Partial Compositeness In Generalized Randall-Sundrum Scenario

A Dissertation Submitted

in Partial Fulfilment of the Requirements

for the Degree of

MASTER OF SCIENCE

in

Physics

by

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IISER THIRUVANANTHAPURAM

to

SCHOOL OF PHYSICS

INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH

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January 2025

DECLARATION

I, **Nabeel Thahir (Roll No: IMS20189)**, hereby declare that, this report entitled **"Partial Compositeness In Generalized Randall-Sundrum Scenario"** submitted to Indian Institute of Science Education and Research Thiruvananthapuram towards the partial requirement of **Master of Science** in **Physics**, is an original work carried out by me under the supervision of **Dr. Mathew Arun Thomas** and has not formed the basis for the award of any degree or diploma, in this or any other institution or university. I have sincerely tried to uphold academic ethics and honesty. Whenever a piece of external information or statement or result is used, then that has been duly acknowledged and cited.

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CERTIFICATE

This is to certify that the work contained in this project report entitled **"Partial Compositeness In Generalized Randall-Sundrum Scenario"** submitted by **Nabeel Thahir** (**Roll No: IMS20189**) to Indian Institute of Science Education and Research, Thiruvananthapuram towards the partial requirement of **Master of Science** in **Physics** has been carried out by him under my supervision and that it has not been submitted elsewhere for the award of any degree.

Thiruvananthapuram - 695 551 | Dr. Mathew Arun Thomas

January 2025 | Project Supervisor

ACKNOWLEDGEMENT

I thank everyone who has assisted me in seeing this project through to its completion. I would like to first express my profound gratitude and deepest regards to Dr. Mathew Arun Thomas, IISER Thiruvananthapuram, and sincerely wish to acknowledge his vision, guidance, valuable feedback and constant support throughout the duration of this project.

I am lastly grateful to the Indian Institute of Science Education and Research Thiruvananthapuram for providing the necessary resources and facilities to complete this project to the best of my ability.

Thiruvananthapuram - 695 551 **Nabeel Thahir**

January 2025

ABSTRACT

Name of the student: **Nabeel Thahir** Roll No: **IMS20189**

Degree for which submitted: **M.Sc.** Department: **School of Physics**

Thesis title: **Partial Compositeness In Generalized Randall-Sundrum Scenario**

Thesis supervisor: **Dr. Mathew Arun Thomas**

Date of thesis submission: **January 2025**

This thesis uses an induced cosmological constant[1] to the Randall-Sundrum (RS) model. We then utilize the holographic basis to decompose the bulk fields not on the usual Kaluza-Klein basis but rather into a holographic basis of 4D fields corresponding to purely elementary source or CFT composite fields. Finally, we numerically show the partial compositeness for the massless mode in the 5D theory and how the induced cosmological constant changes the mixing fraction.

Keywords:

Randall-Sundrum model, Holography, Extra Dimensions, Partial Compositeness.

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Chapter 1

Introduction

This thesis builds on the foundations of the Randall-Sundrum model [2]. We introduce an induced cosmological constant and find a more generalized metric that incorporates a variable warp factor; we then proceed to analyze the dynamics of bulk scalar fields propagating within a slice of AdS in Chapter (3) and explicitly show the massless and massive modes.

Building upon this foundation, In Chapter (4), We define the holographic basis, which decomposes the bulk field into elementary source fields and composite states, which we use to describe the mixing between the elementary (source) and composite CFT sectors through the Kaluza-Klien mass eigenstates. Finally, using the eigenvectors, we describe the partial compositeness of fields using the holographic expansion and numerically show the relationship between the compositeness of the zero-mode and the induced cosmological constant.

Chapter 2

Generalized Randall-Sundrum Scenario

In the Randall-Sundrum scenario(RS1), it was proposed that our universe is five-dimensional, as described by the metric.

