Ans-6(c)

A-6)c) Given: v_i is an eigenvector of Q & $u_i = \frac{A^T v_i}{\|A^T v_i\|_2}$

To prove: $A u_i = \gamma_i v_i$, for some real, non-negative γ_i

Proof: $Q v_i = \lambda v_i$
$\Rightarrow A A^T v_i = \lambda v_i$ — ① ($\because$ v_i is an eigenvector of Q).

$$A u_i = \frac{A A^T v_i}{\|A^T v_i\|_2}$$

(from ①)

$$= \frac{\lambda v_i}{\|A^T v_i\|_2}$$

Let $\gamma_i = \frac{\lambda}{\|A^T v_i\|_2}$, because $\lambda \geq 0$, $\|A^T v_i\|_2 \geq 0$

hence $\gamma_i \geq 0$ & is real.

$\therefore$ $A u_i = \gamma v_i$

Hence $A u_i = \gamma v_i$ for some real, non-negative γ_i.