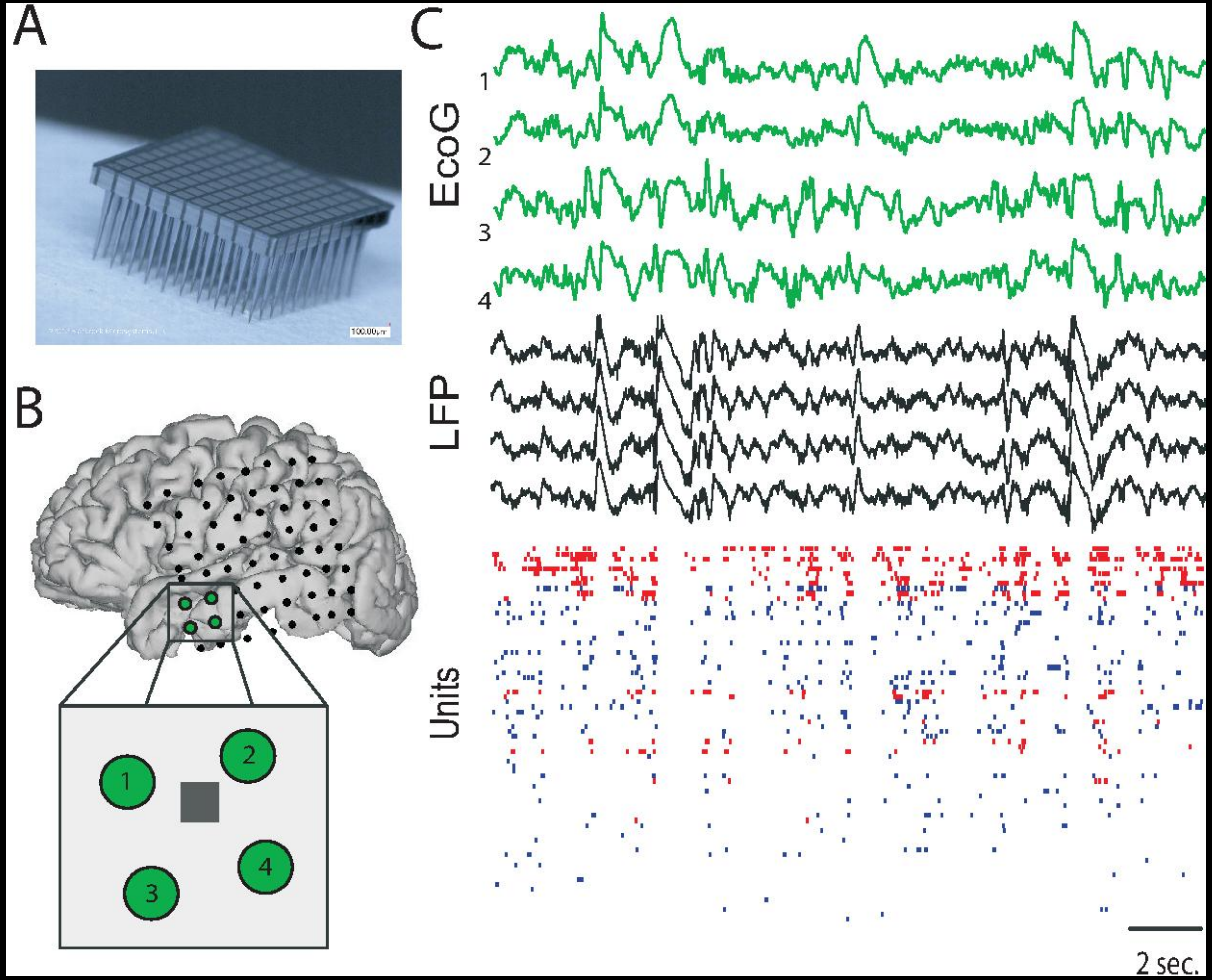
Modeling the Local Field Potential

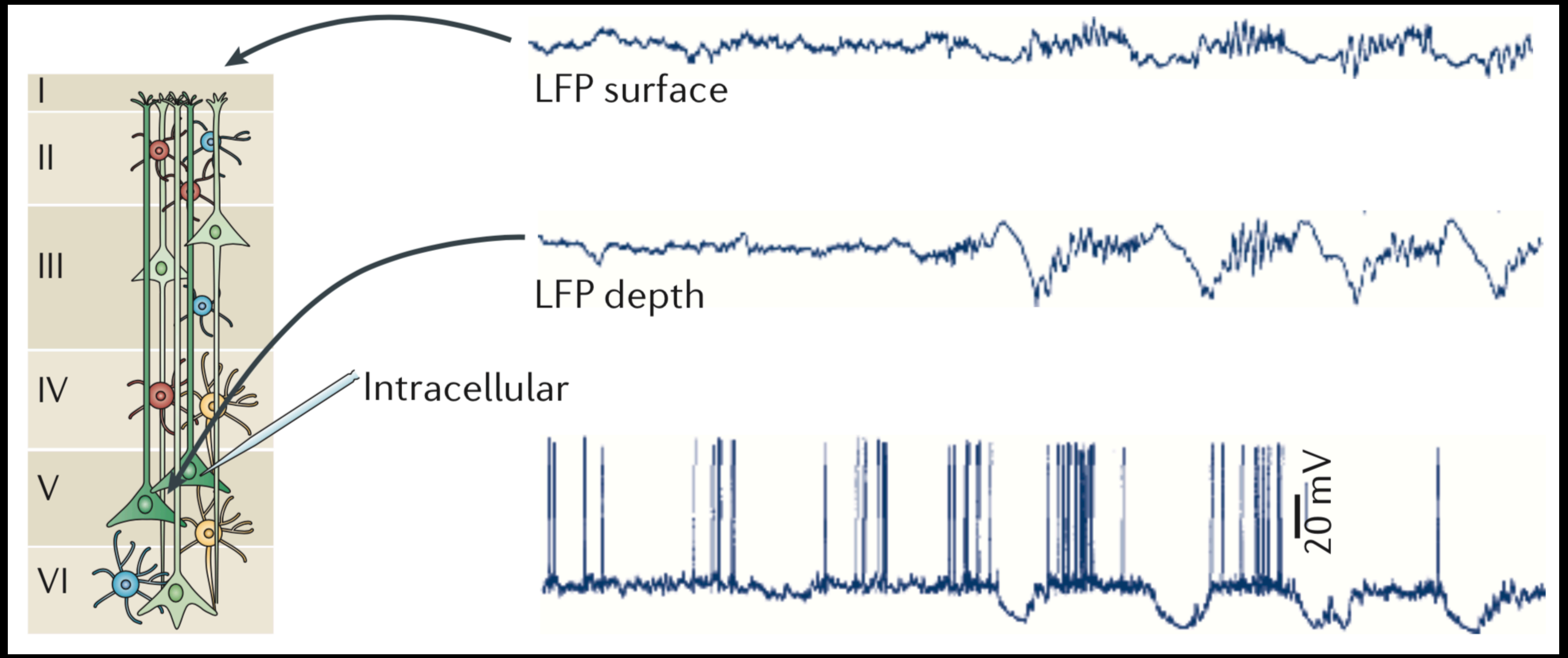
Kai Chen

Nov. 23, 2018

Local Field Potential



modified from Peyrache et al., 2012



Contreras D. & Steriade M. Cellular basis of EEG slow rhythms: a study of dynamic corticothalamic relationships. *J. Neurosci.* **51**, 604–622 (1995).

LFP record the local neuronal activity

Decay as neuron far away from the recording cite

Can record the activity of neurons in deep layer

LFPy: a tool for biophysical simulation of extracellular potentials generated by detailed model neurons

Henrik Lindén[1,2†], Espen Hagen[1†], Szymon Łęski[1,3], Eivind S. Norheim[1], Klas H. Pettersen[1,4] and Gaute T. Einevoll[1]*

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[2] *Department of Computational Biology, School of Computer Science and Communication, Royal Institute of Technology (KTH), Stockholm, Sweden*
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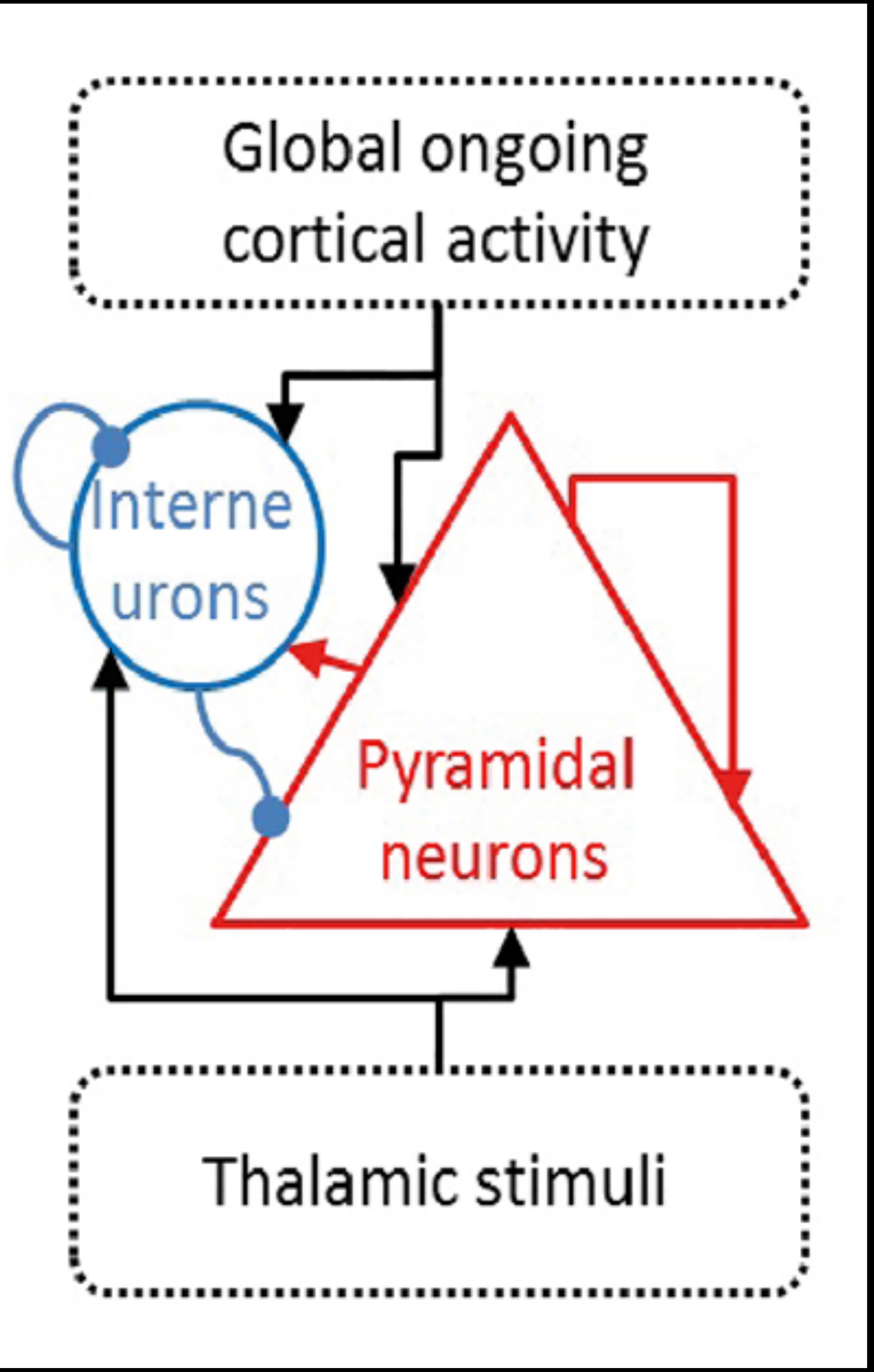
RESEARCH ARTICLE

Computing the Local Field Potential (LFP) from Integrate-and-Fire Network Models

Alberto Mazzoni[1,2☯]*, Henrik Lindén[3,4☯], Hermann Cuntz[5,6,7], Anders Lansner[4], Stefano Panzeri[2], Gaute T. Einevoll[8,9]*

Mazzoni, A., Lindén, H., Cuntz, H., Lansner, A., Panzeri, S., & Einevoll, G. T. (2015). Computing the local field potential (LFP) from integrate-and-fire network models. *PLoS computational biology*, *11*(12), e1004584.

