Application of TDMI on Analyzing Neural Data

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Outline

- Concepts and definitions
  - Self-information
  - Entropy
  - Mutual information(MI) and time-delayed mutual information(TDMI)
- Related works about TDMI
- TDMI between Gaussian random variables
- TDMI between spike train and local field potential
  - TDMI estimation
  - Data generated by integrate-and-fire neuronal model

Self-information

- **Define:**

$$X = \{x_1, x_2, \dots, x_n\} \Longrightarrow Pr(X = x_i) = p_i, \quad \sum_i^n p_i = 1$$

  - $If\ p_1 > p_2, then\ f(p_1) < f(p_2)$
  - $If\ p_i = 1, then\ f(p_1) = 0$
  - $If\ p_i = 0, then\ f(p_1) = \infty$
  - $If\ x_1\ and\ x_2\ are\ independent,\ then\ f(p_1, p_2) = f(p_1 p_2) = f(p_1) + f(p_2)$
- **Define:**

$$f(x_i) = log \frac{1}{p_i}$$

Entropy

- **Define:**

$$H(X) = \sum_{x \in X} p(x) \log \frac{1}{p(x)}$$

A measure of the uncertainty of a random variable

- $H(X) = H(p_X)$ is continues on P_X
- $H(X_N)$ is monotonically increasing on N, if X_N is uniformly distributed
- Additivity: $H(p_1, p_2, p_3, \dots, p_n)$

$$= H(p_1 + p_2, p_3, \dots, p_n) + (\mathrm{p}_1 + \mathrm{p}_2) H\left(\frac{p_1}{p_1 + p_2}, \frac{p_2}{p_1 + p_2}\right)$$

Mutual information(MI)

- **Define:**

$$I(X;Y) = \sum_{x \in X} \sum_{y \in Y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

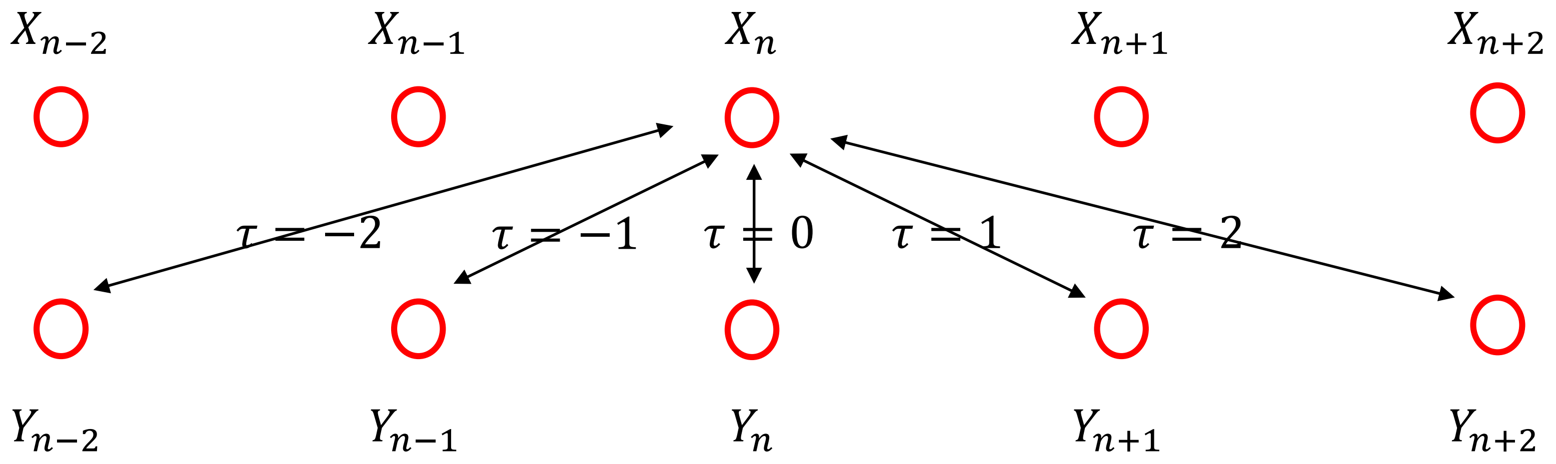
- $I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$
- $I(X;Y) = H(X) + H(Y) - H(X,Y)$
- $I(X;Y) = 0$ if X and Y are independent
- $I(X;X) = H(X)$

Time-delayed Mutual information(TDMI)

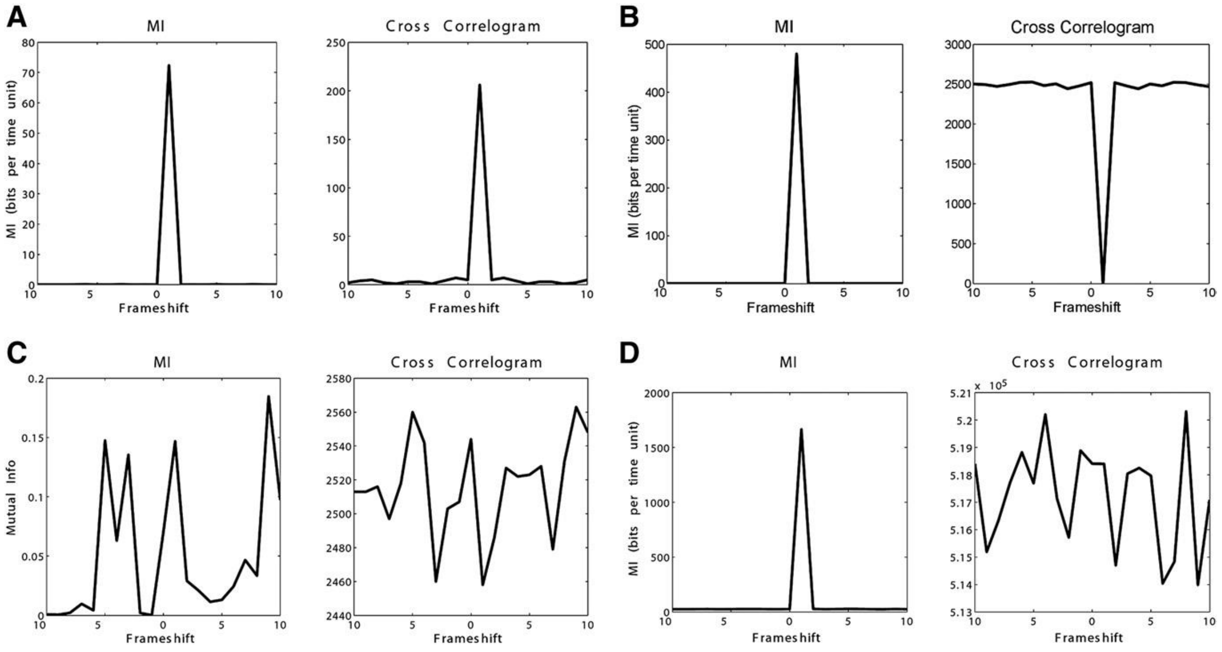
$$I(\tau) = I\big(X(t); Y(t+\tau)\big) = \sum_{x \in X(t)} \sum_{y \in Y(t+\tau)} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

For random series X and Y,

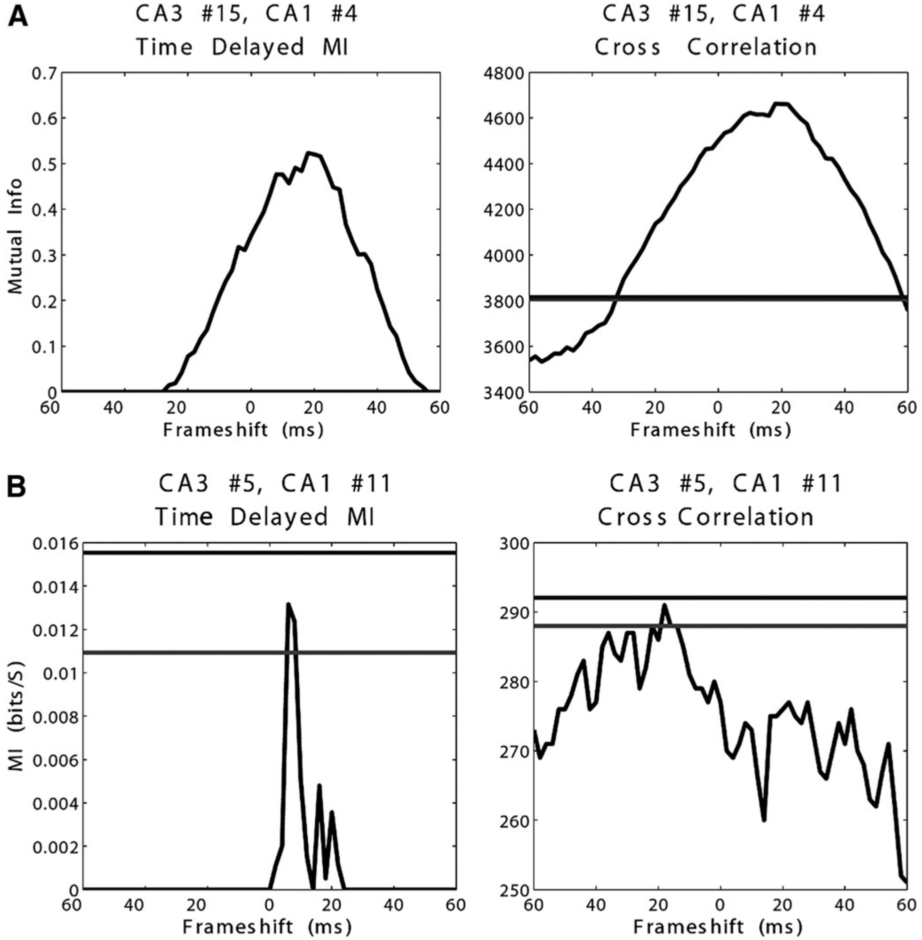
$$\begin{cases} X_i = f(X_{i-1}, Y_{i-1}) \\ Y_i = g(X_{i-1}, Y_{i-1}) \end{cases}$$



TDMI between spike trains

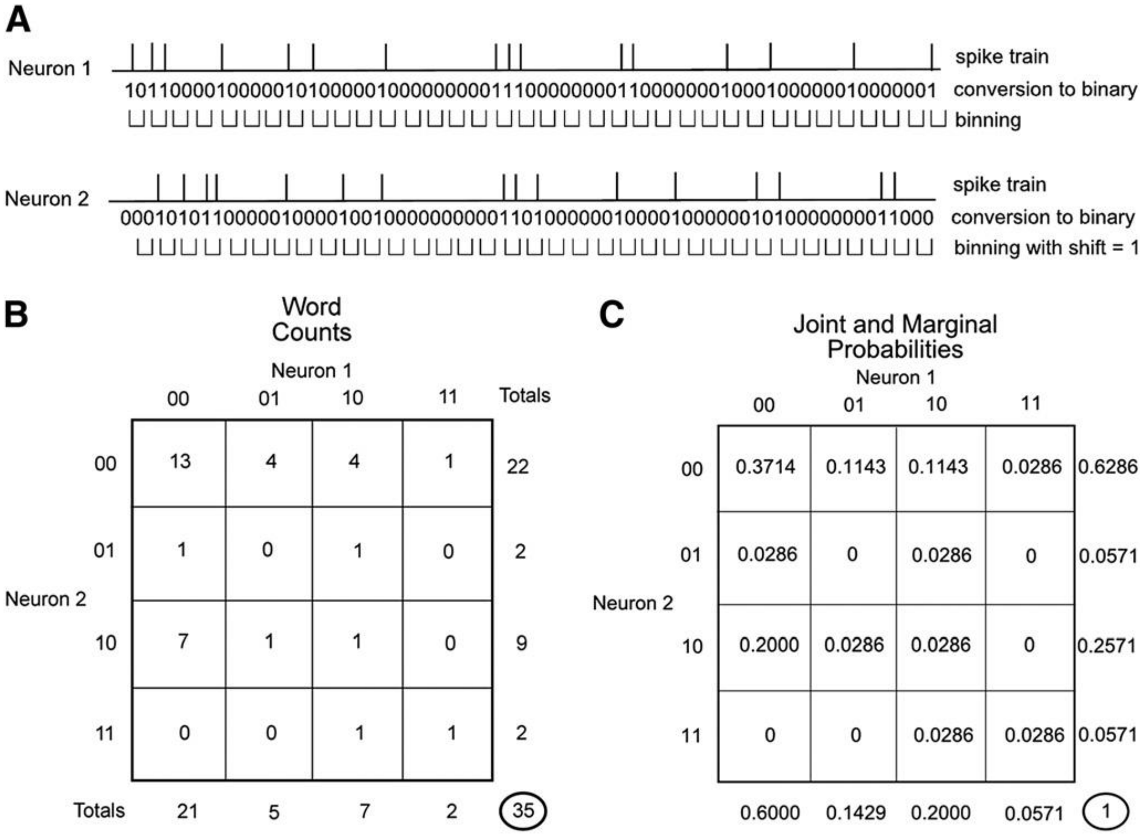


Spike trains generated by probabilistic model



Taghva, A., Song, D., Hampson, R. E., Deadwyler, S. A., & Berger, T. W. (2012). Determination of Relevant Neuron–Neuron Connections for Neural Prosthetics Using Time-Delayed Mutual Information: Tutorial and Preliminary Results. *World neurosurgery*, *78*(6), 618-630.

