Data Structure:
AVL Tree

Binary Search Tree

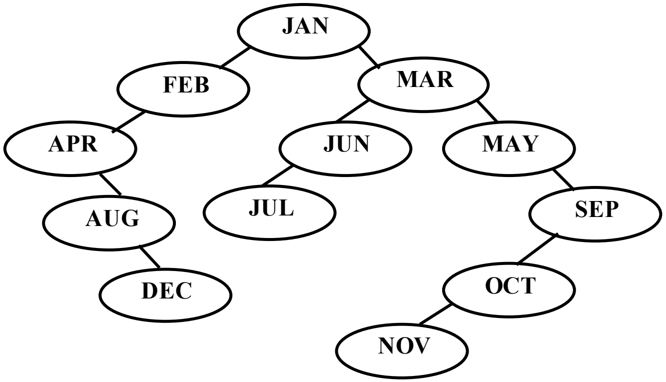
- for every node X in the tree,
  - the values of all the keys in its left subtree are smaller than the key value in X
  - the values of all the keys in its right subtree are larger than the key value in X

Binary Search Tree

Build binary search tree with Jan … Dec

Binary Search Tree

Build binary search tree with Jan … Dec

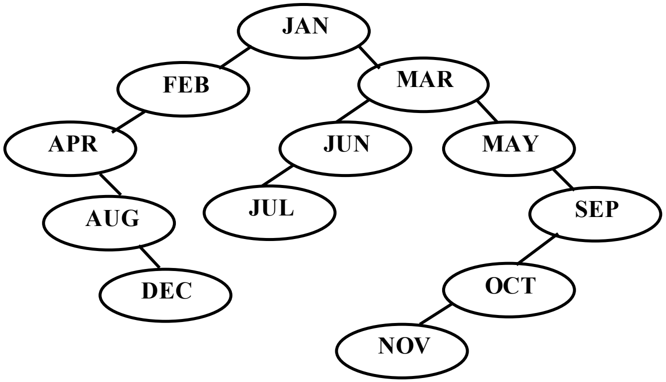


How many comparison do you need to search NOV?

what is the average number of comparisons?

Binary Search Tree

Build binary search tree with Jan … Dec



How many comparison do you need to search NOV?

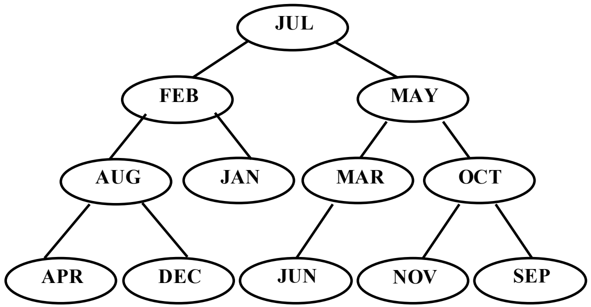
What is the average number of comparisons?

1 (JAN) + 2 (FEB) + 2 (MAR) + 3 (APR) + 3 (MAY) + 3 (JUN) +
4 (JUL) + 4 (AUG) + 4 (SEP) + 5 (OCT) + 6 (NOV) + 5 (DEC) = 42

42 / 12 = 3.5

Binary Search Tree

Insert JUL, FEB, MAY, AUG, DEC, MAR, OCT, APR, JAN, JUN, SEP, and NOV



0 1
2 2 3
3 4 5 6 7
4 8 9 10 11 12

What is the maximum number of comparison?

What is the average number of comparisons?

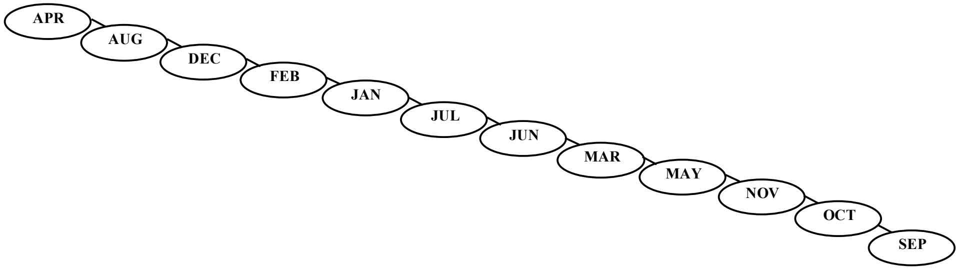
1 + 4 + 12 + 20

3 (JAN) + 2 (FEB) + 3 (MAR) + 4 (APR) + 2 (MAY) + 4 (JUN) +
1 (JUL) + 3 (AUG) + 4 (SEP) + 3 (OCT) + 4 (NOV) + 4 (DEC) = 37 37

37 / 12 = 3.1

Binary Search Tree

What if you insert the key in lexicographical order?



What is the maximum number of comparison?

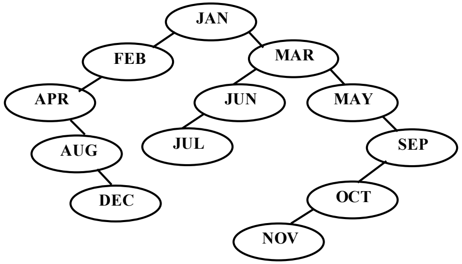
What is the average number of comparisons?

$1 + 2 + \ldots + 12 = 78$

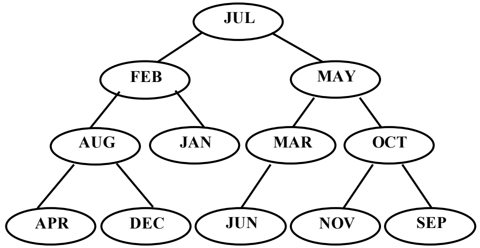
$78 / 12 = 6.5$

Binary Search Tree

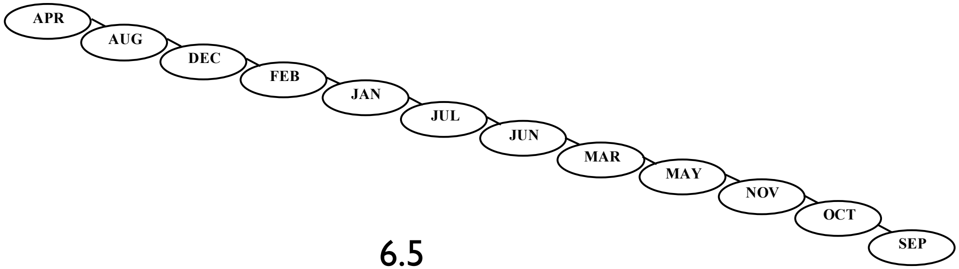
If equal probability, the average search and insertion time is O(logn)



3.5



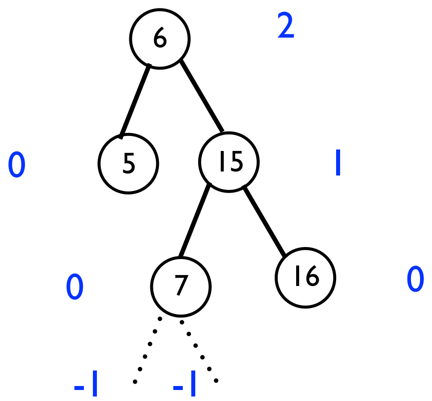
3.1



6.5

AVL (Adelson-Velskii and Landis) Tree

- binary search tree
- for every node in the tree, the heights of its left subtree and right subtree differ by at most 1.
  - the height of a null subtree is -1
  - the height of a subtree with one node is 0



h = height of the subtree

AVL (Adelson-Velskii and Landis) Tree

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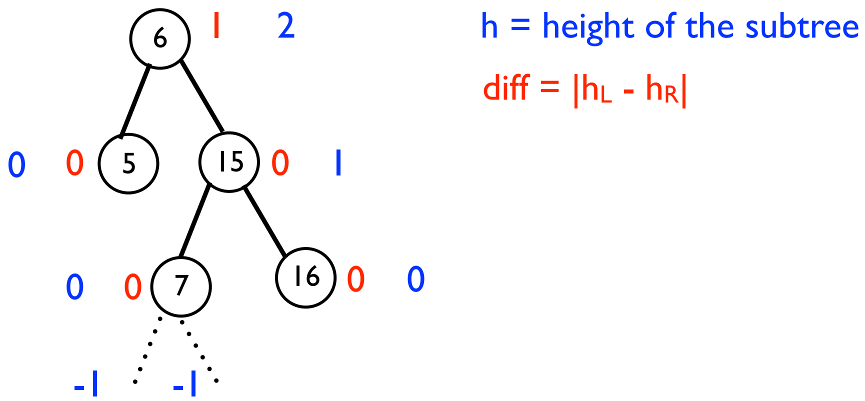


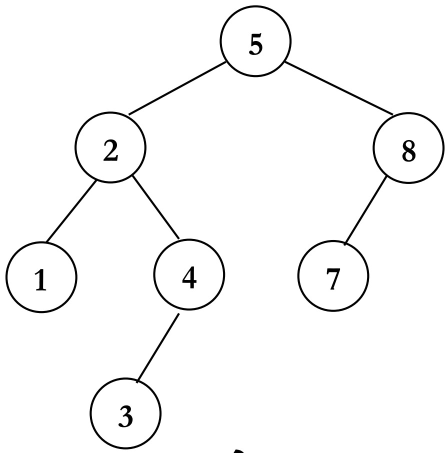
AVL (Adelson-Velskii and Landis) Tree

- An empty tree is height-balanced
- If T is a nonempty binary tree with T_L and T_R as its left and right subtree , T is height-balanced iff