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{2.1}$$

Where Greek indices $\mu, \nu, \ldots$ run over four observed dimensions, y signifies the coordinate on the extra space-like dimension. We define Λ as the bulk cosmological constant and e^{2ky} as the warp factor. To avoid experimental detection, the fifth dimension y must have a finite extent, and the simplest possibility is to assume a periodic geometry, such as a circle,S^1 of radius R, where $y \leftrightarrow y + 2\pi R$. We also identify the opposite points of the circle leading to an Z_2 orbifold (S^1/Z_2) with Z_2 representing the identification $y \leftrightarrow -y$. Two 3-branes, the UV and IR, are located at the endpoints of the orbifold y = 0 and πR.

In [2], it was also shown that the cosmological constant induced on the IR brane is zero. Now we consider a more general warp factor, such that the metric is given by:

$$ds^2 = e^{-2A(y)}g_{\mu\nu}dx^\mu dx^\nu + r^2dy^2 \tag{2.2}$$

The visible brane can have a negative or a positive cosmological constant for the above metric. Now we start with the metric (2.2) and evaluate A(y) which extrimises the action:

$$S = \int d^5x\sqrt{-G}(M^3R - \Lambda) + \int d^4x\sqrt{-g_i}\mathcal{V}_i \tag{2.3}$$

Where Λ the bulk cosmological constant R is the bulk Ricci scalar, and V_i the i^{th} brane $g_{\mu\nu}$ is the four-dimensional metric.

The resulting Einstein equations are:

$${}^4G_{\mu\nu} - g_{\mu\nu}e^{-2A}\left[-6A'^2 + 3A''\right] = -\frac{\Lambda}{2M^3}g_{\mu\nu}e^{-2A} \tag{2.4}$$

$$-\frac{1}{2}e^{2A}R + 6A^2 = -\frac{\Lambda}{2M^3} \tag{2.5}$$

with the boundary conditions

$$[A'(y)]_i = \frac{\epsilon_i}{12M^3}\mathcal{V}_i \tag{2.6}$$

where $\epsilon_{UV} = -\epsilon_{IR} = 1$. Also, ${}^4G_{\mu\nu}$ and ${}^4R$ are the four Einstein tensor and Ricci scalar, respectively, defined concerning $g_{\mu\nu}$. Now, we equate an arbitrary constant Ω to terms while isolating the y dependence shown below.

$${}^4G_{\mu\nu} = -\Omega g_{\mu\nu} \tag{2.7}$$

$$e^{-2A}\left[-6A'^2 + 3A'' - \frac{\Lambda}{2M^3}\right] = -\Omega \tag{2.8}$$

we now simplify the above to get the set of equations leading to a simplified expression for A”(2.10)

$$6A'^2 = -\frac{\Lambda}{2M^3} + 2\Omega e^{2A} \tag{2.9}$$

$$3A'' = \Omega e^{2A} \tag{2.10}$$

The above corresponds to a constant curvature brane spacetime, unlike a Ricci flat spacetime. For $\Omega > 0$ and $\Omega < 0$, the metric corresponds to dS-Schwarzchild and Ads-Schwarzschild spacetimes, respectively.

For Ads bulk($\Lambda < 0$), we first consider the regime for which the induced cosmological constant Ω on the IR brane is negative. For the equations (2.9)(2.10) we find the warp factor as:

$$\boxed{e^{-A(y)} = \omega \cosh\left(\ln\frac{\omega}{c_1} + ky\right) \tag{2.11}}$$

where $w^2 \equiv -\Omega/3k^2 \geq 0$. We can normalize the warp factor to unity at the orbifold fixed point $y = 0$ with $c_1 = 1 + \sqrt{1 - w^2}$. The usual Randall-Sundrum solution A = ky is recovered for w $\rightarrow$ 0.

