NATIONAL UNIVERSITY OF SINGAPORE

CS1231 - DISCRETE STRUCTURES

(SEMESTER 2 AY 2017/2018)

Time allowed: 2 hours

INSTRUCTIONS TO CANDIDATES

1. This assessment paper contains **FIVE** questions and comprises **EIGHT** printed pages, including this page.
2. Answer **ALL** questions within the space in this booklet.
3. This is a Closed Book assessment.
4. Candidates are allowed to bring in an A4-sized help sheet, written on both sides.
5. Calculators are allowed.
6. Please write your Student Number below. Do not write your name.

Student NO: ____________________

Question	Marks	Remarks
A(1-6, Pg 2)		
A(7-10, Pg 2)		
A(Pg 3)		
B		
C		
D		
E		
Total		

Question A [40 marks]. *For each of the following, just write down the answers in the spaces provided. Detailed workings are not required. Also numerical answers are to be written as integers or powers of a single integer. For example, you can write 2300 or* 3^{27} *but neither* $\binom{5}{1}\binom{3}{1}$ *nor* 3!.

(1) Find -5295 **Div** 29.

(2) Find an integer x so that $0 < x < 104$ and $9x \equiv 1 \bmod 104$.

(3) Find the coefficient of x^2 in the expansion of $\left(x - \frac{1}{\sqrt{x}} + \frac{1}{x}\right)^5$.

(4) Find the number of integers in $\{1, 2, 3, \ldots, 2018\}$ which are

(i) multiples of 3 and 4.

(ii) multiples of 4 or 6 but not 5.

(5) How many ways are there to choose 5 integers x, y, z, t, w from the set $\{1, 2, \ldots, 40\}$ so that $x < y < z < t < w$ and $y - x \geq 10$, $z - y \geq 9$, $t - z \geq 8$, $w - t \geq 7$?

(6) An integer n is a perfect square if $n = k^2$ for some $k \in \mathbb{Z}$. Is there an integer that is both a multiple of 2 and a perfect square but not a multiple of 4?

Yes / No

(7) Consider functions from a set with 6 elements to a set with 3 elements.

(i) How many one-to-one functions are there?

(ii) How many onto functions are there?

(8) Let G be a simple graph with 101 vertices.

(i) Is it possible that 50 vertices are of degree 100 and 51 vertices are of degree 20?

Yes / No

(ii) If G has exactly 50 vertices of degree 100, then is it true that such a G must have an Euler circuit?

Yes / No

(9) Find the number of edges in the hypercube Q_5.

(10) Suppose the universal address of a vertex v in a rooted tree is 2.5.2.1.7. Find

(i) The level of v.

(ii) The minimum number of vertices in the tree.

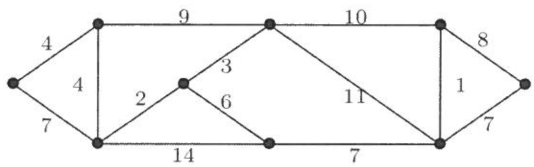
(11) Let T be a full 40-ary tree. How many among the numbers 121, 202, 313, 434, can be the number of vertices of T? (Your answer ranges from 0 to 4.)

(12) How many edges are there in a forest of t trees containing a total of n vertices?

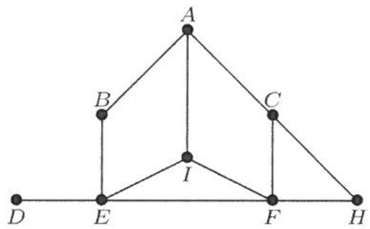
(13) (i) Find the **minimum** values of m if an m-ary tree has at least 600 leaves and height 4.

(ii) Find the **value(s)** of n if a full and balanced n-ary tree has 81 leaves and height 4.

(14) Find the **weight** of a minimum spanning tree in the following graph.



(15) Let G be the following graph. Using the alphabetical ordering, find a spanning tree by **depth first** search. Draw the tree below.



Graph G

Question B [5 marks]. Prove by using mathematical induction that for any integer $n \geq 1$,

$$1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} \leq 2 - \frac{1}{n}.$$

Question C [5 marks]. Prove that for any positive integer n,

$$\sum_{r=0}^{n} \binom{n}{r}^2 = \binom{2n}{n}.$$

Question D [5 marks]. Suppose that T_1 and T_2 are spanning trees of a simple graph G with at least 3 vertices. Moreover, suppose that e_1 is an edge in T_1 that is not in T_2. Show that there is an edge e_2 in T_2 that is not in T_1 such that T_1 remains a spanning tree if e_1 is removed from it and e_2 is added to it, and T_2 remains a spanning tree if e_2 is removed from it and e_1 is added to it.

Question E [5 marks]. How many **primes** among the positive integers, written as usual in base 10, are alternating 1's and 0's, beginning and ending with 1? Justify your answer.

—END OF PAPER—