1.) Given

$$g_1 = f_1 + h_2 * f_2 \quad \text{—— (i)}$$

$$g_2 = f_2 + h_1 * f_1 \quad \text{—— (ii)}$$

Taking fourier transform :-

$$G_1 = F_1 + H_2 \circ F_2 \quad \text{—— (iii)}$$

$$G_2 = F_2 + H_1 \circ F_1 \quad \text{—— (iv)}$$

from (iii) & (iv)

$$G_2 = F_2 + H_1 (G_1 - H_2 F_2)$$

$$\Rightarrow G_2 = F_2 (1 - H_1 H_2) + H_1 G_1$$

$$\Rightarrow F_2 = \left(\frac{G_2 - H_1 G_1}{1 - H_1 H_2} \right) \quad \text{—— (v)}$$

from (iii) & (v)

$$F_1 = G_1 - \frac{H_2 (G_2 - H_1 G_1)}{(1 - H_1 H_2)} \quad \text{— (vi)}$$

Taking IFT of (v) & (vi)

$$f_2 = F^{-1}(F_2) = F^{-1} \left(\frac{G_2 - H_1 G_1}{1 - H_1 H_2} \right) \quad \text{—— (vii)}$$

$$f_1 = F^{-1}(F_1) = F^{-1} \left(G_1 - \frac{H_2 (G_2 - H_1 G_1)}{(1 - H_1 H_2)} \right) \quad \text{—— (viii)}$$

~~from (vii) & (viii)~~ :

from (vi), if $H_1 H_2 \longrightarrow 1$,

$$\cancel{\left[G_1 - \frac{(H_2 G_2 - H_2 H_1 G_1)}{1 - (H_1 H_2)} \right]}$$