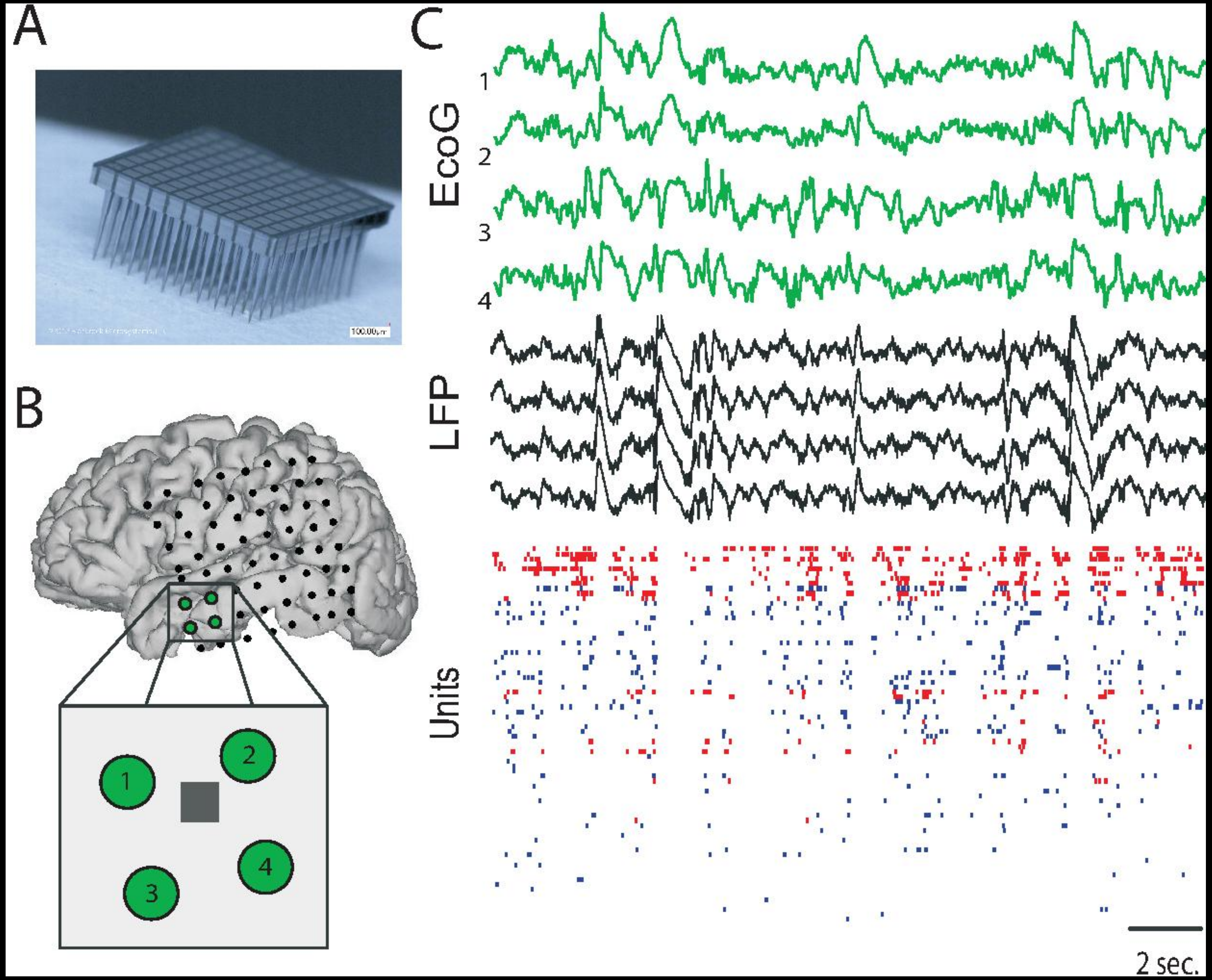
LFP Analysis using Time-delayed Mutual Information

(About My Recent Results)

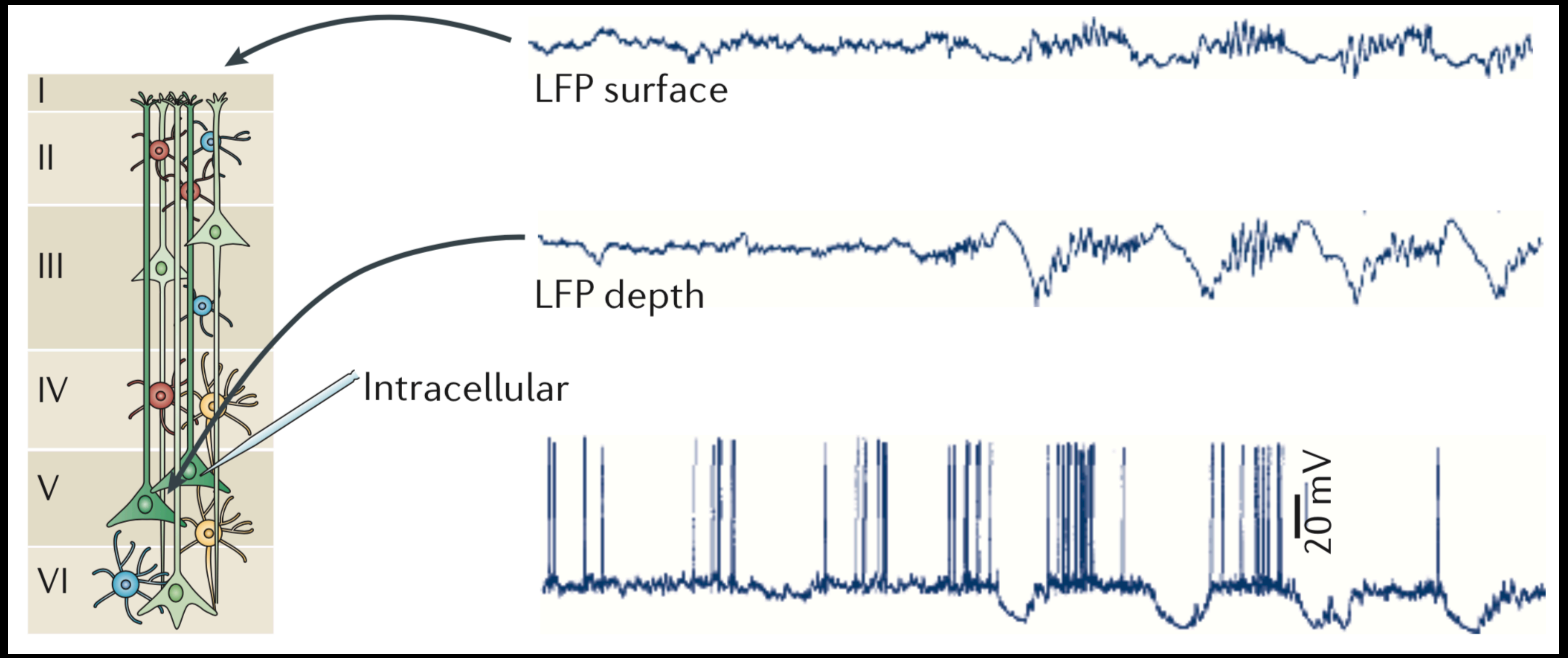
Kai Chen

Mar. 13, 2019

Local Field Potential



modified from Peyrache et al., 2012



Contreras D. & Steriade M. Cellular basis of EEG slow rhythms: a study of dynamic corticothalamic relationships. *J. Neurosci.* **51**, 604–622 (1995).

LFP record the local neuronal activity

Decay as neuron far away from the recording cite

Can record the activity of neurons in deep layer

Experimental Results

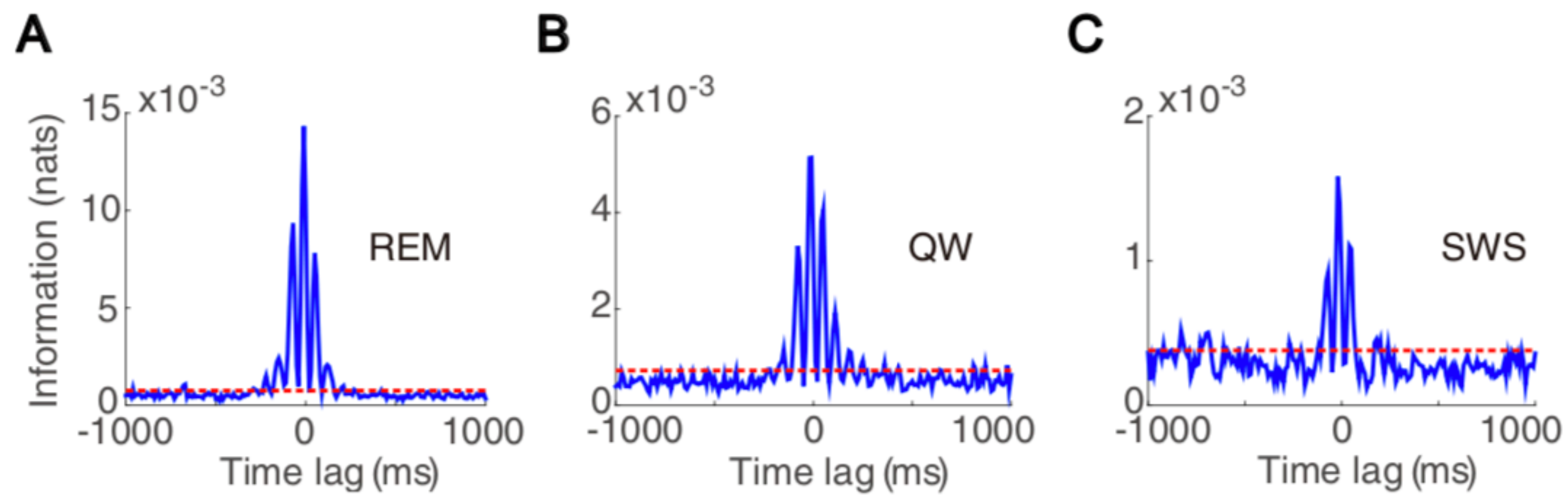


Figure 4. Interaction between a theta-driving neuron and the theta wave over different behavioral states. Time-delayed mutual information (blue) between the theta wave and the firing activity of a theta-driving neuron in the state of (**A**) REM, (**B**) QW, and (**C**) SWS. The red dashed line indicates the significance level.

Li, S., Xu, J., Chen, G., Lin, L., Zhou, D., & Cai, D. (2017). The characterization of hippocampal theta-driving neurons—a time-delayed mutual information approach. *Scientific reports*, *7*(1), 5637.

Model System

Integrate-and-Fire Neuronal Model

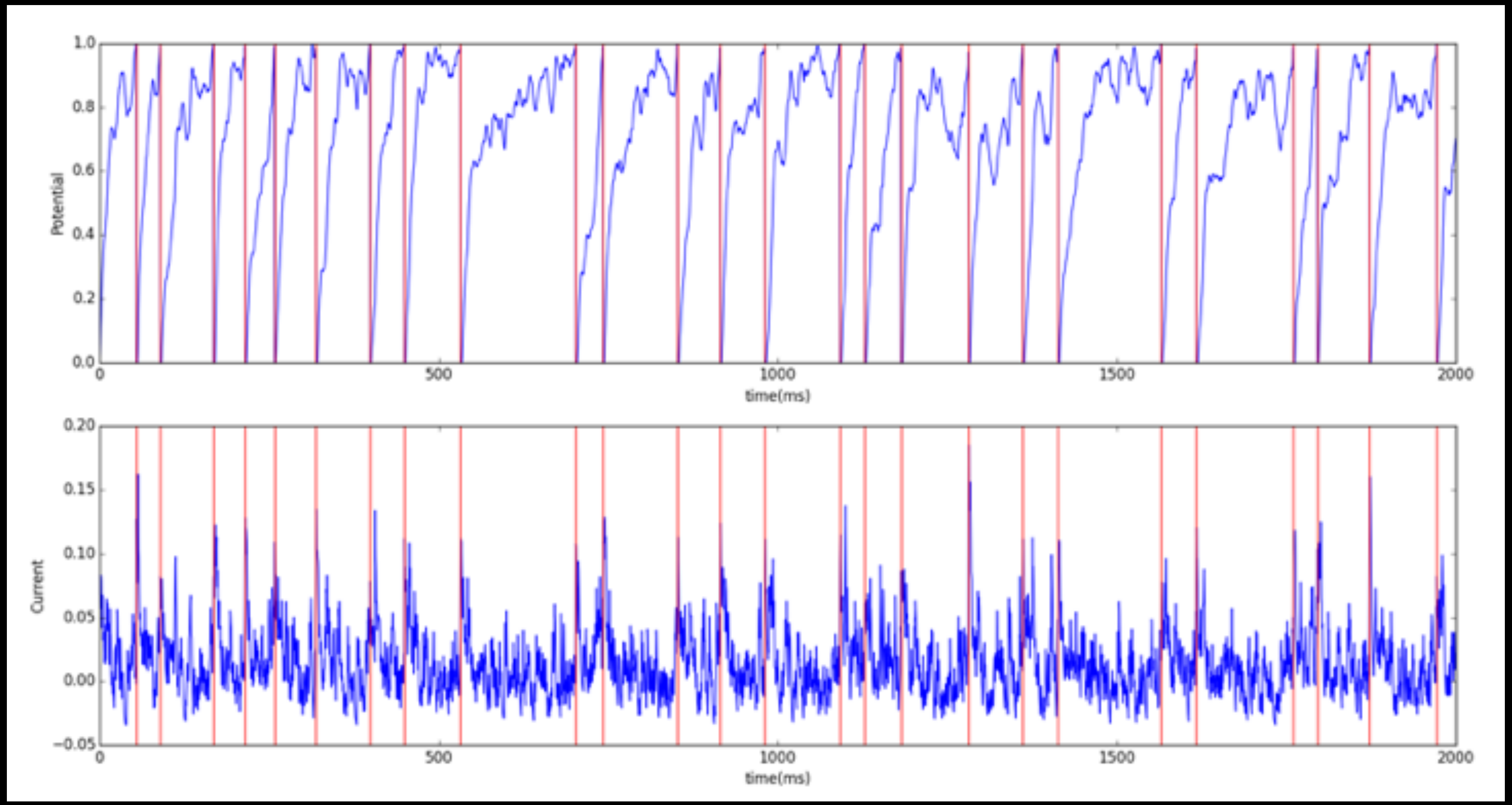
Current based Integrate and fire neuronal model

$$C_m \frac{dV}{dt} = -\, g_l(V - \varepsilon_l) - g_Q(V - \varepsilon_Q)$$

$$g_Q = S_Q \sum_{j, t \geq t_j} \exp\left(-\frac{t - t_j}{\tau_Q} \right)$$

$$Q \in \{E, I\}$$

When $V(t) \geq V_\theta$ **, V reset to resting potential, and maintains for a refractory period**

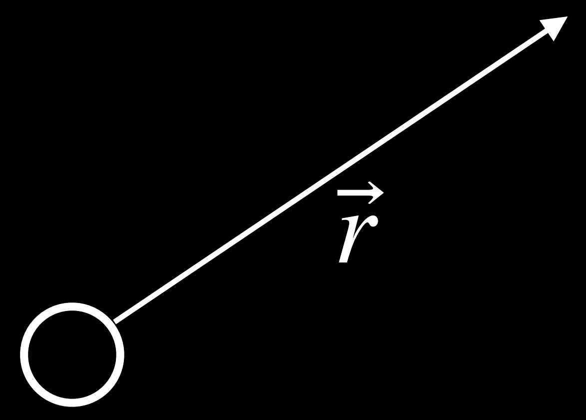


LFP Model

$$\phi(\mathbf{r}, t) = \frac{1}{4\pi\sigma} \sum_{i=1}^{N} \frac{I_i(t)}{|\mathbf{r} - \mathbf{r}_i|}$$

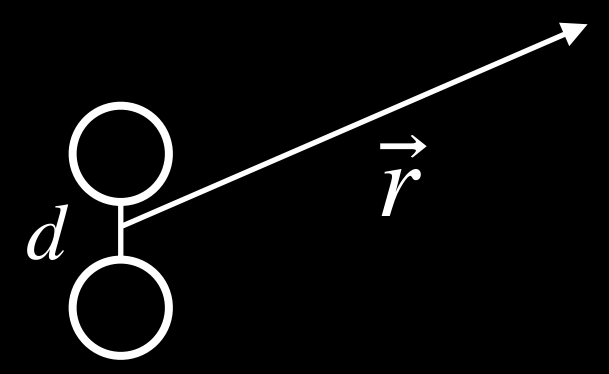
$$\phi(\mathbf{r}, t) = \sum_{n=1}^{N} \frac{I_n(t)}{4\pi\sigma} \frac{1}{\Delta s_n} log \left| \frac{\sqrt{h_n^2 + \rho_n^2} - h_n}{\sqrt{l_n^2 + \rho_n^2} - l_n} \right|$$

