CS2100

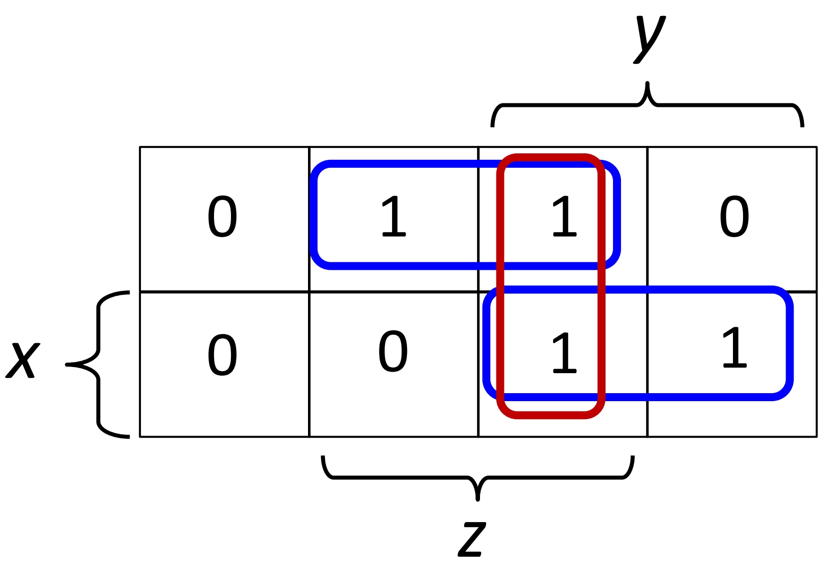
TUTORIAL #6

BOOLEAN ALGEBRA, LOGIC GATES & SIMPLIFICATION

(PREPARED BY: AARON TAN)

Q1. Consensus theorem

$$x\cdot y + x'\cdot z + y\cdot z = x\cdot y + x'\cdot z$$



$$
\begin{aligned}
& x\cdot y + x'\cdot z + y\cdot z \\
&\quad = x\cdot y + x'\cdot z + 1\cdot y\cdot z && \text{[identity law]} \\
&\quad = x\cdot y + x'\cdot z + (x+x')\cdot y\cdot z && \text{[complement law]} \\
&\quad = x\cdot y + x'\cdot z + x\cdot y\cdot z + x'\cdot y\cdot z && \text{[distributive law]} \\
&\quad = x\cdot y + x\cdot y\cdot z + x'\cdot z + x'\cdot y\cdot z && \text{[commutative law]} \\
&\quad = (x\cdot y + x\cdot y\cdot z) + (x'\cdot z + x'\cdot y\cdot z) && \text{[associative law]} \\
&\quad = x\cdot y + x'\cdot z && \text{[absorption theorem 1]}
\end{aligned}
$$

Q2(a) $F(x,y,z) = (x+y\cdot z')\cdot\boxed{(y'+y)} + x'\cdot(y\cdot z'+y)$

$= \boxed{(x+y\cdot z')\cdot 1} + x'\cdot(y\cdot z'+y)$ (by the complement law)

$= (x+y\cdot z') + x'\cdot\boxed{(y\cdot z'+y)}$ (by the identity law)

$= x + \boxed{y\cdot z' + x'\cdot y}$ (by absorption theorem 1)

$= \boxed{x + x'\cdot y} + y\cdot z'$ (by the commutative law)

$= x + \boxed{y + y\cdot z'}$ (by absorption theorem 2)

$= x + y$ (by absorption theorem 1)

Q2(b)

$5 = (0101)_2;\ 9 = (1001)_2;\ 13 = (1101)_2$

$$G(p,q,r,s) = \Pi M(5, 9, 13)$$

$$= \boxed{(p+q'+r+s')} \cdot (p'+q+r+s') \cdot \boxed{(p'+q'+r+s')}$$ (by definition of maxterms)

$$= ((p \cdot p') + (q'+r+s')) \cdot (p'+q+r+s')$$ (by the distributive law)

$(p+(q'+r+s')) \cdot (p'+(q'+r+s'))$
$= (p \cdot p') + (q'+r+s')$

Distributive law:

$A + (B \cdot C) = (A+B) \cdot (A+C)$

or $(B \cdot C) + A = (B+A) \cdot (C+A)$

Q2(b)

$5 = (0101)_2$; $9 = (1001)_2$; $13 = (1101)_2$

$G(p,q,r,s) = \Pi M(5, 9, 13)$

$= \boxed{(p+q'+r+s')} \cdot (p'+q+r+s') \cdot \boxed{(p'+q'+r+s')}$ (by definition of maxterms)

$= (\boxed{(p \cdot p')} + (q'+r+s')) \cdot (p'+q+r+s')$ (by the distributive law)

$= \boxed{(0 + (q'+r+s'))} \cdot (p'+q+r+s')$ (by the complement law)

$= \boxed{(q'+r+s') \cdot (p'+q+r+s')}$ (by the identity law)

$= \boxed{(q' \cdot (p'+q))} + (r+s')$ (by the distributive law)

$= p' \cdot q' + r + s'$ (by absorption 2)

Absorption theorem 2:
$A \cdot (A'+B) = A \cdot B$

$q' \cdot (p'+q) = q' \cdot (q+p') = q' \cdot p' = p' \cdot q'$

Q3.

