Astron. Astrophys. 358, L59–L62 (2000)

ASTRONOMY AND ASTROPHYSICS

LETTER

Letter to the Editor

Optimizing Doppler estimates for extrasolar planet detection

I. A specific algorithm for shifted spectra

A. Chelli

Laboratoire d'Astrophysique, Observatoire de Grenoble, B.P. 53, 38041 Grenoble Cedex 9, France

Received 15 March 2000 / Accepted 8 May 2000

Abstract. We present an optimized algorithm to estimate from shifted stellar spectra the variations of radial velocity caused by a rotating planet. This algorithm is based on a rigorous approach in the spectral Fourier space that uses a weighted analysis of the cross spectrum phase between the high resolution spectra of the object and an appropriate reference. We give a simple method to calculate the best value of the stellar radial velocity in the presence of photon noise from a non linear least square fit, and we derive a formal expression for the velocity error. We show experimentally that the Fourier approach gives much better results than the classical cross-correlation technique.

Key words: methods: data analysis – techniques: spectroscopic

1. Introduction

The search for extrasolar planets is an astronomical field in rapid development. Five years after the detection of a Jupiter-like body rotating around the star 51 Peg (Mayor & Queloz 1995), more than 30 extrasolar planets candidates have been found (e.g. Marcy & Butler 1996; Cochran et al. 1997; Noyes et al. 1997; Delfosse et al. 1998; Fischer et al. 1999; Kürster et al. 2000; see Marcy & Butler 1998, for a review). The usual method for the detection consists in measuring the variations of the stellar radial velocity due to the orbiting body. The experimental approaches to this problem, and hence the data reduction model, differ depending on how the wavelength is calibrated. This calibration is done simultaneously to the observations, either separately from the stellar spectrum (Baranne et al. 1996) or superposed on it (Butler et al. 1996).

We are not concerned here with calibration problems, but with a rather simple theoretical question: what is the best method to estimate the distance between two identical spectral energy distributions shifted with respect to each other? This question applies exclusively to the experimental approach of Baranne et al. (1996), which is the only one producing strictly translated spectra. To find the wavelength shift Baranne et al. model the cross correlation, between the high resolution stellar spectrum and a synthetic spectrum made of box-shaped emission lines of equal amplitude, with a gaussian function. This method is far to be optimum because: i) the use of a synthetic spectrum for the reference reduces the advantages of a differential approach, ii) part of the information, like low frequency structures and spectral discontinuities, is discarted (to force the cross correlation to mimic a gaussian function?) and iii) to the knowledge of the author, there is no apparent reason for the cross correlation to be a gaussian function.

In this paper, we propose to derive the variations of the stellar radial velocity from a rigorous approach using all the spectral information. It is based on a straightforward data analysis in the spectral Fourier space, similar to that developed for differential speckle interferometry (Petrov et al. 1986; Chelli 1989; Chelli & Petrov 1995).

Send offprint requests to: Alain Chelli

2. The formalism

Let us consider a reference spectrum $S_{\rm r}(\lambda)$ and its Doppler shifted version $S(\lambda) = S_{\rm r}(\lambda - \lambda\frac{u}{c})$, where u is the radial velocity associated to $S(\lambda)$. We assume that the optical bandwidth is small enough for the Doppler shift to be considered, in a first approximation, constant within the bandwidth. And we write $\lambda\frac{u}{c} \approx \lambda_0\frac{u}{c}$, where λ_0 is the central wavelength.

Our aim is to derive the best estimate v of the radial velocity u in the presence of photon noise. A formal solution to this problem has been given by Petrov et al. (1986). They suggest to compute the Fourier cross spectrum $\widehat{I}(\nu) = \widehat{S}_{\rm r}(\nu)\widehat{S}^*(\nu)$, given by:

$$\widehat{I}(\nu) = {\rm e}^{2i\pi\nu\lambda_0\frac{u}{c}}|\widehat{S}_{\rm r}(\nu)|^2, \tag{1}$$

and to perform a linear least square fit of its phase. However, this procedure suffers from a limitation in the usable frequencies due to the 2π ambiguity affecting the phase estimate if its error is larger than typically 1 radian. In the present case, the Fourier cross spectrum is a rapidly oscillating function which may have values close to zero, and experimentally it is not easy to eliminate the unwanted points. To overcome these difficulties we should avoid working with phases (ϕ). On the contrary, Chelli & Petrov (1995) proposed to use phasors ($\mathrm{e}^{i\phi}$) to warrant the phase continuity and to look for the velocity v which minimizes in a least square sense the imaginary part $\mathrm{Im}[\widehat{C}(\nu)]$ of the quantity:

$$\widehat{C}(\nu) = \mathrm{e}^{-2i\pi\nu\lambda_0\frac{v}{c}}\widehat{I}(\nu). \tag{2}$$

In this context any point of the cross spectrum can be used independently on the associated error.