Current Source Monopole

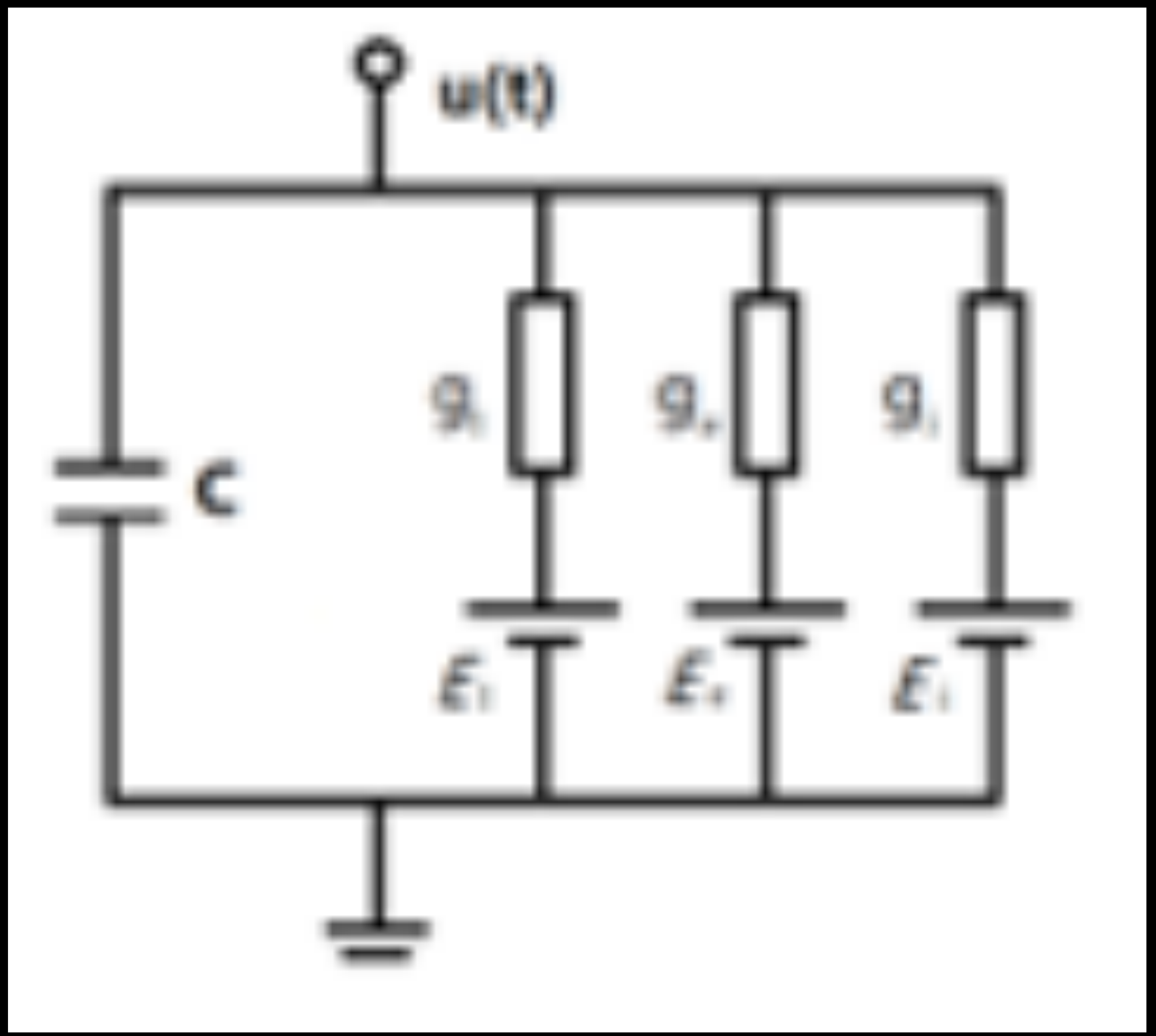


$$\phi(\mathbf{r}, t) = \frac{1}{4\pi\sigma} \sum_{i=1}^{N} \frac{I_i(t)}{|\mathbf{r} - \mathbf{r}_i|}$$

Current Source Dipole



$$\phi(\mathbf{r}, t) = \frac{1}{4\pi\sigma} \sum_{i=1}^{N} \frac{I_i(t)d}{|\mathbf{r} - \mathbf{r}_i|^2}$$



Lindén, H., Hagen, E., Leski, S., Norheim, E. S., Pettersen, K. H., & Einevoll, G. T. (2014). LFPy: a tool for biophysical simulation of extracellular potentials generated by detailed model neurons. *Frontiers in neuroinformatics*, *7*, 41.

LFP Model

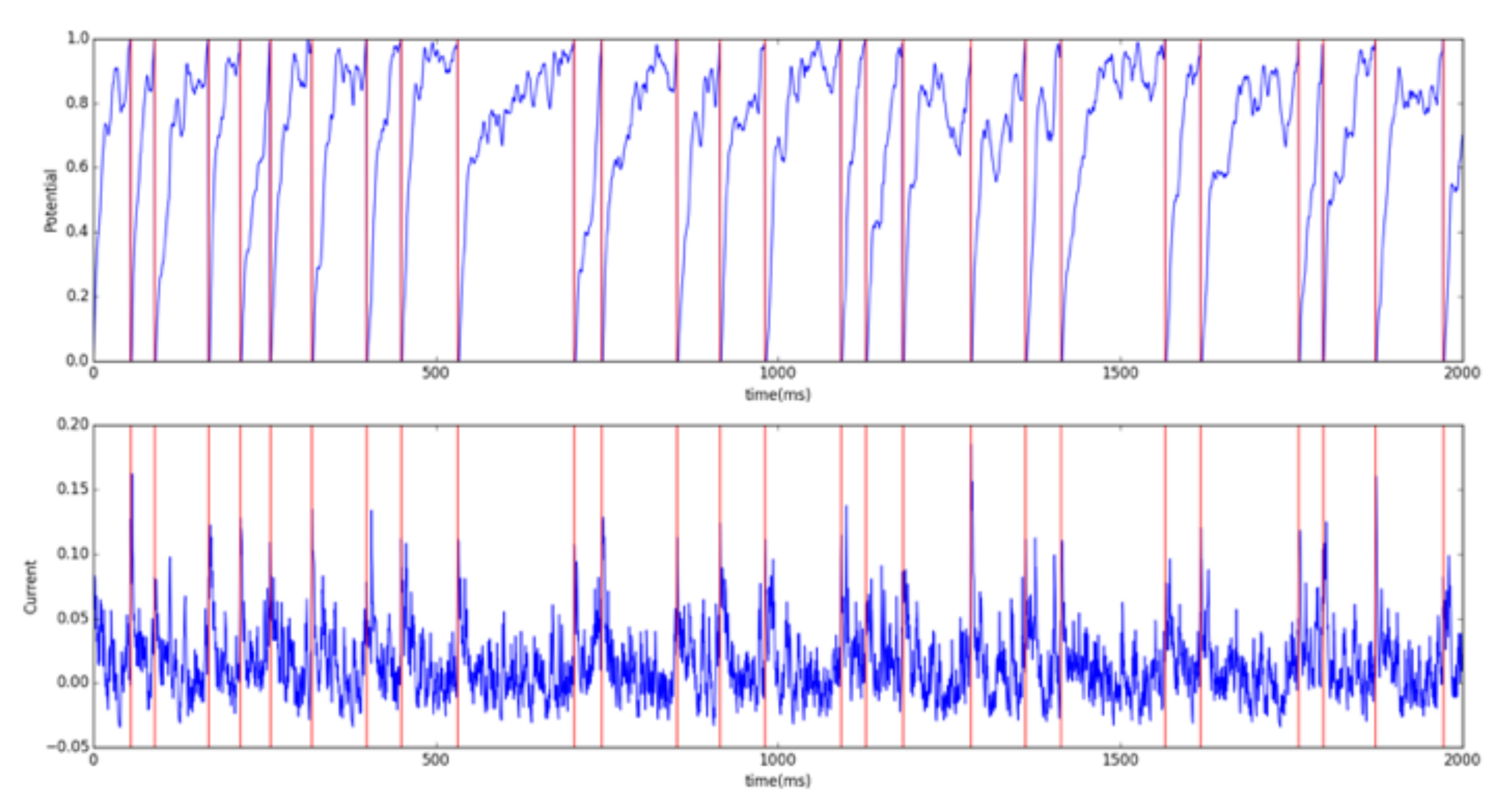
Current based Integrate and fire neuronal model

$$C_m \frac{dV}{dt} = -g_l(V - \varepsilon_l) - g_Q(V - \varepsilon_Q)$$

$$g_Q = S_Q \sum_{j, t \geq t_j} \exp\left(-\frac{t - t_j}{\tau_Q}\right)$$

$$Q \in \{E, I\}$$

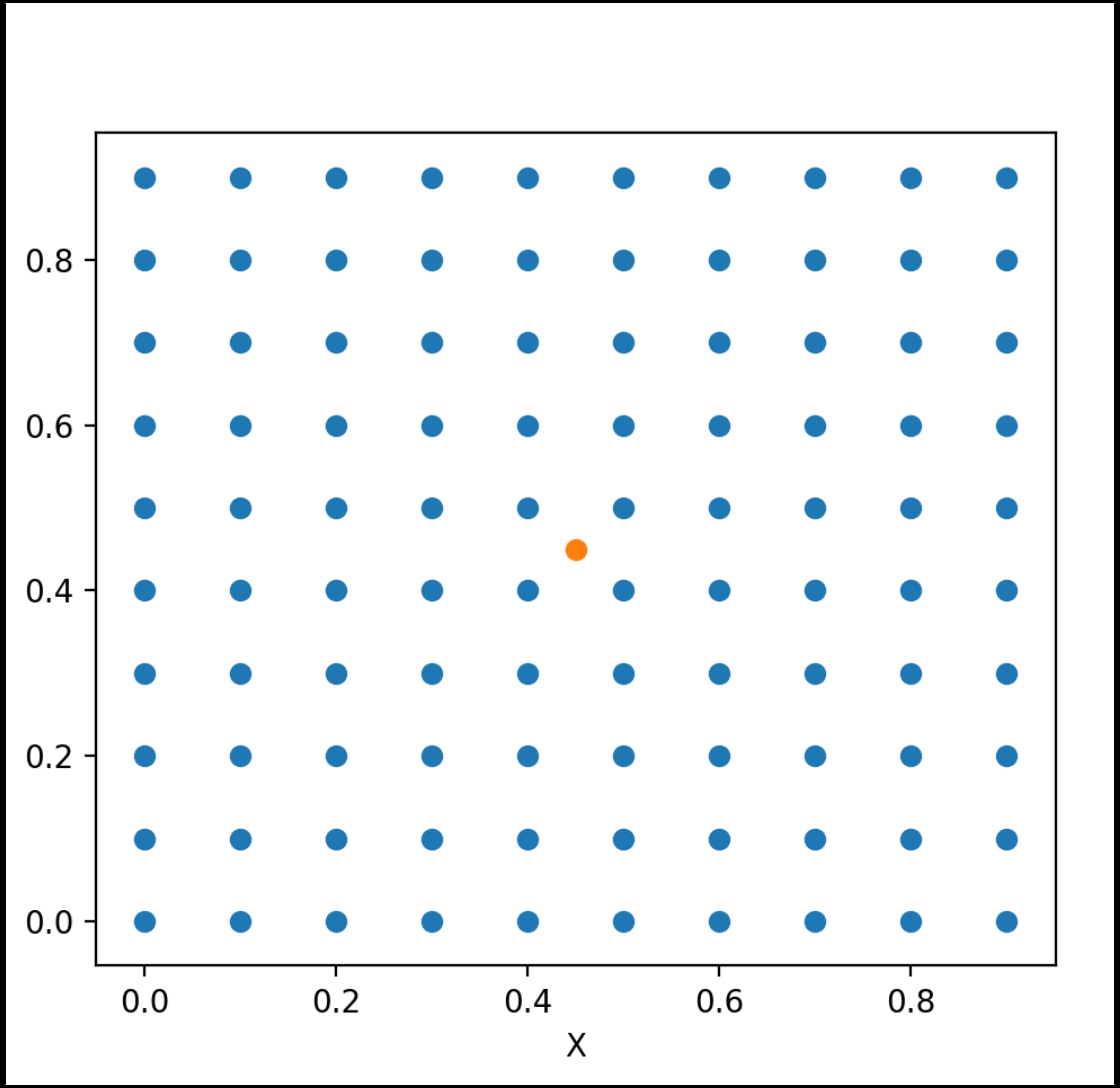
When $V(t) \geq V_\theta$ **, V reset to resting potential, and maintains for a refractory period**



$$I(t) = -g_l(V - \varepsilon_l) - g_Q(V - \varepsilon_Q)$$

2-D I&F neural network

- 100 I&F neurons embedded in 10 by 10 grid
- Measuring electrode placed at the center
- Neurons randomly connected with certain probability
- 80% excitatory neurons
- 20% inhibitory neurons



