

Ans-6) a)

Ans-6) a) Proving $y^T P y \geq 0$, knowing $P = A^T A$, $Q = A A^T$

$$y^T P y = y^T A^T A y = (A y)^T A y = \|A y\|_2^2 \geq 0$$

$$\text{Hence } y^T P y \geq 0$$

Proving $z^T Q z \geq 0$

$$z^T Q z = z^T A A^T z = (A^T z)^T (A^T z) = \|A^T z\|_2^2 \geq 0$$

$$\text{Hence } z^T Q z \geq 0$$

Proving eigenvalues of P and Q are non negative. For any eigenvalue λ of P

$$P y = \lambda y \Rightarrow y^T P y = \lambda y^T y$$

$$\text{Since we know } y^T P y \geq 0 \text{ \& } y^T y \geq 0$$

$$\text{Hence } \lambda \geq 0$$

Similarly for Q ,

$$Q y = \lambda_2 y \Rightarrow y^T Q y = \lambda_2 y^T y$$

$$\text{Since Hence } y^T Q y \geq 0 \text{ \& } y^T y \geq 0$$

$$\text{Hence } \lambda_2 \geq 0$$