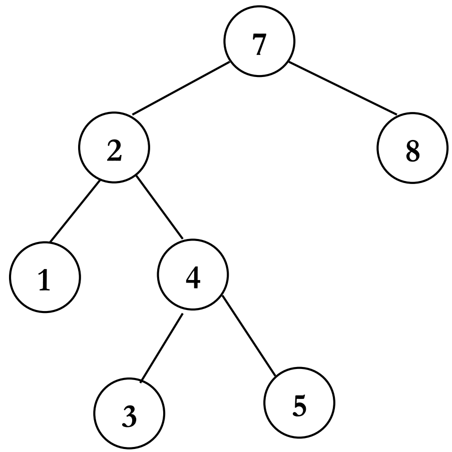
  (1) T_L and T_R are height-balanced and

  (2) $|h_L - h_R| <= 1$ where h_L and h_R are the heights of T_L and T_R

AVL (Adelson-Velskii and Landis) Tree

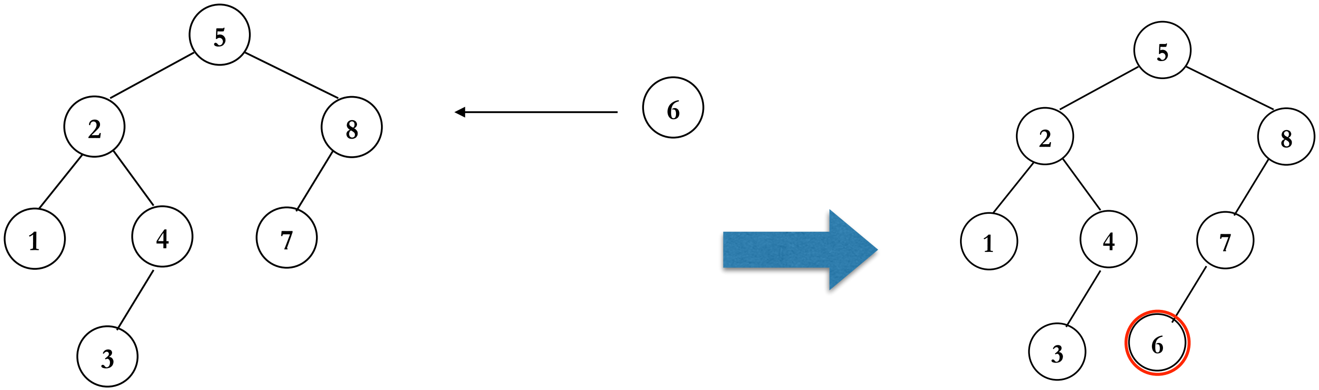


?

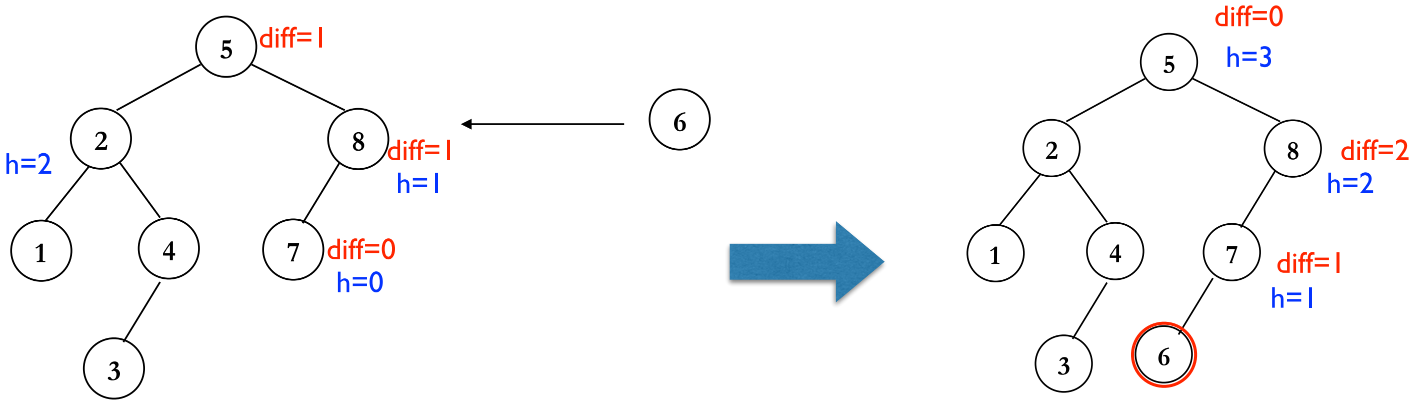


?

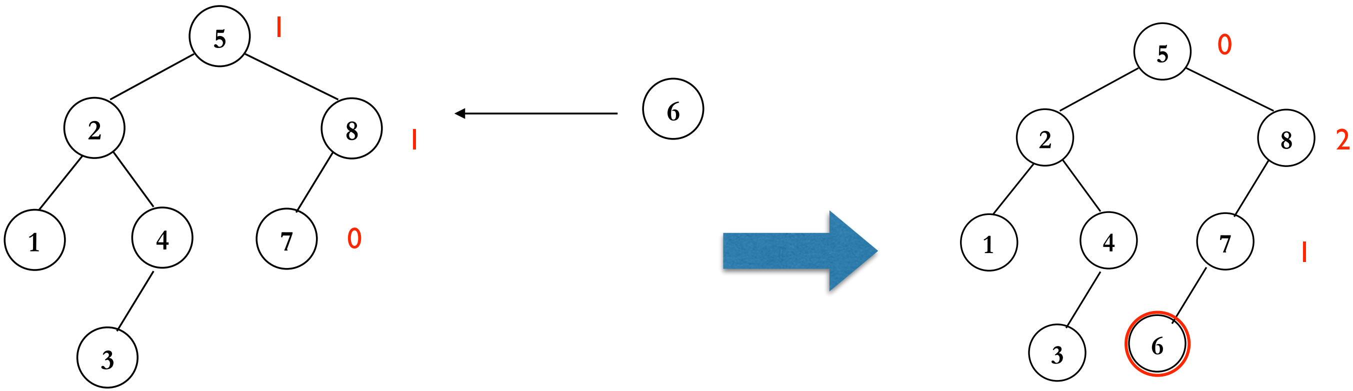
AVL Tree: insertion



AVL Tree: insertion



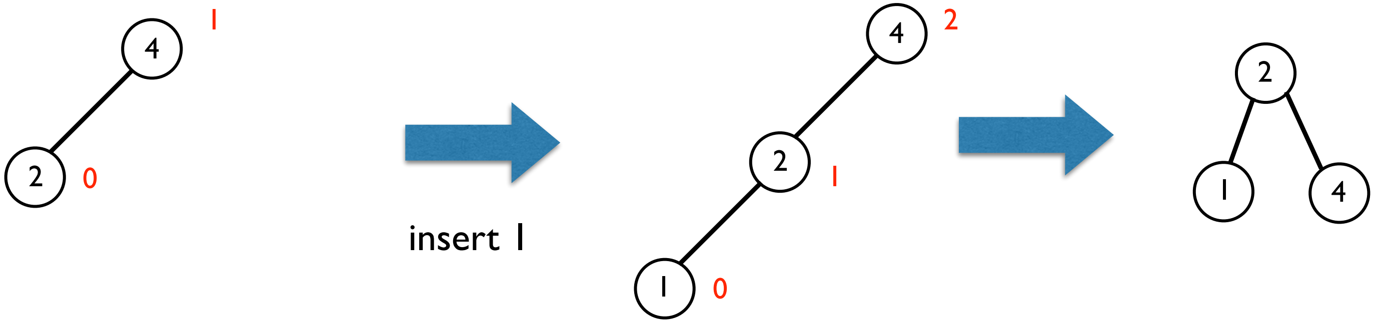
AVL Tree: insertion



- case 1: an insertion into the left subtree of the left child
  - single rotation

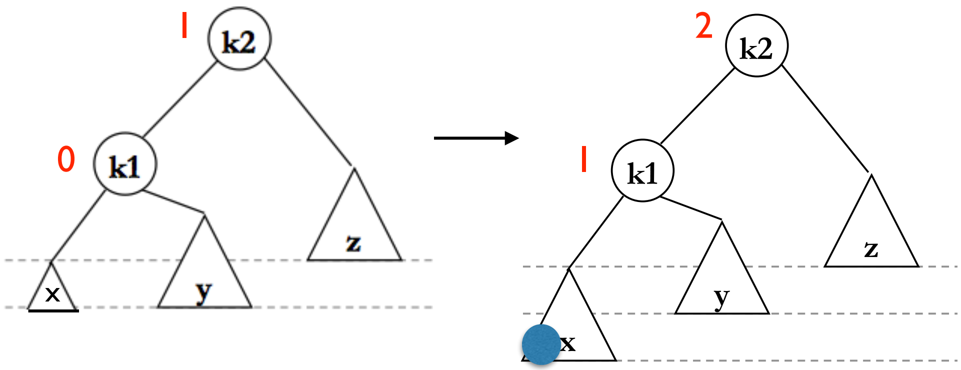
AVL Tree: insertion

- case 1: an insertion into the left subtree of the left child
  - single rotation