Tools for Causal Inference

Shannon Entropy and Mutual Information

- For a random variable X

$$H(X) = \sum_{x \in X} p(x) log \frac{1}{p(x)}$$

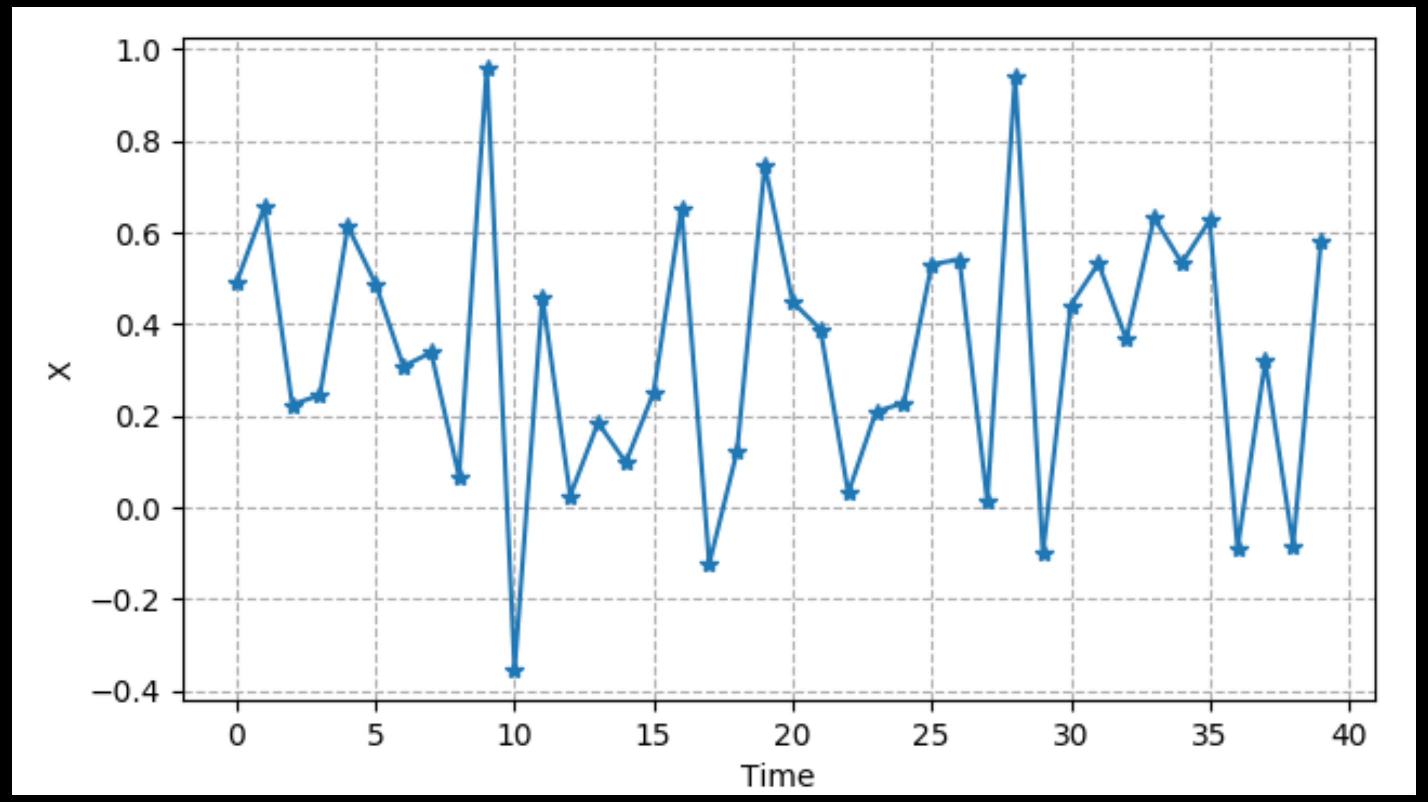
- For two random variable X and Y

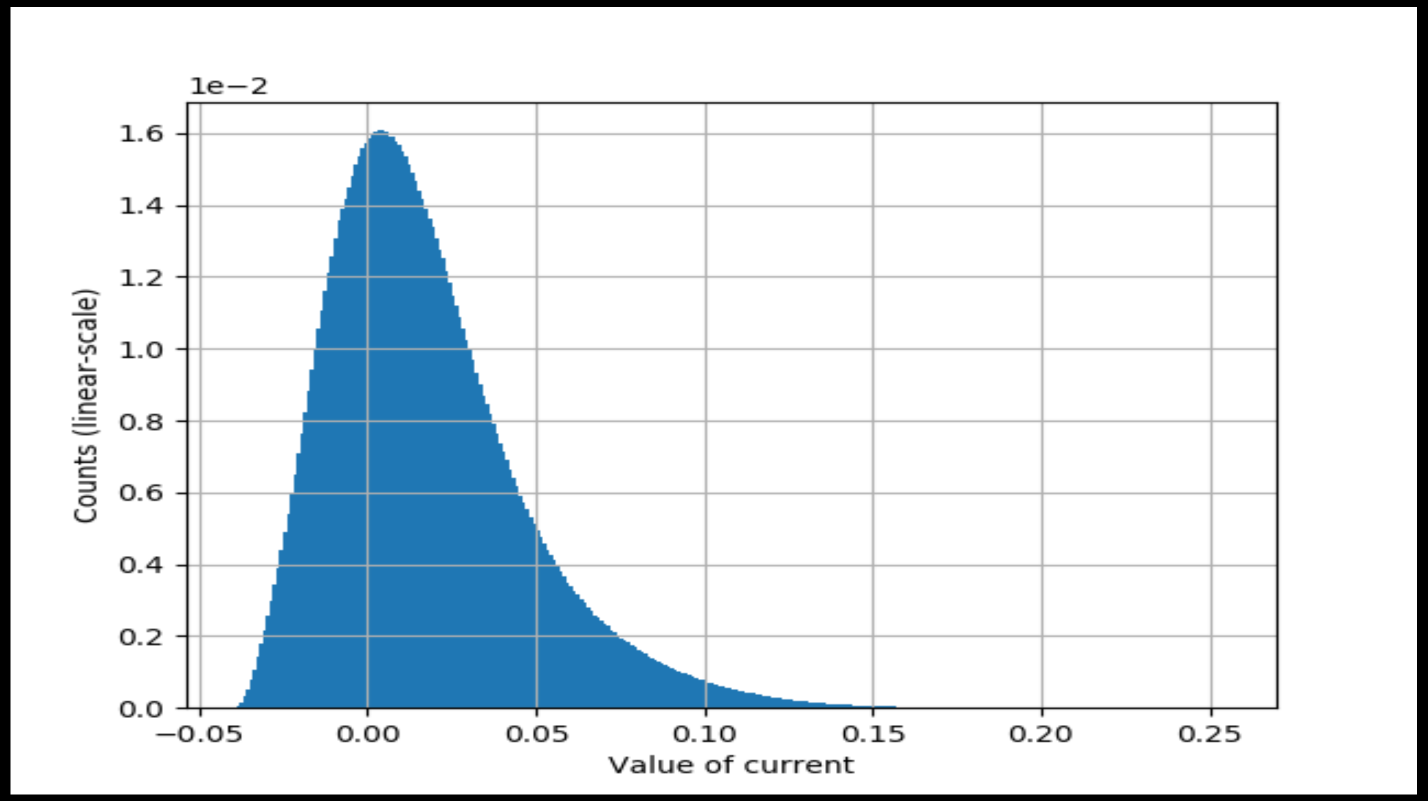
$$I(X;Y) = \sum_{x \in X} \sum_{y \in Y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

- I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) = H(X) + H(Y) - H(X,Y)

Mutual Information and Time Series

- For a time series X(t), **wide-sense stationary**
- Calculate its distribution function
- Mutual information between two time series

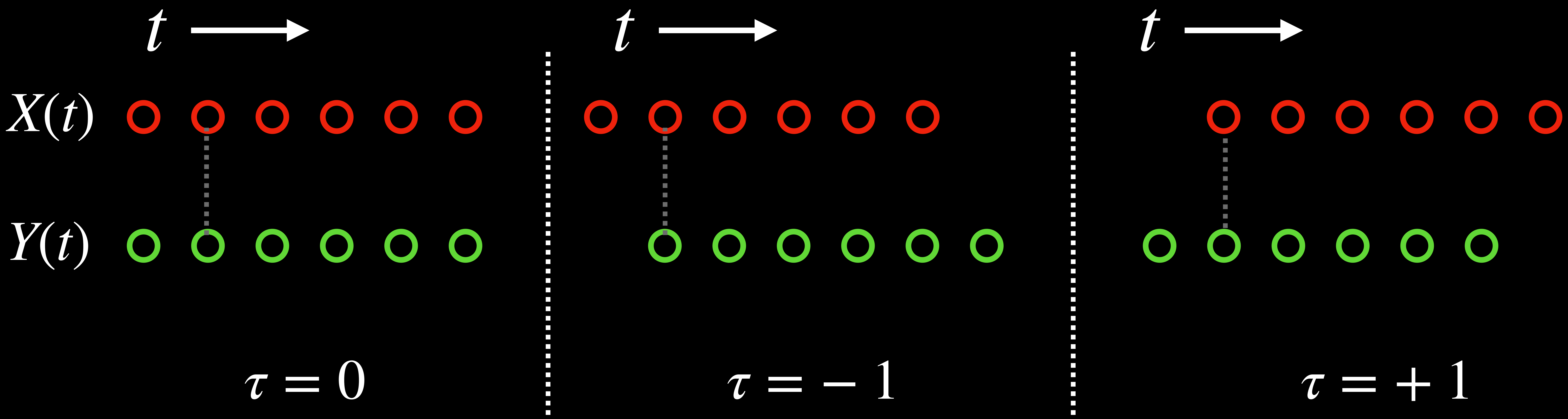




$$I\left(X(t); Y(t)\right) = \sum_{x \in X(t)} \sum_{y \in Y(t)} p(x, y) \log \frac{p(x, y)}{p(x)p(y)}$$

Mutual Information and Time Series

- How to infer the causal relation between X(t) and Y(t)



$$I(X;Y,\tau) = I\left(X_t;Y_{t+\tau}\right) = \sum_{x\in X_t}\sum_{y\in Y_{t+\tau}} p(x,y)\log\frac{p(x,y)}{p(x)p(y)}$$

Mutual Information

$$I(X;Y,\tau) = \sum_{x \in X_t} \sum_{y \in Y_{t+\tau}} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

- Statistical point of view
- Capture the common information (entropy) shared between two time series
- Both linear and nonlinear mapping

Cross-correlation

$$(f \star g)(\tau) \triangleq \int_{-\infty}^{\infty} f(t) g(t+\tau) dt$$

- Similarity between two time series
- Only linear correlation

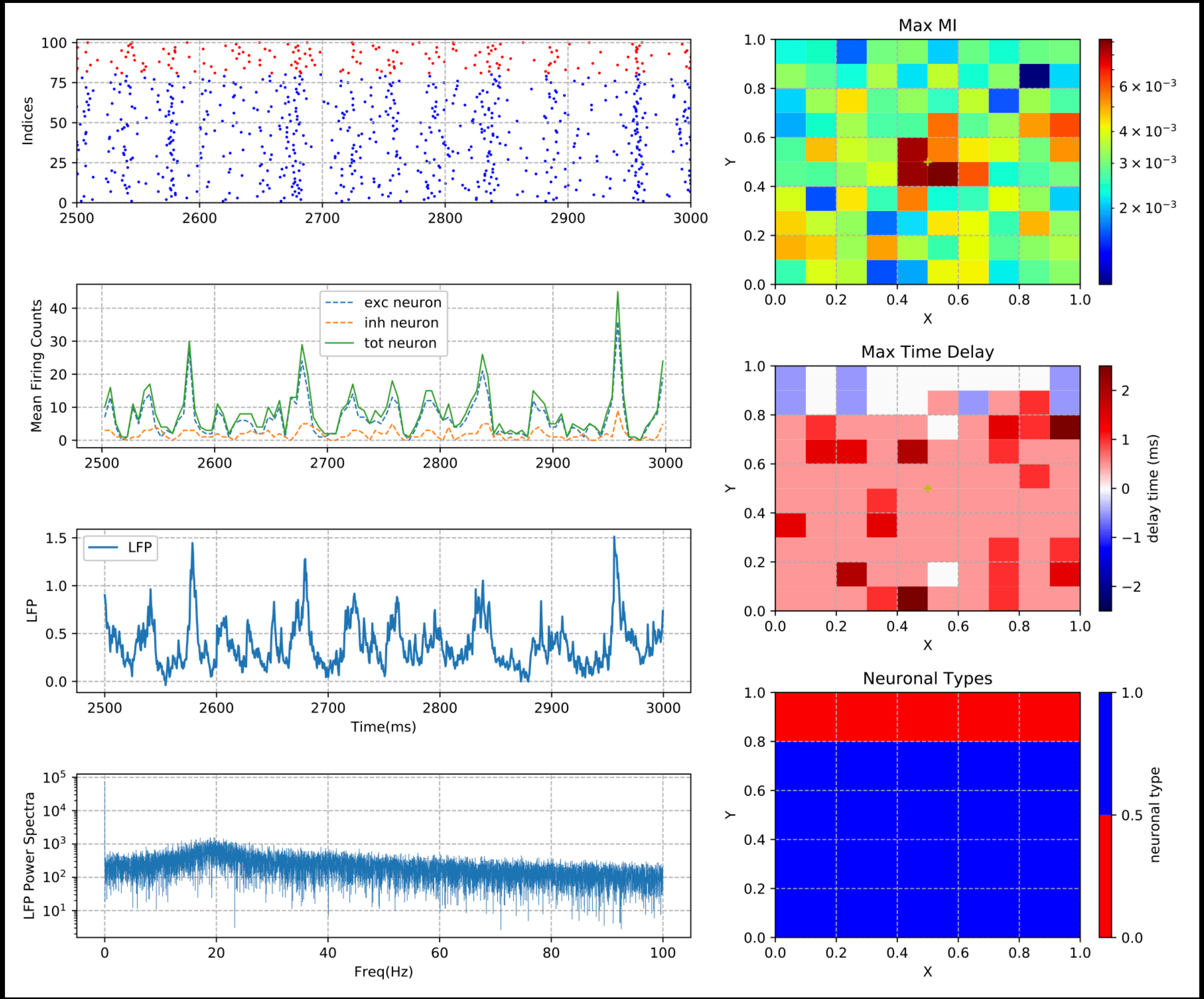
Figure Demos

Raster plot of neurons

Total firing events of network

Local field potential

Power Spectrum of LFP



Color map of max mutual information

Color map of the time delay of max MI

Color map of neuronal types (blue for excitatory neurons, red for inhibitory neurons)

Synchronous State

Results

Parameters:

Neuron number = 100

Connecting Probability = 0.1

Synaptic Strength (E/I) = 8e-3(0.8mV)

Poisson Rate = 2 kHz

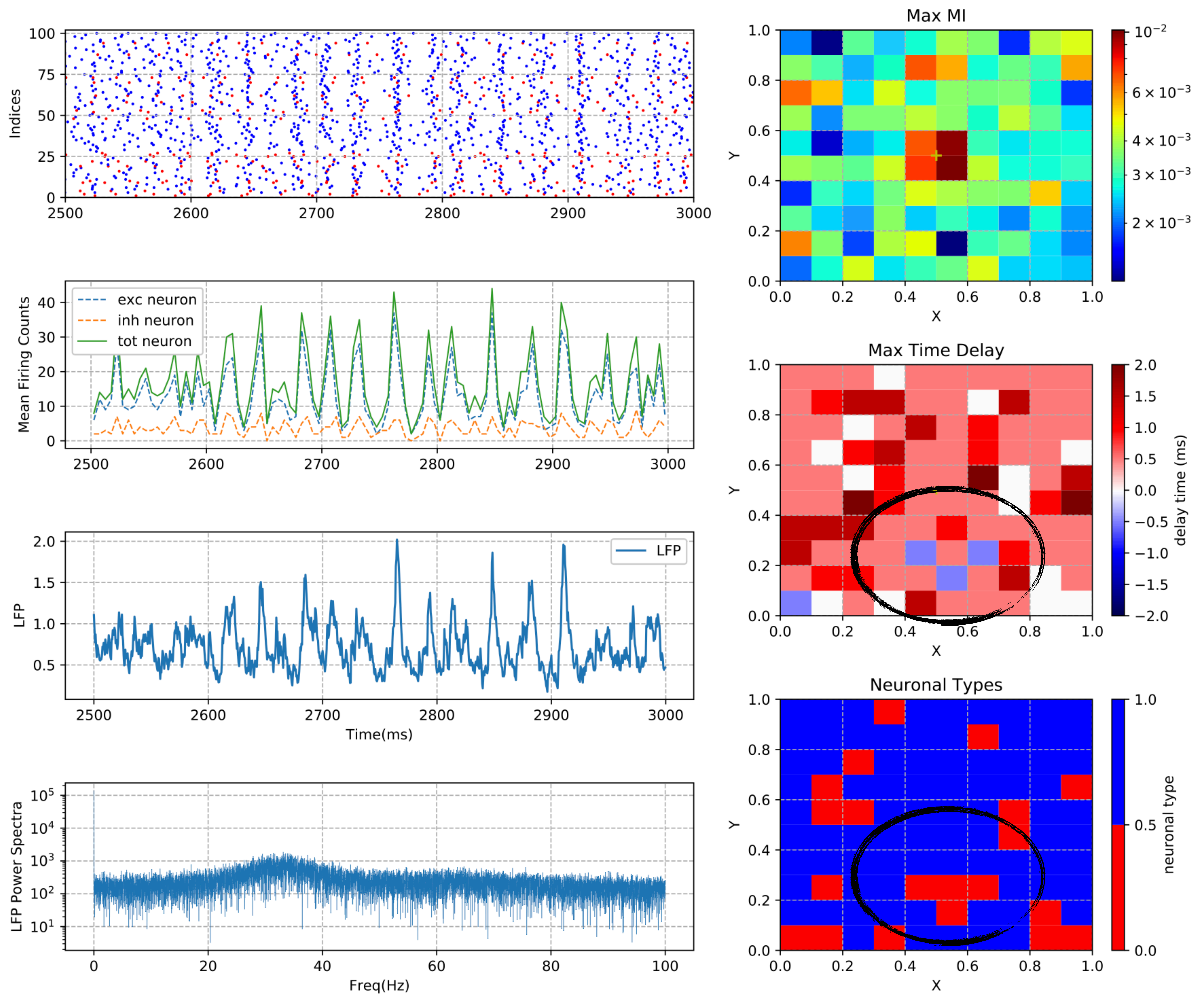
Poisson Strength = 3.5e-3(0.35mV)

Simulation Time = 1e5 ms

Electrode Position = (0.5, 0.5)

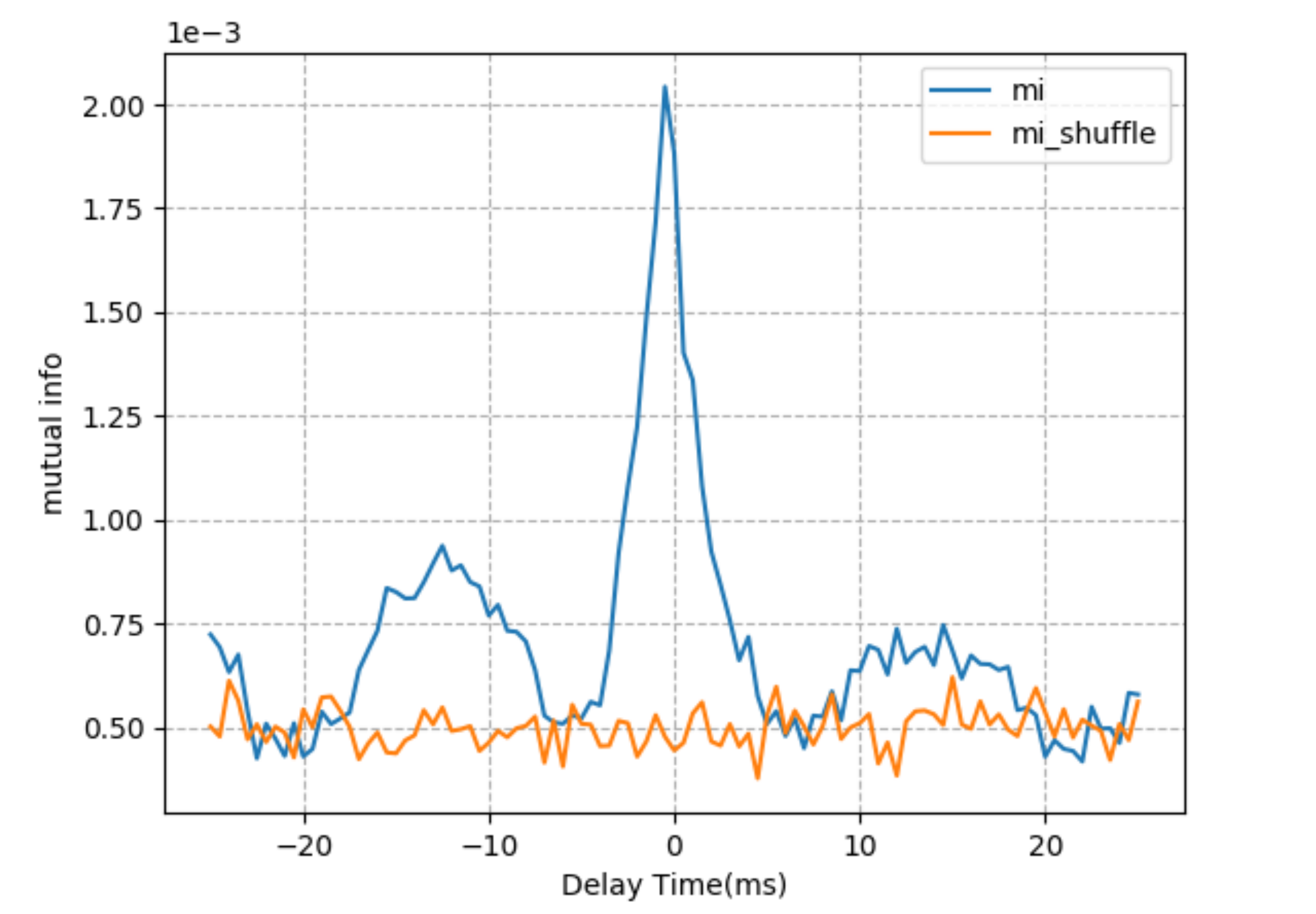
Comments:

- Neurons are synchronized
- The dominant frequency of LFP is around 33 Hz
- Most neurons reach the phase lock states with LFP.
- Exc. neurons always have proceeding phase respect to LFP, while phases of some inh. neurons fall behind.