We assume now that the frequency ν is regularly sampled at the points $j = 1, ...n$. To completely solve the problem of velocity derivation, we need to determine the covariance matrix in the presence of photon noise for the $\{\mathrm{Im}[\widehat{C}(\nu_j)]\}$. It can be deduced from the general calculations of Chelli (1989). The results at the convergence point, i.e. $v \approx u$, are given in appendix A. They show that the $\{\mathrm{Im}[\widehat{C}(\nu_j)]\}$ are poorly correlated. Hence, in order to simplify the problem, we neglect the non diagonal covariance coefficients. This will not significantly change neither the calculated velocity nor the associated formal error. The quantity to be minimized then becomes:

$$Q = \sum_{j=0}^{j=n} \frac{\mathrm{Im}^2[\widehat{C}(\nu_j)]}{\sigma^2(\nu_j)}, \tag{3}$$

where $\sigma^2(\nu_j)$ is the variance of $\mathrm{Im}[\widehat{C}(\nu_j)]$ at the convergence point. It holds (see Petrov et al. 1986, Chelli 1989 and appendix A):

$$\sigma^2(\nu_j) \approx \frac{1}{2}\left[KK_\mathrm{r} + (K + K_\mathrm{r})|\widehat{C}(\nu_j)|\right], \tag{4}$$

where K and K_r are the total number of photoevents in the spectra $S(\lambda)$ and $S_\mathrm{r}(\lambda)$, respectively. Given the precision on the velocity required for extrasolar planet detection, the values of K and K_r are normally large to very large. At such count levels, the statistics of the $\{\mathrm{Im}[\widehat{C}(\nu_j)]\}$ is close to gaussian and the solution to the least square fit equation practically corresponds to the maximum likelihood estimate.

From standard non linear least square fit techniques, the correct value of the velocity v can be obtained via an iterative procedure in which the increment Δv to give to the previous value of the velocity is:

$$\Delta v = -\frac{\sum_{j=0}^{j=n} \frac{\mathrm{Im}[\widehat{C}(\nu_j)]}{\sigma^2(\nu_j)} \frac{\partial \mathrm{Im}[\widehat{C}(\nu_j)]}{\partial v}}{\sum_{j=0}^{j=n} \frac{1}{\sigma^2(\nu_j)} \left(\frac{\partial \mathrm{Im}[\widehat{C}(\nu_j)]}{\partial v}\right)^2}, \tag{5}$$

or explicitly:

$$\Delta v = \frac{c\Delta\lambda}{2\pi\lambda_0} \frac{\sum_{j=0}^{j=n} j \frac{\mathrm{Re}[\widehat{C}(\nu_j)]\mathrm{Im}[\widehat{C}(\nu_j)]}{\sigma^2(\nu_j)}}{\sum_{j=0}^{j=n} j^2 \frac{\mathrm{Re}^2[\widehat{C}(\nu_j)]}{\sigma^2(\nu_j)}}, \tag{6}$$

where $\Delta\lambda$ is the optical bandwidth and Re stands for the real part. Note that $\widehat{C}(\nu)$ must be updated after each iteration and that there are no limitations on the maximum usable frequency point n. At the end of the iteration process, the variance $\sigma^2(v)$ on the derived velocity v is given by:

$$\sigma^2(v) = \frac{1}{\sum_{j=0}^{j=n} \frac{1}{\sigma^2(\nu_j)} \left(\frac{\partial \mathrm{Im}[\widehat{C}(\nu_j)]}{\partial v}\right)^2}, \tag{7}$$

which holds:

$$\sigma^2(v) = \frac{\frac{1}{2}\left(\frac{c\Delta\lambda}{2\pi\lambda_0}\right)^2}{\sum_{j=0}^{j=n} j^2 \frac{\mathrm{Re}^2[\widehat{C}(\nu_j)]}{KK_\mathrm{r} + (K+K_\mathrm{r})|\widehat{C}(\nu_j)|}}. \tag{8}$$

