COMP/ENGN8535 Homework Assignment #1

1. Frobenius Norm of a matrix $H \in R^{n \times m}$ is defined as

$$\|H\|_F = \left(\sum_{i=1}^{n} \sum_{j=1}^{m} H_{ij}^2\right)^{1/2}$$

prove the following:

(a) $\|H\|_F = \sqrt{tr(HH^T)}$

proof: Let H be a matrix that $H \in R^{n \times m}$

$\|H\|_F = \left(\sum_{i=1}^{n} \sum_{j=1}^{m} H_{ij}^2\right)^{1/2}$ where H_{ij} is the (i,j) th entry of matrix H.

using the summation definition of matrix multiplication we have $HH^T(i,j) = \sum_{k=1}^{m} H_{ik} H^T_{kj}$

Since the trace of a matrix is the sum of its diagonal elements.

therefore, The trace of HH^T is defined as follows

$$tr(HH^T) = \sum_{i=1}^{n} \left(\sum_{j=1}^{m} H_{ij} H^T_{ji} \right)$$

note that $H^T_{ji} = H_{ij}$ in matrix transpose

the above equation can be simplified to

$$tr(HH^T) = \sum_{i=1}^{n} \sum_{j=1}^{m} H_{ij}^2$$

Hence, $\|H\|_F = \sqrt{tr(HH^T)}$ ▨

(b) Any orthogonal matrix $P \in R^{n\times n}$ or $Q \in R^{m\times m}$ preserves the Frobenius norm

$$\|H\|_F = \|PH\|_F = \|HQ\|_F$$

proof: Let H be a matrix of $R^{n\times m}$ and let $P \in R^{n\times n}$ and $Q \in R^{m\times m}$ be two orthogonal matrices.

Based on the fact that P and Q are orthogonal,

we have $Q^TQ = I \quad P^TP = I$

where $Q^T = Q^{-1} \quad P^T = P^{-1}$ ①

For a matrix X, let X_{ij} denote the (i,j)th entry of X

Let $A \in R^{m\times n}$ and $B \in R^{n\times m}$

then
$$\begin{aligned} tr(AB) &= \sum_{i=1}^{n} [AB]_{ii} \\ &= \sum_{i=1}^{n}\sum_{j=1}^{m} A_{ij} \cdot B_{ji} \\ &= \sum_{j=1}^{m}\sum_{i=1}^{n} B_{ji} \cdot A_{ij} \\ &= \sum_{j=1}^{m} [BA]_{jj} \\ &= tr(BA) \end{aligned}$$

therefore $tr(HH^T) = tr(H^TH)$ ②

According to the equation we proved in (a) we can then derive the following equation

$$\|H\|_F = \sqrt{tr(HH^T)} = \sqrt{tr(H^TH)} \quad ③ \quad \cdots\cdots (a)$$

For proving $\|H\|_F = \|PH\|_F$:

Based on ② ③, we can infer that

$$\|PH\|_F = \sqrt{tr(PH(PH)^T)} = \sqrt{tr((PH)^T PH)}$$

in which according to matrix transpose

$$(AB)^T = B^TA^T \quad ④$$

we can then derive

$$\|PH\|_F = \sqrt{\cancel{tr}((PH)^T PH)}$$

$$= \sqrt{tr(H^TP^TPH)} \quad \cdots\cdots ④$$

$$= \sqrt{tr(H^TP^{-1}PH)} \quad \cdots\cdots ①$$

$$= \sqrt{tr(H^TI\,H)} \quad \cdots\cdots ①$$

$$= \sqrt{tr(H^TH)}$$

$$= \sqrt{tr(HH^T)} = \|H\|_F \quad \cdots\cdots ③$$

For proving $\|H\|_F = \|HQ\|_F$:

We have $\|HQ\|_F = \sqrt{tr(HQ(HQ)^T)} \quad \cdots\cdots (a)$

$$= \sqrt{tr(HQQ^TH^T)} \quad \cdots\cdots ④$$

$$= \sqrt{tr(HQQ^{-1}H^T)} \quad \cdots\cdots ①$$

$$= \sqrt{tr(HIH^T)} \quad \cdots\cdots ①$$

$$= \sqrt{tr(HH^T)} = \|H\|_F \quad \cdots\cdots ③$$

Above all, we can prove that

$$\|H\|_F = \|PH\|_F = \|HQ\|_F$$

for some orthogonal matrices $P \in R^{n\times n}$ and $Q \in R^{m\times m}$

$\blacksquare$

(c) Prove that $\|H\|_F$ is equal to the root sum square of H's singular values, that is

$$\|H\|_F = \sqrt{\sum_i \sigma_i^2}$$

proof: Let H be a matrix that $H \in R^{n\times m}$ with singular values $\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_k$, where k is the rank of H ordered such that $\sigma_1 \geq \sigma_2 \geq \sigma_3 \cdots \geq \sigma_k \geq 0$

Then the Frobenius norm of H is defined as

$$\|H\|_F = \left(\sum_{i=1}^{n}\sum_{j=1}^{m} H_{ij}^2\right)^{1/2}$$ definition of Frobenius Norm (1)

where H_{ij} is the (i,j)th entry of H.

The root sum square of H's singular values is defined as

$$\sqrt{\sum_{i=1}^{k} \sigma_i^2}$$

We use the singular value decomposition of H

$$H = U\Sigma V^T$$ SVD decomposition (2)

where $U \in R^{n\times k}$, $\Sigma \in R^{k\times k}$, $V \in R^{m\times k}$

U and V are orthogonal

Σ is a diagonal matrix with non-negative diagonal entries $\sigma_1, \sigma_2, \sigma_3 \cdots \sigma_k$

we can then write $\|H\|_F$ as

$$\|H\|_F = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m} H_{ij}^2} \quad \cdots\cdots (1)$$

$$= \sqrt{tr(H^T H)} \quad \cdots\cdots \text{proof in (a) and (b) ②}$$

$$= \sqrt{tr(V\Sigma U^T U \Sigma V^T)} \quad \cdots\cdots \text{transpose rule}$$

$$= \sqrt{tr(V \Sigma^2 V^T)} \quad \cdots\cdots \text{orthogonal matrix } U^T U = I$$

$$= \sqrt{\sum_{i=1}^{k} (\sigma_i^2\, tr(V_i V_i^T))} \quad \cdots\cdots \text{orthogonal matrix } V$$

V_i is the ith column of V

$$= \sqrt{\sum_{i=1}^{k} \sigma_i^2} \quad \cdots\cdots tr(V_i V_i^T) = 1 \text{ for each } i$$

therefore, we showed that

$$\|H\|_F = \sqrt{\sum_{i=1}^{k} \sigma_i^2}$$ for k is the rank of H

$\blacksquare$

2. (a) Answer: SVD decomposition is $A = U\Sigma V^T$

given $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

1. Compute AA^T

$$AA^T = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

2. Compute the eigenvalues and eigenvectors of AA^T

$\det(A^TA - \lambda I) = 0 \qquad \det\left(\begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{bmatrix} \right) = 0$

$-\lambda(\lambda-3)(\lambda-1) = 0$ thus $\lambda_1 = 3 \quad \lambda_2 = 1 \quad \lambda_3 = 0$

① $\lambda_1 = 3 \quad A^TA - \lambda I = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 0 \\ 1 & 0 & -2 \end{bmatrix}$ row-echelon form is $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

Solving $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ gives $U_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

② $\lambda_2 = 1 \quad A^TA - \lambda I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ row-echelon form is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Solving $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ gives $U_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

③ $\lambda_3 = 0 \quad A^TA - \lambda I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ row-echelon form is $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

Solving $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ gives $U_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

3. Find the square roots of the nonzero eigenvalues (σ_i) to compute Σ

$\sigma_1 = \sqrt{3}$ diagonal matrix Σ is $\begin{bmatrix} \sqrt{3} & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\sigma_2 = 1$