Sample figures of TDMI

24



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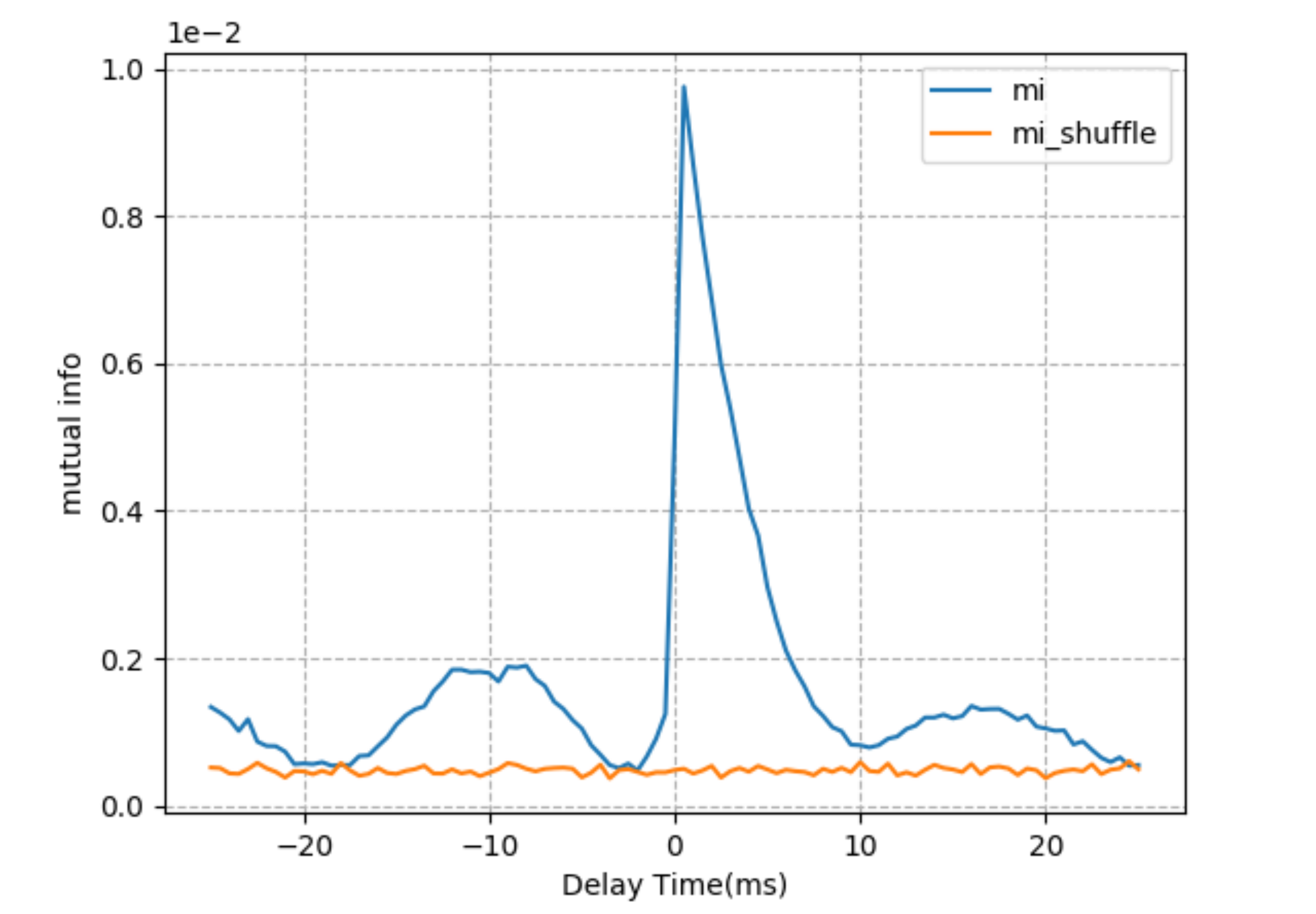

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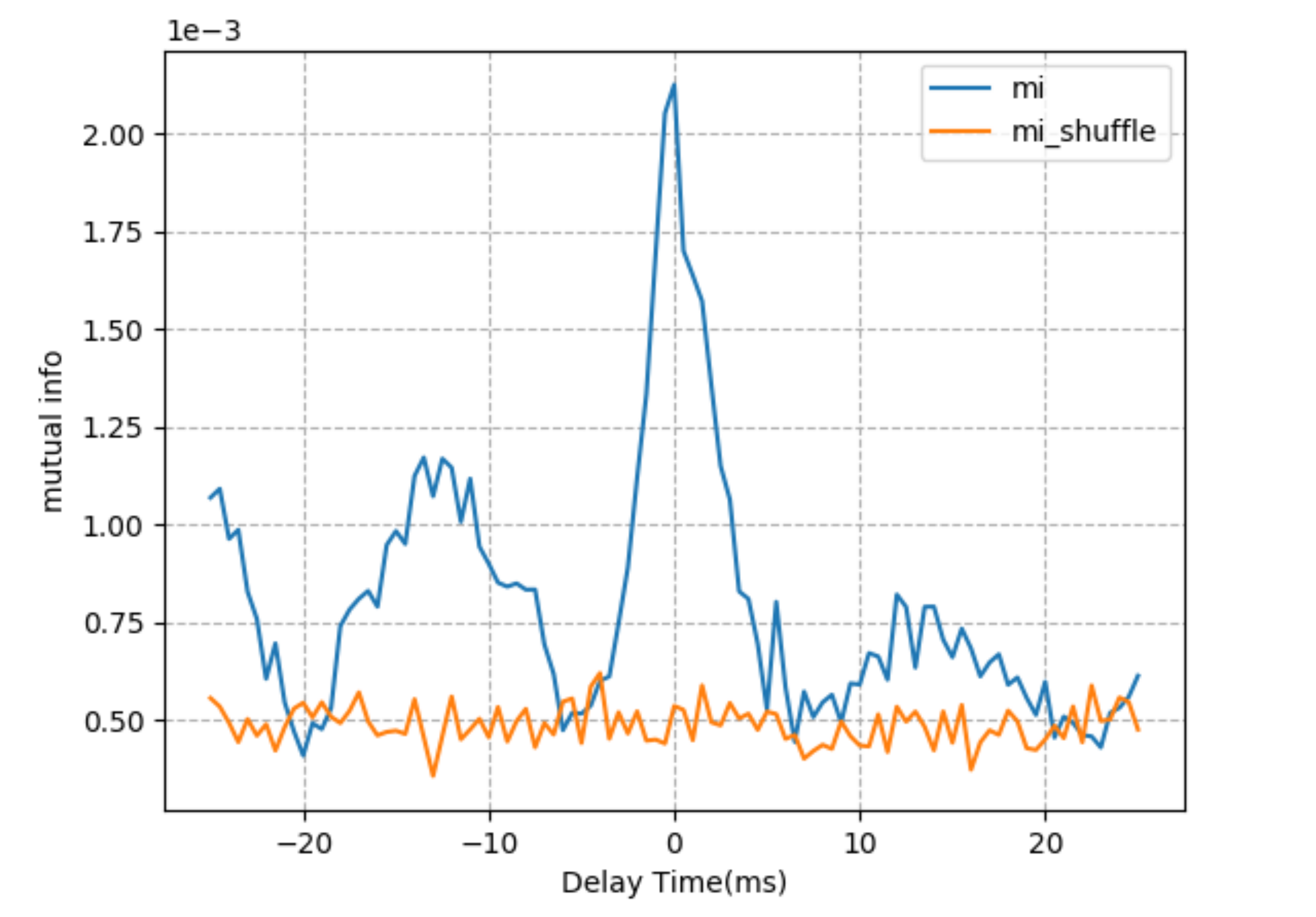

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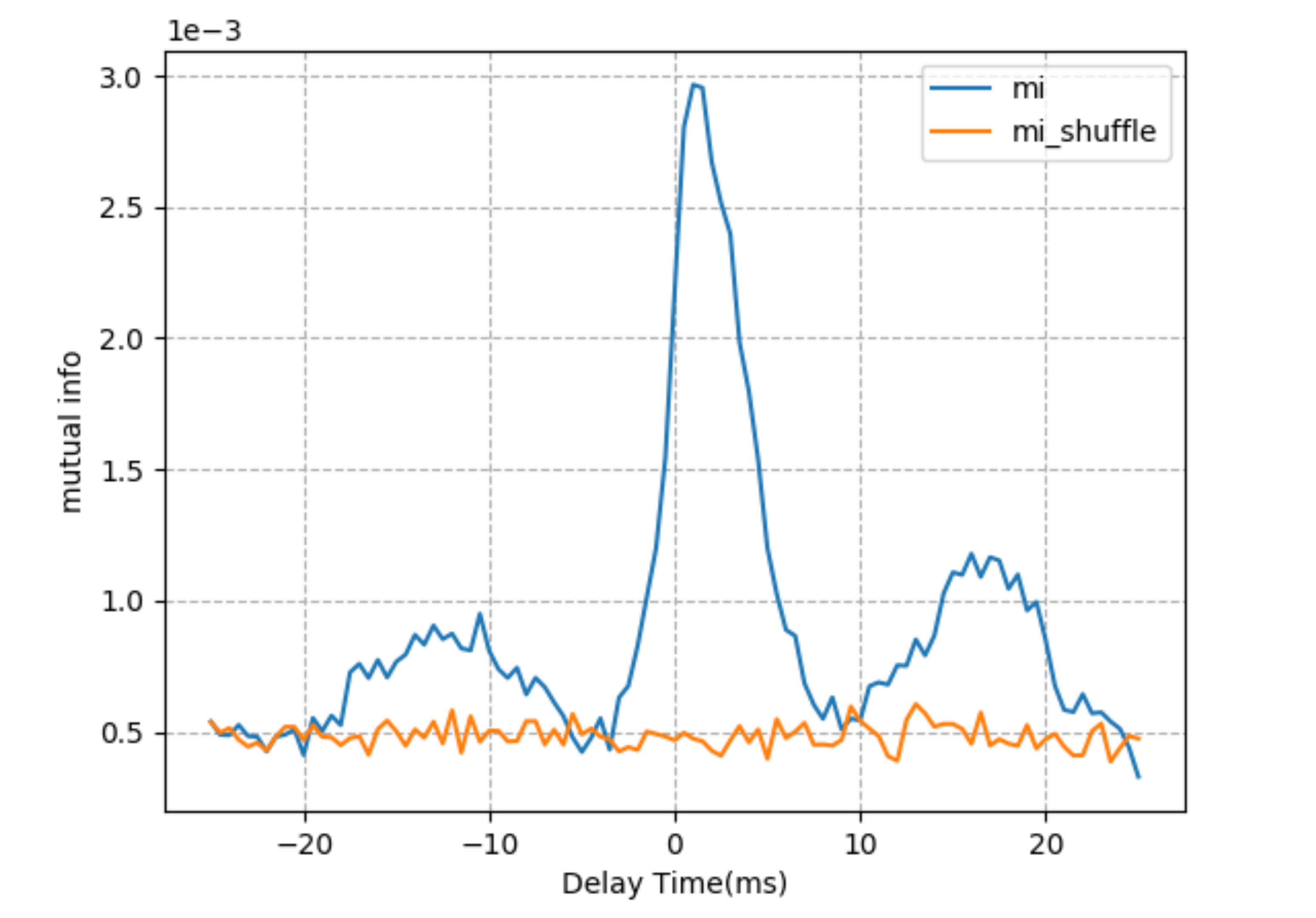


Possible Guesses

- Poisson process
- Data length
- Oscillating frequencies
- Intrinsic neuronal dynamics
- Measuring site
- Neuron numbers in population

Identical Setting with different Poisson Seed

Results

Parameters:

Neuron number = 100

Connecting Probability = 0.1

Synaptic Strength (E/I) = 8e-3(0.8mV)

Poisson Rate = 2 kHz

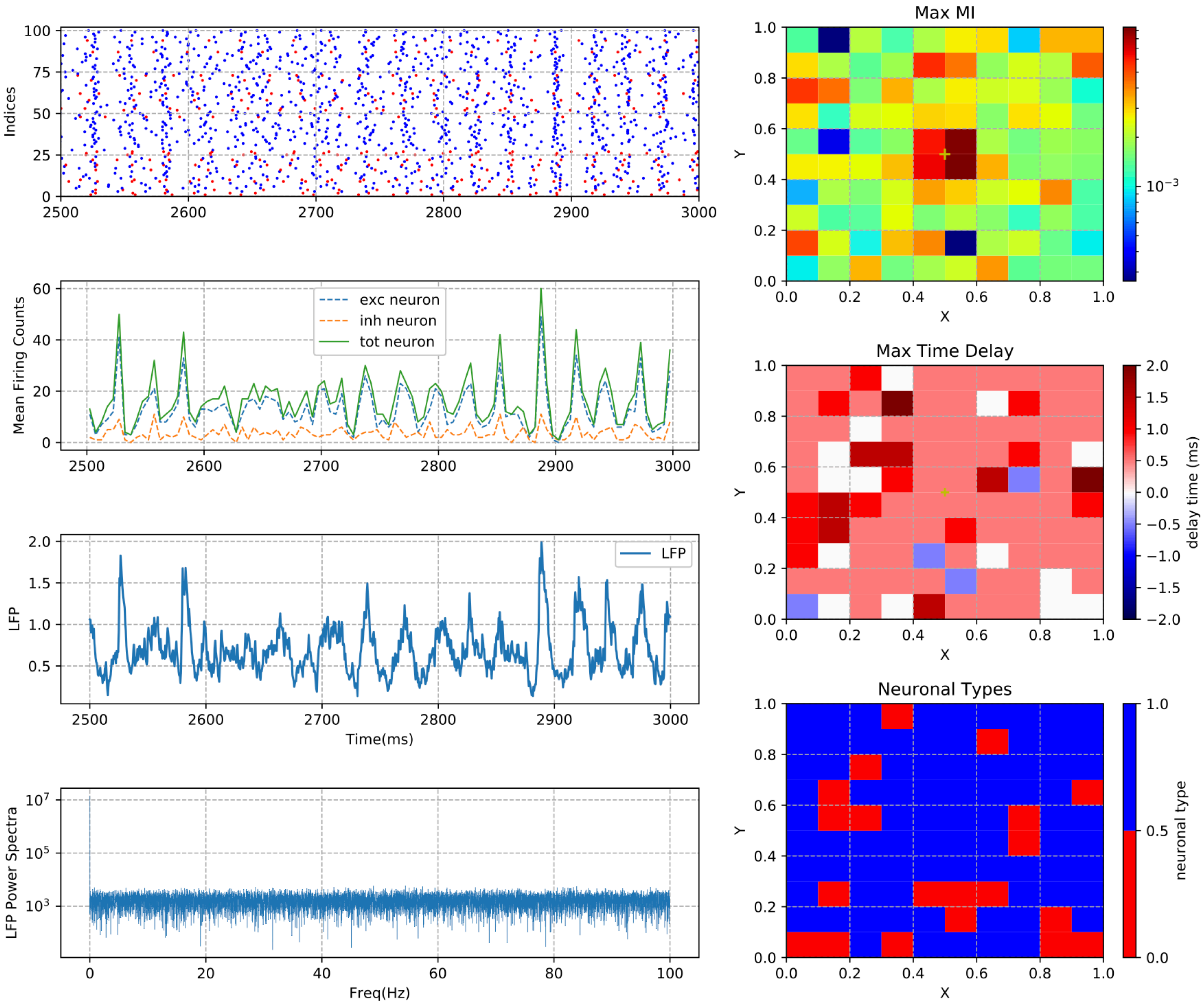
Poisson Strength = 3.5e-3(0.35mV)

Simulation Time = 1e7 ms

Electrode Position = (0.5, 0.5)

Comments:

- The results does't change as we increase the length of data.



Results

Parameters:

Neuron number = 100

Connecting Probability = 0.1

Synaptic Strength (E/I) = 8e-3(0.8mV)

Poisson Rate = 2 kHz

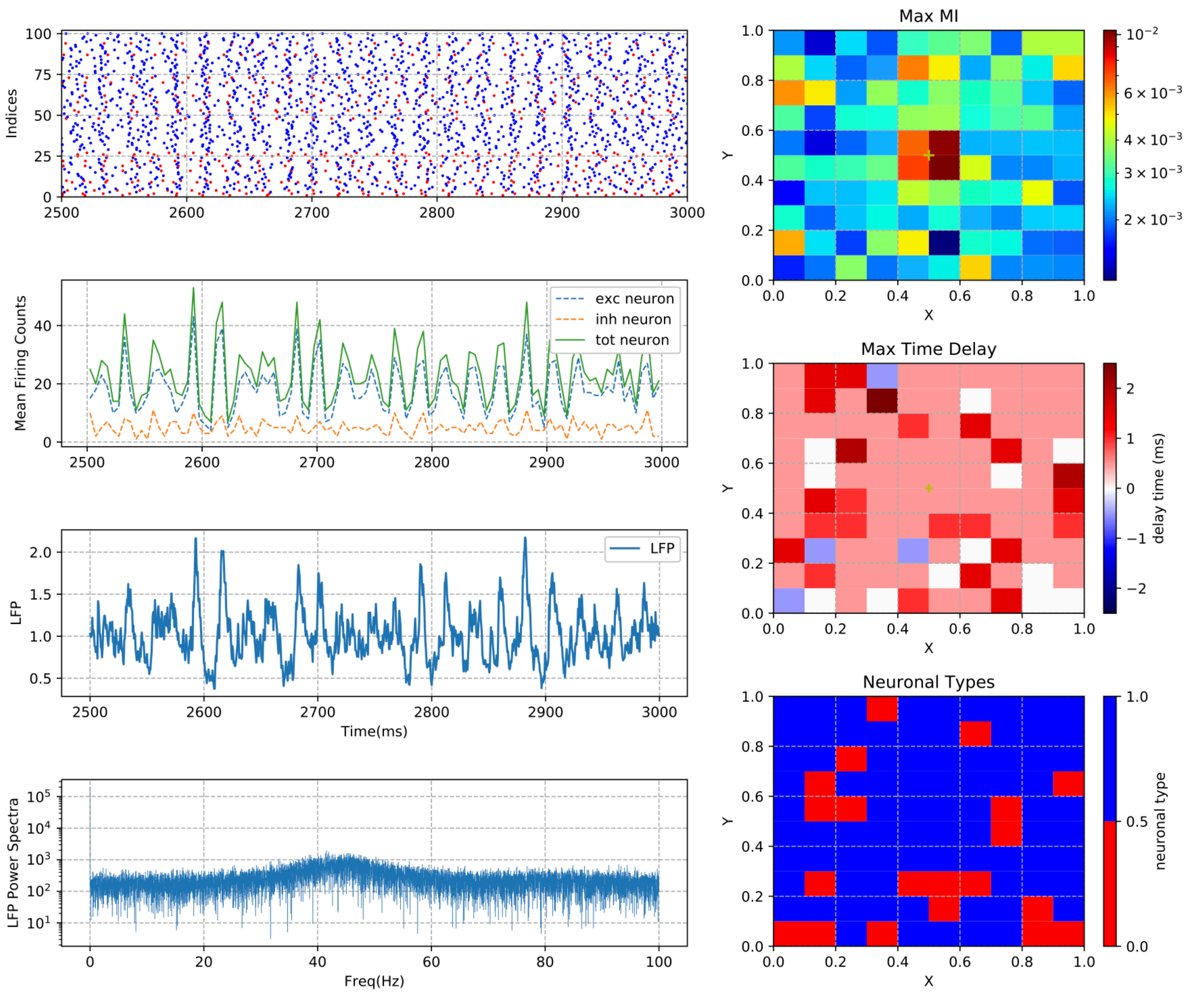
Poisson Strength = 4e-3(0.4mV)

Simulation Time = 1e5 ms

Electrode Position = (0.5, 0.5)

Comments:

- Neurons are synchronized
- The dominant frequency of LFP is around 43 Hz
- All neurons reach the LFP phase lock states with proceeding phase.



Results

Parameters:

Neuron number = 100

Connecting Probability = 0.15

Synaptic Strength (E/I) = 8e-3(0.8mV)

Poisson Rate = 2 kHz

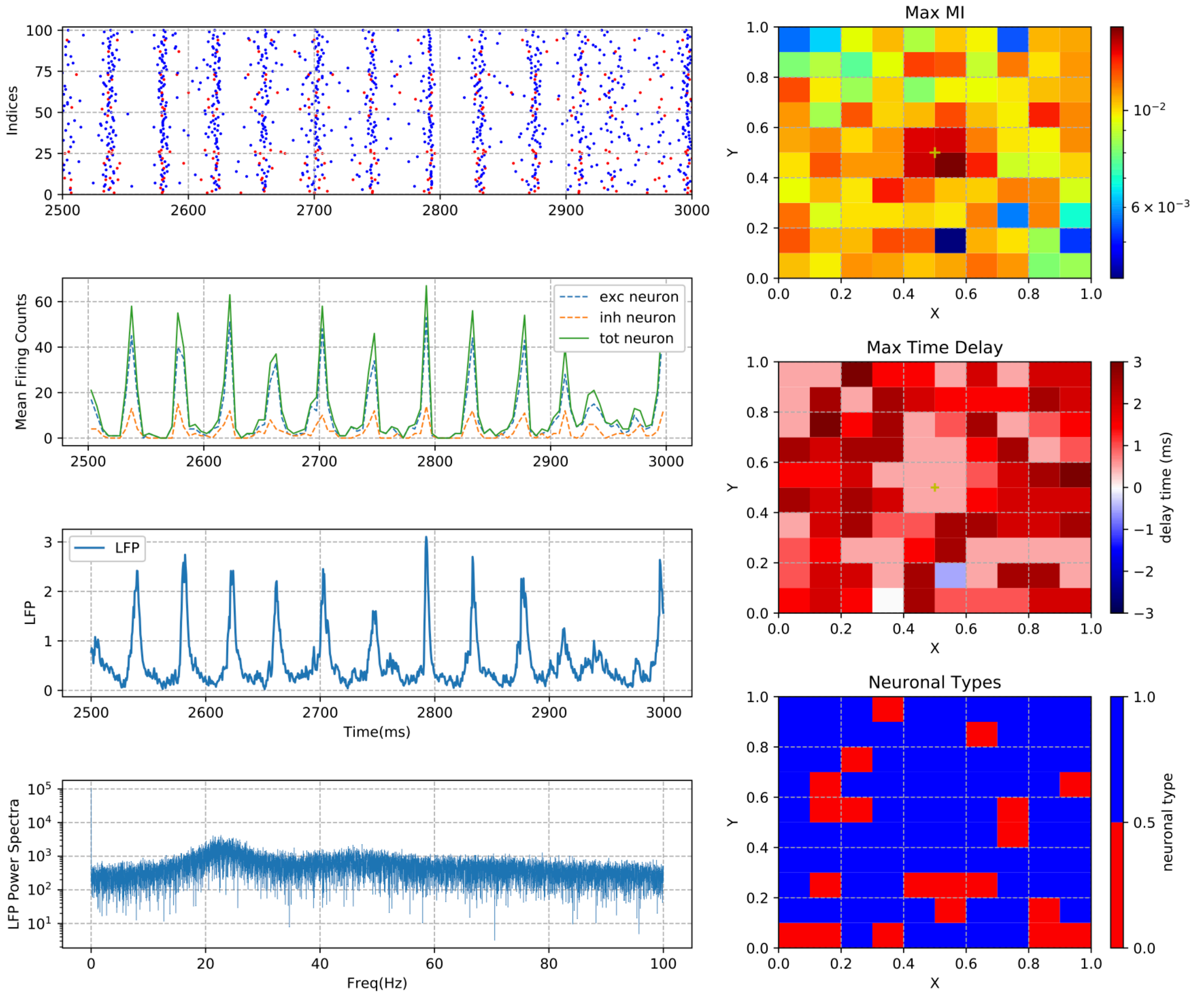
Poisson Strength = 3e-3(0.3mV)

Simulation Time = 1e5 ms

Electrode Position = (0.5, 0.5)

Comments:

- Neurons are highly synchronized
- The dominant frequency of LFP is around 23 Hz
- All neurons reach the LFP phase lock states with proceeding phase.



Results

Parameters:

Neuron number = 400

Connecting Probability = 0.1

Synaptic Strength (E/I) = 2e-3(0.2mV)

Poisson Rate = 2 kHz

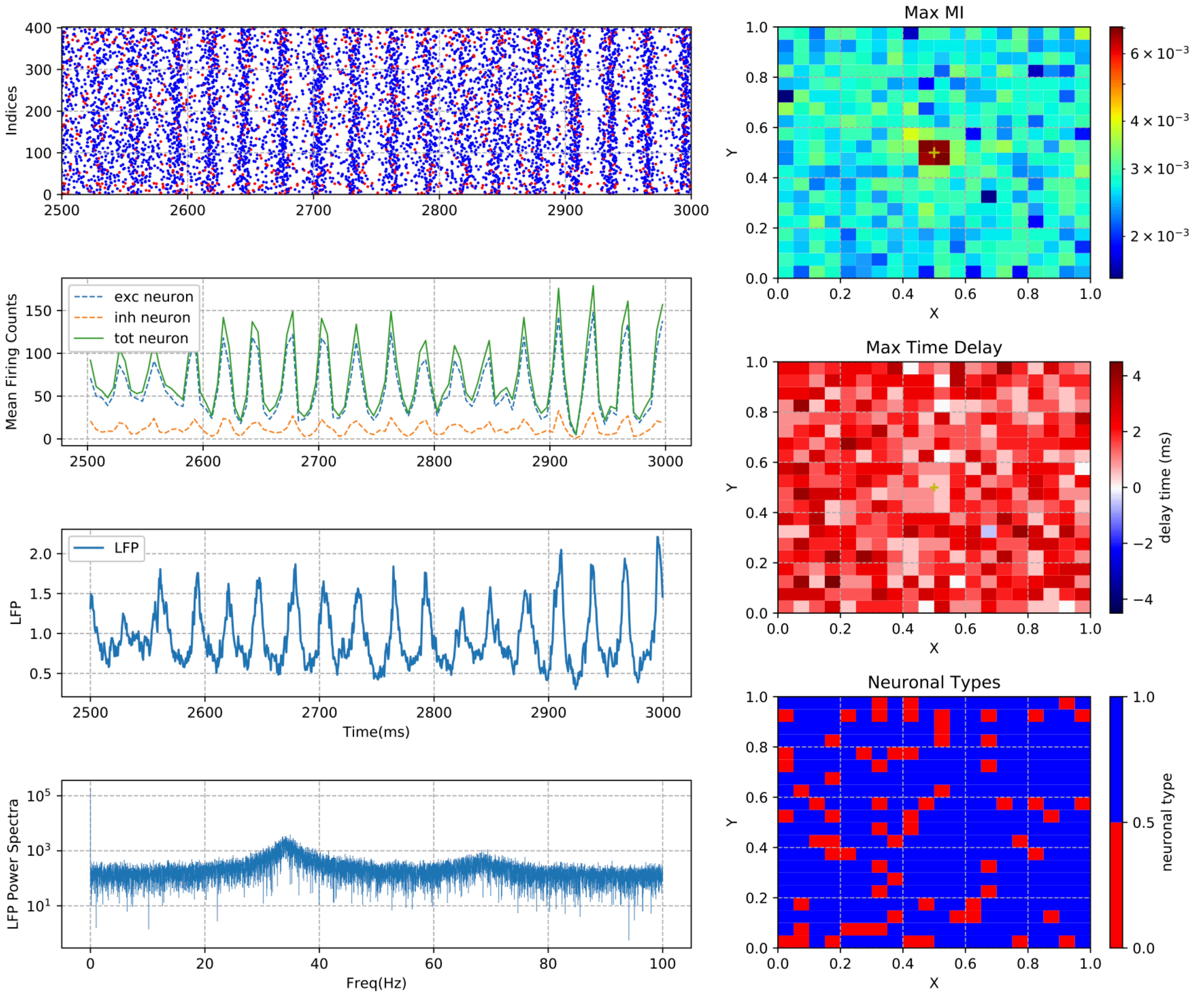
Poisson Strength = 3.5e-3(0.35mV)

Simulation Time = 1e5 ms

Electrode Position = (0.5, 0.5)

Comments:

- Neurons are synchronized
- The dominant frequency of LFP is around 34 Hz
- All neurons reach the LFP phase lock states with proceeding phase.



Results

Parameters:

Neuron number = 100

Connecting Probability = 0.1

Synaptic Strength (E/I) = 8e-3(0.8mV)

Poisson Rate = 2 kHz

Poisson Strength = 3.5e-3(0.35mV)

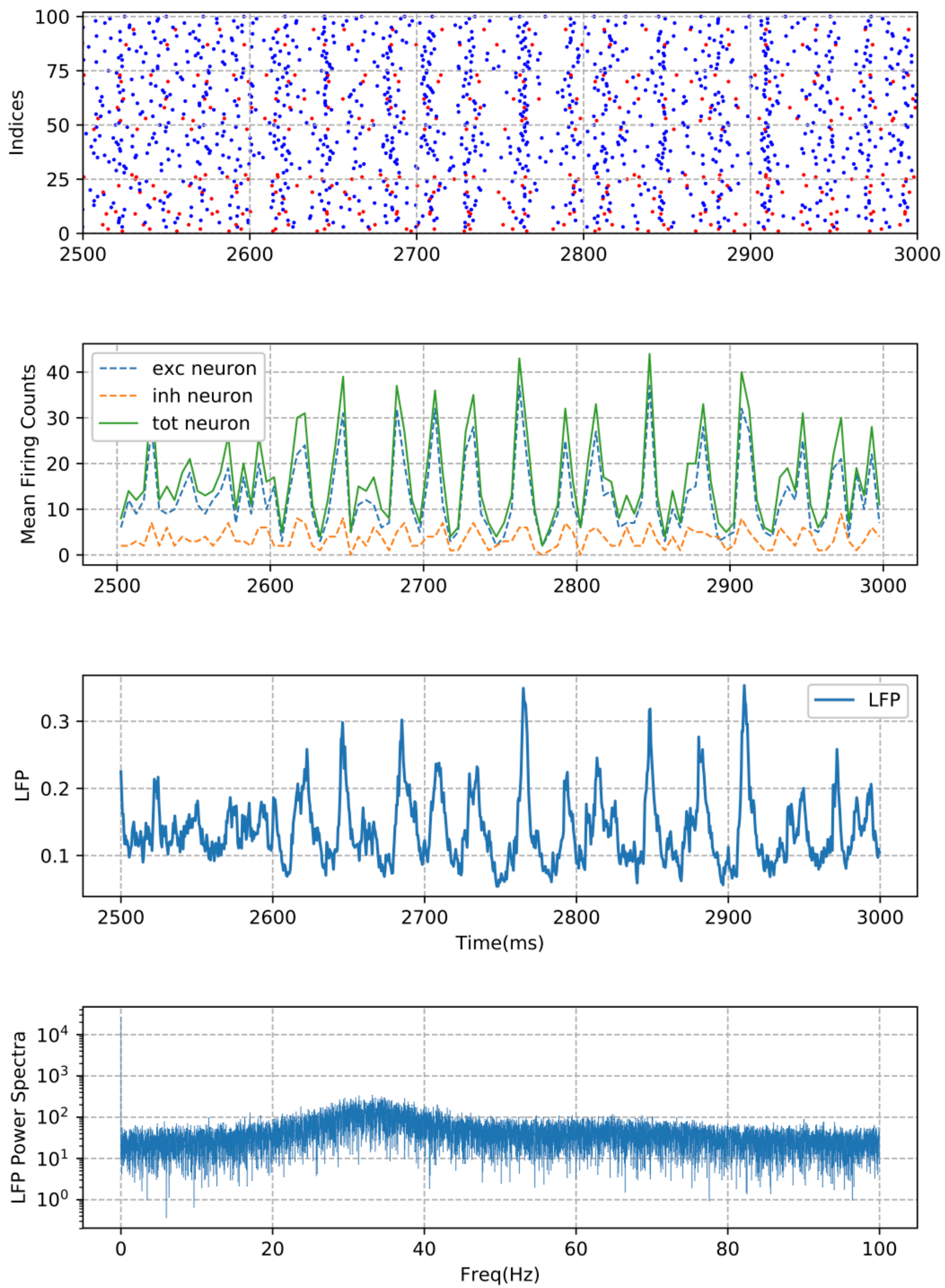
Simulation Time = 1e5 ms

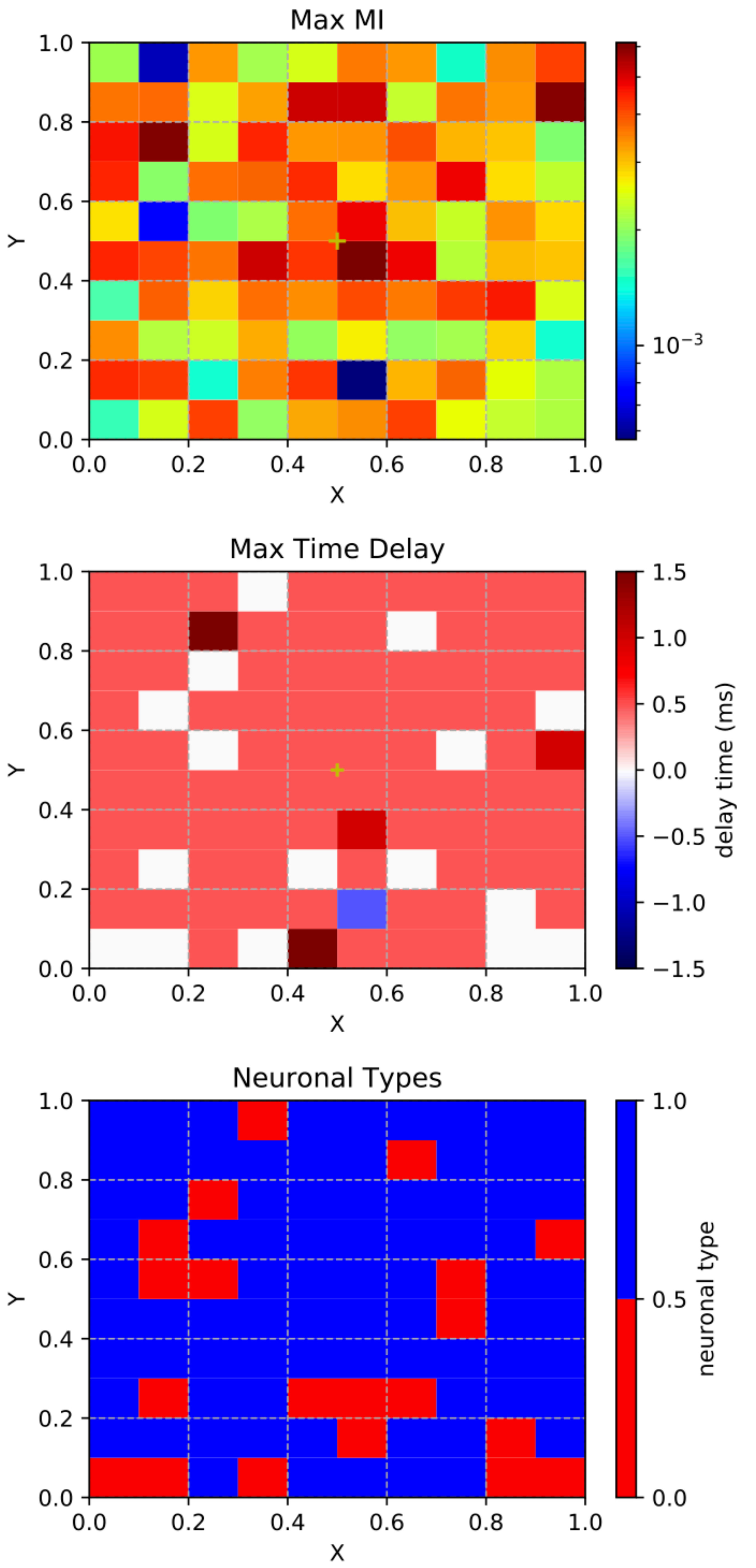
Electrode Position = (0.5, 0.5)

Current Monopole Model

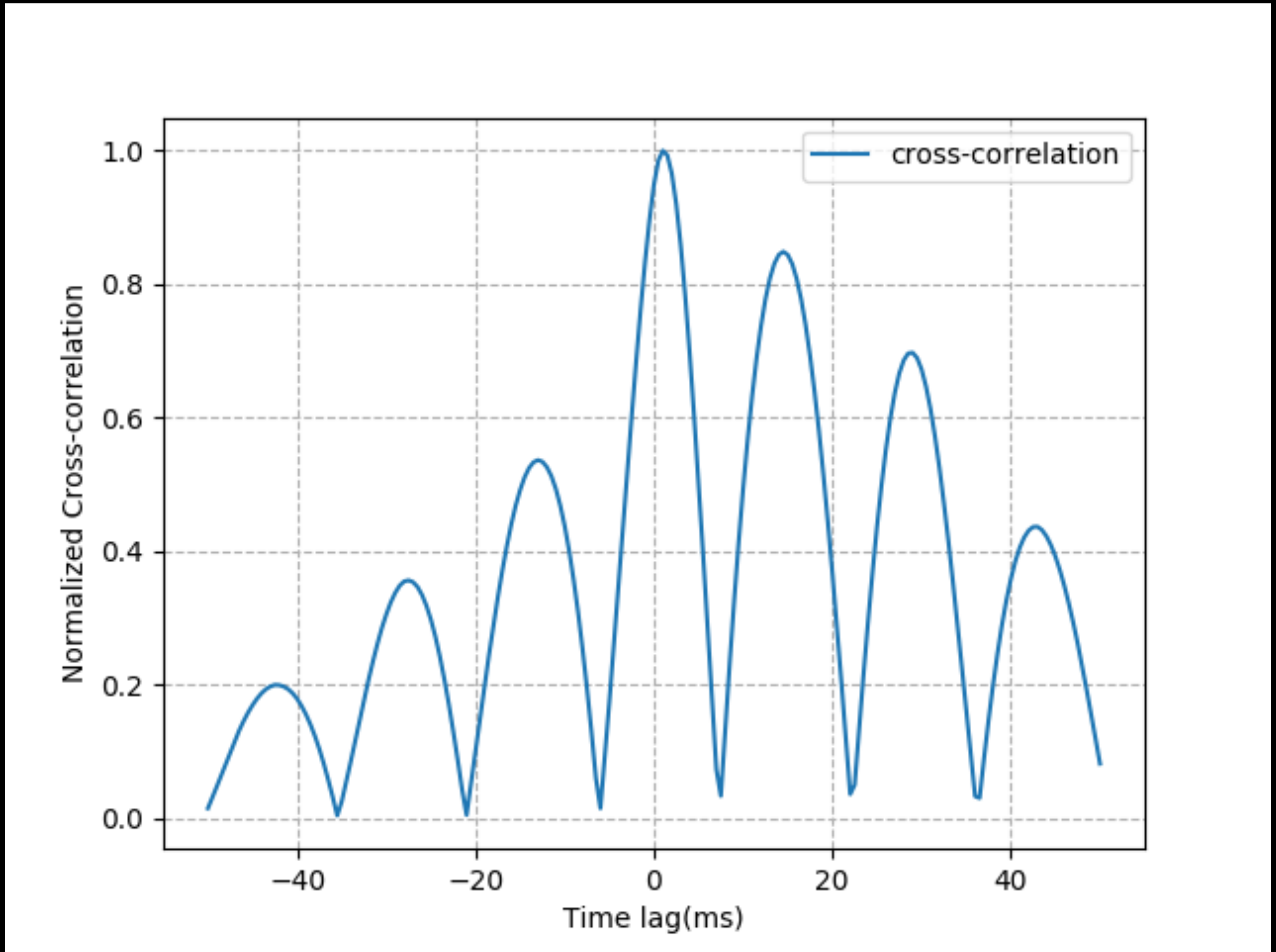
Comments:

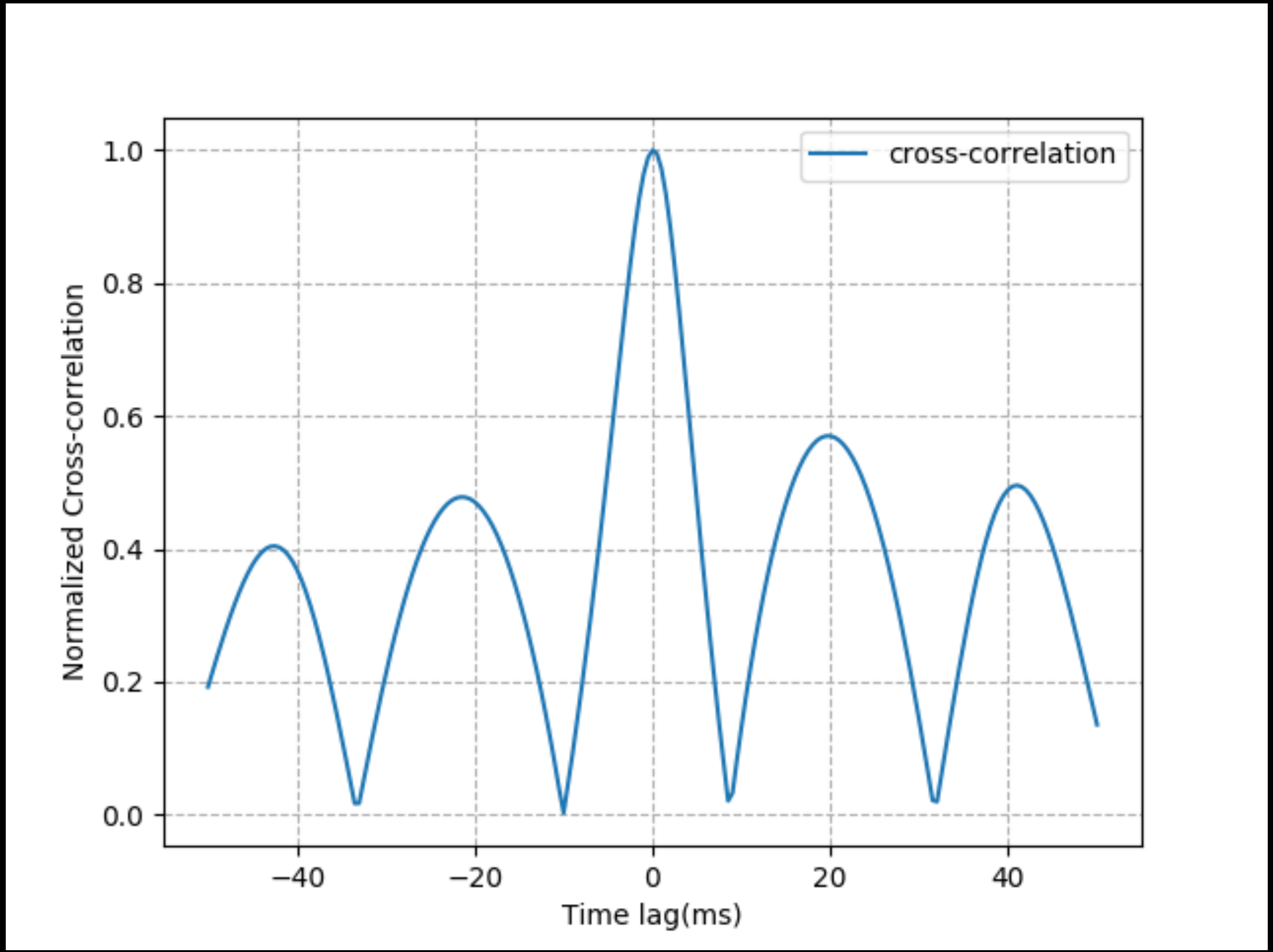
- LFP has larger spatial reach in current monopole model





Cross-correlation between Exc. and Inh. Neurons





Results

Parameters:

Neuron number = 100

Connecting Probability = 0.1

Synaptic Strength (E/I) = 8e-3(0.8mV)

Poisson Rate = 2 kHz

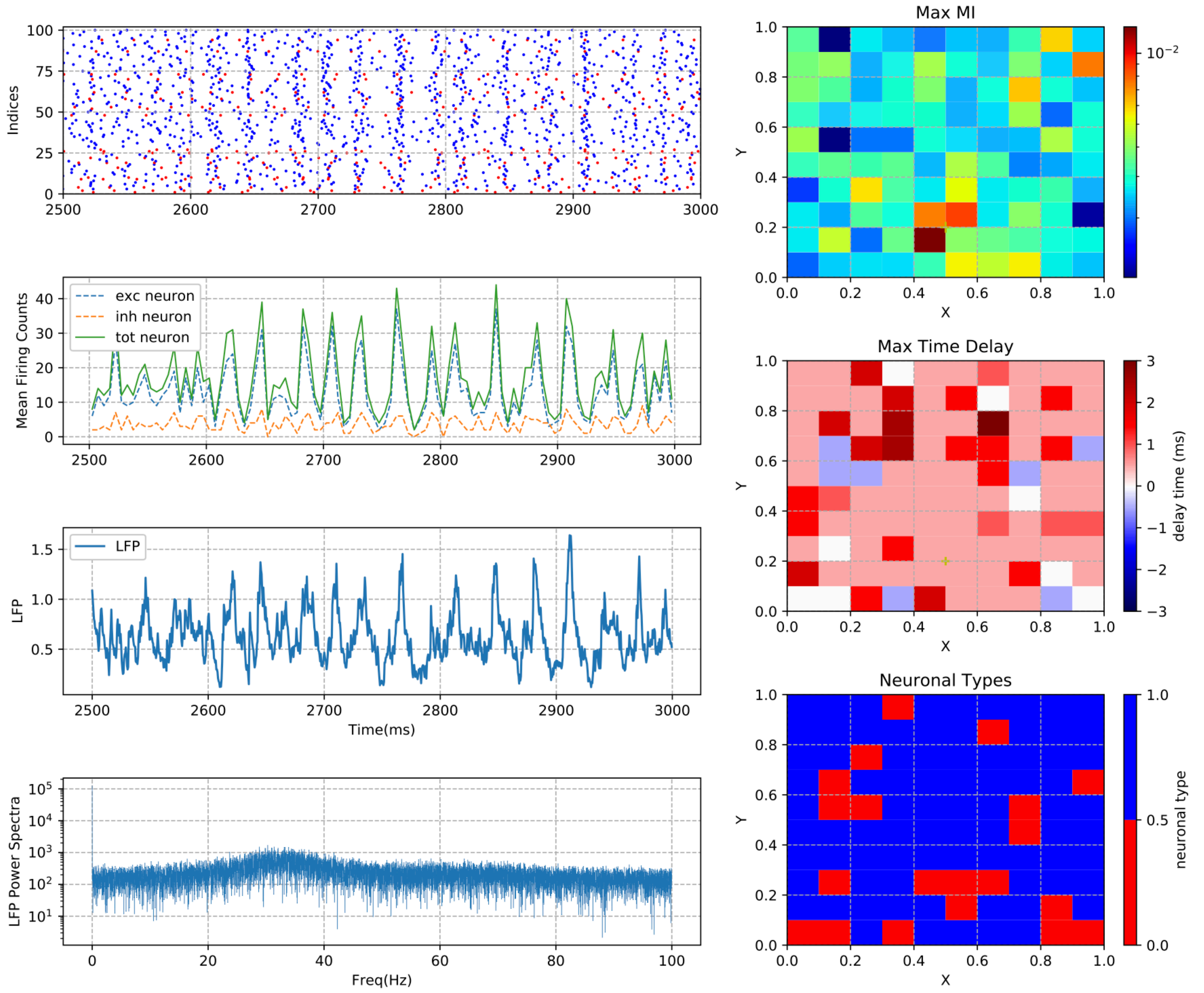
Poisson Strength = 3.5e-3(0.35mV)

Simulation Time = 1e5 ms

Electrode Position = (0.5, 0.2)

Comments:

- Inh. neurons near the electrode present positive delays in TDMI calculation



Synchronous State

Tuning model parameters

Results

Parameters:

Neuron number = 100

Connecting Probability = 0.1

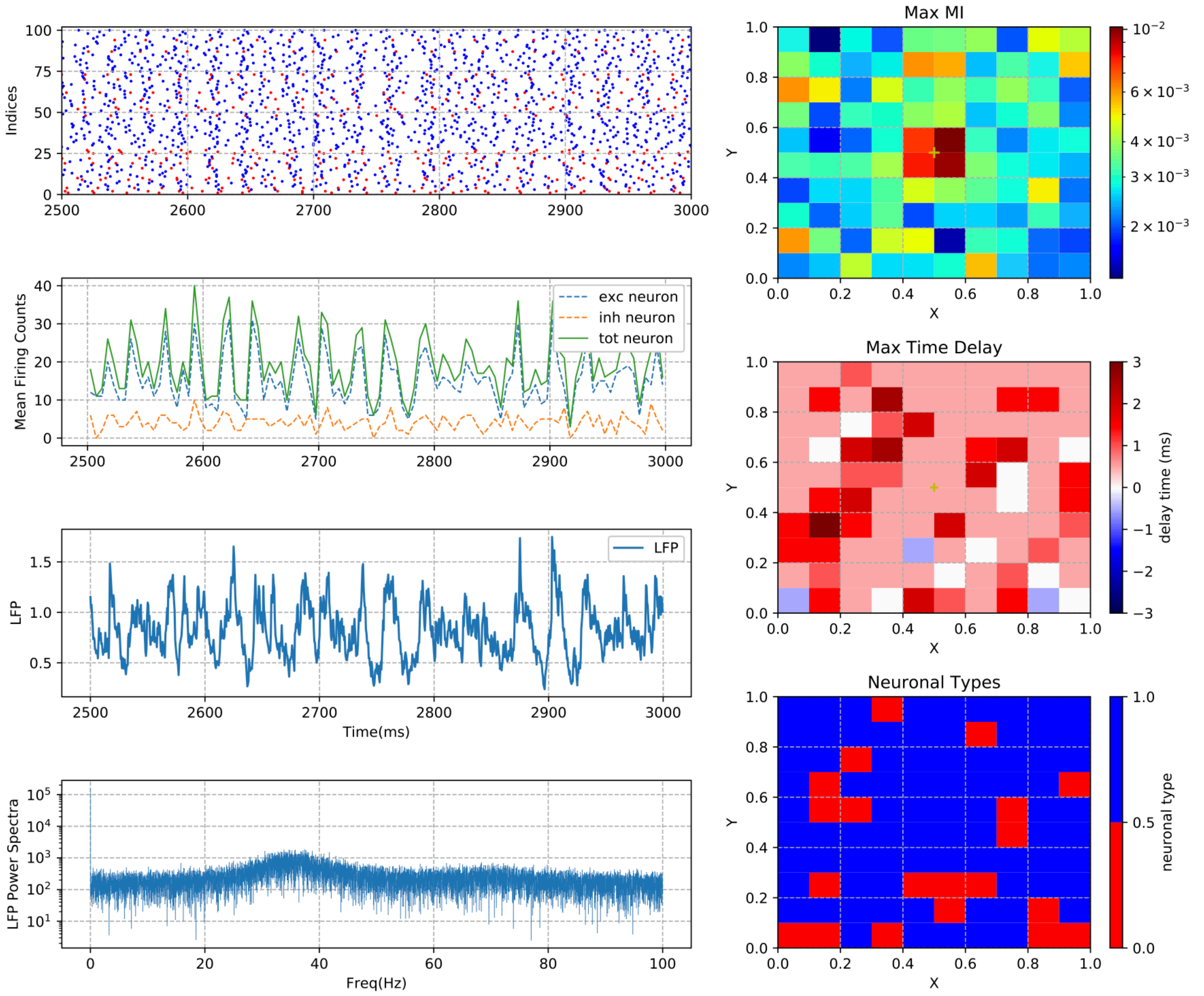
Synaptic Strength (E/I) = 8e-3(0.8mV)

Poisson Rate = 2 kHz

Poisson Strength = 3.5e-3(0.35mV)

Simulation Time = 1e5 ms

Electrode Position = (0.5, 0.5)



Comments:

- Set the time constant of exc. and inh. conductance identical, 2 ms.

Results

Parameters:

Neuron number = 100

Connecting Probability = 0.2/0.1

Synaptic Strength (E/I) = 8e-3(0.8mV)

Poisson Rate = 2 kHz

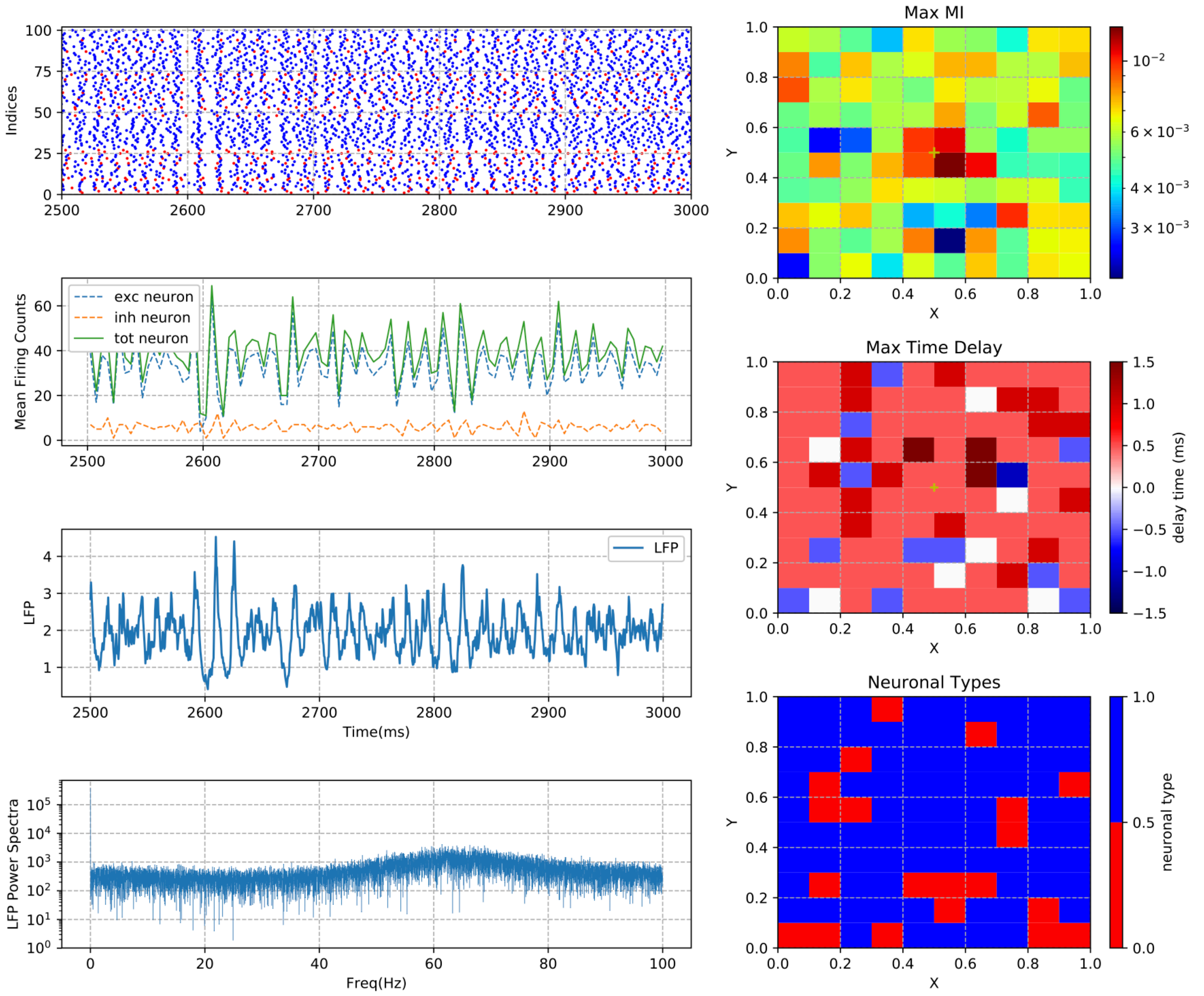
Poisson Strength = 3.5e-3(0.35mV)

Simulation Time = 1e5 ms

Electrode Position = (0.5, 0.5)

Comments:

- Set the time constant of exc. and inh. conductance identical, 2 ms.
- Set the reversal potential of inh. synapse equal to -14/3



Asynchronous State

Result

Parameters:

Neuron number = 100

Connecting Probability = 0.1

Synaptic Strength (E/I) = 3e-3(0.3mV)

Poisson Rate = 2 kHz

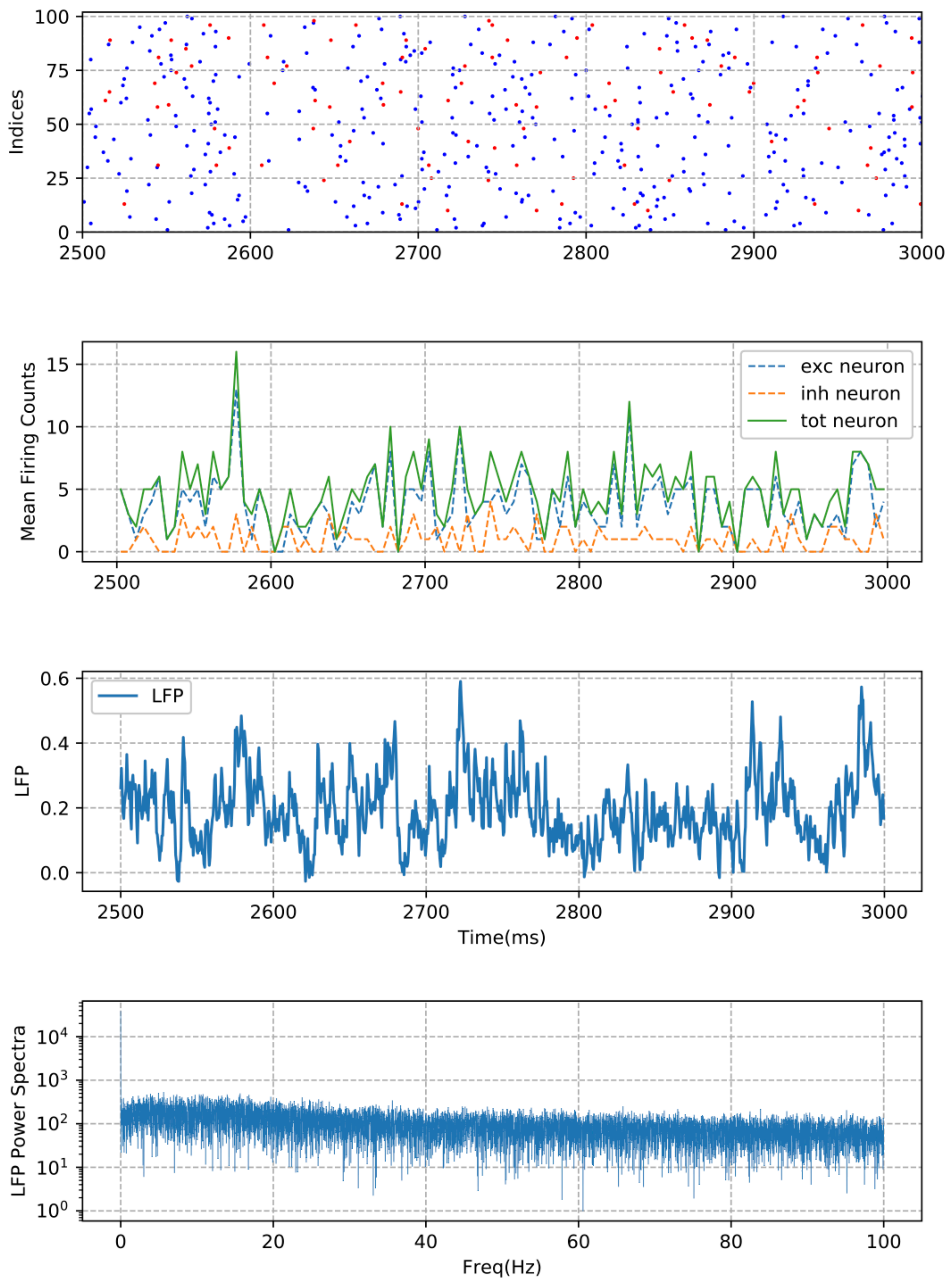
Poisson Strength = 3e-3(0.3mV)

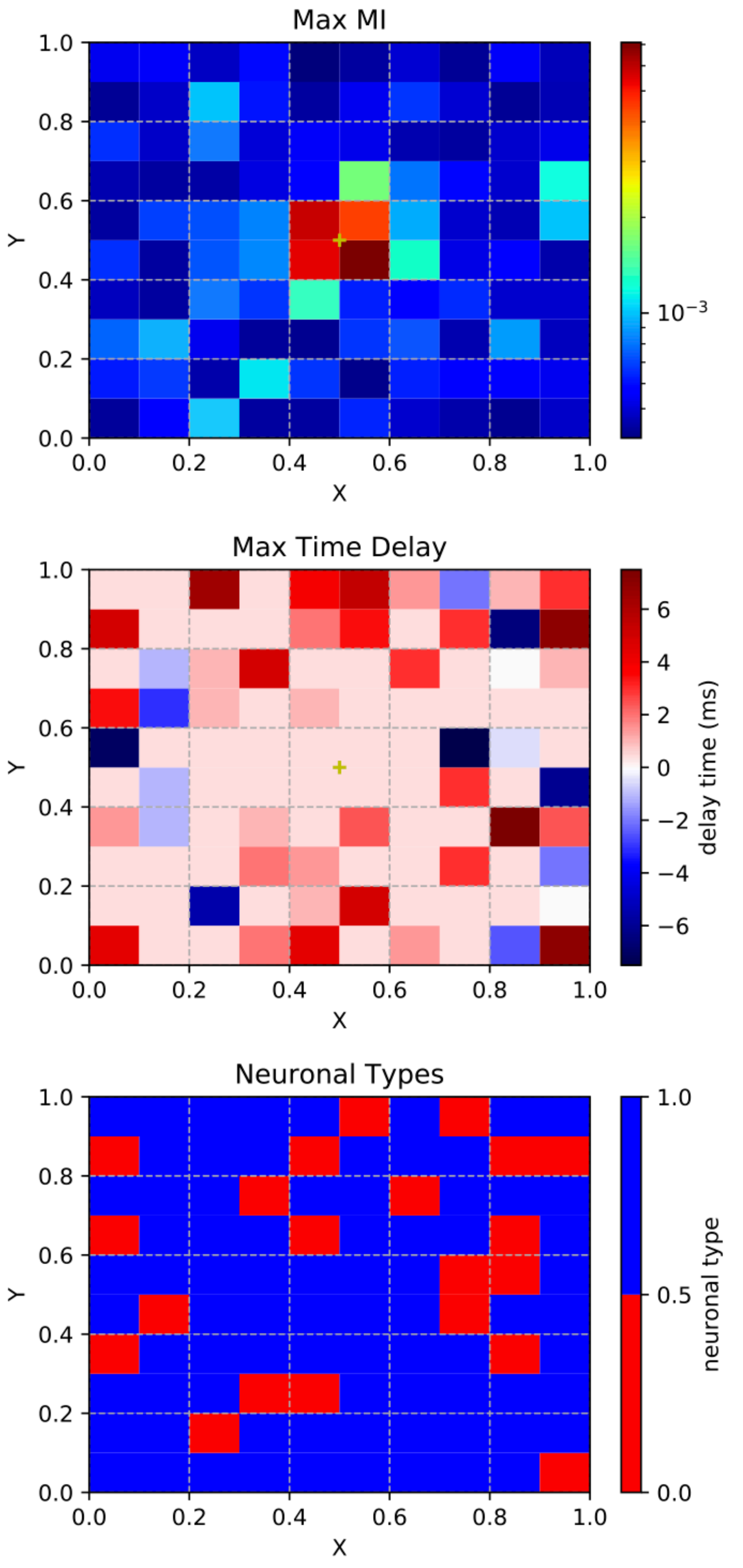
Simulation Time = 1e5 ms

Electrode Position = (0.5, 0.5)

Comments:

- Neurons are synchronized
- LFP is driven by the localized neurons around the electrode
- Exc. neurons always have proceeding phase respect to LFP, while phases of some inh. neurons fall behind.





Result

Parameters:

Neuron number = 100

Connecting Probability = 0.1

Synaptic Strength (E/I) = 3e-3(0.3mV)

Poisson Rate = 2 kHz

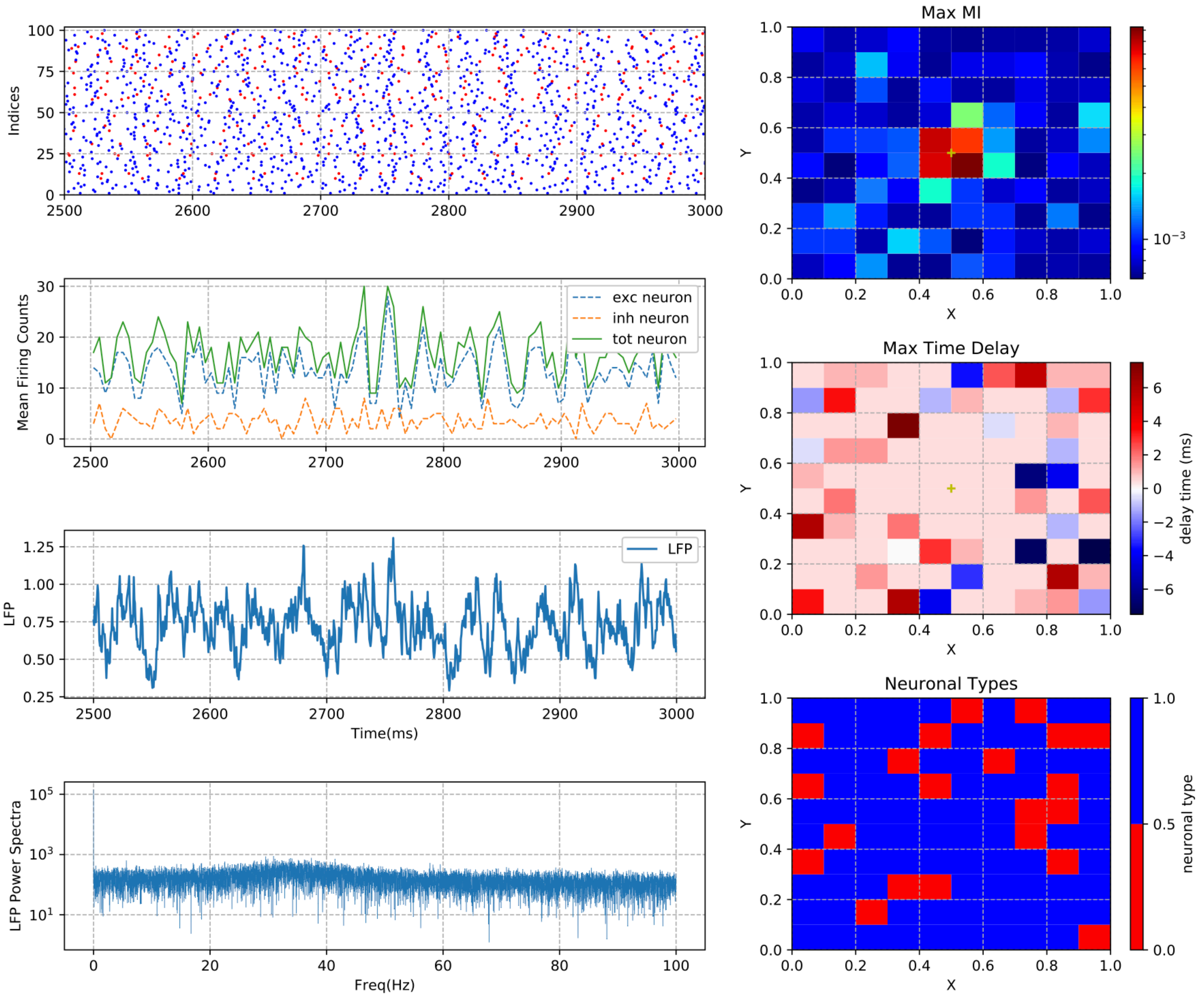
Poisson Strength = 4e-3(0.4mV)

Simulation Time = 1e5 ms

Electrode Position = (0.5, 0.5)

Comments:

- Neurons are asynchronized
- Most neurons reach the phase lock states with LFP.
- Exc. neurons always have proceeding phase respect to LFP, while phases of some inh. neurons fall behind.



Synchronous State

synaptic interaction with synaptic delay

Result

Parameters:

Neuron number = 100

Connecting Probability = 0.1

Synaptic Strength (E/I) = 8e-3(0.8mV)

Spike Transmitting Speed = 0.3

Poisson Rate = 2 kHz

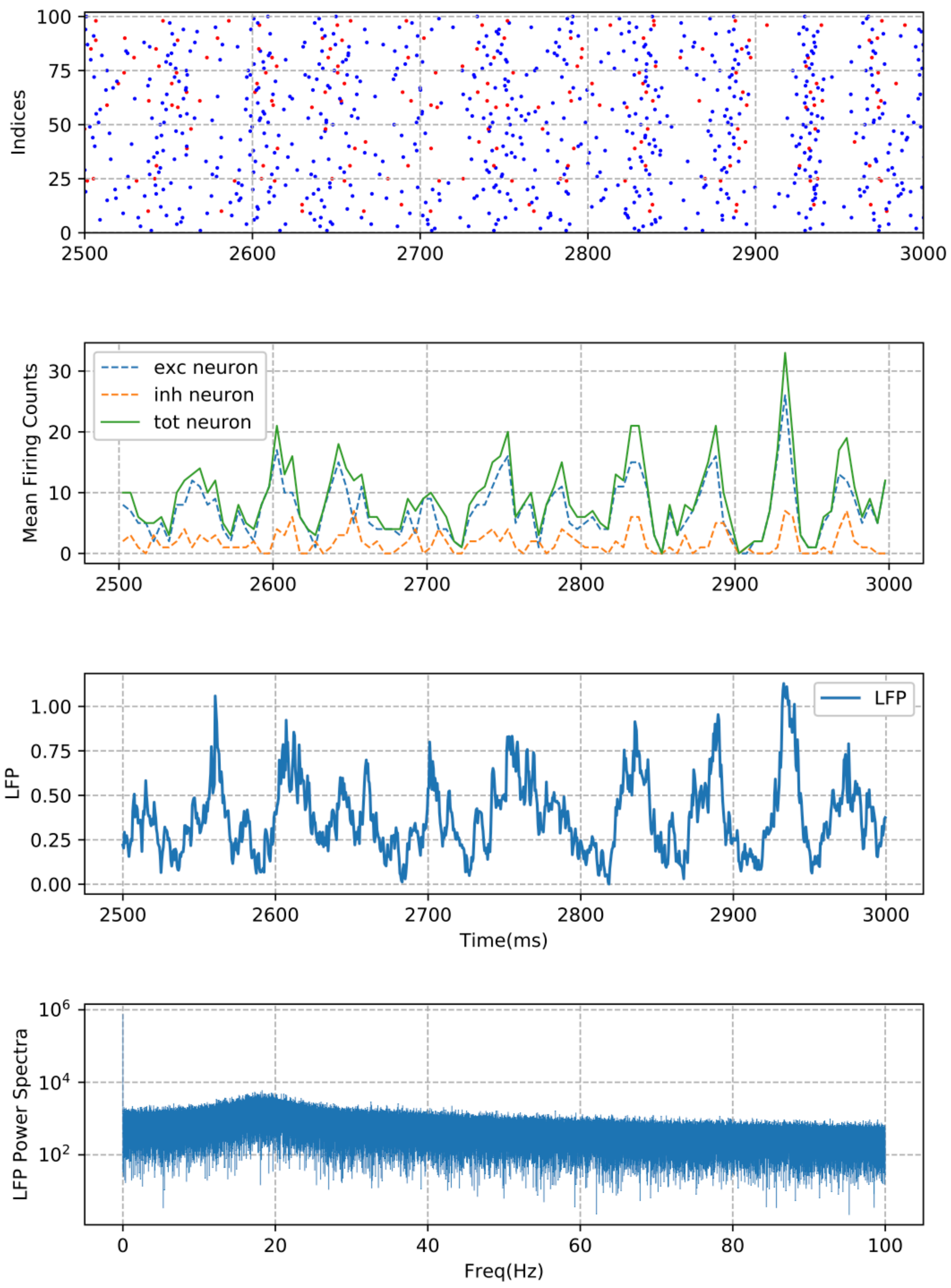
Poisson Strength = 3e-3(0.3mV)

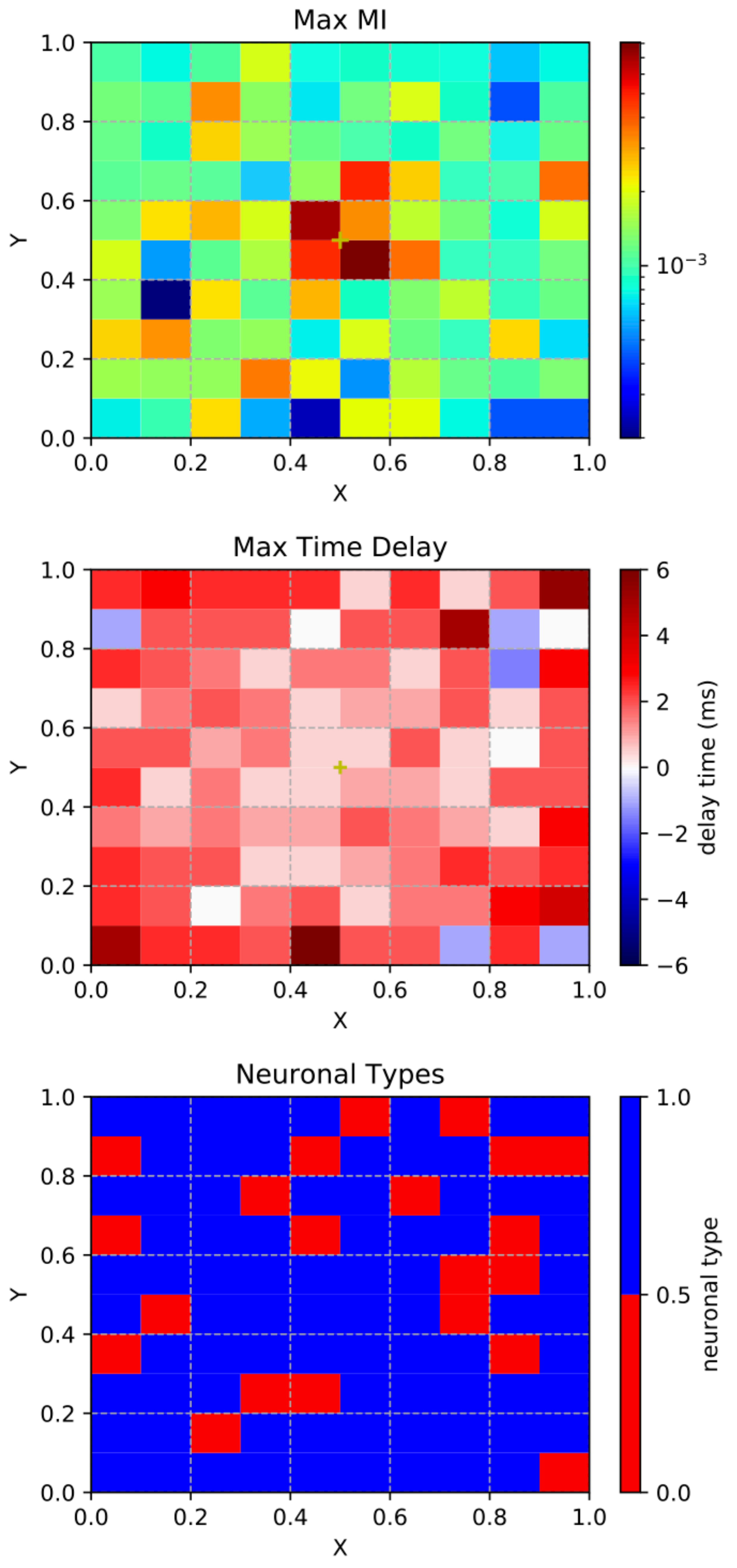
Simulation Time = 1e6 ms

Electrode Position = (0.5, 0.5)

Comments:

- Neurons are still synchronized
- The dominant frequency of LFP is around 19 Hz
- Most neurons reach the phase lock states with LFP.
- Exc. neurons always have proceeding phase respect to LFP, while phases of some inh. neurons fall behind.





Result

Parameters:

Neuron number = 100

Connecting Probability = 0.1

Synaptic Strength (E/I) = 8e-3(0.8mV)

Spike Transmitting Speed = 0.3

Poisson Rate = 2 kHz

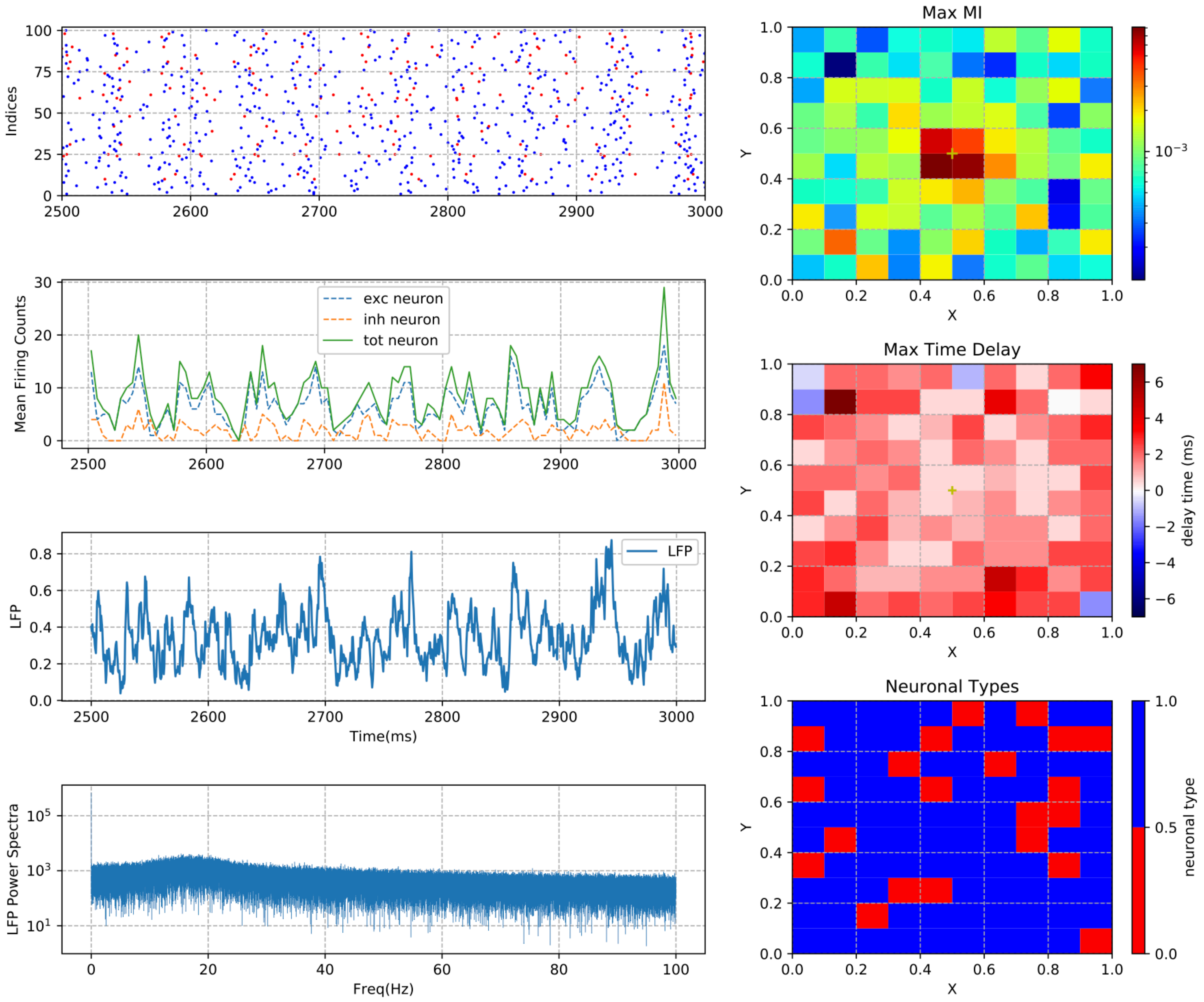
Poisson Strength = 3e-3(0.3mV)

Simulation Time = 1e6 ms

Electrode Position = (0.5, 0.5)

Comments:

- Most neurons reach the phase lock states with LFP.
- Exc. neurons always have proceeding phase respect to LFP, while phases of some inh. neurons fall behind.
- Those phenomenons are depends on network structures.



Conclusions

- In asynchronized state, LFP is only driven by few neurons neighboring the measuring electrode.
- In synchronized state, majority of neurons become LFP driving neuron.
- For LFP driving neurons, excitatory neurons have positive phase (positive time delay of max. MI in TDMI) related the LFP signal, while some of the inhibitory neurons have negative phase (with time delay around -0.5 ms). Usually those inh. neurons locate a bit far away from the electrode.
- This phenomenon results from the different behaviors in synchronized network. Exc. neurons usually process ascending phase related to inh. neurons.

Thanks you