3. Noise analysis

Let us try to simplify Eq. (8). In practice, the number of photoevents in the reference channel is much larger than that of the working channel, hence $K_\mathrm{r} + K \approx K_\mathrm{r}$. Taking into account that at the convergence point, $\mathrm{Re}[\widehat{C}(\nu_j)] \approx |\widehat{C}(\nu_j)|$, and writing $|\widehat{C}(\nu)| \approx KK_\mathrm{r}\widehat{W}(\nu)$, where $\widehat{W}(\nu)$ is the (unbiased, i.e. equal to zero beyond the cut-off frequency) spectral density of $S(\lambda)$ or $S_\mathrm{r}(\lambda)$ normalized to the zero frequency, it holds:

$$\sigma^2(v) \approx \frac{\frac{1}{2}\left(\frac{c\Delta\lambda}{2\pi\lambda_0}\right)^2}{K\sum_{j=0}^{j=n} j^2 \frac{K_\mathrm{r}\widehat{W}^2(\nu_j)}{1+K_\mathrm{r}\widehat{W}(\nu_j)}}. \tag{9}$$

Assuming that K_r is large enough for the useful parts of $\widehat{W}(\nu)$, i.e. those containing most of the velocity information, verify $K_\mathrm{r}\widehat{W}(\nu) \gg 1$, we obtain:

$$\sigma^2(v) \approx \frac{\frac{1}{2}\left(\frac{c\Delta\lambda}{2\pi\lambda_0}\right)^2}{K\sum_{j=0}^{j=n} j^2\widehat{W}(\nu_j)}. \tag{10}$$

Eq. (10) is now very simple to handle as it does not depend any more on the number of photoevents K_r in the reference channel, but only on the object and intrumental characteristics. It is beyond the scope of this work, but it may advantageously be used to study the influence of the stellar spectral type and the influence of the spectral resolution on the velocity error via the term $\sum_{j=0}^{j=n} j^2\widehat{W}(\nu_j) = \Delta\lambda^3 \int \nu^2\widehat{W}(\nu)d\nu$.

To calculate the expected precision on the stellar radial velocity, we use a very good quality high resolution spectrum of the K7V star HD 201092 (GJ 820B) with visible magnitude $m_v = 6$. It was obtained under average seeing conditions, in a 15 minutes integration time, at the 2 m telescope of the Observatoire de Haute Provence (OHP, France) with the instrument ELODIE (see Baranne et al. 1996). This instrument produces high resolution spectra (≈ 40000) formed by 67 spectral orders, each of a few tens of Angstroms width, covering the wavelength range [3900 Å, 6800 Å]. For each order, the observed number of photoevents K and the calculated normalized spectral density $\widehat{W}(\nu)$ were inserted into Eq. (10) to derive the expected error on the radial velocity as a function of the wavelength. The results are shown in Fig. 1. Clearly, most of the useful velocity information is contained beyond 4400 Å, where the error is quasi independent of the spectral order. Properly combining the 67 velocity estimates (i.e. weighted by the inverse of their variance), we derive for an equivalent integration time of 1 hour, a global error of 0.45 ms^{-1}. For the same integration time, Fig. 2 shows this global error as a function of the visible magnitude. The symbolic error of 1 ms^{-1} is reached for a K7V star of magnitude 8.

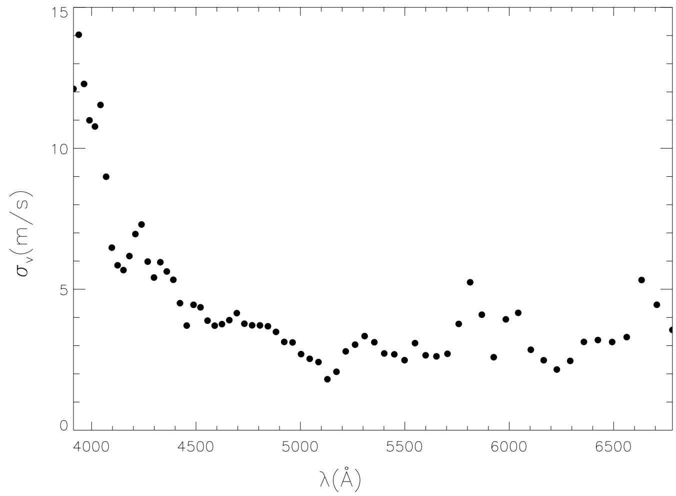

Fig. 1. Expected error on the radial velocity as a function of the wavelength (spectral order) for a K7V star of visible magnitude 6, observed during an equivalent integration time of 1 hour with the ELODIE spectrograph installed on the 2 m telescope of the OHP. Note that the main velocity information is contained beyond 4400 Å, where the error stabilizes around 3 ms^{-1} (see text).

