

# Tutorial 07 — BST

CS2040S Semester 1 2023/2024

*By Wu Biao, adapted from previous slides*

## Set real display name



<https://pollev.com/rezwanarefin430>

# General Properties of BST

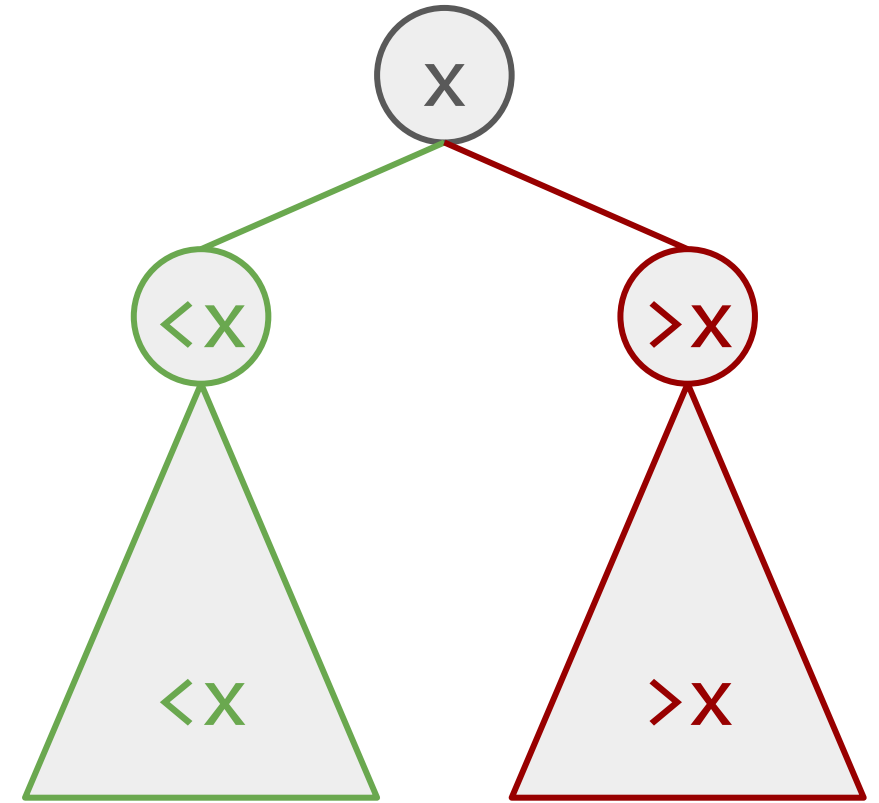
Non-Balanced

# Definition

A Binary Search Tree (BST) is a *binary tree* where for every vertex **X**:

- It has **at most** 2 children
- Every vertex in left subtree is lesser than **X**
- Every vertex in right subtree is greater than **X**

Note: We assume no duplicates for now

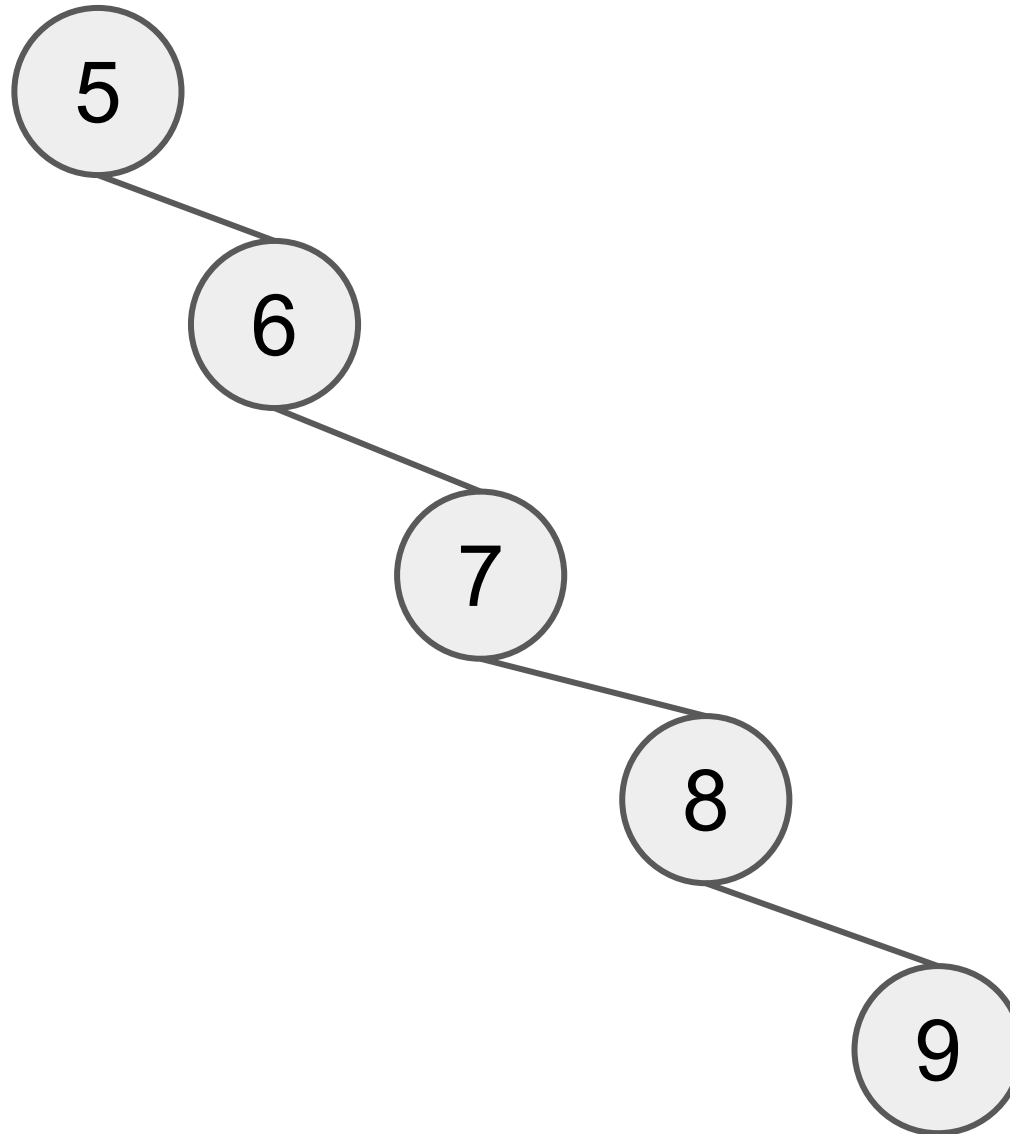


# Test yourself!

In the next few slides we quickly shall test your understanding of BSTs!

For the sake of brevity, we shall denote a Binary Search Tree rooted at vertex  $x$  to be  $BST_x$

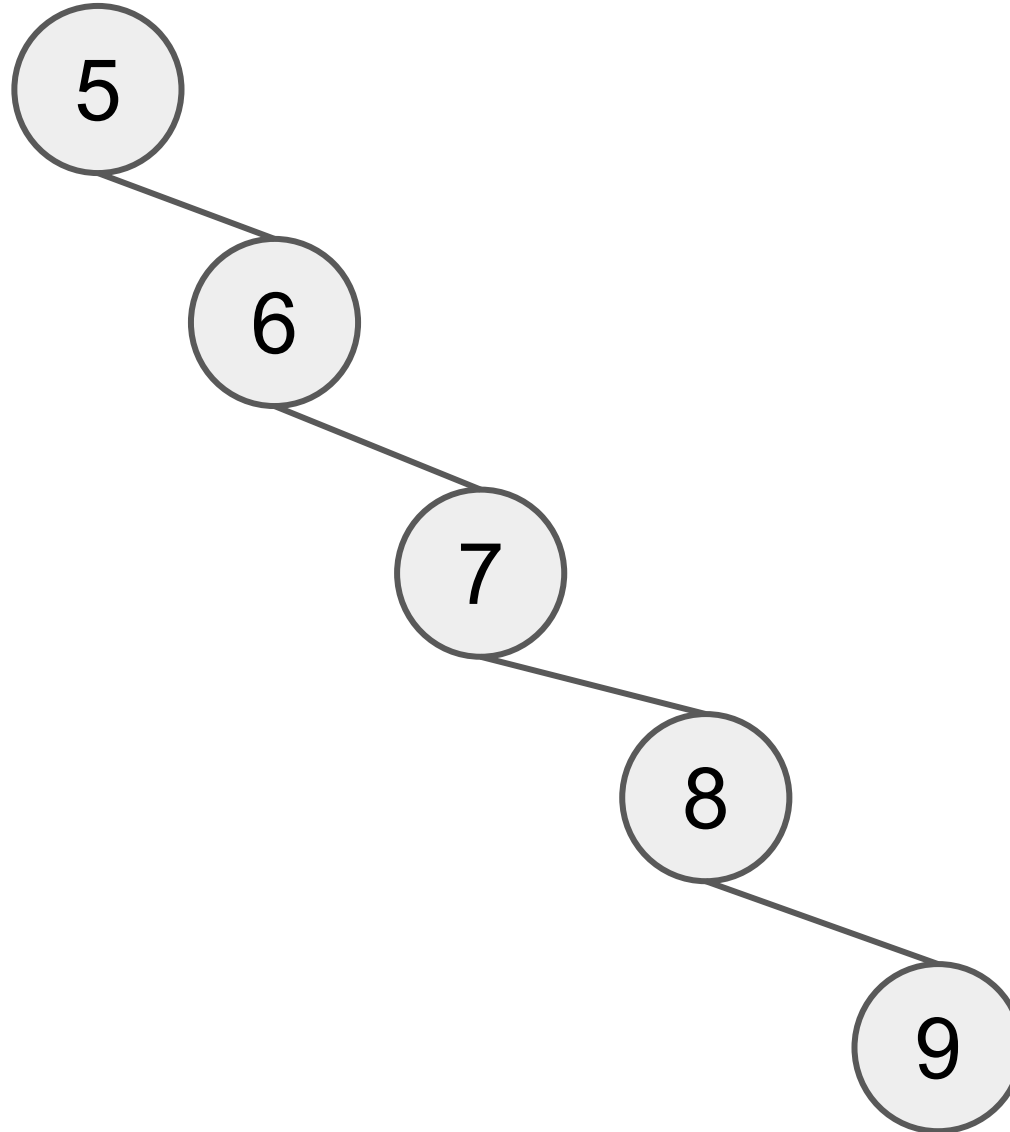
Is this a BST?



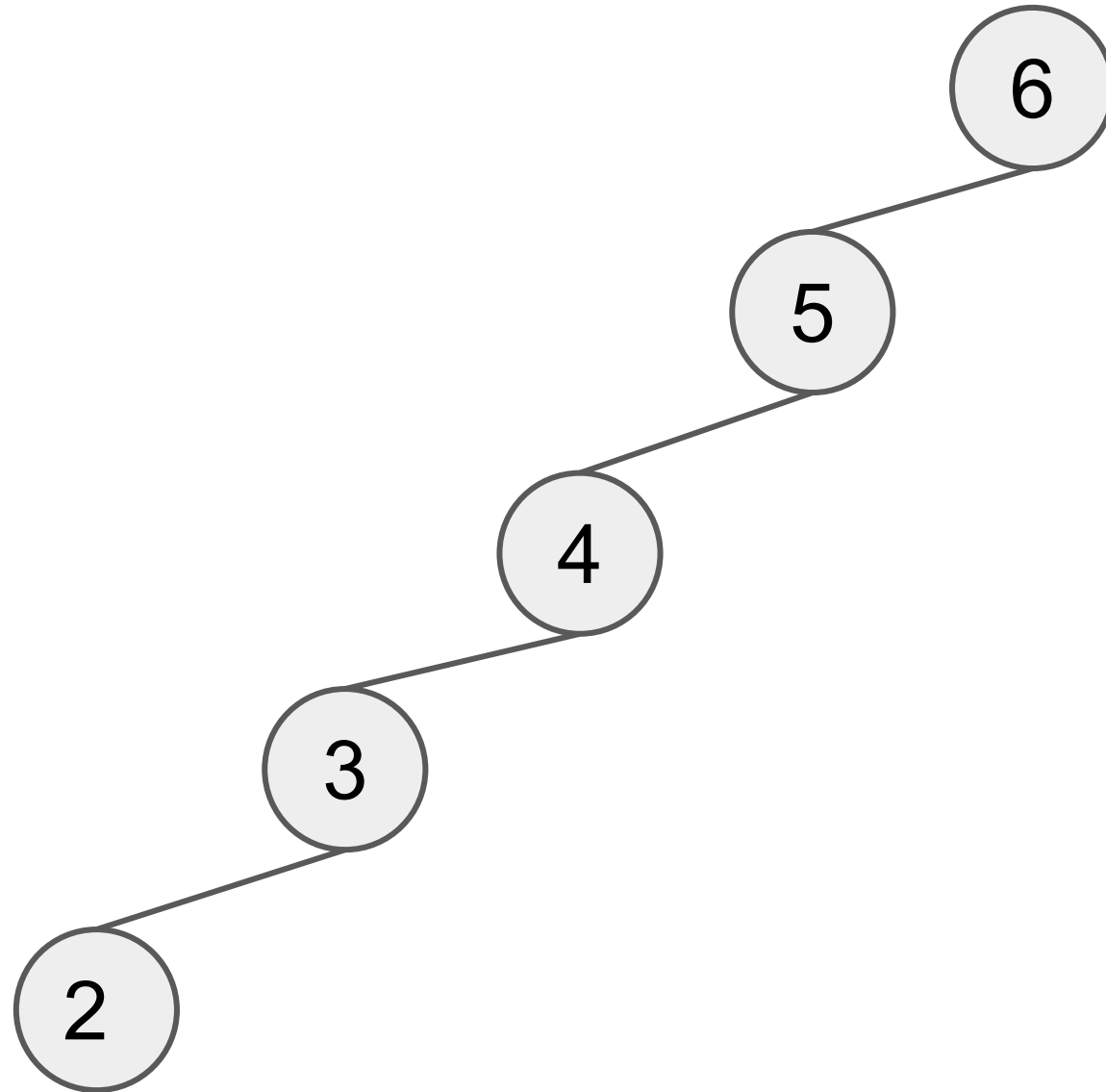
# Is this a BST?

YES!

Unlike in binary heap, a BST vertex can have right child without left child



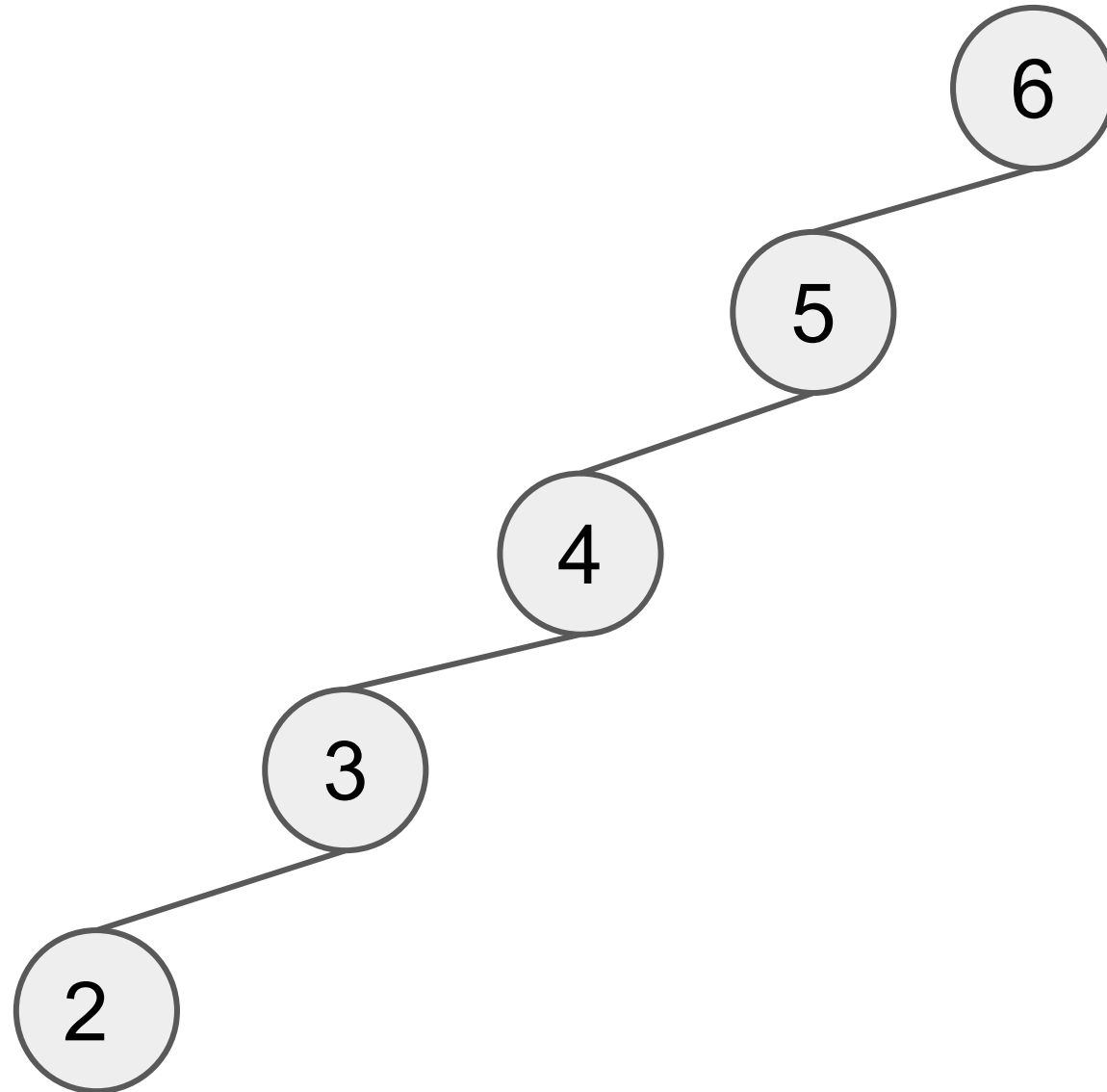
Is this a BST?





Is this a BST?

YES!



Is this a BST?



# Is this a BST?

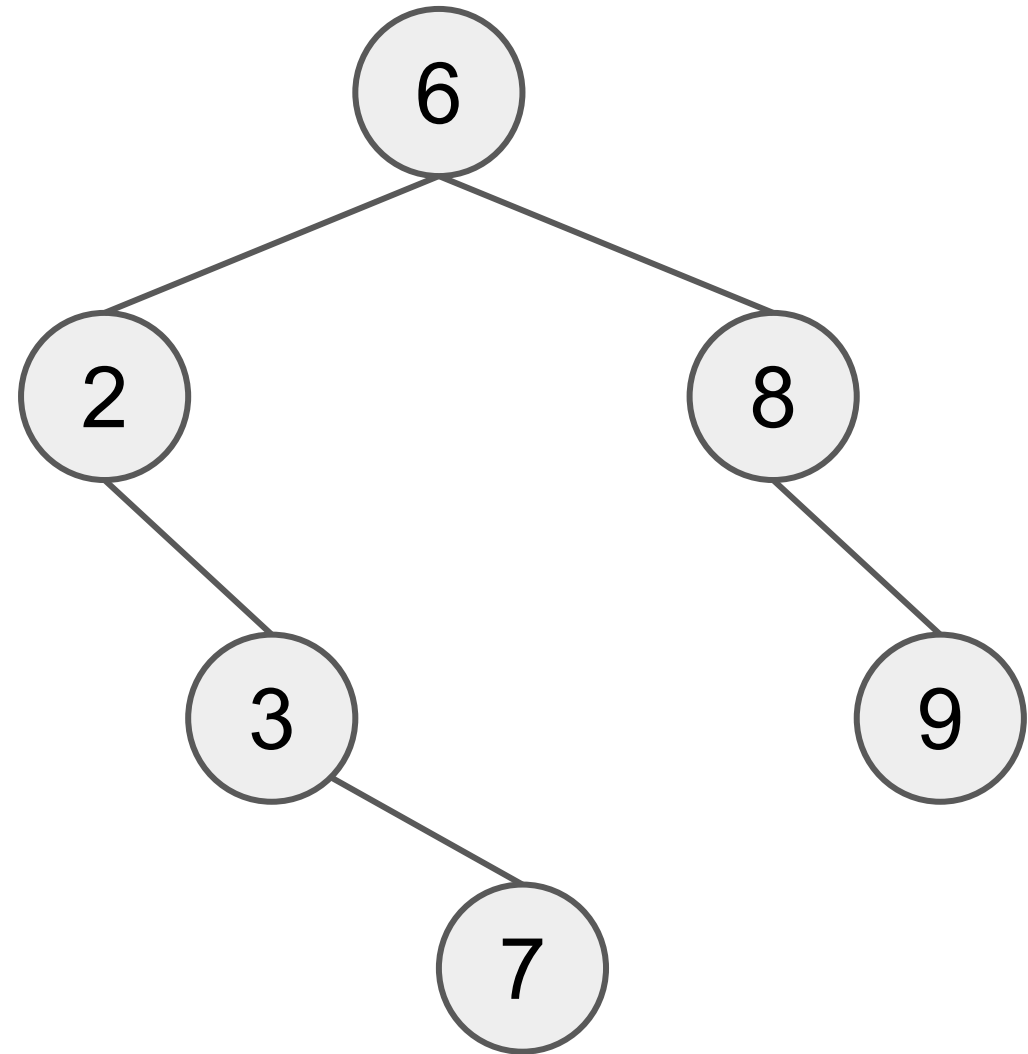
YES!

A BST vertex can have no children.

In fact we need this condition for binary trees to allow for leaves!  
:O



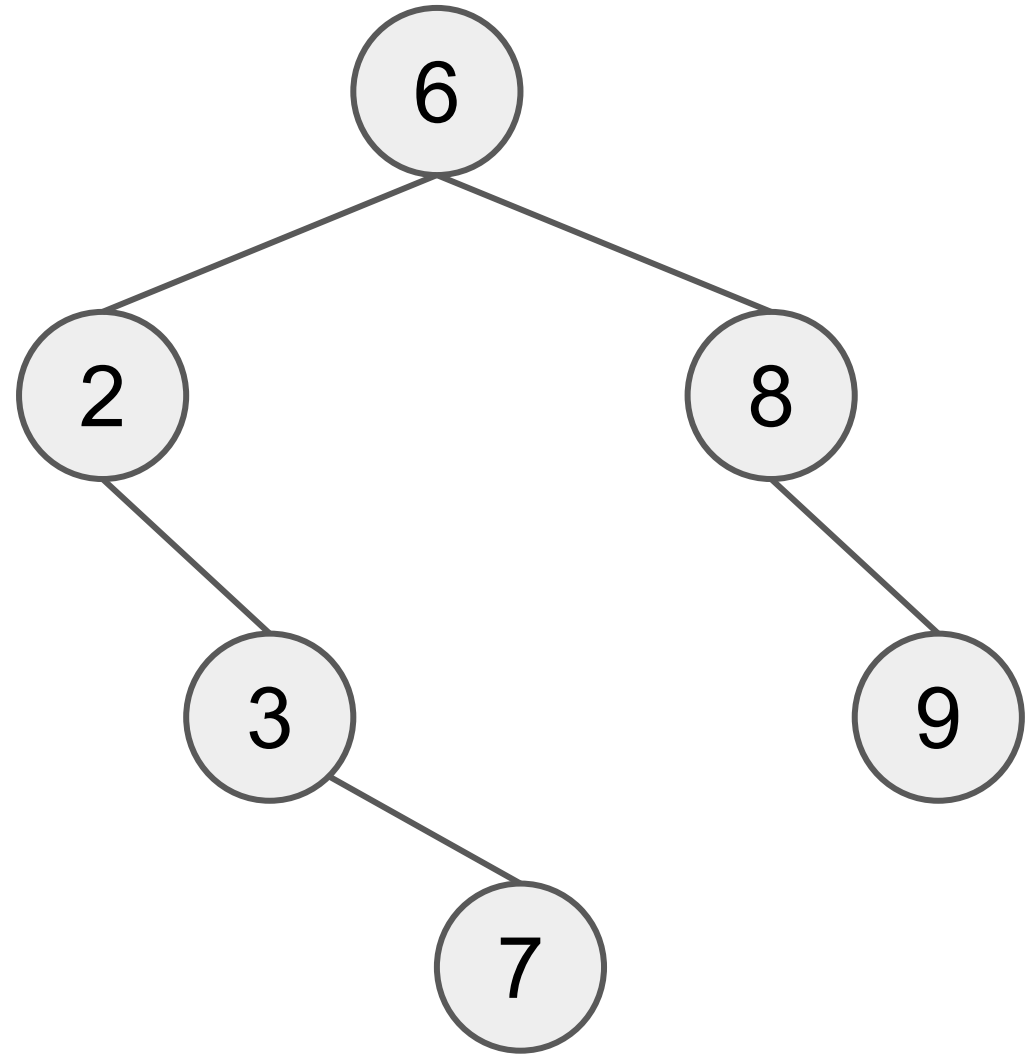
Is this a BST?



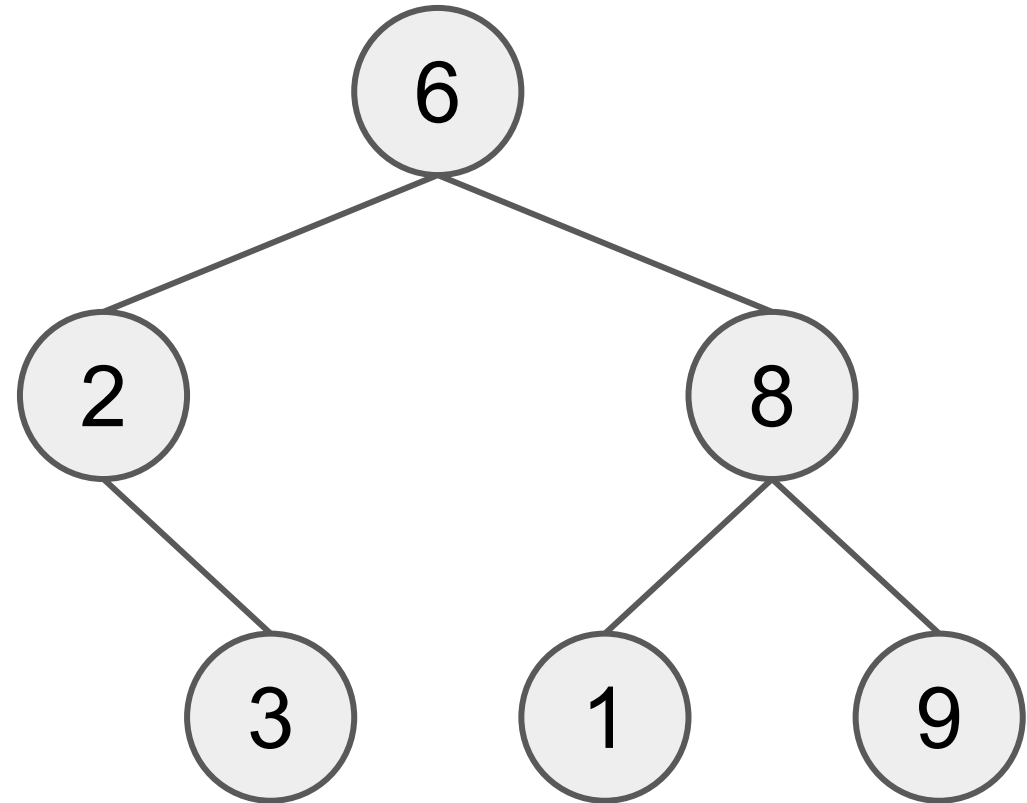
Is this a BST?

NO!

$BST_7$  violates  $BST_6$



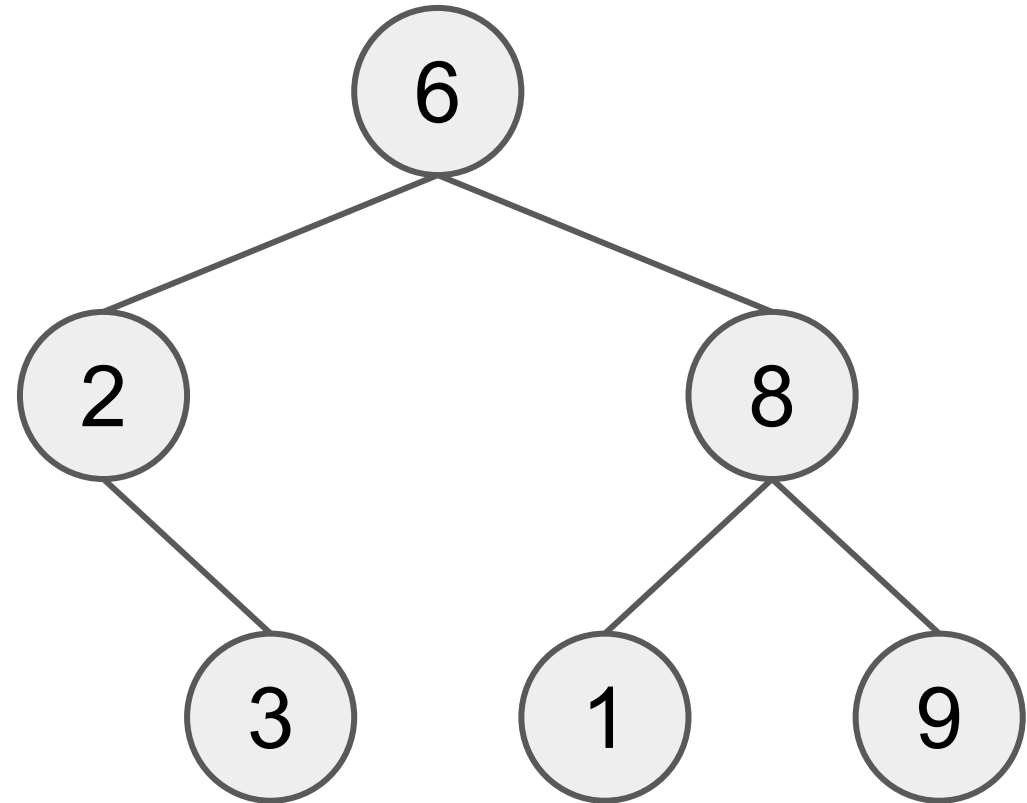
Is this a BST?



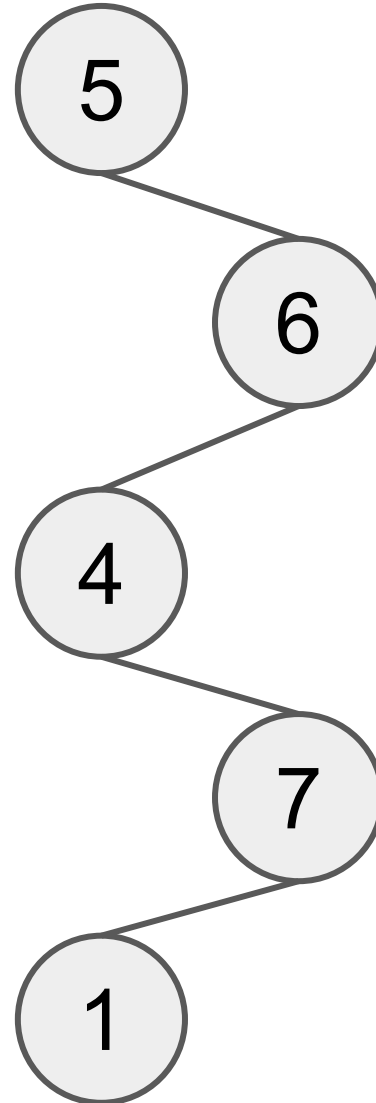
Is this a BST?

NO!

$BST_1$  violates  $BST_6$



Is this a BST?