AVL Tree: insertion

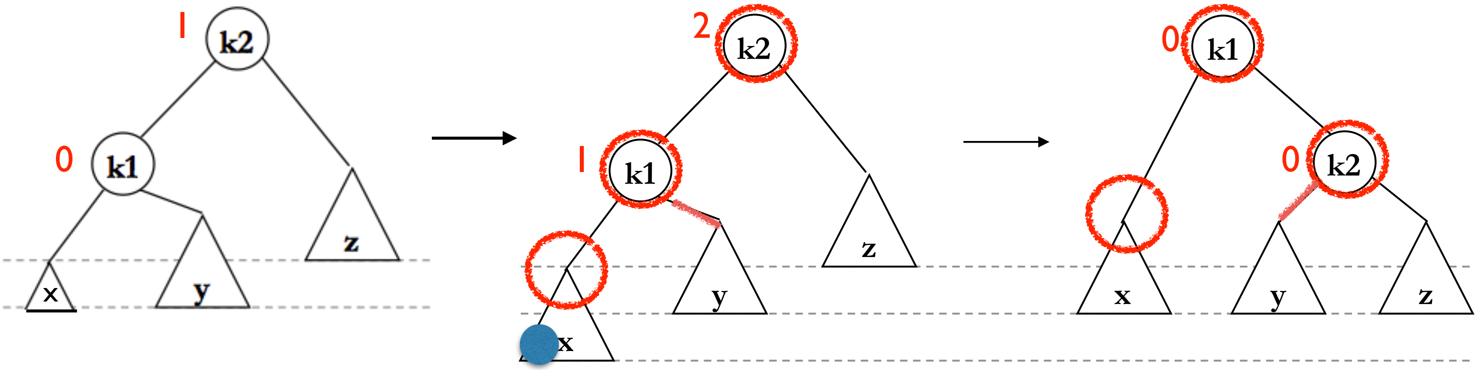
- case 1: an insertion into the left subtree of the left child
  - single rotation



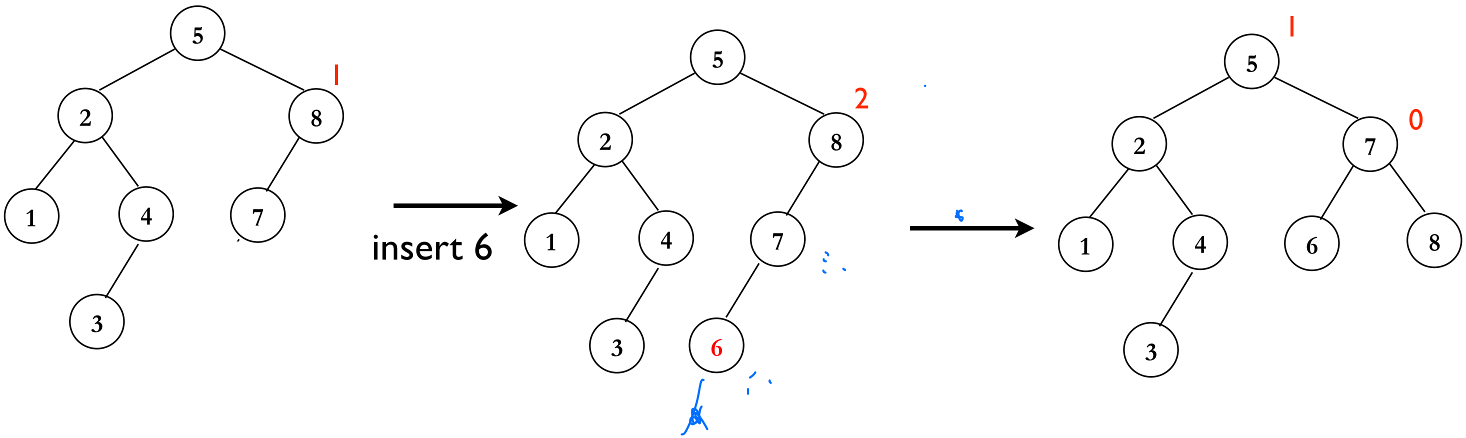
결과 tree의 height는 insertion 이전 tree의 height와 같음. 따라서 k2가 더 큰 tree 안에 embed 되어 있어도 다른 변경은 필요 없음.

AVL Tree: insertion

- case 1: an insertion into the left subtree of the left child
  - single rotation