(a)

B

m0	m1	m5	m4
m2	m3	m7	m6
m10	m11	m15	m14
m8	m9	m13	m12

C

A=1

D

m1: 0001 = A’·B’·C’·D

Q3.

(b) $T(A,B,C,D) = \prod M(3,7,8,10,12,13) \cdot X(6,11,14,15).$

$T(A,B,C,D) = \sum m(0,1,2,4,5,9) + X(6,11,14,15).$

(a)

			B	*B*	
	m0	m1	m5	m4	
	m2	m3	m7	m6	*C*
A	m10	m11	m15	m14	*C*
A	m8	m9	m13	m12	
		D	*D*		

(c)

			B	*B*	
	1	1	1	1	
	1	0	0	X	*C*
A	0	X	X	X	*C*
A	0	1	0	0	
		D	*D*		

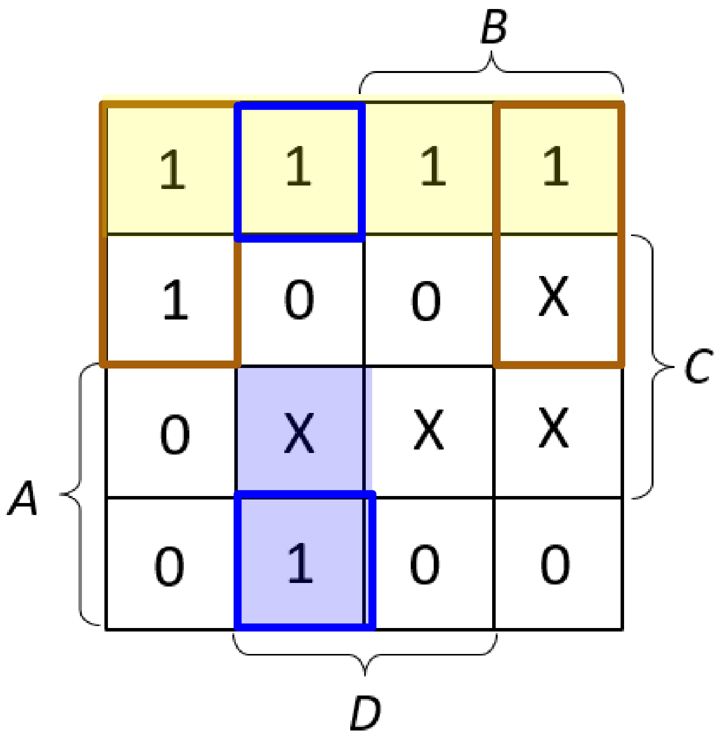
Q3.

(d) How many PIs: 4

A’.D’, A’.C’, B’.C’.D, A.B’.D

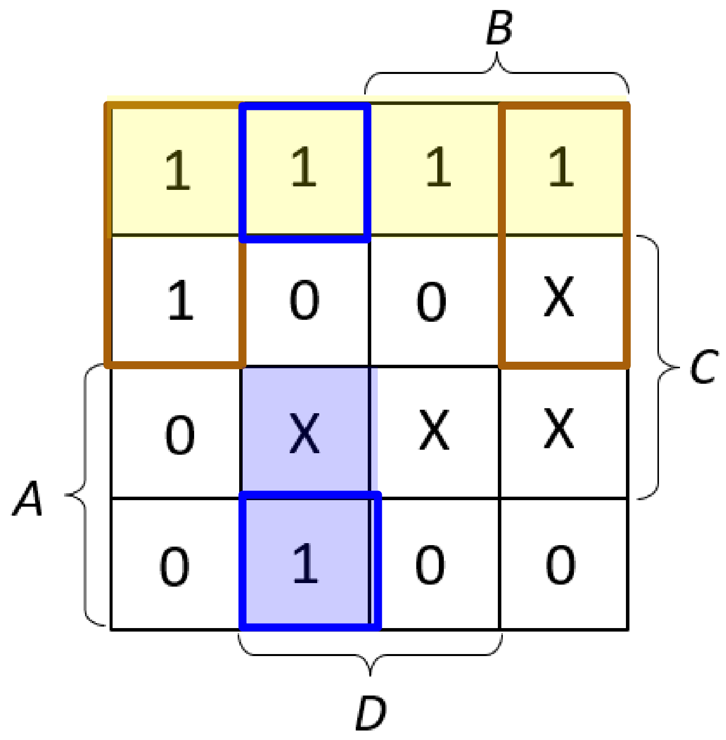
(e) How many EPIs: 2

A’.D’, A’.C’