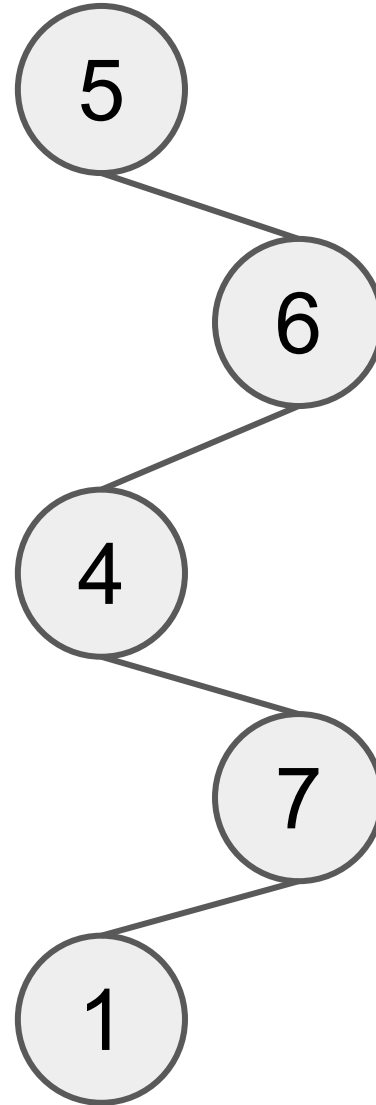




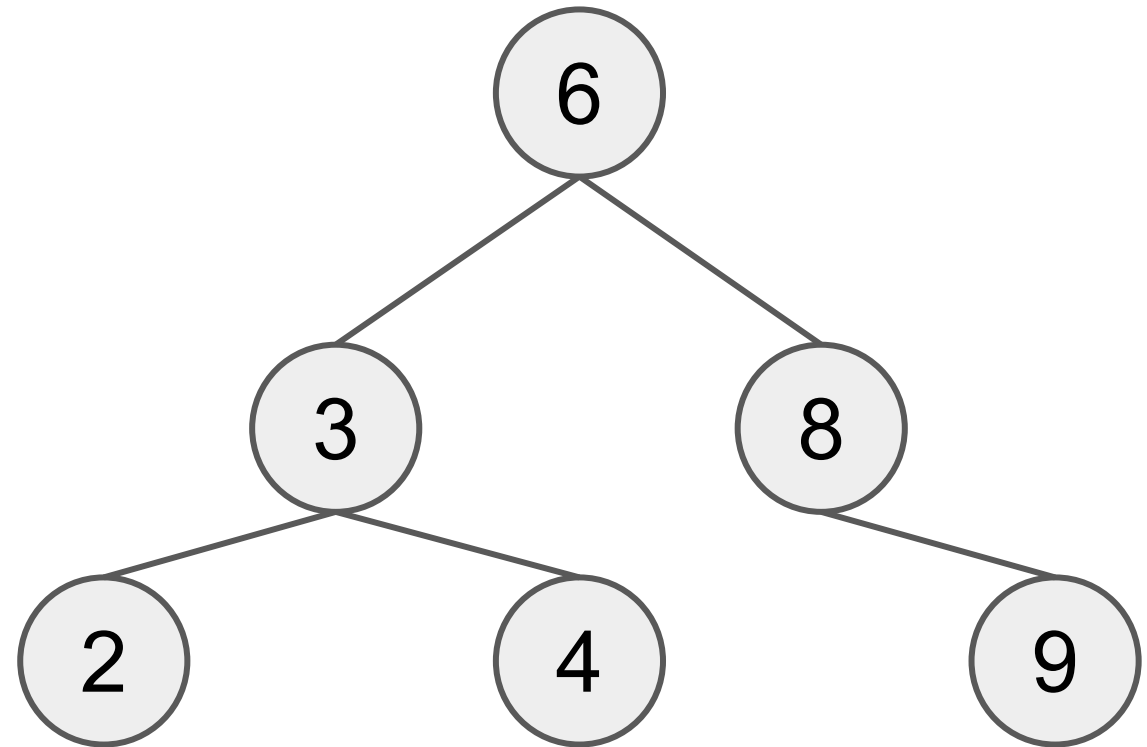
# Is this a BST?

**NO!**

$BST_4$  violates  $BST_5$   
 $BST_1$  violates  $BST_5$   
 $BST_7$  violates  $BST_6$   
 $BST_1$  violates  $BST_4$

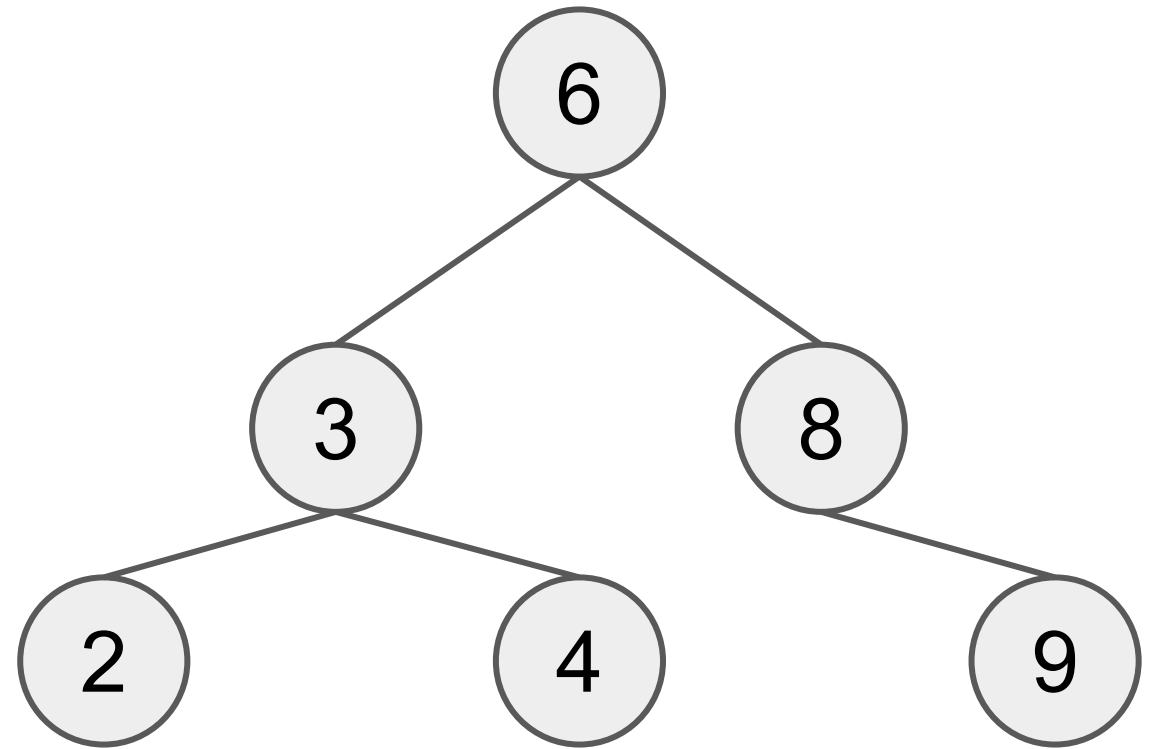


Is this a BST?



Is this a BST?

YES!



# Test yourself!

Is a BST is always a *complete binary tree*?

What about a **balanced** BST?

*Hint: see previous slide*



# Test yourself!

- What is the **min/max** height of a BST with **31** vertices?
- Height = number of edges from root to deepest leaf.
- How did you get those values?



# Test yourself!

- What is the **min/max** height of a BST with **31** vertices?
- Height = number of edges from root to deepest leaf.
- How did you get those values?

## Answer:

- Minimum:  $31 \leq 2^h - 1 \Rightarrow h \geq \lceil \log_2(31+1) \rceil = 5$ .  
Maximum:  $h \leq 30$  (just a chain is also BST).
- Also see VisuAlgo BST slides [13-5](#) and [13-7](#)

# In-order Traversal

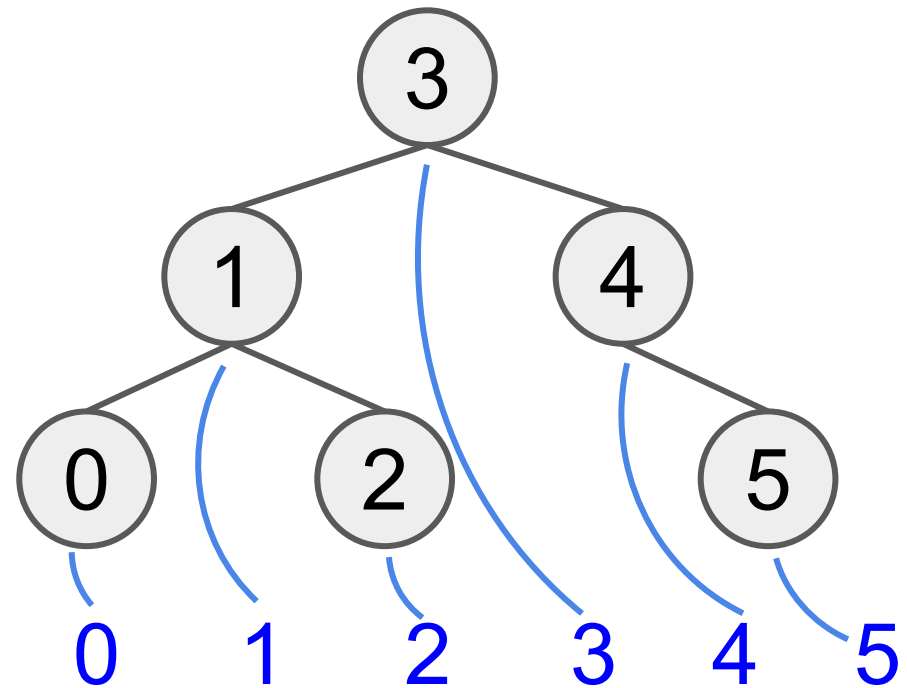
Recall from Tutorial 5 self-read slides:

- In-order traversal performs operations on the vertex after completing the left subtree, but before commencing the right subtree
- When performed on a BST, In-order traversal will *operate on* each vertex “in order” (i.e. in sorted order)

# In-order Traversal

1. Recurse left
2. Current operation
3. Recurse right

```
void in_order(vertex v) {  
    if (v) {  
        in_order(v.left);  
        cout << v.value << " ";  
        in_order(v.right);  
    }  
}
```

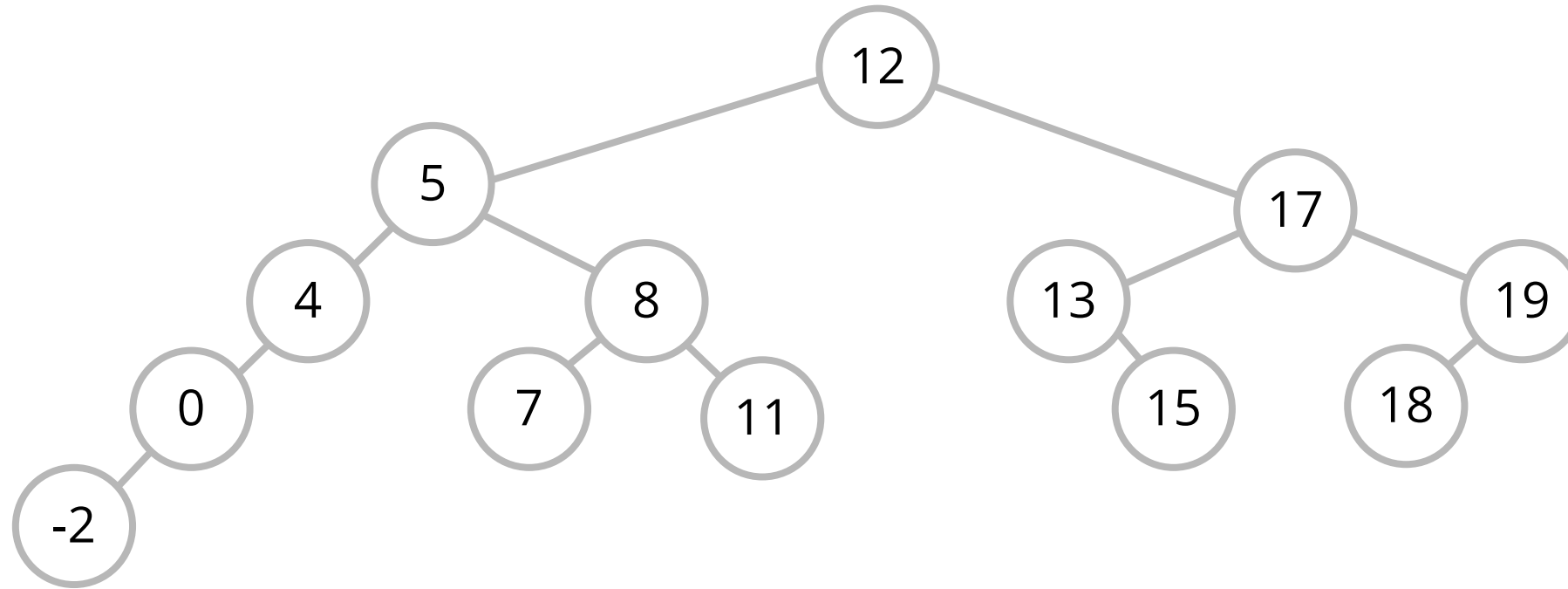


Print order in blue

Resembles “downward projection” or  
“flattening” of the tree!

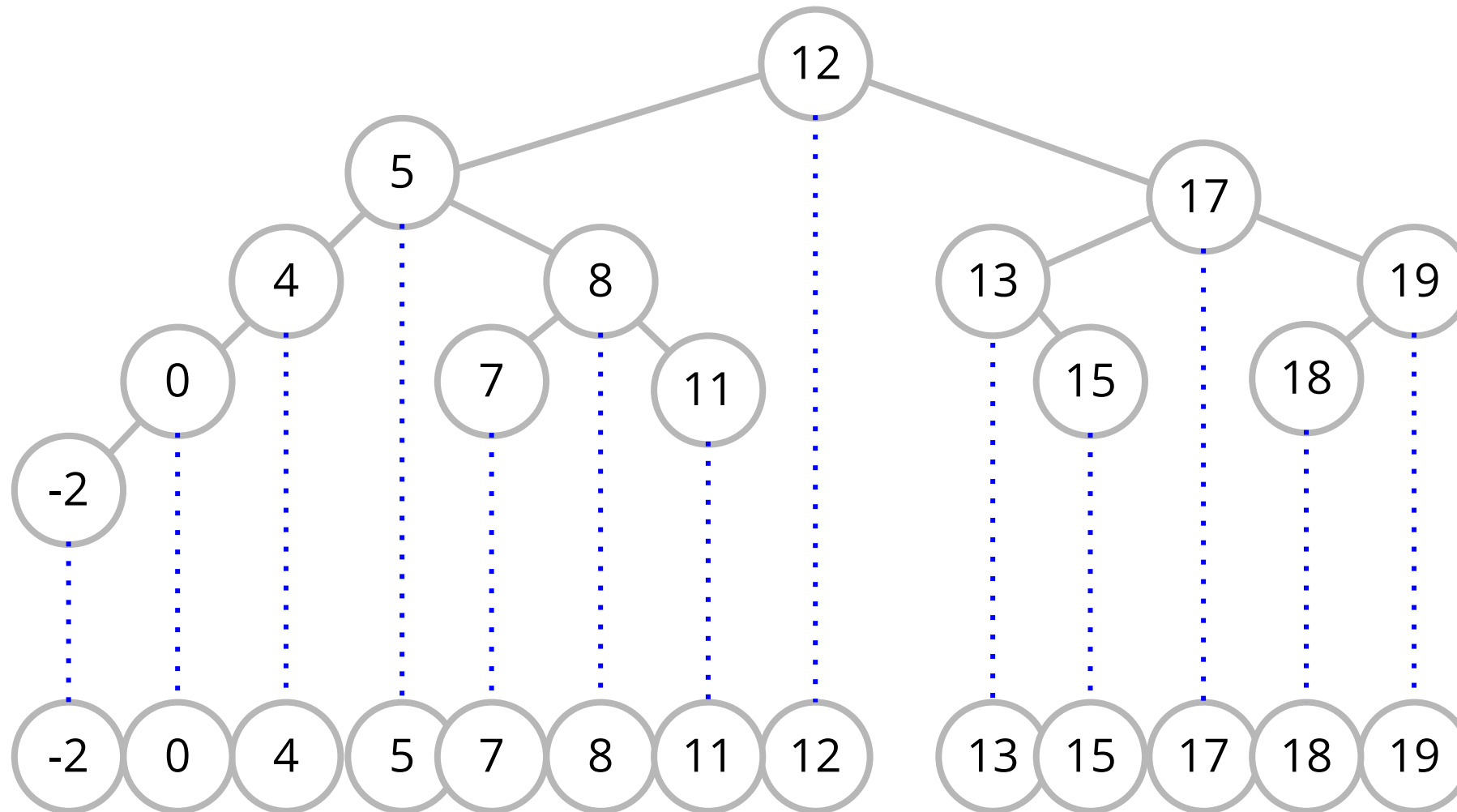


# Test yourself!



What's the sequence of operations by in-order traversal on this BST?

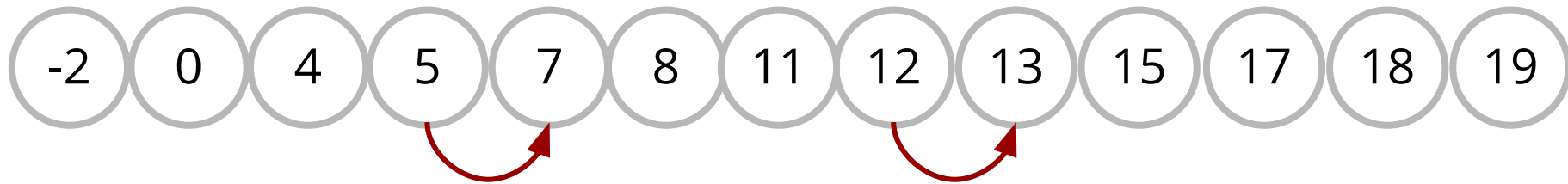
# Test yourself!



# Finding successor

To find **successor** of a key **k**, it's equivalent to finding:

- The *next highest* key
- The *next* vertex after **k** in in-order traversal sequence

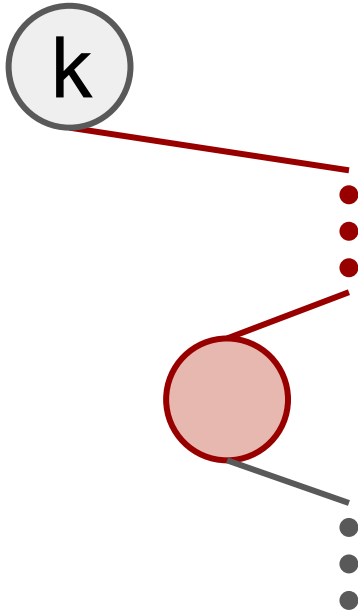


Finding the **successor** of keys 5 and 12 respectively

# Finding successor

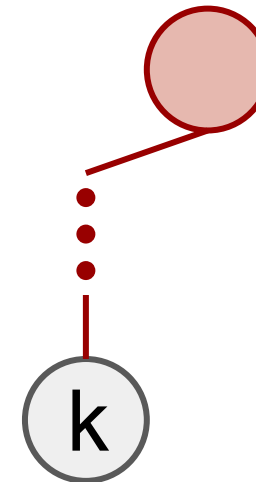
Case 1: Has **right** child:

- **Leftmost** vertex in **right** subtree

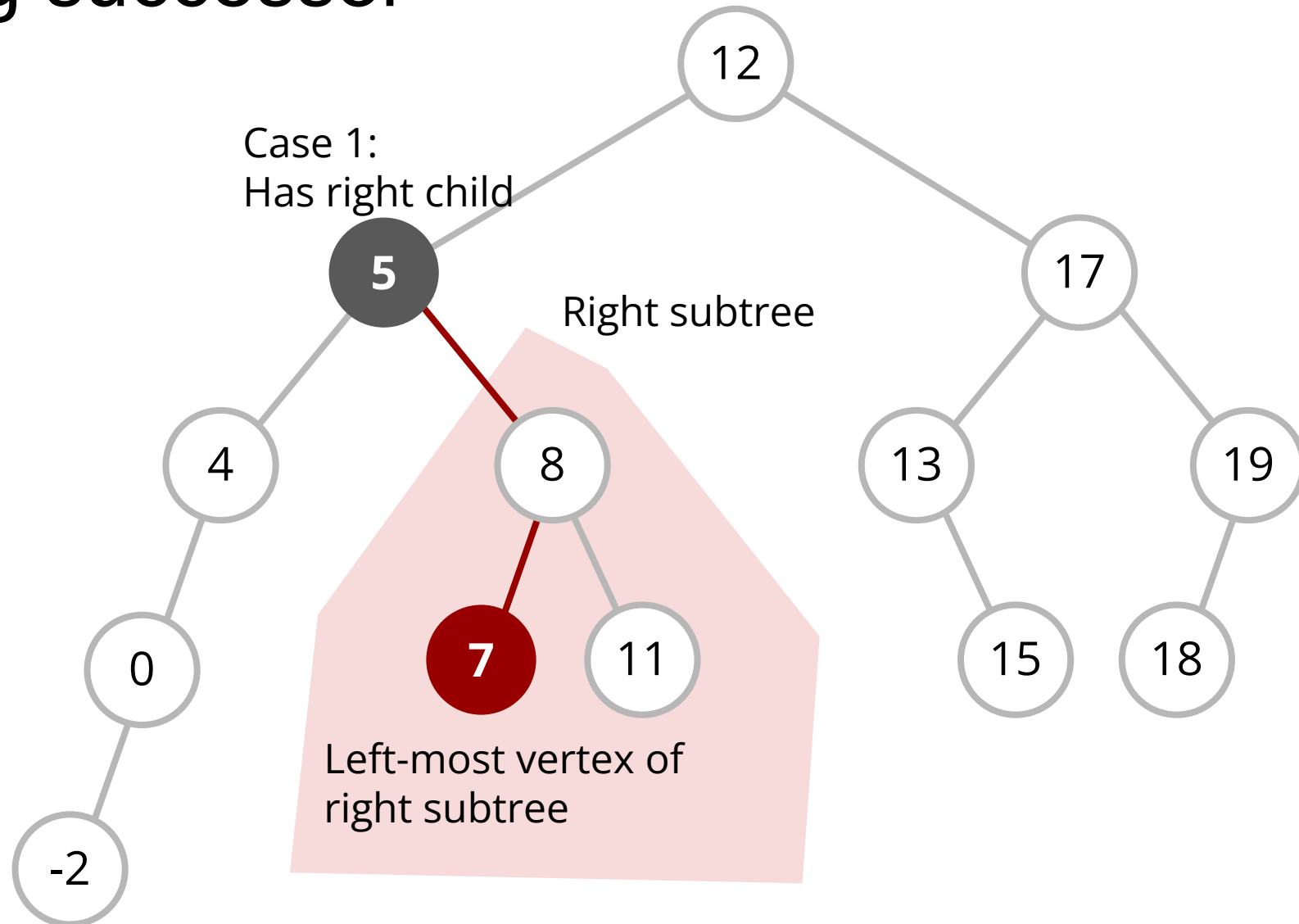


Case 2: No **right** child

- First **right** parent
- What if there isn't a **right** parent?



# Finding successor

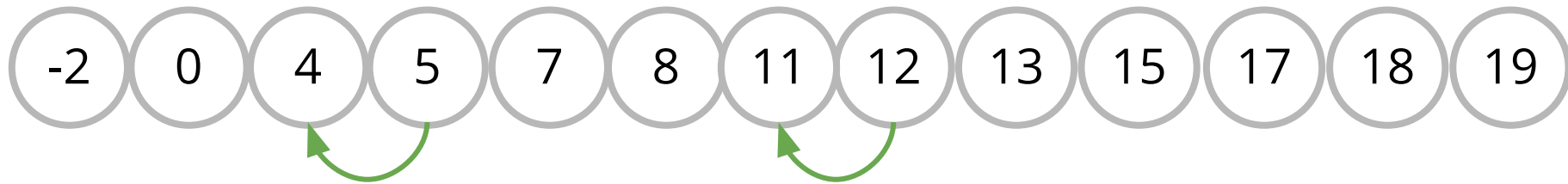




# Finding predecessor

To find **predecessor** of a key **k**, it's equivalent to finding:

- The *previous largest* key
- The *previous* vertex of **k** in in-order traversal sequence

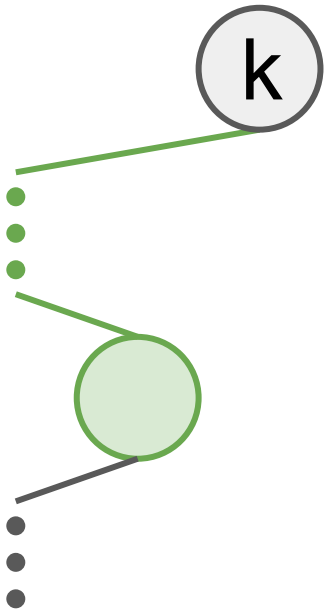


Finding the **predecessor** of keys 5 and 12 respectively

# Finding predecessor

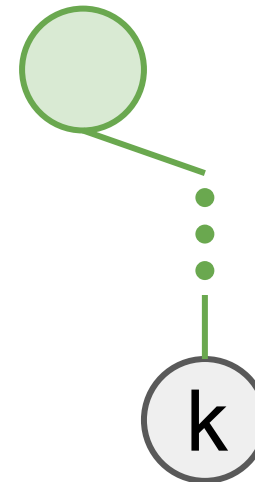
Case 1: Has **left** child:

- **Rightmost** vertex in **left** subtree



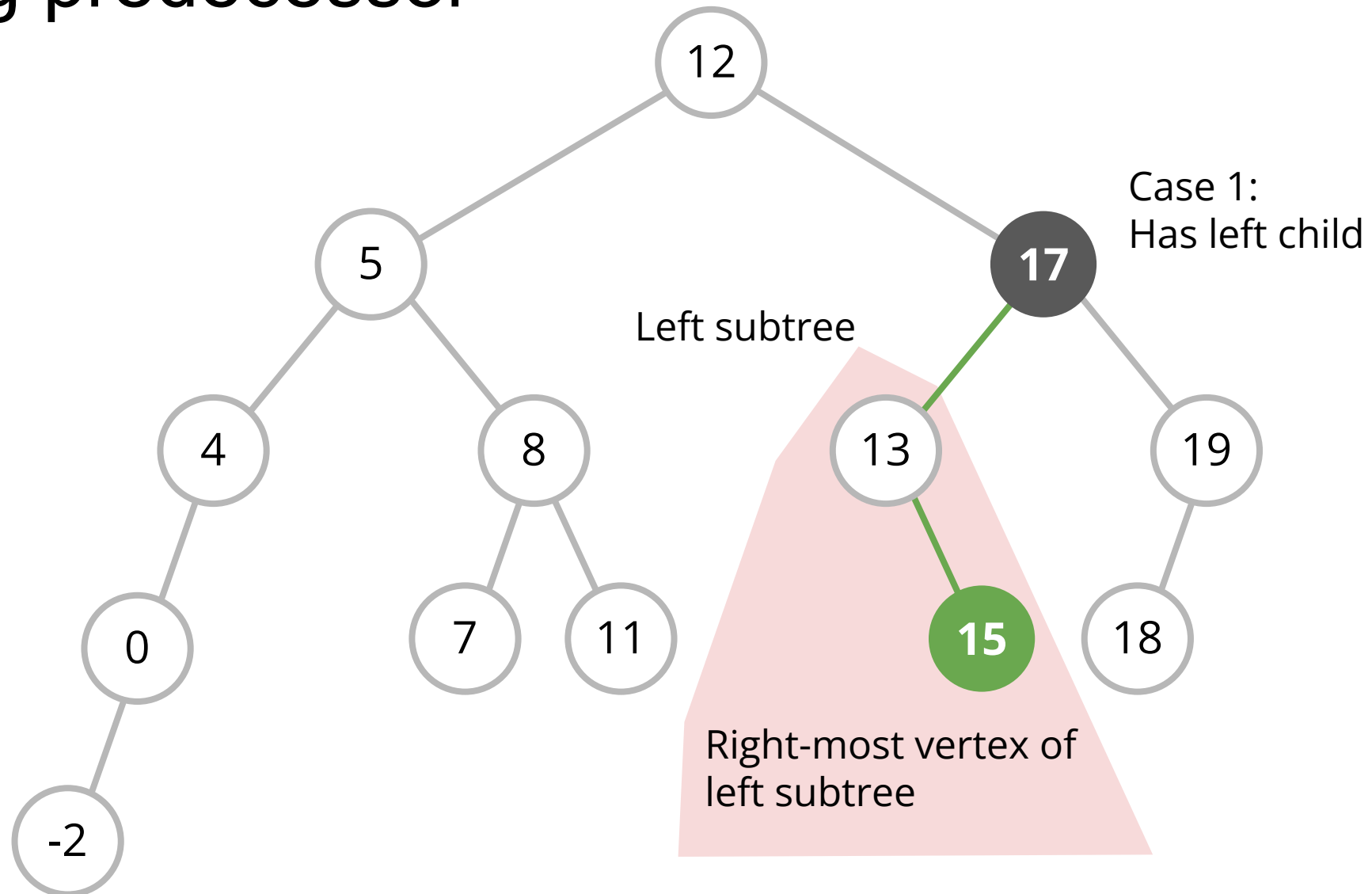
Case 2: No **left** child

- First **left** parent
- What if there isn't a **left** parent?

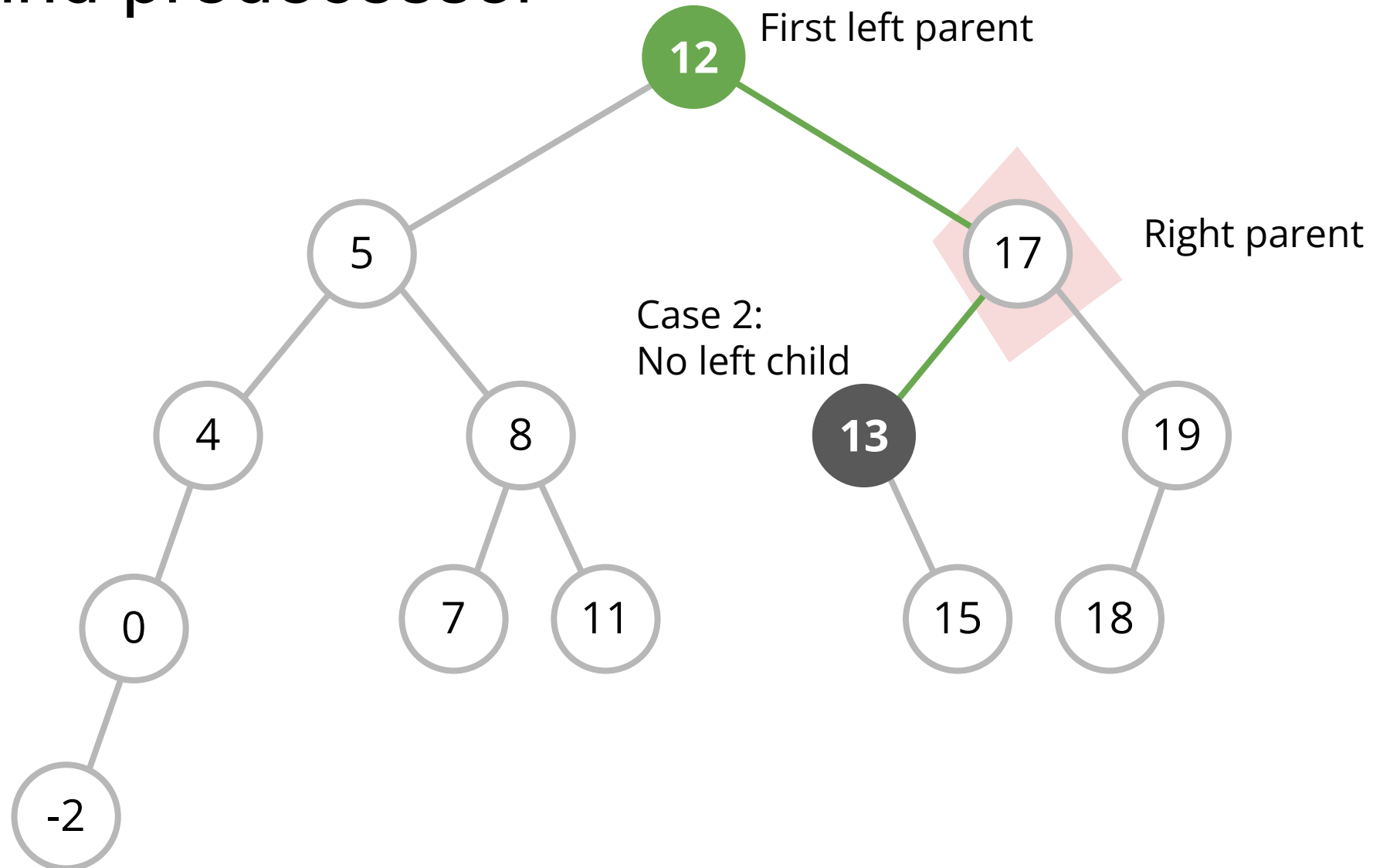




# Finding predecessor



# BST: Find predecessor





# In-order traversal??

Assume `TreeSet<Integer> set` contains  $N$  elements.

What is the code below doing besides printing all the numbers? What's the time complexity?  $O(N)$  or  $O(N \log N)$ ?

```
for (Integer x : set)
    System.out.println(x);
```



# In-order traversal??

Assume `TreeSet<Integer> set` contains  $N$  elements.

What is the code below doing besides printing all the numbers? What's the time complexity?  $O(N)$  or  $O(N \log N)$ ?

```
for (Integer x : set)
    System.out.println(x);
```

Each iteration is bounded by  $O(\log N)$  – finding next element.  
Total complexity is  $O(N)$  – the tree is iterated on once in total.