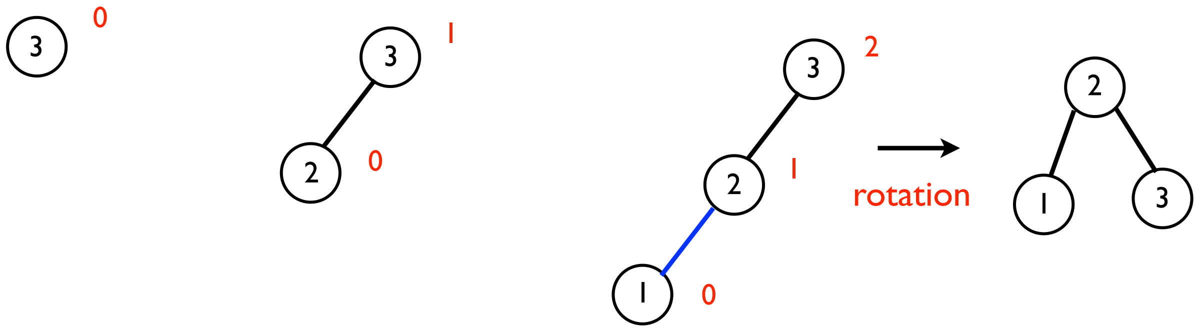


AVL Tree: insertion



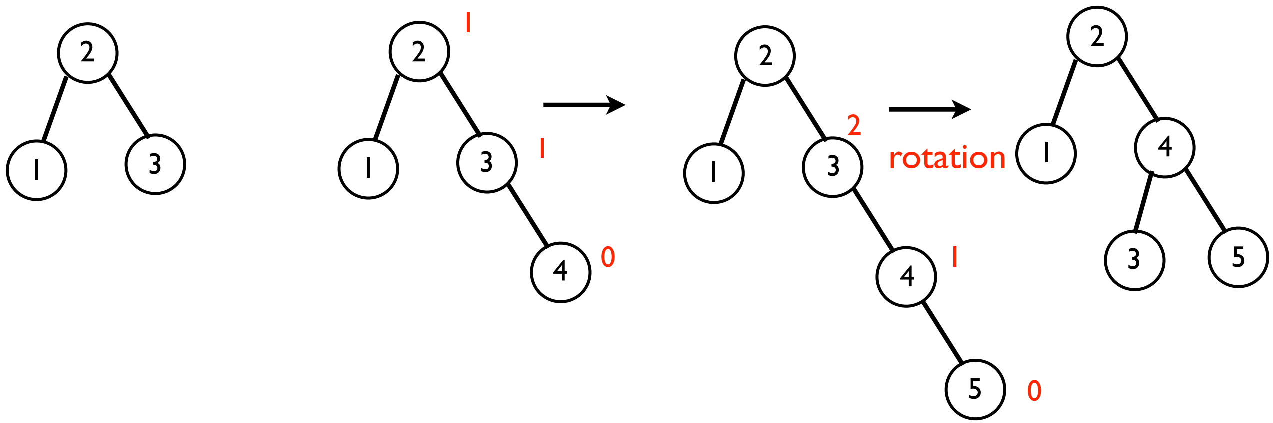
AVL Tree: insertion

insert 3, 2, 1, 4, 5, 6



AVL Tree: insertion

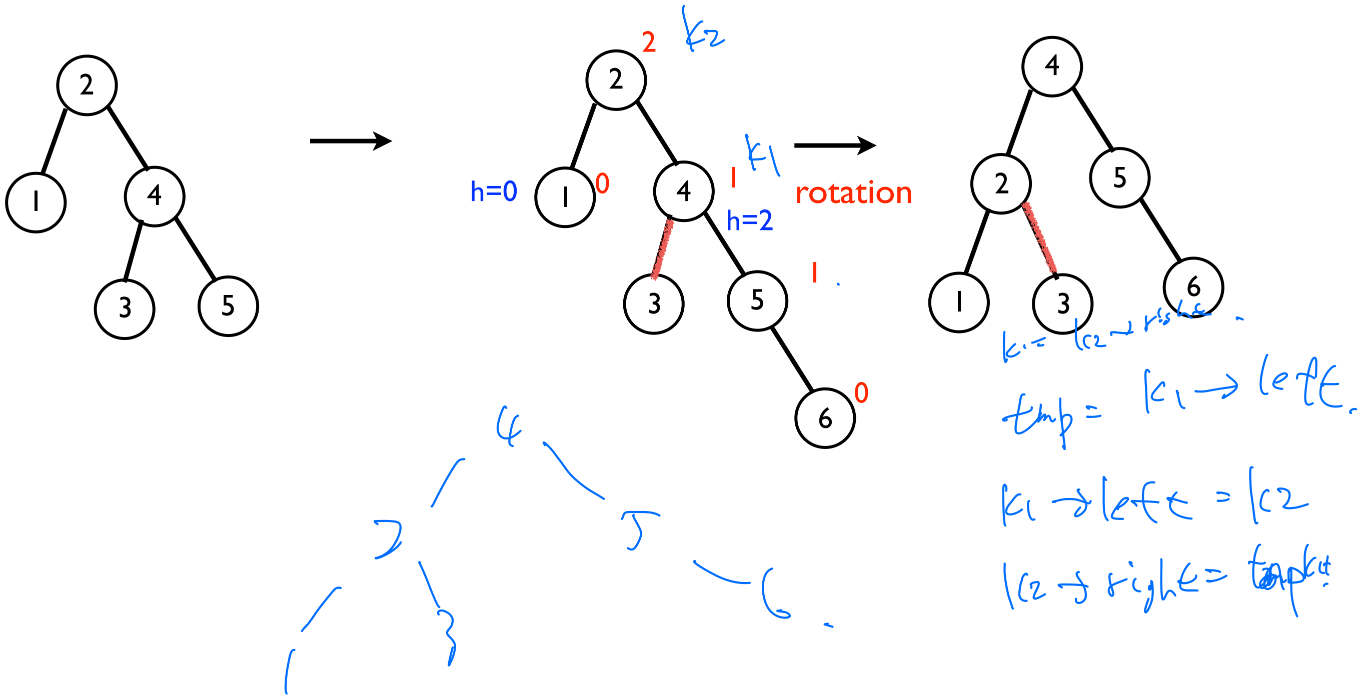
insert 3, 2, 1, 4, 5, 6



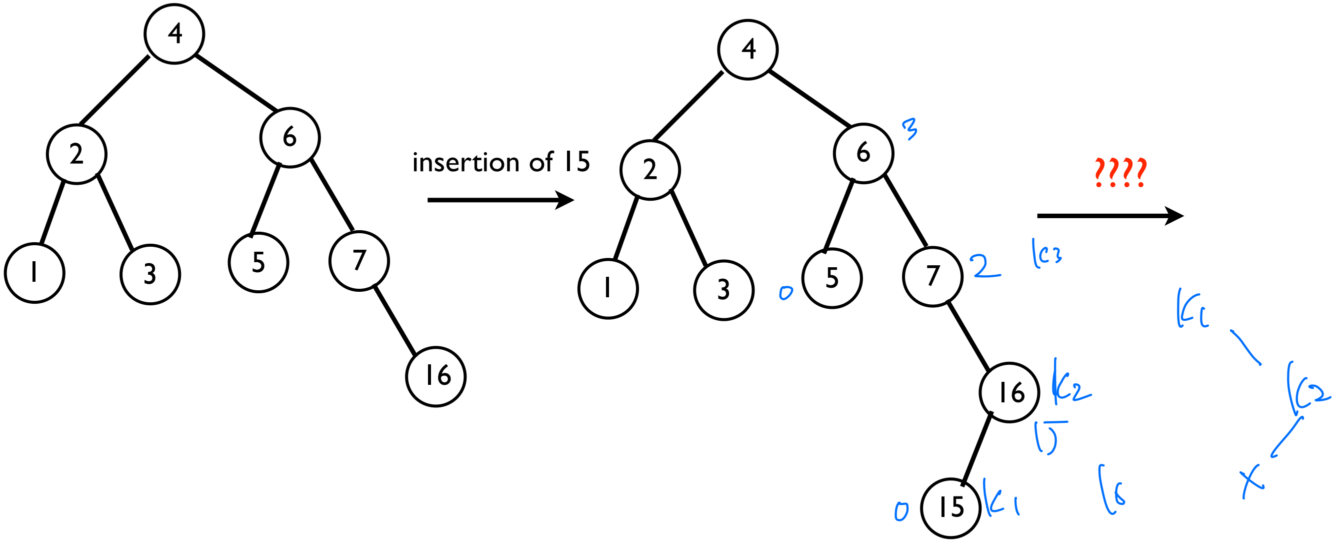
- case 2: an insertion into the right subtree of the right child of the node A
  - single rotation

AVL Tree: insertion

insert 3, 2, 1, 4, 5, 6

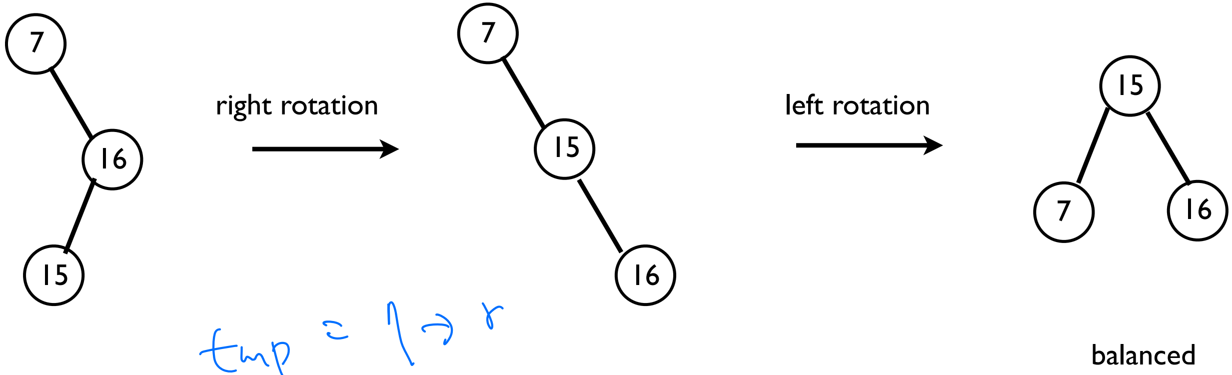


AVL Tree: insertion



AVL Tree: insertion

- case 3: an insertion into the left subtree of the right child of the unbalanced node
  - double rotation

