CS1231 Discrete Structures
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Tutorial 1 Summary Sheet

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Consultation Hours: Flexible through Telegram
To be discussed: 1.1, 1.2, 1.3(iii), 1.4, 1.5, 1.6, 1.7

1 Propositions

Definition 1.1. A **proposition** is a declarative sentence that is true or false. Let p and q be propositions. We define the following propositions.

Proposition	$p \wedge q$	$p \vee q$	$\neg p$	$p \to q$	$p \leftrightarrow q$	t	c
Notation	**Conjunction**	**Disjunction**	**Negation**	**Conditional**	**Biconditional**	**Tautology**	**Contradiction**

The truth values of each proposition is summarised as follows.

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$	t	c
T	T	F	T	T	T	T	T	F
T	F	F	F	T	F	F	T	F
F	T	T	F	T	T	F	T	F
F	F	T	F	F	T	T	T	F

Two propositions are **logically equivalent** if their truth values are the same. Order of operations: $(\neg) > (\wedge, \vee) > (\to, \leftrightarrow)$.

Theorem 1.2 (Logical Equivalences)**.** Let p, q, r be propositions. We have the following logical equivalences.

Name	**Commutativity**		**Associativity**		**Distributivity**	
Proposition	$p \wedge q$	$p \vee q$	$p \wedge (q \wedge r)$	$p \vee (q \vee r)$	$p \wedge (q \vee r)$	$p \vee (q \wedge r)$
Equivalent	$q \wedge p$	$q \vee p$	$(p \wedge q) \wedge r$	$(p \vee q) \vee r$	$(p \wedge q) \vee (p \wedge r)$	$(p \vee q) \wedge (p \vee r)$

Name	**Implication**	**Double negation**	**Absorption**		**Negation**	
Proposition	$p \to q$	$\neg(\neg p)$	$p \wedge (p \vee q)$	$p \vee (p \wedge q)$	$p \wedge (\neg p)$	$p \vee (\neg p)$
Equivalent	$\neg p \vee q$	p	p	p	$\mathbf{false}$	$\mathbf{true}$

Name	**de Morgan's**		**Identity**	
Proposition	$\neg(p \wedge q)$	$\neg(p \vee q)$	$p \wedge \mathbf{true}$	$p \vee \mathbf{false}$
Equivalent	$(\neg p) \vee (\neg q)$	$(\neg p) \wedge (\neg q)$	p	p

Definition 1.3. Let $p \to q$ be a conditional statement. We define three related propositions.

Proposition	$\neg p \to \neg q$	$q \to p$	$\neg q \to \neg p$
Name	**Inverse**	**Converse**	**Contrapositive**

Theorem 1.4. We have $(p \to q) \equiv (\neg q \to \neg p)$ and $(\neg p \to \neg q) \equiv (q \to p)$.