Chapter 3

Bulk Fields in a Slice of AdS

We will consider a scalar bulk field propagating in a slice of AdS_5. These fields are assumed to have negligible backreaction on the background geometry (2.2) with the warp factor (2.11). We find the equation of motion for the bulk fields by variation of the action with the generic form:

$$\delta S_5 = \int d^5x\, \delta\phi\, (\mathcal{D}\phi) + \int d^4x\, \delta\phi\, (\mathcal{B}\phi)\Big|_{y*} \tag{3.1}$$

where we require the first term in (3.1) to vanish gives the equation of motion $\mathcal{D}\phi = 0$. However, the second term in (3.1) is evaluated at the boundaries of the fifth dimension y. The vanishing second term leads to the boundary conditions $\delta\phi|_{y^*} = 0$ or $\mathcal{B}\phi\Big|_{y^*} = 0$, also we have assumed the fields to vanish at the 4D boundary $x^\mu \to \pm\infty$.

3.1 Bulk Scalar Field

Consider a bulk scalar field ϕ whose action to quadratic order is given by:

$$S_\Phi = -\int d^5x\sqrt{-g}\left[|\partial_M\Phi|^2 + m_\Phi^2|\Phi|^2\right] \tag{3.2}$$

where $m_\phi^2 = ak^2$ is the bulk mass parameter defined in units of curvature scale k with dimensionless coefficient a. Detailed derivations including Kaluza-Klien decomposition is given in Appendix[A.2]. Here we analyze the Equation of motion of the action(3.2)

3.1.1 Scalar: $m_n = 0$

The general solution for a massless mode ($m_0 = 0$) is given by

$$f_\Phi^{(0)} = \left[c_1\, \mathrm{P}_1^\alpha(\tanh(\tilde{y})) + c_2\, \mathrm{Q}_1^\alpha(\tanh(\tilde{y}))\right] \mathrm{sech}^2(\tilde{y}) \tag{3.3}$$

where $\alpha = \sqrt{4+a}$ and $\tilde{y} = \ln\frac{\omega}{c_1} + ky$ and P and Q are Legendre polynomials. Generally, no massless mode solution exists for either Neumann or Dirichlet boundary conditions. Instead, to obtain a massless mode, we need to modify the boundary action and include boundary mass terms of the form [3]

$$S_{\partial\Phi} = -\int d^5x\sqrt{-g}\, 2bk\,[\delta(y) - \delta(y-\pi R)]\,|\Phi|^2 \tag{3.4}$$

where b is a dimensionless constant parametrising the boundary mass in k units. The Neumann boundary conditions are now modified to $(\partial_5 - bk)f_\Phi^{(0)}|_{0,\pi R}$. We will find that the boundary mass parameter must be tuned to satisfy $b \approx 2 \pm \sqrt{4+a}$ for as $w \to 0$.

3.1.2 Scalar: $m_n \neq 0$

The general solution of the Kaluza-Klein modes for $m_n \neq 0$ is given by:

$$f_\Phi^{(n)} = N_\Phi^{(n)}\left[\mathrm{P}_\gamma^\alpha(\tanh(\tilde{y})) + b_\Phi^{(n)}\,\mathrm{Q}_\gamma^\alpha(\tanh(\tilde{y}))\right]\mathrm{sech}^2(\tilde{y}) \tag{3.5}$$

Where $N_\Phi^{(n)}$ and $b_\Phi^{(n)}$ are arbitrary constants obtained by orthonormal relation (A.9) and boundary conditions respectively. and $\gamma = \frac{1}{2}\left(-1 + \frac{\sqrt{4(mn/k)^2+9w^2}}{w}\right)$.

Chapter 4

Holographic Mixing In A Slice Of AdS_5

We constructed an weakly coupled five-dimensional (5D) holographic dual to QCD[4]. The extra dimension can be utilized as a calculational tool to understand the properties of composite states in the 4D theory. In a slice of 5D anti-de Sitter (AdS) space, the finite ultraviolet (UV) boundary in the warped extra dimension translates into a UV cutoff of the dual conformal field theory (CFT) and, in turn, implies the existence of a dynamical source field. Mixing between the elementary (source) and composite (CFT) sectors produces the mass eigenstates of the theory, corresponding to the Kaluza-Klein fields from the 5D perspective. In other words, the mass eigenstates in the dual theory exhibit partial compositeness.