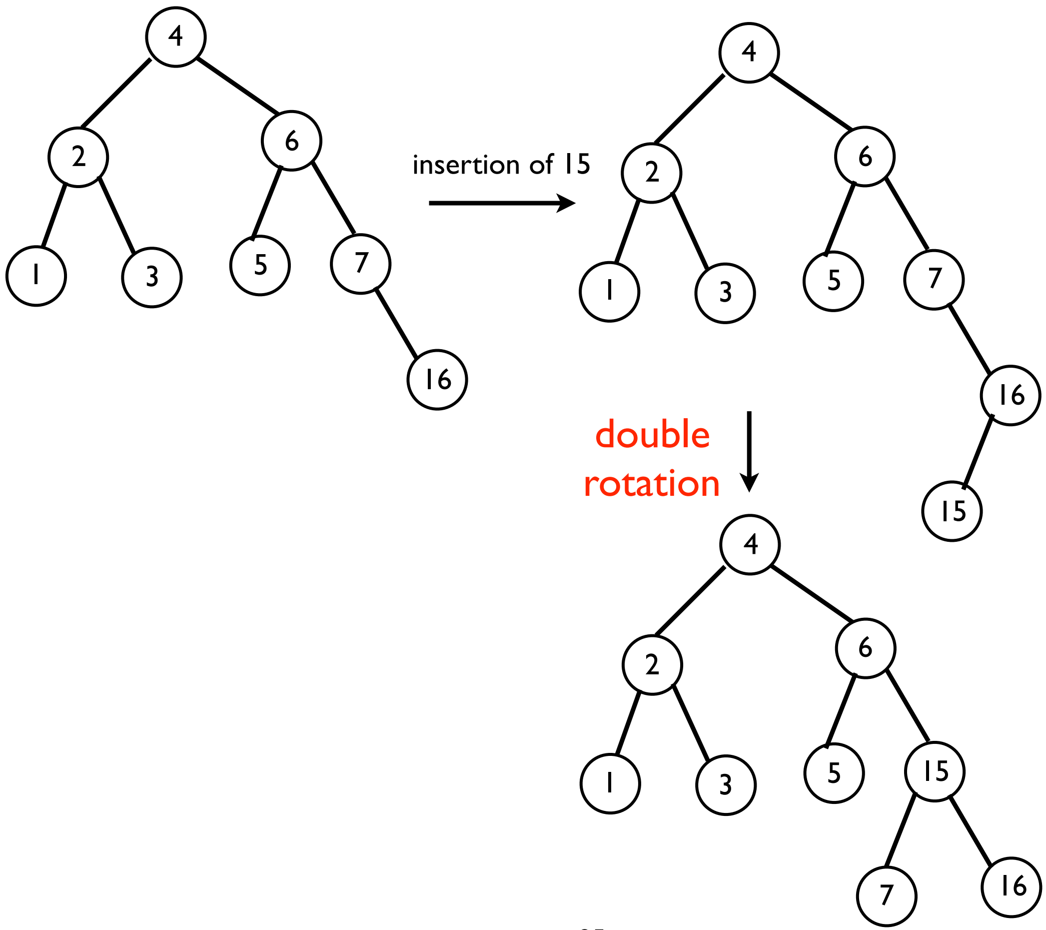


tmp = n→r

n→r = tmp→l

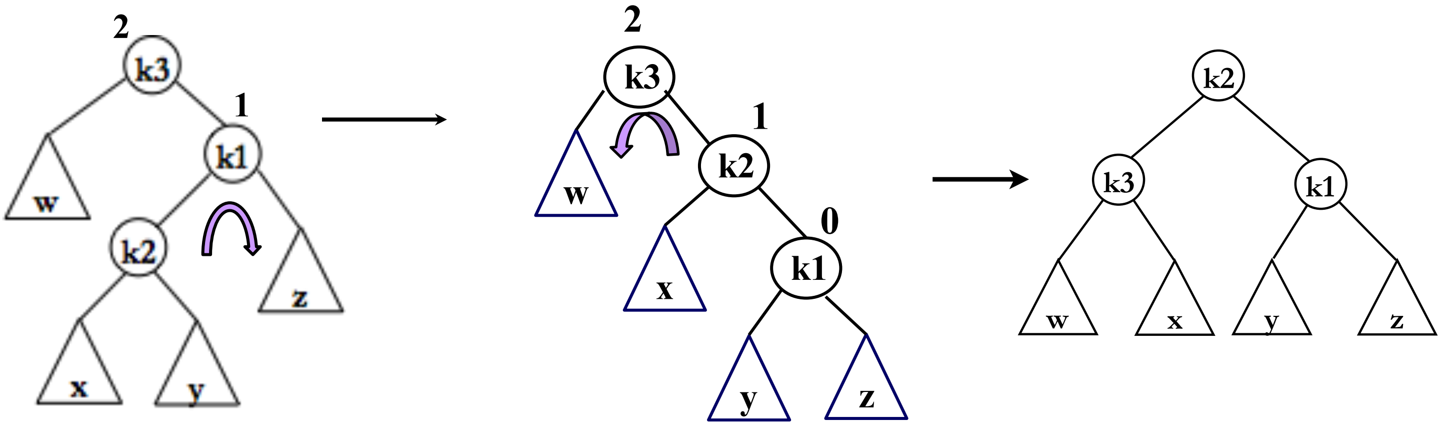
n

AVL Tree: insertion



AVL Tree: insertion

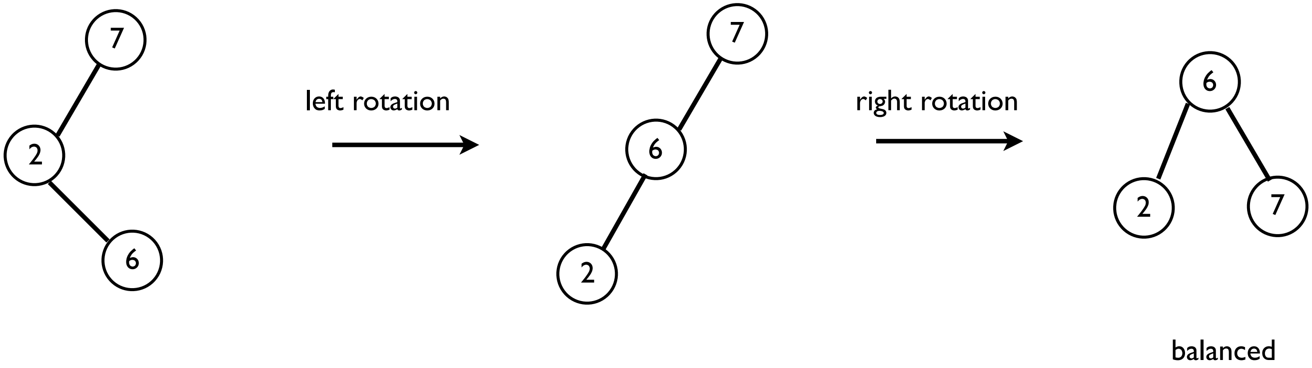
- case 3: an insertion into the left subtree of the right child of the unbalanced node
  - double rotation



right-left double notation

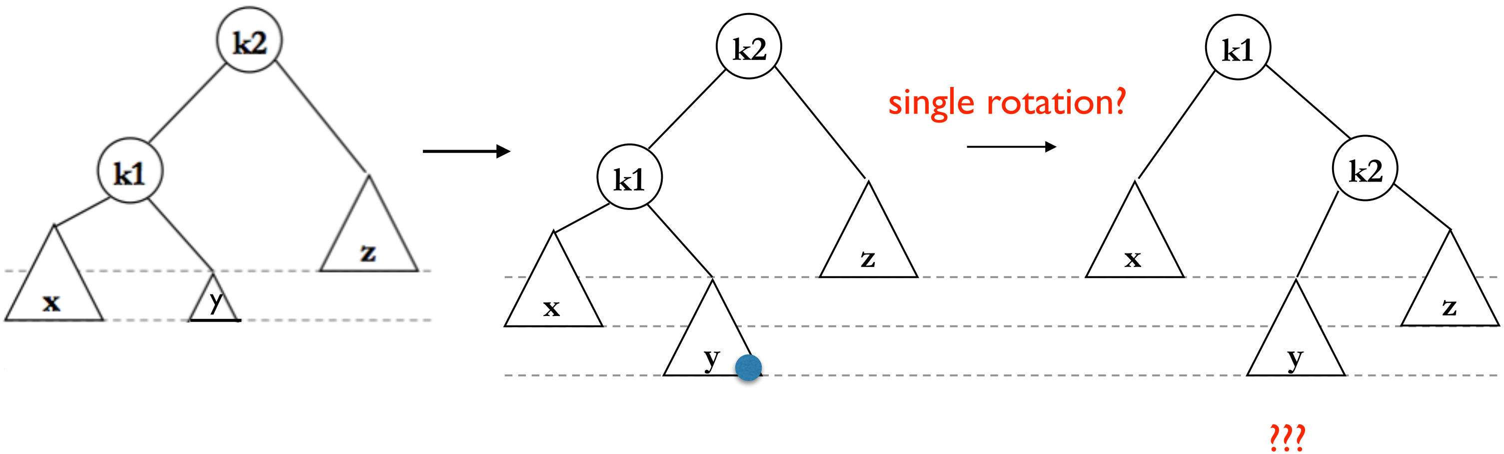
AVL Tree: insertion

- case 4: an insertion into the right subtree of the left child of the unbalanced node
  - double rotation



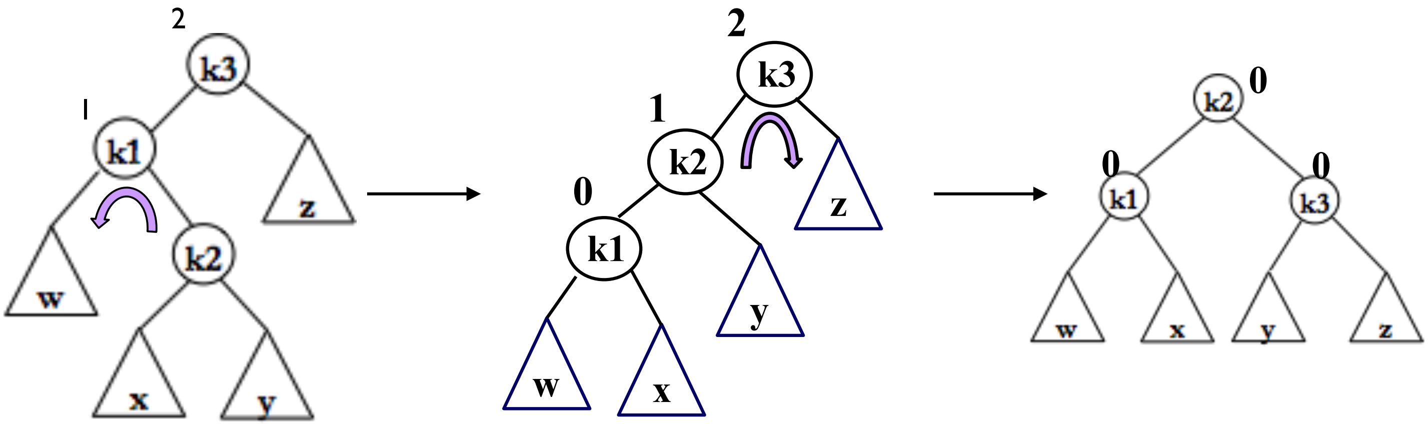
AVL Tree: insertion

- case 4: an insertion into the right subtree of the left child of the unbalanced node
  - double rotation



AVL Tree: insertion

- case 4: an insertion into the right subtree of the left child of the unbalanced node
  - double rotation