TDMI between spike trains

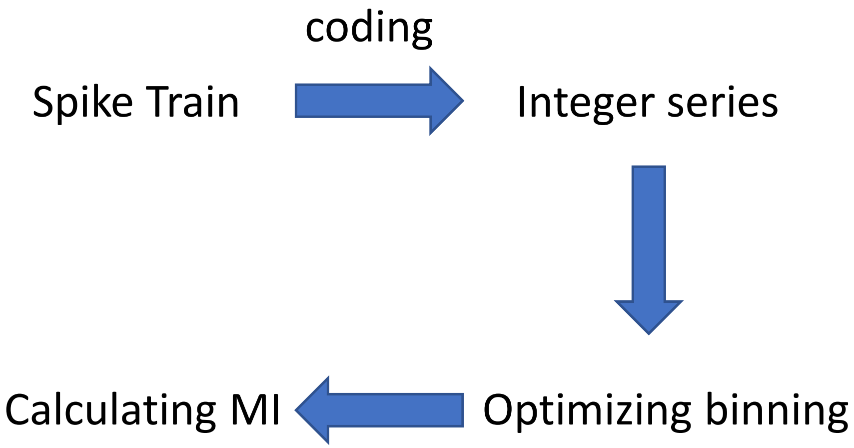


B Word Counts

Neuron 2 \ Neuron 1	00	01	10	11	Totals
00	13	4	4	1	22
01	1	0	1	0	2
10	7	1	1	0	9
11	0	0	1	1	2
Totals	21	5	7	2	35

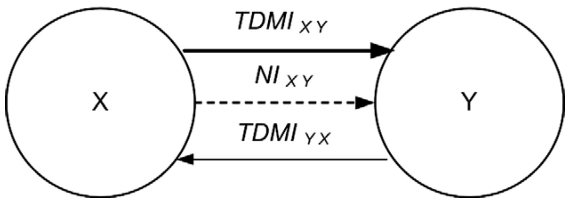
C Joint and Marginal Probabilities

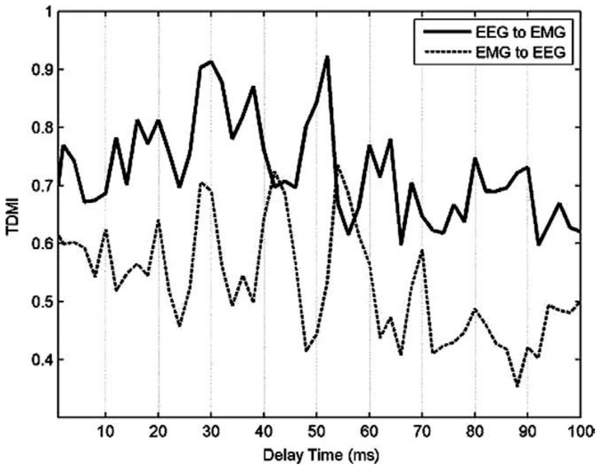
Neuron 2 \ Neuron 1	00	01	10	11	
00	0.3714	0.1143	0.1143	0.0286	0.6286
01	0.0286	0	0.0286	0	0.0571
10	0.2000	0.0286	0.0286	0	0.2571
11	0	0	0.0286	0.0286	0.0571
	0.6000	0.1429	0.2000	0.0571	1

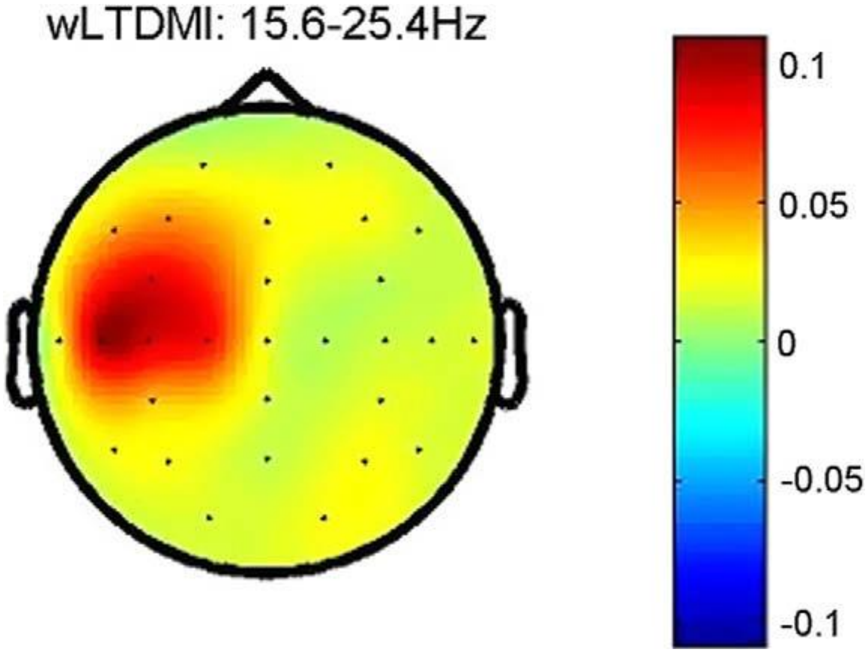


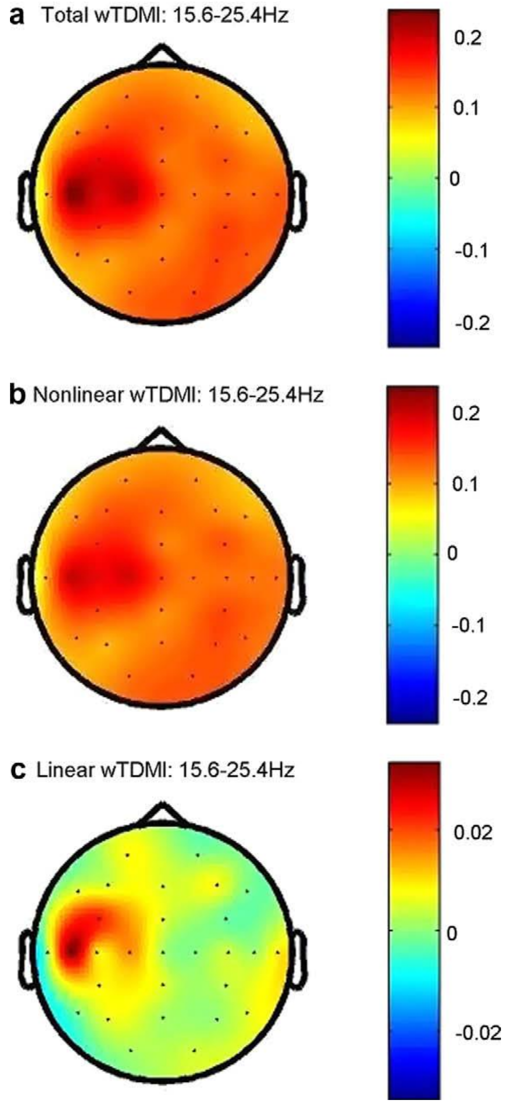
Taghva, A., Song, D., Hampson, R. E., Deadwyler, S. A., & Berger, T. W. (2012).
Determination of Relevant Neuron–Neuron Connections for Neural Prosthetics Using Time-Delayed Mutual Information: Tutorial and Preliminary Results.

TDMI between EEG and sEMG



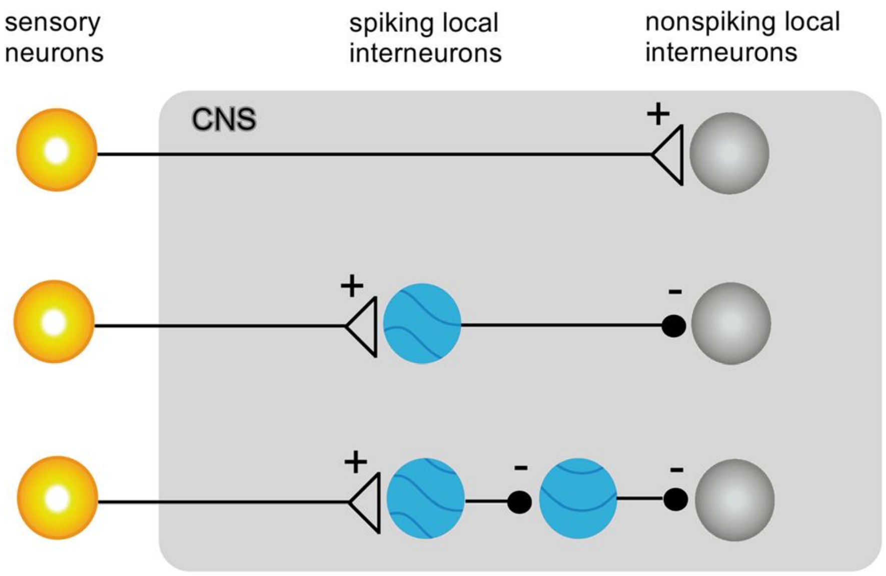


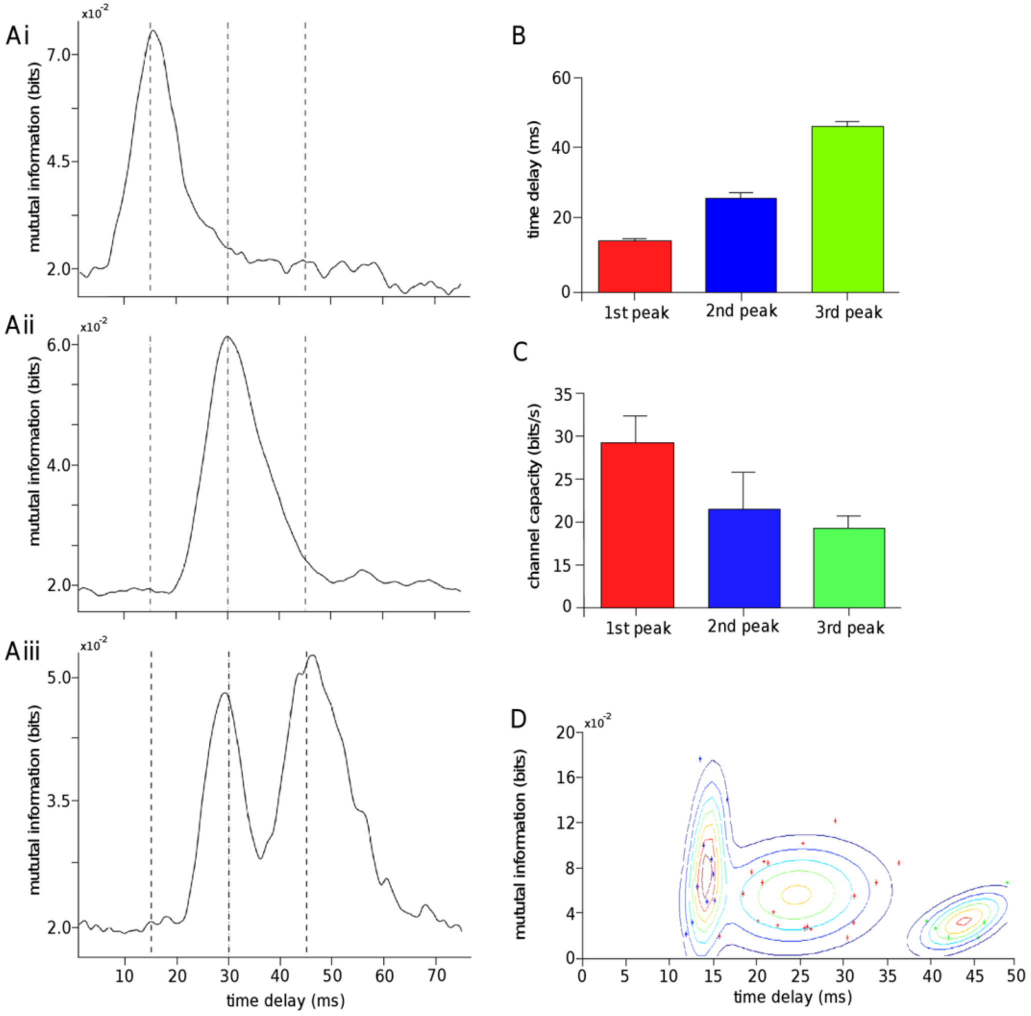




Jin, S. H., Lin, P., & Hallett, M. (2010).
Linear and nonlinear information flow based on time-delayed mutual information method and its application to corticomuscular interaction.

TDMI infers connectivity patterns





Endo, W., Santos, F. P., Simpson, D., Maciel, C. D., & Newland, P. L. (2015). Delayed mutual information infers patterns of synaptic connectivity in a proprioceptive neural network.