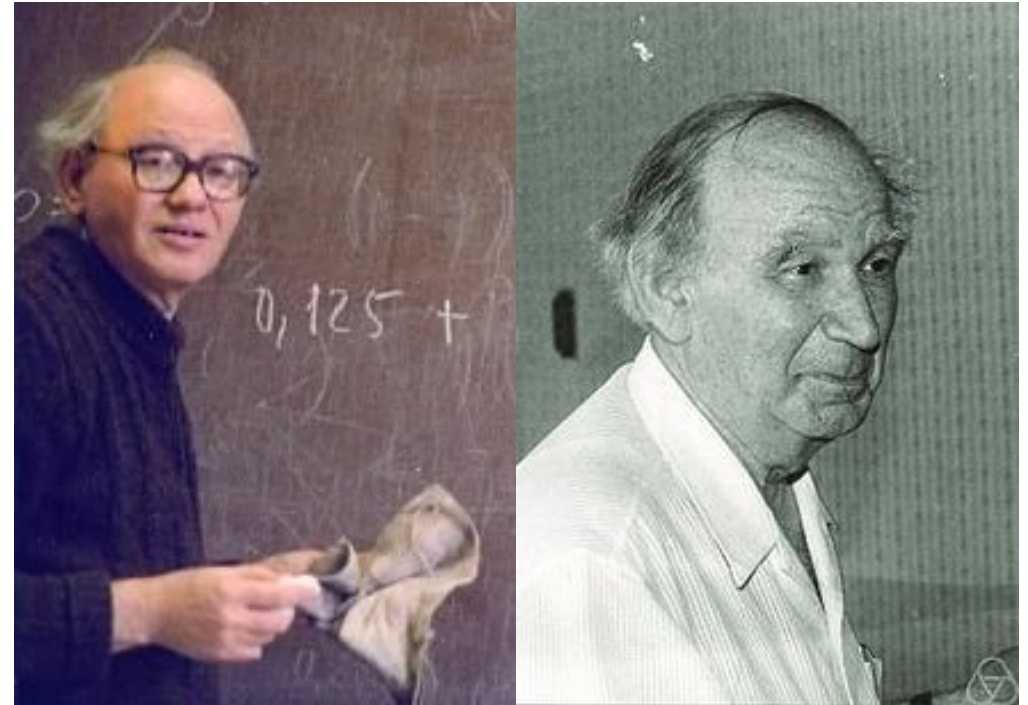
# Balanced BST

AVL Tree

# Did you know?

*The AVL tree is named after its two Soviet inventors, Georgy **Adelson-Velsky** and Evgenii **Landis**, who published it in their 1962 paper "An algorithm for the organization of information"*

Source: [Wikipedia](#)



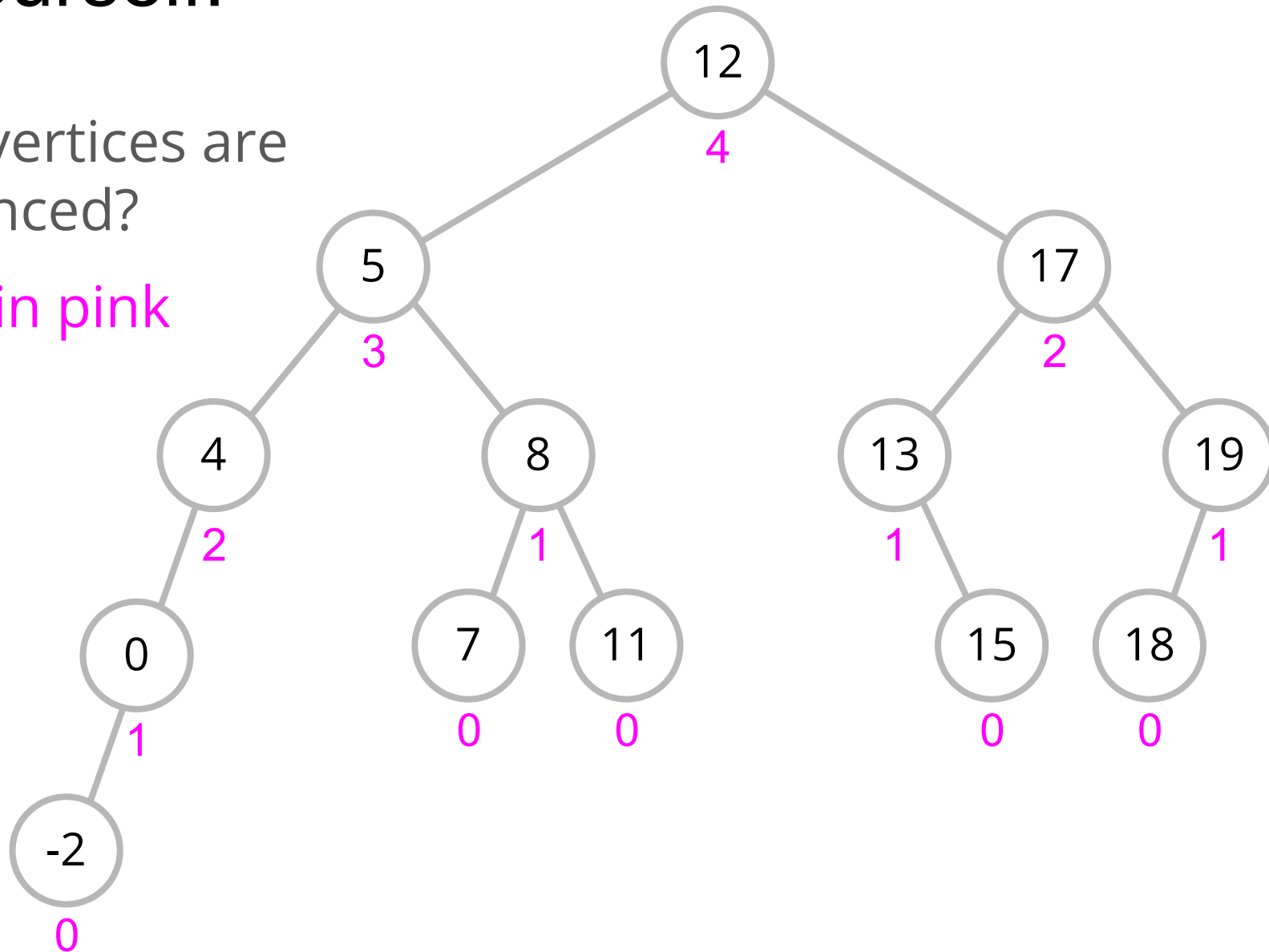
# AVL Tree

- An AVL tree is a *self-balancing* BST where every vertex is *height-balanced*
- A vertex is *height-balanced* if difference in height between left and right subtree is at most 1.

# Test yourself!

Which vertices are unbalanced?

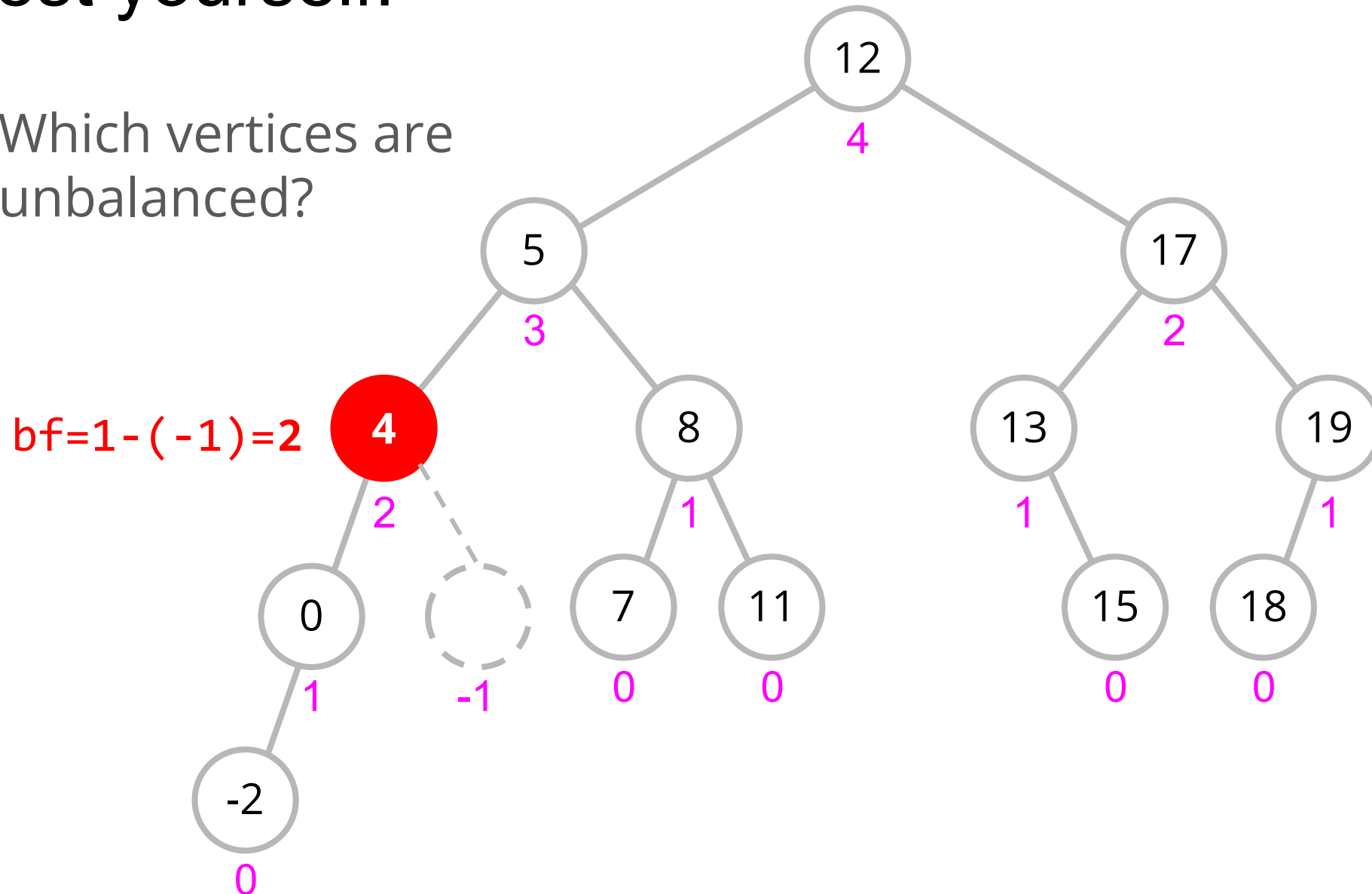
Height in pink





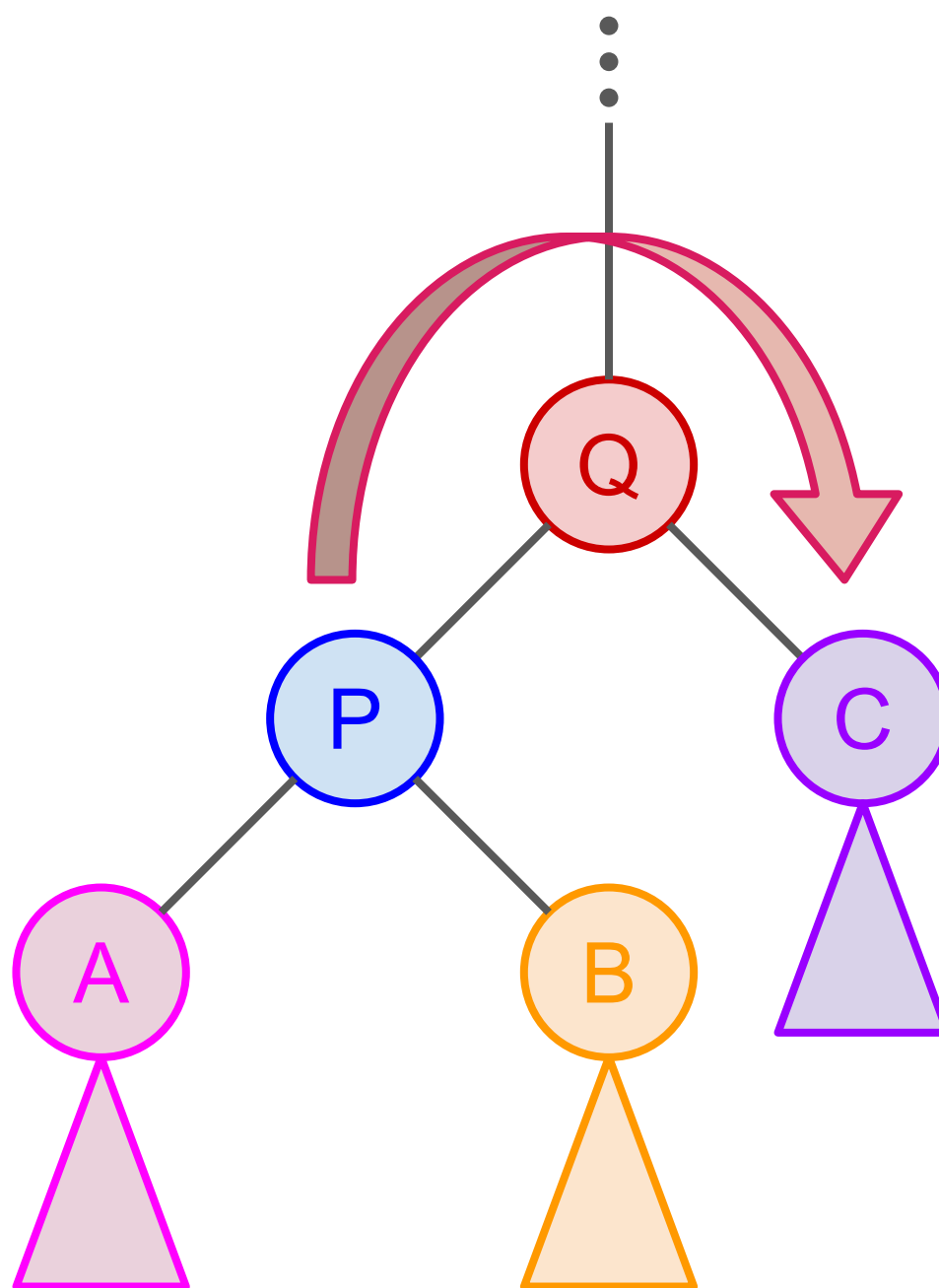
# Test yourself!

Which vertices are unbalanced?



# Right rotate

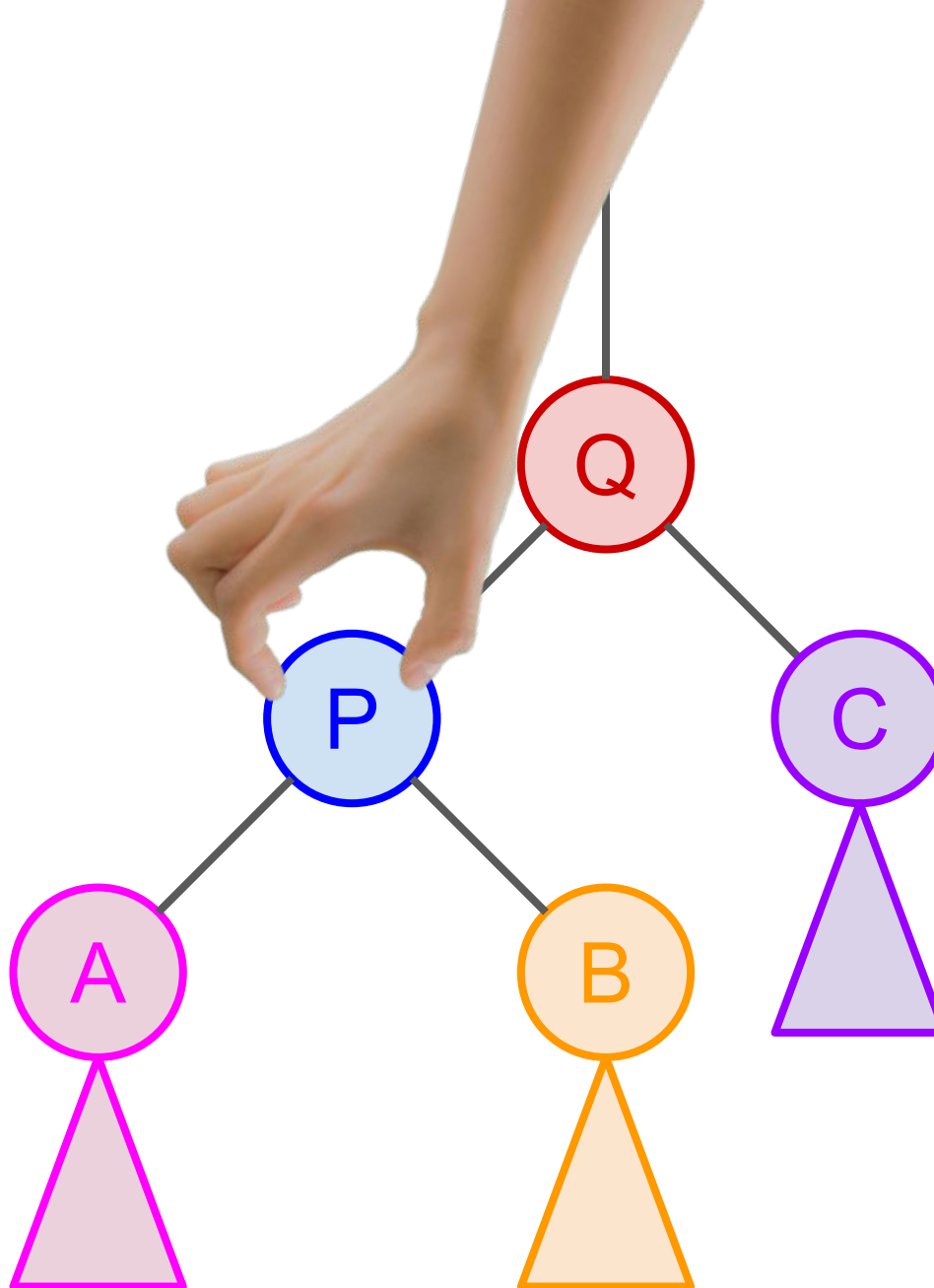
Right rotation  
on Q



# Right rotate

Right rotation  
on Q

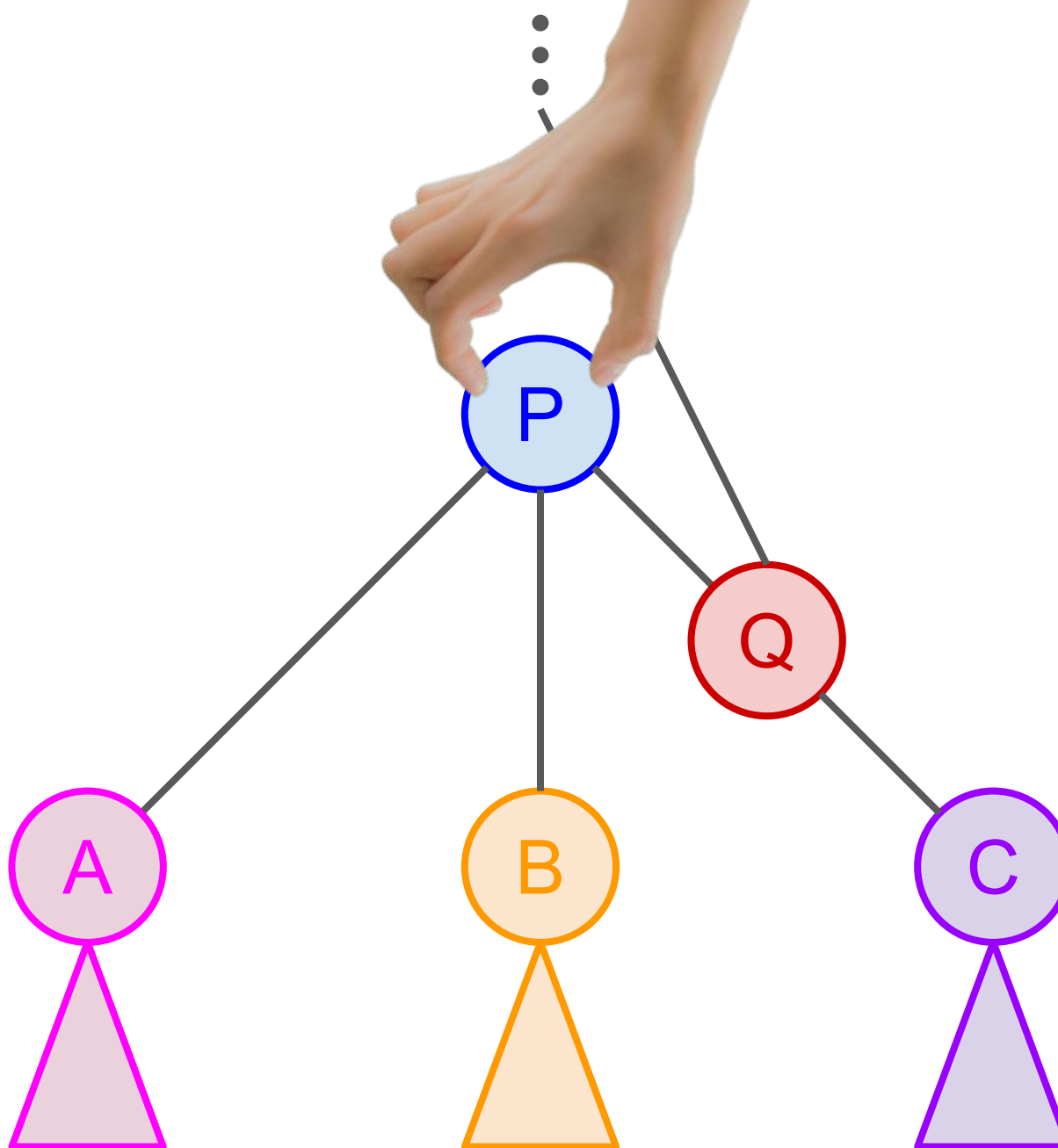
Imagine we pinch  
P and bring it  
**above and over** Q



# Right rotate

Right rotation  
on Q

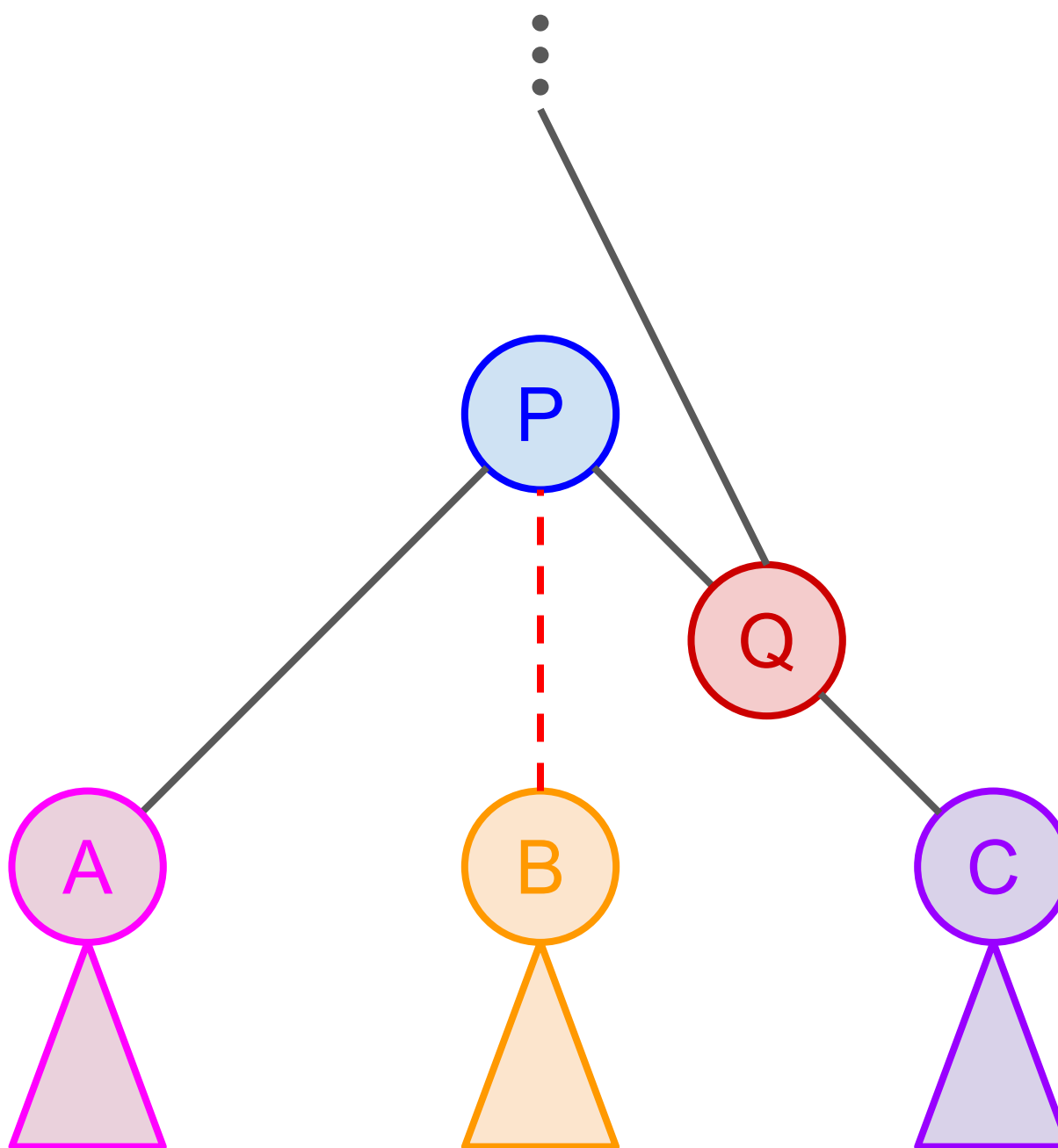
Imagine we pinch  
P and bring it  
**above and over** Q



# Right rotate

Right rotation  
on Q

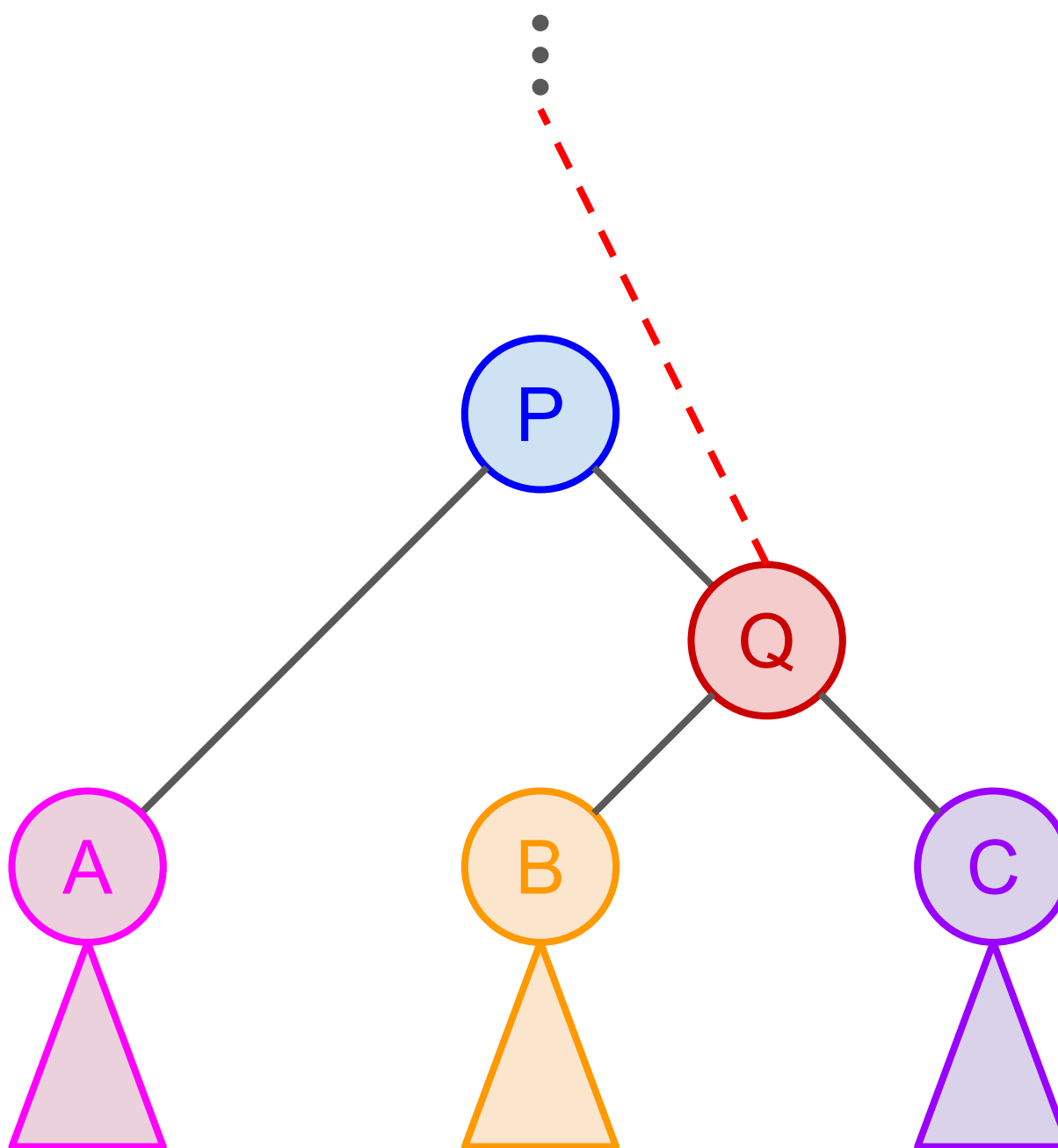
P cannot have 3  
children! So we  
shall make subtree  
B the left subtree  
of Q.



# Right rotate

Right rotation on Q

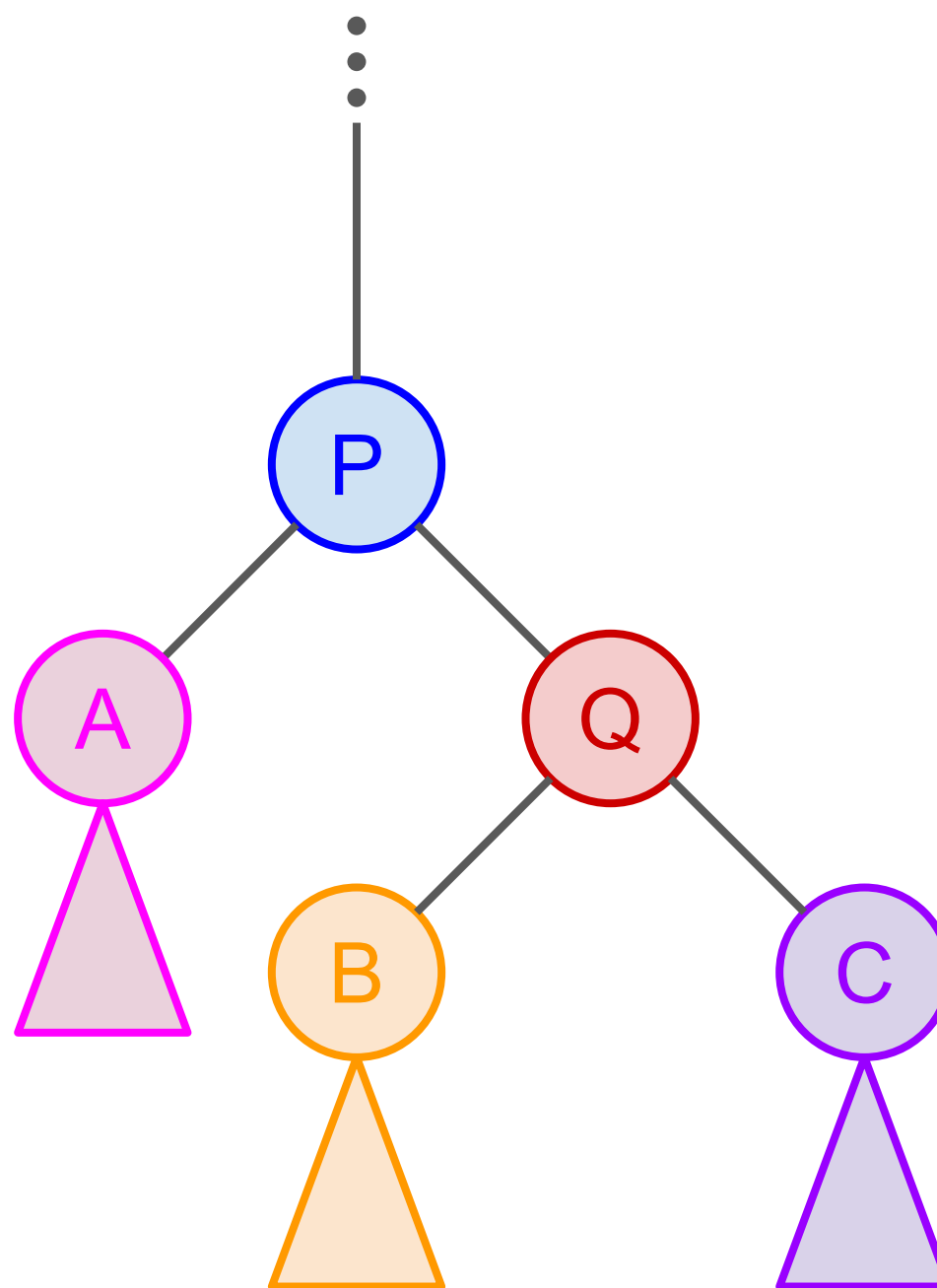
Q cannot have 2 parents and P cannot be without a parent! So we shall make the previous parent of Q the new parent of P instead.