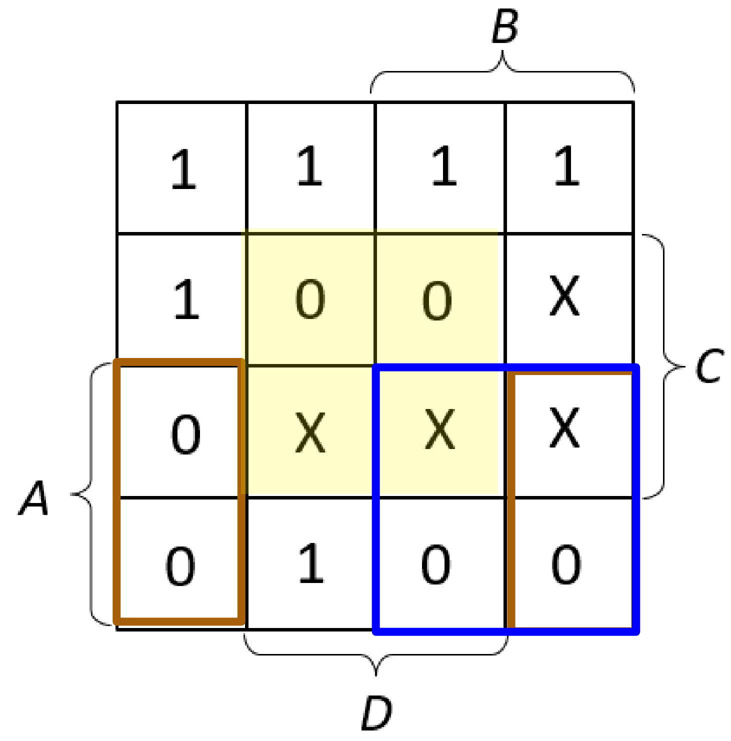


Q3. (f) Simplified SOP: $T(A,B,C,D) = A'.C' + A'.D' + B'.C'.D$

$T(A,B,C,D) = A'.C' + A'.D' + A.B'.D$

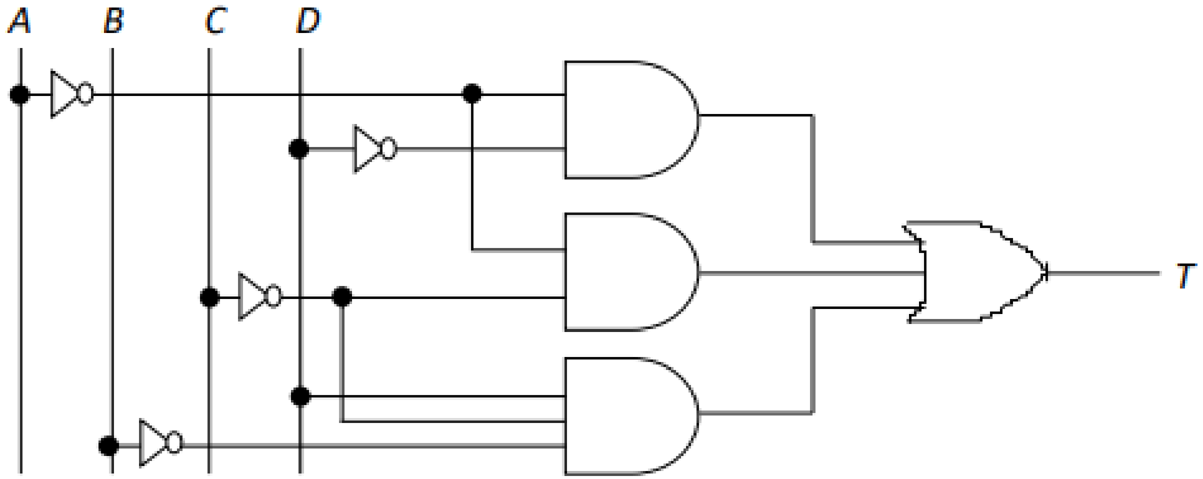


Q3. (g) Simplified POS: $T(A,B,C,D) = (A'+B').(C'+D').(A'+D)$

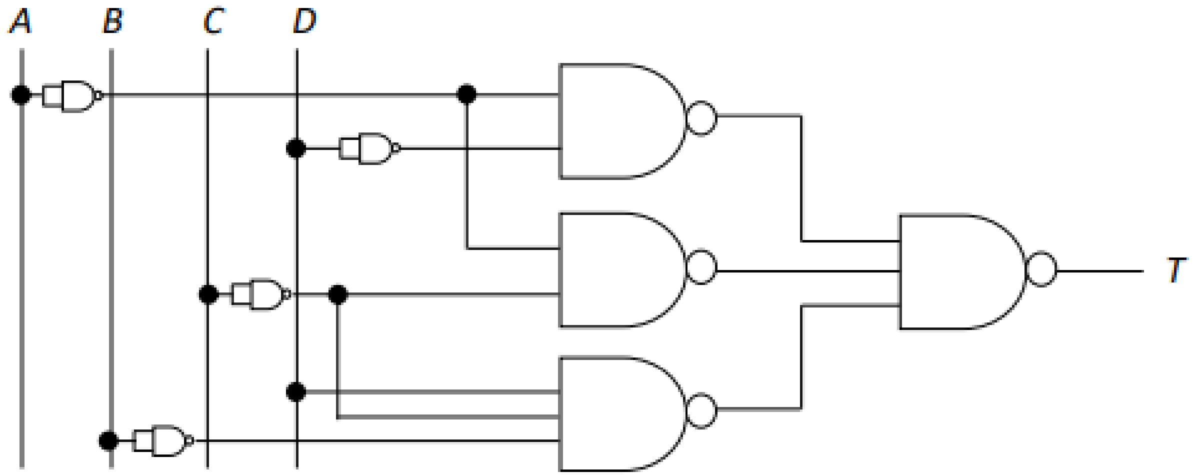


Q3. (h) $T(A,B,C,D)$ = A'.C'+ A'.D' + B'.C'.D

2-level AND-OR circuit:



2-level NAND circuit:



Q4. For output X,

K	*L*	*M*	*N*	*X*	*Y*	*Z*
0	0	0	0	d	d	d
0	0	0	1	0	1	1
0	0	1	0	0	1	1
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	1	1	1
0	1	1	0	0	1	1
0	1	1	1	0	1	1

K	*L*	*M*	*N*	*X*	*Y*	*Z*
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	1	0
1	0	1	1	0	1	0
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	0	0	0
1	1	1	1	1	1	0

(b) ***X***

d	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

M *L* *K* *N*

(c) $X = K' \cdot L \cdot M' \cdot N + K \cdot L' \cdot M \cdot N' + K \cdot L \cdot M \cdot N$

(d) $KLMN = 0000 \rightarrow X = 0$

Q4. (b)(c)(d)

K	*L*	*M*	*N*	*X*	*Y*	*Z*
0	0	0	0	d	d	d
0	0	0	1	0	1	1
0	0	1	0	0	1	1
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	1	1	1
0	1	1	0	0	1	1
0	1	1	1	0	1	1

K	*L*	*M*	*N*	*X*	*Y*	*Z*
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	1	0
1	0	1	1	0	1	0