4.1 The holographic basis

Mass eigenstates in the holographic theory result from mixing between the elementary(source) and composite (CFT) sectors. To represent the mixing taking place between the elementary and composite sectors, we propose to decompose the action by expanding the field $\phi(x, y)$ directly in terms of a source field $\varphi^s(x)$ and a tower of CFT bound states $\varphi^n(x)$, with the associated wavefunctions $g^s(y)$ and $g^n(y)$:

$$\phi(x, y) = \varphi^s(x)g^s(y) + \sum_{n=1}^{\infty} \varphi^n(x)g^n(y) \tag{4.1}$$

This expansion is called the *holographic basis*. The profiles $g^s(y)$ and $g^n(y)$ differ from Kaluza Klein profiles. From [5], the CFT spectrum derived from the correlator $\Sigma(p)$ corresponds to applying a pure Dirichlet condition at the UV boundary, $\phi(x, y = 0) = 0$, and the modified Neumann condition at the IR boundary. We assume therefore that the CFT profiles $g^n(y)$ satisfy the bulk equation of motion (with eigenvalues M_n^2) and the following boundary conditions:

$$g^n(y)|_0 = 0, \quad (\partial_5 - bk)g^n(y)|_{\pi R} = 0. \tag{4.2}$$

Also, we impose a wavefunction normalization analogous to (A.9) to have canonical kinetic terms. Explicitly, the CFT eigenfunctions are given by

$$g_{CFT}^{(n)} = N_{CFT}^{(n)} \Big[\mathrm{P}_{\gamma}^{\alpha}(\tanh(\tilde{y})) + b_{CFT}^{(n)}\, \mathrm{Q}_{\gamma}^{\alpha}(\tanh(\tilde{y})) \Big] \,\mathrm{sech}^2(\tilde{y}) \tag{4.3}$$

We require the bulk field $\phi(x, y)$ to construct the boundary action to behave near the UV boundary as [6]. In (A.3), we have shown that the holographic basis describes the mixing between the elementary (source) and composite CFT sectors by diagonalizing the lagrangian.

4.2 Partial Compositeness

The holographic basis accurately characterizes the mixing of elementary and composite components in the 4D dual theory, resulting in mass eigenstates that consist of both elementary and composite fields. The eigenvectors can be derived directly by setting the Kaluza-Klein expansion (A.7) equal to the holographic expansion (4.1) of the bulk field.

$$\sum_{n=0}^{\infty} \phi^{(n)}(x^{\mu}) f_{\phi}^{(n)}(y) = \varphi^{s}(x^{\mu}) g^{s}(y) + \sum_{n=1}^{\infty} \varphi^{(n)}(x^{\mu}) g_{\varphi}^{(n)}(y) \tag{4.4}$$

Using the orthonormal condtion (A.9), we can write the mass eigenstate in terms of the source and CFT fields:

$$\phi^{(n)}(x^{\mu}) = v^{ns} \varphi^{s}(x^{\mu}) + \sum_{n=1}^{\infty} v^{nm} \varphi^{(m)}(x^{\mu}) \tag{4.5}$$

where,

$$\begin{aligned} v^{ns} &= \int_0^{\pi R} dy\, e^{-2A(y)} f_{\phi}^{(n)}(y) g^{(s)}(y), \\ v^{nm} &= \int_0^{\pi R} dy\, e^{-2A(y)} f_{\phi}^{(n)}(y) g_{\phi}^{(m)}(y). \end{aligned}$$

To define the "compositeness" of a mass eigenstate when the mixing involves an infinite set of composite resonances, let us focus on the massless mode, with eigenvector

$$\phi^0(x) = v^{0s}\varphi^s(x) + \sum_{n=1}^{\infty} v^{0n}\varphi^n \tag{4.6}$$

Mathematically, it would be natural to define the "compositeness" of the zero mode by the following fraction ϵ:

$$\epsilon = \frac{\sum_{n=1}^{\infty}(v^{0n})^2}{(v^{0s})^2 + \sum_{n=1}^{\infty}(v^{0n})^2} \tag{4.7}$$

4.2.1 Numerical Results

In this section, we will show how the compositeness of the zero mode changes to the Induced Cosmological Constant numerically and also the transformation matrix that diagonalized the scalar field action, which is the correspondence between the 5D Ads space and the lower dimensional dual. We have taken the b_+ branch for all numerical calculations.