# Right rotate

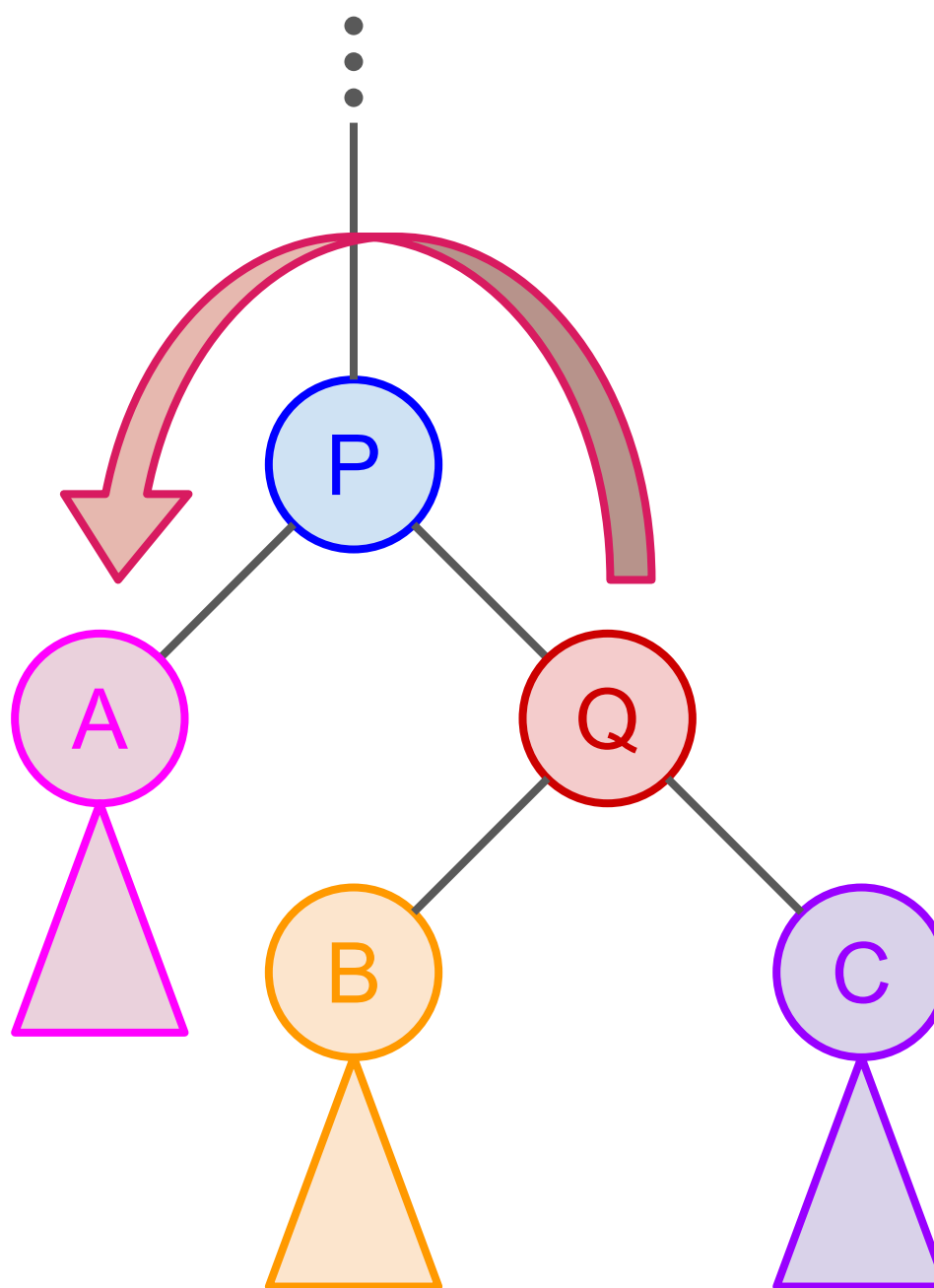
Right rotation  
on Q complete!

Notice A went up 1  
level, C went down  
1 level



# Left rotate

## Left rotation on P

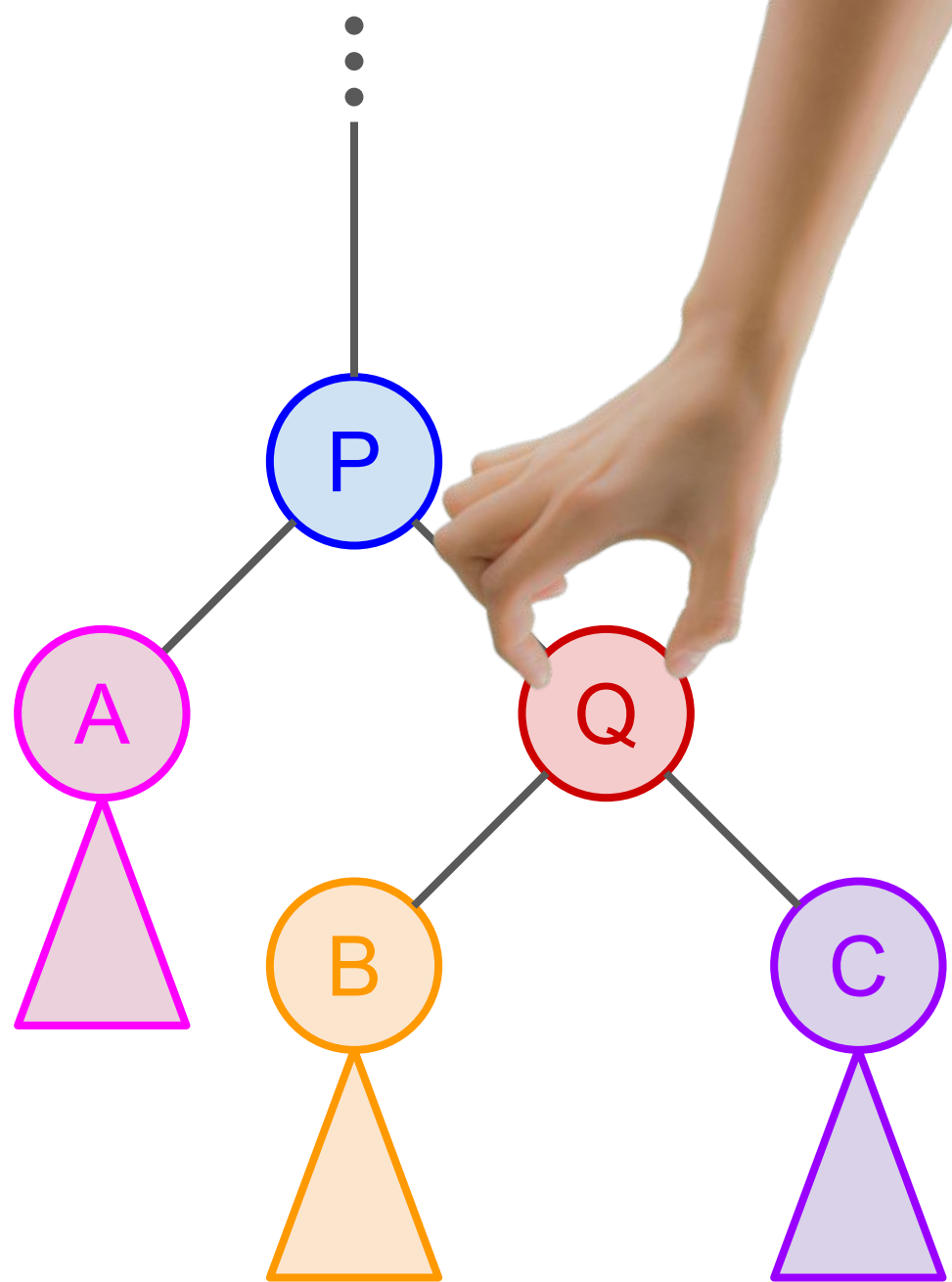




# Left rotate

Left rotation  
on P

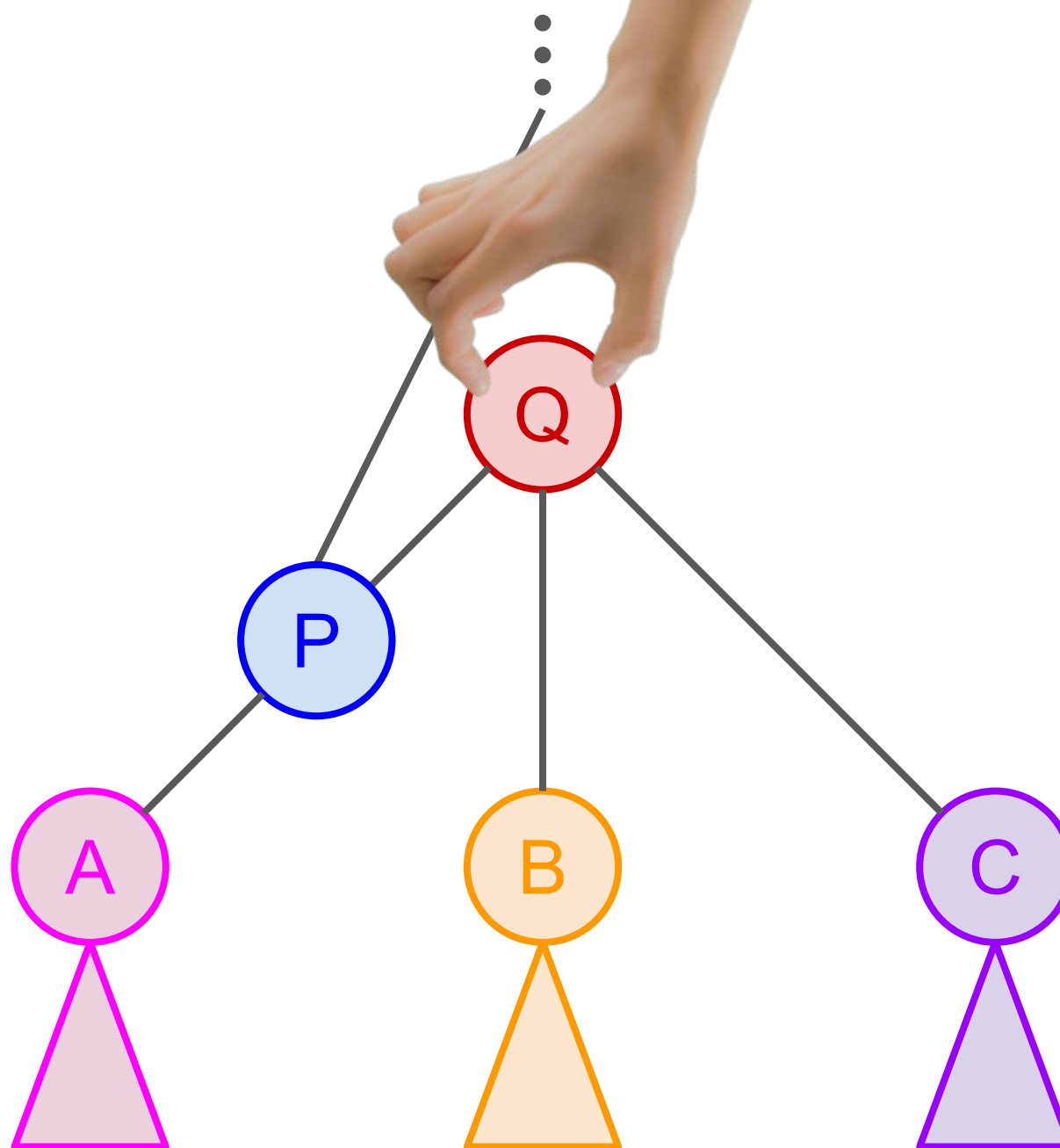
Imagine we pinch  
Q and bring it  
**above and over P**



# Left rotate

Left rotation  
on P

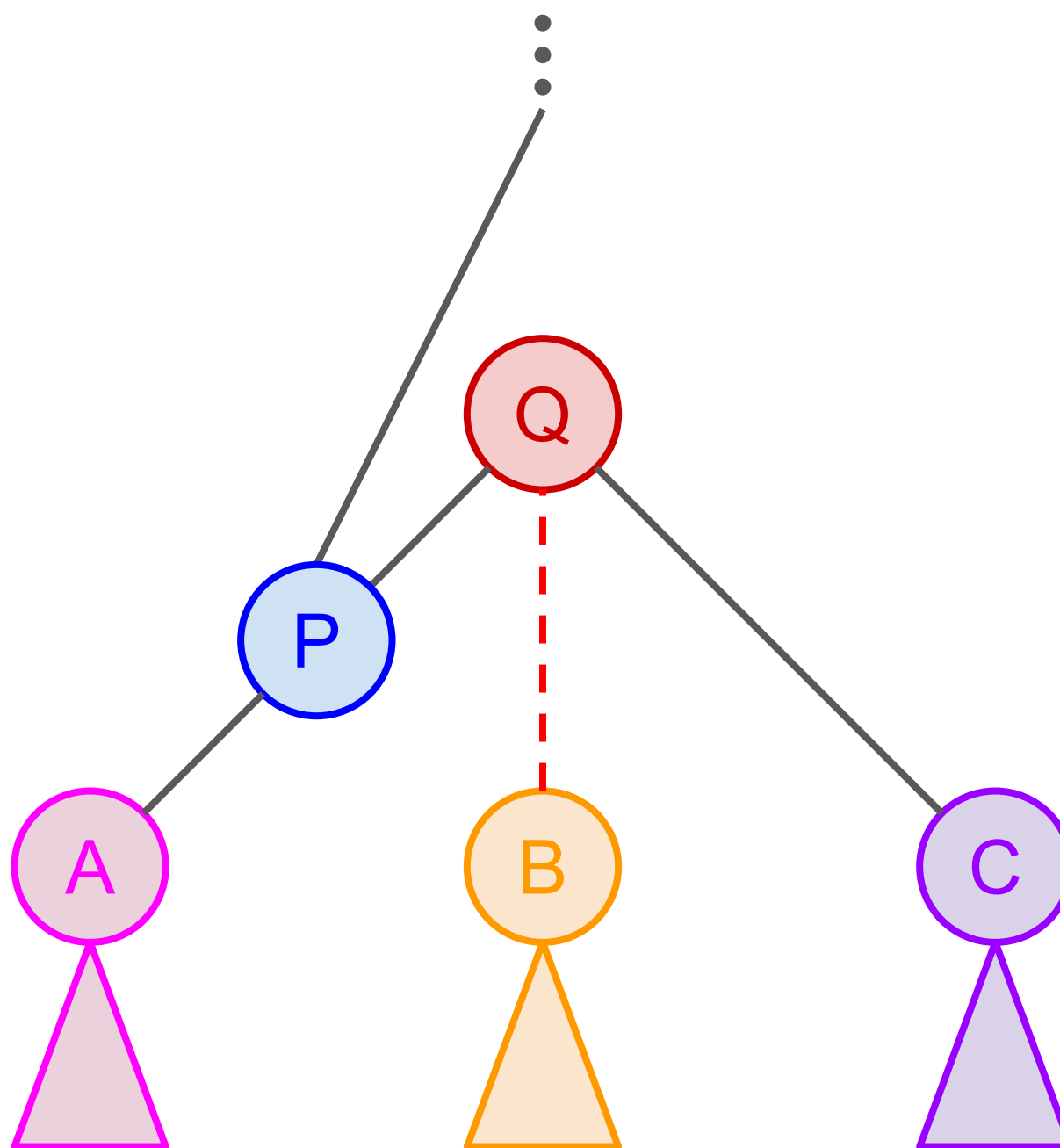
Imagine we pinch  
Q and bring it  
**above and over P**



# Left rotate

## Left rotation on P

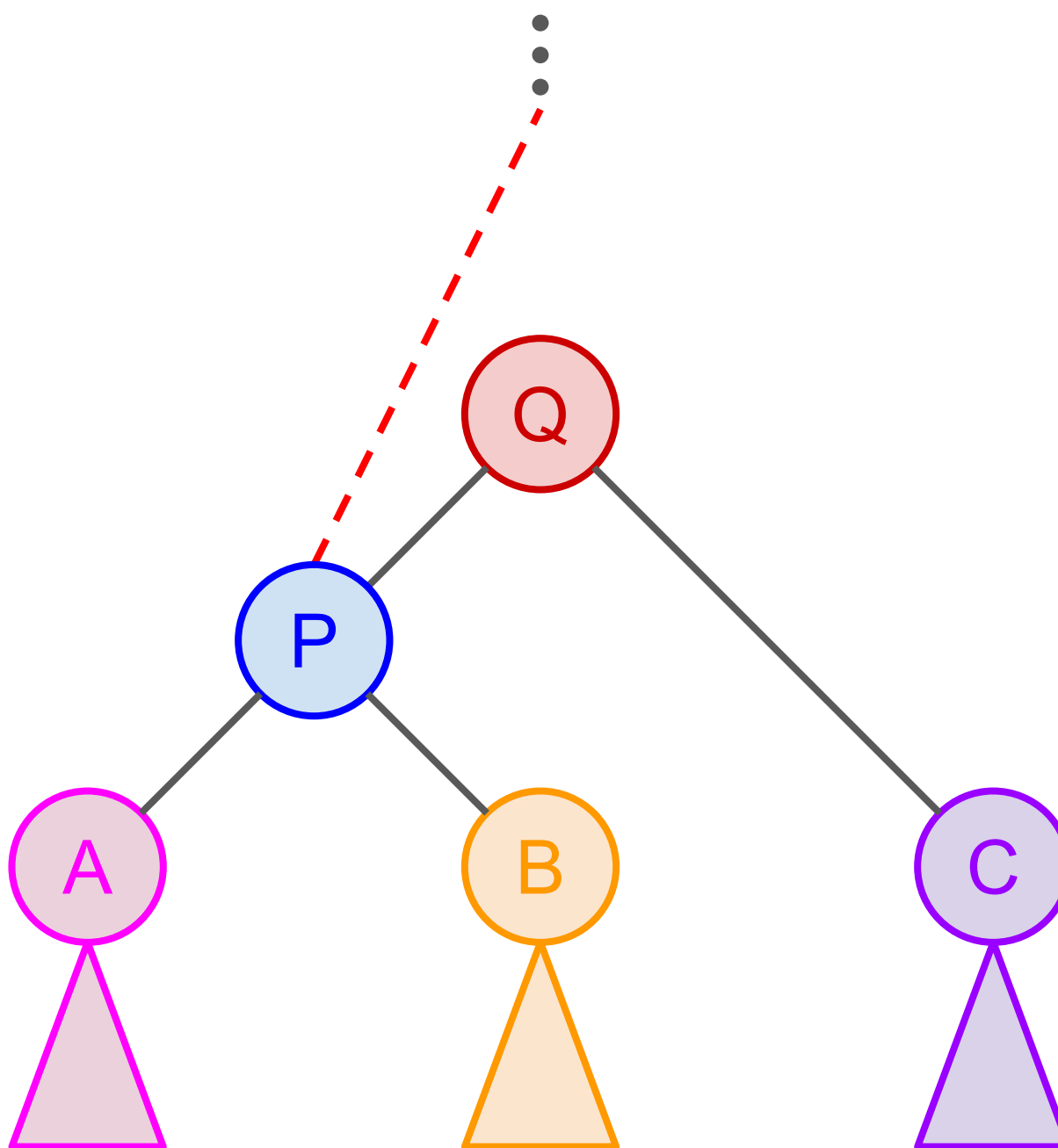
Q cannot have 3  
children! So we  
shall make subtree  
B the right subtree  
of P.



# Left rotate

Left rotation on P

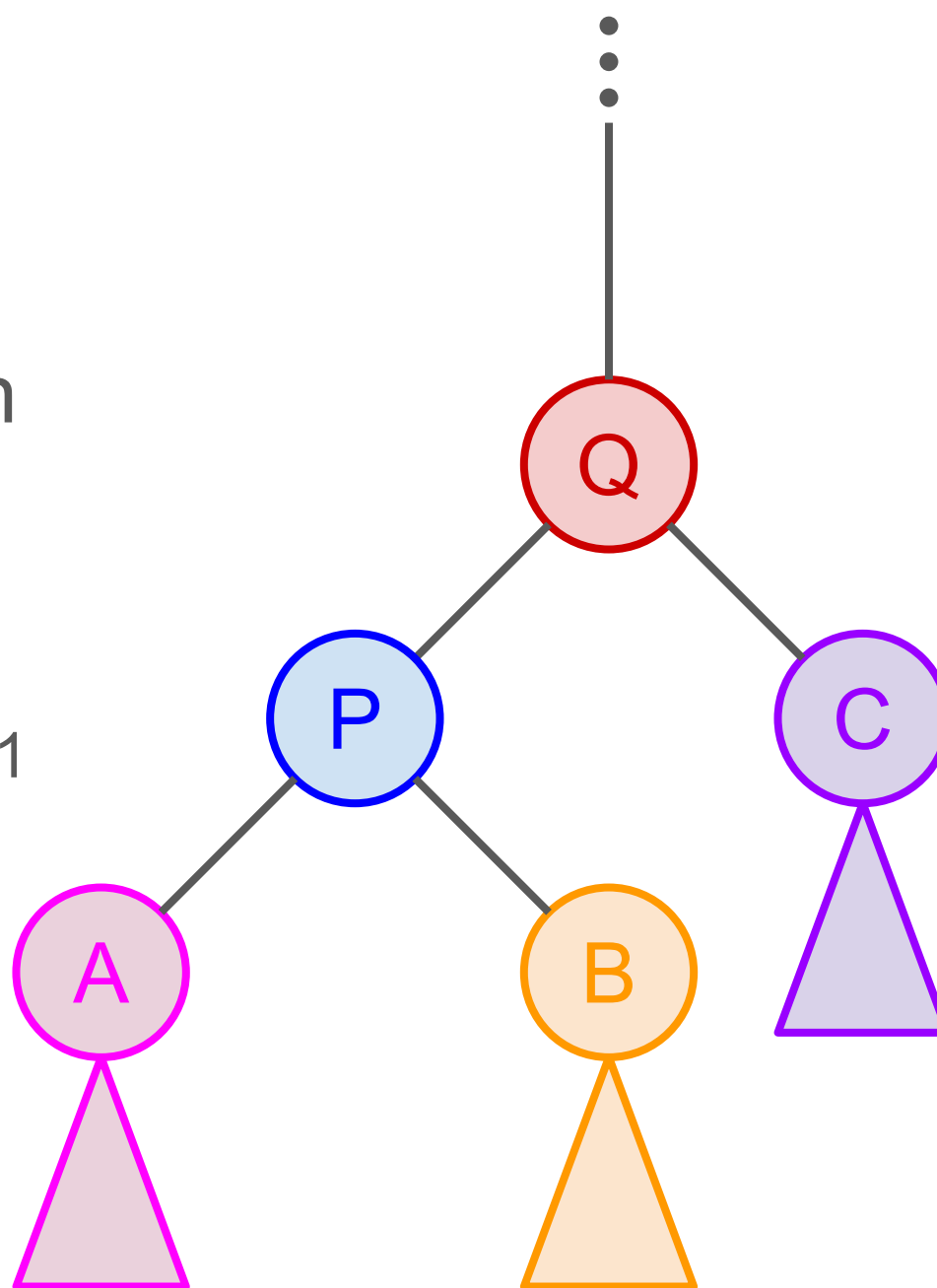
P cannot have 2 parents and Q cannot be without a parent! So we shall make the previous parent of P the new parent of Q instead.



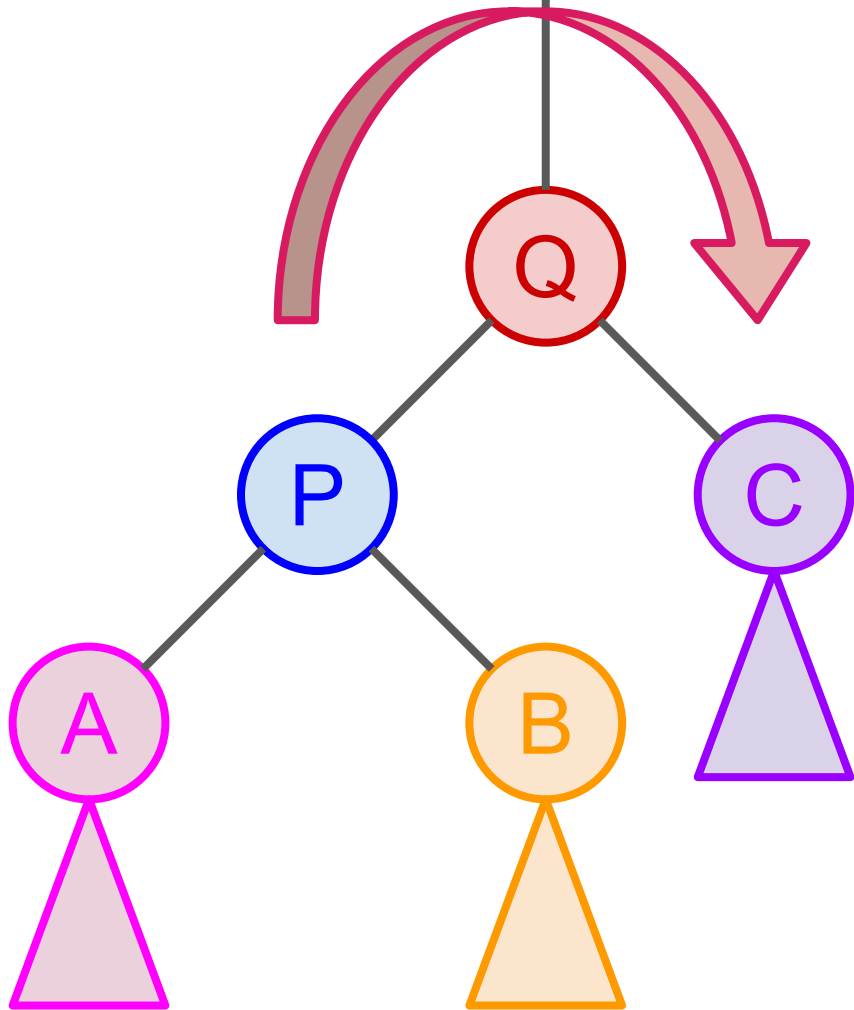
# Left rotate

Left rotation on  
P complete!

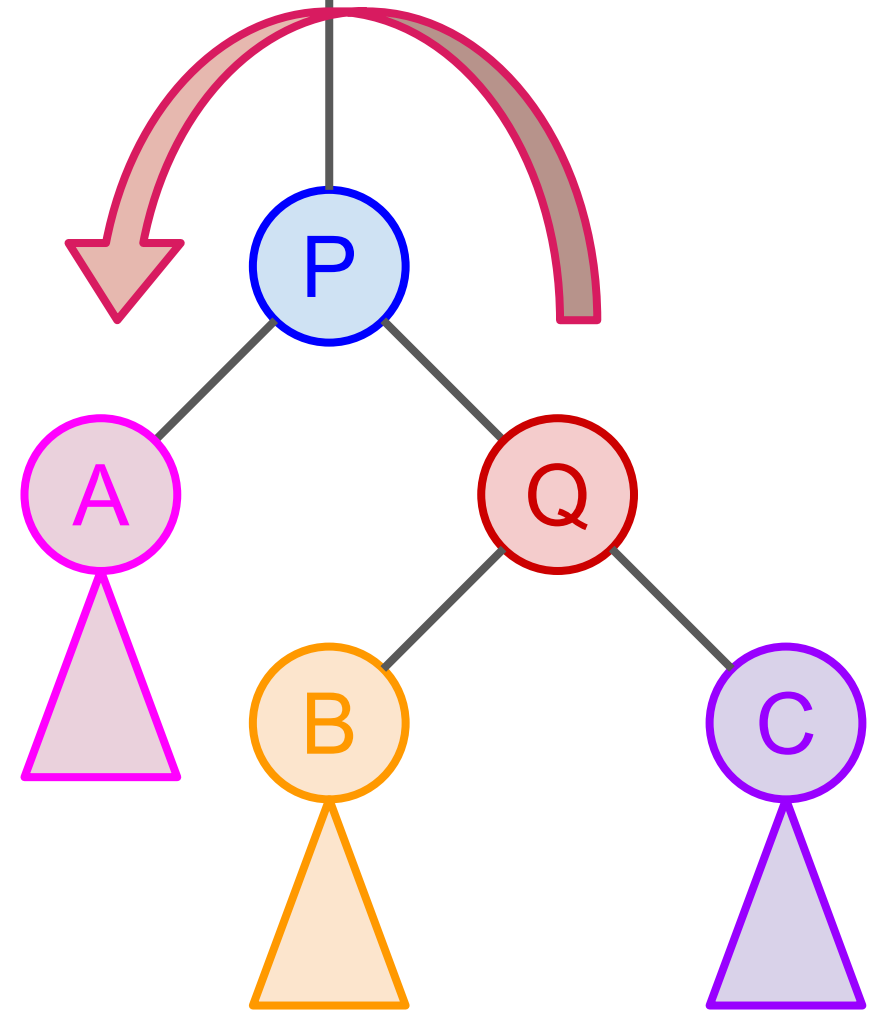
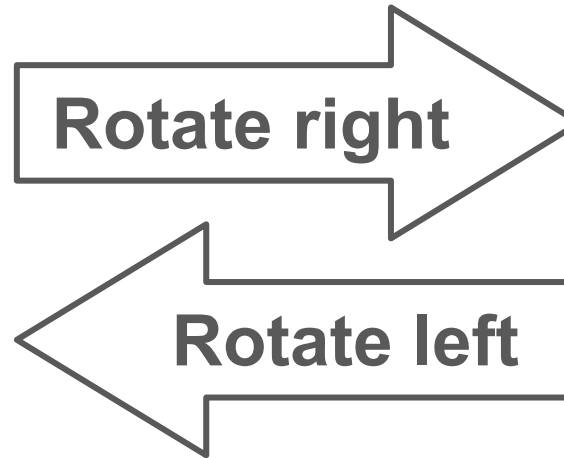
Notice C went up 1  
level, A went down 1  
level



⋮ AVL tree — Rotation summary ⋮



When left subtree is taller



When right subtree is taller

# AVL Tree—Balancing

We define *balance factor* for a vertex  $v$  to be:

$$bf(v) = h(v.left) - h(v.right)$$

$bf(v)$	Balance type
$[-1, 1]$	Balanced
$> 1$	left subtree <i>taller</i> than right subtree
$< -1$	left subtree <i>shorter</i> than right subtree

# AVL Tree Main Idea

- Keep all the nodes balanced after all operation.
- Before an operation: all nodes are balanced.
- After doing an operation, backtrack up the tree, and whenever you see a node with balance = 2 or -2, fix that node.
  - Is it possible to see a node with balance  $> 2$  or  $< -2$ ?
  -
- After an operation: all nodes are balanced.



# AVL Tree Main Idea

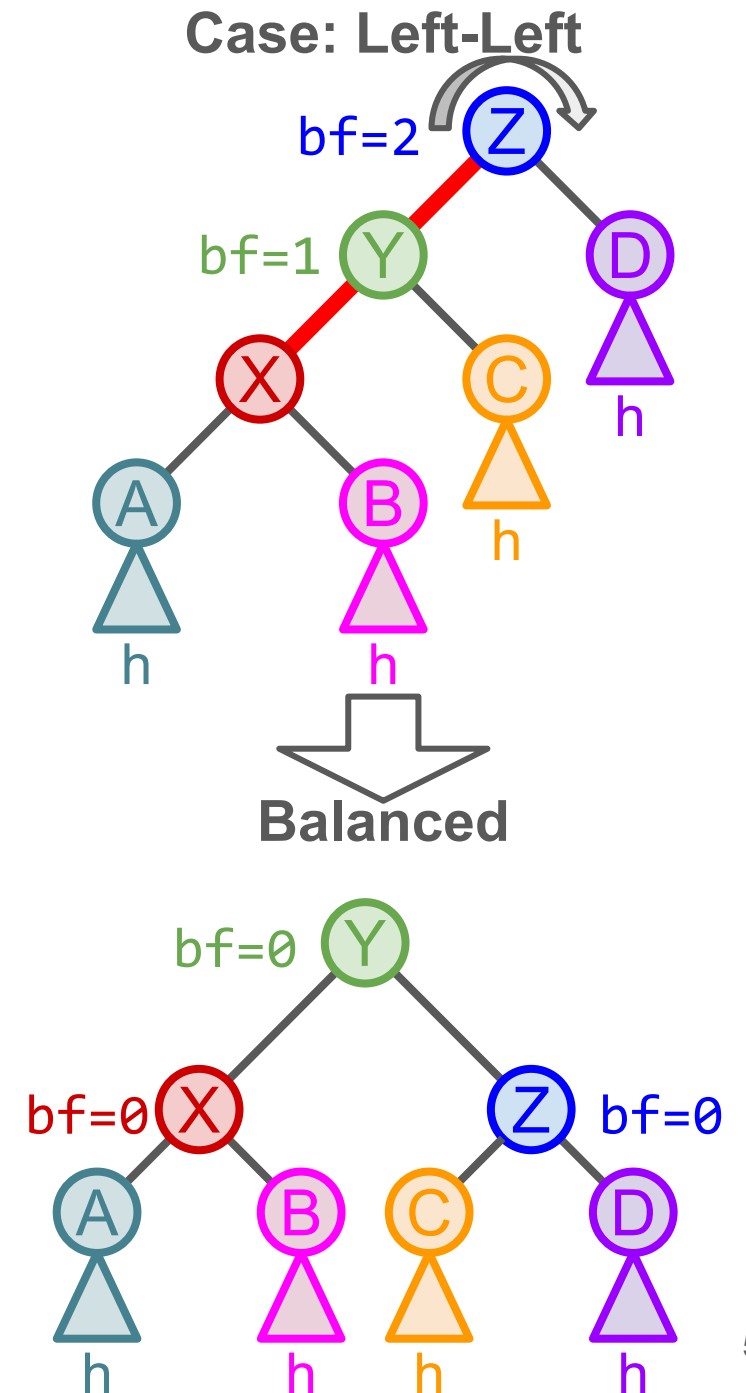
- Keep all the nodes balanced after all operation.
- Before an operation: all nodes are balanced.
- After doing an operation, backtrack up the tree, and whenever you see a node with balance = 2 or -2, fix that node.
  - Is it possible to see a node with balance  $> 2$  or  $< -2$ ?
  - **No! Because one operation only adds/removes one node.**  
**So the first unbalanced node visited will have balance = 2 or -2.**
- After an operation: all nodes are balanced.