LIF Neuronal Network



Ornstein Uhlenbeck process (OU) with zero mean

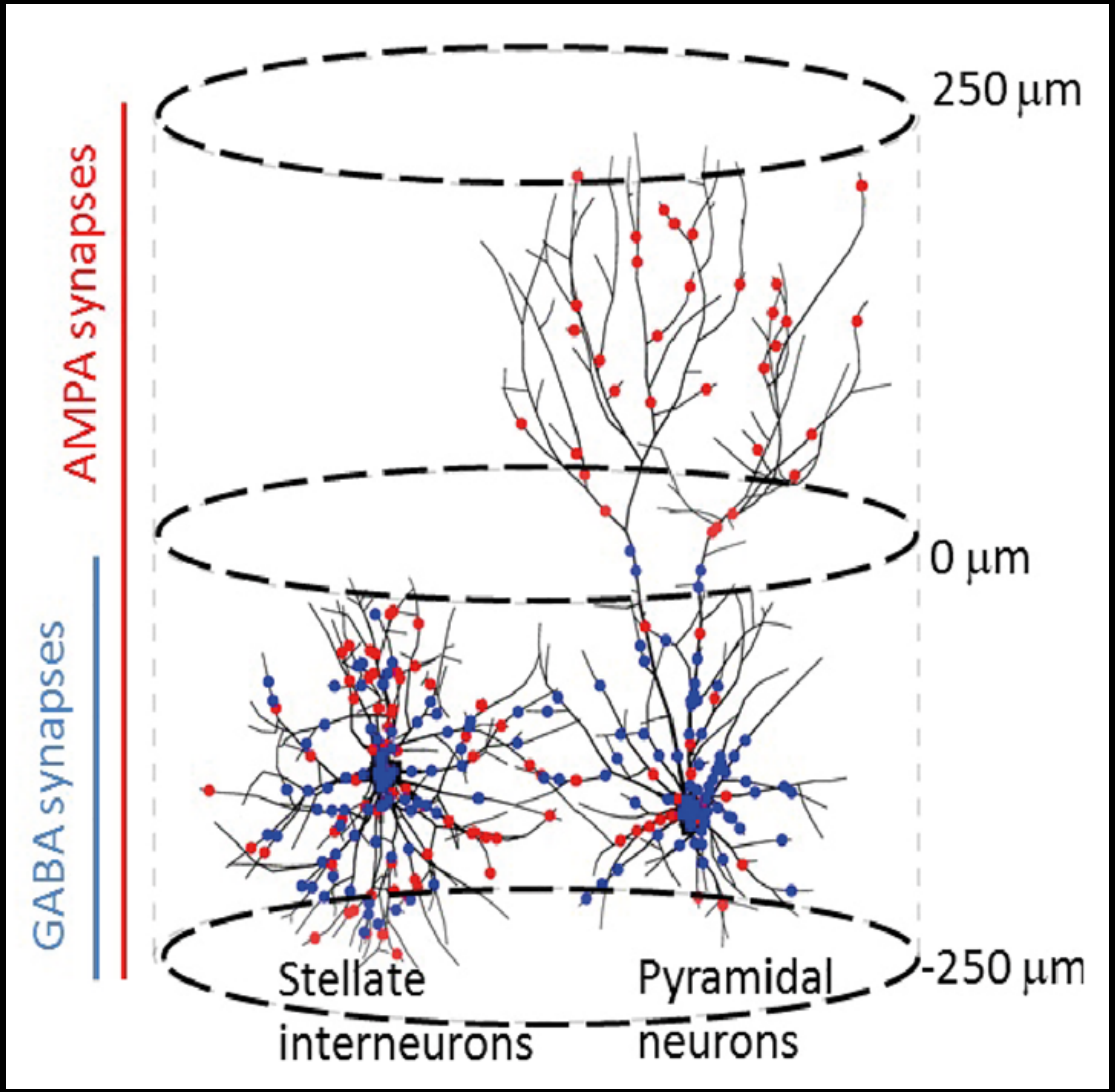
$$\tau_n \frac{d}{dt} OU = -OU + \sigma(2\tau_n)W$$

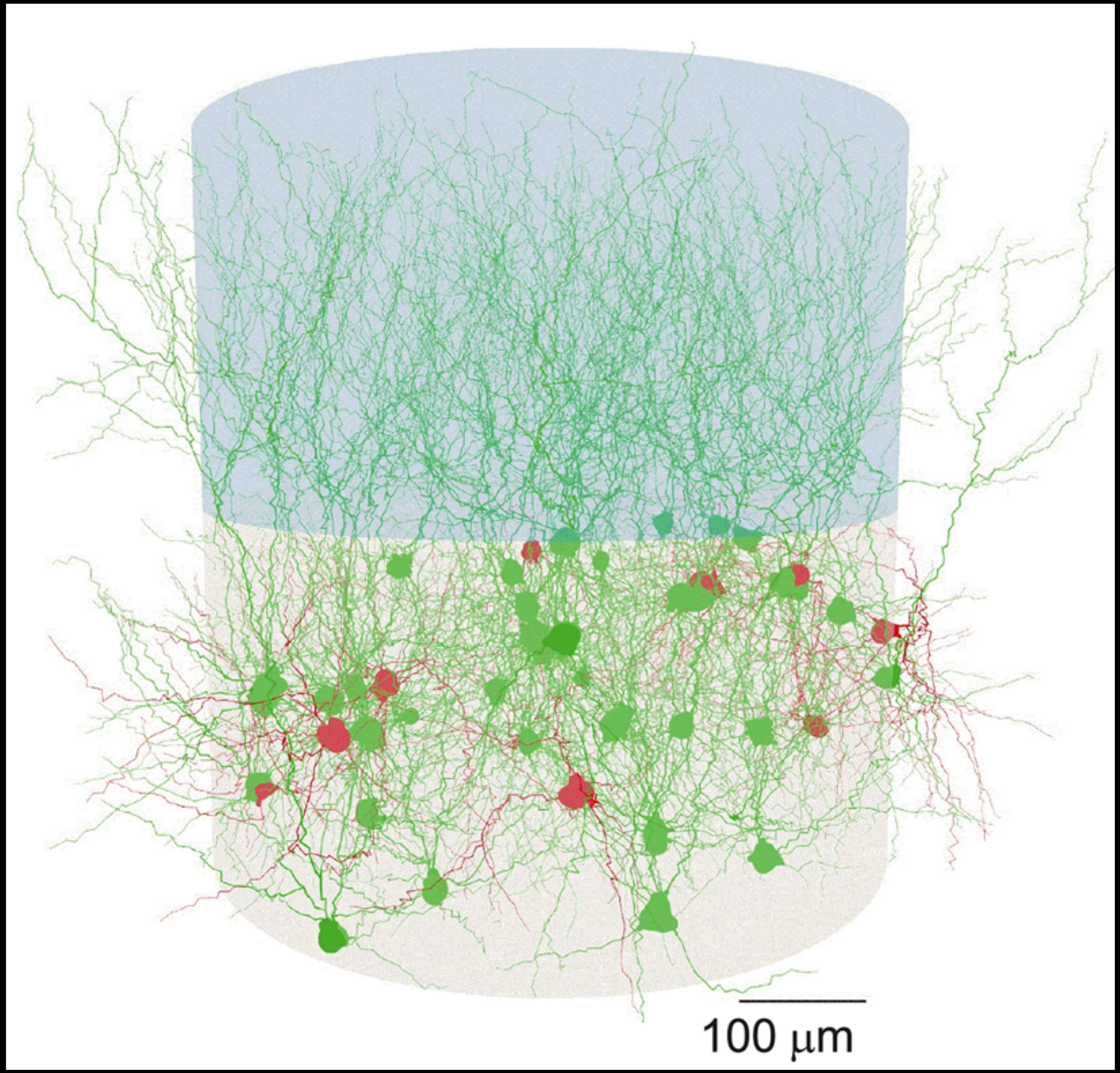
$$\tau_{dsyn} \frac{d}{dt} PSC(t) = -PSC(t) + x(t)$$

$$\tau_{rsyn} \frac{d}{dt} x(t) = -x(t) + \tau_m (J_{syn} \sum_{syn} \delta(t - t_{syn} - \tau_l))$$

Poisson process with instant input rate λ

3D Multi-compartment neuronal network





Connecting two distinct networks

Aims:

- Share same network structure
- Driven by same external drive
- Have same dynamic patterns

LIF net	3D net
4000 pyramidal 1000 interneuron	4000 pyramidal 1000 interneuron
Ongoing cortical inputs Thalamic inputs	Identical to LIF net
Neurons interact with spikes	Inject synaptic inputs according to the spike train of LIF net

- ★ Neurons randomly connect with p = 0.2 in LIF net.
- ★ Presynaptic neurons have single synaptic connection with postsynaptic neurons
- ★ NO inter-neuronal connection in 3D net
- ★ Form of synaptic interactions are identical

Form of post synaptic current in 3D network

$$PSC(t) = J_{syn}A\left[\exp\left(\frac{-\left(t-t_{syn}\right)}{\tau_{decay}}\right) - \exp\left(\frac{-\left(t-t_{syn}\right)}{\tau_{rise}}\right)\right]$$

LFP model in 3D network

Point source model

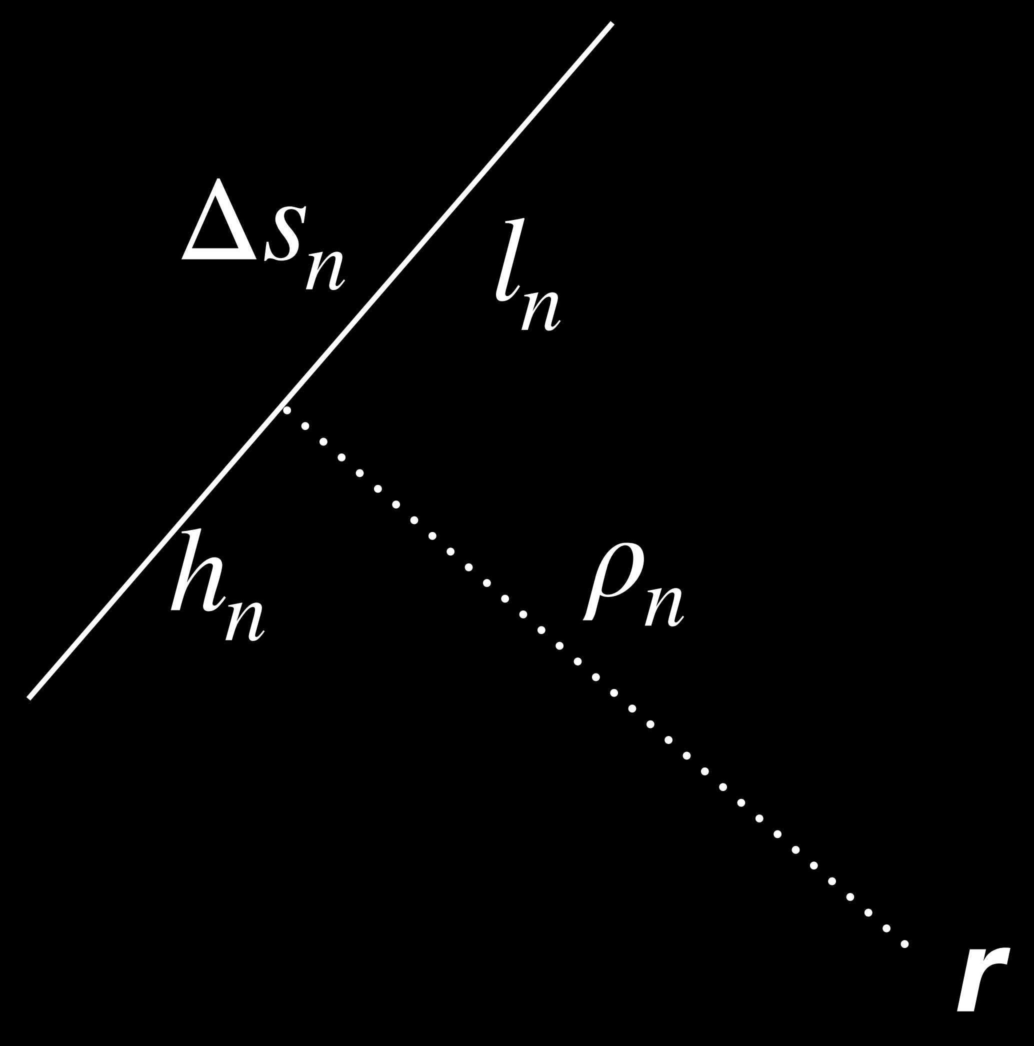
$$\phi(\mathbf{r},t) = \frac{1}{4\pi\sigma}\sum_{n=1}^{N}\frac{I_n(t)}{|\mathbf{r}-\mathbf{r}_n|}$$

Line-source model

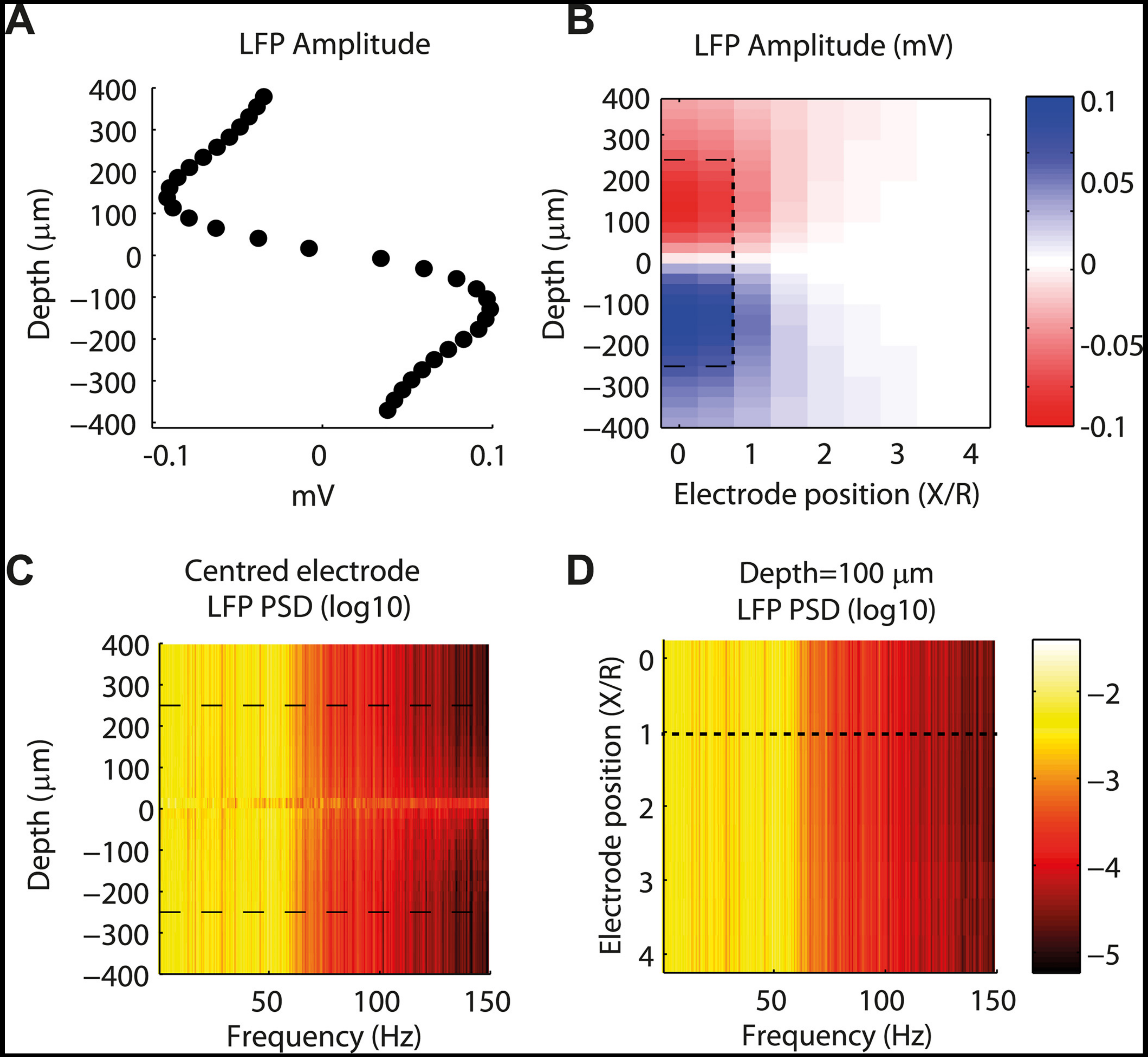
$$\phi(\mathbf{r},t) = \sum_{n=1}^{N}\frac{I_n(t)}{4\pi\sigma\Delta s_n} log\left|\frac{\sqrt{h_n^2+\rho_n^2}-h_n}{\sqrt{l_n^2+\rho_n^2}-l_n}\right|$$