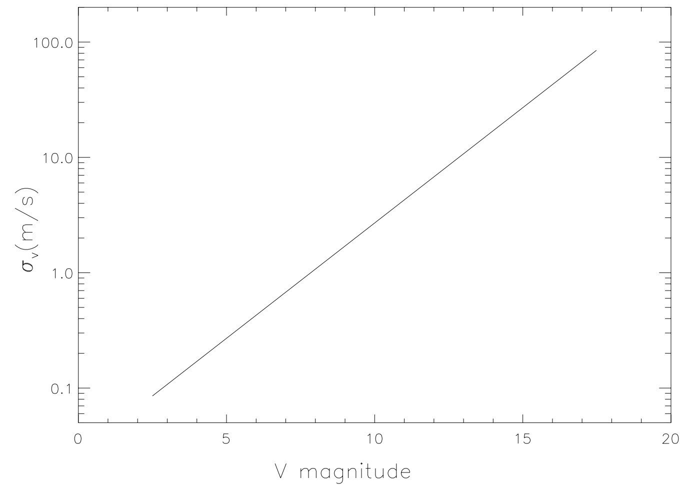

Fig. 2. Expected global error on the radial velocity as a function of the visible magnitude for a K7V star, observed during an equivalent integration time of 1 hour with the ELODIE spectrograph installed on the 2 m telescope of the OHP. The symbolic velocity error of 1 ms^{-1} is obtained for a visible magnitude 8 (see text).

4. Experimental approach

In this section we compare experimentally the performances of the Fourier and the cross-correlation methods on the HD 201092 spectrum used in the previous section. The cross-correlation results have been obtained with the Tacos program currently used to reduce the ELODIE spectra. For each spectral order, the velocity is calculated by comparison with a synthetic reference derived from a K0 spectrum, and then corrected for the zero point. The zero points are the mean velocity per order, derived with Tacos from a library of thirteen spectra of HD 201092 obtained during routine observations with ELODIE (Delfosse et al. 1999). The results are plotted in Fig. 3 (white points) as a function of the wavelength.

For the Fourier approach, an appropriate reference spectrum has first to be derived. This is achieved from the mentioned thirteen spectra of HD 201092. Each spectrum is processed as follows: i) all the orders are regularly resampled with a 0.03 Å wavelength step, ii) they are corrected by Fourier transform for the Earth velocity and the stellar velocity (calculated as a first guess with for example the Tacos program), iii) their end parts corresponding to the extreme observed wavelength shifts (Earth + stellar) are canceled. The reference spectrum is then just obtained by summing up, order by order, all the individual resulting spectra. In practice, for HD 201092 we correct the thirteen individual spectra only for the Earth velocity as the observed stellar velocity amplitude is smaller than ± 100 ms^{-1} which represents less than 2% of the spectral resolution. The residual radial velocity of the selected spectrum of HD 201092 calculated from the Fourier approach is plotted in Fig. 3 (black points) as a function of the wavelength.

As expected (see Fig. 1), the velocity values are highly dispersed before 4400 Å. They become stable between 4400 Å and 5500 Å, and they start to vary in a somewhat non random way beyond 5500 Å. Given such a behavior, the variance of the data is not a good way to estimate the velocity dispersion. To quantify the dispersion we prefer to build an histogram of the velocities for each set of results. The resulting histograms are shown in Fig. 4 with a binsize of 15 ms^{-1}, and are fitted with a gaussian function. The width (variance of the gaussian fit) of the Fourier distribution is 10 ms^{-1}, against 34 ms^{-1} for the cross-correlation distribution. Taking into account the surface of the gaussian functions, the internal error for the mean velocity becomes 4.7 ms^{-1} for the cross-correlation method and 1.5 ms^{-1}, that is to say 3 times smaller, for the Fourier method. The obtained 1.5 ms^{-1} compares well with the 0.90 ms^{-1} theoretical error computed from the experimental spectrum and Eq. (10).