# AVL Tree—Balancing

## Observation

A right rotate will *balance* a subtree if:

- The *left* child has a taller left subtree
- We call it *left-left* case because that's the 2 turns we take to traverse down to the taller subtree
- $\text{bf}(\text{v.left}) \geq 0$

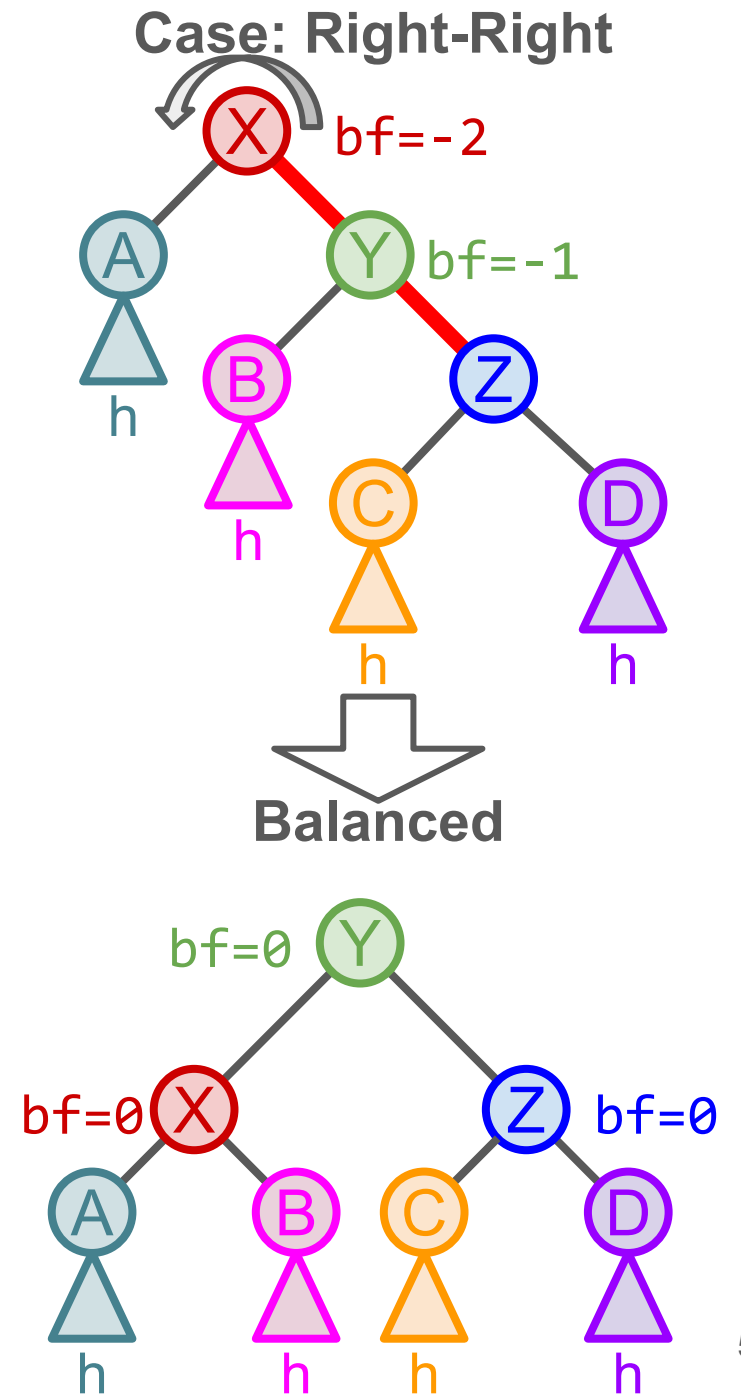


# AVL Tree—Balancing

## Observation

A left rotate will *balance* a subtree if:

- The *right* child has a taller right subtree
- We call it *right-right* case because that's the 2 turns we take to traverse down to the taller subtree
- $\text{bf}(v.\text{right}) \leq 0$



# AVL Tree—Balancing

## Observation

In essence, the previous 2 balancing tricks applies *iff* `bf(v)` and `bf(v.taller_child())` *do not* have opposing signs!

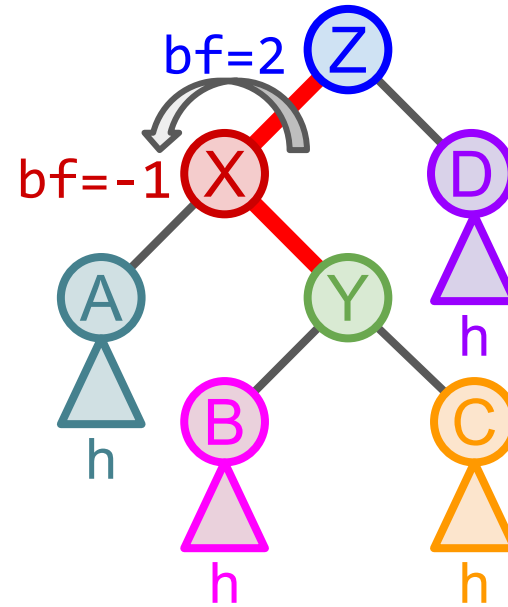
We regard 0 as neither positive or negative so they do not oppose any sign.

# AVL Tree—Balancing

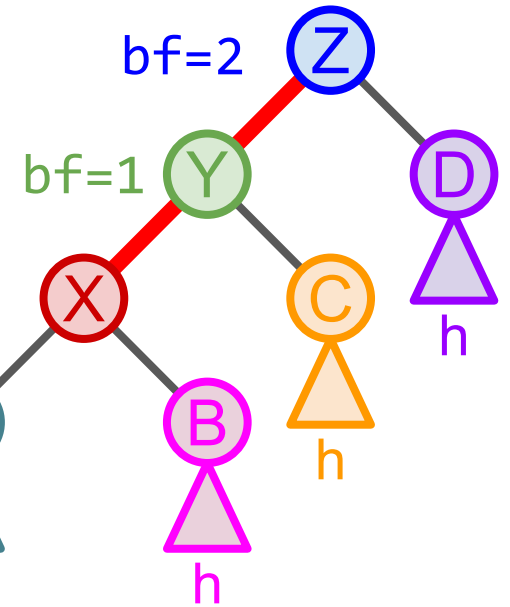
## Observation

If `bf(v.taller_child())` has an opposite sign to `bf(v)`, we can **rotate** `v.taller_child()` to transform its `bf` into the desired sign!

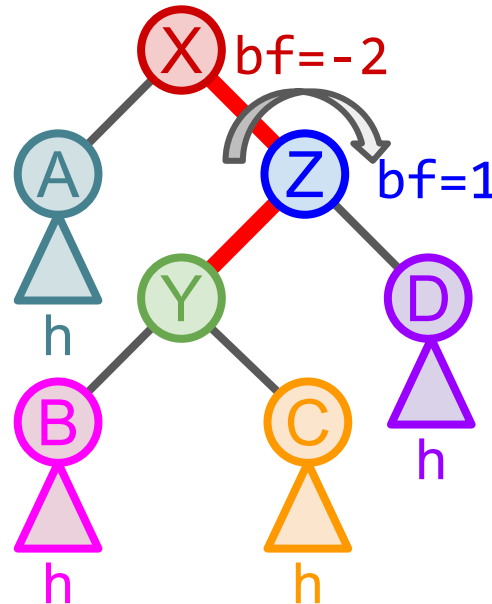
Case: Left-Right



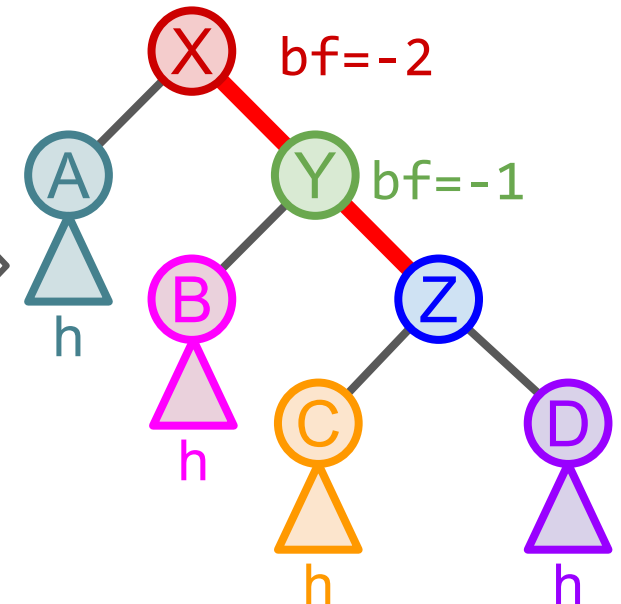
Case: Left-Left



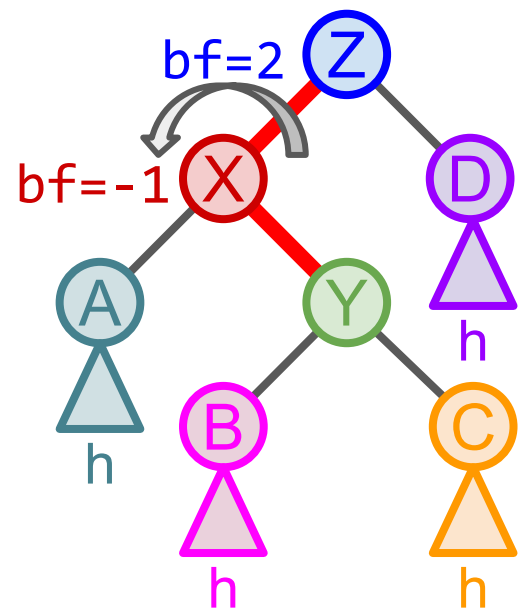
Case: Right-Left



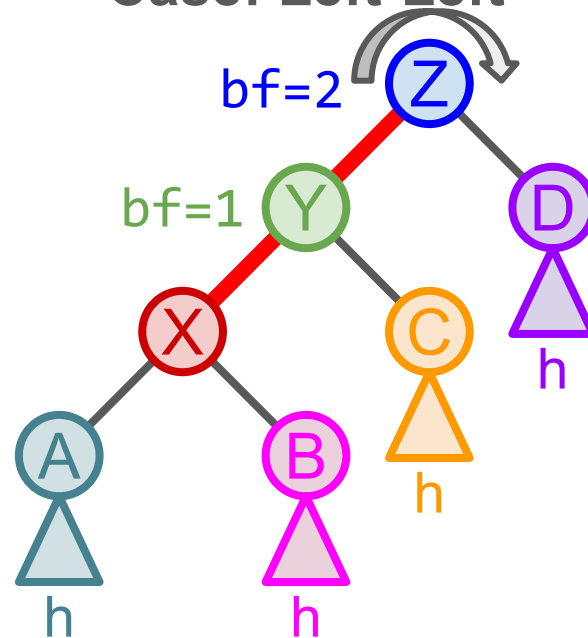
Case: Right-Right



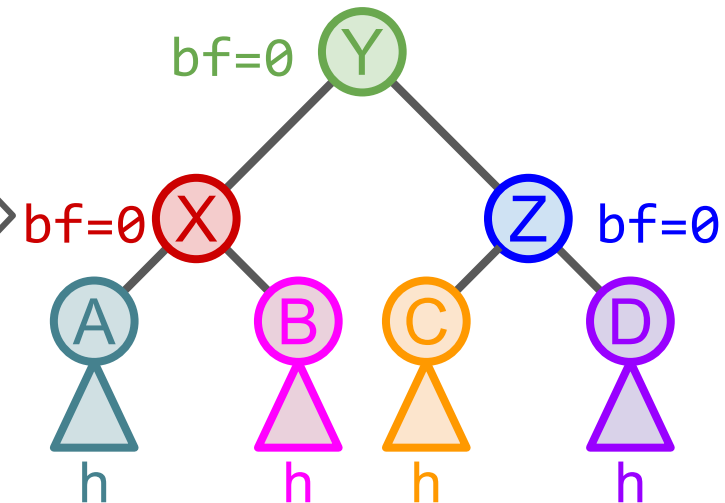
Case: Left-Right



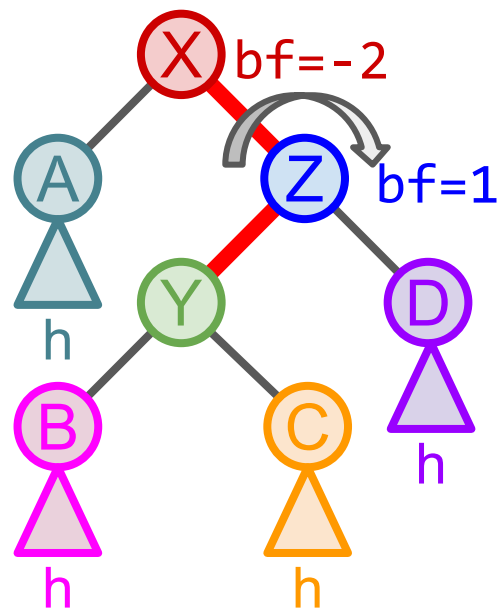
Case: Left-Left



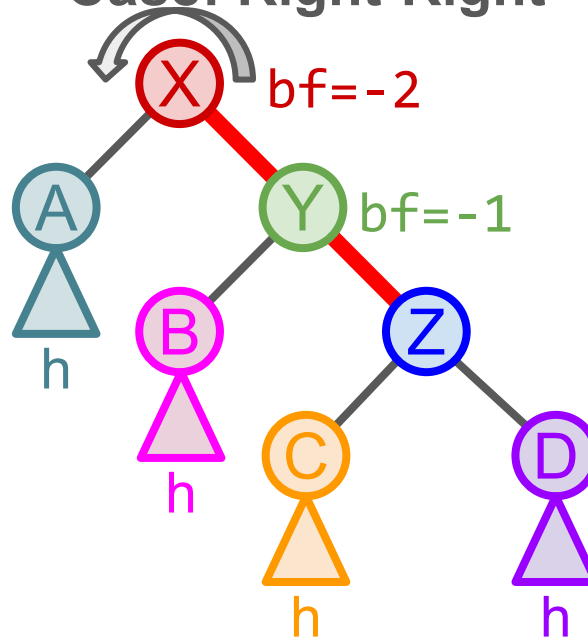
Balanced



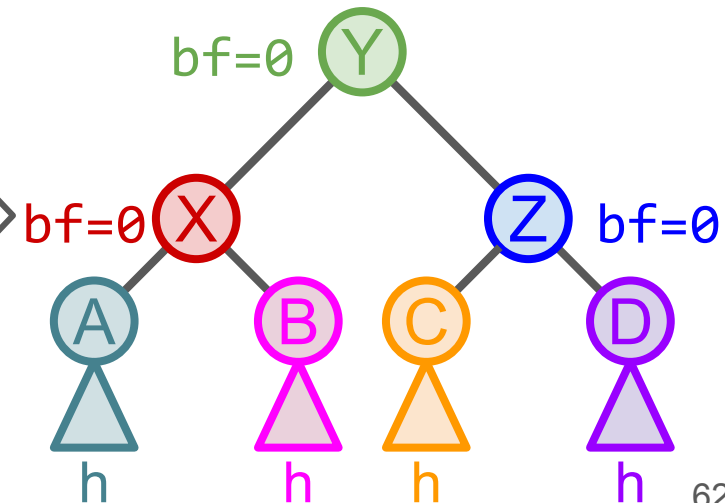
Case: Right-Left



Case: Right-Right



Balanced



# AVL Tree—Balancing summary

$bf(v)$	$bf(v.left)$	$bf(v.right)$	Case	Rotation
$[-1, 1]$	Don't care	Don't care	Balanced	NIL
2	$\geq 0$	Don't care	Left-Left	<code>rotate_right(v)</code>
	$< 0$		Left-Right	<code>rotate_left(v.left)</code>
-2	Don't care	$\leq 0$	Right-Right	<code>rotate_left(v)</code>
		$> 0$	Right-Left	<code>rotate_right(v.right)</code>

# Question 2



# Problem statement

Draw a valid AVL Tree and nominate a vertex to be deleted such that if that vertex is deleted:

- a. No rotation happens
- b. Exactly one of the four rotation cases happens
- c. Exactly two of the four rotation cases happens (you cannot use the sample given in VisuAlgo which is <https://visualgo.net/en/bst?mode=AVL&create=8,6,16,3,7,13,19,2,11,15,18,10>, delete vertex 7; think of your own test case)

# Test yourself!

Recall that an unbalanced subtree is the condition for rebalancing rotation(s).

After a subtree has been re-balanced with rotation(s), which other vertices do we have to check for imbalances?

# Question 2: Deletion

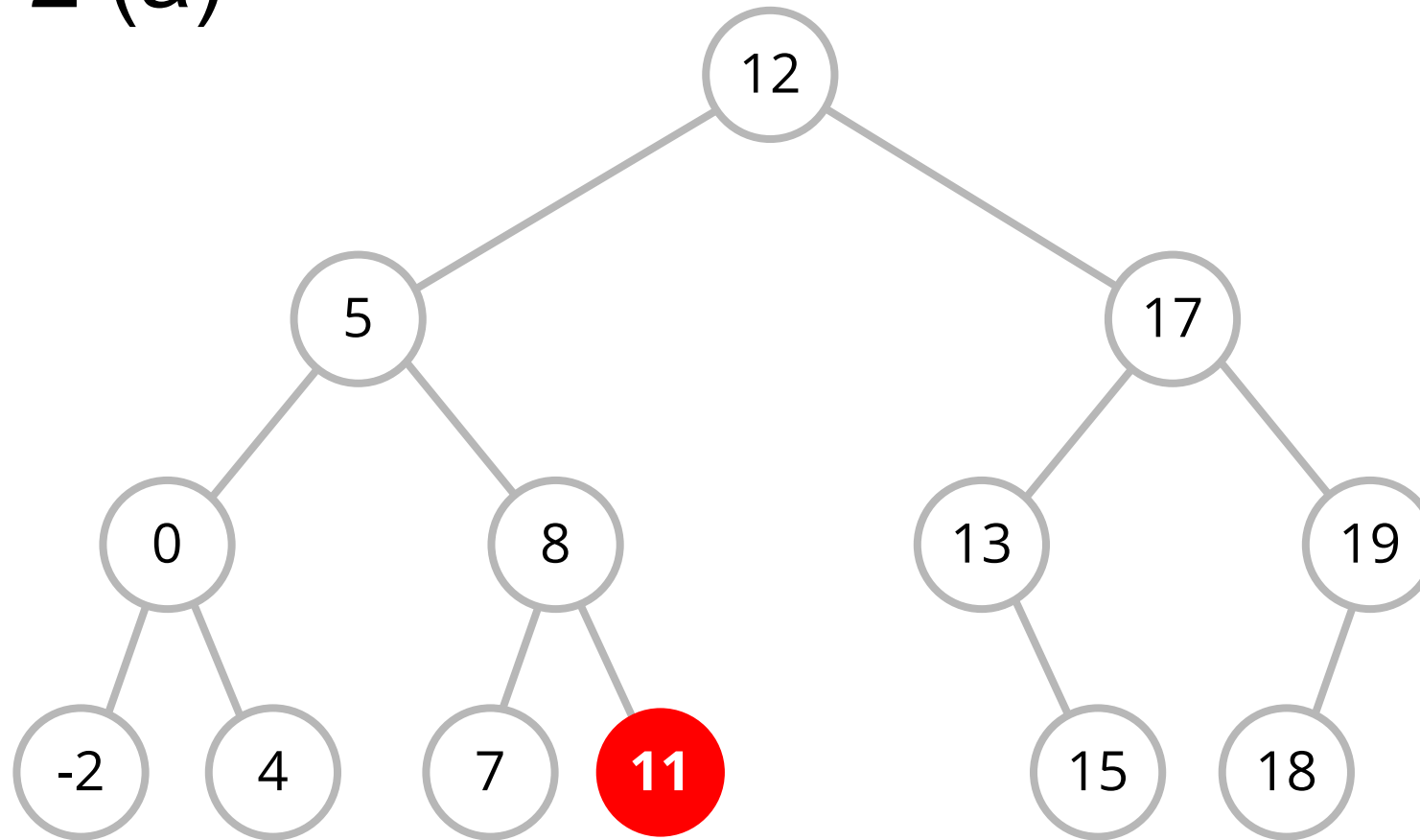
**What are the 3 cases for deletion?**

- Case 1: Is leaf vertex
  - Just remove it
- Case 2: Has one child
  - Connect the child subtree to the deleted vertex's parent
- Case 3: Has 2 children
  - Replace with successor, delete successor instead
  - Can also use predecessor

## Question 2: Rotation

- a. No rotation happens
  - No imbalance
- b. Only one rotation case
  - Imbalance that can be *resolved* with one rotation
- c. Exactly two rotation cases.
  - Multiple sequential rotations
  - “Skewed” BBST
- d. Exactly three rotation cases.

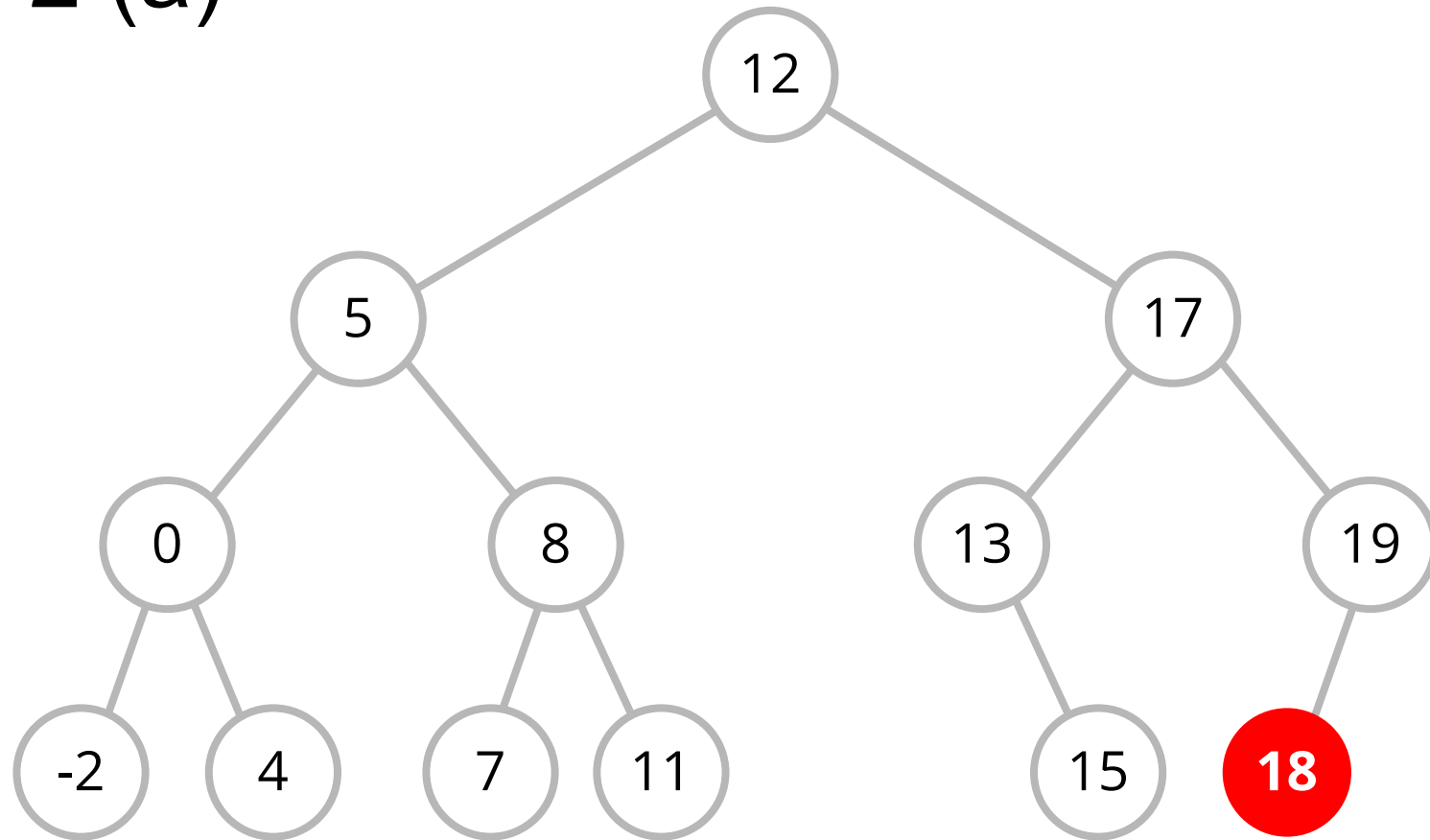
## Question 2 (a)



a. No rotation

<https://visualgo.net/en/bst?create=12,5,17,0,8,13,19,-2,4,7,11,15,18&mode=AVL>

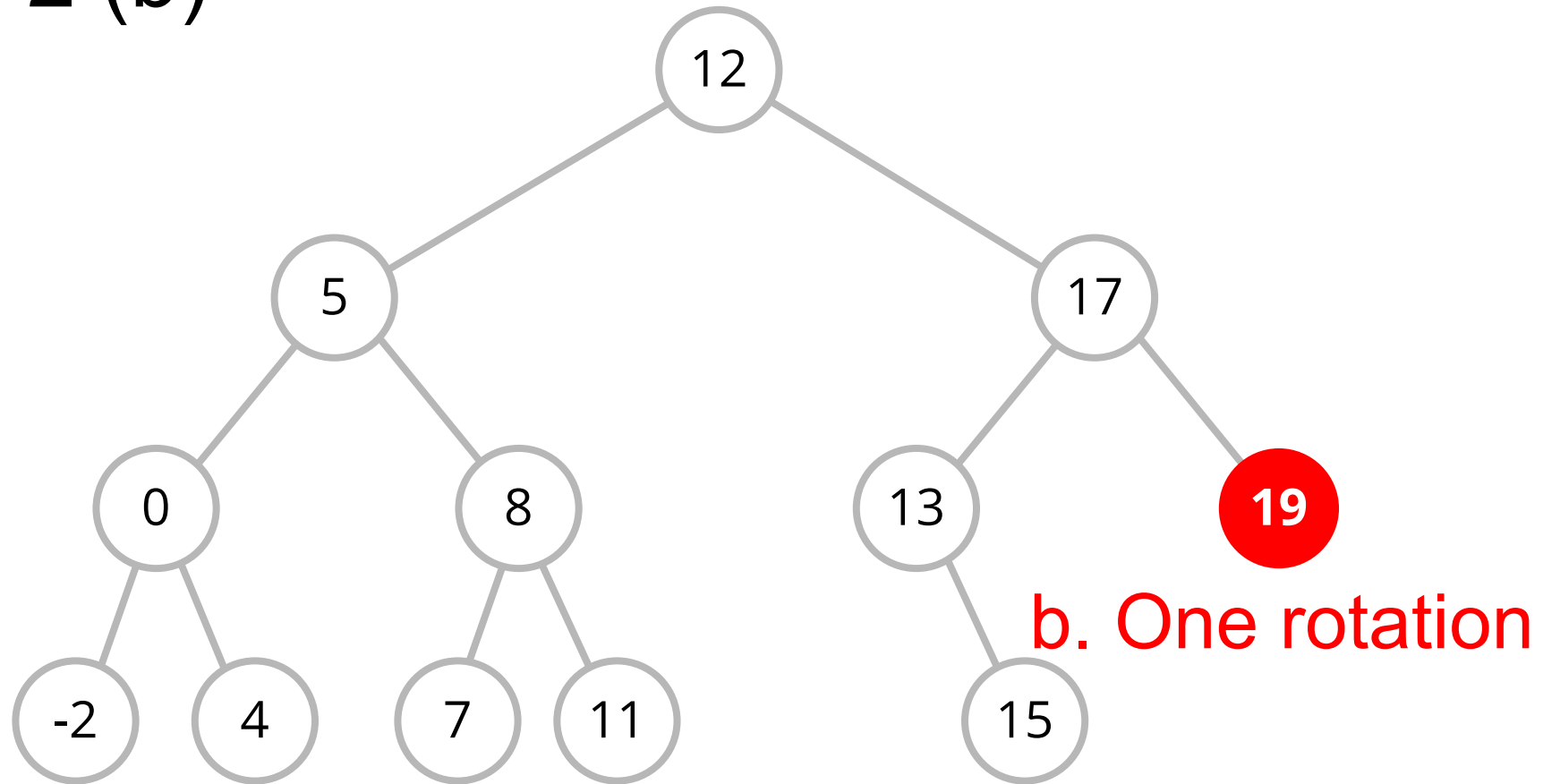
## Question 2 (a)



a. No rotation

<https://visualgo.net/en/bst?create=12,5,17,0,8,13,19,-2,4,7,11,15,18&mode=AVL>

## Question 2 (b)



<https://visualgo.net/en/bst?create=12,5,17,0,8,13,19,-2,4,7,11,15&mode=AVL>

## Question 2 (c)

### Observation

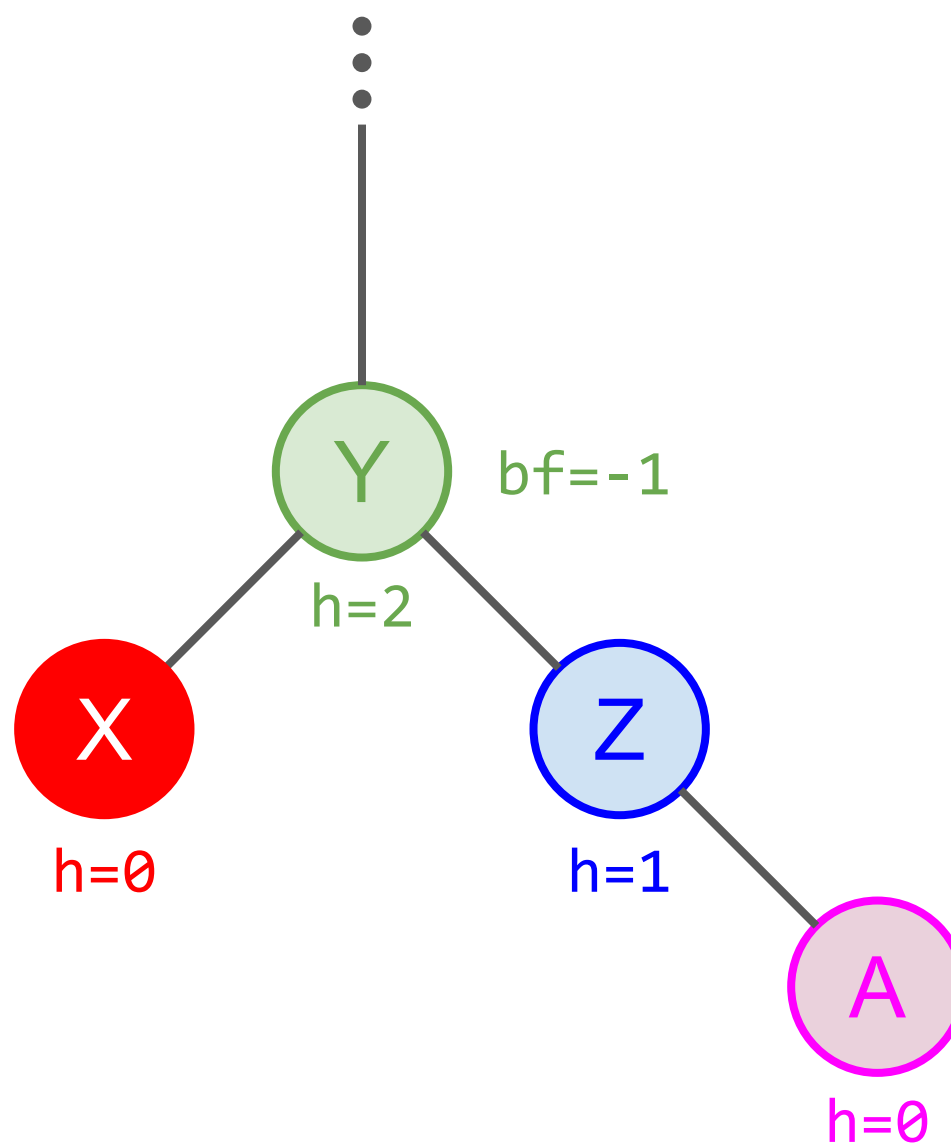
- Rebalancing a subtree can reduce its height by **at most 1**
- To get *2 rotations*, you must make it such that when you remove the vertex, *2 subtrees* will be imbalanced in the process



## Question 2 (c)

We first show a scenario where rebalancing reduces the subtree's height by 1.