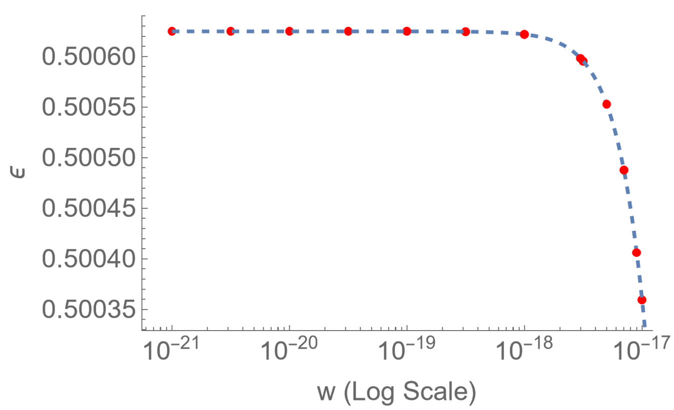

Figure 4.1: Plot of Compositeness of Zero Mode($\phi^0(x)$) vs w

From (4.1) we see that as $w \to 0$ the compositeness converges to the RS value.

From A.3, we can write the mass eigenstates in terms of the source and CFT fields to see precisely how much each mass eigenstate is elementary and composite. Defining $\bar{\phi}^T = (\phi^0, \phi^1, \phi^2, ...)$, we have

$$\bar{\phi} = VT^{-1}U\bar{\varphi} \tag{4.8}$$

The below represents the Transformation matrix that diagonalized the action (A.11) for $w \to 0$ case:

$$\bar{\phi} = \begin{pmatrix} 0.999 & 1.00 & 0.00125 & 0.000879 & 0.000245 & 0.000502 \\ 0.0468 & -0.00163 & 1.00 & 0.00136 & 0.000954 & 0.000305 \\ 0.0105 & -0.000496 & -0.00175 & 1.00 & 0.00142 & 0.000993 \\ 0.00999 & -0.000626 & -0.000571 & -0.00181 & 1.00 & 0.00147 \\ 0.00473 & 0.000180 & -0.000206 & 0.000173 & -0.000212 & 0.000167 \\ -0.0000259 & -0.000119 & -0.000686 & -0.000611 & -0.00186 & 1.00 \end{pmatrix} \bar{\varphi}$$

For $w \approx 10^{-21}$ the below was the computed transformation matrix:

$$\bar{\phi} = \begin{pmatrix} 0.999 & 1.00 & 0.00144 & 0.000688 & 0.000436 & 0.000311 & 0.000238 \\ 0.0470 & -0.00144 & 1.00 & 0.00155 & 0.000763 & 0.000496 & 0.000361 \\ 0.0103 & -0.000688 & -0.00156 & 1.00 & 0.00162 & 0.000802 & 0.000527 \\ 0.0102 & -0.000435 & -0.000763 & -0.00162 & 1.00 & -0.00166 & -0.000829 \\ 0.00455 & -0.000310 & -0.000495 & 0.000803 & -0.00167 & 1.00 & 0.00170 \\ 0.00486 & 0.000237 & 0.000361 & 0.000527 & 0.000829 & 0.00170 & 1.00 \end{pmatrix} \bar{\varphi}$$

The above transformations diagonalize the system [A.3] and get the lagrangian's mass term to match Kaluza-Klien's mass eigenstates. This provides a nontrivial confirmation that the holographic basis indeed describes the mixing between the elementary (source) and composite CFT sectors.