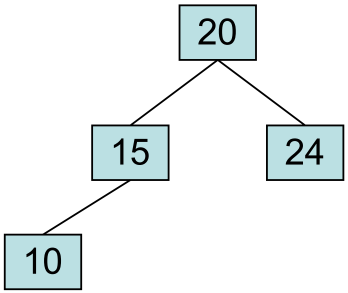
Left-right double notation

AVL Tree: insertion

The node "A" ($|h_L - h_R| > 1$) needs to be rebalanced, when

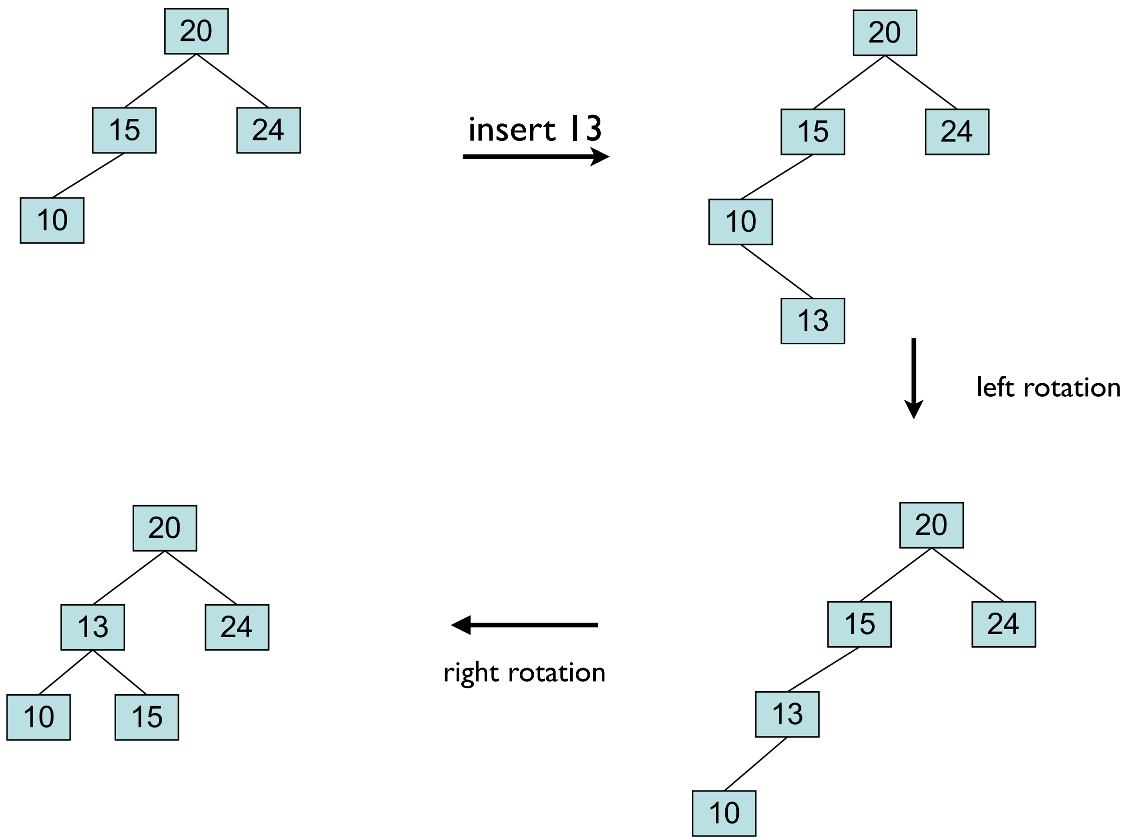
- case 1: an insertion into the left subtree of the left child of the node A
  - single rotation (LL)
- case 2: an insertion into the right subtree of the right child of the node A
  - single rotation (RR)
- case 3: an insertion into the right subtree of the left child of the node A
  - double rotation (LR)
- case 4: an insertion into the left subtree of the right child of the node A
  - double rotation (RL)