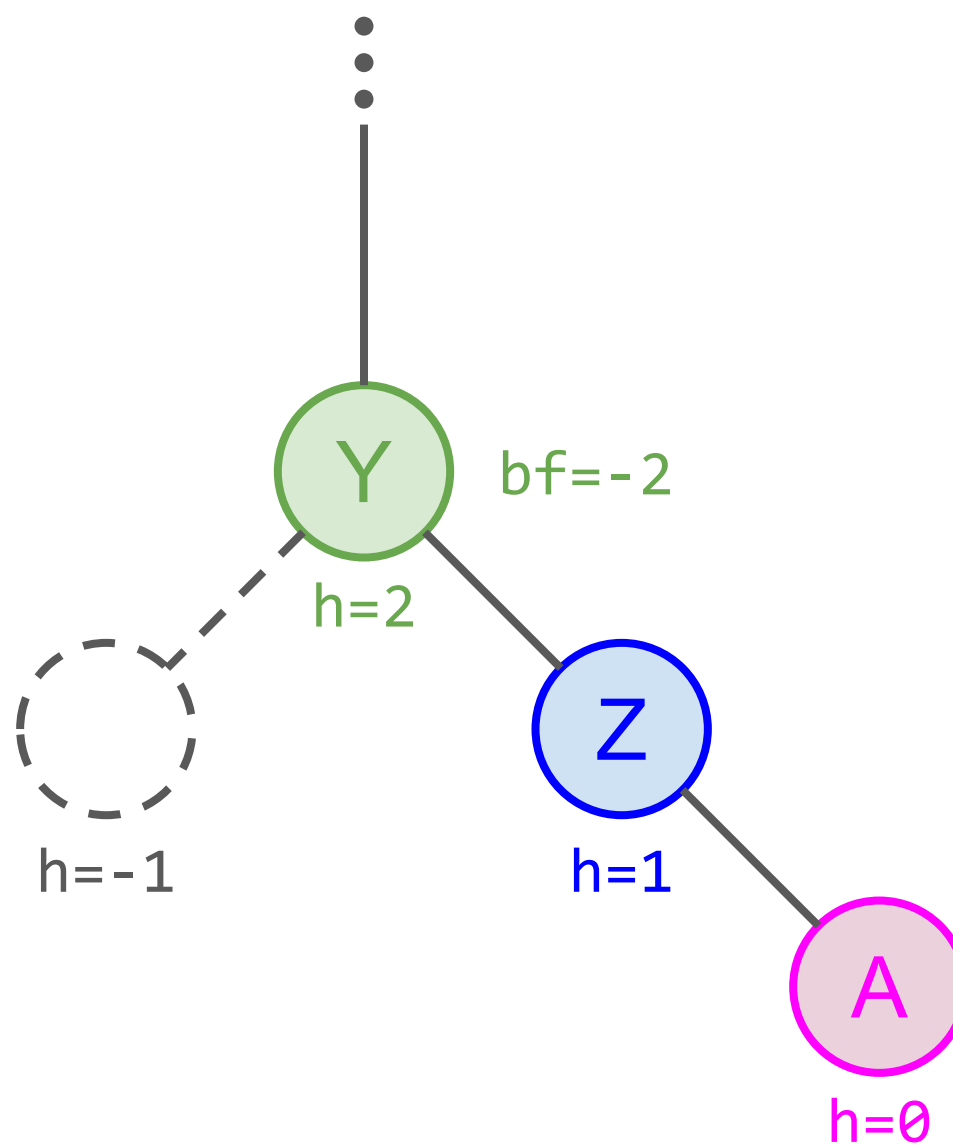
We shall remove  $X$  from this subtree  $Y$  with height 2.



## Question 2 (c)

X removed.

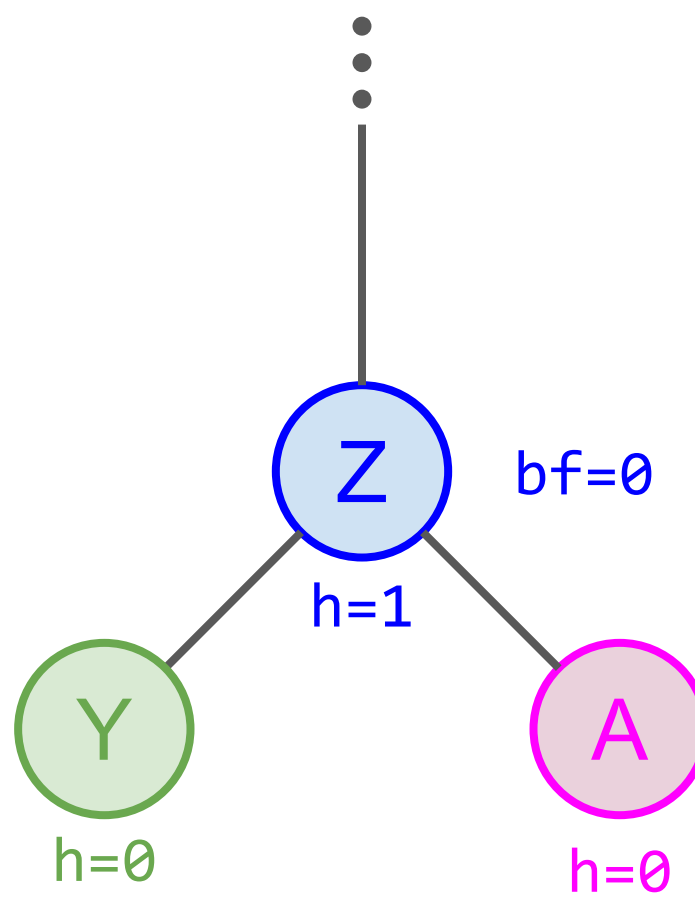
$bf(Y)$  is upset.



## Question 2 (c)

After rotation, this subtree has height 1.

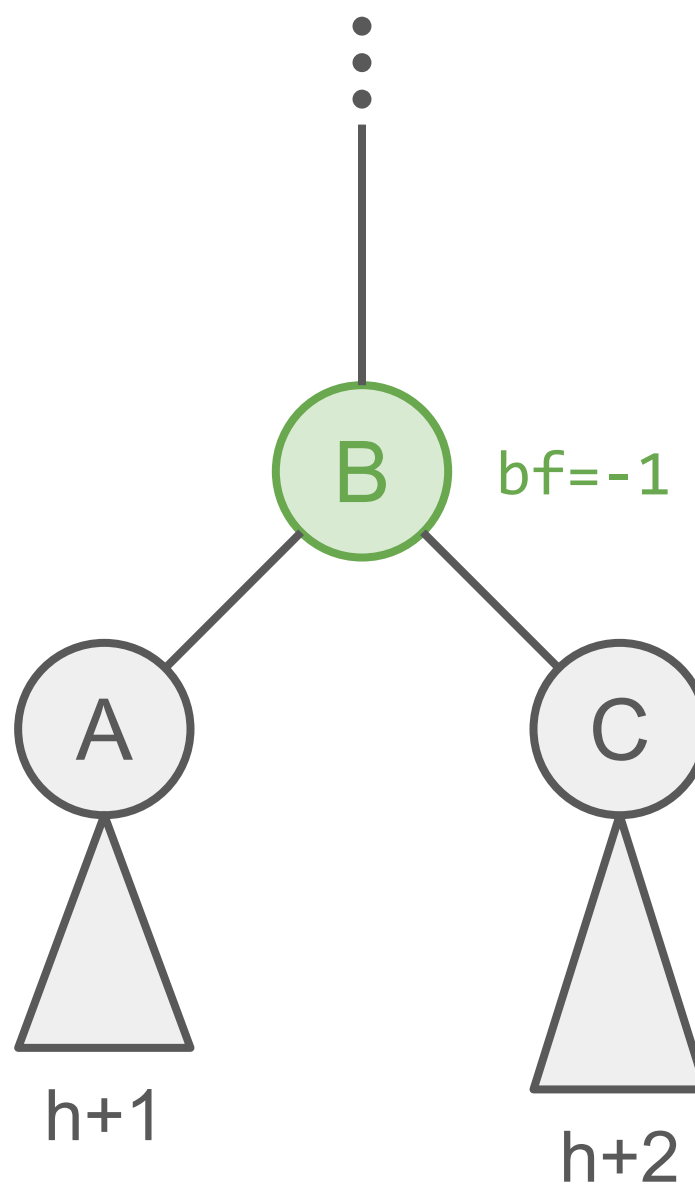
Realize the height will also drop by 1 if we simply removed A instead of X, in which case no rotation would be required.



## Question 2 (c)

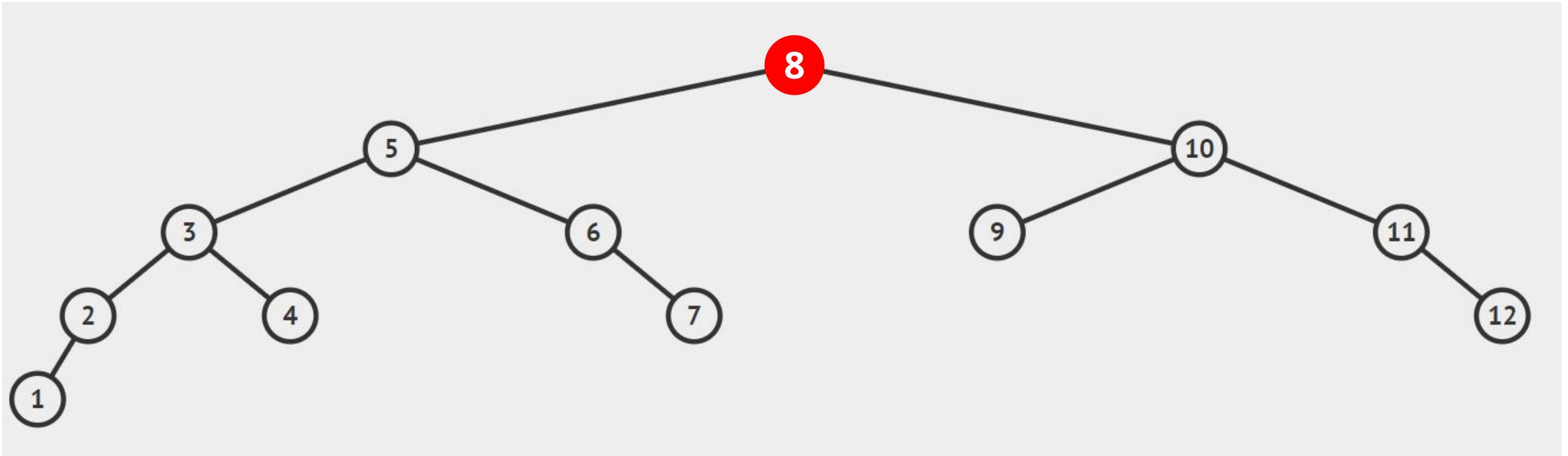
So here's a general strategy, given a subtree of this form:

To cause at least 2 rotations, we can simply find a vertex to remove in A such that A's height will be decreased by 1 after a rebalancing rotation within.



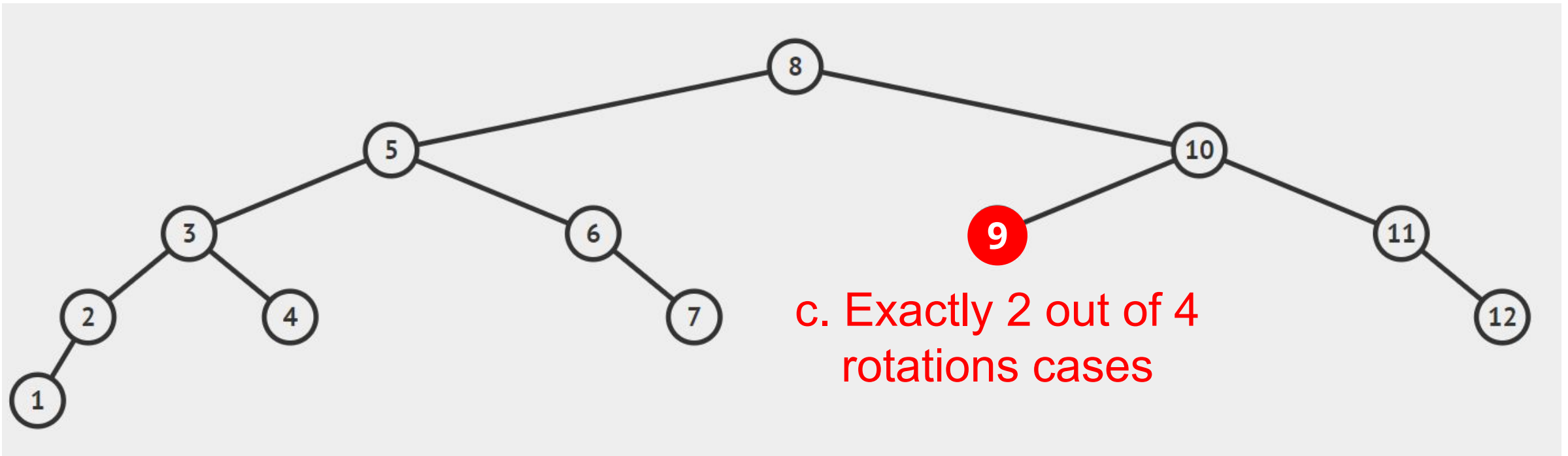
## Question 2 (c)

c. Exactly 2 out of 4 rotations cases



<https://visualgo.net/en/bst?create=8,5,10,3,6,9,11,2,4,7,12,1&mode=AVL>

## Question 2 (c)

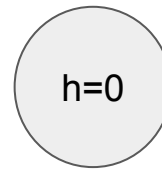


<https://visualgo.net/en/bst?create=8,5,10,3,6,9,11,2,4,7,12,1&mode=AVL>

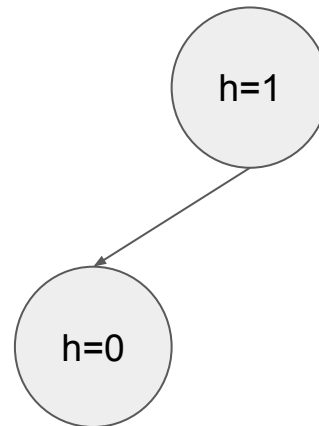
# Minimum number of nodes in AVL tree of height $h$

Suppose  $N_h$  is the minimum number of nodes.

Then,  $N_0 = 1$ :



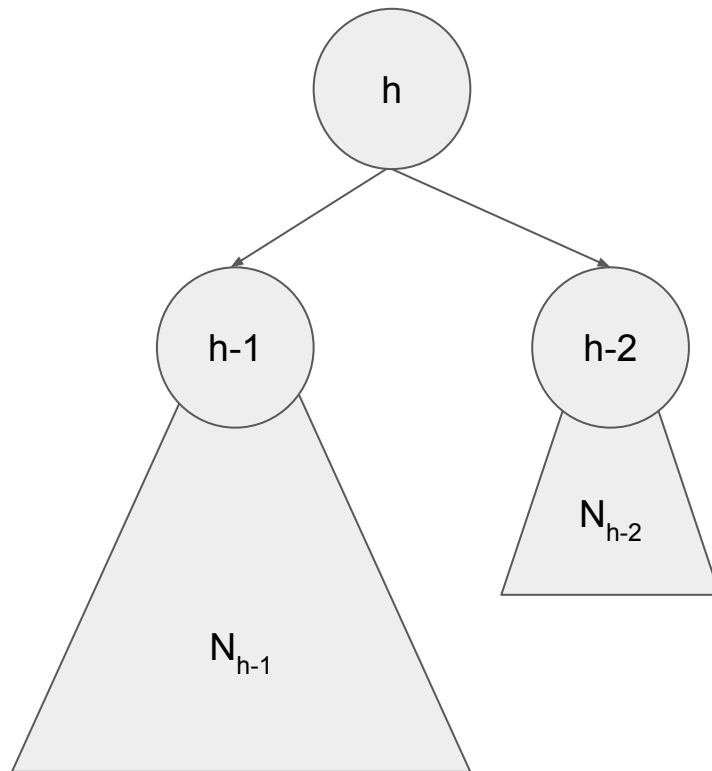
Then  $N_1 = 2$ :



# Minimum number of nodes in AVL tree of height h

Suppose  $N_h$  is the minimum number of nodes.

Then,  $N_h = 1 + N_{h-1} + N_{h-2}$ :







# Test yourself!

- What is the **min/max** height of a AVL Tree with **33** vertices?
- How did you get those values?



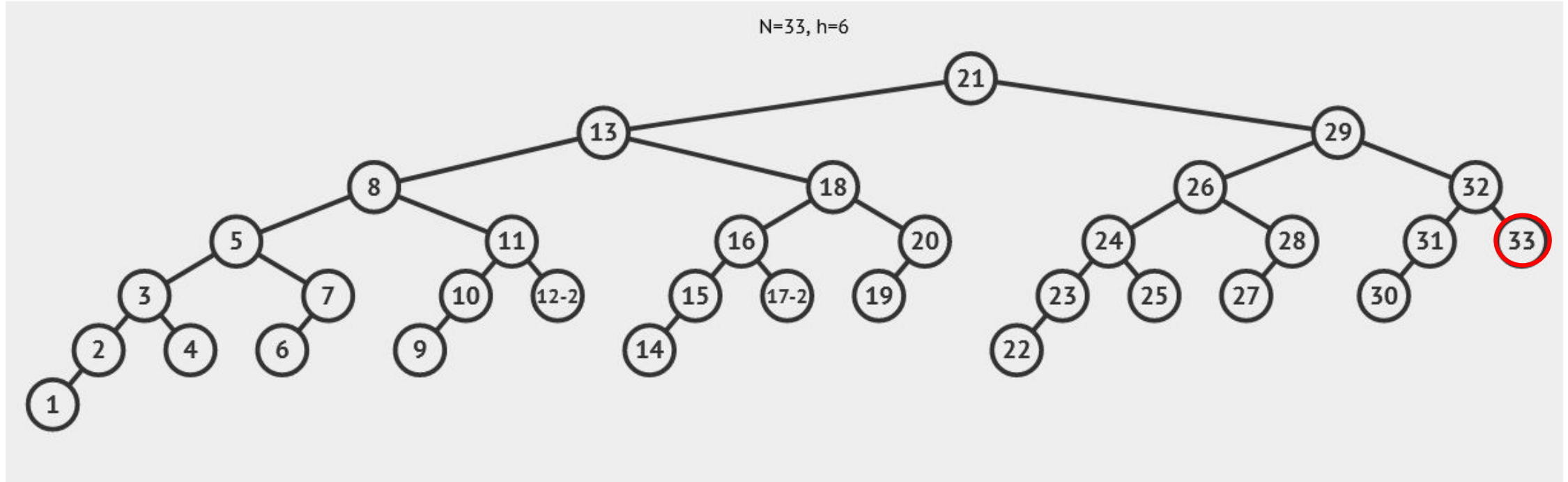
# Test yourself!

- What is the **min/max** height of a AVL Tree with **15** vertices?
- How did you get those values?

## Answer:

- **Minimum:**  $\lfloor \log_2(15+1) \rfloor = 4$ ; as compact as possible.
- **Maximum:**  $N_4 = 12$ ,  $N_5 = 20$ . Need 20 nodes for height 5. So 15 nodes can have maximum height 4.

## Question 2 (d)



<https://visualgo.net/en/bst?mode=AVL&create=21,13,29,8,18,26,32,5,11,16,20,24,28,31,33,3,7,10,12,15,17,19,23,25,27,30,2,4,6,9,14,22,1>

# Question 3

Augmented BBST

# Problem statement

There are two important BST operations: *Rank* and *Select* that are not included in VisuAlgo yet

(Overview at <https://visualgo.net/en/bst?slide=5-1>) but can be quite useful for some order statistics problems. Please discuss on how to implement these two operations efficiently

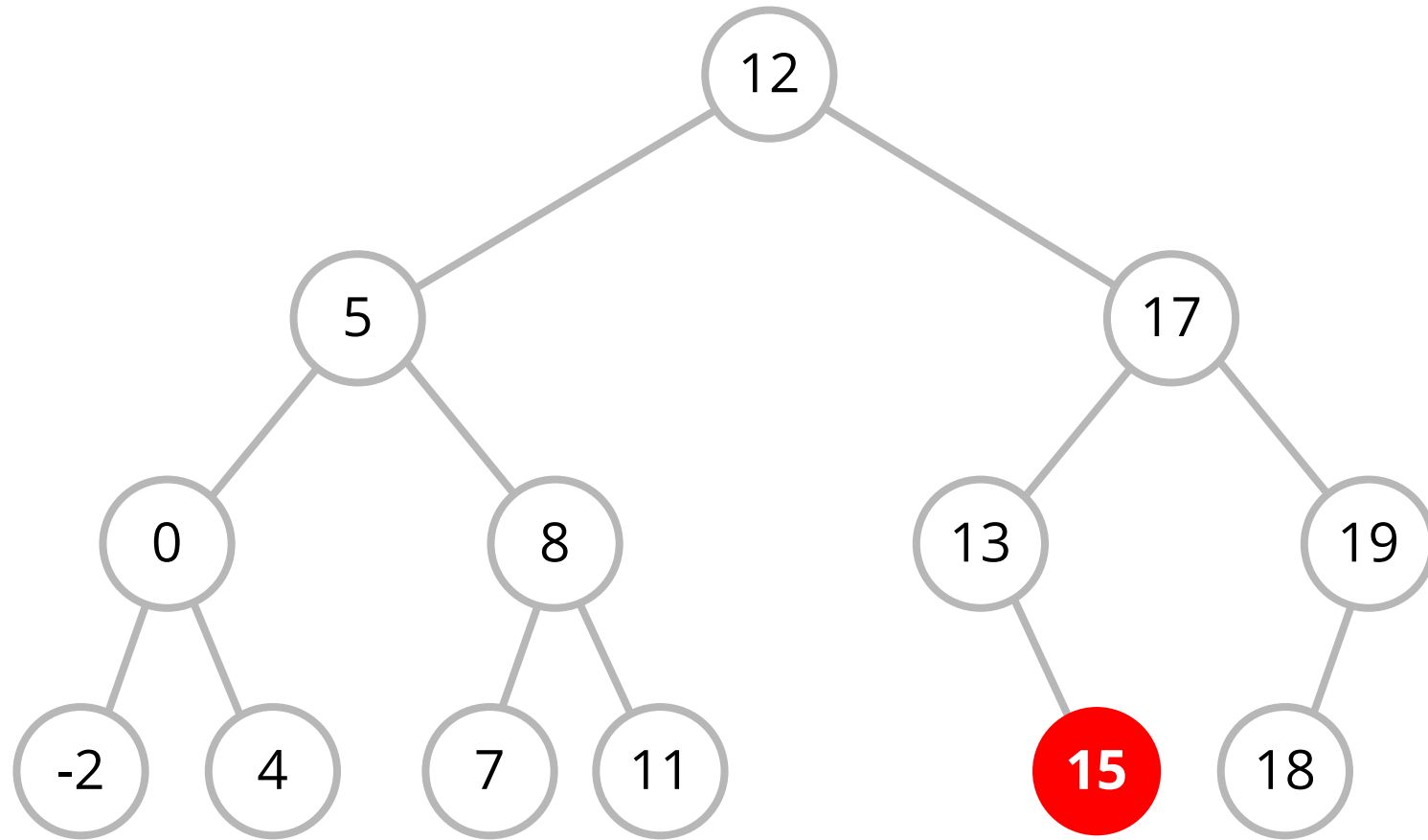
## Question 3: Rank

`rank(key)` returns the 1-based index of `key` in the sorted ordering of all keys in the BST.

E.g. `rank(5)` on a BST containing `{2, 1, 7, 5, 0}` is `4` because it belongs to that 1-based index position in the sorted sequence `{0, 1, 2, 5, 7}`.

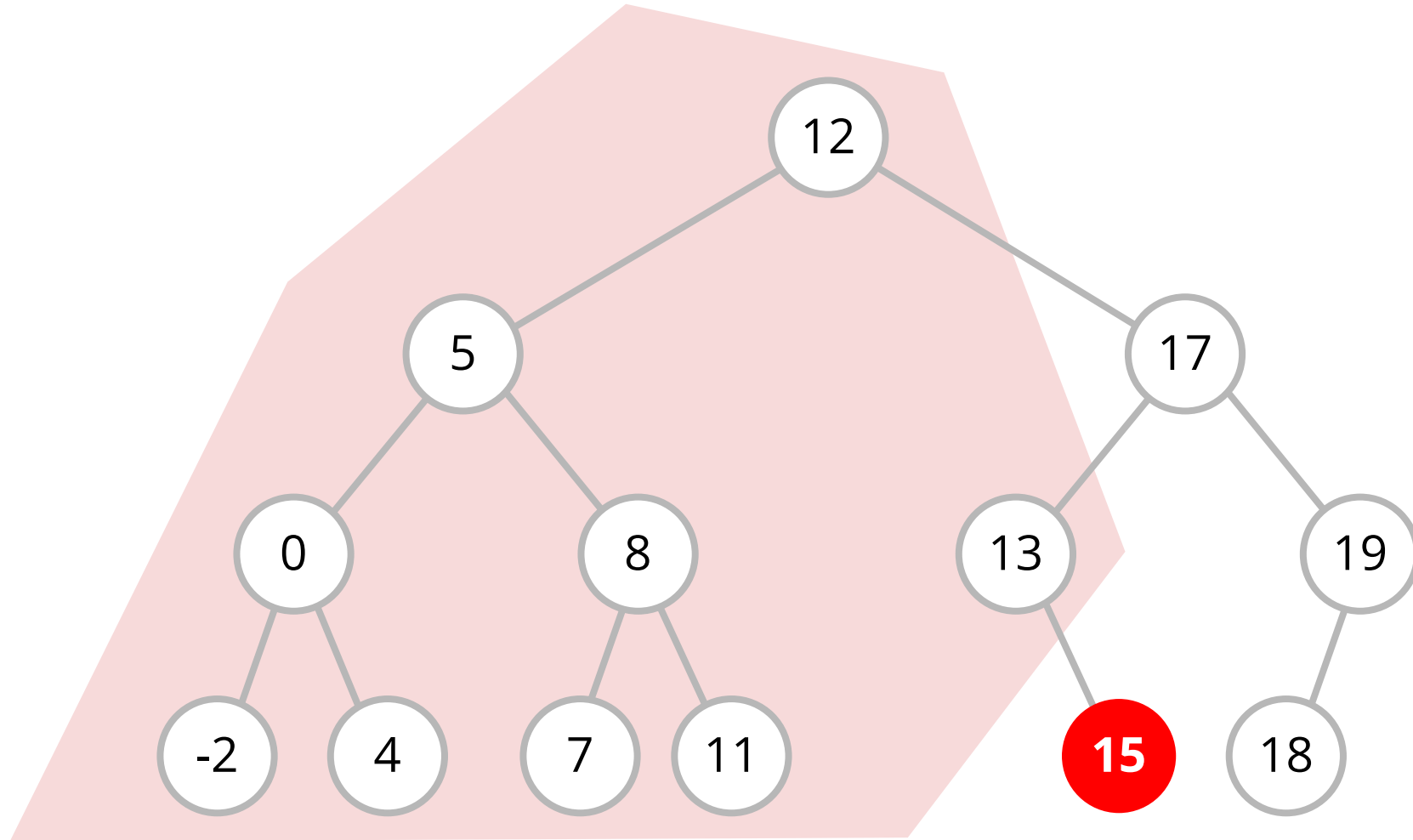
Realize, this is simply  $1 +$  the number of keys in the BST that are *smaller than* `key`.

## Question 3: Rank



Get rank of 15.

## Question 3: Rank



Get rank of 15:  
Rank 10.



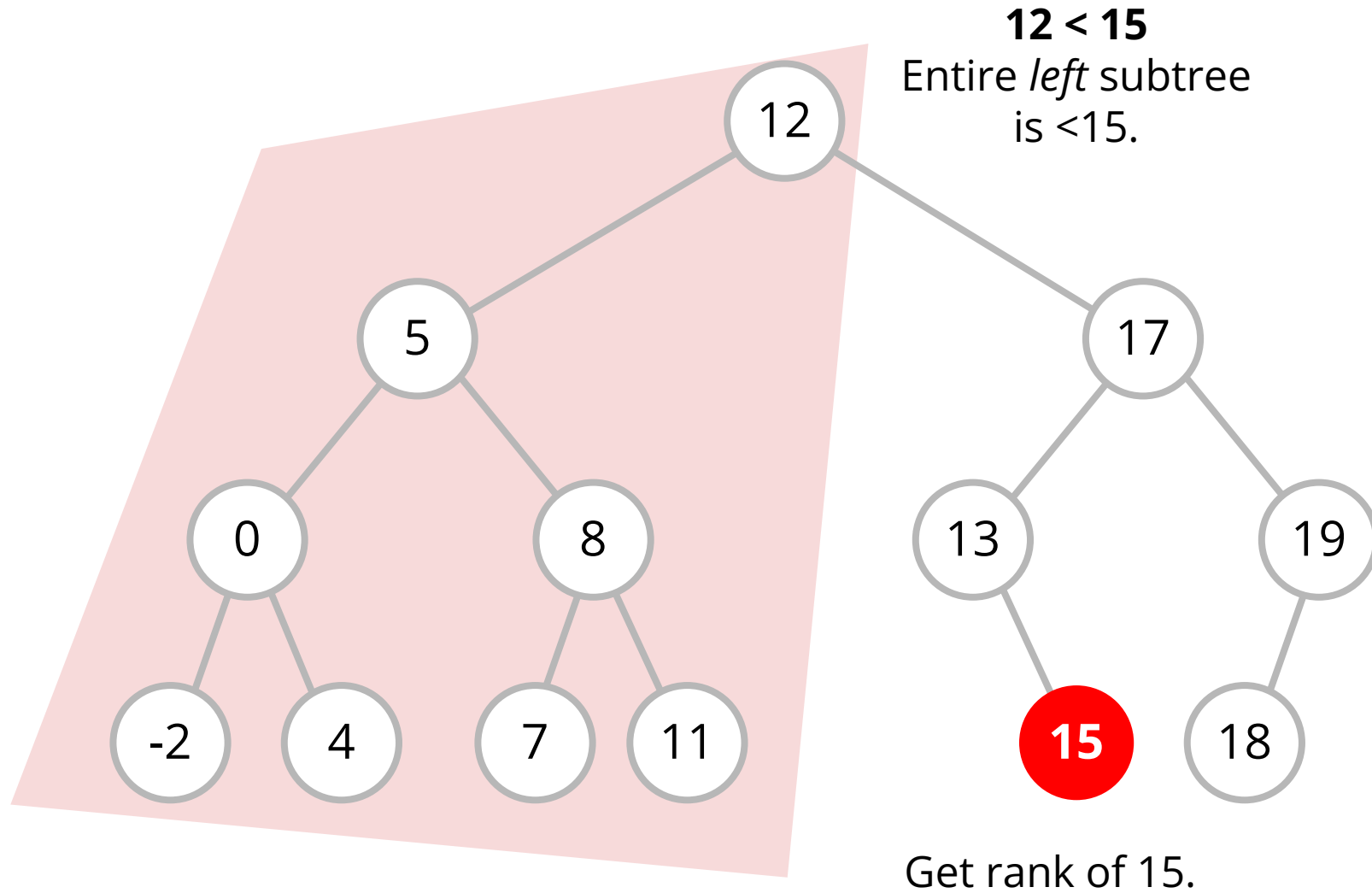
## Question 3: Rank

### **Storing Subtree Size**

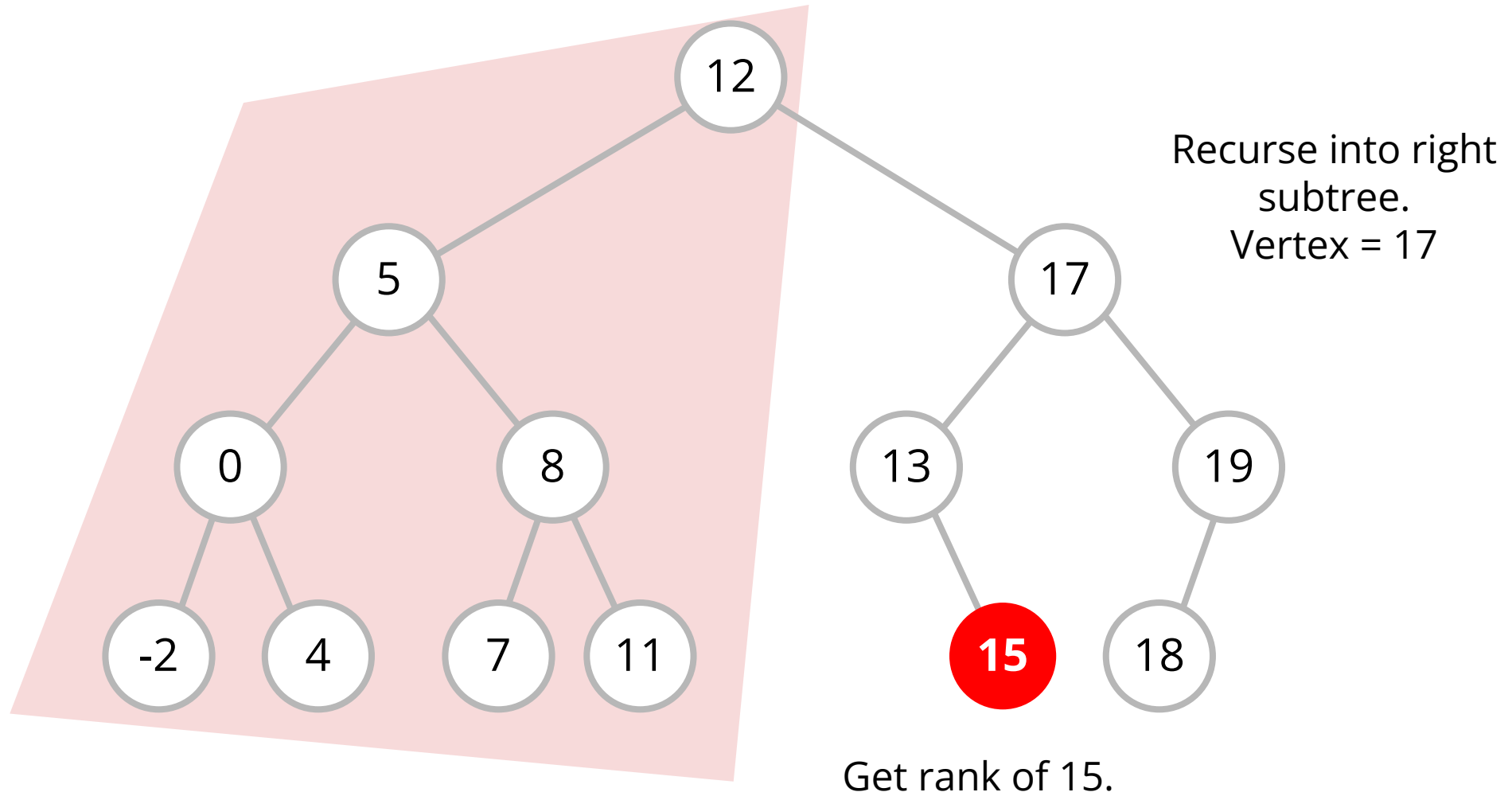
To implement this efficiently, at every vertex, we need to store number of vertices in the subtree (i.e size) rooted at that vertex.