4. Find U column of U are normalized eigenvectors

$U_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \quad \bar{U}_1 = \begin{bmatrix} \sqrt{6}/3 \\ \sqrt{6}/6 \\ \sqrt{6}/6 \end{bmatrix} \quad U_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \quad \bar{U}_2 = \begin{bmatrix} 0 \\ -\sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$

$U_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \quad \bar{U}_3 = \begin{bmatrix} -\sqrt{3}/3 \\ \sqrt{3}/3 \\ \sqrt{3}/3 \end{bmatrix}$ therefore we can construct $U = \begin{bmatrix} \sqrt{6}/3 & 0 & -\sqrt{3}/3 \\ \sqrt{6}/6 & -\sqrt{2}/2 & \sqrt{3}/3 \\ \sqrt{6}/6 & \sqrt{2}/2 & \sqrt{3}/3 \end{bmatrix}$

5. Find V using formula $V_i = \frac{1}{\sigma_i} A^T \cdot u_i$

$$V_1 = \frac{1}{\sigma_1}\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}^T \cdot u_1 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} \sqrt{6}/3 \\ \sqrt{6}/6 \\ \sqrt{6}/6 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$V_2 = \frac{1}{\sigma_2}\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}^T \cdot u_2 = \frac{1}{1}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -\sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix} = \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$$

therefore $V = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

the matrices U, Σ, V are such that

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = U \Sigma V^T$$

(b) $A = \begin{bmatrix} 0 & 6 & 8 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \end{bmatrix}$ $\quad V = \begin{pmatrix} 16 \\ 4 \\ 1 \end{pmatrix}$ $\quad W = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$

Answer : only V is the eigenvector of A

as it satisfies the equation $AV = \lambda V$

$$\begin{bmatrix} 0 & 6 & 8 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \end{bmatrix}\begin{bmatrix} 16 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 32 \\ 8 \\ 2 \end{bmatrix} = \lambda V$$

therefore $\lambda = 2$

3. (a) Sylvester's criterion : a symmetric matrix is PD iff all its upper-left sub-matrices have a positive determinant

Answer : ① $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ $\quad A^T = A$ therefore A is symmetric

The upper-left sub-matrices of A are $\begin{bmatrix} 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

In A $\det([1]) = 1 > 0$, $\det\left(\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\right) = 1 - 4 = -3 < 0$

Hence A is not PD

② $B = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ $\quad B^T = B$ so B is symmetric

The upper-left sub-matrices of B are $[3]$, $\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

In B, $\det([3]) = 3 > 0$ $\det\left(\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\right) = 6 - 1 = 5 > 0$

$$\det\left(\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\right) = 3 \times \det\left(\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\right) - (-1)\det\left(\begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}\right)$$

$$+ 0 \cdot \det\left(\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}\right)$$

$$= 3 + (-1) = 2 > 0$$

So B is PD ∎

(b) Find range of b value, for the following matrix to be PD.

$$\begin{bmatrix} 3 & -1 & b \\ -1 & 2 & -1 \\ b & -1 & 1 \end{bmatrix}$$

this matrix has upper-left sub-matrices

$[3]$, $\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 3 & -1 & b \\ -1 & 2 & -1 \\ b & -1 & 1 \end{bmatrix}$

where the first two are the same as B

from (a), we have $\det([3]) = 3$ $\det\left(\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\right) = 5$

$$\det\left(\begin{bmatrix} 3 & -1 & b \\ -1 & 2 & -1 \\ b & -1 & 1 \end{bmatrix}\right) = 3\det\left(\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\right) - (-1)\det\left(\begin{bmatrix} -1 & -1 \\ b & 1 \end{bmatrix}\right) + b\det\left(\begin{bmatrix} -1 & 2 \\ b & -1 \end{bmatrix}\right)$$