AVL Tree: insertion



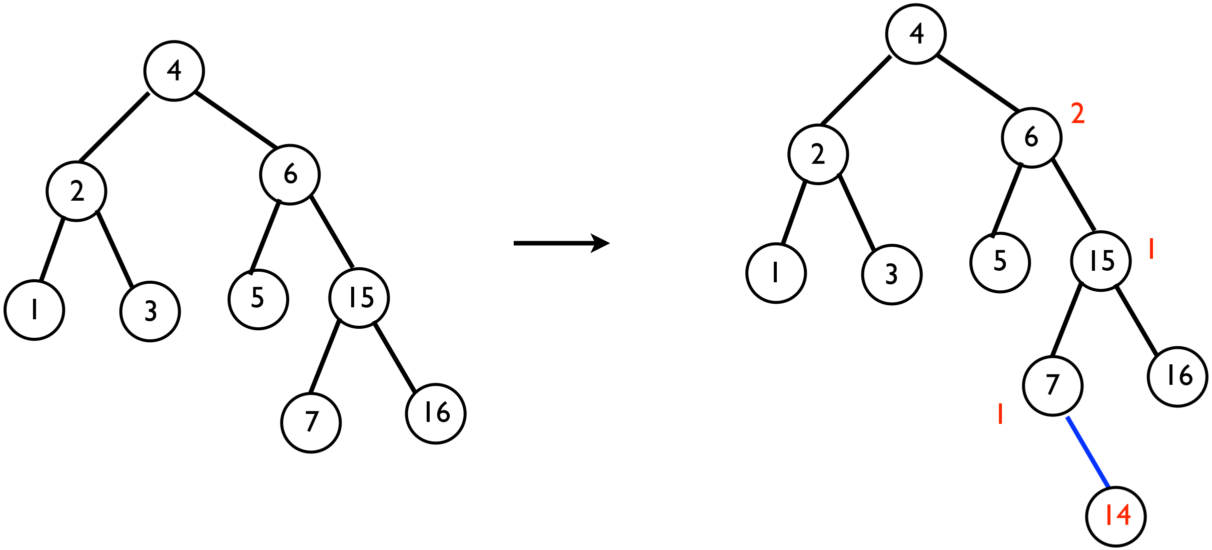
insert 13

AVL Tree: insertion

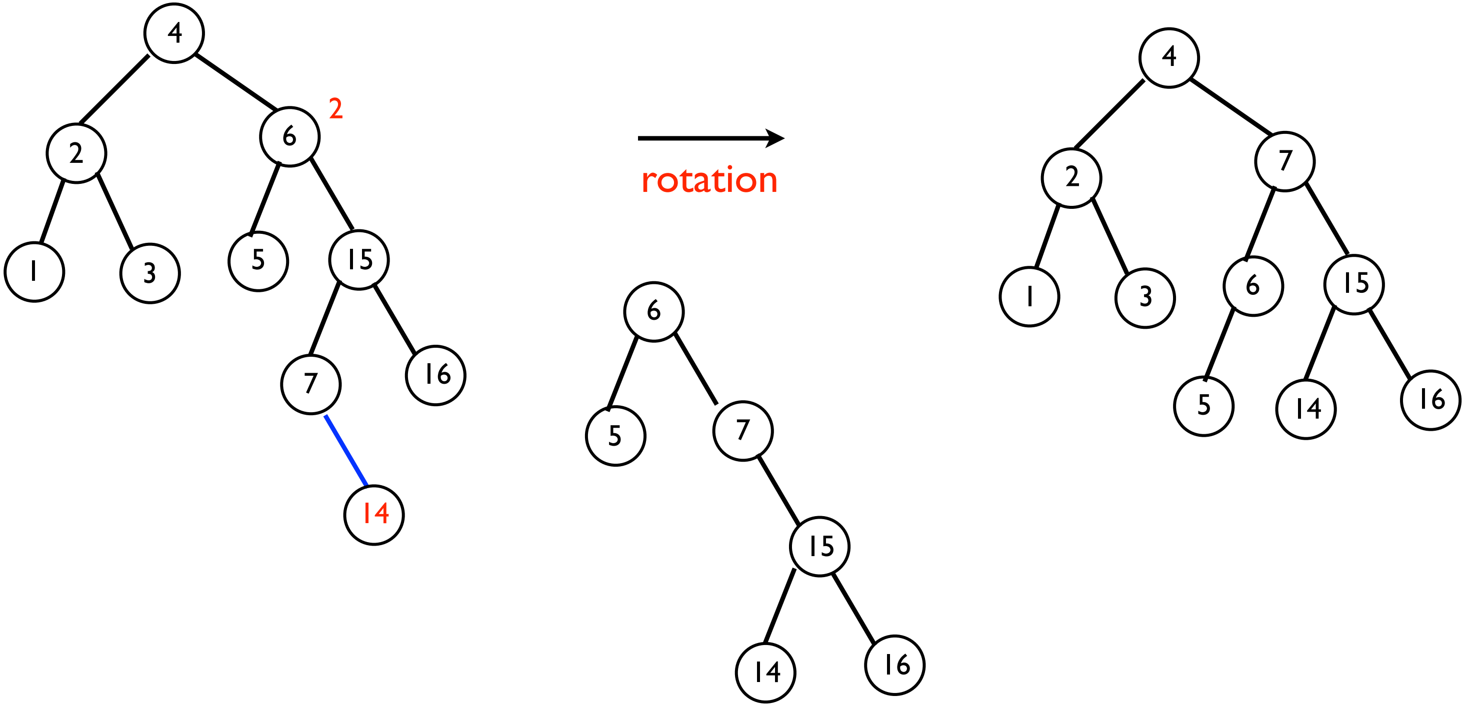


AVL Tree: insertion

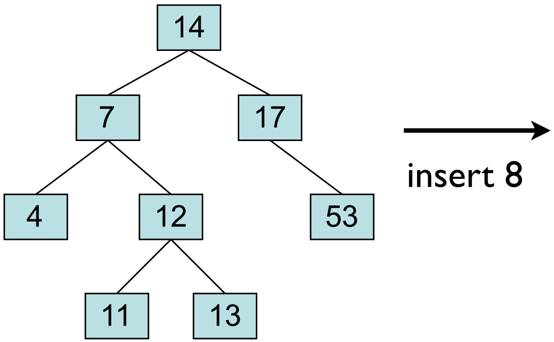
insert 14



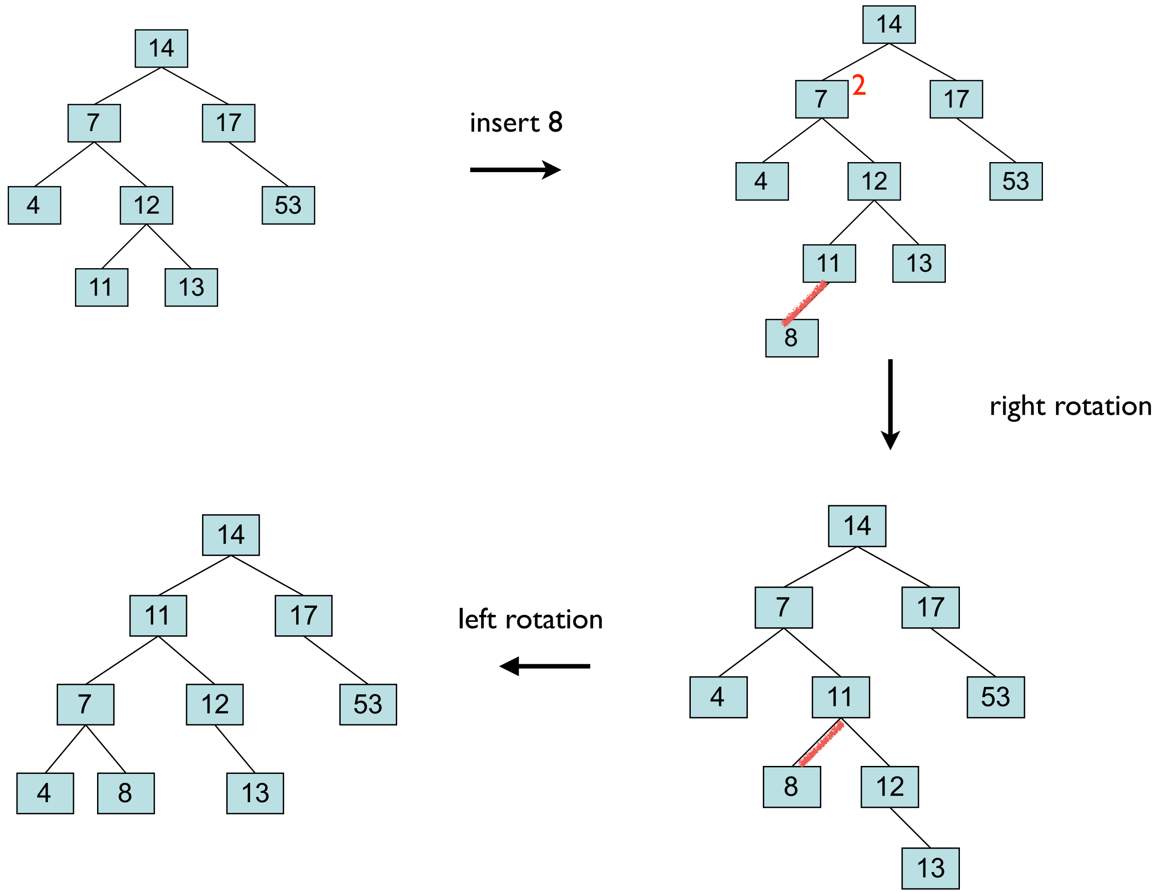
AVL Tree: insertion



AVL Tree: insertion



AVL Tree: insertion



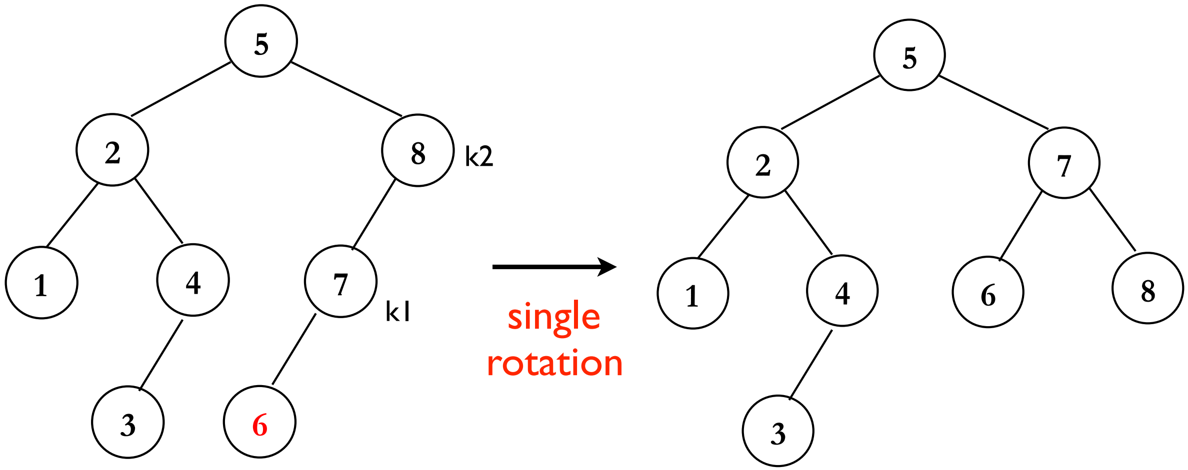
AVL Tree: exercise

Insert sequence: 2, 1, 4, 5, 9, 3, 6, 7
Insert sequence: 3, 2, 1, 4, 5, 6, 7, 16, 15, 14, 13, 12, 11, 10, 8, 9

AVL Tree

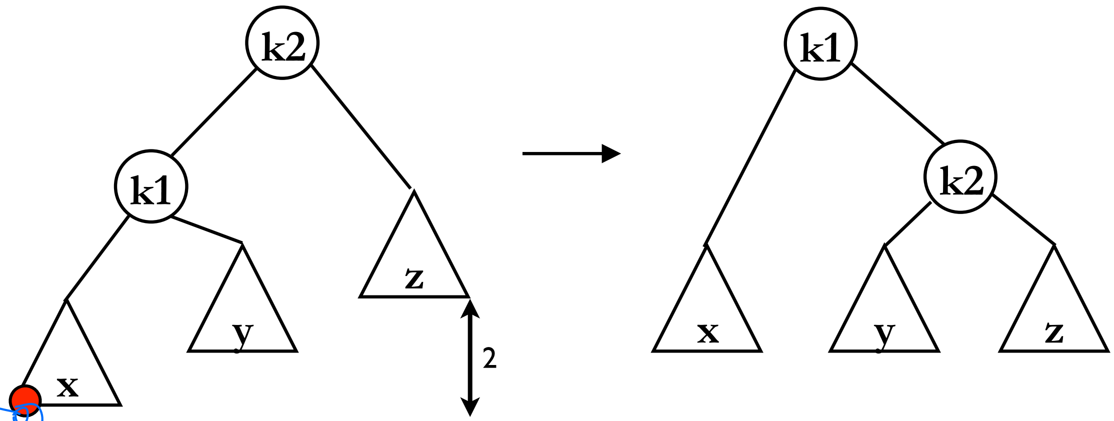
```
struct AVLNode;
typedef struct AVLNode *Position;
typedef struct AVLNode *AVLTree;

struct AVLNode
{
       ElementType Element;
       AVLTree Left;
       AVLTree Right;
       int Height;
}

int Height(Position P)
{
     if (P == NULL)
          return –1;
     else
          return P->Height;
}
```

AVL Tree: insertion



K1->Right = K2;

AVL Tree: insertion



K2->Left = K1->Right;

K1->Right = K2;

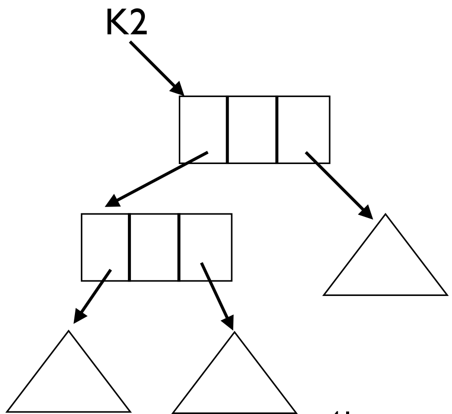
k1 = k2->left

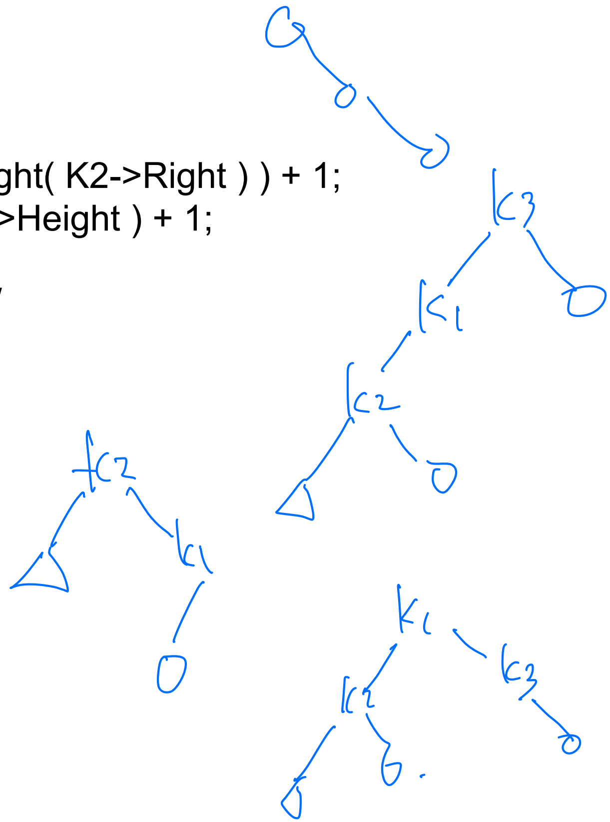
AVL Tree: rotation

k1 = k2->left

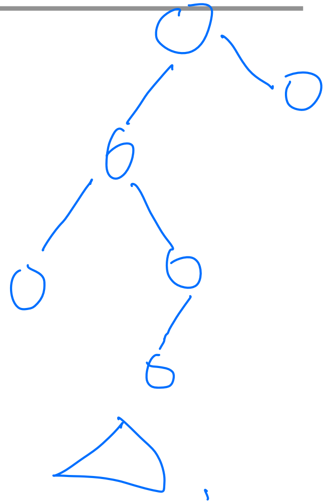
k1->right = k2

```
Position  SingleRotateWithLeft( Position K2 )  /* LL */
{
    Position K1;

    K1 = K2->Left;
    K2->Left = K1->Right;              /* Y */
    K1->Right = K2;

    K2->Height = Max( Height( K2->Left ), Height( K2->Right ) ) + 1;
    K1->Height = Max( Height( K1->Left ), K2->Height ) + 1;

    return K1;                         /* New root */
}
```