Current function

$$I_n(t) = I_0\frac{t}{\tau_n}exp(1-t/\tau_n)\theta(t)$$

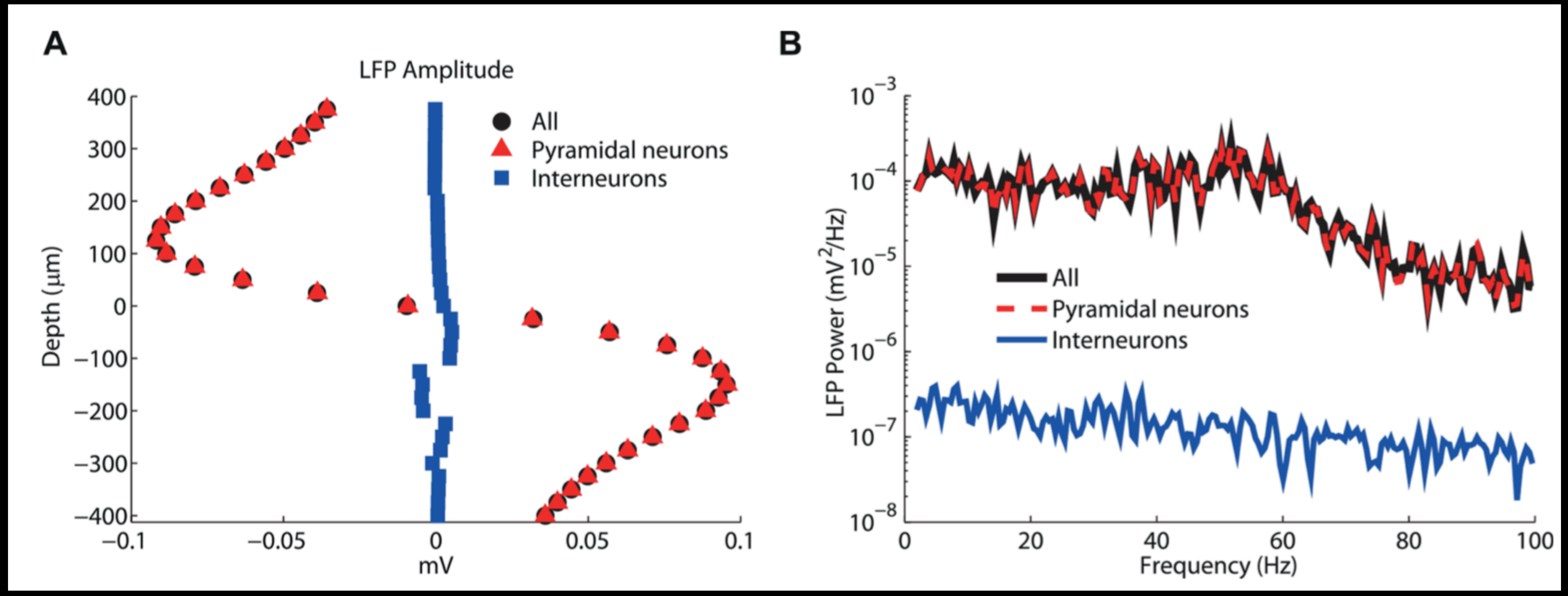


LFP Models

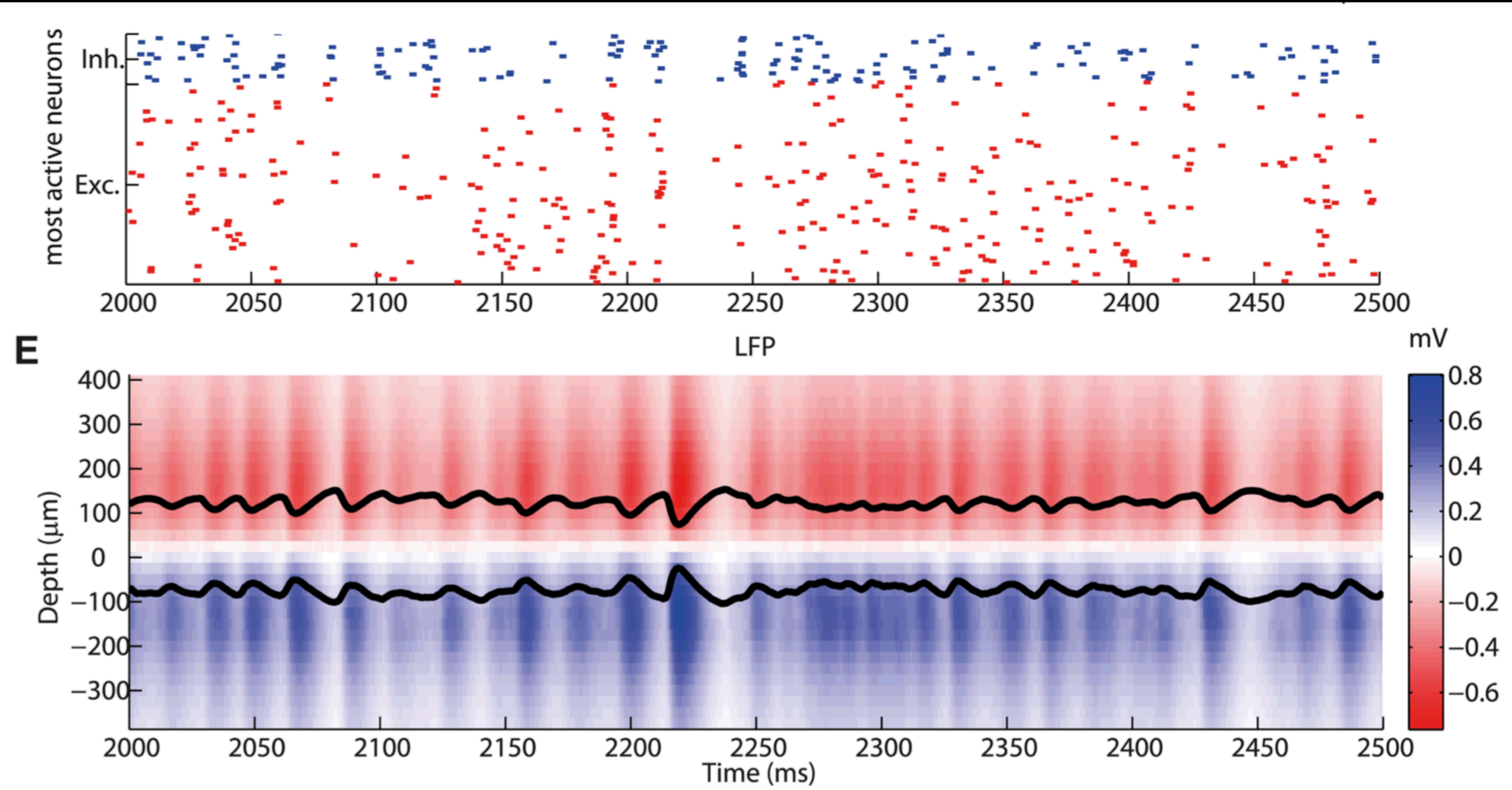


$$LFP_{proxy}(r, d, t) = f_{proxy}(r, d) * g_{proxy}(t)$$

Spatial course **Time course**



How to determine the spatial course of LFP



Thalamic stimulation := 1.5 spikes/ms (Poisson Rate)

J_AMPA_th	J_GAMA_th
0.55mV	0.95mV

LFP Time Course

$$LFP_{FR}(r,d,t) = f_{FR}(r,d) * R(t)$$

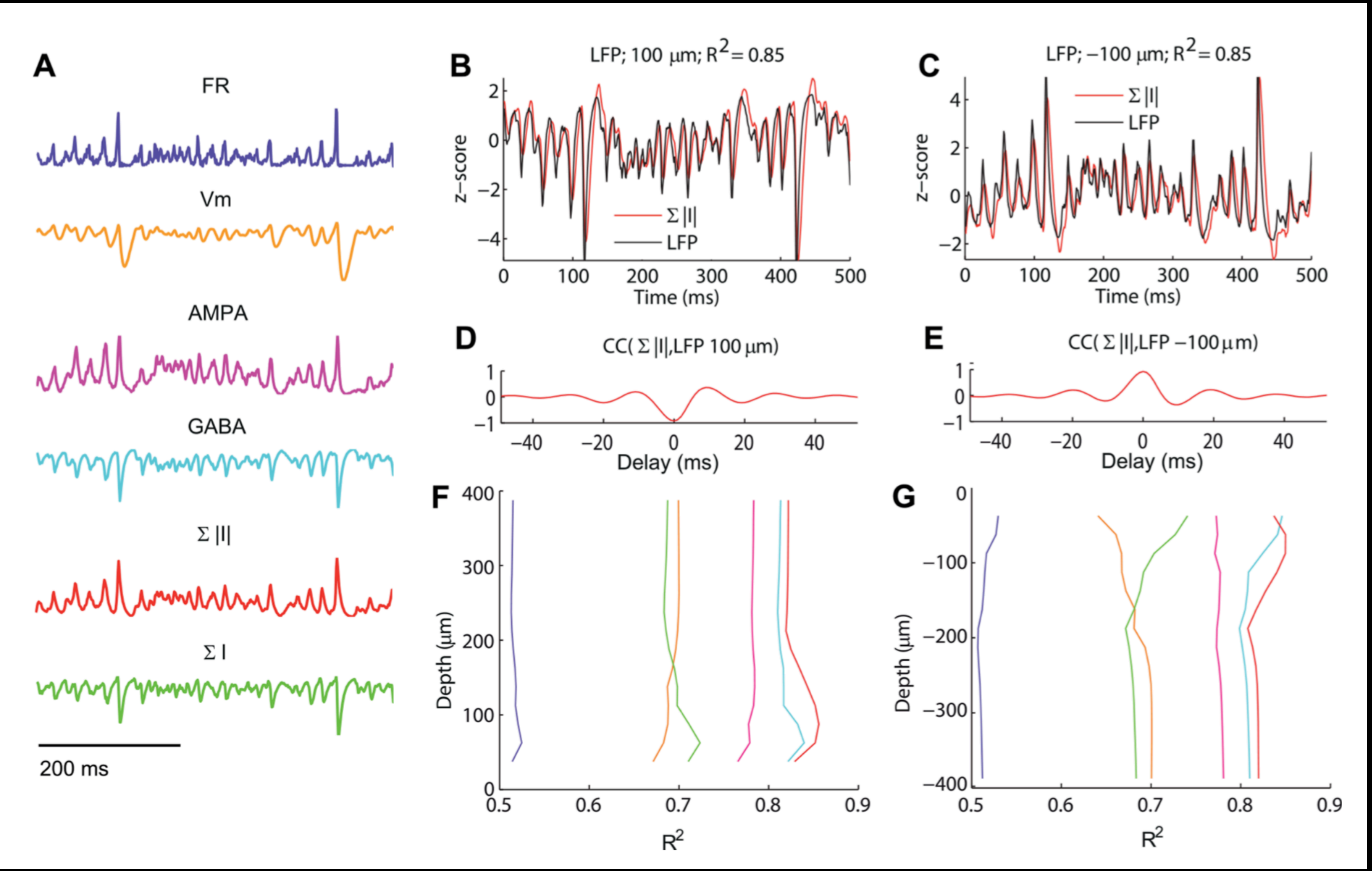
$$LFP_{V_m}(r,d,t) = f_{V_m}(r,d) * V(t)$$

$$LFP_{AMPA}(r,d,t) = f_{AMPA}(r,d) * \sum_{pyr} AMPA(t-\tau)$$

$$LFP_{GABA}(r,d,t) = f_{GABA}(r,d) * \sum_{pyr} GABA(t-\tau)$$

$$LFP_{\Sigma_I}(r,d,t) = f_{\Sigma_I}(r,d) * Norm\left[\sum_{pyr} AMPA(t-\tau) + \sum_{pyr} GABA(t-\tau)\right]$$

$$LFP_{\Sigma_{|I|}}(r,d,t) = f_{\Sigma_{|I|}}(r,d) * Norm\left[\sum_{pyr} AMPA(t-\tau) - \sum_{pyr} GABA(t-\tau)\right]$$

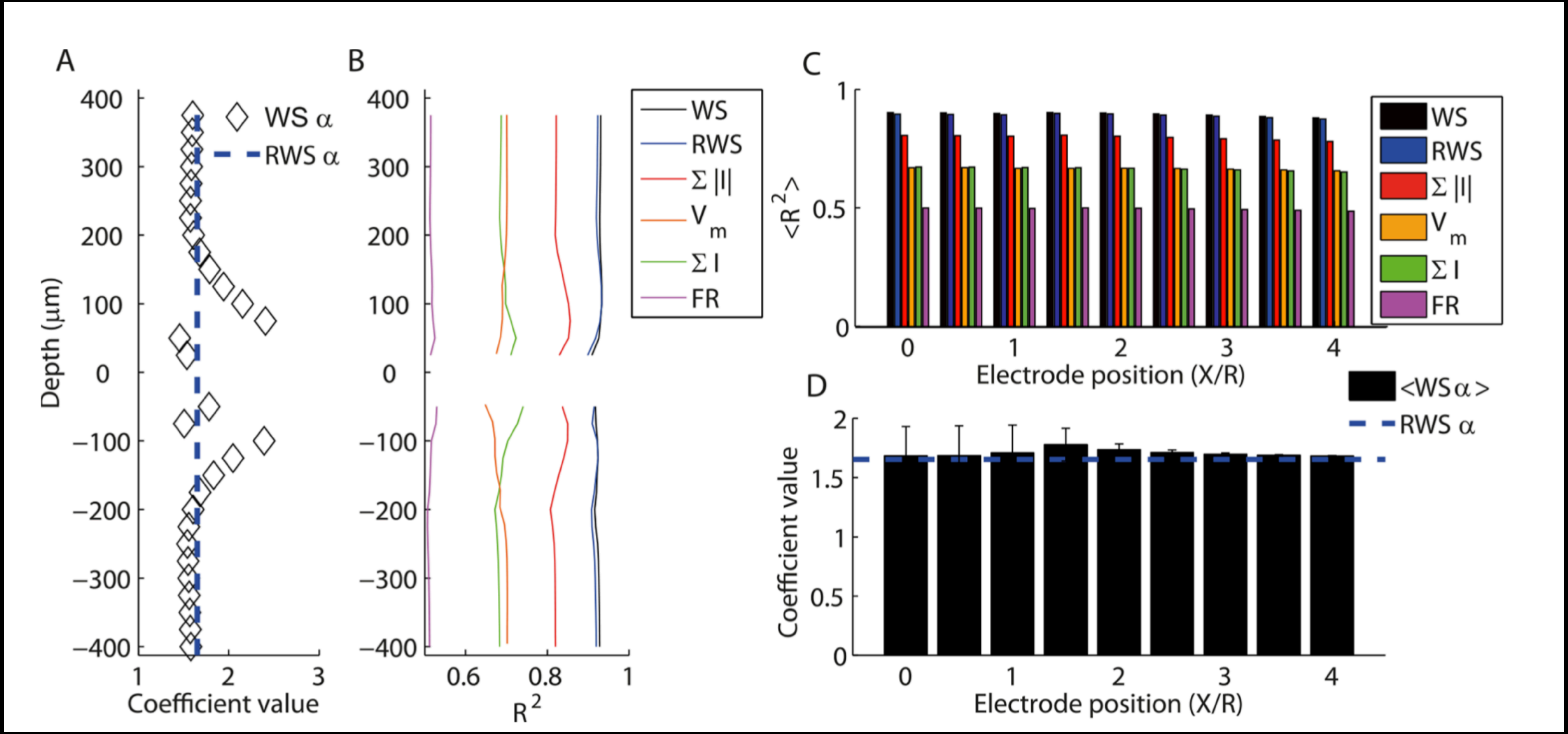


New LFP Proxy

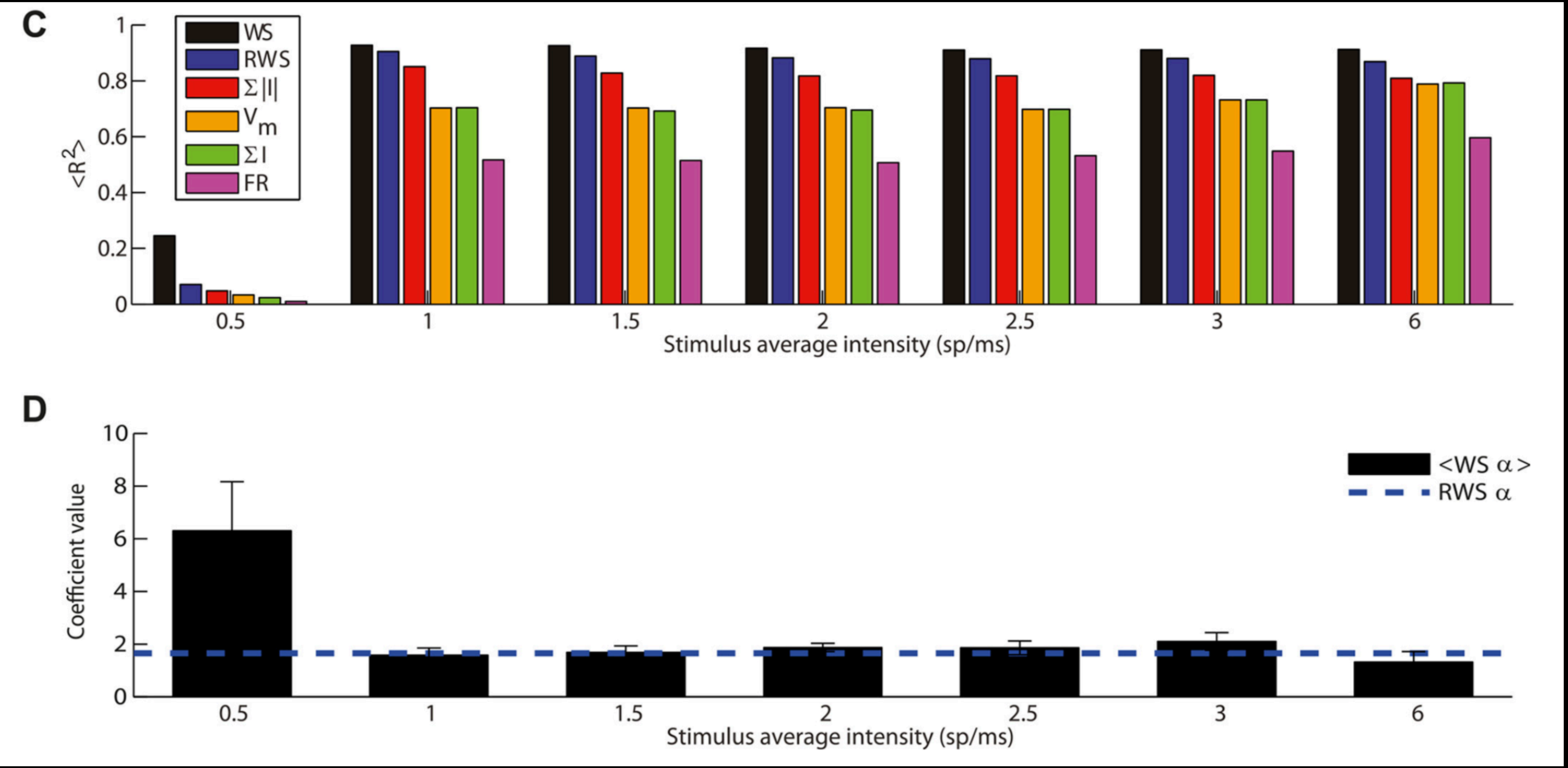
$$LFP_{WS}(r,d,t) = f_{WS}(r,d) * Norm\left[\sum_{pyr} AMPA\left(t-\tau_{AMPA}\right) - \alpha\left(\sum_{pyr} GABA\left(t-\tau_{GABA}\right)\right)\right]$$

Proxy RWS

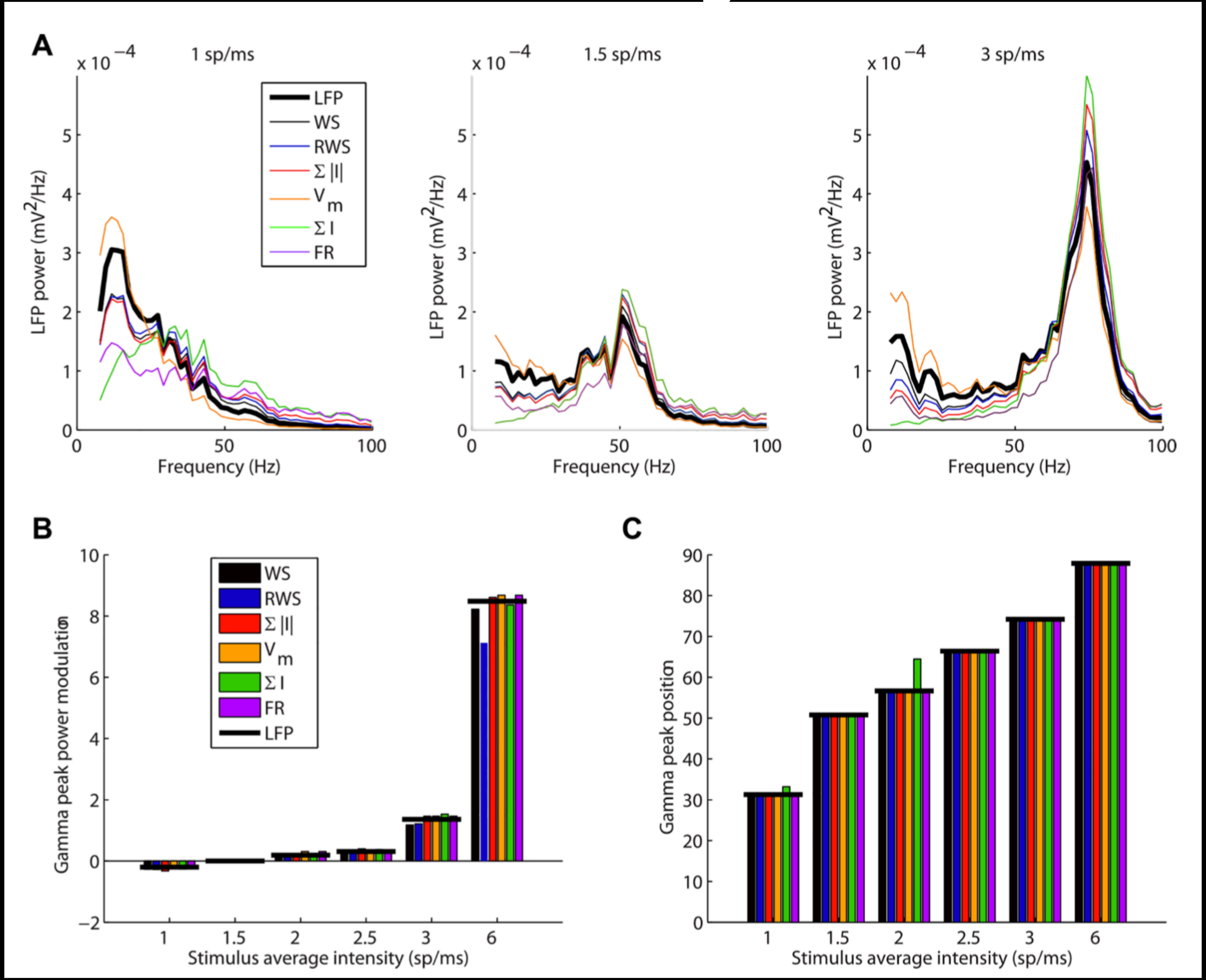
$$\tau_{AMPA} = 6ms$$
$$\tau_{GABA} = 0ms$$
$$\alpha = 1.65$$



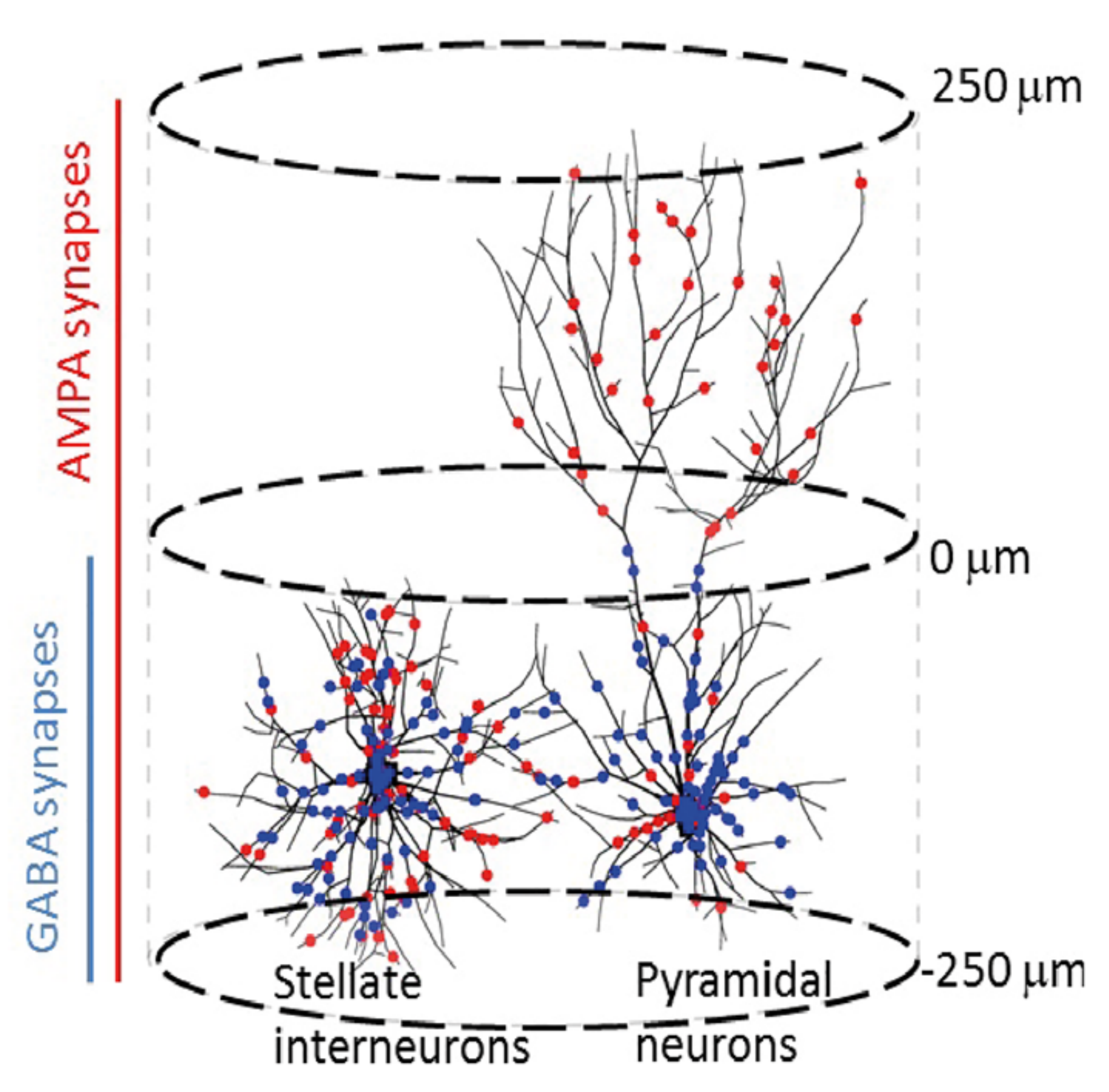
Dependence of Dynamic States

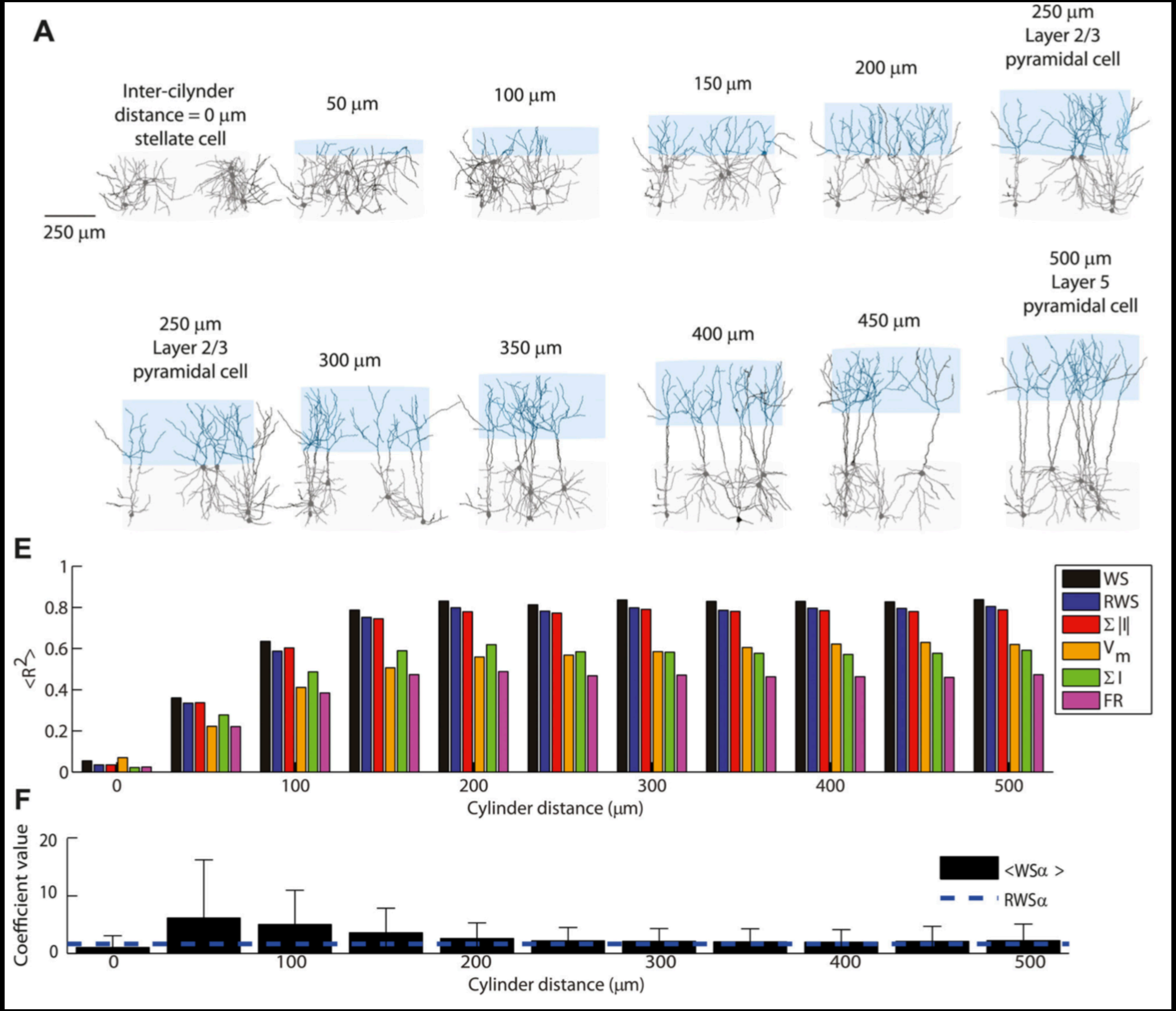


Dependence of Dynamic States

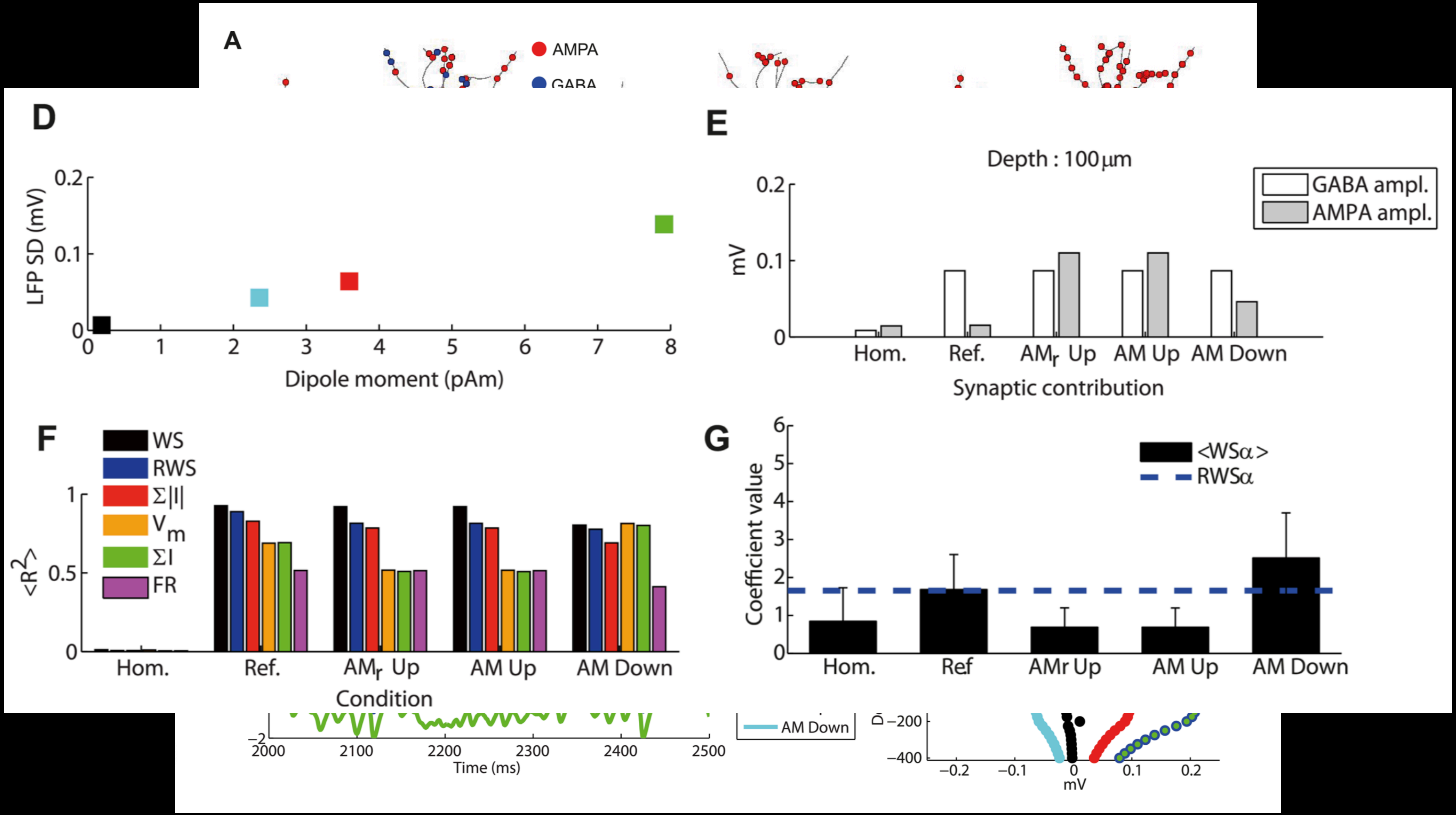


Dependence of Dendritic Structure





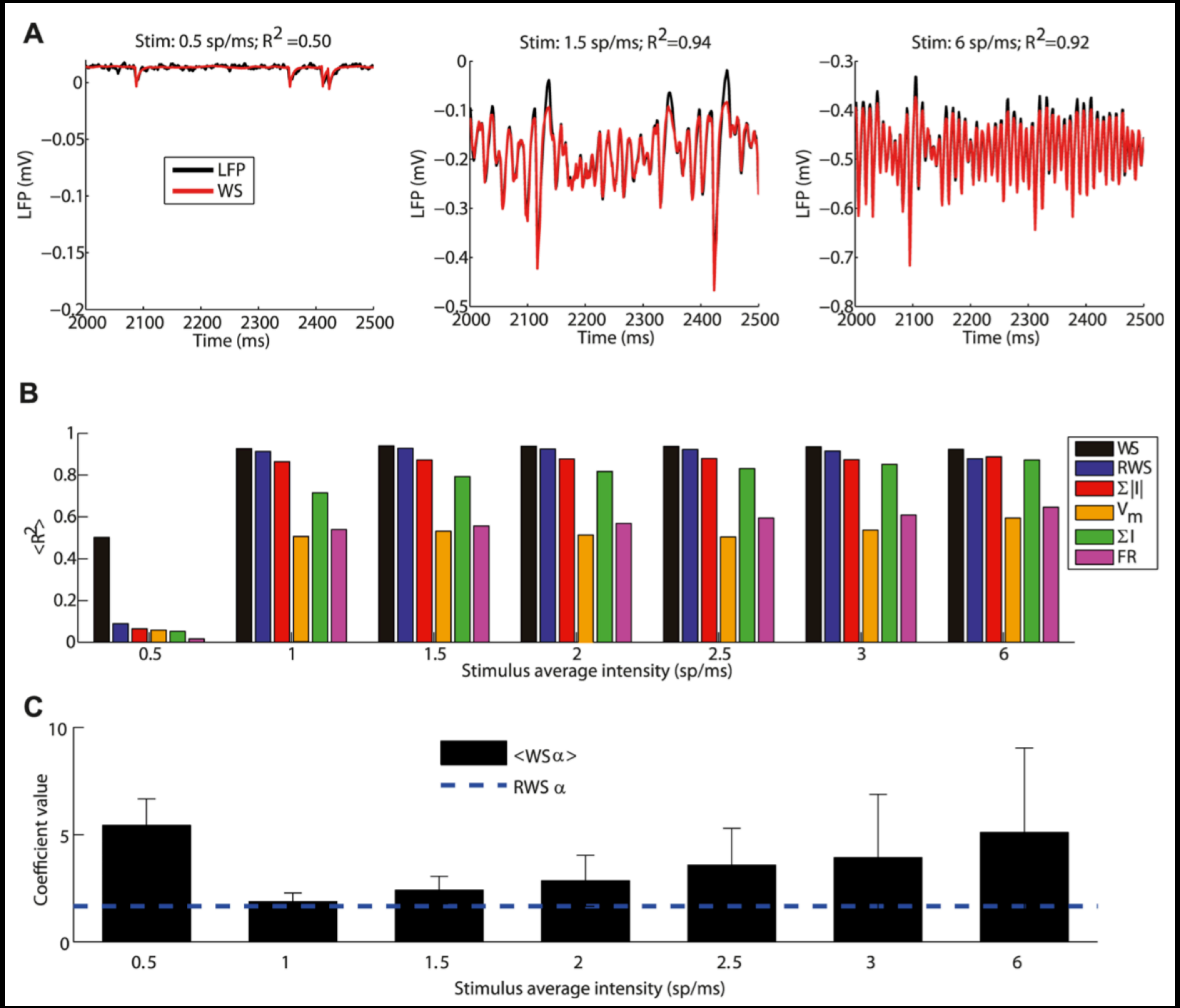
Dependence of the Distribution of Synapses



Dependence of the Type of Synapses

Form of post synaptic current in 3D network

$$PSC(t) = G\left(V(t) - E_{syn}\right) A \left[\exp\left(\frac{-\left(t - t_{syn}\right)}{\tau_{decay}}\right) - \exp\left(\frac{-\left(t - t_{syn}\right)}{\tau_{rise}}\right)\right]$$

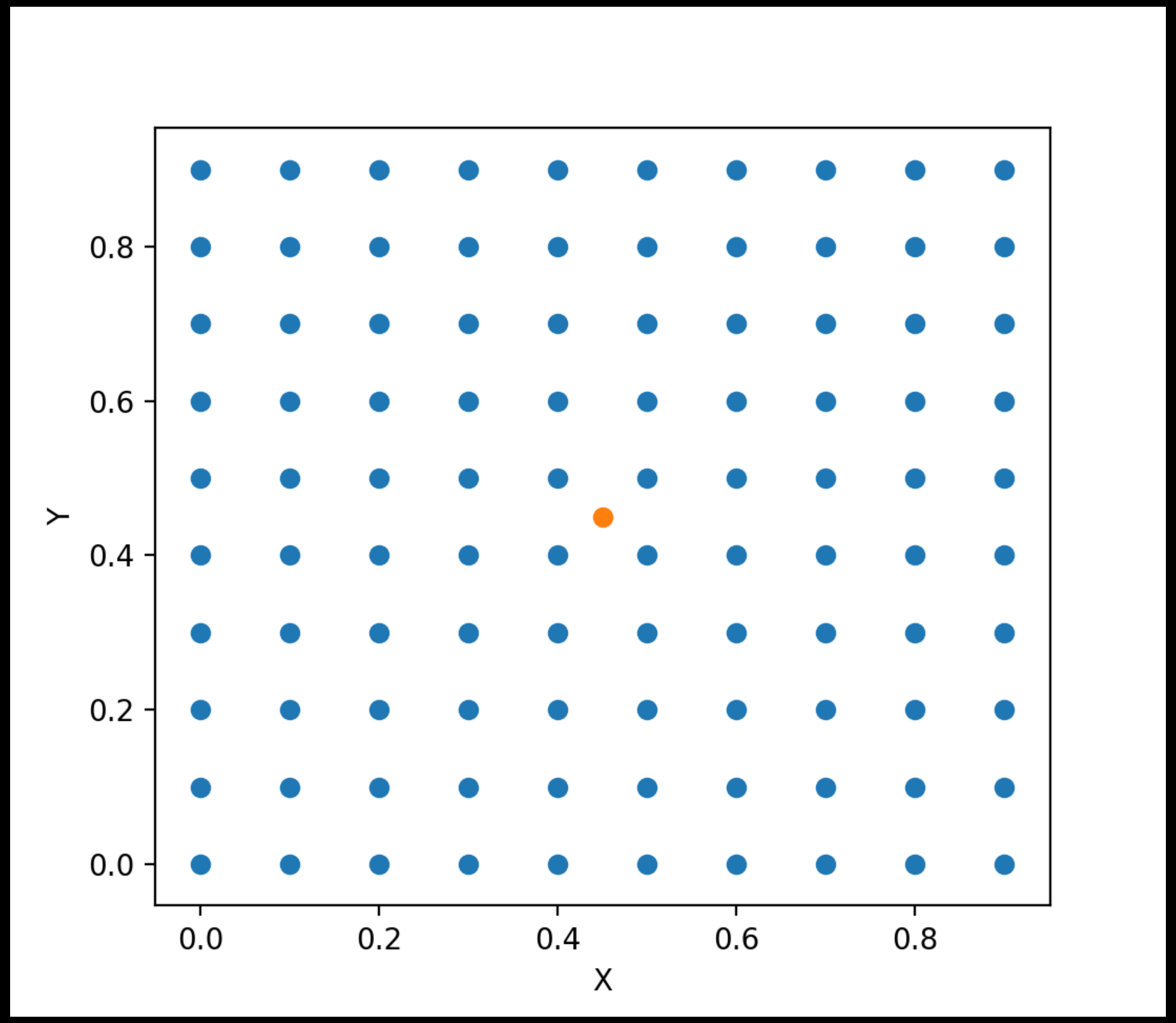


Conclusion

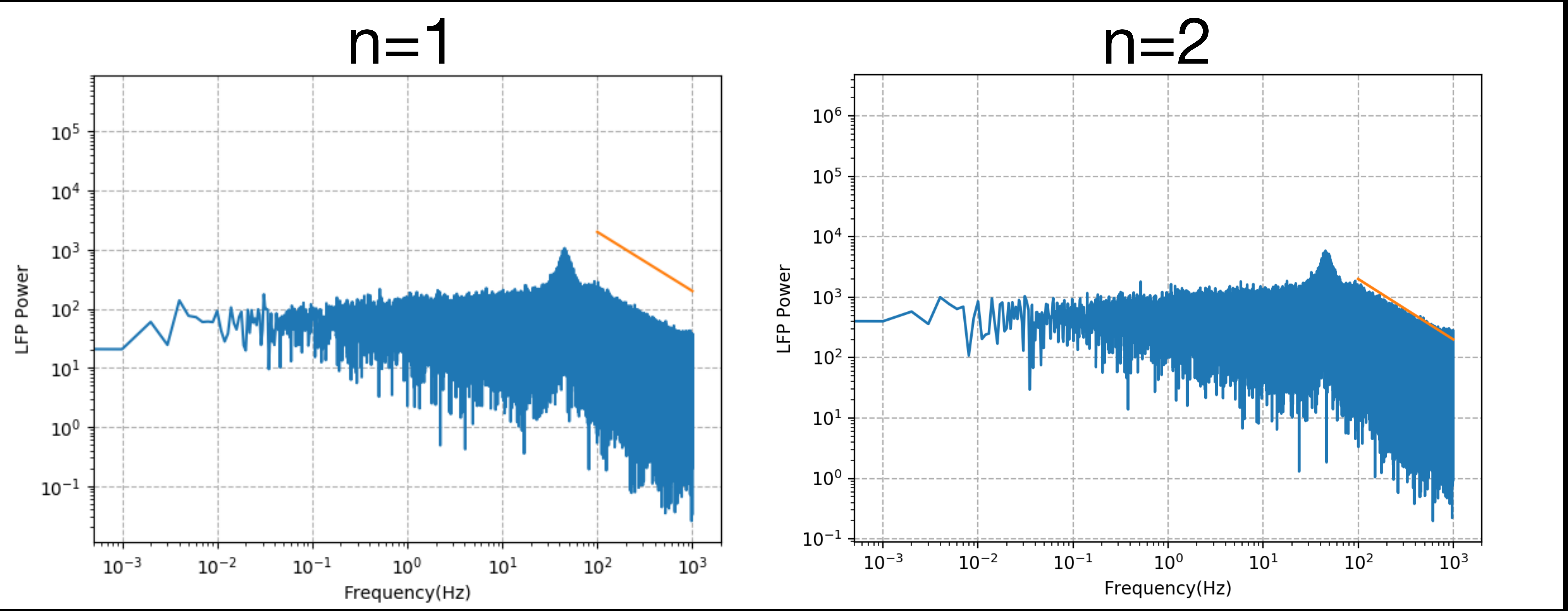
- For a given network structure in LIF network, proxy RWS is a good approximation of LFP, which is relatively independent towards the structure of 3D network.
- However, in order to make it applicable, we need to setup a 3D net, to optimize the fitting parameter and spatial course of the LFP proxy.

Calculating MI between Spike train and LFP

$$p_{con} = 0.1$$



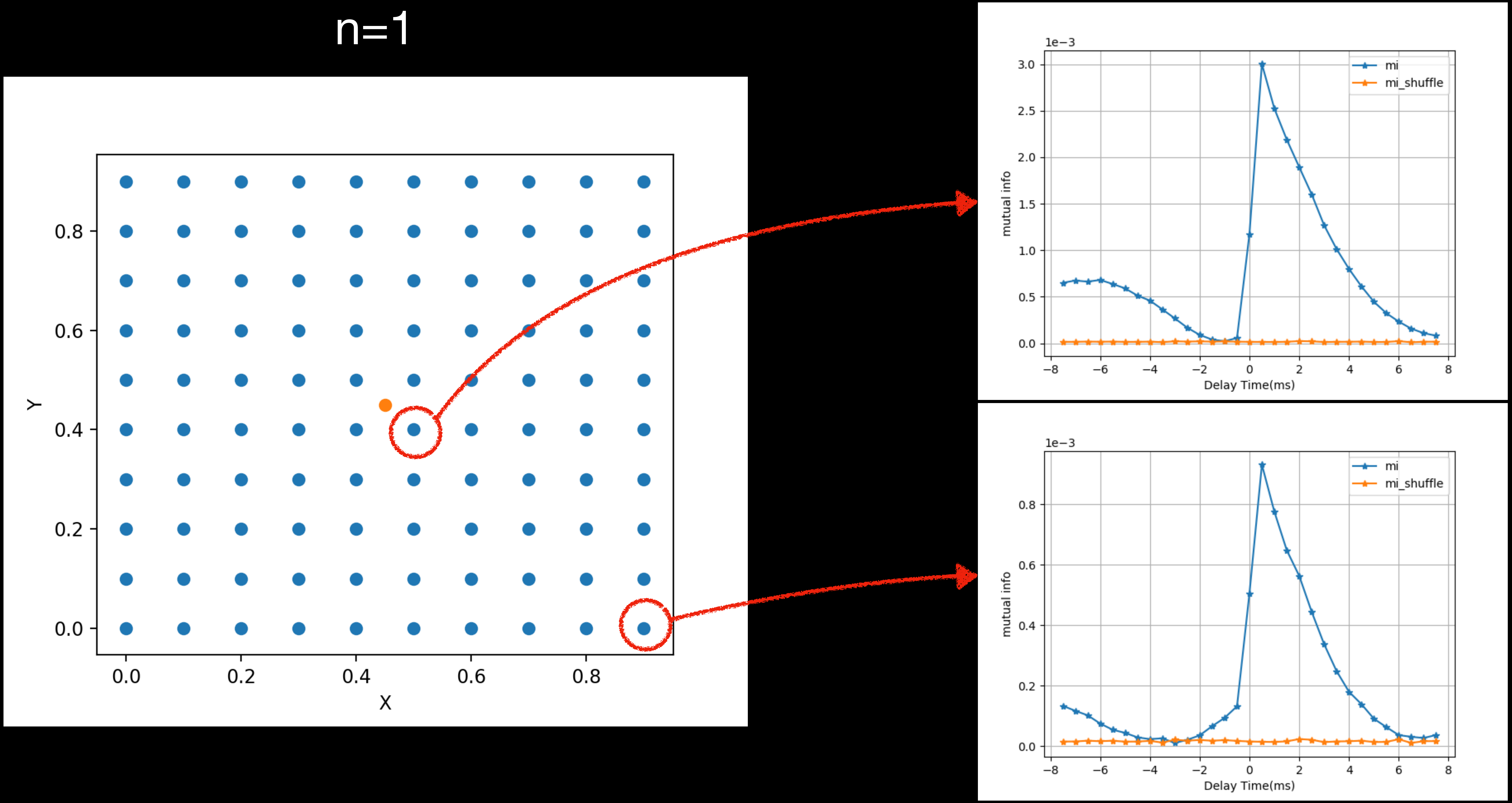
$$\phi(\mathbf{r}, t) = \frac{1}{4\pi\sigma} \sum_{i=1}^{N} \frac{I_i(t)}{|\mathbf{r} - \mathbf{r}_i|^n}$$



Neuron Model	LIF_G
Neuron Number	100 exc
Poisson Rate	2 kHz
Poisson Strength	5e-3 (0.5mV)
Synaptic Strength	1e-3 (0.1mV)
Mean Firing rate	47 Hz
Simulation Period	1e6 ms

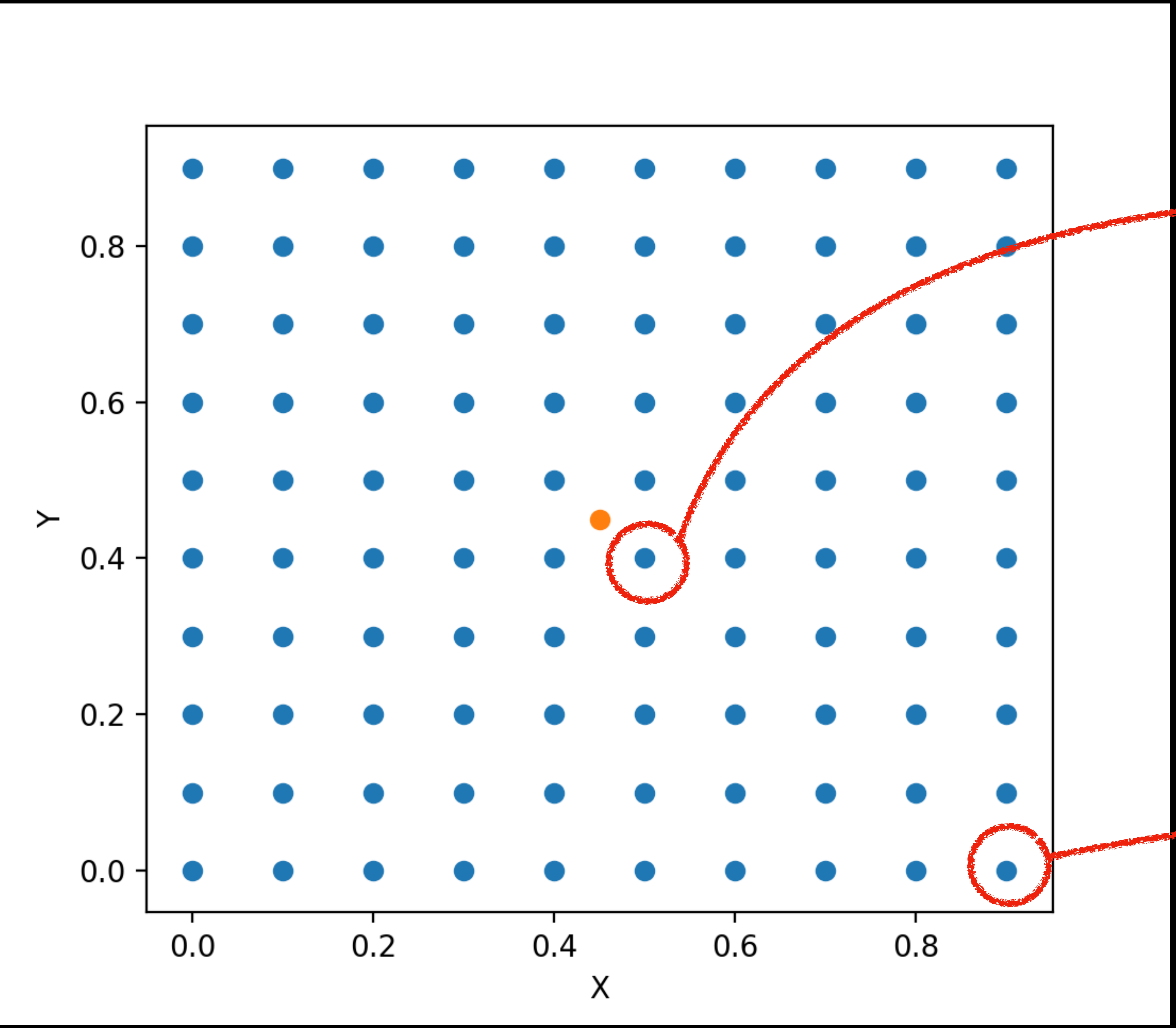
Calculating MI between Spike train and LFP

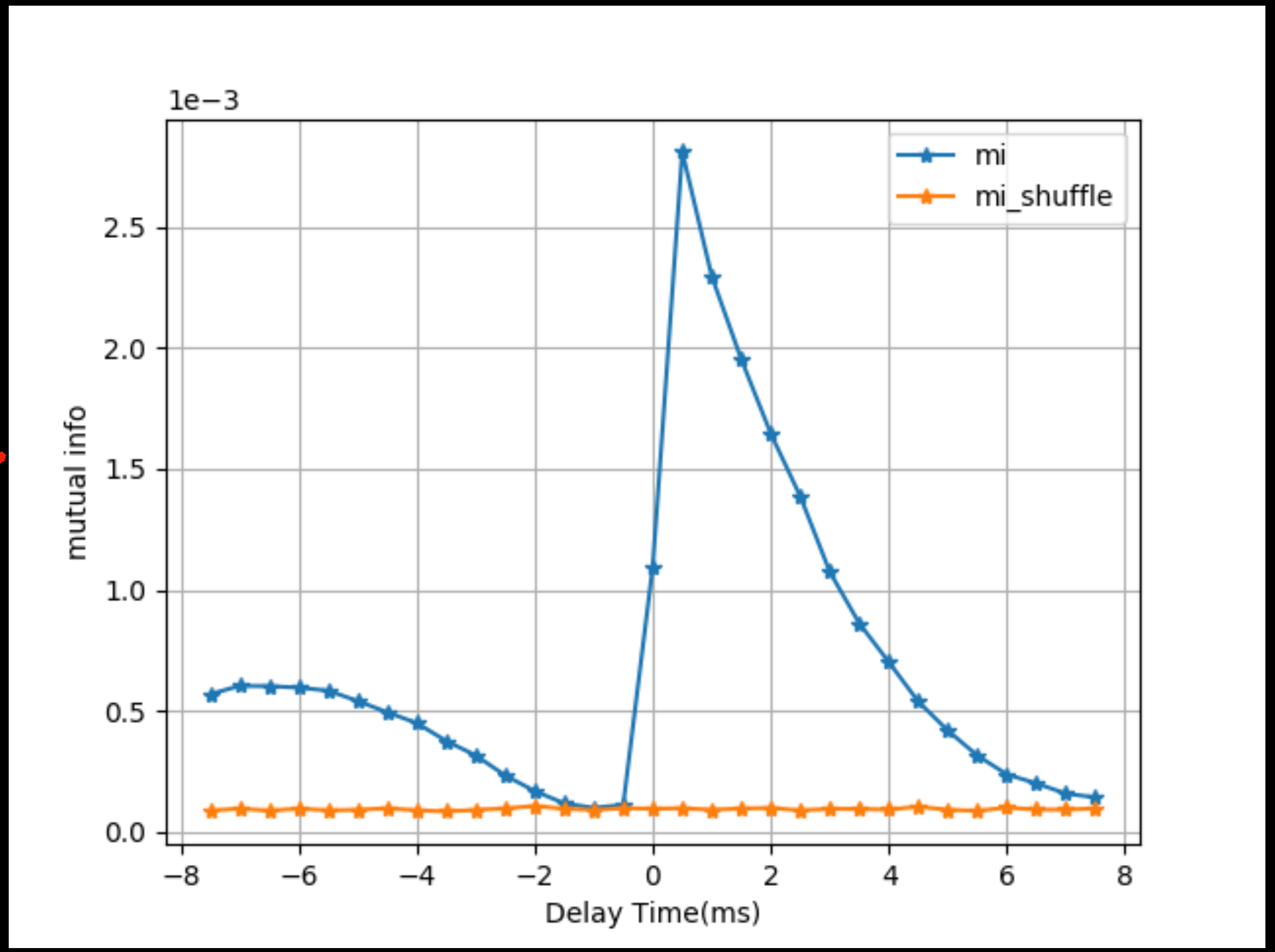
n=1

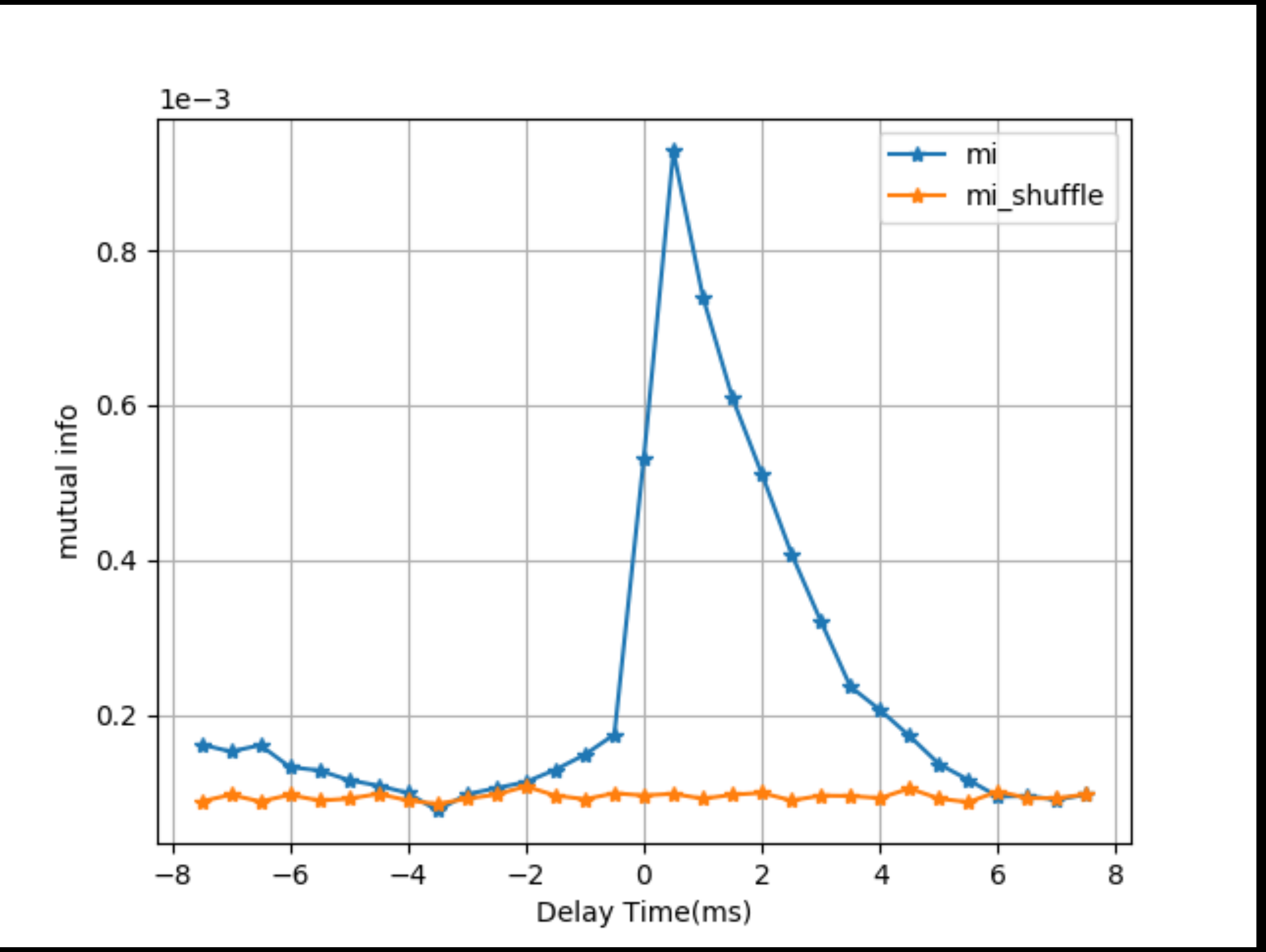


Calculating MI between Spike train and LFP

n=2

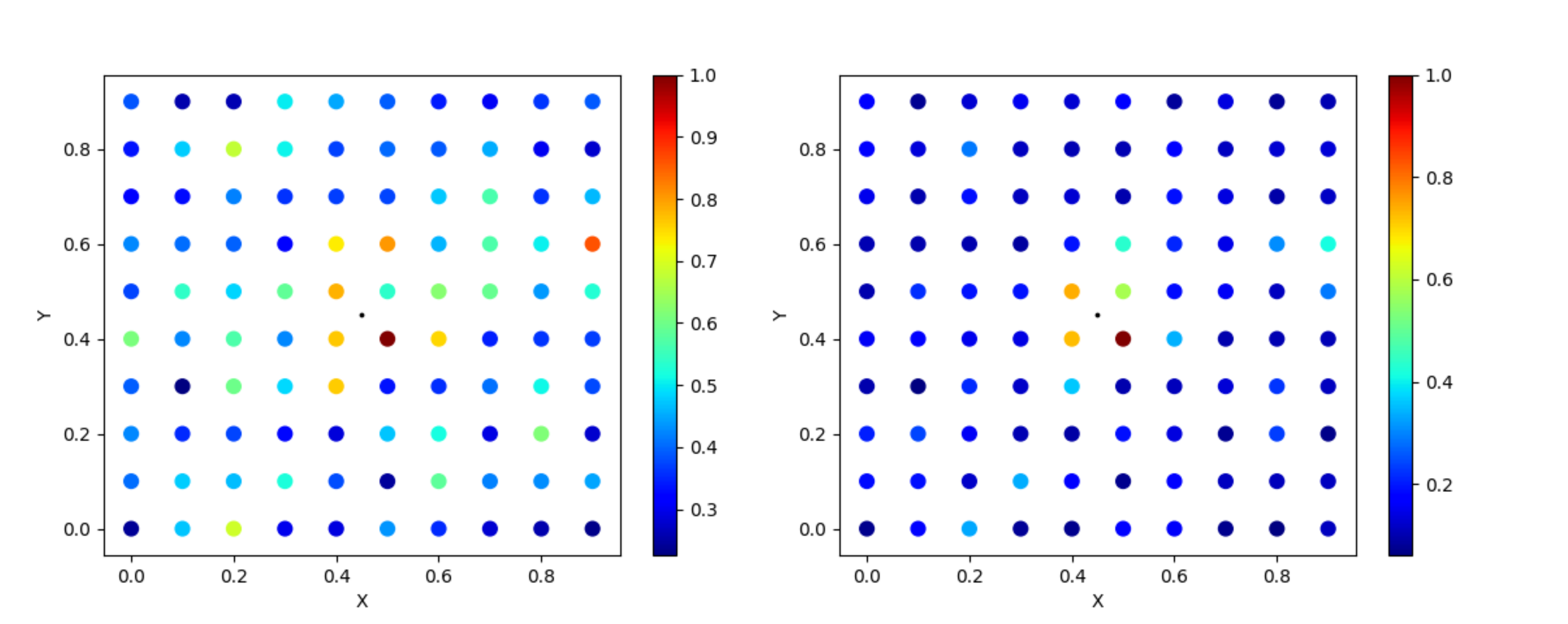




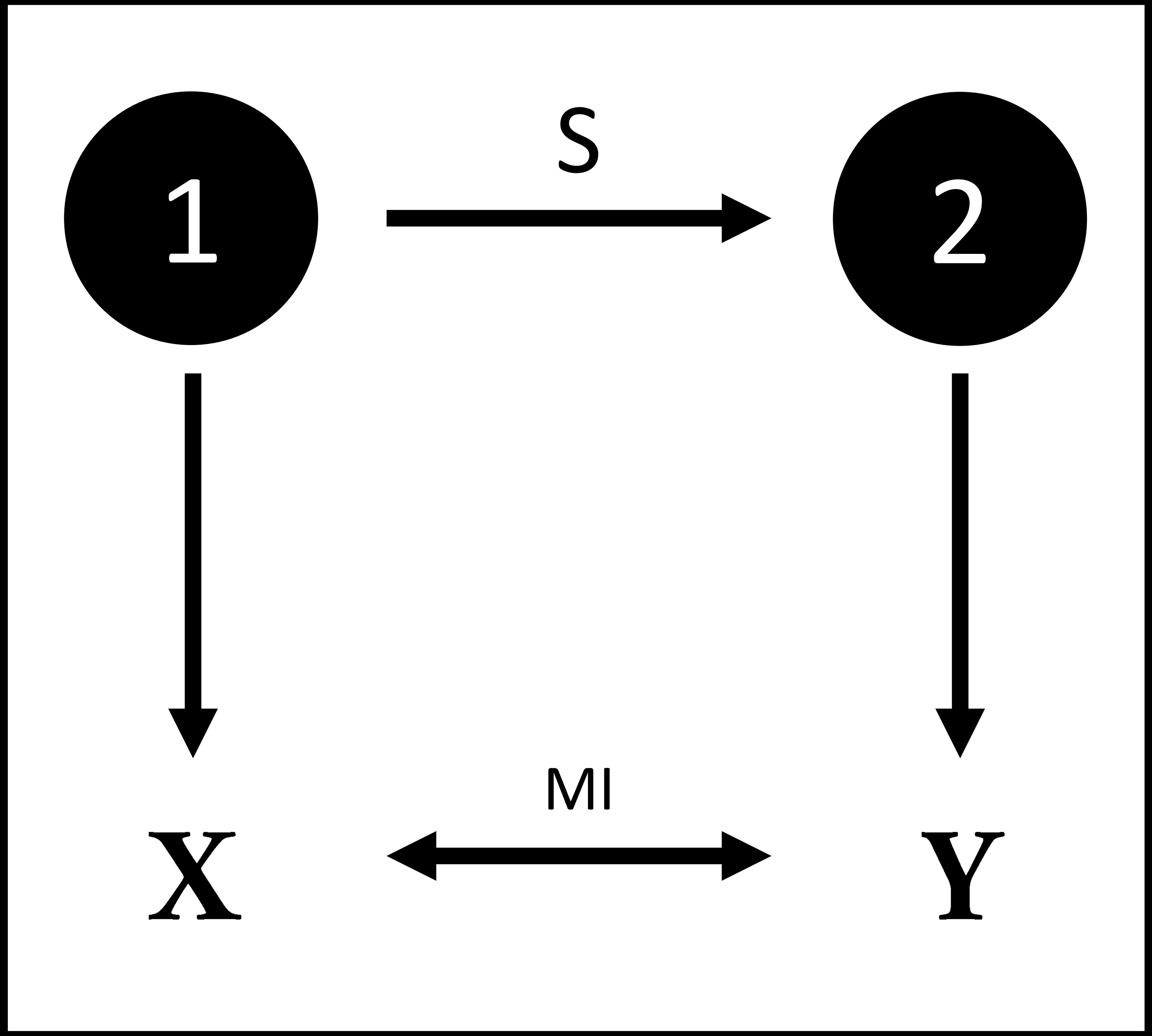


Calculating MI between Spike train and LFP

n=1 n=2



Mutual information



$$I(X,Y) = \sum_{Y}\sum_{X} P(X,Y)\log\left[\frac{P(X,Y)}{P(X)P(Y)}\right]$$

For weakly interacting regime:

$$P(X,Y) \approx P(X)P(Y)$$

Define:

$$P(X,Y) = P(X)P(Y) + \Delta P(X,Y)$$

$$where : \Delta P(X,Y) \ll P(X)P(Y)$$

Mutual information

$$I(X,Y) = \sum_Y \sum_X (P(X)P(Y) + \Delta P(X,Y)) \log\left(1 + \frac{\Delta P(X,Y)}{P(X)P(Y)}\right)$$

$$Denote : \delta p_{XY} = \frac{\Delta P(X,Y)}{P(X)P(Y)}$$

$$I(X,Y) = \sum_Y \sum_X (P(X)P(Y) + \Delta P(X,Y))\left(\frac{\Delta P(X,Y)}{P(X)P(Y)} - \frac{1}{2}\frac{\Delta P(X,Y)}{P(X)P(Y)}^2 + O(\delta p_{XY}^3)\right)$$

$$\boxed{I(X,Y) = \underbrace{\sum_Y \sum_X \Delta P(X,Y)}_{0} + \frac{1}{2P(X)P(Y)}\Delta P(X,Y)^2 + O\left(\delta p_{XY}^3\right)}$$

Second order term dominant

This approx. holds for weakly interacting system, independent with model.

MI—Spike train & Local field potential

X := spike train (binary series)

Y := local field potential (continues series)

$P(x) := P(X = 1)$

$P(\bar{x}) := P(X = 0)$

$$I(X,Y) \approx \frac{1}{2}\int dy \left[\frac{1}{P(x)P(y)}\Delta P(x,y)^2 + \frac{1}{P(\bar{x})P(y)}\Delta P(\bar{x},y)^2\right]$$

Preserve 2nd order of δp_{XY}

$$\Delta P(x,y) = P(x,y) - P(x)P(y) \qquad \Delta P(\bar{x},y) = \left(P(y) - P(x,y)\right) - \left(1 - P(x)\right)P(y) = -\Delta P(x,y)$$

$$I(X,Y) \approx \frac{1}{2P(x)P(\bar{x})}\int dy \frac{1}{P(y)}\Delta P(x,y)^2$$

Find the expression of $\Delta P(x,y)$

MI—Spike train & Local field potential

$$I(X,Y) \approx \frac{1}{2P(x)P(\bar{x})}\int dy \frac{1}{P(y)}\Delta P(x,y)^2$$

By definition: $$P(y\,|\,x) - P(y) = \frac{P(x,y) - P(x)P(y)}{P(x)} = \frac{\Delta P(x,y)}{P(x)}$$

$$\boxed{P(y\,|\,x) - P(y) \approx P(y - \Delta y) - P(y) = -P'(y)\Delta y + O\left(\Delta y^2\right)}$$

$$\Delta P(x,y) \approx P(x)\Big(-P'(y)\Delta y + O\left(\Delta y^2\right)\Big)$$

$$I(X,Y) \approx \frac{P(x)}{2P(\bar{x})}\int dy \frac{\Big(-P'(y)\Delta y + O\left(\Delta y^2\right)\Big)^2}{P(y)} = \frac{P(x)}{2P(\bar{x})}\int dy \left[\frac{P'(y)^2}{P(y)}\Delta y^2 + O\left(\Delta y^3\right)\right]$$

Synaptic Strength vs Δ y

$$I(X,Y) \approx \frac{P(x)}{2P(\bar{x})}\int dy \left[\frac{P'(y)^2}{P(y)}\Delta y^2 + O\left(\Delta y^3\right)\right]$$

For current based I&F model

$$C_m\frac{dv_i}{dt} = -g_m v_i + \sum_j I_i^j(t) = I_i^{resistive}(t)$$

$$\tau_I\frac{d}{dt}I_i^j(t) = -I_i^j(t) + S_{ij}\sum_k \delta(t-t_k)$$

$$\Downarrow$$

$$\Delta I_i^{resistive}(t) = S_{ij} = \Delta y$$

$$I(X,Y) \approx S^2\frac{P(x)}{2P(\bar{x})}\int \frac{P'(y)^2}{P(y)}dy + O\left(S^3\right)$$

$$\boxed{I(X,Y) \propto S^2}$$

For conductance based I&F model

$$C_m\frac{dv_i}{dt} = -g_m v_i - g_i^e(v_i-\epsilon_e) - g_i^i(v_i-\epsilon_i) = I_i^{resistive}(t)$$

$$\tau_e\frac{d}{dt}g_i^e(t) = -g_i^e(t) + \sum_j S_{ij}^e\delta(t-t_j)$$

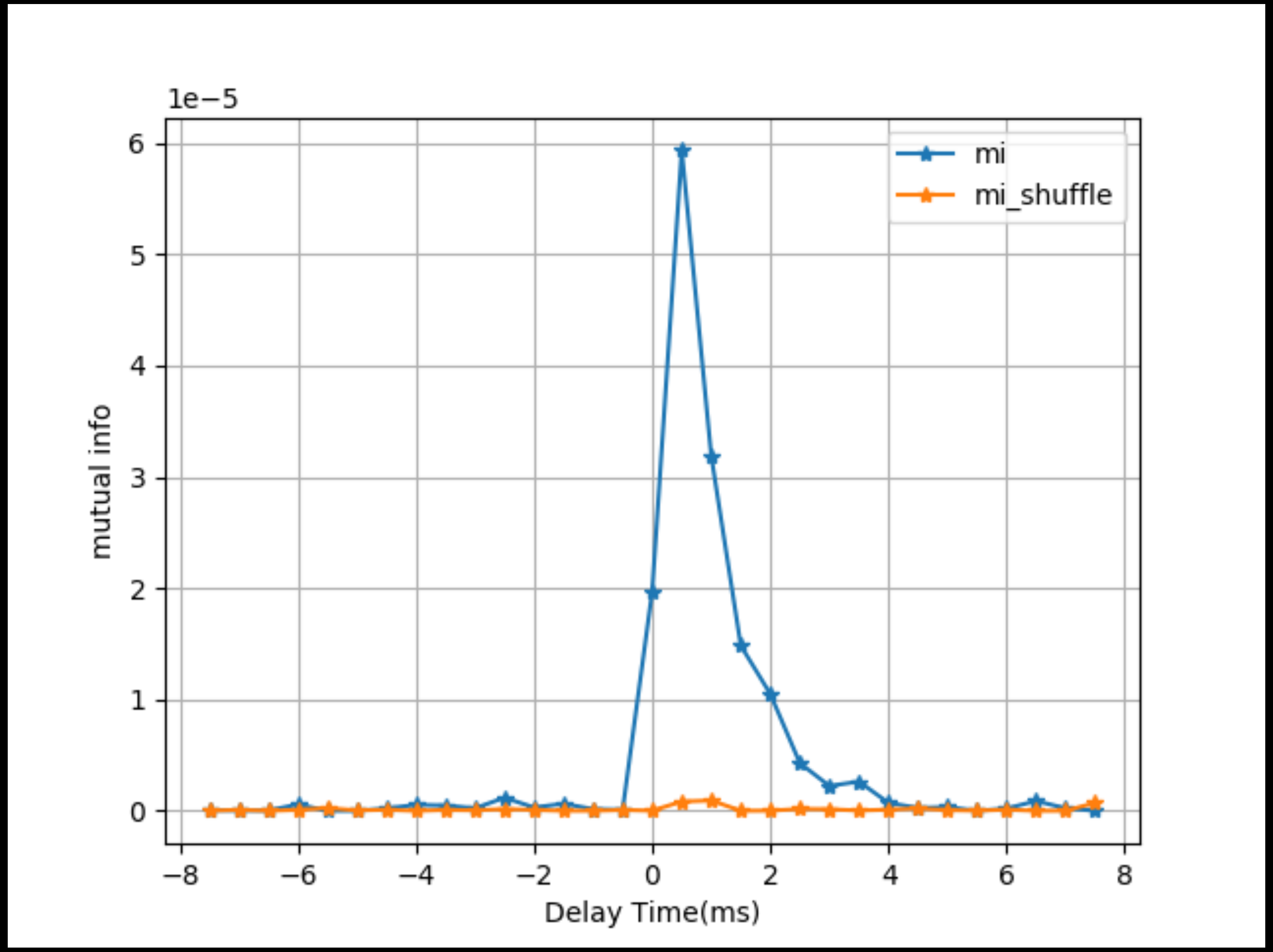
$$\Downarrow$$

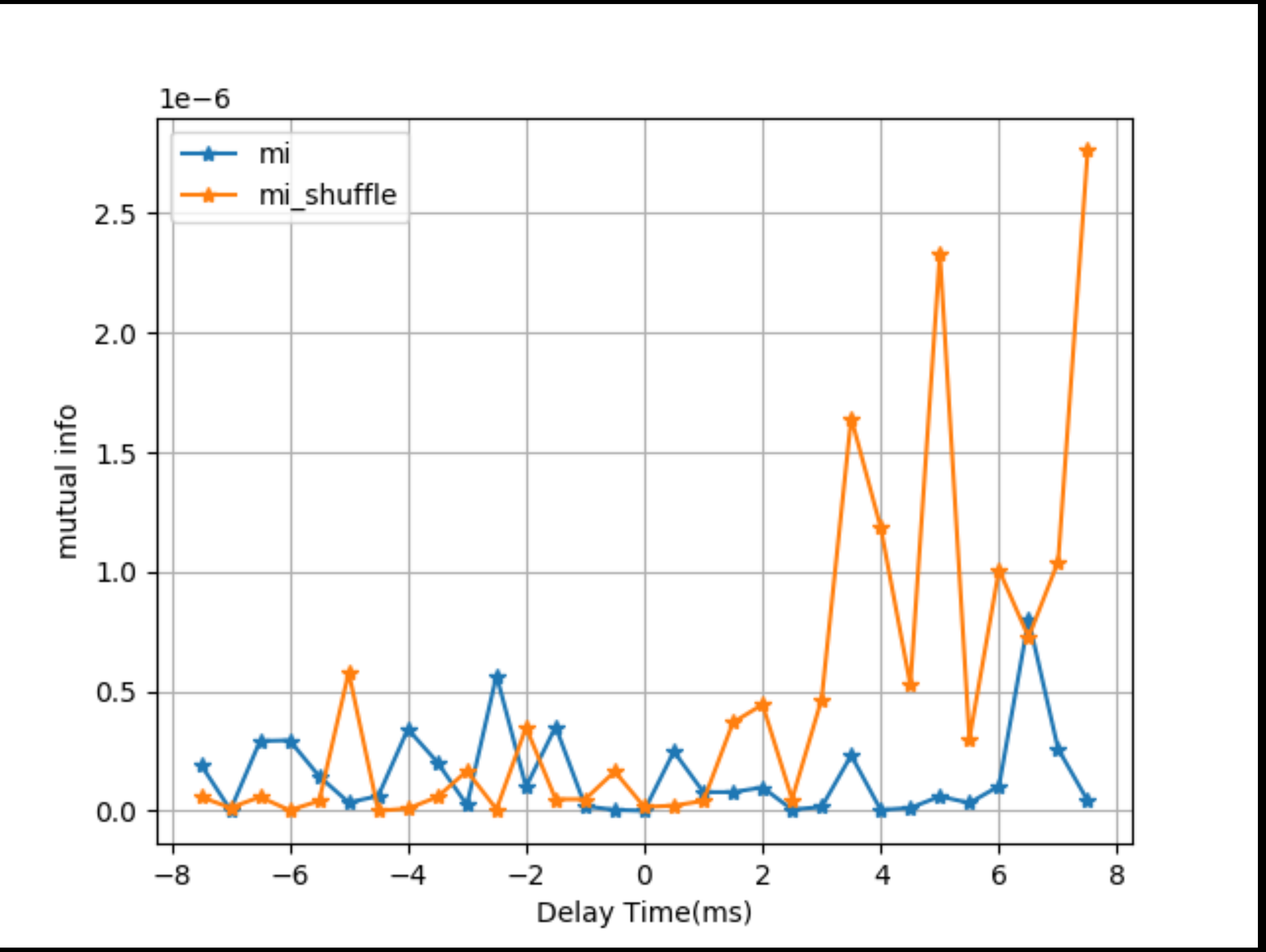
$$\Delta I_i^{resistive}(t) = S_{ij}^e(v_i-\epsilon_e) = \Delta y(y,S) = -S(\frac{1}{g_m}y+\epsilon_e)$$

$$I(X,Y) \approx S^2\frac{P(x)}{2P(\bar{x})}\int \frac{P'(y)^2}{P(y)}(\frac{1}{g_m}y+\epsilon_e)^2 dy + O\left(S^3\right)$$

$$\boxed{I(X,Y) \propto S^2}$$

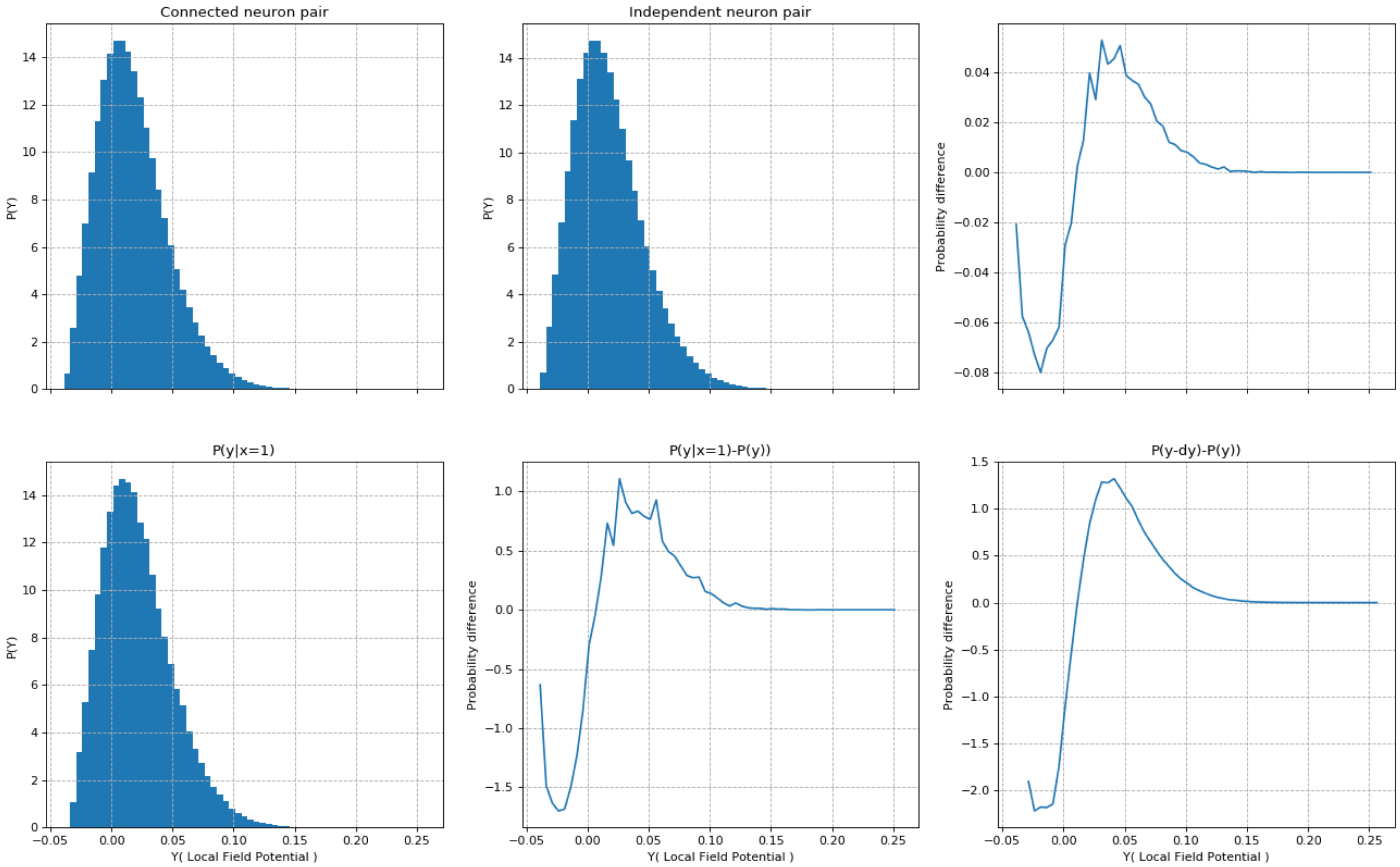
Numerical Test





Current based LIF model

Poisson Rate	Poisson Strength	Synaptic Strength	Mean Firing rate
1 kHz	2.5e-2 (0.38mV)	5e-3 (0.072mV)	15 Hz



Connected neuron pair
Independent neuron pair
P(y|x=1)
P(y|x=1)-P(y))
P(y-dy)-P(y))
P(Y)
Probability difference
Y(Local Field Potential)

Thanks you