Appendices

Appendix A

Supplementary On Warped 5D Geometry

A.1 Gauge Hierarchy Problem

We will use the warped geometry to explain the gauge hierarchy problem, i.e., why $m_{Higgs} << M_p$. In RS1, the standard model particles are confined to the IR brane. Consider H be a complex scalar field, representing the Higgs 4 doublet, with the action(A.1):

$$S_H = -\int d^5x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu H^\dagger \partial_\nu H - M_5^2|H|^2 + \lambda|H|^4\right]\delta(y-\pi R) \tag{A.1}$$

$$= -\int d^4x\left[e^{-2\pi kR}\eta^{\mu\nu}\partial_\mu H^\dagger \partial_\nu H - M_5^2 e^{-4\pi kR}|H|^2 + \lambda e^{-4\pi kR}|H|^4\right] \tag{A.2}$$

In the slice of Ads_5, the Higgs mass, M_5 represents a value near the 5D cutoff scale, as expected for a scalar field. The second line in (A.2) is obtained by using the metric (2.1) and performing the y integration. The result is the usual 4D action for the Higgs field, except that the kinetic term is not canonically normalized to one. This can be achieved by rescaling the field $H \Rightarrow e^{\pi kR}H$ leading to:

$$S_H = -\int d^4x\left[\eta^{\mu\nu}\partial_\mu H^\dagger \partial_\nu H - (M_5 e^{-\pi kR})^2|H|^2 + \lambda|H|^4\right]. \tag{A.3}$$

The Higgs mass parameter is now identified as $M_5 e^{-\pi kR}$. The original mass parameter has been scaled down or redshifted by an amount $e^{-\pi kR}$ because the Higgs boson is confined to the IR brane at y = πR. For kr $\simeq$ 11.727, one gets m $\approx$ 1 TeV. Thus, in this picture, the origin of a small Higgs mass lies in the warped geometry of five-dimensional spacetime.

A.2 Bulk Scalar Field Solution

Consider a bulk scalar field ϕ whose action to quadratic order is given by:

$$S_\Phi = -\int d^5x\sqrt{-g}\left[|\partial_M\Phi|^2 + m_\Phi^2|\Phi|^2\right] \tag{A.4}$$

where $m_\phi^2 = ak^2$ is the bulk mass parameter defined in units of curvature scale k with dimensionless coefficient a. We get the equation of motion as:

$$\Box\Phi + e^{2A(y)}\partial_5(e^{-4A(y)}\partial_5\Phi) - m_\phi^2 e^{-2A(y)}\Phi = 0 \tag{A.5}$$

where $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$, $\partial_5 = \frac{\partial}{\partial y}$ and we have taken A(y) from the warp factor (2.11) .The boundary terms vanish provided

$$(\delta\Phi^*\partial_5\Phi)\Big|_{0,\pi R} = 0 \tag{A.6}$$

To solve (A.5) we assume a separation of variables

$$\Phi(x^\mu, y) = \sum_{n=0}^{\infty}\Phi^{(n)}(x^\mu)f_\Phi^{(n)}(y) \tag{A.7}$$

where $\Phi^{(n)}(x^\mu)$ are the 4D Kaluza-Klein modes s satisfying the Klein-Gordon equation $\Box\Phi^{(n)} = m_n^2\Phi^n$ with masses m_n, and $f_\Phi^{(n)}(y)$ is the bulk profile of the Kaluza-Klein mode, which on substitution we get:

$$-\partial_5\left(e^{-4A(y)}\partial_5 f_\Phi^{(n)}\right) + m_\Phi^2 e^{-4A(y)}f_\Phi^{(n)} = m_n^2 e^{-2A(y)}f_\Phi^{(n)} \tag{A.8}$$

The above differential equation (A.8) has the form of a Sturm-Liouville equation where the eigenfunctions form a complete set of orthonormal relations with $w(y) = e^{-2A(y)}$.

$$\int_0^{\pi R} dy\, w(y)\, f_\Phi^{(n)}\, f_\Phi^{(m)} = \delta_{nm} \tag{A.9}$$

The general solution of the Kaluza-Klein modes for $m_n \neq 0$ is given by:

$$f_{\Phi}^{(n)} = N_{\Phi}^{(n)} \left[\mathrm{P}_{\gamma}^{\alpha}(\tanh(\tilde{y})) + b_{\Phi}^{(n)}\, \mathrm{Q}_{\gamma}^{\alpha}(\tanh(\tilde{y})) \right] \mathrm{sech}^2(\tilde{y}) \tag{A.10}$$

Where $N_{\Phi}^{(n)}$ and $b_{\Phi}^{(n)}$ are arbitrary constants obtained by orthonormal relation (A.9) and boundary conditions respectively. and $\gamma = \frac{1}{2}\left(-1 + \frac{\sqrt{4(mn/k)^2+9w^2}}{w}\right)$. The Code is made available here.