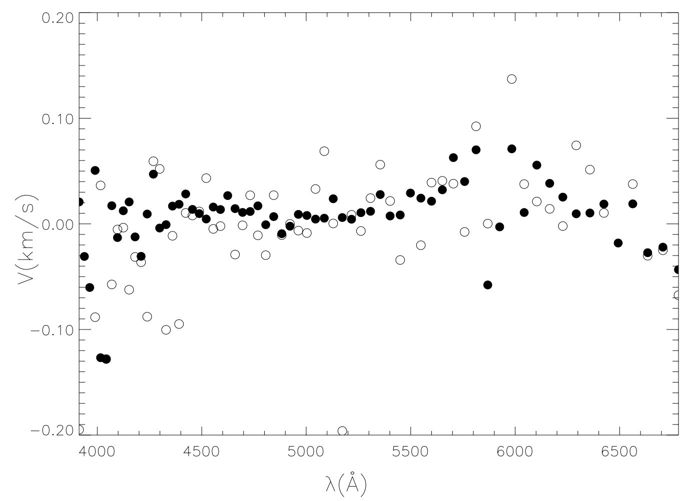

Fig. 3. Stellar radial velocity for HD 201092 as a function of the wavelength (spectral order) calculated with the cross-correlation method (white points) and with the Fourier method (black points).

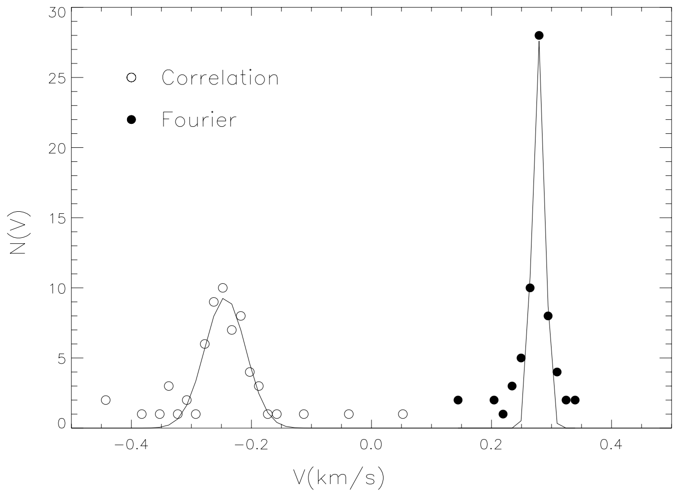

Fig. 4. Histograms of the radial velocities of Fig. 3, shifted for clarity along the x-axis. The gaussian fits (continuous lines) indicate a velocity dispersion and an internal mean velocity error of: 34 ms^{-1} and 4.7 ms^{-1} for the cross-correlation method (white points), 10 ms^{-1} and 1.5 ms^{-1} for the Fourier method (black points).

5. Concluding remarks

The expected error on the radial velocity obviously depends on the seeing angle which is not apparent in the previous calculation as we used an observed spectrum. Hence, the calculated limiting performances have to be taken only as indicative. Given the level of precision required for extrasolar planet detection, it is crucial to derive the reference spectrum from the object itself to keep the advantages of a full differential approach. When this is fulfilled, there is *a priori* no need of a window function to supress regions of spectral mismatches between the reference and the object spectra. In addition, the biases inherent to the comparison of different spectra can be avoided or minimized.

We have shown experimentally that the Fourier approach for stellar radial velocity calculation gives much better results than the classical cross-correlation technique with a synthetic spectrum. A more extensive comparison including tens of spectra and a large range of stellar magnitudes will be reported in a forthcoming paper.

Appendix A: The covariance matrix

In this section, we give the expression of the covariance coefficients $\mu(\nu_j, \nu_k)$ at the frequencies ν_j and ν_k, of the imaginary part of the Fourier cross spectrum $\widehat{C}(\nu)$, in the presence of photon noise. From the work of Chelli (1989), at the convergence point (i.e. $v \approx u$, see Sect. 2), it is given by:

$$\mu(\nu_j,\nu_k) = \frac{\overline{K}_\mathrm{r}\overline{K}}{2}\left[|\widehat{T}(\nu_j-\nu_k)|^2 - |\widehat{T}(\nu_j+\nu_k)|^2\right]$$
$$+\frac{\overline{K}_\mathrm{r}\overline{K}}{4}(\overline{K}+\overline{K}_\mathrm{r})\widehat{T}^*(\nu_j)\widehat{T}(\nu_k)\widehat{T}(\nu_j-\nu_k) + cc$$
$$-\frac{\overline{K}_\mathrm{r}\overline{K}}{4}(\overline{K}+\overline{K}_\mathrm{r})\widehat{T}(\nu_j)\widehat{T}(\nu_k)\widehat{T}^*(\nu_j+\nu_k) - cc, \qquad \text{(A1)}$$