$$= 3 - (-1)(-1 + b) + b(1 - 2b)$$

$$= 3 - (1 - b) + b - 2b^2$$

$$= 2 + 2b - 2b^2$$

In order to make this matrix PD

we want $\det\left(\begin{bmatrix} 3 & -1 & b \\ -1 & 2 & -1 \\ b & -1 & 1 \end{bmatrix}\right) > 0$ as well

therefore $2 + 2b - 2b^2 > 0$ $\quad 1 + b - b^2 > 0$

$f(x) = 1 + b - b^2$ $f(x)$ is a quadratic function that has a maxima

by setting $f'(x) = -2b + 1 = 0$, we find $f(x)_{max} = 1.25 > 0$ at $b = 0.5$

$1 + b - b^2 = 0$ can be solve by quadratic formula that

$$b_1 = \frac{1+\sqrt{5}}{2} \quad b_2 = \frac{1-\sqrt{5}}{2}$$

Hence b has to satisfy $b_2 < b < b_1$

that is $\frac{1-\sqrt{5}}{2} < b < \frac{1+\sqrt{5}}{2}$

so that the matrix is PD.

4. (a) X, Y be real-valued random variable

X and Y are statistically independent i.e. $p(X,Y) = p(X)p(Y)$

we want to show that $p(X \mid Y) = p(X)$

proof: X and Y are independent if $p(X,Y) = p(X)p(Y)$

the conditional pmf or pdf of X given Y is $p(X \mid Y) = \frac{p(X,Y)}{p(Y)}$

therefore substituting $p(X,Y)$ into $p(X \mid Y)$

we have $p(X \mid Y) = \frac{p(X)p(Y)}{p(Y)} = p(X)$ ∎

(b) A and B are real-valued random variables

$0 < p(B) < 1$

proof:

two random variables A and B are independent iff $p(A,B) = p(A)p(B)$

So we need to show that if $P(A|B) = P(A)$ then $P(A, B) = P(A)P(B)$

According to the definition of conditional probability

$$P(A,B) = P(A|B)\,P(B)$$

since $0 < P(B) < 1$ and $P(A|B) = P(A)$

$$P(A,B) = P(A)\,P(B)$$

therefore, it shows that A and B are statistically independent. $\blacksquare$

5. Let D be the event that Tom has the disease

Let T be the event that Tom tests positive for the disease

The objective is to find the probability of D given T that is $P(D|T)$

using Bayes' theorem, we have

$$P(D|T) = P(T|D)\,P(D) / P(T)$$

we know that the probability of testing positive given that someone has the disease is 0.99, therefore $P(T|D) = 0.99$

$P(D)$ is $1/100{,}000$ as the disease strikes only one in 100,000 people.

To find $P(T)$, the probability of Tom tests positive,

we use the law of total probability: $P(T) = P(T, D) + P(T, \text{not} D)$

$$P(T) = P(T|D)\,P(D) + P(T|\text{not}D)\,P(\text{not}D)$$

where not D is the complement of D, that is the event that Tom does not have the disease. Thus $P(\text{not}D) = 1 - P(D) = \frac{99{,}999}{100{,}000}$

since the test is 99% accurate, $P(T|\text{not}D) = 1 - P(T|D) = 0.01$

substituting the values, we got

$$P(T) = 0.99 \times \frac{1}{100{,}000} + 0.01 \times \frac{99{,}999}{100{,}000} = 0.00000099 + 0.0099999$$
$$= 0.0100098$$

Now $P(D|T) = P(T|D)\, P(D) \big/ P(T)$

$$= \left(0.99 \times \frac{1}{100{,}000}\right) \Big/ 0.0100098$$

$$= 0.00009803$$

$$\approx 0.0009 \approx 0.001$$

So the chances that Tom actually has the disease is 0.001

∎

6. (a) Optimize $\underset{f(x,y)}{z = x(y+4)}$ with constraint $\underset{g(x,y)}{x+y=8}$

To use Lagrange multiplier method

step ① we form the lagrangian function $L(x,y,\lambda) = x(y+4) - \lambda(x+y-8)$