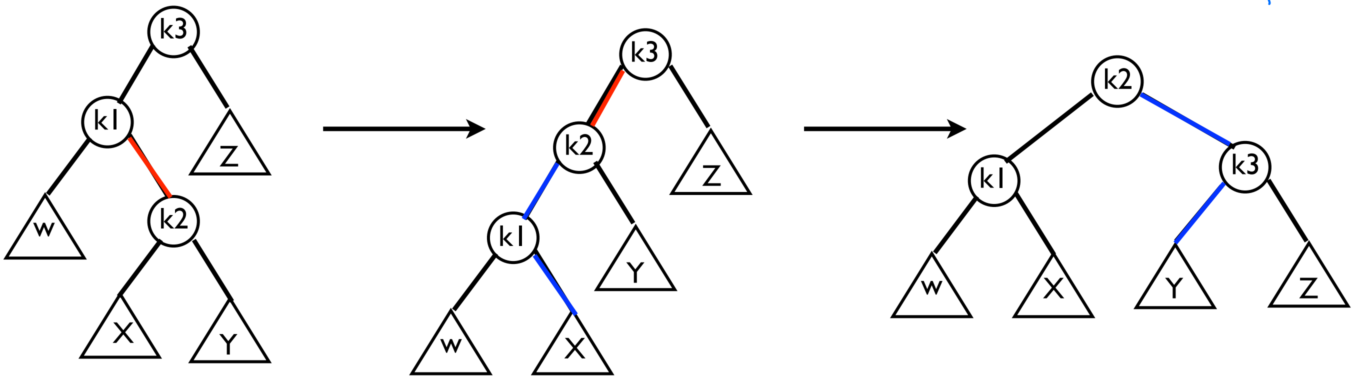




AVL Tree: rotation



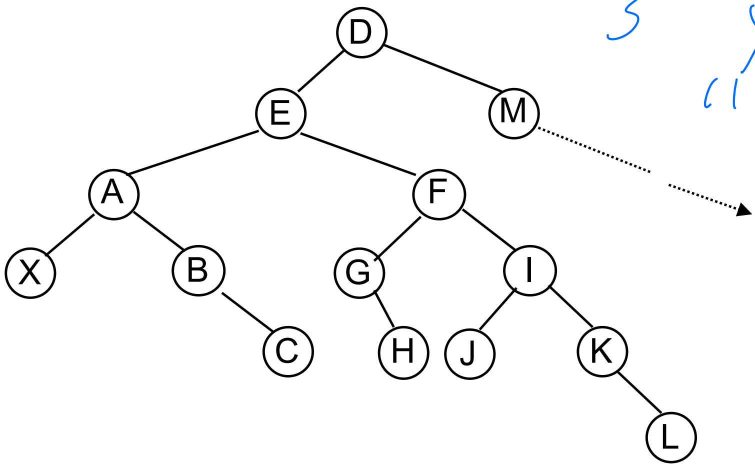
```
static Position DoubleRotateWithLeft  ( Position K3 )     /*LR */
{
     /* rotate between K1 and K2  */
     K3->Left = SingleRotateWithRight( K3->Left );   /* k2 *//

     /* rotate between K3 and K2   */
     return SingleRotateWithLeft( K3 );                /* K2  */
}
```

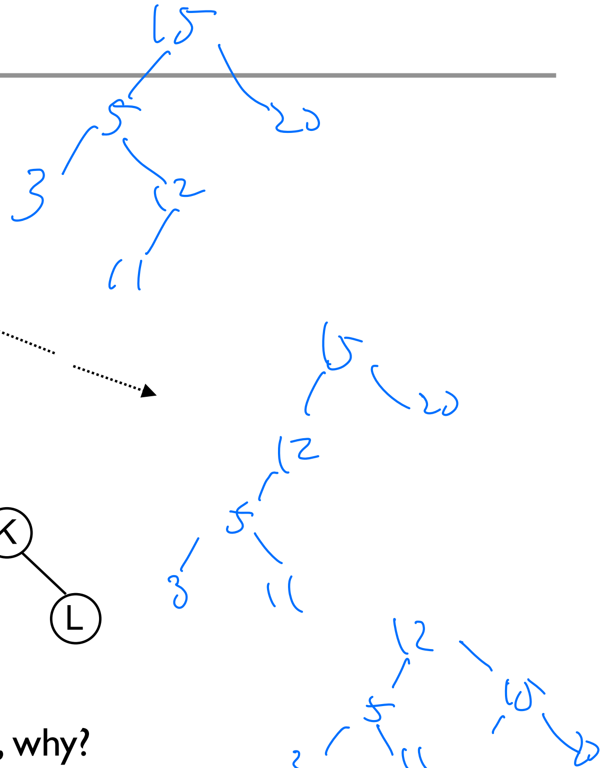


AVL Tree: insertion

```
AVLTree Insert( ElementType X, AVLTree T )   {

    if( T == NULL ) {                              /* found the right place for insertion*/
        T = malloc( sizeof( struct AVLNode ) );
        if( T == NULL )
            FatalError( "Out of space!!!" );
        else  {
            T->Element = X; T->Height = 0;
            T->Left = T->Right = NULL;
        }
    }  else if ( X < T->Element )  {
        T->Left = Insert( X, T->Left );                /* BST*/
        if( Height( T->Left ) – Height( T->Right ) == 2 )
            if( X < T->Left->Element )
                T = SingleRotateWithLeft( T );
            else
                T = DoubleRotateWithLeft( T );
    } else if( X > T->Element )   {
      T->Right = Insert( X, T->Right );
      if( Height( T->Right ) – Height( T->Left ) == 2 )
          if( X > T->Right->Element )
            T = SingleRotateWithRight( T );
          else
            T = DoubleRotateWithRight( T );
          }
    }
    T->Height = Max( Height( T->Left ), Height( T->Right ) ) + 1;
    return T;
}
```

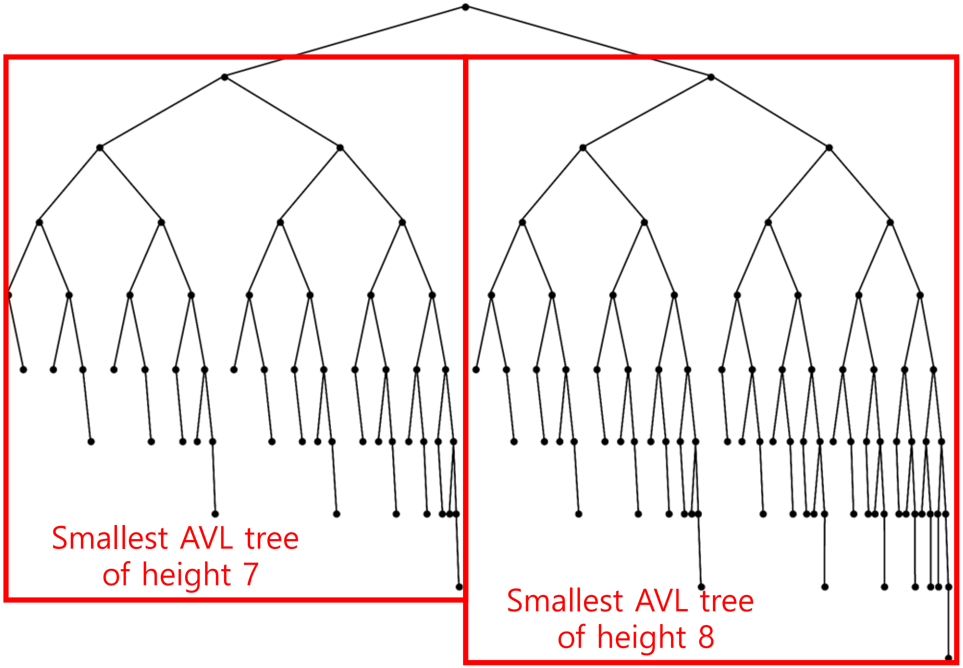
AVL Tree: deletion





- Restructuring occurs only once by insertion, why?
- How should we apply rotations for deletion of X in the AVL tree shown above, for example?
- How many restructuring do we need for deletion?

Smallest AVL Tree of height 9



Height of AVL Tree

Denote N_h the minimum number of nodes in an AVL tree of height h

$N_0 = 1$, $N_1 = 2$ (base)

$N_h = N_{h-1} + N_{h-2} + 1$ (recursive definition)

$N > N_h = N_{h-1} + N_{h-2} + 1$

$> 2 \times N_{h-2} > 4 \times N_{h-4} > \ldots > 2^i \times N_{h-2i}$

If h is even, let $i = h/2 - 1$.

The equation becomes $N > 2^{h/2-1} N_2$, $N > 2^{h/2-1} \times 4$. $h = O(\log N)$

If h is odd, let $i = (h - 1)/2$.

The equation becomes $N > 2^{(h-1)/2} N_1$, $N > 2^{(h-1)/2} \times 2$. $h = O(\log N)$

Thus, many operations (i.e. searching) on an AVL tree will take $O(\log N)$ time.