A.3 The Eigenvalue Problem

Inserting the expansion (4.1) into the action (3.2), gives

$$S = S(\varphi^s) + S(\varphi^{(n)}) + S_{\text{mix}} \tag{A.11}$$

where,

$$\begin{aligned} \mathcal{S}(\varphi^s) &= \int d^4x \left[-\frac{1}{2}(\partial_\mu \varphi^s)^2 - \frac{1}{2} M_s^2 (\varphi^s)^2 \right], \\ \mathcal{S}(\varphi^{(n)}) &= \int d^4x \sum_{n=1}^{\infty} \left[-\frac{1}{2}(\partial_\mu \varphi^{(n)})^2 - \frac{1}{2} M_n^2 (\varphi^{(n)})^2 \right], \\ \mathcal{S}_{\text{mix}} &= \int d^4x \sum_{n=1}^{\infty} \left[-z_n \partial_\mu \varphi^s \partial^\mu \varphi^{(n)} - \mu_n^2 \varphi^s \varphi^{(n)} \right]. \end{aligned} \tag{A.12}$$

We see that the two sectors mix in a nontrivial way via kinetic mixing z_n and mass mixing μ_n^2, both of which can be computed from wavefunction overlap integrals:

$$z_n = \int_0^{\pi R} dy\, e^{-2A(y)} g^s g_{\varphi}^{(n)}, \tag{A.13}$$

$$\mu_n^2 = \int_0^{\pi R} dy\, e^{-4A(y)} \left[\partial_s g^s \partial_s g_{\varphi}^{(n)} + g^s g_{\varphi}^{(n)} \left(ak^2 + 2bk \left(\delta(y) - \delta(y - \pi R) \right) \right) \right], \tag{A.14}$$

The kinetic mixing $z_n \neq 0$ means that the functions$g^s(y)$ and $g^n(y)$ form a nonorthogonal basis. The system can also be represented more compactly in matrix notation:

$$\mathcal{L} = \frac{1}{2}\vec{\varphi}^{\,\mathrm{T}}\mathbf{Z}\Box\vec{\varphi} - \frac{1}{2}\vec{\varphi}^{\,\mathrm{T}}\mathbf{M}^2\vec{\varphi} \tag{A.15}$$

where the mixing matrices are defined as

$$\mathbf{Z} = \begin{pmatrix} 1 & z_1 & z_2 & z_3 & \cdots \\ z_1 & 1 & 0 & 0 & \cdots \\ z_2 & 0 & 1 & 0 & \cdots \\ z_3 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{A.16}$$

$$\mathbf{M}^2 = \begin{pmatrix} M_s^2 & \mu_1^2 & \mu_2^2 & \mu_3^2 & \cdots \\ \mu_1^2 & M_1^2 & 0 & 0 & \cdots \\ \mu_2^2 & 0 & M_2^2 & 0 & \cdots \\ \mu_3^2 & 0 & 0 & M_3^2 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \ddots \end{pmatrix} \tag{A.17}$$

we now diagonalize the system as in [7] and get the mass term of the lagrangian to match Kaluza-Klien mass eigenstates exactly:

$$\mathbf{m}^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ 0 & m_1^2 & 0 & 0 & \cdots \\ 0 & 0 & m_2^2 & 0 & \cdots \\ 0 & 0 & 0 & m_3^2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{A.18}$$

This provides a nontrivial confirmation that the holographic basis indeed describes the mixing between the elementary (source) and composite CFT sectors.

A.4 Code Availability

All codes used in this thesis is made available Here

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