Same idea as “caching” height of subtree in AVL Tree

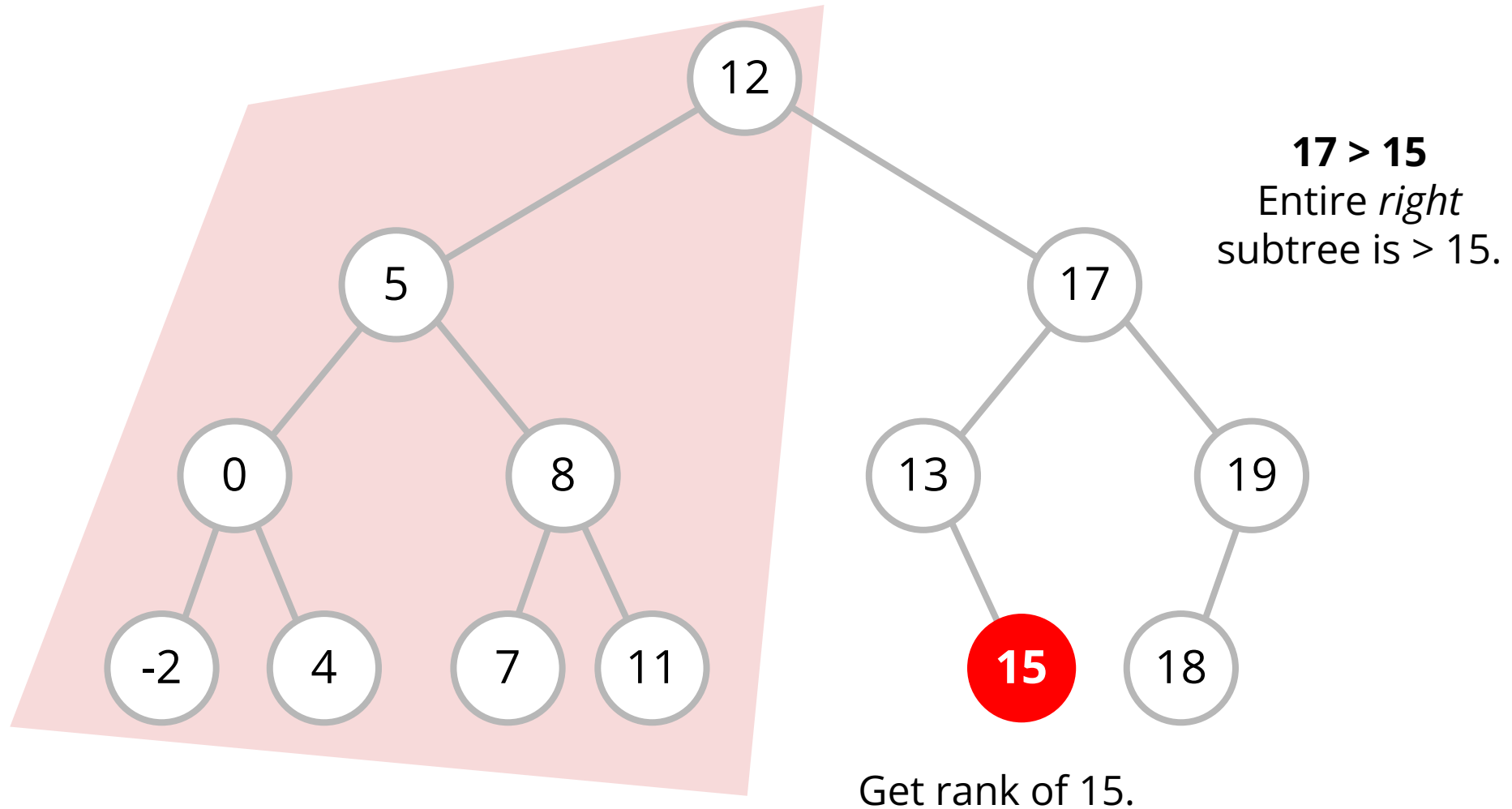
# Question 3: Rank



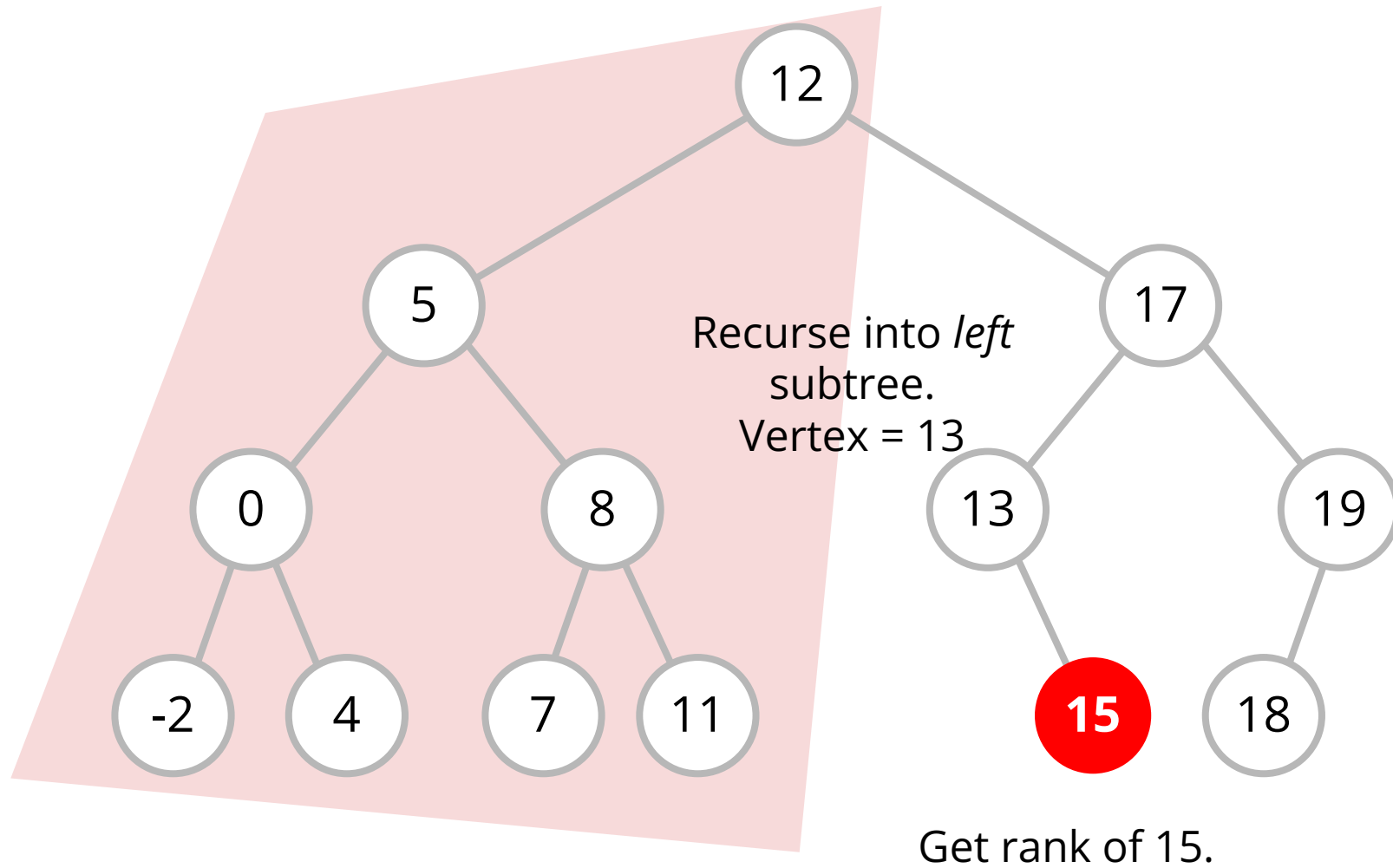
# Question 3: Rank



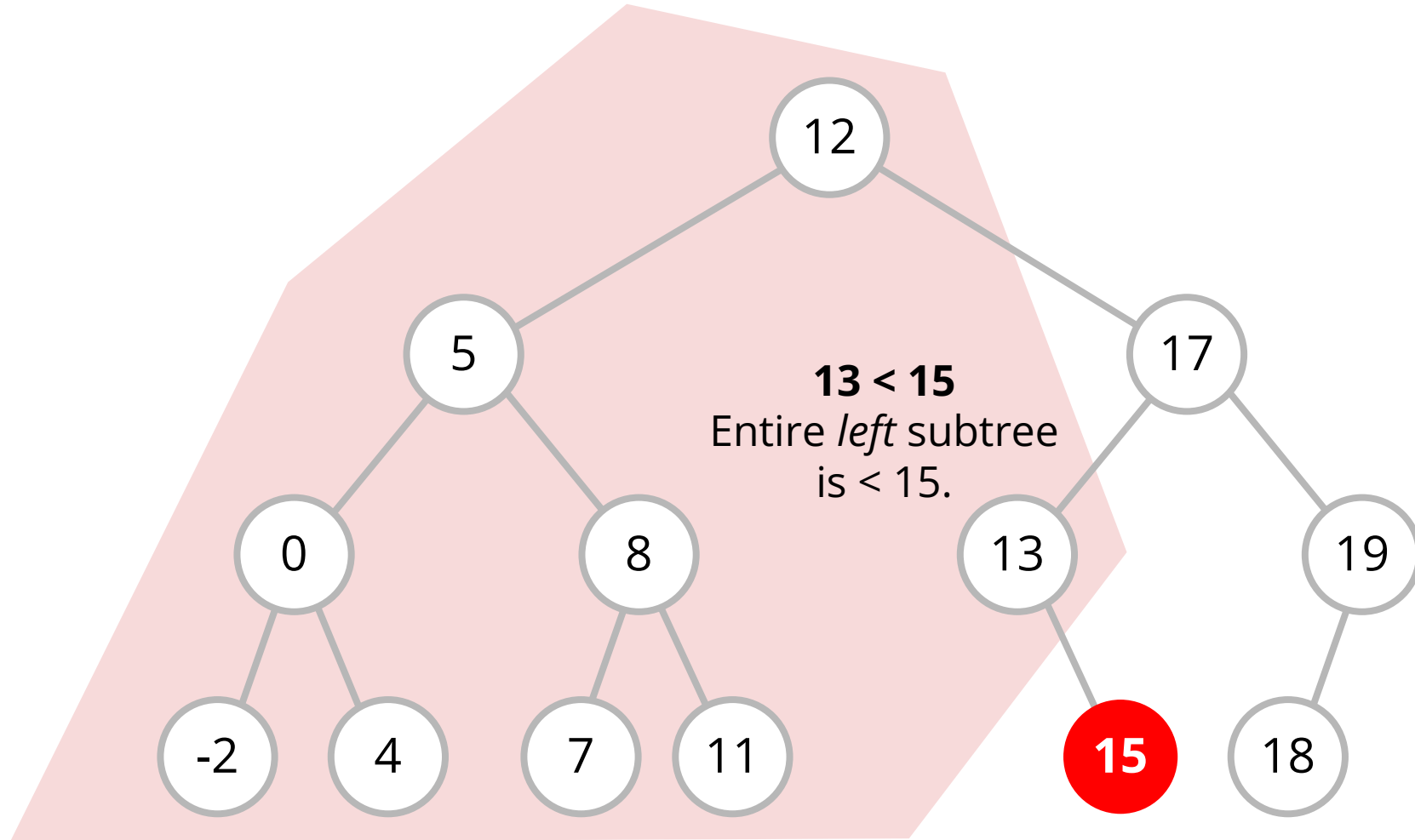
## Question 3: Rank



## Question 3: Rank

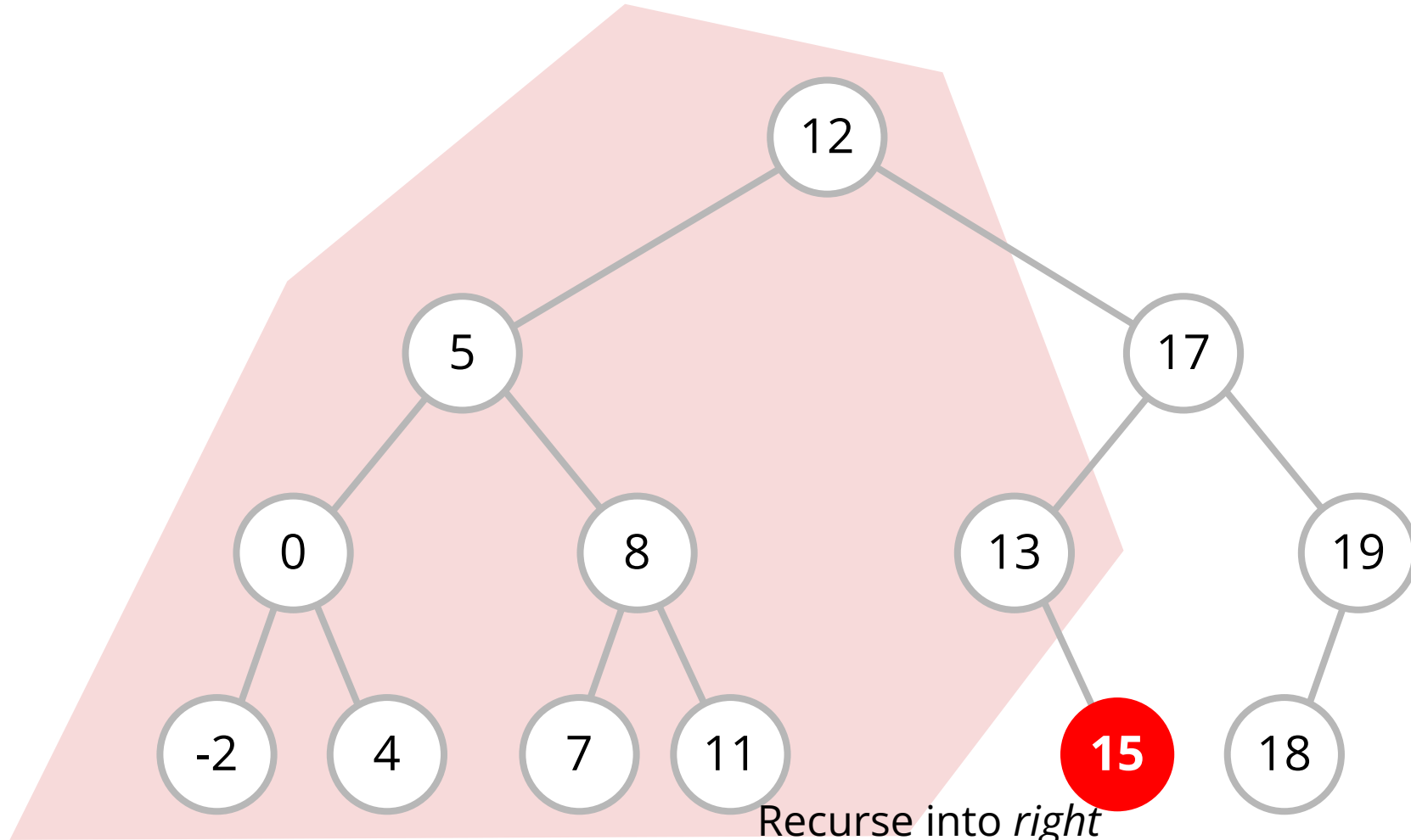


## Question 3: Rank



Get rank of 15.

## Question 3: Rank



Recurse into *right*  
subtree.

Vertex = 15. **Stop!**

# Question 3: Rank

## Solution 1: Tail-recursion

```
int rank(vertex v, int key, int accum) {  
    if (v.key == key)           // Base case: at key  
        return v.left.size + accum + 1;  
    else if (v.key < key)       // Recurse right  
        return rank(v.right, key, accum + v.left.size + 1);  
    else                        // Recurse left  
        return rank(v.left, key, accum);  
}
```



# Question 3: Rank

## Solution 2

```
int rank(vertex v, int key) {  
    if (v.key == key)          // Base case: at key  
        return v.left.size + 1;  
    else if (v.key < key)      // Recurse right  
        return v.left.size + 1 + rank(v.right, key);  
    else                       // Recurse left  
        return rank(v.left, key);  
}
```

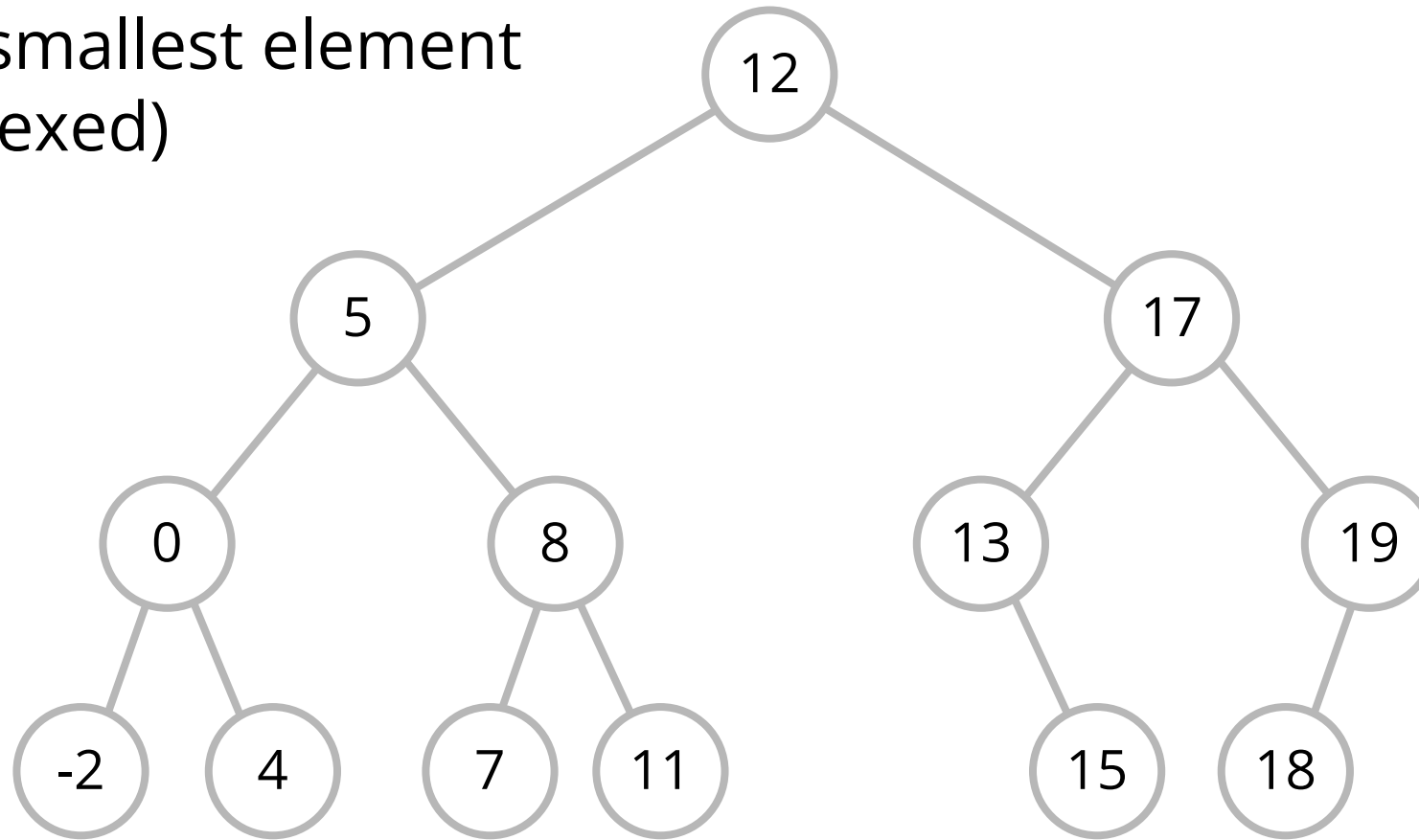
## Question 3: Select

`select(k)` returns the  $k^{\text{th}}$  **smallest** key in the BST.

This is same as retrieving the key which is of **rank k**.

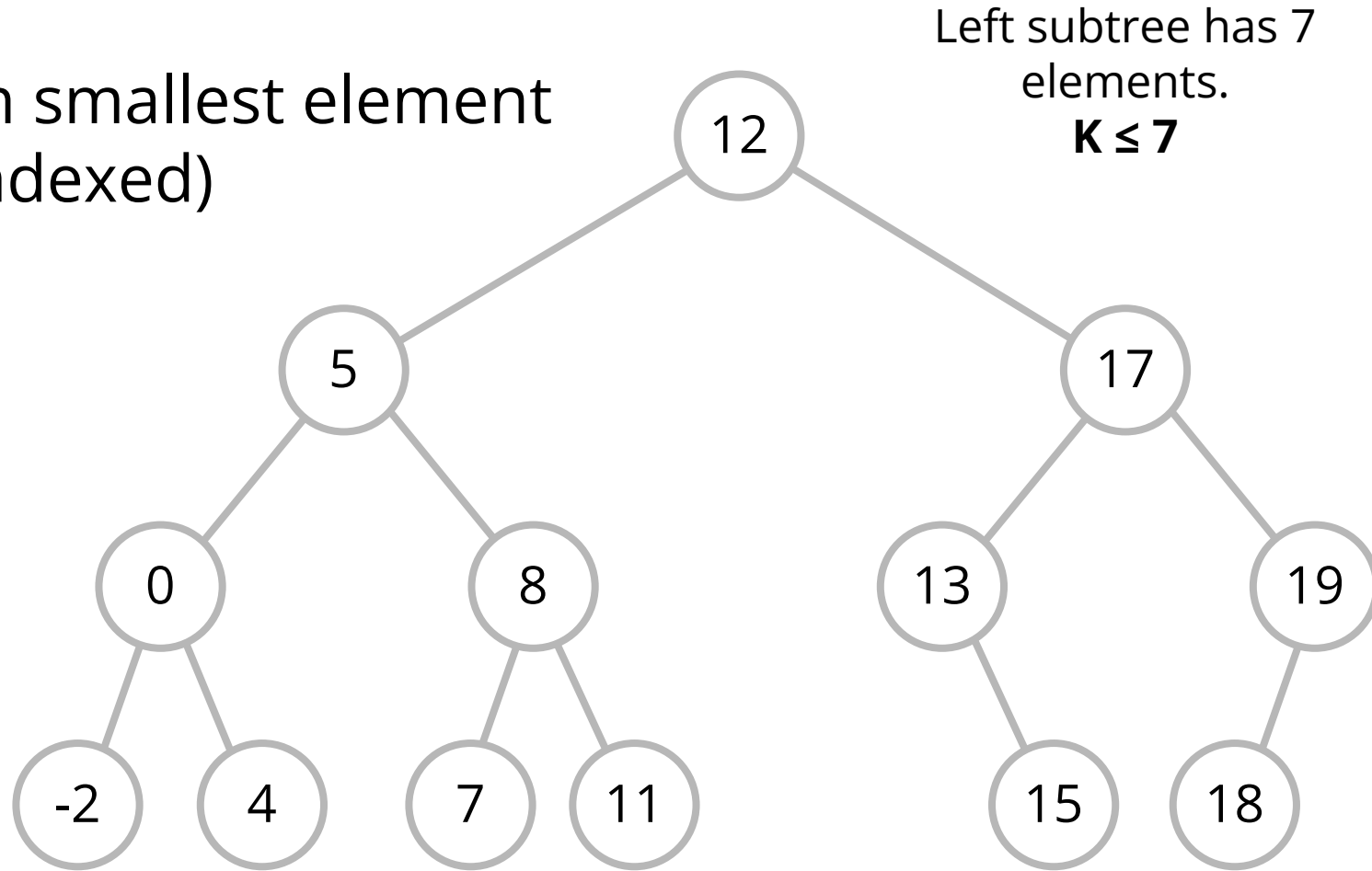
## Question 3: Select

Select **K**-th smallest element  
**K = 5** (1 indexed)



## Question 3: Select

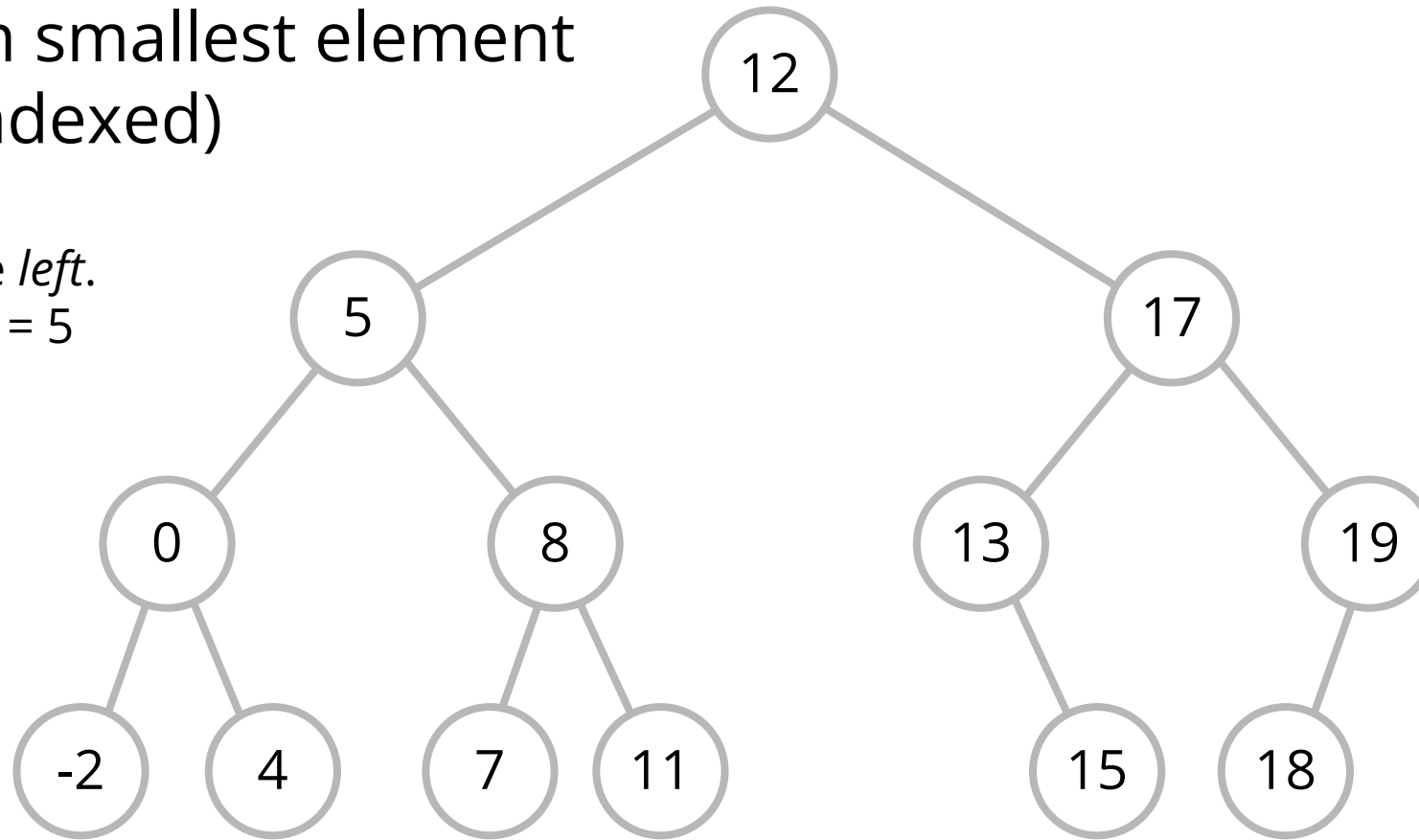
Select **K**-th smallest element  
**K = 5** (1 indexed)



## Question 3: Select

Select **K**-th smallest element  
**K = 5** (1 indexed)

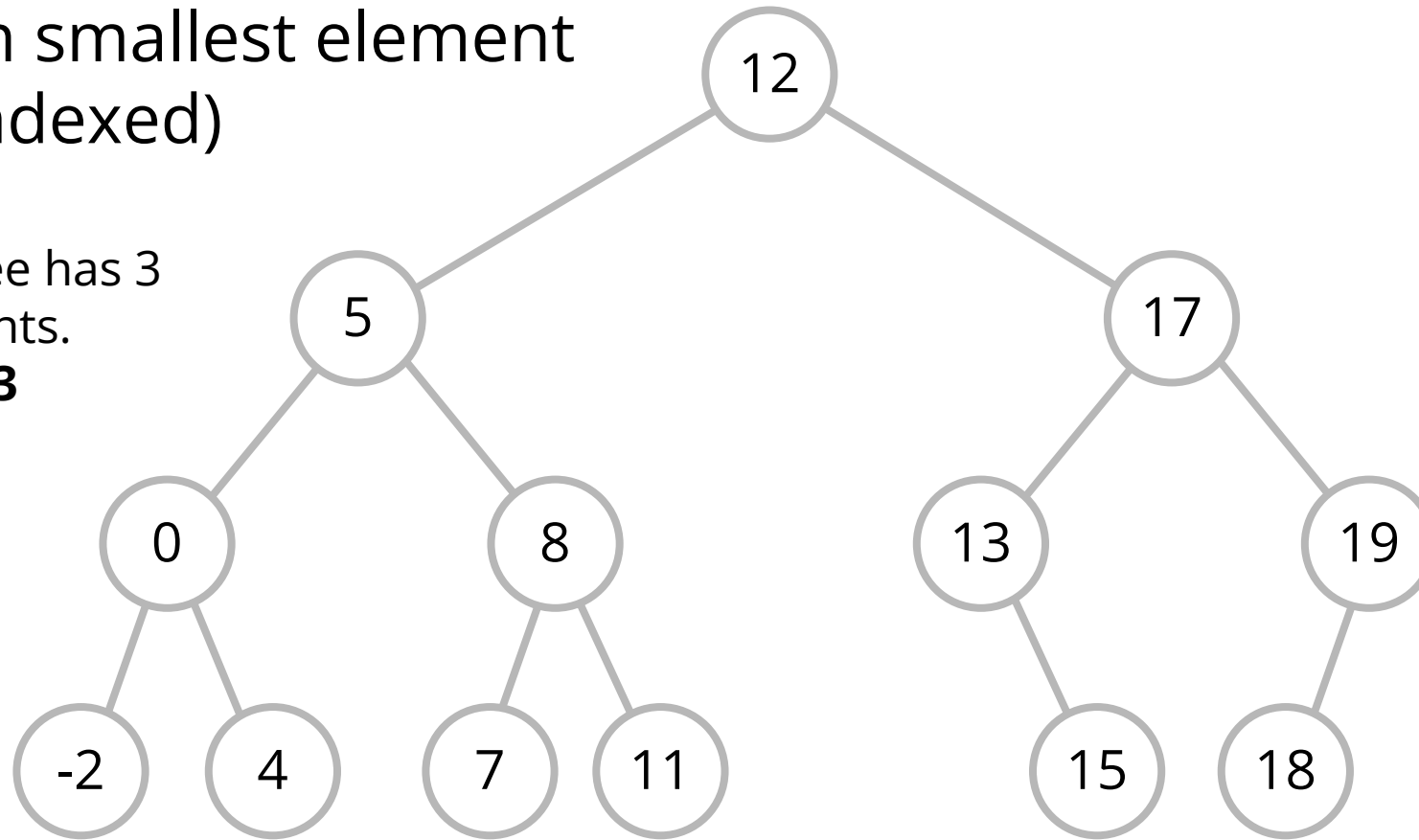
Recurse *left*.  
Vertex = 5



## Question 3: Select

Select **K**-th smallest element  
**K = 5** (1 indexed)