where the upper bar over K and K_r stands for the expected value, cc denotes the complex conjugate of the preceeding term, and $\widehat{T}(\nu)$ represents the non noisy spectral Fourier transform of $S(\lambda)$ or $S_\mathrm{r}(\lambda)$ normalized to the zero frequency (note that $\widehat{W}(\nu) = |\widehat{T}(\nu)|^2$, see Sect. 3). Setting $\nu_j = \nu_k = \nu$, we derive the variance $\sigma^2(\nu)$ of Im$[\widehat{C}(\nu)]$:

$$\sigma^2(\nu) = \frac{\overline{K}_\mathrm{r}\overline{K}}{2}(1-|\widehat{T}(2\nu)|^2) + \frac{\overline{K}_\mathrm{r}\overline{K}}{2}(\overline{K}+\overline{K}_\mathrm{r})|\widehat{T}(\nu)|^2$$
$$-\frac{\overline{K}_\mathrm{r}\overline{K}}{4}(\overline{K}+\overline{K}_\mathrm{r})\left[\widehat{T}^2(\nu)\widehat{T}^*(2\nu)+\widehat{T}^{*2}(\nu)\widehat{T}(2\nu)\right]. \qquad \text{(A2)}$$

Neglecting the double frequencies and the bispectra, it holds:

$$\sigma^2(\nu) = \frac{\overline{K}_\mathrm{r}\overline{K}}{2}\left[1+(\overline{K}+\overline{K}_\mathrm{r})|\widehat{T}(\nu)|^2\right]. \qquad \text{(A3)}$$

Experimentally, we have not access to $\overline{K}$, $\overline{K}_\mathrm{r}$ and $\widehat{T}(\nu)$, but only to their estimates. Hence, making the approximations $\overline{K} \approx K$, $\overline{K}_\mathrm{r} \approx K_\mathrm{r}$ and $\overline{K}_\mathrm{r}\overline{K}|\widehat{T}(\nu)|^2 \approx |\widehat{C}(\nu)|$, the variance writes:

$$\sigma^2(\nu) \approx \frac{1}{2}\left[KK_\mathrm{r} + (K+K_\mathrm{r})|\widehat{C}(\nu)|\right]. \qquad \text{(A4)}$$

Note that the correlation coefficient between Im$[\widehat{C}(\nu_j)]$ and Im$[\widehat{C}(\nu_k)]$ is of the order $|\widehat{T}(\nu_j-\nu_k)|$. For a K7V star, at a spectral resolution of about 40000, we generally speak of a correlation degree of a few percent or much less, which means that the $\{\mathrm{Im}[\widehat{C}(\nu_j)]\}$ are poorly correlated.

Acknowledgements. The author would like to thank Drs. X. Delfosse, T. Forveille and C. Perrier for helpful discussions and for providing him the data prior to their publication. Thanks also to the referee, Dr. F. Pepe, for insightful comments and suggestions which improved the scientific content of this work.

References

Baranne A. et al., 1996, A&AS 119, 373
Butler R.P., Marcy G.W., Williams E., McCarthy C., Dosanjh P., Vogt S.S., 1996,, PASP 108, 500
Chelli A., 1989, A&A 225, 277
Chelli A., Petrov R.G., 1995, A&AS. 109, 389
Cochran W.D., Hatzes A.P., Butler R.P., Marcy G.W., 1997, ApJ 483, 457
Delfosse X., Forveille T., Mayor M., Perrier C., Naef D., Queloz D., 1998, A&A 338, L67
Delfosse X., Forveille T., Beuzit J.L., Udry S., Mayor M, Perrier C., 1999, A&A 344, 897
Fischer D.A., Marcy G.W., Butler R.P., Vogt S.S., Apps K., 1999, PASP 111, 50
Kürster M., Endl M., Els S., Hatzes A.P., Cochran W.D., Döbereiner S., Dennerl K., 2000, A&A 353, L33
Marcy G.W, Butler R.P., 1996, ApJ 464, L147
Marcy G.W, Butler R.P., 1998, ARAA 36, 57
Mayor M., Queloz D., 1995, Nature 378, 355
Noyes R.W., Jha S., Korzennik S.G., Krockenberger M., Nisenson P., Brown T.M., Kenelly E.J., Horner S.D., 1997, ApJ 483, L111
Petrov R.G., Roddier F., Aime C., 1986, J. Opt. Soc. Am A3, 634