NATIONAL UNIVERSITY OF SINGAPORE

CS1231 - DISCRETE STRUCTURES

(SEMESTER 2 AY 2018/2019)

Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This assessment paper contains **FIVE** questions and comprises **EIGHT** printed pages, including this page.
2. Answer **ALL** questions within the space in this booklet.
3. This is a Closed Book assessment.
4. Candidates are allowed to bring in an A4-sized help sheet.
5. Calculators are allowed.
6. Please write your Student Number below. Do not write your name.

Student NO: ______________________________

Question	Marks	Remarks
A(1)-(3) (Pg2)		
A(4)-(7) (Pg2)		
A(8)-(10) (Pg3)		
A(11)-(14) (Pg3)		
B		
C		
D		
E		
Total		

Question A [40 marks]. *For each of the following, just write down the answers in the spaces provided. Detailed workings are not required. Also numerical answers are to be written as integers. For example, you can write 2300, but not* $\binom{5}{1}\binom{3}{1}$ *or* $2^5 \cdot 3^{27}$.

(1) Find -9876 **Mod** 101.

(2) Is 2029 a prime number? Yes / No

(3) (i) Find the number of different ways one can travel in the xy-plane (i.e. Cartesian plane) from $(1,2)$ to $(5,9)$ if each move is one of the following types (R) and (U):

$$(R) : (x,y) \to (x+1,y) \qquad (U) : (x,y) \to (x,y+1)$$

(ii) Find the number of different ways one can travel in the xy-plane from $(1,2)$ to $(5,9)$, if a third (diagonal) move

$$(D) : (x,y) \to (x+1,y+1)$$

is also allowed (i.e. each move is one of the types (R), (U) and (D).)

(4) Is the following true? Yes / No

$$\forall n \in \mathbb{Z}^+, \sum_{i=1}^{n} i\binom{n}{i} = n \cdot 2^{n-1}$$

(5) In the complete expansion of $(x+3y-2z)^5$,

(i) find the coefficient of x^2yz^2

(ii) find the number of distinct terms

(iii) find the sum of all coefficients

(6) In how many ways can the letters in MISSISIPPI be arranged in a row such that no two letters of "I" are next to each other?

(7) Three married couples are to be seated in a row. In how many ways can they be arranged so that no wife sits next to her husband?

(8) (i) Is there a graph G with 12 vertices and 23 edges in which the degree of each vertex is either 3 or 6?

Yes / No

(ii) Let V be a set of vertices, E_1, E_2 be sets of edges, and

$$G_1 = (V, E_1), \quad G_2 = (V, E_2) \quad G = (V, E_1 \cup E_2)$$

be three simple connected graphs such that $H = (V, E_1 \cap E_2)$ is disconnected. If $|V| \geq 3$, then is it true that G must have a cycle?

Yes / No

(9) In the hypercube Q_5, find the **length** of the shortest simple path from 01011 to 11000.

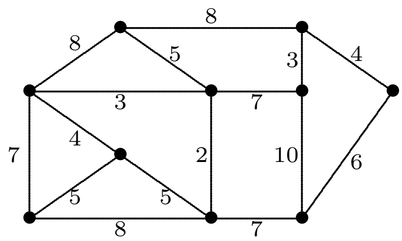
(10) Let T be a tree with at least 2 vertices of degree 5. Find the **minimum** number of vertices of degree 1 in T.

(11) (i) Find the height of a full balanced binary tree with 4095 vertices.

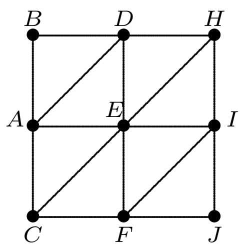
(ii) Find the number of vertices of a full 5-ary tree with 100 internal vertices.

(12) Suppose the universal address of a vertex v in a rooted tree is 7.2.3.7.4.8.3. Find the minimum number of siblings of v.

(13) Find the weight of a minimum spanning tree in the following graph.



(14) Let G be the graph below. Using the alphabetical ordering, find a spanning tree by **depth first** search and by **breadth first** search. Draw the trees below.



Depth First Search

Breadth First Search

Question B [5 marks]. Prove by using mathematical induction that for any **odd positive** integer n,

$$n(n^2 - 1) \text{ is divisible by } 24.$$

Question C [5 marks]. The serial numbers on the tram tickets range from 000000 to 999999 (both inclusive). For example, 720362 is a serial number. Let us call a ticket with number $ABCDEF$ *unlucky* if $A + B + C \neq D + E + F$. For example, consider the ticket with the serial number 720362, $A = 7, B = 2, C = 0, D = 3, E = 6, F = 2$ and $7 + 2 + 0 \neq 3 + 6 + 2$, therefore the ticket with the serial number 720362 is unlucky. Find the largest number of **consecutive** unlucky tickets and justify your answer.

Question D [5 marks]. (i) Show that in every simple graph G, if v is a vertex of odd degree in G, then there is another vertex u of **odd degree** in G, such that there is a path in G from u to v. (Here, the graph G could be connected or not connected.)

(ii) Draw a full m-ary tree with 16 leaves and height 3, where m is a positive integer, or show that no such tree exists.

Question E [5 marks]. Let the public key of a RSA cryptosystem be $(n, e) = (4307, 41)$.

(i) Encrypt the message "GO" using 01 for A, 02 for B, etc.

(ii) Find the Decryption key.

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