By setting
$$\begin{cases} \frac{\partial L}{\partial x} = y+4-\lambda = 0 \\ \frac{\partial L}{\partial y} = x - \lambda = 0 \\ \frac{\partial L}{\partial \lambda} = 8 - x - y = 0 \end{cases}$$

we get the following solution

$x = 6 \quad y = 2 \quad \lambda = 6$

substituting them into the objective function.

we can find the optimum value of z

step ②

$$z = x(y+4) = 36$$

To determine whether it is a minimum or maximum

we construct the bordered Hessian matrix BH

$$BH = \begin{bmatrix} 0 & g_x & g_y \\ g_x & L_{xx} & L_{xy} \\ g_y & L_{yx} & L_{yy} \end{bmatrix}$$ where $g_x = \frac{d g(x,y)}{dx} = 1$, $g_y = \frac{d g(x,y)}{dy} = 1$

calculating the second partial derivative $L_{xx} = \frac{\partial^2 L}{\partial x^2} = 0$

of L with respect to x, y

$$L_{yy} = \frac{\partial^2 L}{\partial y^2} = 0$$

$$L_{xy} = \frac{\partial^2 L}{\partial x \partial y} = 0$$

$$L_{yx} = \frac{\partial^2 L}{\partial y \partial x} = 0$$

Step 3

Then $BH = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ at critical point $(x, y, \lambda) = (6, 2, 6)$

$$\det(BH) = 0 \cdot \det\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - 1 \cdot \det\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} + 1 \cdot \det\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = 0$$

therefore we cannot determine whether the solution is a minimum or maximum.

■

(b) Denote the three sides of a triangle as a, b, c

where $a \le b \le c$

the perimeter constraint can be written as $a + b + c = 2$

the area of the triangle $A = \sqrt{s(s-a)(s-b)(s-c)}$

where s is the semi-perimeter $\frac{a+b+c}{2} = 1$

The objective is to maximize the area A subject to constraint

$a + b + c = 2$, we can form the Lagrangian function

$$L(a, b, c, \lambda) = A + \lambda(2 - a - b - c)$$

$$= \sqrt{s(s-a)(s-b)(s-c)} + \lambda(2 - a - b - c)$$

where λ is the Lagrangian multiplier

To find the critical points, we will solve the following

$\frac{\partial L}{\partial a} = 0$ $\frac{\partial L}{\partial b} = 0$ $\frac{\partial L}{\partial c} = 0$ and $\frac{\partial L}{\partial \lambda} = 0$

taking partial derivatives, we get:

$$\frac{\partial L}{\partial a} = \frac{1}{2} s(s-a)(s-b)(s-c)^{-\frac{1}{2}} \times (-2a+2s) - \lambda = 0$$

$$\frac{\partial L}{\partial b} = \frac{1}{2} s(s-a)(s-b)(s-c)^{-\frac{1}{2}} \times (-2b+2s) - \lambda = 0$$

$$\frac{\partial L}{\partial c} = \frac{1}{2} s(s-a)(s-b)(s-c)^{-\frac{1}{2}} \times (-2c+2s) - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 2-a-b-c = 0$$

through the first 3 equation we get

$$s-a = \lambda\left(1/\sqrt{s(s-a)(s-b)(s-c)}\right)$$

$$s-b = \lambda\left(1/\sqrt{s(s-a)(s-b)(s-c)}\right)$$

$$s-c = \lambda\left(1/\sqrt{s(s-a)(s-b)(s-c)}\right)$$

substracting these 3 equation pairwisely we get

$$a-b=0$$

$$b-c=0$$

$$c-a=0$$

therefore $a=b=c$

Since $a+b+c=2$, $a=b=c=\frac{2}{3}$

the critical point of L corresponds to an equilateral triangle with sides of length $\frac{2}{3}$.

The solution can be found using the Lagrange method

To show it is a maximum, we can verify that the Bordered Hessian matrix's determinant is negative at this point.

(negative definite)

$\blacksquare$