Mutual information of Gaussian random variables

$$\begin{cases} X_n = \alpha X_{n-1} + \varepsilon_n \\ Y_n = \beta Y_{n-1} + \xi X_{n-1} + \eta_n \end{cases}$$

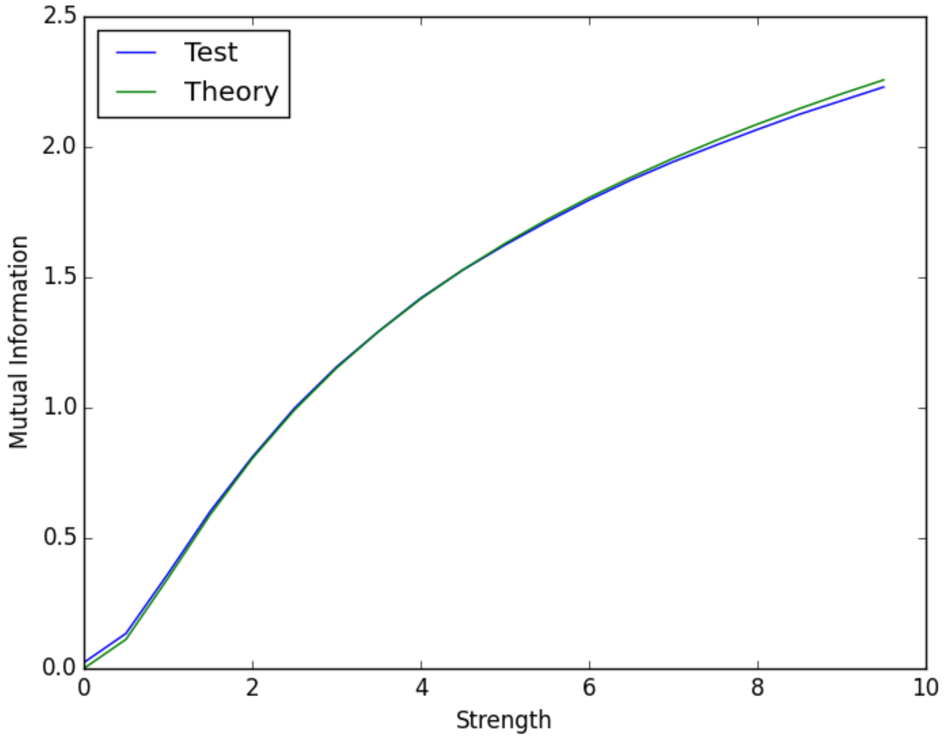
$\alpha = 0.01$， $\beta = 0.01$ #bin=50 T= 300000

$$I(X,Y) = -\frac{1}{2}\log(1-\rho^2)$$

$$\rho = \rho(\xi,\alpha,\beta) = \frac{Cov(X,Y)}{\sigma_X\sigma_Y} = \frac{E(XY) - \mu_X\mu_y}{\sigma_X\sigma_Y}$$

α and β are smaller than 1, suppose $n \gg 1$,

$$\rho(\xi)^2 = \begin{cases} \dfrac{\xi^2}{(1-\alpha^2)^2 + \xi^2(1+\alpha^2)} & \alpha = \beta \\ \dfrac{\xi^2(1-\beta^2)}{(1-\alpha\beta)^2(1-\alpha^2) + \xi^2(1-(\alpha\beta)^2)} & \alpha \neq \beta \end{cases}$$



Mutual information of Gaussian random variables

$$\begin{tikzcd}
X_{n-1} \arrow[r, "\alpha"] \arrow[dr, "\xi"] & X_n \arrow[r, "\alpha"] \arrow[dr, "\xi"] & X_{n+1} \\
Y_{n-1} \arrow[r, "\beta"] & Y_n \arrow[r, "\beta"] & Y_{n+1}
\end{tikzcd}$$

If $0 < |\alpha|, |\beta| \ll 1$, then, $\rho^2 = \dfrac{\xi^2}{1+\xi^2}$

$$\boxed{I(X,Y) = \frac{1}{2}\log(1+\xi^2)}$$

$$\boxed{\rho = \frac{E(XY)}{\sigma_X \sigma_Y}}$$

If $0 < |\alpha| \ll |\beta| < 1$, then, $\rho^2 = \dfrac{\xi^2(1-\beta^2)}{(1-2\alpha\beta)+\xi^2}$

$$\boxed{I(X,Y) = -\frac{1}{2}\log\left(\frac{(1-2\alpha\beta)+\xi^2\beta^2}{(1-2\alpha\beta)+\xi^2}\right)}$$

When $\xi \ll 1$, $I(X,Y)$ approaches $O(\xi^2)$.
When $\xi \gg 1$, $I(X,Y) = -\log\beta$

If $0 < |\beta| \ll |\alpha| < 1$, then, $\rho^2 = \dfrac{\xi^2}{(1-2\alpha\beta)(1-\alpha^2)+\xi^2}$

$$\boxed{I(X,Y) = -\frac{1}{2}\log\left(\frac{(1-2\alpha\beta)(1-\alpha^2)}{(1-2\alpha\beta)(1-\alpha^2)+\xi^2}\right)}$$

When $\xi \ll 1$, $I(X,Y)$ approaches $O(\xi^2)$.

When $\xi \gg 1$, $I(X,Y) = \frac{1}{2}\log\left(1+\frac{\xi^2}{(1-2\alpha\beta)(1-\alpha^2)}\right)$

Mutual information of Gaussian random variables

$$\begin{tikzcd}
X_{n-1} \arrow[r, "\alpha"] \arrow[dr, "\xi"] & X_n \arrow[r, "\alpha"] \arrow[dr, "\xi"] & X_{n+1} \\
Y_{n-1} \arrow[r, "\beta"] & Y_n \arrow[r, "\beta"] & Y_{n+1}
\end{tikzcd}$$

If $|\alpha|, |\beta| < 1$, then,

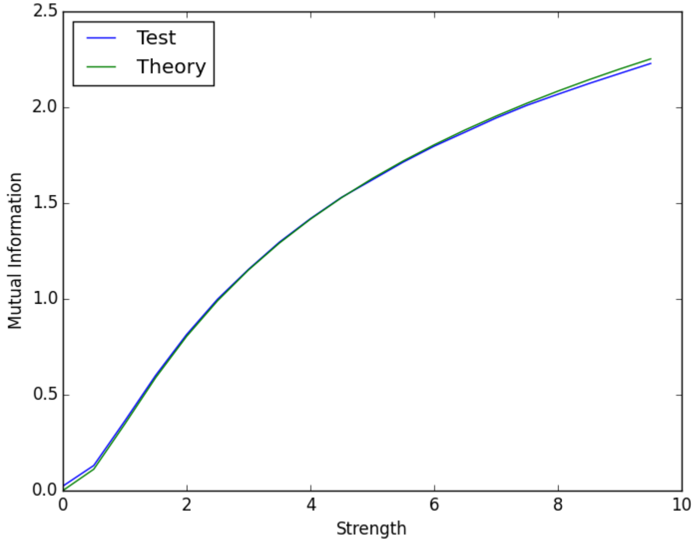
$$\boxed{I(X,Y) = \begin{cases} -\frac{1}{2} log\left(\frac{(1-\alpha^2)^2 + \xi^2\alpha^2}{(1-\alpha^2)^2 + \xi^2(1+\alpha^2)}\right) & \alpha = \beta \\ -\frac{1}{2} log\left(\frac{(1-\alpha\beta)^2(1-\alpha^2) + \xi^2\beta^2(1-\alpha^2)}{(1-\alpha\beta)^2(1-\alpha^2) + \xi^2(1-(\alpha\beta)^2)}\right) & \alpha \neq \beta \end{cases}}$$

When $\xi \ll 1$, $I(X,Y)$ approaches $O(\xi^2)$.

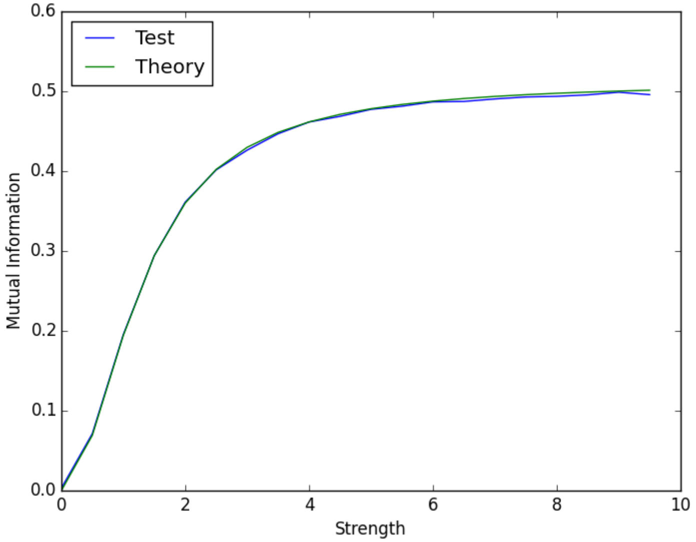
When $\xi \gg 1$, $I(X,Y) = \begin{cases} -\frac{1}{2} log\left(\frac{\alpha^2}{1+\alpha^2}\right) & \alpha = \beta \\ -\frac{1}{2} log\left(\frac{\beta^2(1-\alpha^2)}{1-(\alpha\beta)^2}\right) & \alpha \neq \beta \end{cases}$

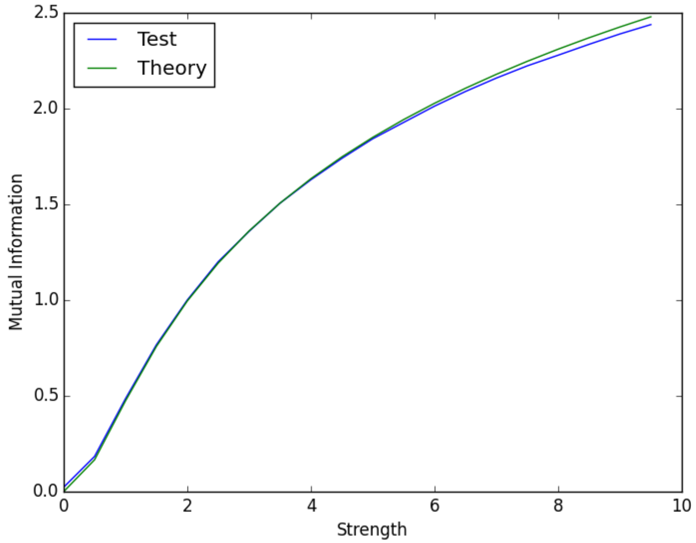
$$\boxed{\rho = \frac{E(XY)}{\sigma_X \sigma_Y}}$$

$\alpha = 0.01$, $\beta = 0.01$ #bin=150 T= 300000

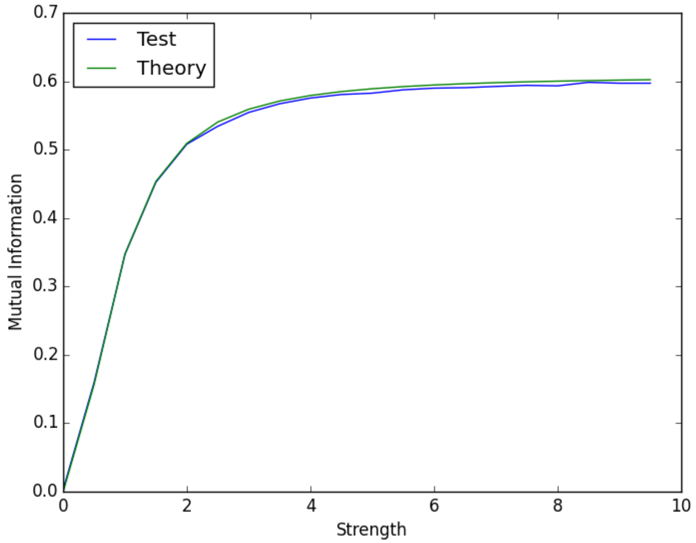


$\alpha = 0.01$, $\beta = 0.6$ #bin=50 T= 300000

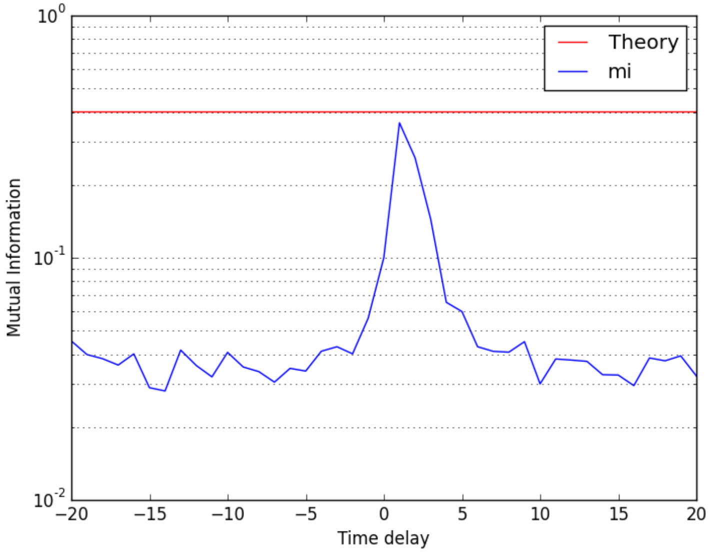


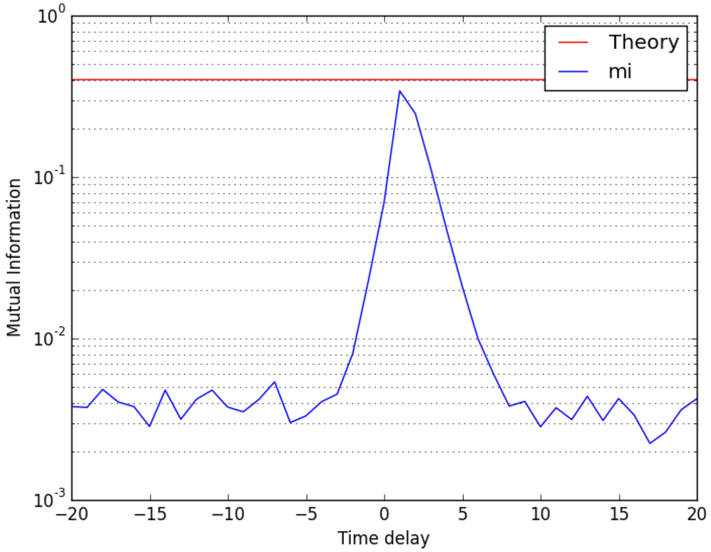


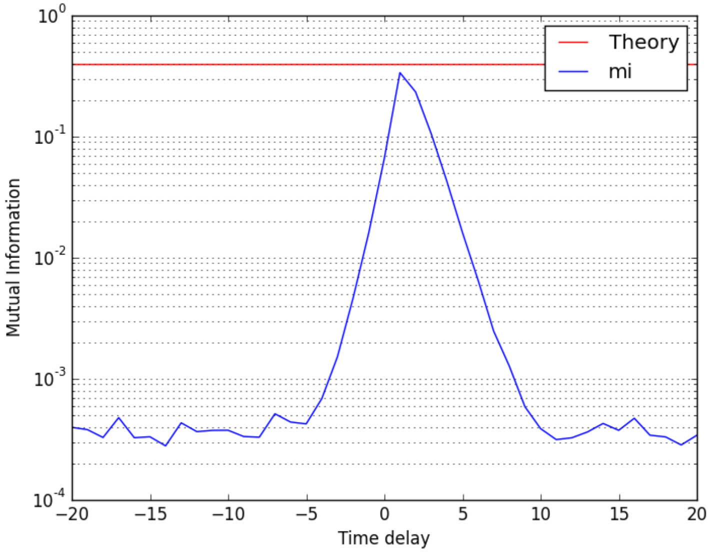
$\alpha = 0.6$, $\beta = 0.01$ #bin=150 T= 300000



$\alpha = 0.5$, $\beta = 0.6$ #bin=50 T= 300000







1k trials

10k trials

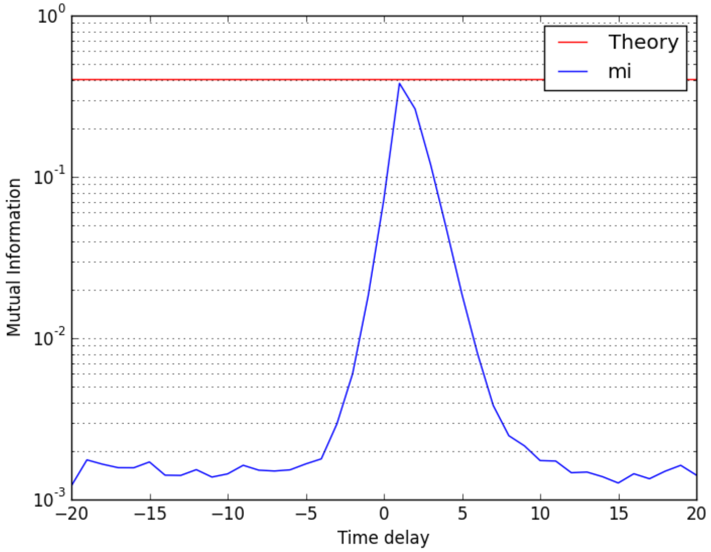
100k trials

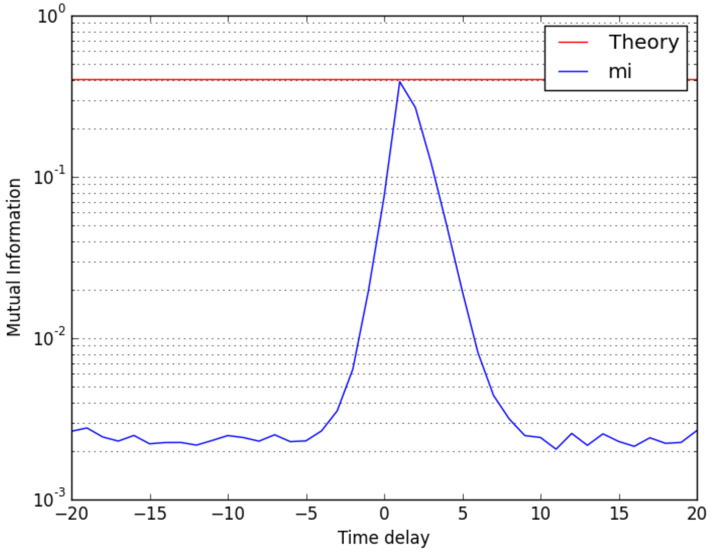
$$\alpha = \beta = 0.5, \xi = 1, \#bins = 10$$

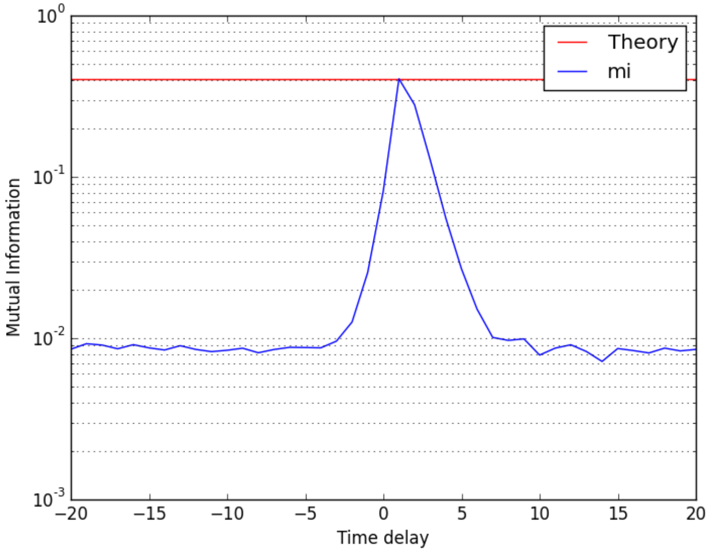
20 bins

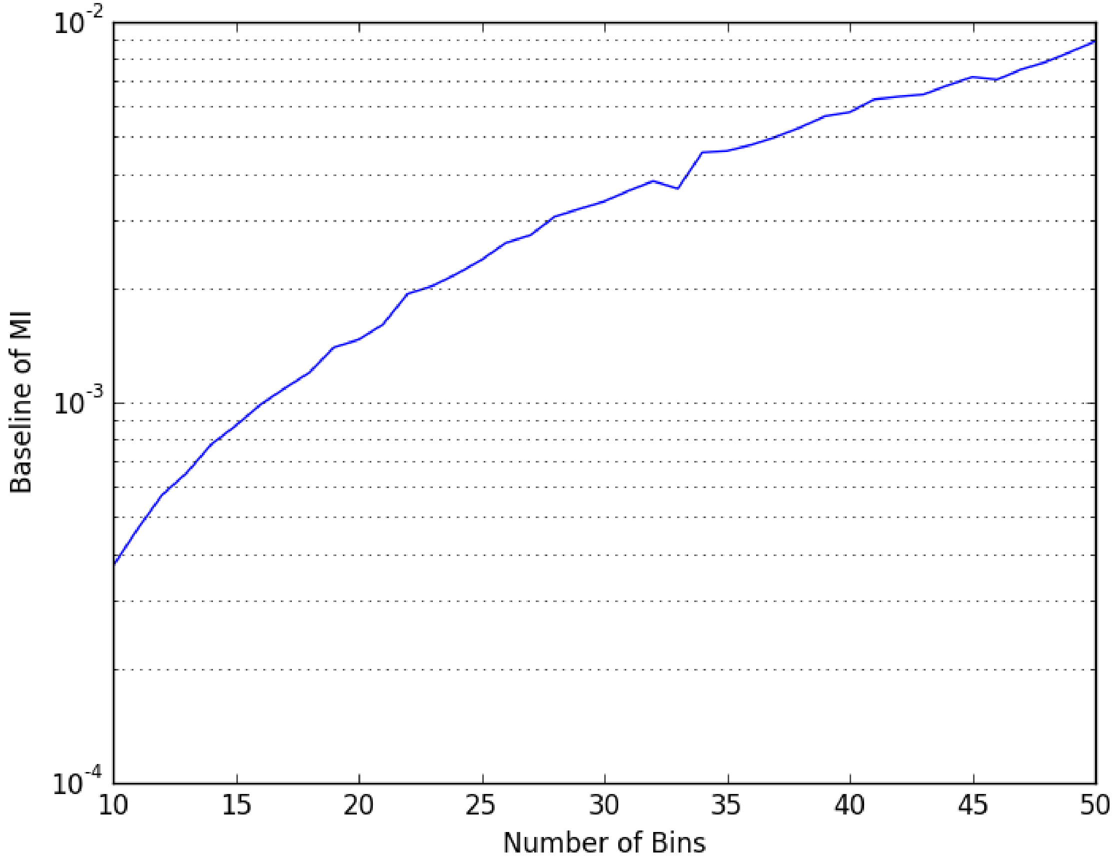
25 bins

50 bins



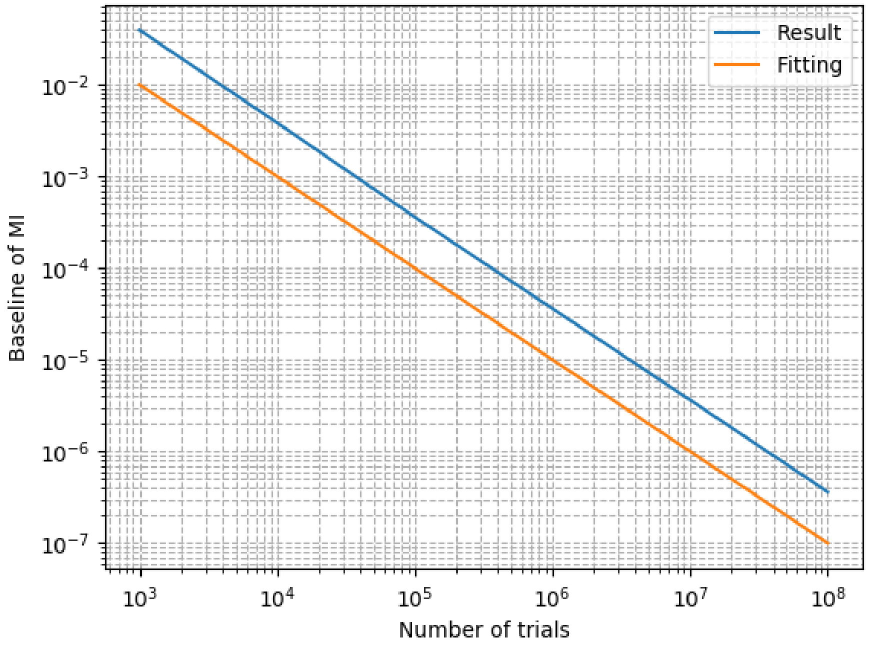






#trials = 100 k

$$Baseline \propto \frac{1}{Length\ of\ data\ set}$$



#bins = 10

TDMI between spike train and local field potential(LFP)

Spike Train

dt

0 0 0 0 1 0 0 0 1 0 0 0 0 1 0 1 $\longrightarrow$ X

Local Field Potential

$I_{t_1}, I_{t_2}, I_{t_3}, I_{t_4}, I_{t_5}, I_{t_6}, \dots, I_{t_n}$ $\longrightarrow$ Y

$$I(X;Y,\tau) = \sum_{x \in X} \sum_{y \in Y(\tau)} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

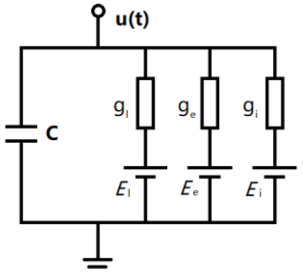
$$I_{real} \cong I_{obs} + \frac{B_X + B_Y - B_{XY} - 1}{2N}$$

Roulston, M. S. (1999).
Estimating the errors on measured entropy and mutual information.

Conductance-based Integrate-and-fire model:

$$C\frac{dv}{dt} = -g_l(v - \epsilon_l) - g_Q(v - \epsilon_Q) \quad Q \in \{e, i\}$$

$$g_Q = S_Q \sum_{j, t \geq t_j} \exp(-\frac{t - t_j}{\tau_Q})$$



When $v(t = t_i) \geq v_\theta$,
$v(t) = v_r$, $\quad v \in [t_i, t_i + \tau_{ref})$

'Point source' current model of local field potential:

$$V = \frac{1}{4\pi\sigma} \sum_i \frac{I_i}{\boldsymbol{r_0} - \boldsymbol{r_i}}$$

$$I = -g_l(v - \epsilon_l) - g_Q(v - \epsilon_Q) \quad Q \in \{e, i\}$$

TDMI Estimation

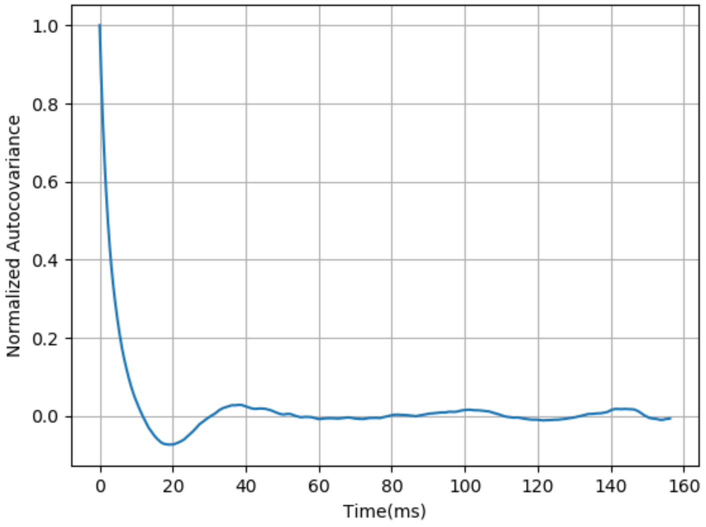
$$I(X;Y,\tau) = \sum_{x\in X}\sum_{y\in Y(\tau)} p(x,y)\log\frac{p(x,y)}{p(x)p(y)} \longrightarrow p(x,y)\text{ estimation}$$

$$X(t) = \{X_{t_1}, X_{t_2}, \dots, X_{t_n}\} \longrightarrow P(X) \rightarrow \boxed{I(X;Y,\tau)}$$

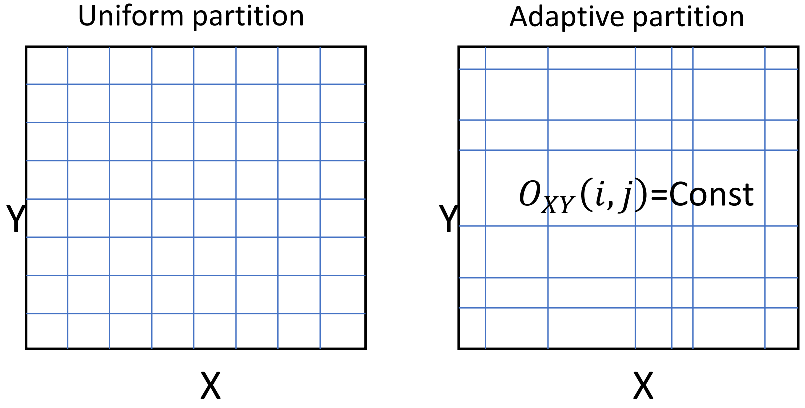
- Treated time series of LFP as WSS signal.
- Neglect the autocovariance length of LFP.

$$X' = \{X_{t^1}, X_{t^2}, \dots, X_{t^m}\},\ \mathrm{m} < \mathrm{n}$$

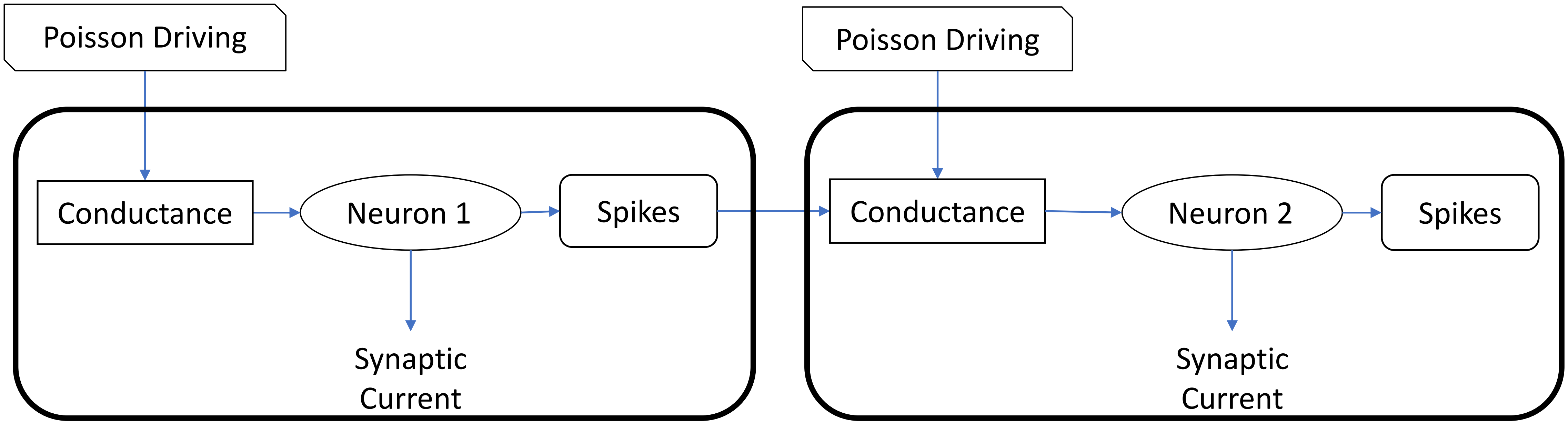
$$X_{t^i} = \{X_{t_i}, X_{t_{i+m}}, \dots, X_{t_{i+km}}\} \rightarrow P(X_{t^i}) \rightarrow \boxed{I\left(X_{t^i}; Y_{t^{i+\tau}}\right)}$$



Remark: $p(x,y)$



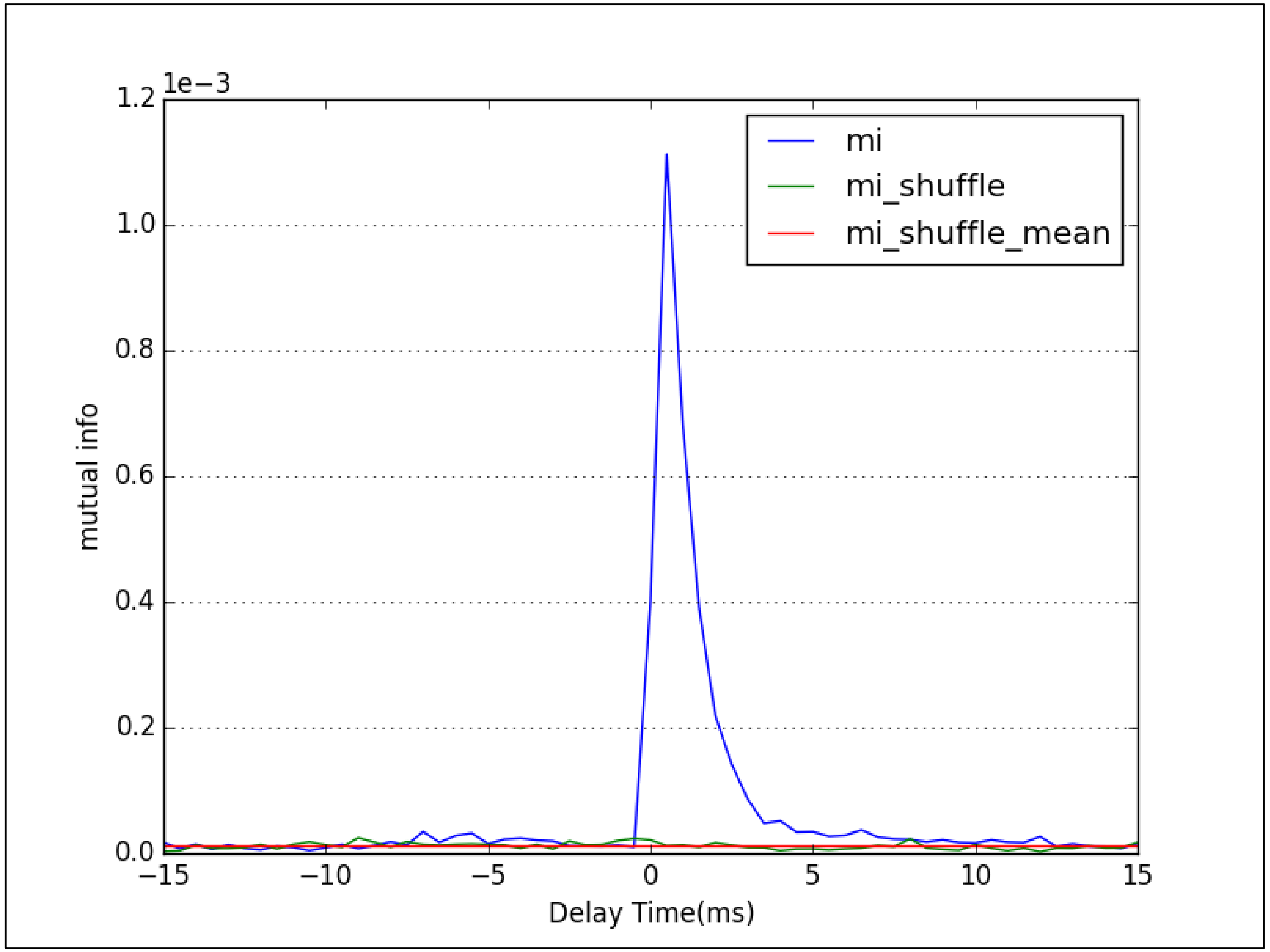
Paradigm of simulation



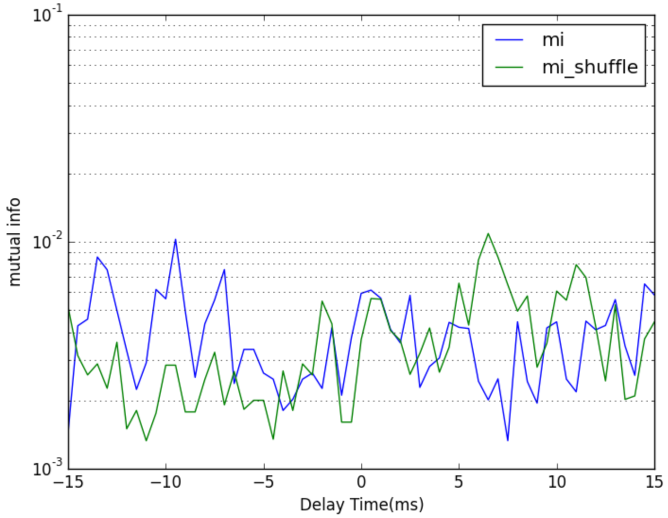
Sample Figure

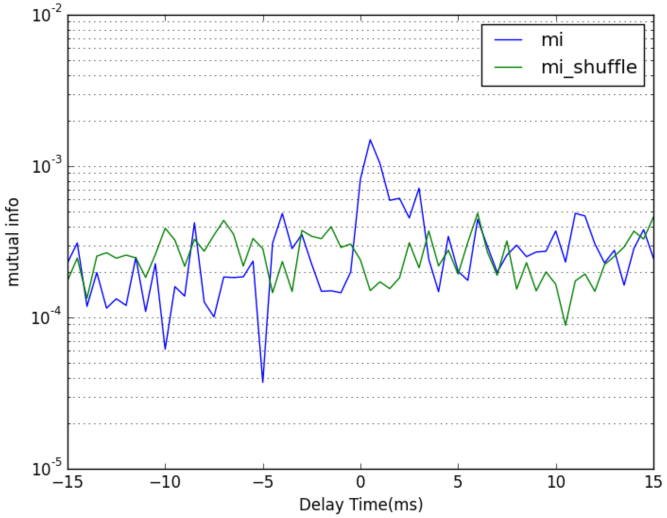
The peak lying on the positive side of the graph indicates the same direction of neuronal information as the physical connection does.

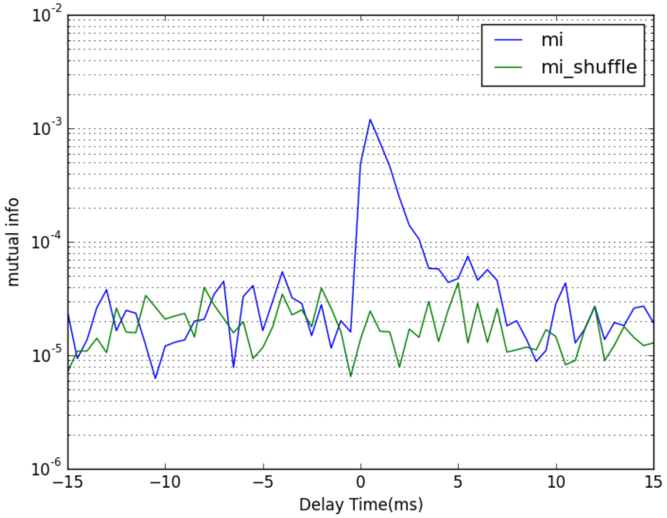
dt	0.5 ms
#bins	10
Poisson Rate	1.3kHz
Forward Strength	0.005
Synaptic Strength	0.005
T	200 s



#bins=10







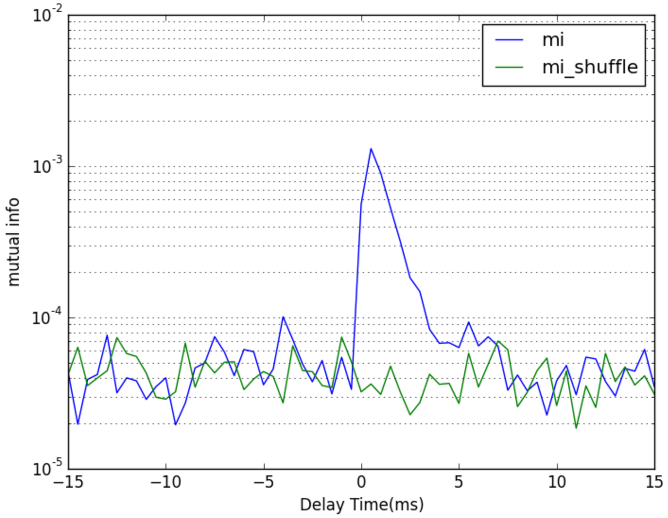
Length of time series

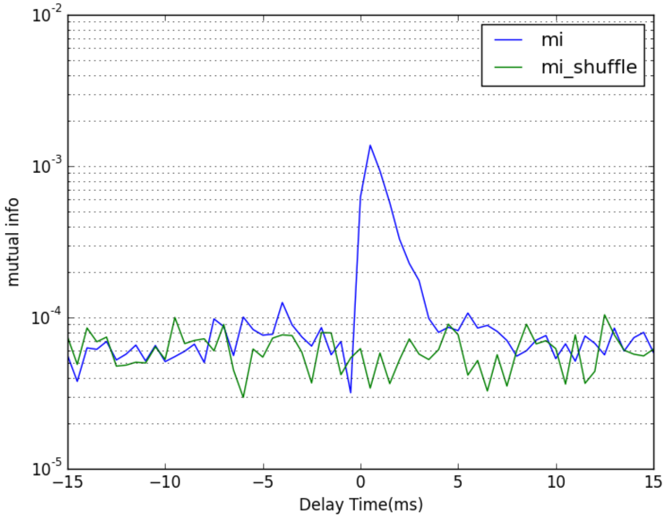
1k ms

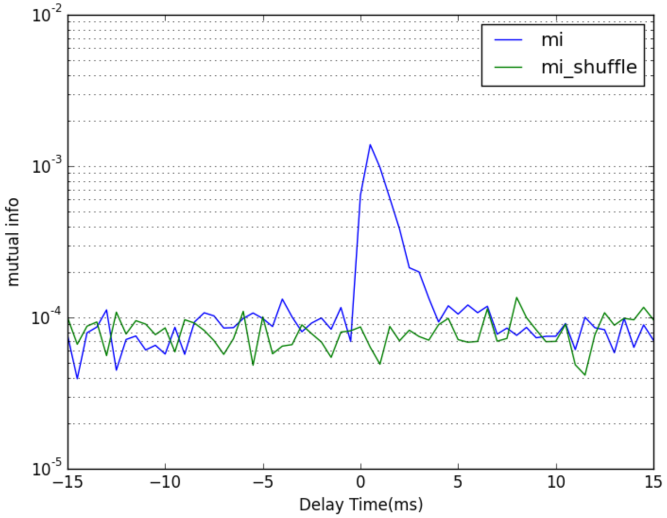
10k ms

100k ms

100k ms





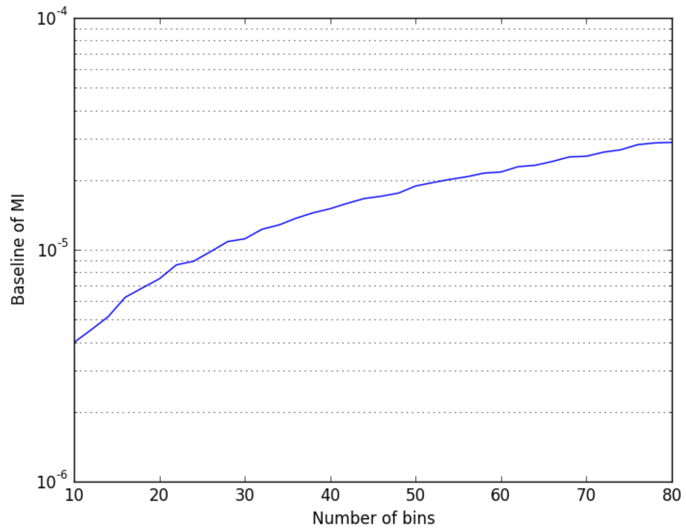


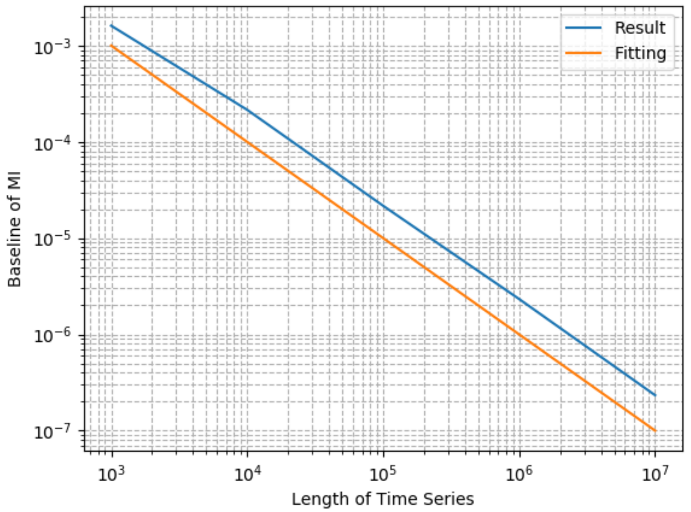
#bins=20

#bins=30

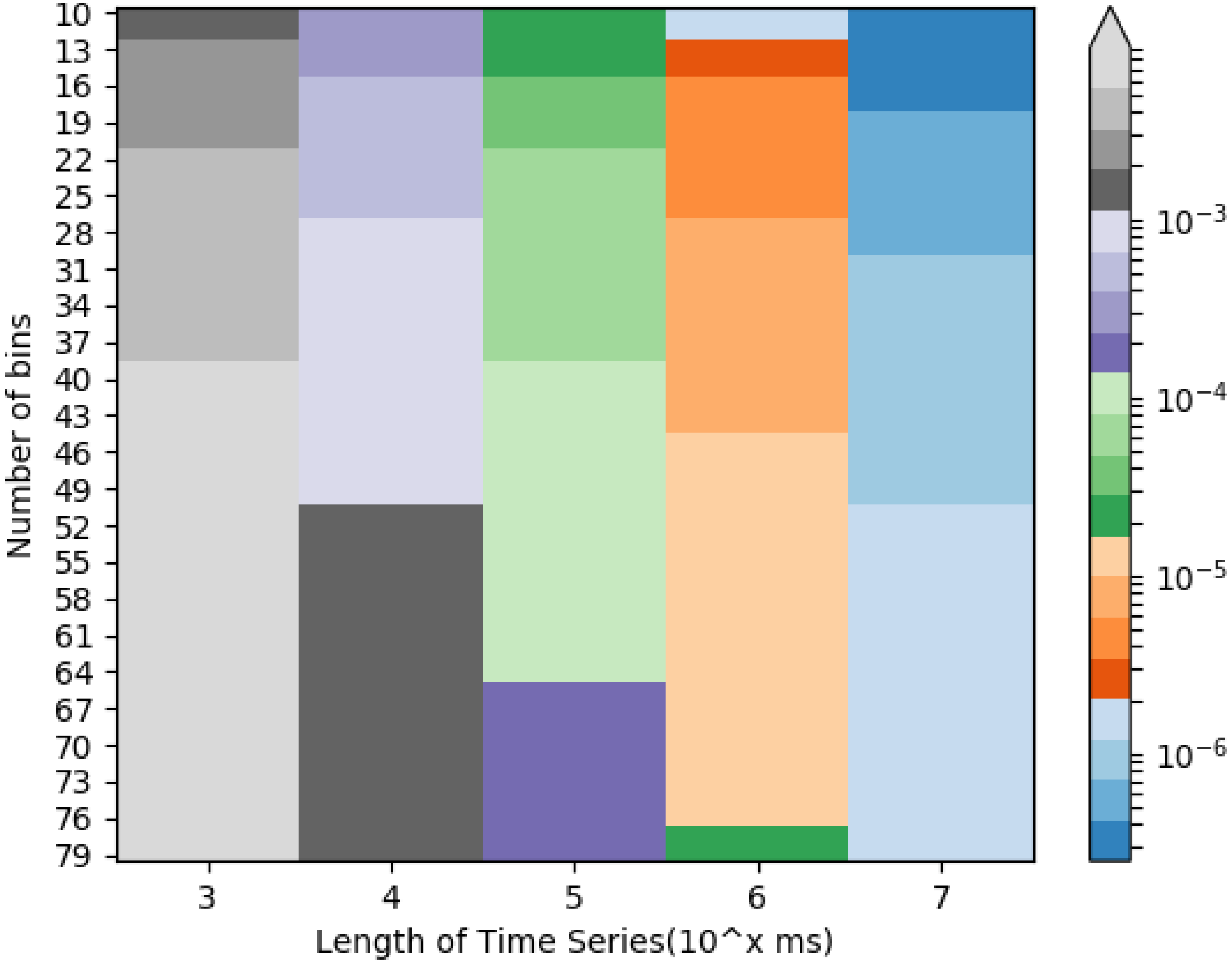
#bins=40

#bins = 10



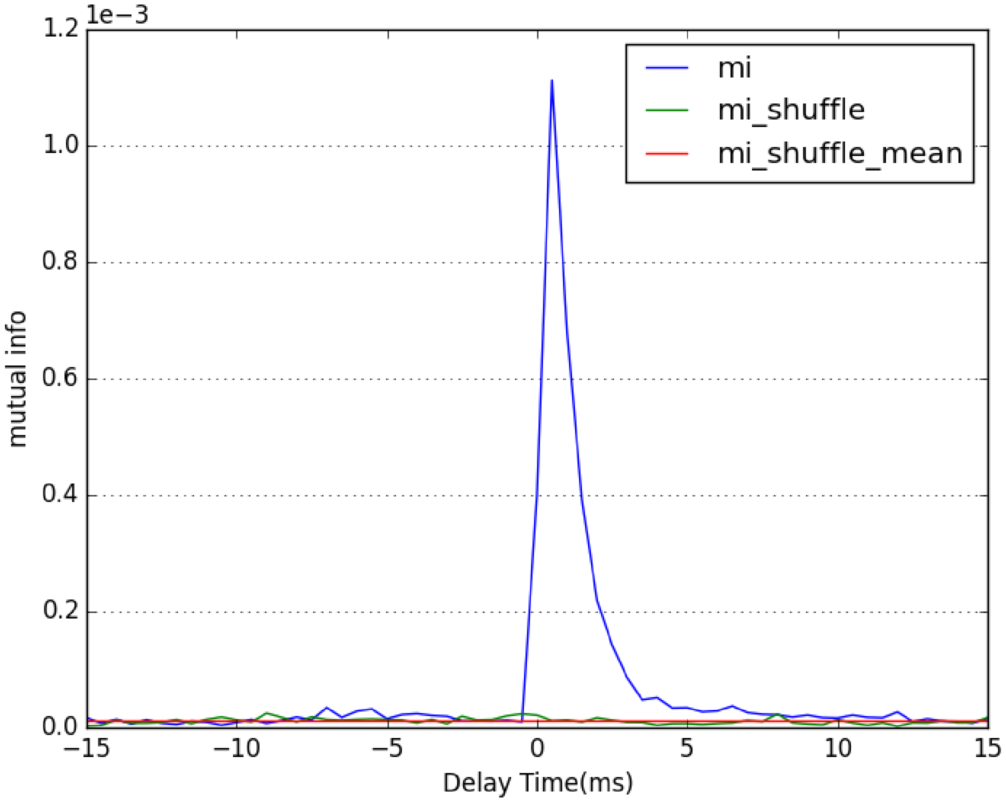


Length of time series = 600 s

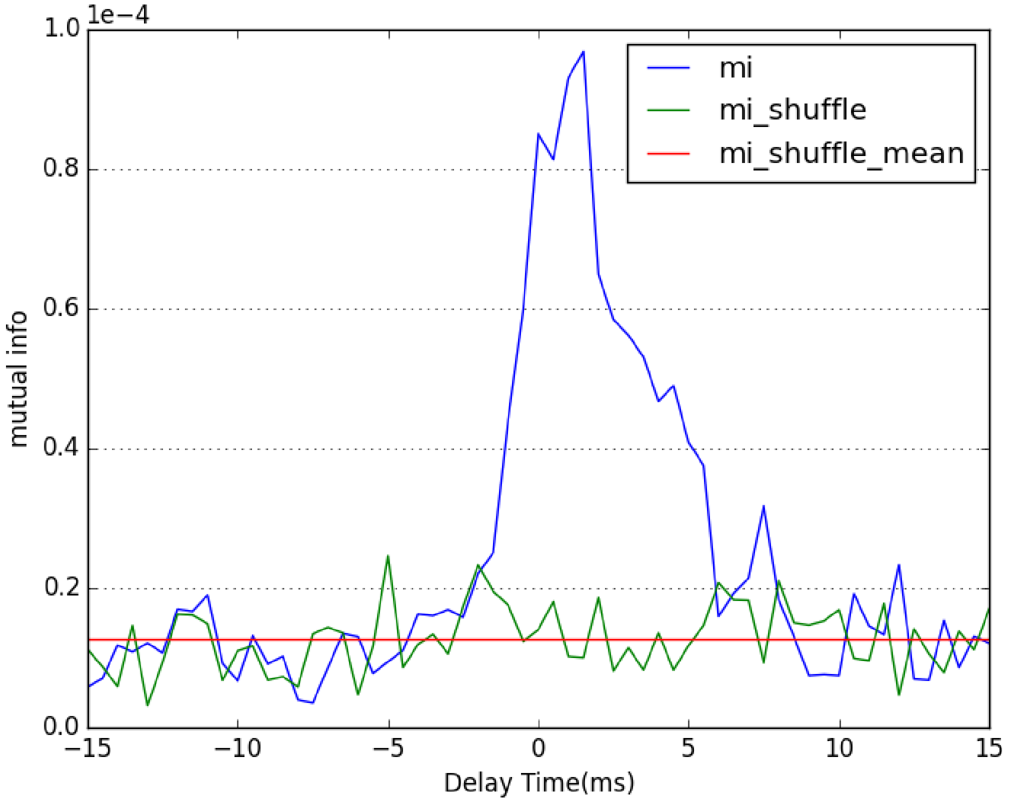


One-way Connection

Poisson Rate	1.3 kHz	dt	0.5 ms
S	0.005	#bins	10
F	0.005	T	200 s

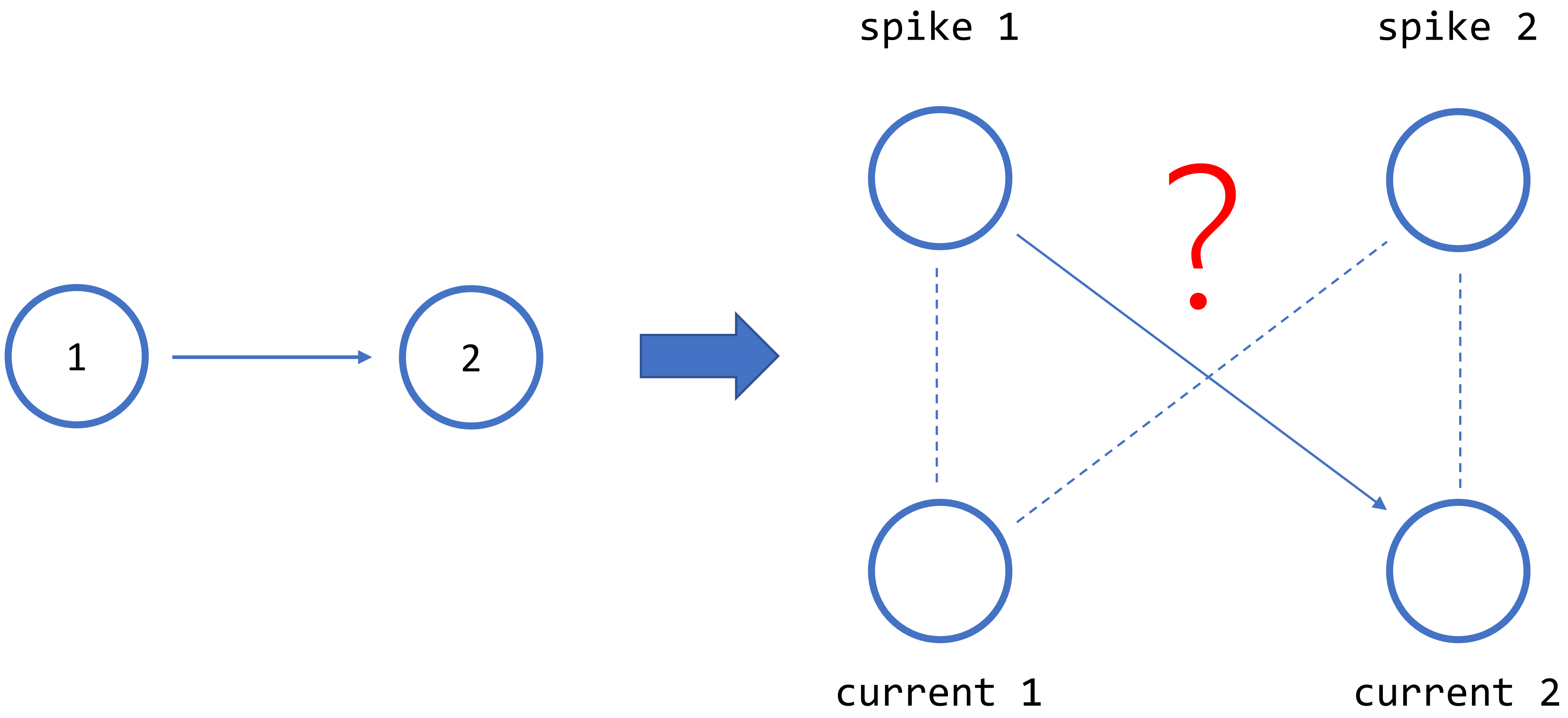


From 1 to 2



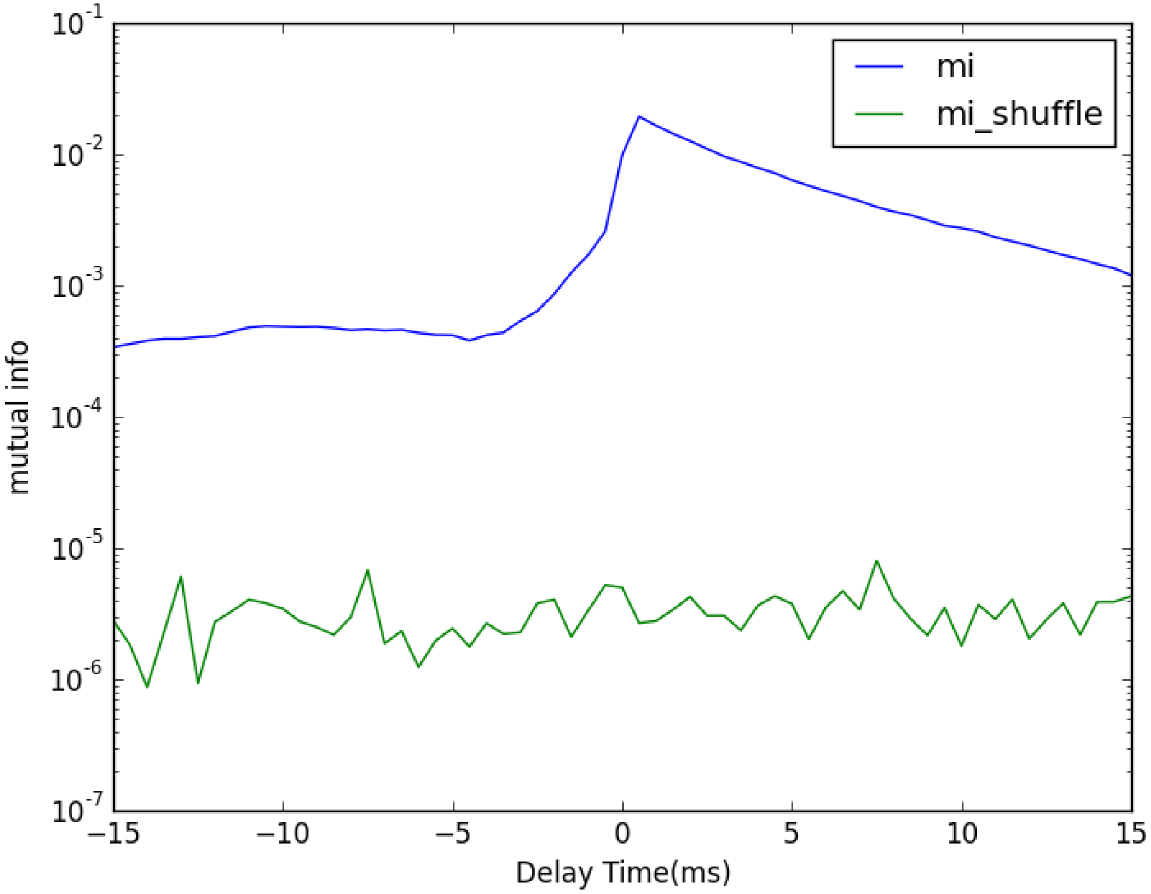
From 2 to 1

Neuronal Interaction layout

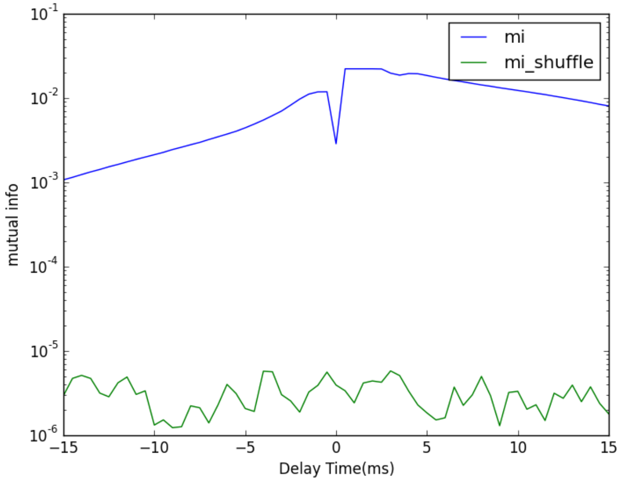


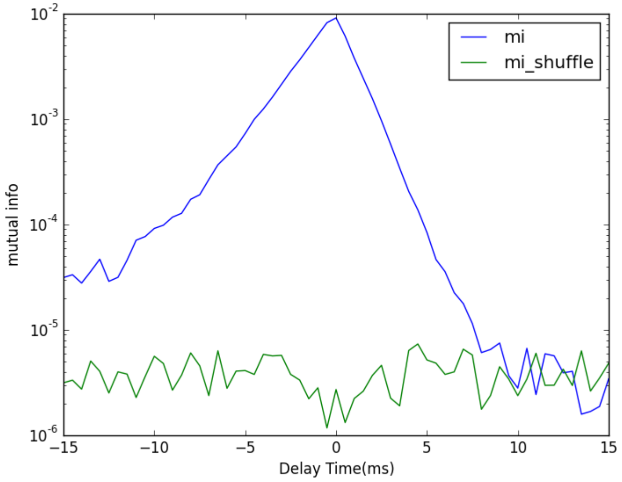
TDMI between spike and its own current, voltage and conductance respectively

I(spike, current)



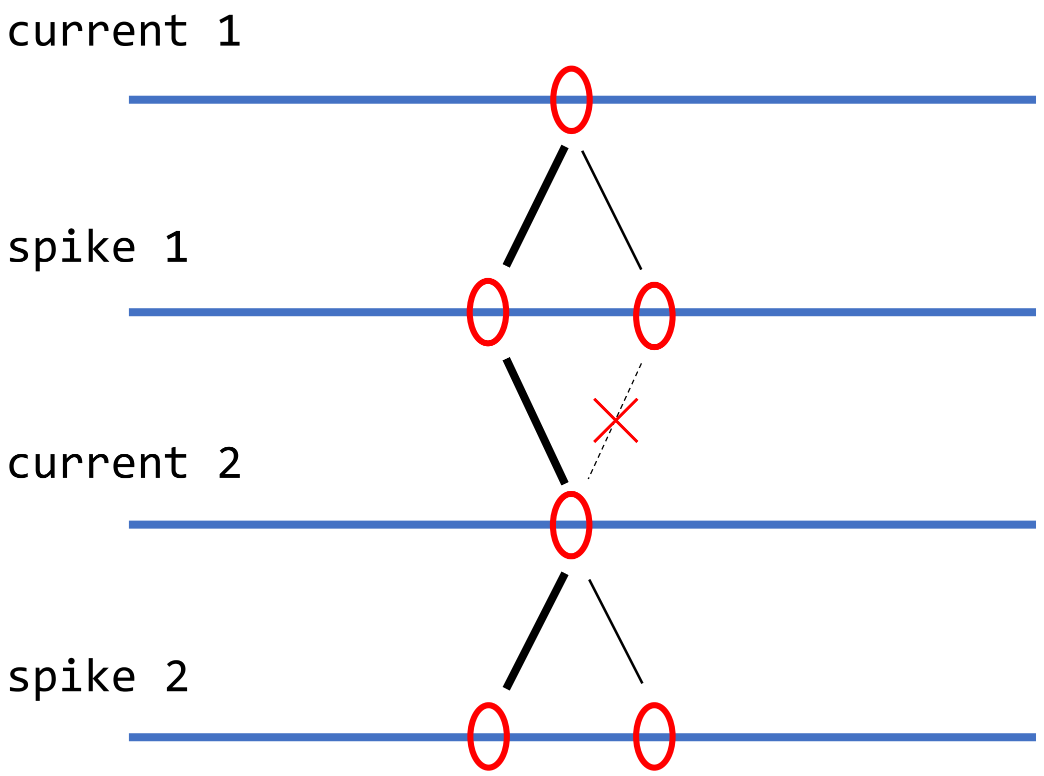
I(spike, voltage)

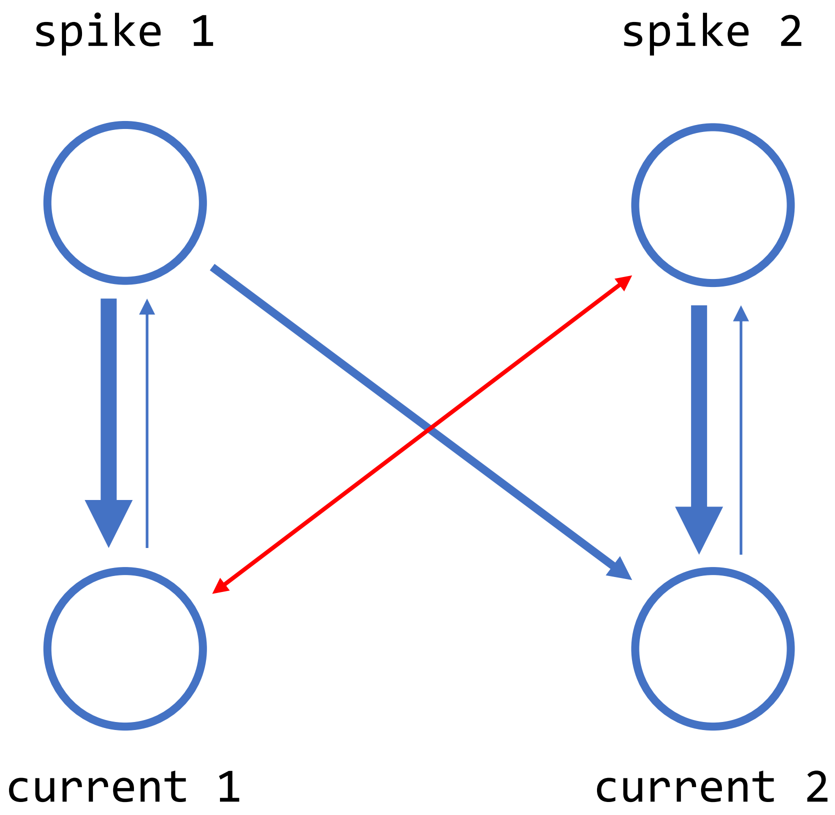




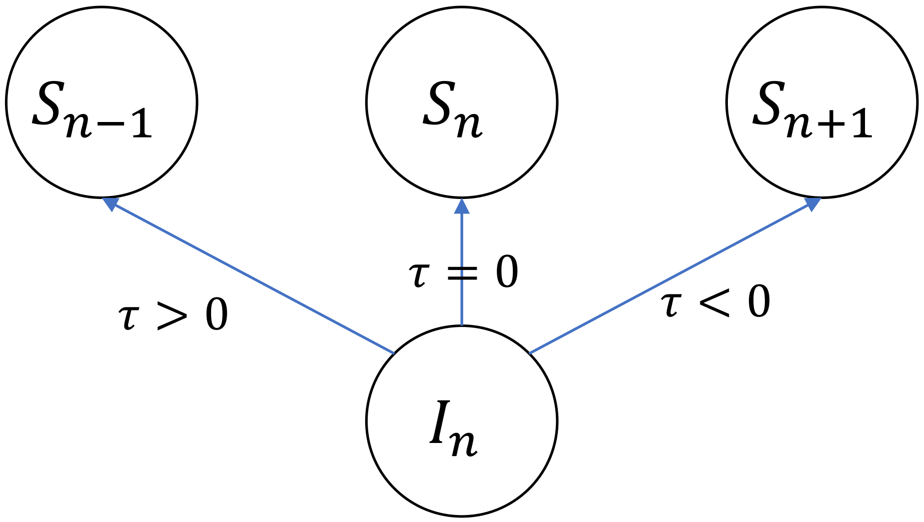
I(spike, conductance)

Neuronal Interaction layout

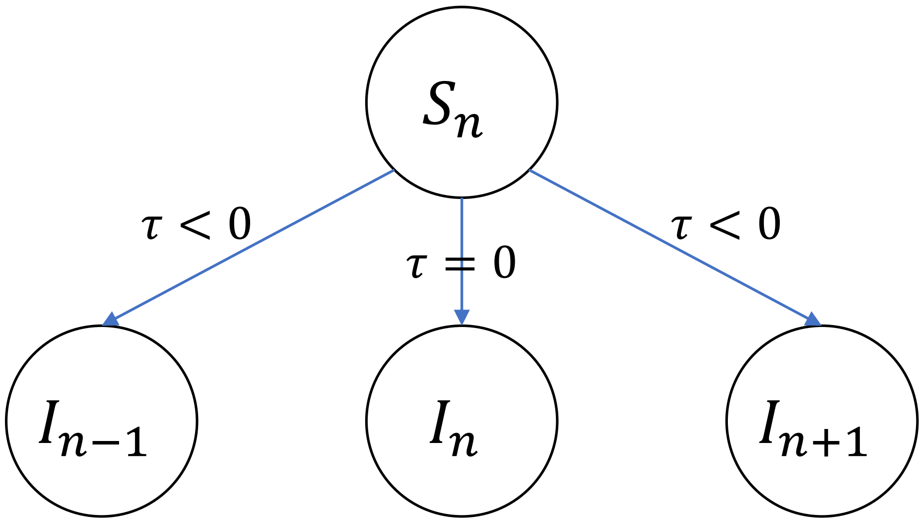




Mutual information calculation with different shifting scheme



Shifting spike train



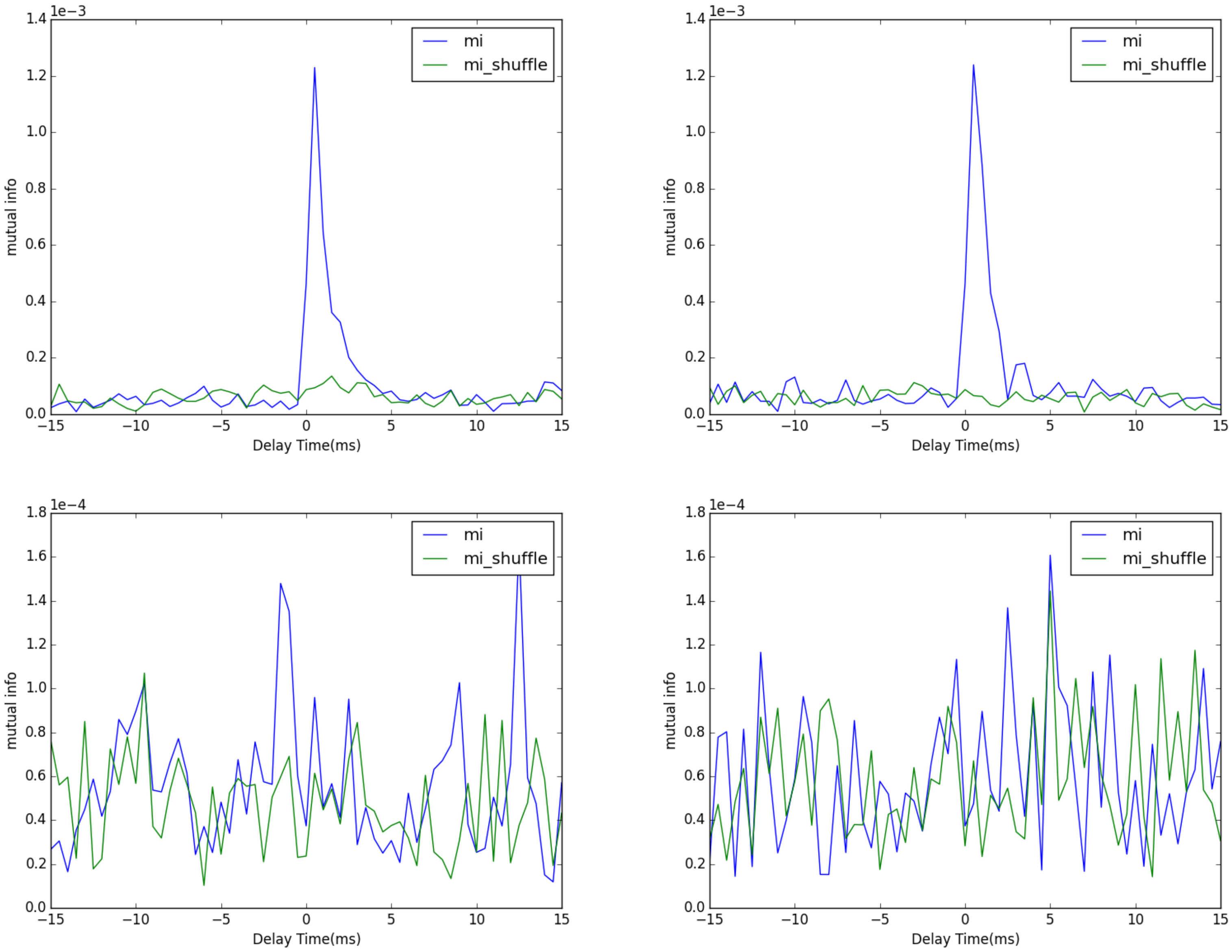
Shifting current

Mutual information calculation with different shifting scheme

Poisson Rate	1.3 kHz
Poisson S	0.005
Synaptic S	0.005
dt / dτ	0.5 ms
#bins	10
T	50mins
Delay	0 ms
Firing Rate	14 Hz
#data points	75k

Shifting spike train

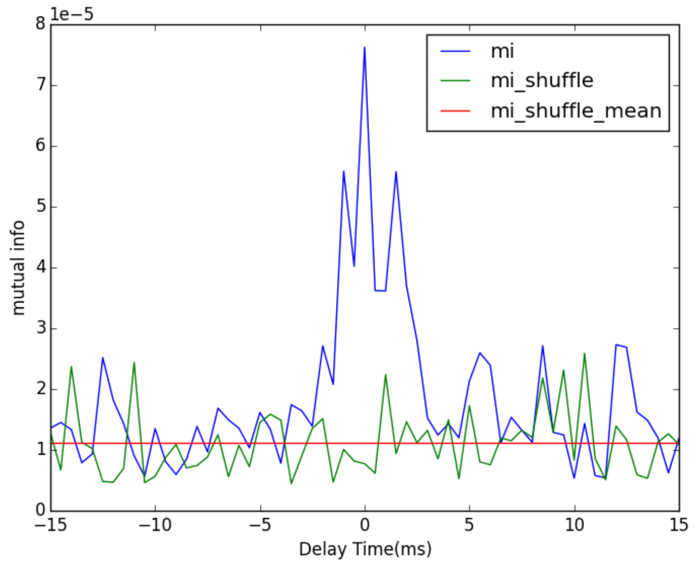
Shifting current



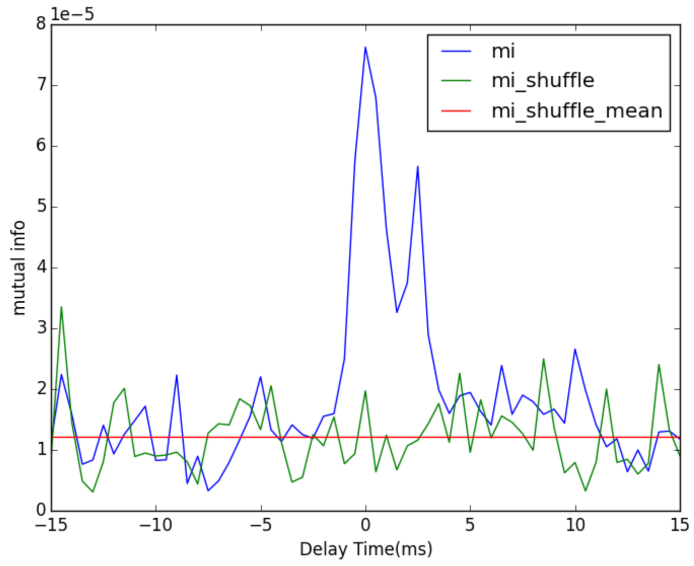
Mutual information calculation with different shifting scheme

Poisson Rate	1.3 kHz
Poisson S	0.005
Synaptic S	0.005
dt / dτ	0.5 ms
#bins	10
T	4.4h
Delay	0 ms
Firing Rate	14 Hz
#data points	400k

Shifting spike train

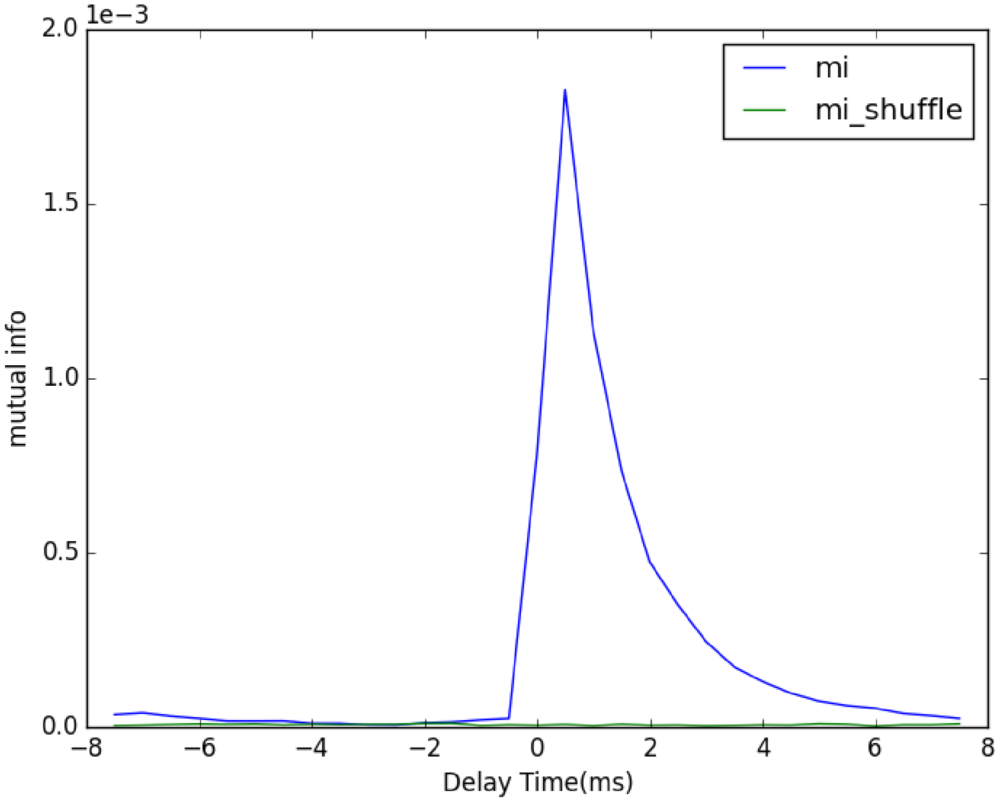


Shifting current

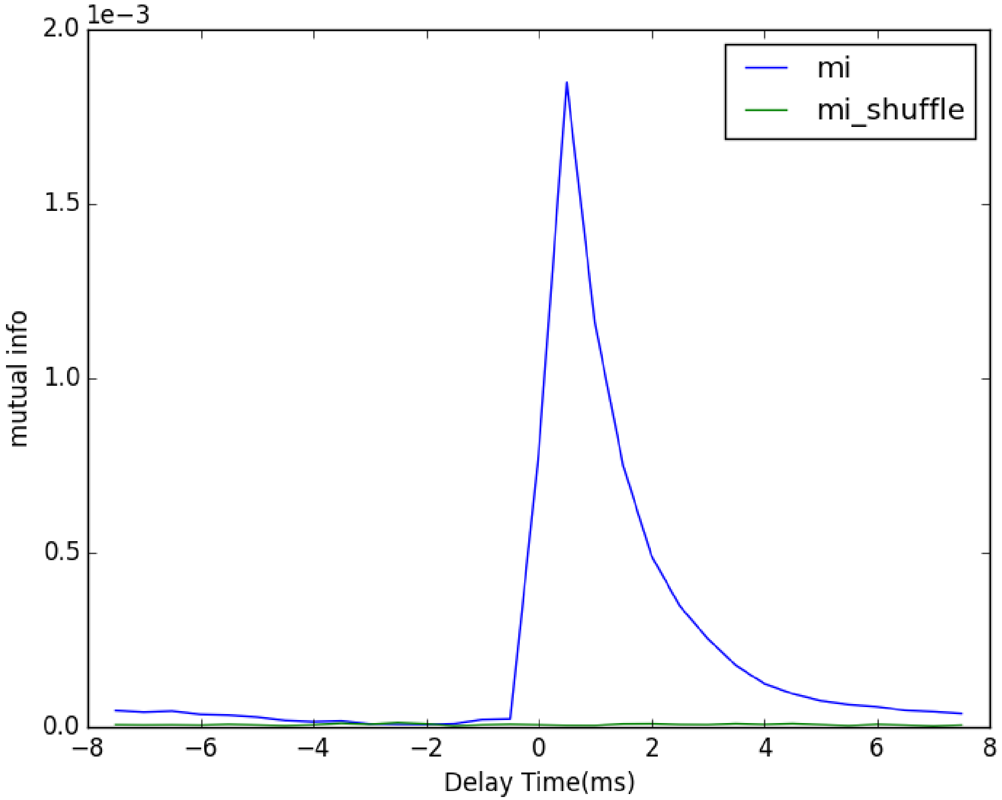


Bi-directed Connection

Poisson Rate	1.5 kHz	dt	0.5 ms
S	0.005	#bins	20
F	0.005	T	600 s



From 1 to 2



From 2 to 1

Summary

- Concepts of self-information, entropy and mutual information
- TDMI between spike-spike interaction, EEG-sEMG correlation, and its inference of neuronal connecting pattern
- TDMI between Gaussian random variables
- TDMI between spike train and local field potential

Thanks for your attention