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	0	0	0
1	1	1	1	1	1	0

(b) ***Y***

			M	*M*	
	d	1	1	1	
	0	1	1	1	*L*
K	0	0	1	0	*L*
K	0	0	1	1	
		N	*N*		

(c)

$Y = M{\cdot}N + K'{\cdot}N + K'{\cdot}M + L'{\cdot}M$

(d) $KLMN = 0000 \rightarrow Y = 0$

Q4. (b)(c)(d)

K	*L*	*M*	*N*	*X*	*Y*	*Z*
0	0	0	0	d	d	d
0	0	0	1	0	1	1
0	0	1	0	0	1	1
0	0	1	1	0	1	1
0	1	0	0	0	0	1
0	1	0	1	1	1	1
0	1	1	0	0	1	1
0	1	1	1	0	1	1

K	*L*	*M*	*N*	*X*	*Y*	*Z*
1	0	0	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	1	0
1	0	1	1	0	1	0
1	1	0	0	0	0	0
1	1	0	1	0	0	0
1	1	1	0	0	0	0
1	1	1	1	1	1	0

(b) ***Z***

d	1	1	1
1	1	1	1
0	0	0	0
0	0	0	0

M, *L*, *K*, *N*

(c) $Z = K'$

(d) $KLMN = 0000 \rightarrow Z = 1$

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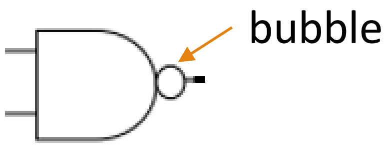
All the best for Mid-terms

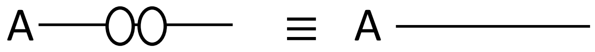
Bubble Pushing for Converting between Complete Sets of Logic

Notation: Use a bubble to represent an inverter/NOT gate



Rule 1: Can create a <u>pair</u> of bubbles at any input/output of a gate.

Reason: Involution Theorem (A')' ≡ A



Rule 2: Can push a bubble through a gate by changing the gate from AND to OR and OR to AND

Reason: De Morgan's Theorem A'B' ≡ (A + B)' and A' + B' ≡ (AB)'