov_FG = EM(8,foreground,200)
    for idx_BG in range(M):
        print('BG '+ str(idx_BG))
        pi_BG,mu_BG,cov_BG = EM(8,background,200)
        error_list = []
        for cur_dim in dim:
            print('dimension ' + str(cur_dim))
            mu_FG_cur,cov_FG_cur = mu_FG[:,:cur_dim],cov_FG[:,:cur_dim,:
cur_dim]
            mu_BG_cur,cov_BG_cur = mu_BG[:,:cur_dim],cov_BG[:,:cur_dim,:
cur_dim]
            A = Gaussian_mixture_BDR(pi_FG,mu_FG_cur,cov_FG_cur,pi_BG,mu
_BG_cur,
                                     cov_BG_cur,foreground,background,im_arr
ay,cur_dim)
            error = prob_error(A.flatten(),im_test_array.flatten())
            error_list.append(error)
        error_list = np.array(error_list)
        label = 'FG' + str(idx_FG) + 'BG' + str(idx_BG)
        error_dic0[label] = error_list
```

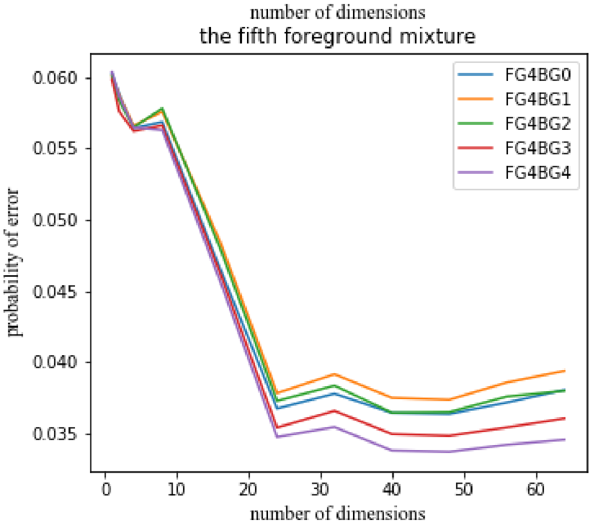
In [198]:
```
fig=plt.figure(figsize=(12,16))
voc = ['first','second','third','forth','fifth']
for idx_FG in range(M):
    fig.add_subplot(3,2,idx_FG+1)
    plt.plot(dim,error_dic0['FG'+str(idx_FG)+'BG0'],label = 'FG'+str(idx
_FG)+'BG0')
    plt.plot(dim,error_dic0['FG'+str(idx_FG)+'BG1'],label = 'FG'+str(idx
_FG)+'BG1')
    plt.plot(dim,error_dic0['FG'+str(idx_FG)+'BG2'],label = 'FG'+str(idx
_FG)+'BG2')
    plt.plot(dim,error_dic0['FG'+str(idx_FG)+'BG3'],label = 'FG'+str(idx
_FG)+'BG3')
    plt.plot(dim,error_dic0['FG'+str(idx_FG)+'BG4'],label = 'FG'+str(idx
_FG)+'BG4')
    plt.title('the '+ voc[idx_FG] + ' foreground mixture')
    plt.legend(loc='upper right')
    plt.xlabel('number of dimensions', fontdict={'family' : 'Times New R
oman', 'size'   : 12})
    plt.ylabel('probability of error', fontdict={'family' : 'Times New R
oman', 'size'   : 12})
```


the first foreground mixture
probability of error
number of dimensions
FG0BG0
FG0BG1
FG0BG2
FG0BG3
FG0BG4


the second foreground mixture
probability of error
number of dimensions
FG1BG0
FG1BG1
FG1BG2
FG1BG3
FG1BG4


the third foreground mixture
probability of error
number of dimensions
FG2BG0
FG2BG1
FG2BG2
FG2BG3
FG2BG4


the forth foreground mixture
probability of error
number of dimensions
FG3BG0
FG3BG1
FG3BG2
FG3BG3
FG3BG4


the fifth foreground mixture
probability of error
number of dimensions
FG4BG0
FG4BG1
FG4BG2
FG4BG3
FG4BG4

The plot of POE with respect to different number of dimensions is shown above. As we can see from the result, the combination of different foreground and background mixture models will generate different results(probability of error). This is because of the random initialization during the EM training. Also we can observe that with the increase of the number of dimensions, the difference between the results of different mixture models increases.