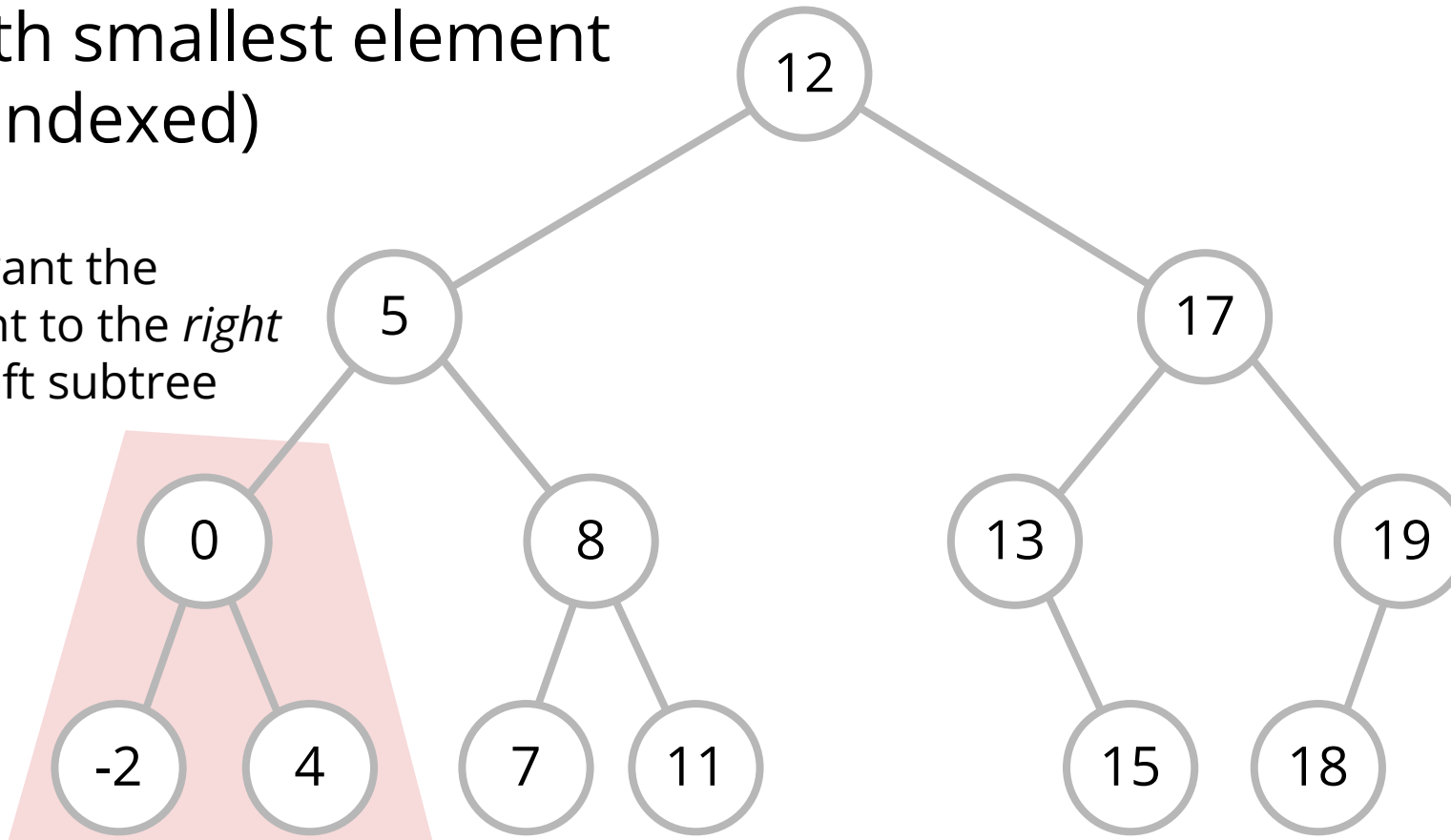
Left subtree has 3  
elements.  
**K > 3**



# Question 3: Select

Select **K**-th smallest element  
**K = 5** (1 indexed)

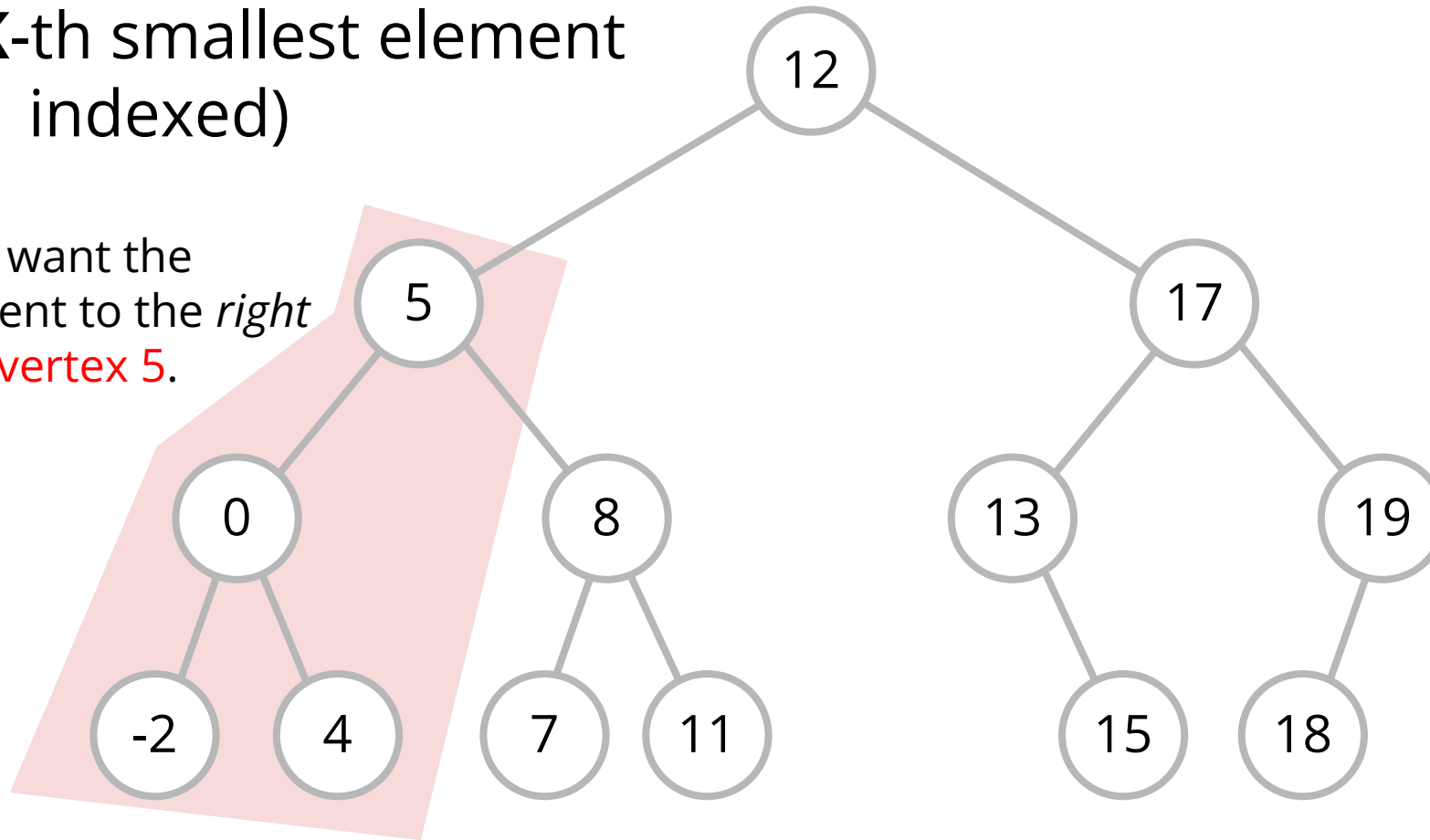
We want the  
**K-3** element to the *right*  
of the left subtree



# Question 3: Select

Select **K**-th smallest element  
**K = 5** (1 indexed)

We want the  
**K-4** element to the *right*  
of **vertex 5**.

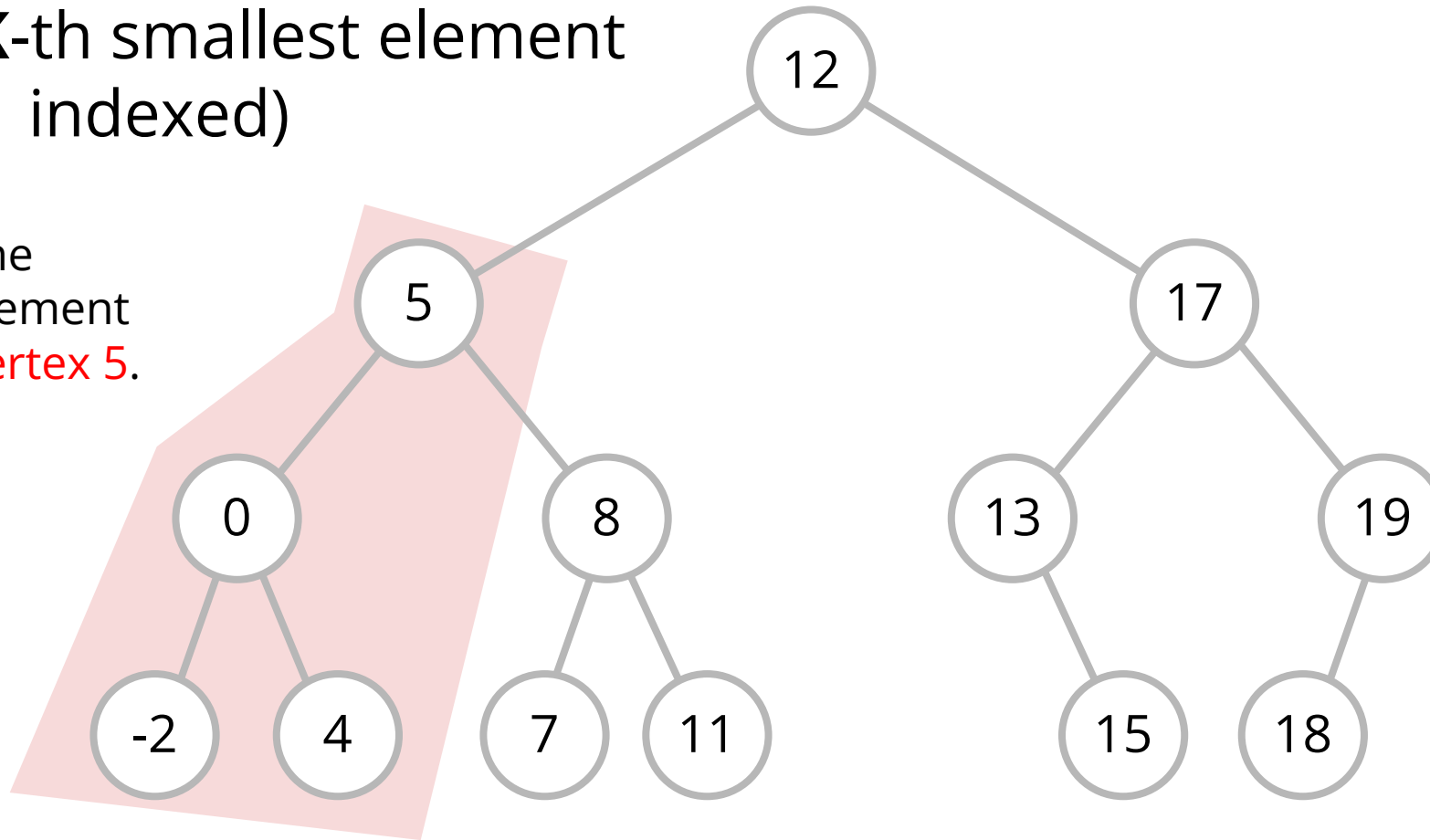




# Question 3: Select

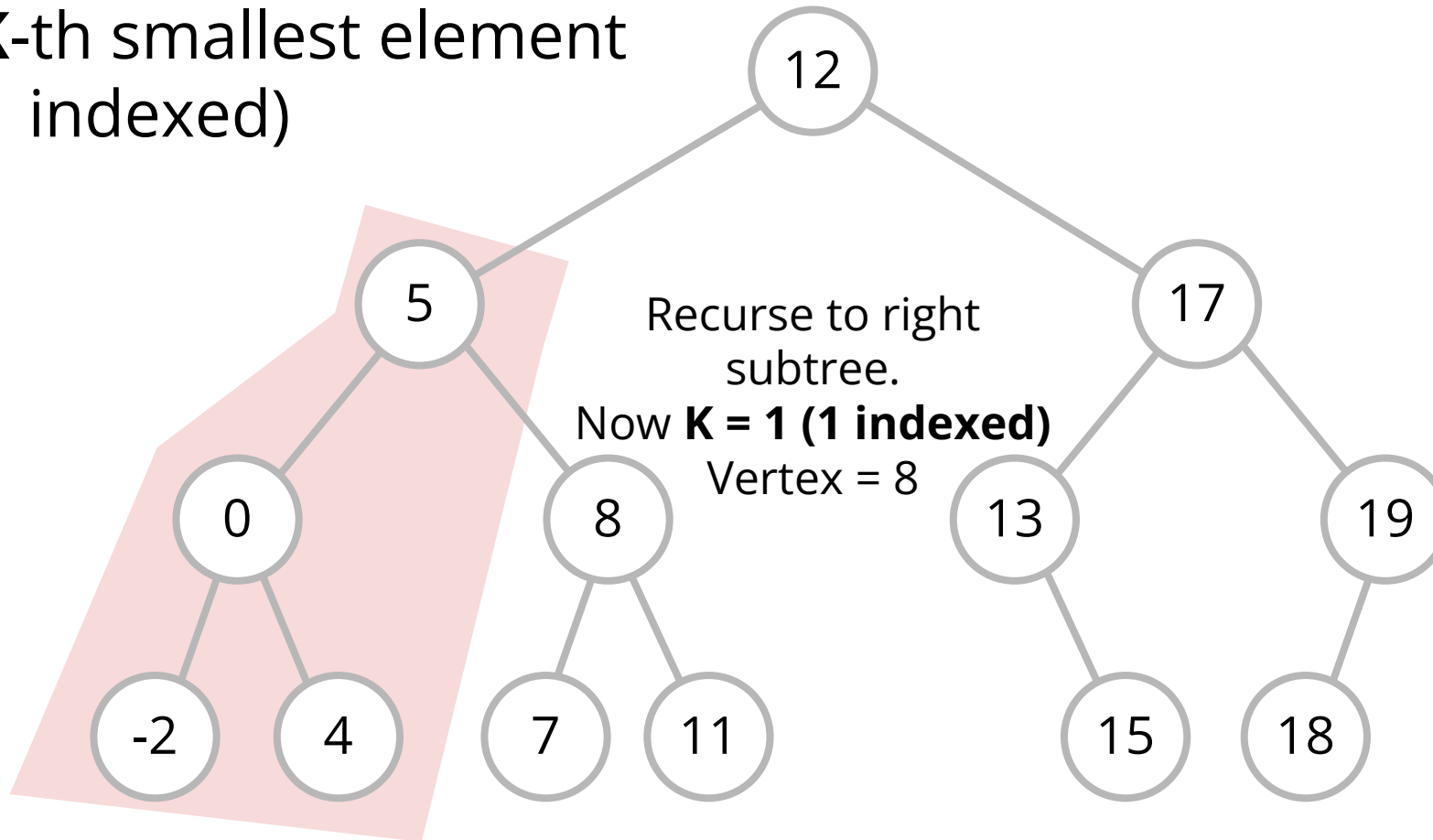
Select **K**-th smallest element  
**K = 5** (1 indexed)

We want the  
**1st smallest** element  
to the *right* of **vertex 5**.



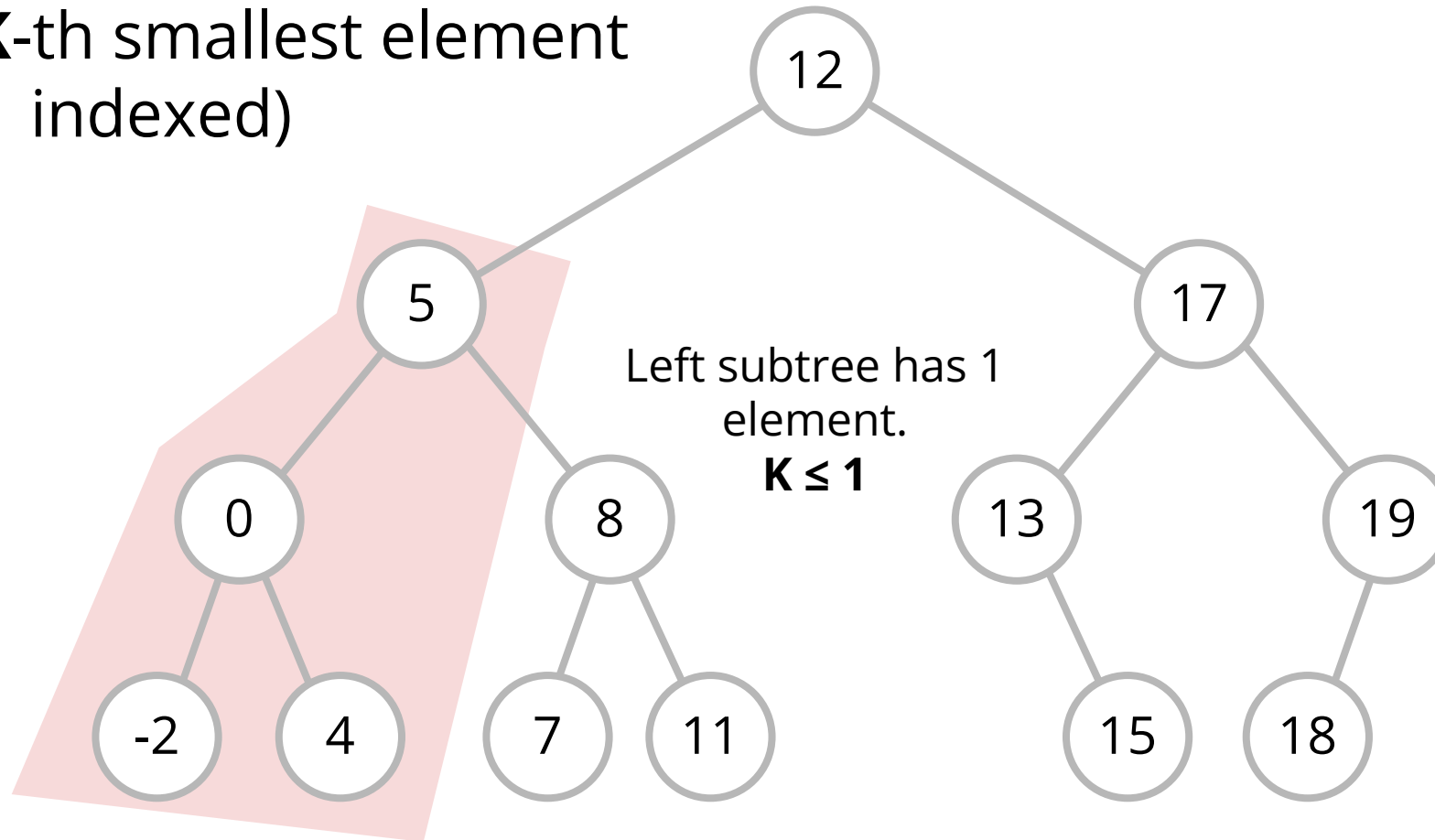
## Question 3: Select

Select **K**-th smallest element  
**K = 5** (1 indexed)



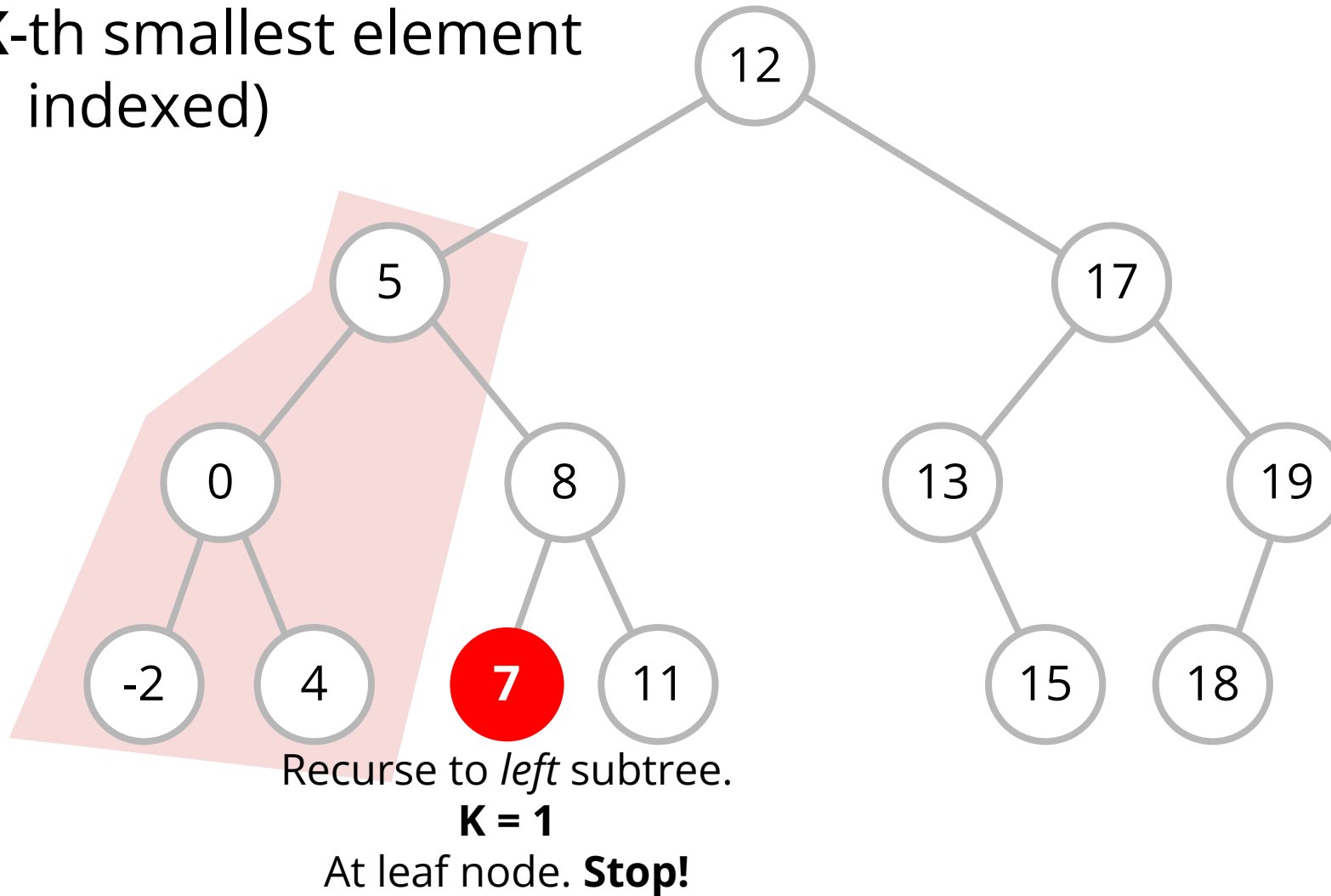
## Question 3: Select

Select **K**-th smallest element  
**K = 5** (1 indexed)



## Question 3: Select

Select **K**-th smallest element  
**K = 5** (1 indexed)



# Question 3: Select

## Solution

```
int select(vertex v, int k) {  
    // Base case: found it  
    if (k == v.left.size + 1)  
        return v.key;  
    // If in left subtree: recurse left  
    else if (k < v.left.size + 1)  
        return select(v.left, k);  
    // If in right subtree: recurse right with updated k  
    else  
        return select(v.right, k - v.left - 1);  
}
```

## Question 4: Handling duplicates

- So far for simplicity sake we assumed the values stored in BST are unique.
- How shall we handle duplicate values?
- One reasonable approach is to include an additional attribute for each vertex to **count** the number of times the value was encountered. i.e. keep a frequency counter for every vertex.

# Question 5

Priority Queue ADT  
Table ADT

# Problem statement

We mentioned that Binary (Max) Heap can be used to implement Priority Queue ADT

How can we modify the implementation such that for  $n$  elements

- Both `ExtractMax()` and `ExtractMin()` done in  $O(\log n)$  time
- Every other Priority Queue related-operations, especially `insert/enqueue` retains the same  $O(\log n)$  time

Hint: What is the topic of this tutorial?



# ADT Recap

## Analogy

Company is looking to hire a new employee.

Employee needs to be able to perform certain tasks:

- `enqueue()`
- `ExtractMax()`
- `top()`

# ADT Recap

## Analogy

Many potential candidates applied for the job:

- Array
- Binary Heap
- Hash Table
- Binary Search Tree
- Balanced Binary Search Tree

# ADT Recap

## Analogy

All of them *technically* can perform the task.

But some do it more *efficiently* than others.

Job Specifications ↔ Abstract Data Type

Candidate ↔ DS implementations

# Question 5: Modified Priority Queue ADT

## Original Functions

- `enqueue()`
- `ExtractMax()`
- `top()`

## New Functions

- `ExtractMin()`

## Question 5: Modified Priority Queue ADT

We can use a BBST to implement PQ ADT with all the desired functions aforementioned

Operations	BBST Implementation
<code>enqueue()</code>	Insert into BBST. $O(\log n)$
<code>top()</code>	Find the maximum vertex. $O(\log n)$
<code>ExtractMax()</code>	Find the maximum vertex and remove it. $O(\log n)$
<code>ExtractMin()</code>	Find the minimum vertex and remove it. $O(\log n)$

# Test yourself!

With a BBST implementation for Priority Queue ADT, how can you achieve `top()` in  $O(1)$  time? Can you employ the same strategy to peek at the minimum value in  $O(1)$  time as well?

# Question 5

# Problem statement

- Follow up from Question 4
- Let's revisit Question 3 of Tutorial 04 (on right)
- Would you answer that question differently?

There are two interesting features of Binary Heap data structure that are not available in C++ STL priority queue and Java PriorityQueue yet: `Increase/Decrease/UpdateKey(old k, new k)` and `DeleteKey(k)` where `v` is not necessarily the max element. These two operations are not yet included in VisuAlgo .



## Question 5: DeleteKey(k)

- Lazy deletion
  - Spoiler given 2 tutorials ago
  - Can keep track of deleted elements using another priority queue
- BBST
  - Delete any element in  $O(\log N)$

## Question 5: UpdateKey(old k, new k)

- Lazy update
  - Spoiler given 2 tutorials ago
  - Use lazy deletion approach to find and mark old k as invalid, then enqueue new k as new element
- BBST
  - Find key in  $O(\log N)$ , delete key in  $O(\log N)$ , finally insert new k as new element in  $O(\log N)$

## Relevant discussion: BBST for Table ADT

- Up till now, we have only seen examples of BBST storing simple values
- Realize that BBST vertices can also store (Key, Value) pairs (i.e. Table ADT's satellite data)!
- This allows us to effectively implement Table ADT using BBST!

## Relevant discussion: BBST for Table ADT

- With a BBST implementation for Table ADT, the comparison between vertices is based on **Key**. i.e. Successor to a vertex represents the next largest key
- We can thus conduct *binary search* for keys!
- Therefore no hash function is necessary, and consequently collision resolution is also not required
- How might we handle (Key, Value) pairs sharing the same key?  
We can use employ separate chaining idea where each vertex is a now a bucket!

# Some remarks

- Priority Queue ADT
  - Doesn't *necessarily* have to be a Binary Heap!
  - For exposure: There are many other heaps as well
- Table ADT
  - Doesn't *necessarily* have to be a Hash Table!
- We have seen how BBST can serve as a reasonable implementation for both ADTs

# Question 7

# Problem statement

As of now, you have been exposed with both possible implementations of Table ADT:

1. Hash Table (and its variations)
2. BST (including Balanced BST like AVL Tree)

Now write down **4 potential usage scenarios** of Table ADT

- Two scenarios should favor the usage of Hash Table
- The other two scenarios should favor using Balanced BST

## Question 6: When to use Hash Table

1. Pure key-to-value mapping without ordering of keys
2. No order statistic queries needed
3. Time limit is tight: Need to choose  $O(1)$  over  $O(\log N)$
4. Hashing of keys is easier/well-known
5. Occasional re-hashing is tolerated



## Question 6: When to use BBST

1. Key-to-value mapping with **keys in sorted order**
2. Need to support order statistic queries:  
`min/max/lower_bound/upper_bound/select/rank` etc.
3. Keys are harder to be hashed (but easier to compare, like a tuple)
4. Memory constraints imposed where a Hash Table would be considered wasteful in comparison
5. Consistent and deterministic time complexity requirements where re-hashing in Hash Table cannot be tolerated

# Java

Container	Data	Allow duplicates?	DS / ADT
TreeSet	Key	No	BBST
TreeMap	(Key, Value)	No	BBST
HashSet	(Key, Value)	No	Hash Table
HashMap	Key	No	Hash Table

Questions  
Break  
Attendance

# PS5 Discussion

## PS5 [/kannafriendship](#)

- Given some intervals  $[l_i, r_i]$ .
- Find the total length they cover.
- The two types of queries can be interleaved.

## PS5 [/kannafriendship](#)

- Given some intervals  $[l_i, r_i]$ .
- Find the total length they cover.
- The two types of queries can be interleaved.

Solution:

- Maintain *disjoint* intervals in a BBST.
- Figure out how to add a new interval to the BBST.
  - Find intersecting intervals.
  - Remove them.
  - Insert the union of the removed intervals and the new interval.
  - Update global counter of total length covered.

## PS5 [/traveltheskies](#)

Reading comprehension problem.

Bruteforce algorithm with some Maps will pass.

# Java TreeSet and TreeMap



# Java TreeSet and TreeMap

[https://visualgo.net/training?diff=Medium&n=5  
&tl=5&module=bst](https://visualgo.net/training?diff=Medium&n=5&tl=5&module=bst)

Hands-on session: [/coursescheduling](#)

# Hands-on session: </coursescheduling>

Given (student, course) pairs. One pair might be given multiple times. For each course in sorted order, print how many students are taking it.

**Hint 5 min:**

**Hint 10 min:**

**Hint 15 min:**

# Hands-on session: </coursescheduling>

Given (student, course) pairs. One pair might be given multiple times. For each course in sorted order, print how many students are taking it.

**Hint 5 min:** How about a Map for each course?

**Hint 10 min:**

**Hint 15 min:**

# Hands-on session: </coursescheduling>

Given (student, course) pairs. One pair might be given multiple times. For each course in sorted order, print how many students are taking it.

**Hint 5 min:** How about a Map for each course?

**Hint 10 min:** Map course to Set of students taking that course.

**Hint 15 min:**

# Hands-on session: </coursescheduling>

Given (student, course) pairs. One pair might be given multiple times. For each course in sorted order, print how many students are taking it.

**Hint 5 min:** How about a Map for each course?

**Hint 10 min:** Map course to Set of students taking that course.

**Hint 15 min:** In the end, sort courses and print the size of the mapped set.

# Thank You!

Anonymous Feedback:

<https://forms.gle/MkETeXdUT53Vhh896>