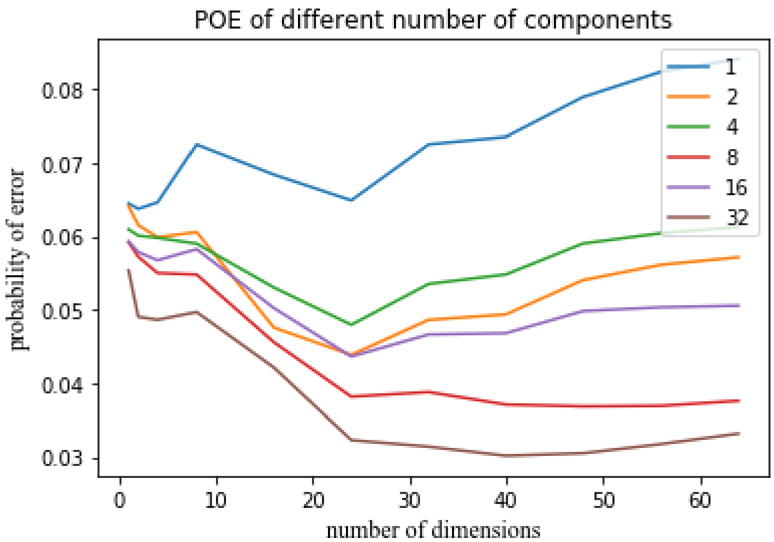
However, we can observe that the general trend of all 25 models are really similar. With more dimensions, the probablility of error generally decreases. However, as shown in homework 2, if we select the best 8 features, the result will be better than using all 64 features. As we can see from the result of this homework, for all 25 models the POE doesn't reach its minimum when we use all 64 dimensions. The number of dimensions that have the best probability of error varies within all 25 models, but generally it is betwenn 24 to 48.

b) Mixture models with different numbers of components:

In [210]:

```
error_dic1 = {}
dim = np.array([1,2,4,8,16,24,32,40,48,56,64])
mixtures = np.array([1,2,4,8,16,32])
for c in mixtures:
    error_list = []
    pi_FG,mu_FG,cov_FG = EM(c,foreground,200)
    pi_BG,mu_BG,cov_BG = EM(c,background,200)
    for cur_dim in dim:
        mu_FG_cur,cov_FG_cur = mu_FG[:,:cur_dim],cov_FG[:,:cur_dim,:cur_
dim]
        mu_BG_cur,cov_BG_cur = mu_BG[:,:cur_dim],cov_BG[:,:cur_dim,:cur_
dim]
        A = Gaussian_mixture_BDR(pi_FG,mu_FG_cur,cov_FG_cur,pi_BG,mu_BG_
cur,
                                 cov_BG_cur,foreground,background,im_arr
ay,cur_dim)
        error = prob_error(A.flatten(),im_test_array.flatten())
        error_list.append(error)
    error_list = np.array(error_list)
    label = str(c)
    error_dic1[label] = error_list
```

In [211]:
```
for i in range(mixtures.shape[0]):
    plt.plot(dim,error_dic1[str(2**(i))],label = str(2**(i)))
plt.title('POE of different number of components')
plt.legend(loc='upper right')
plt.xlabel('number of dimensions', fontdict={'family' : 'Times New Roma
n', 'size'   : 12})
plt.ylabel('probability of error', fontdict={'family' : 'Times New Roma
n', 'size'   : 12})
plt.show()
```



As we can see from the result, the POE of the mixture model with only 1 component is strictly larger than the results of the rest of the models. Thus we can conclude that, using only 1 component cannot best fit the true distribution of $P_{X|Y}(x|i)$, in other words $P_{X|Y}(x|i)$ is not just a simple multivariate Gaussian distribution. With more components the trend of different curves are generally the same. In this experiment, using 32 components can give us the best probability of error. However, the optimal number of components cannot be revealed from this single experiment.

In [ ]: