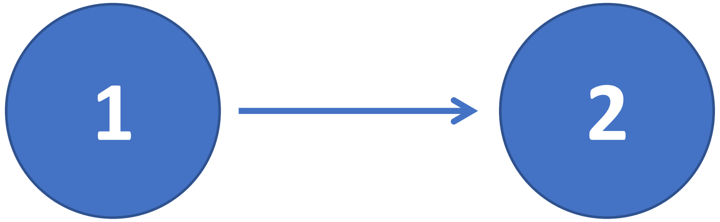
Recent numerical results related to neuronal network simulation

Kyle Chen Nov. 20, 2017

Paradigm of simulation

Poisson Driving Rate	1.5 kHz
Synaptic Strength	0.005 (changing during test)
Feedforward Strength	0.005
Total time duration	600 s
Number of bins of histogram calculation	Variable

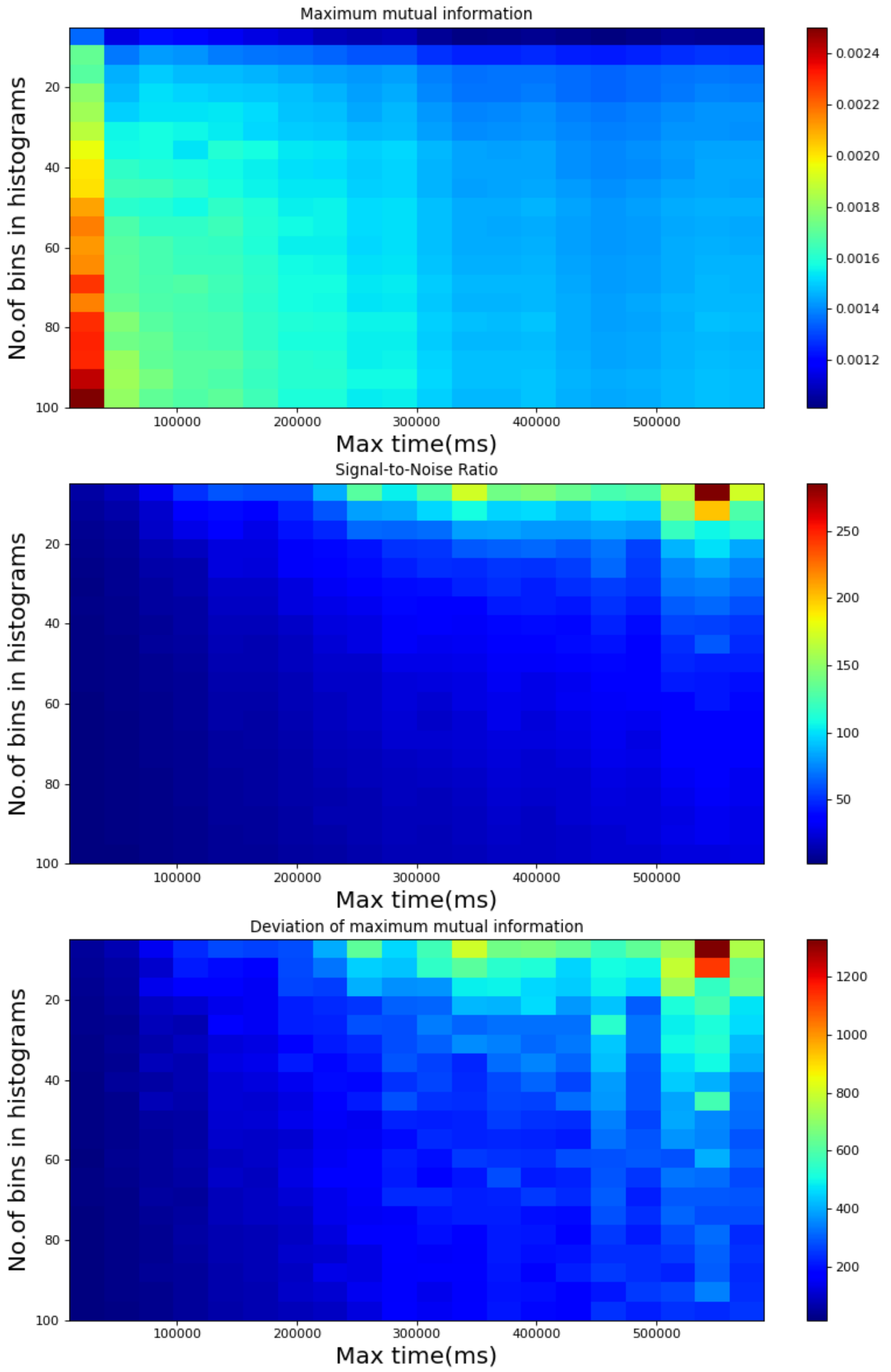


Comparison between three strategy to calculate mutual information

Direct calculation

- Treated time series of LFP as WSS signal.
- Neglect the autocovariance length of LFP.

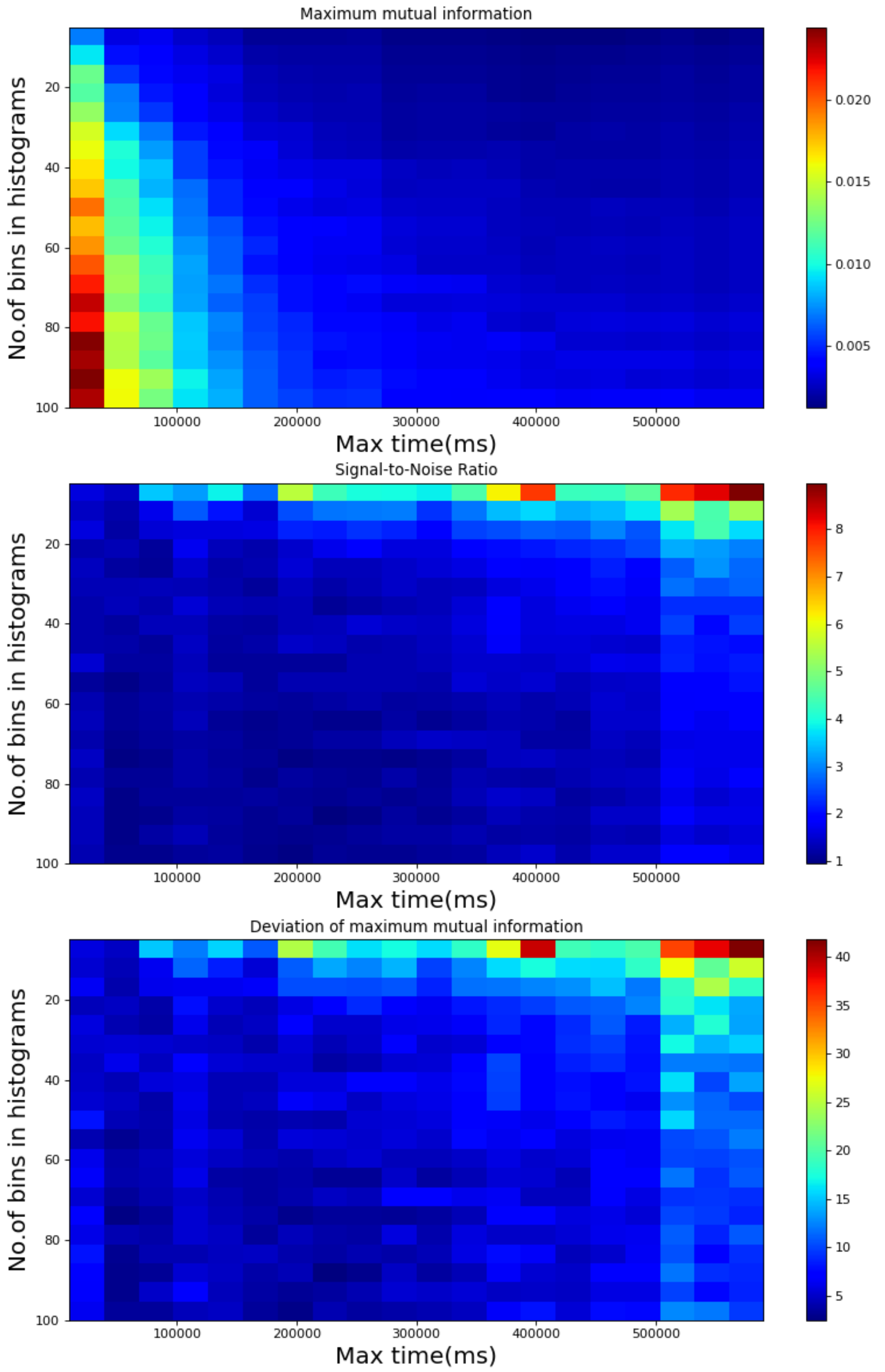
dt = 0.5 ms



Autocovariance-based calculation without average

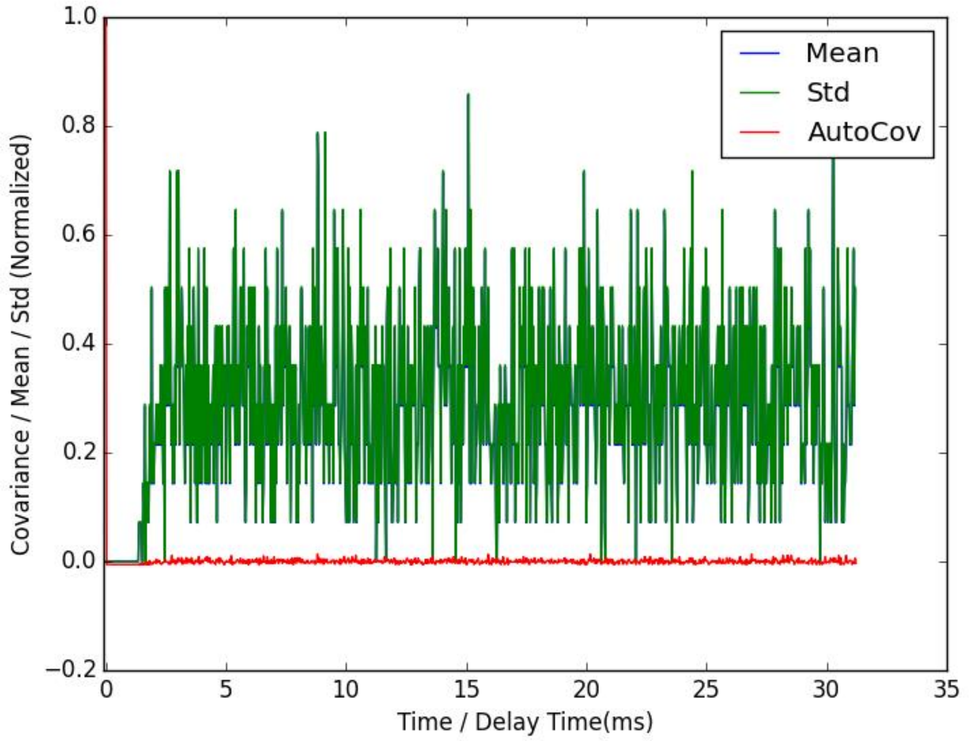
- Find the autocovariance length of LFP, which is 20 ms here.
- Break time series of LFP in pieces with length 20 ms, and treat all those pieces as a statistical ensemble.
- Calculate the PDF at every time point in such 20 ms.
- Do the same operates to the time series of binarized spike trains
- Randomly pick one time point within 20 ms for the spike train, and calculate the TDMI between the spike train at this time point and LFP at other time points

dt = 0.5 ms
Index of spike train = 21

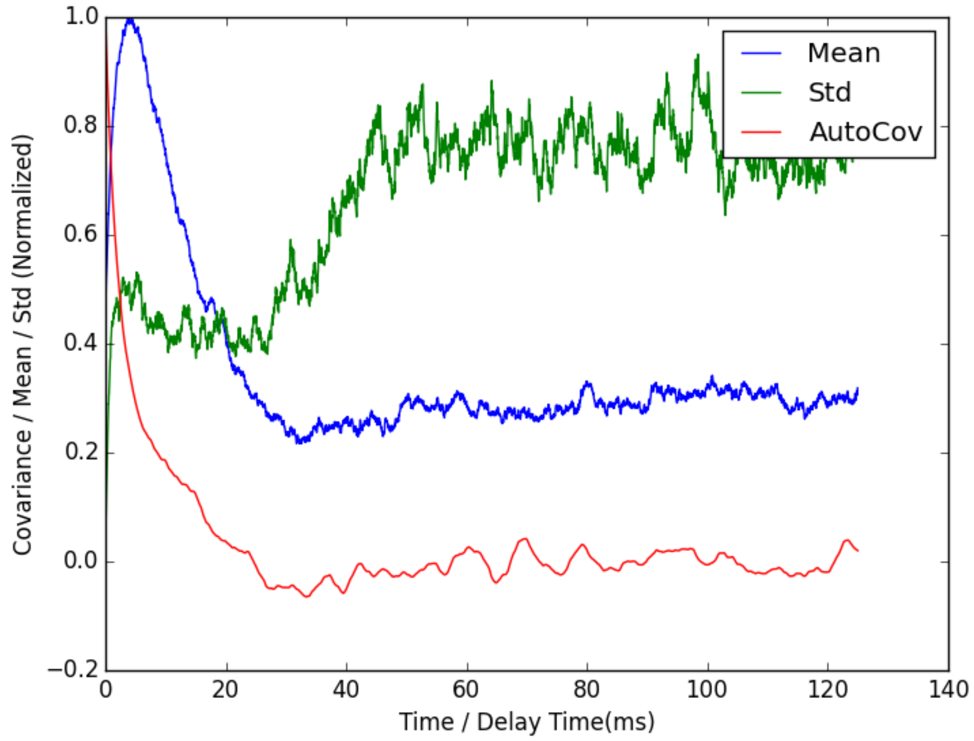


Details in mutual info measurements

S = 0.0050



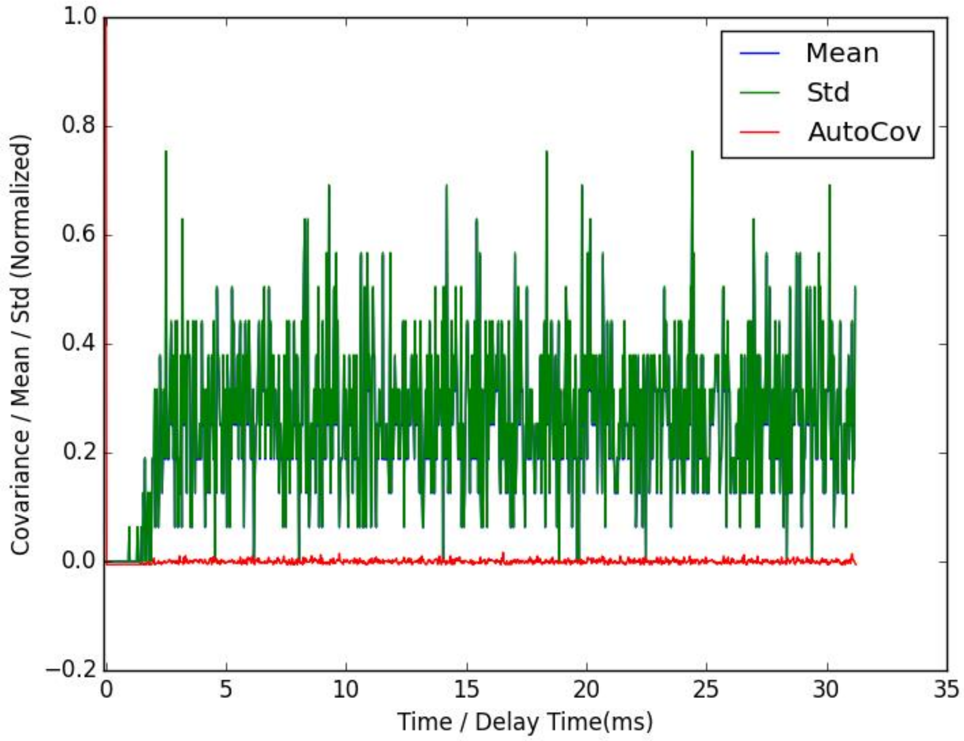
Autocovariance of spike train of Neuron 1



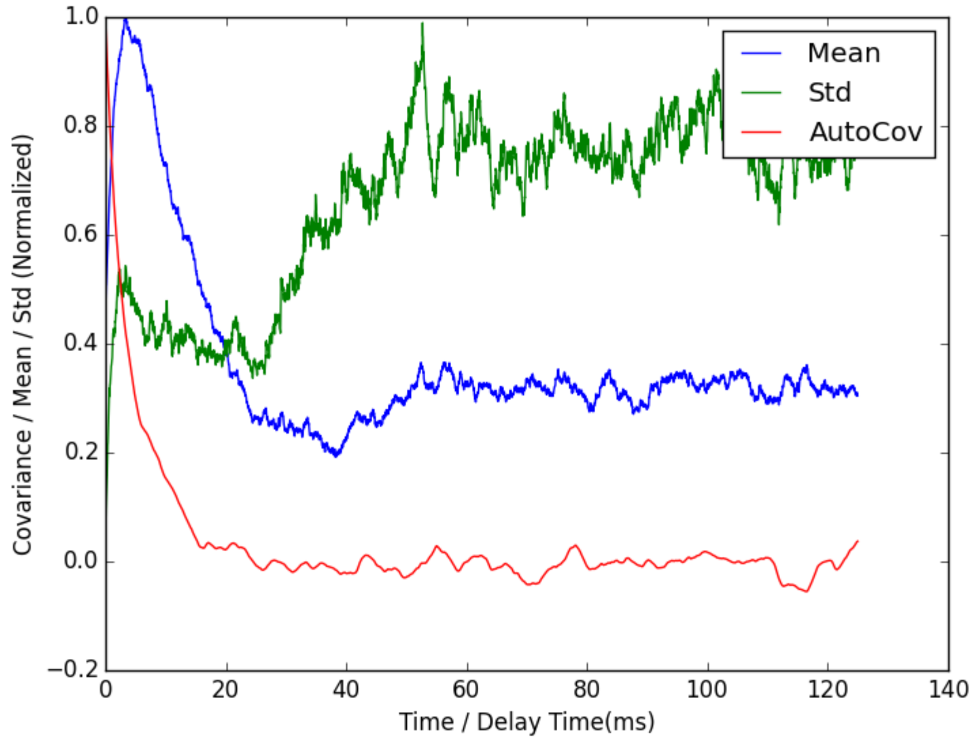
Autocovariance of LFP of Neuron 1

Details in mutual info measurements

S = 0.0145



Autocovariance of spike train of Neuron 1

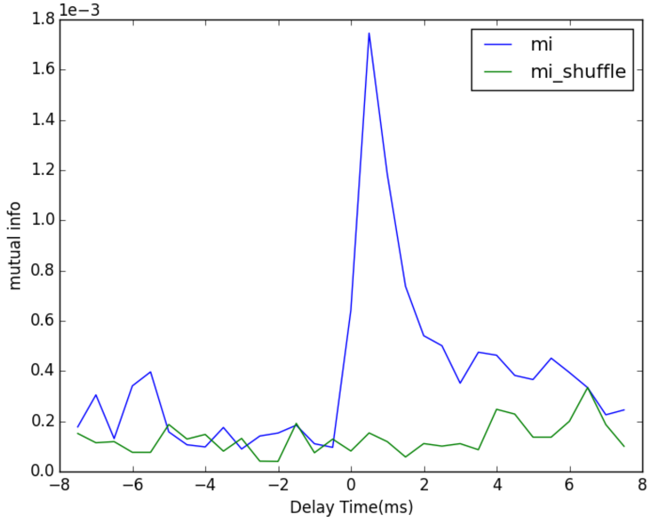


Autocovariance of LFP of Neuron 1

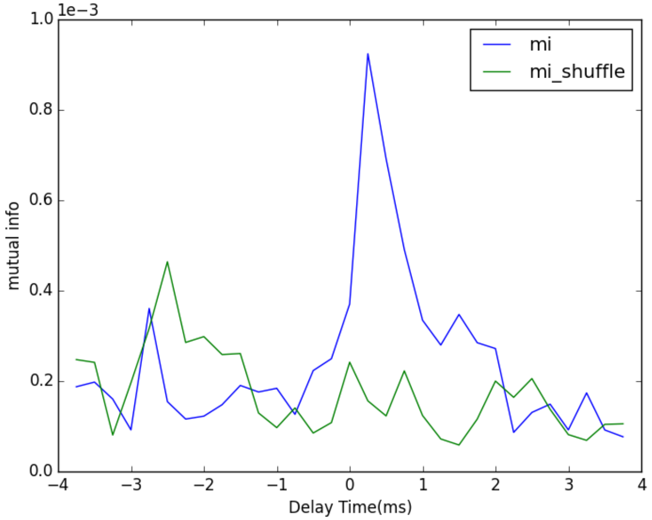
Different Timing Steps

Poisson Rate	1.5 kHz
S	0.005
F	0.005
#bins	10
T	59.5 s

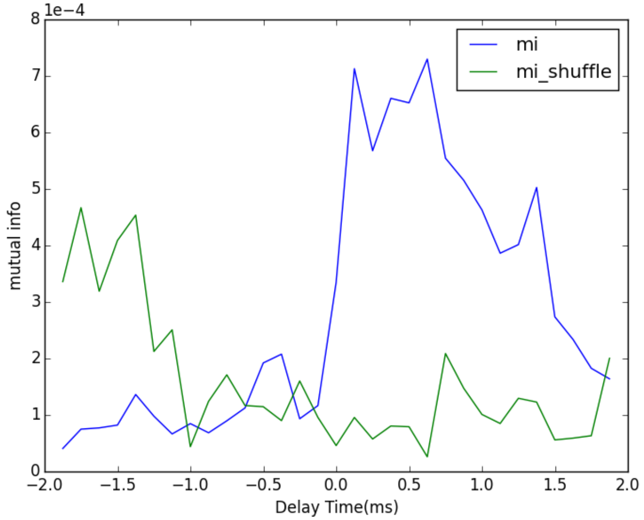
0.5ms

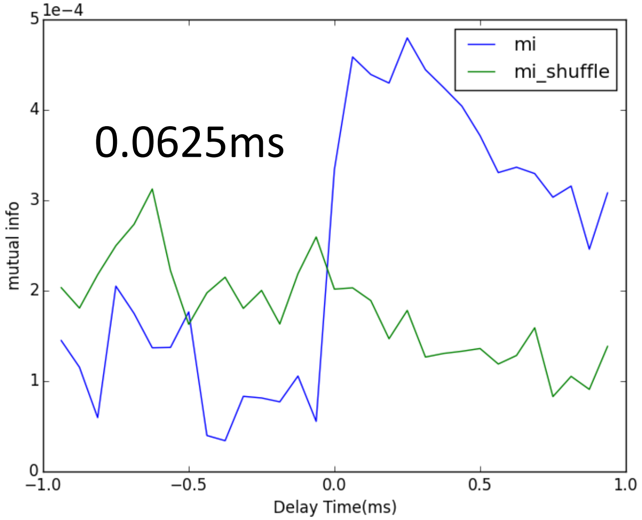


0.25ms



0.125ms

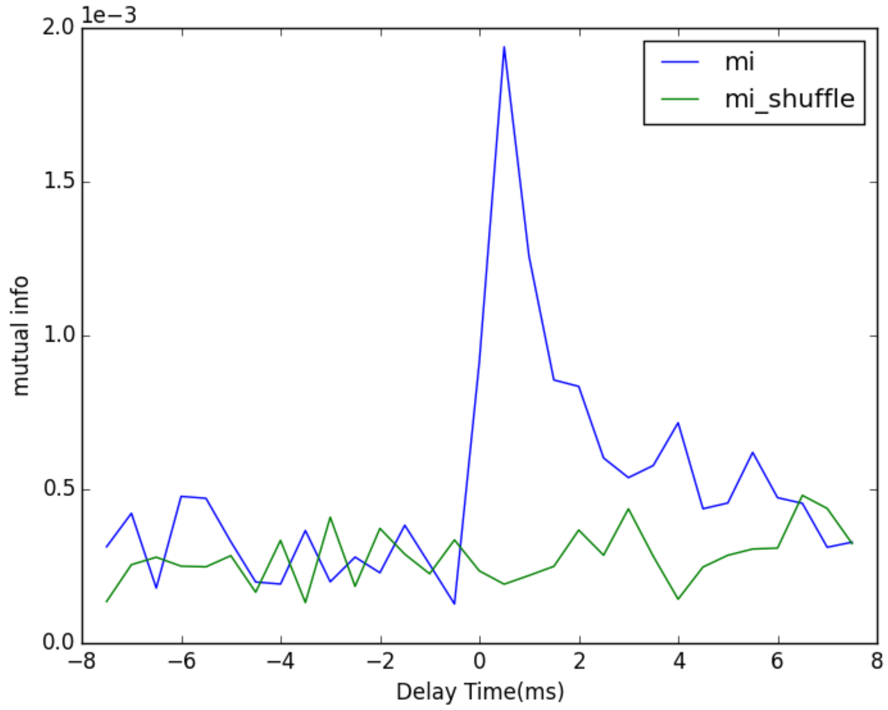




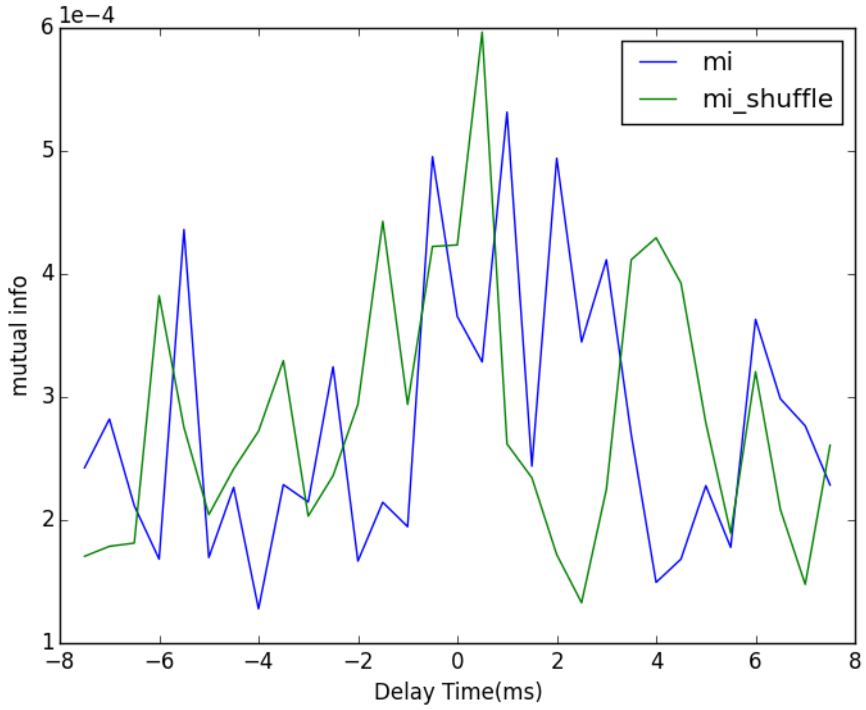
Mono-direction

Poisson Rate	1.5 kHz
S	0.005
F	0.005
dt	0.5 ms
#bins	20
T	59.5 s

From 1 to 2



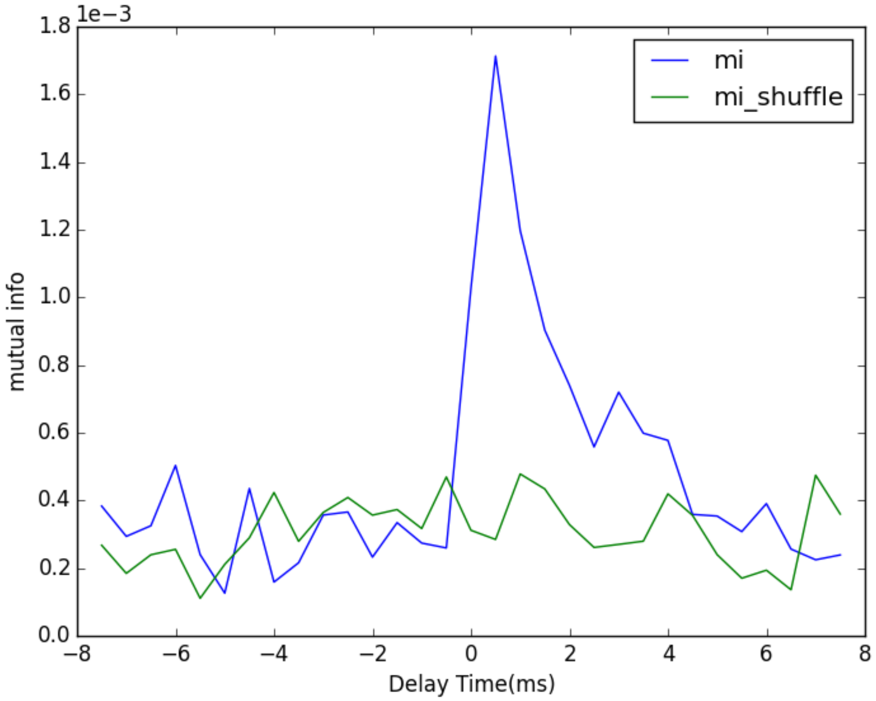
From 2 to 1



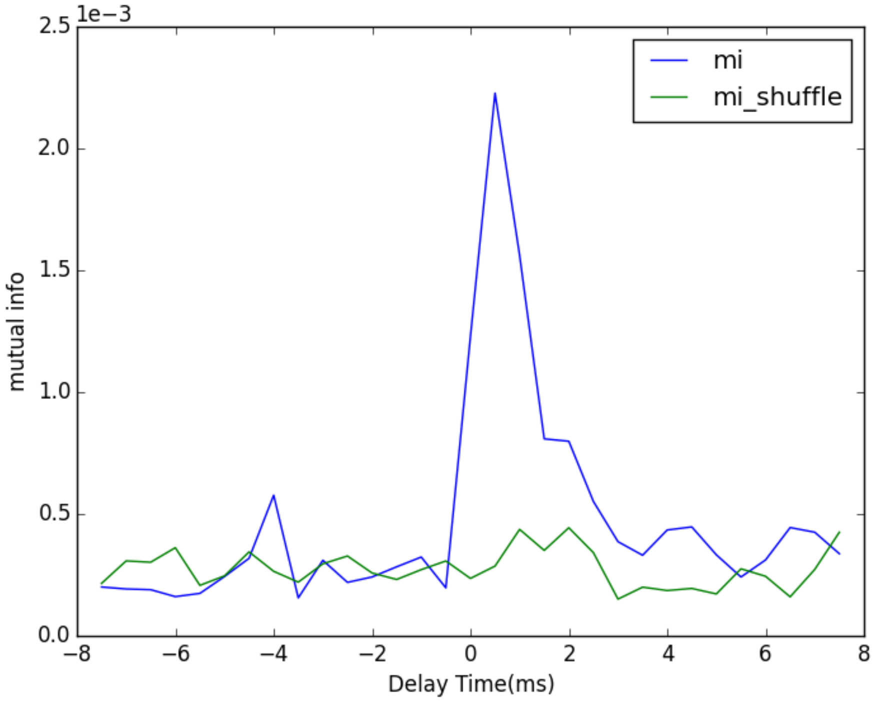
Bi-direction

Poisson Rate	1.5 kHz
S	0.005
F	0.005
dt	0.5 ms
#bins	20
T	59.5 s

From 1 to 2

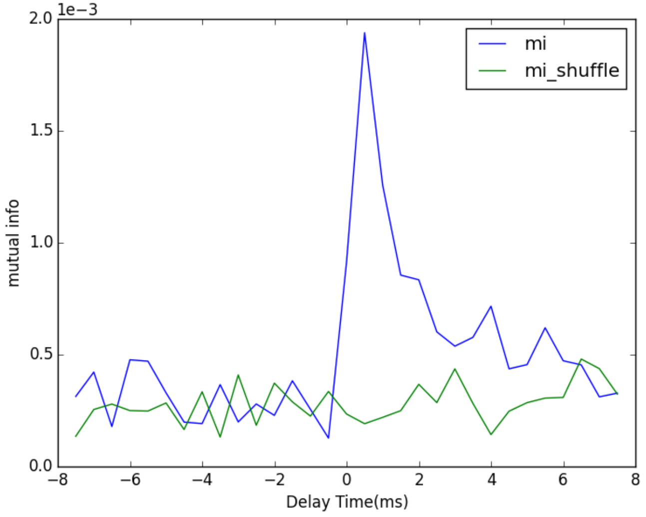


From 2 to 1

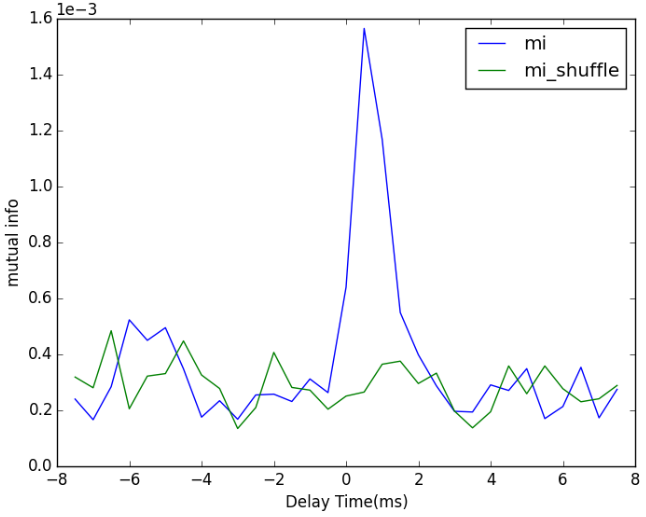


Robustness

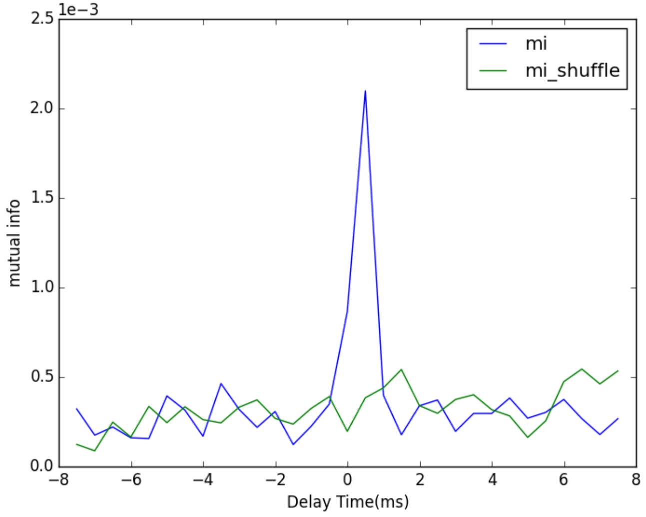
Fix the product of Poisson driving rate and feedforward strength. Change the Poisson driving rate and investigate the change of mutual information pattern



1.5 kHz



0.5 kHz

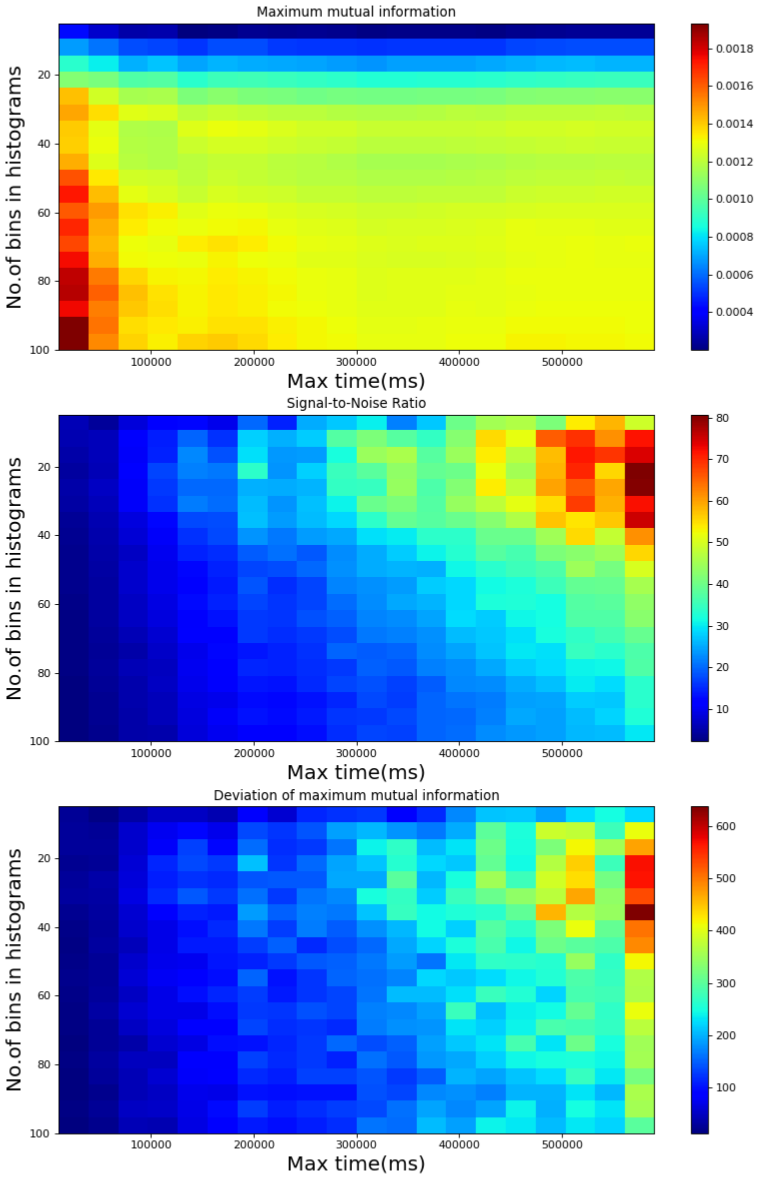


0.1 kHz

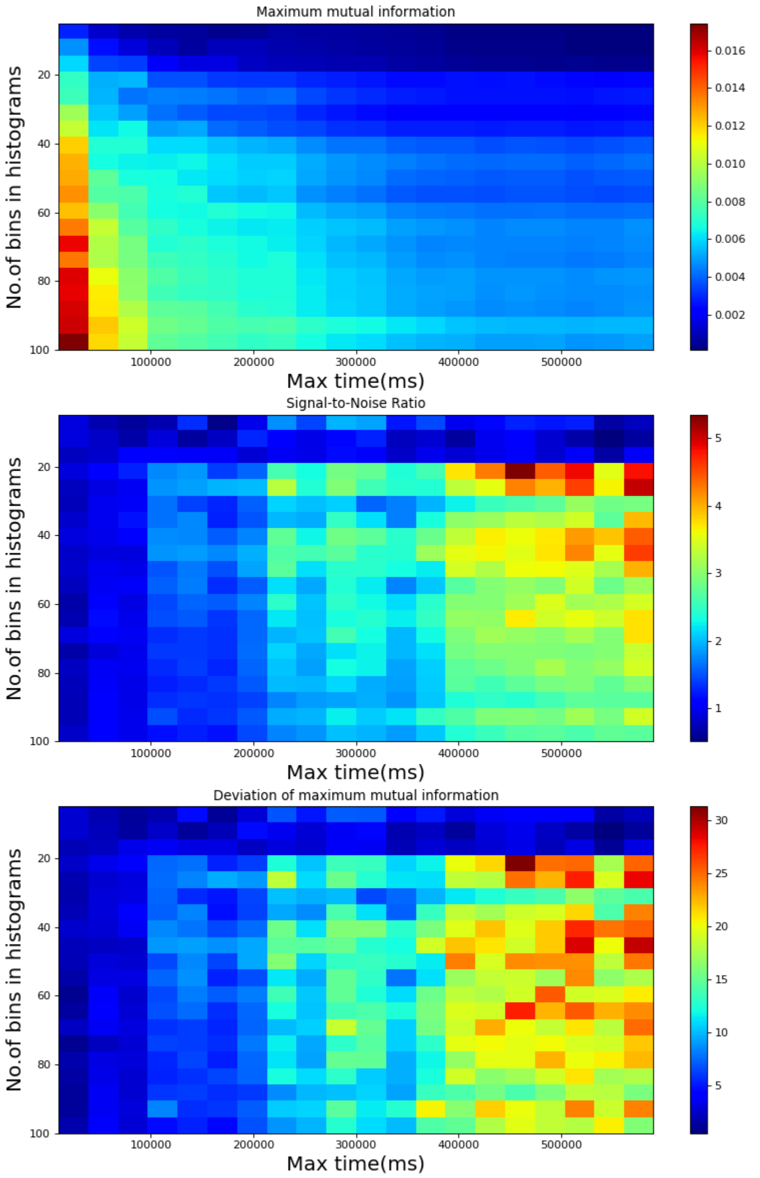
dt	0.5 ms
#bins	30
T	59.5 s

Full method

S = 0.015

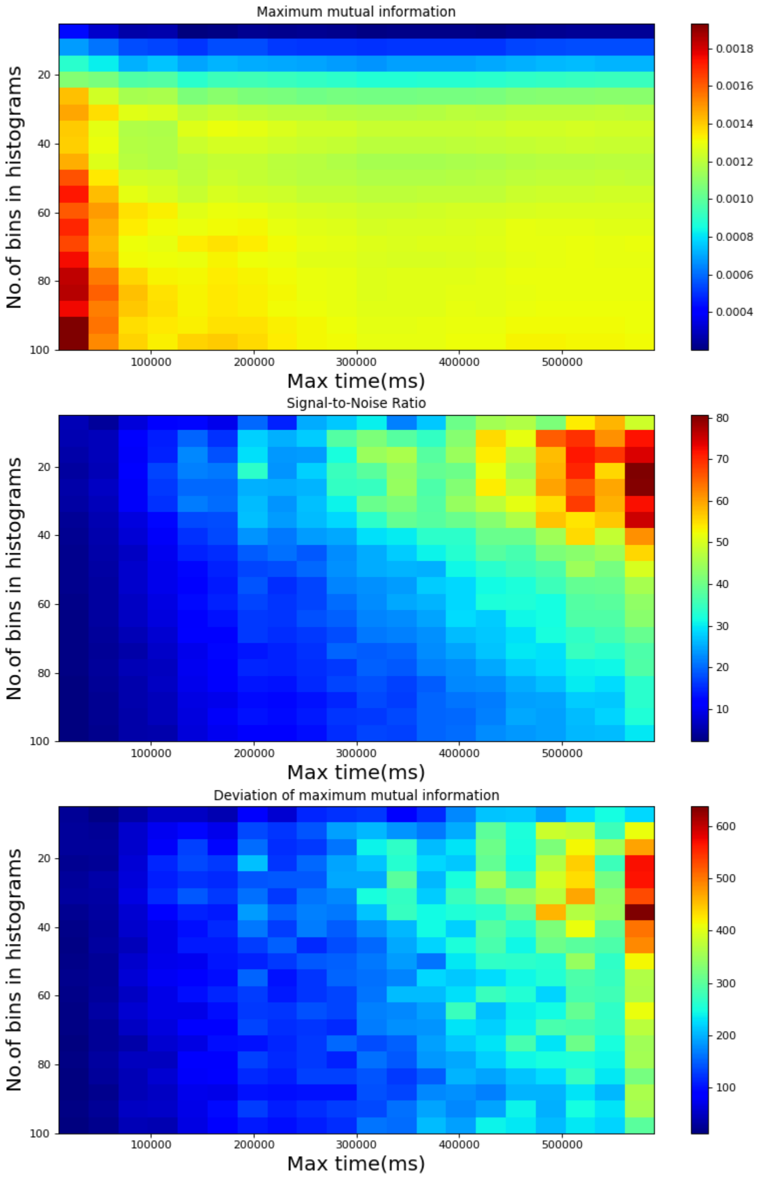


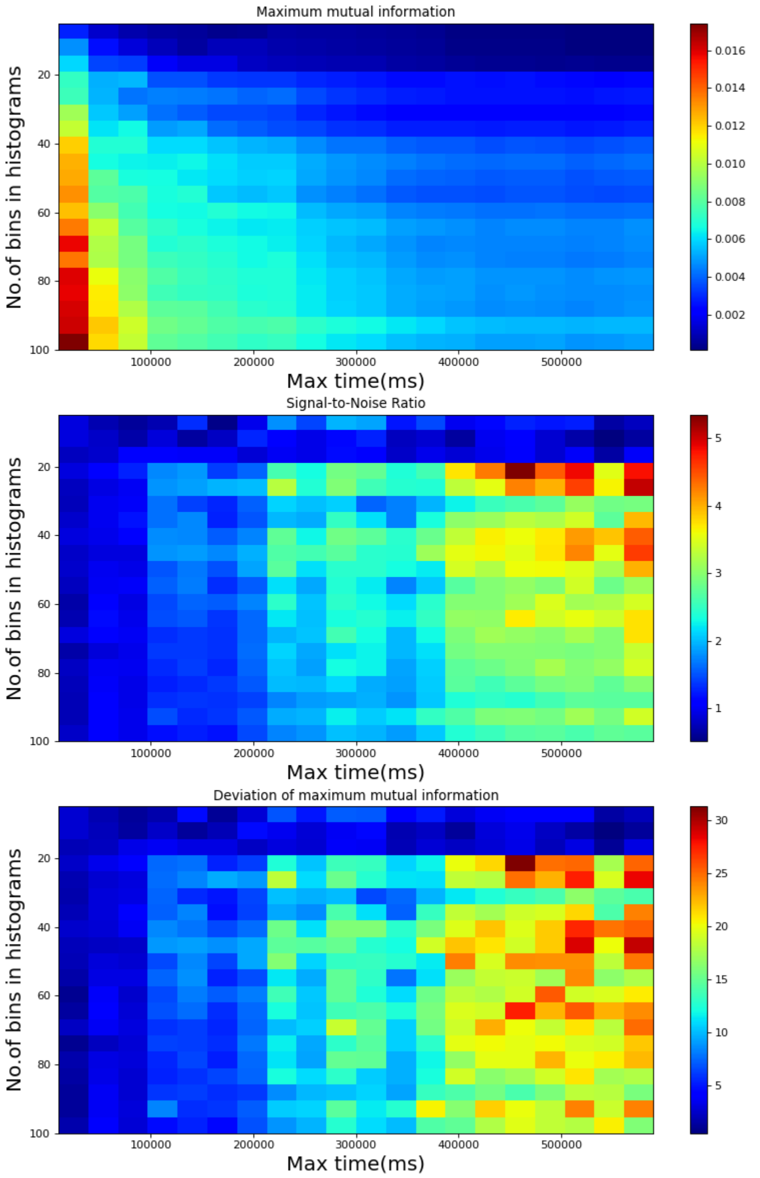
S = 0.075



partial method

S = 0.015

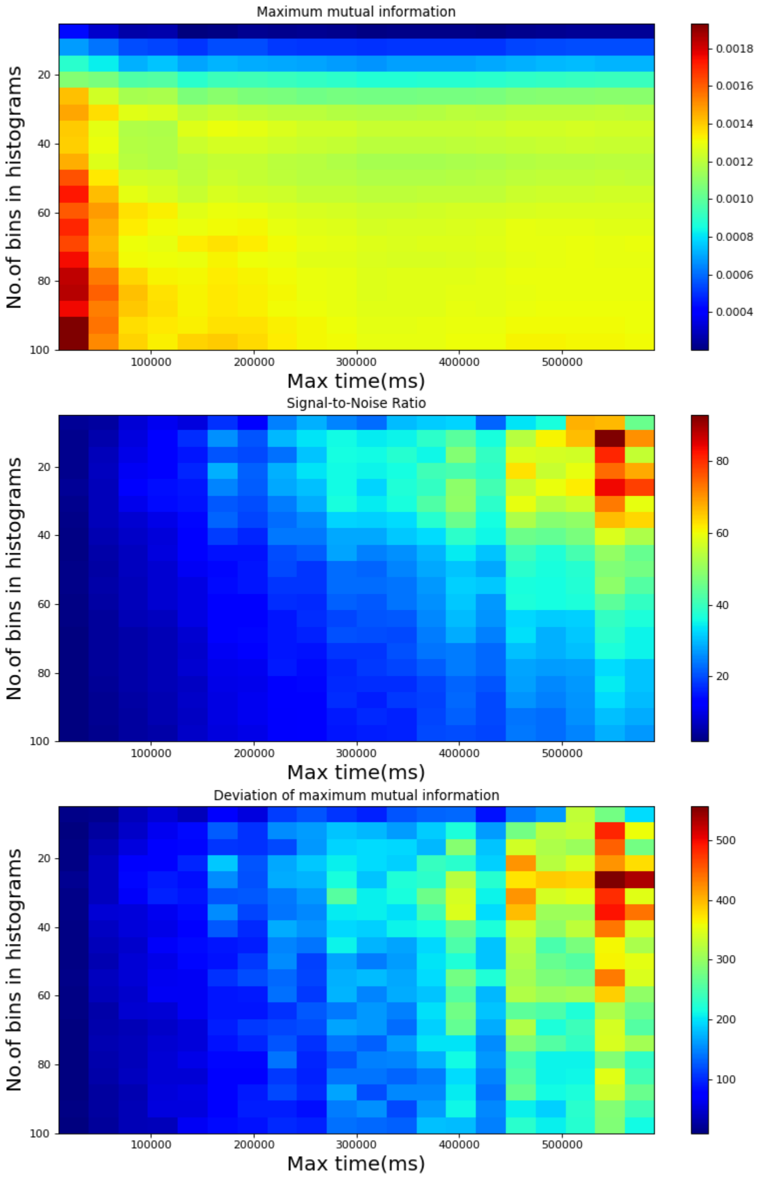


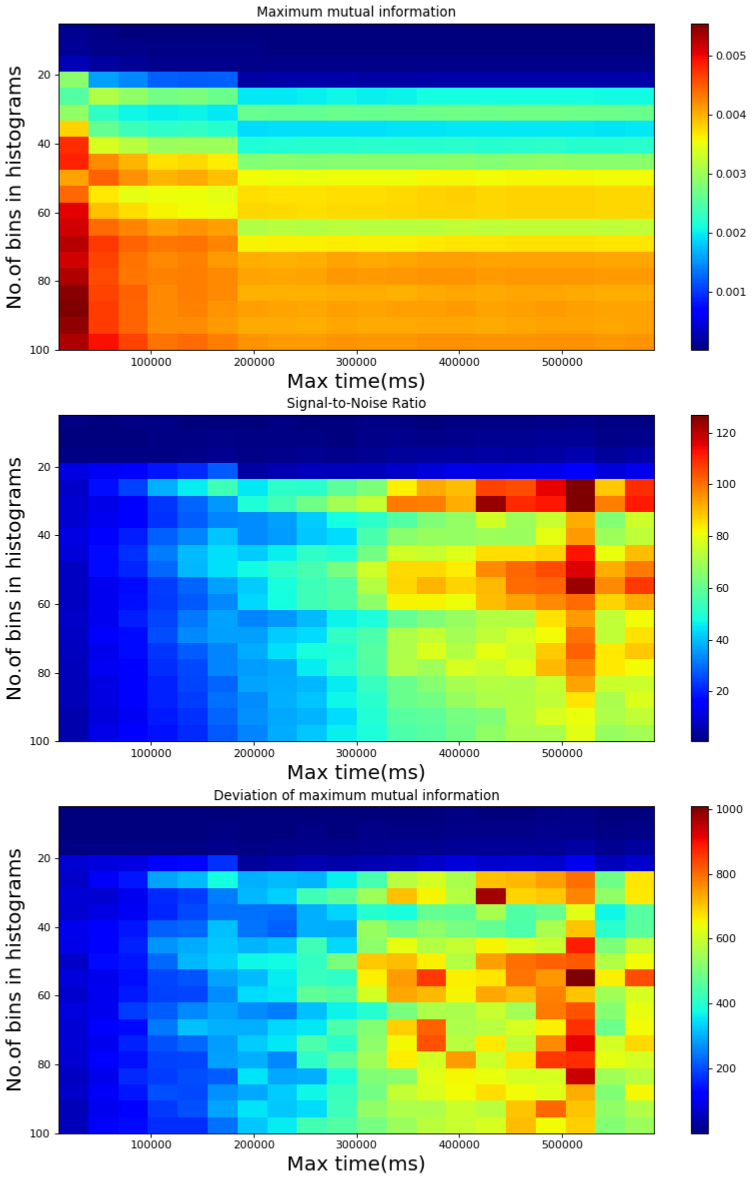


S = 0.075

Direct method

S = 0.015



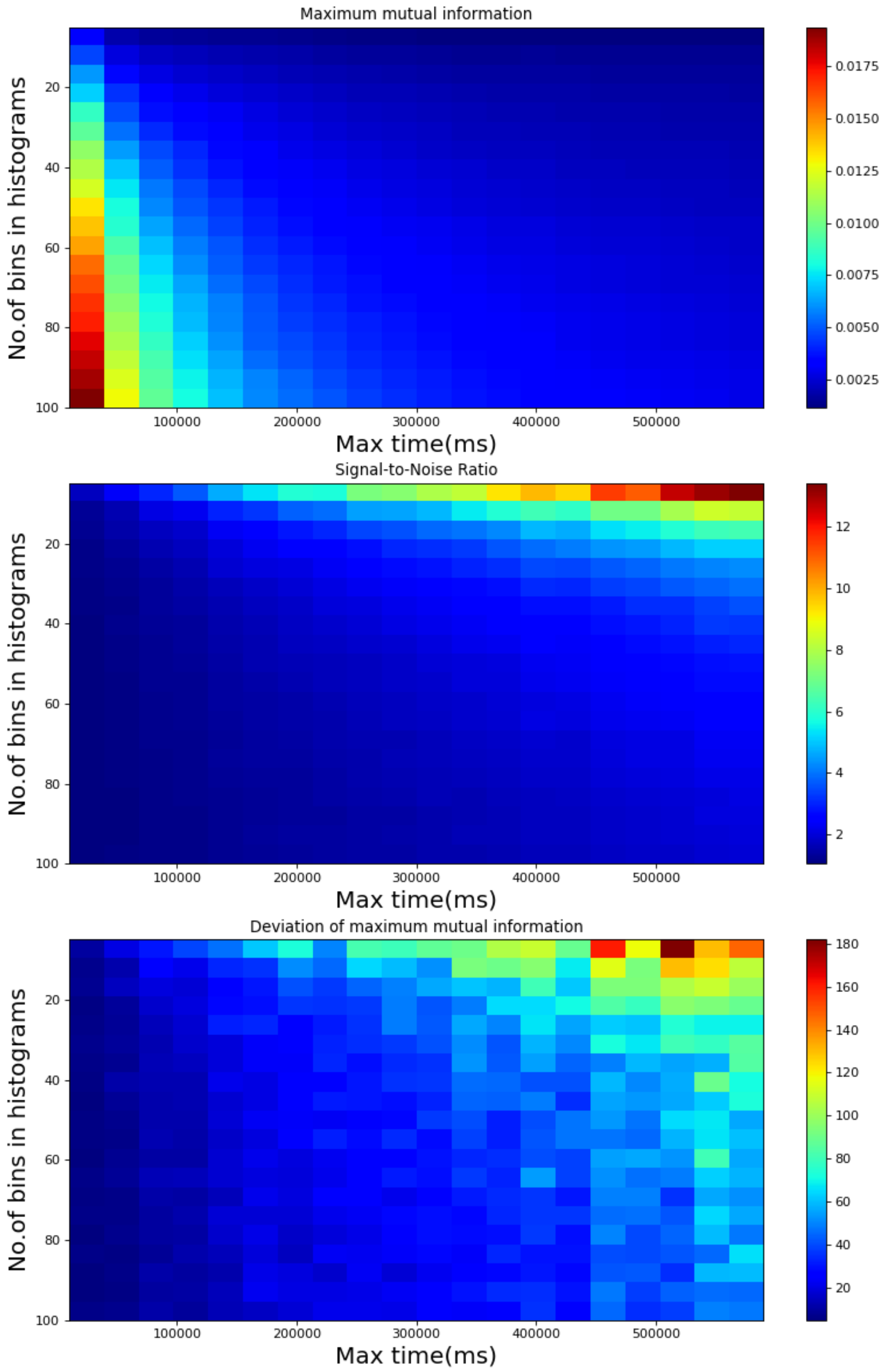


S = 0.075

Autocovariance-based calculation with average

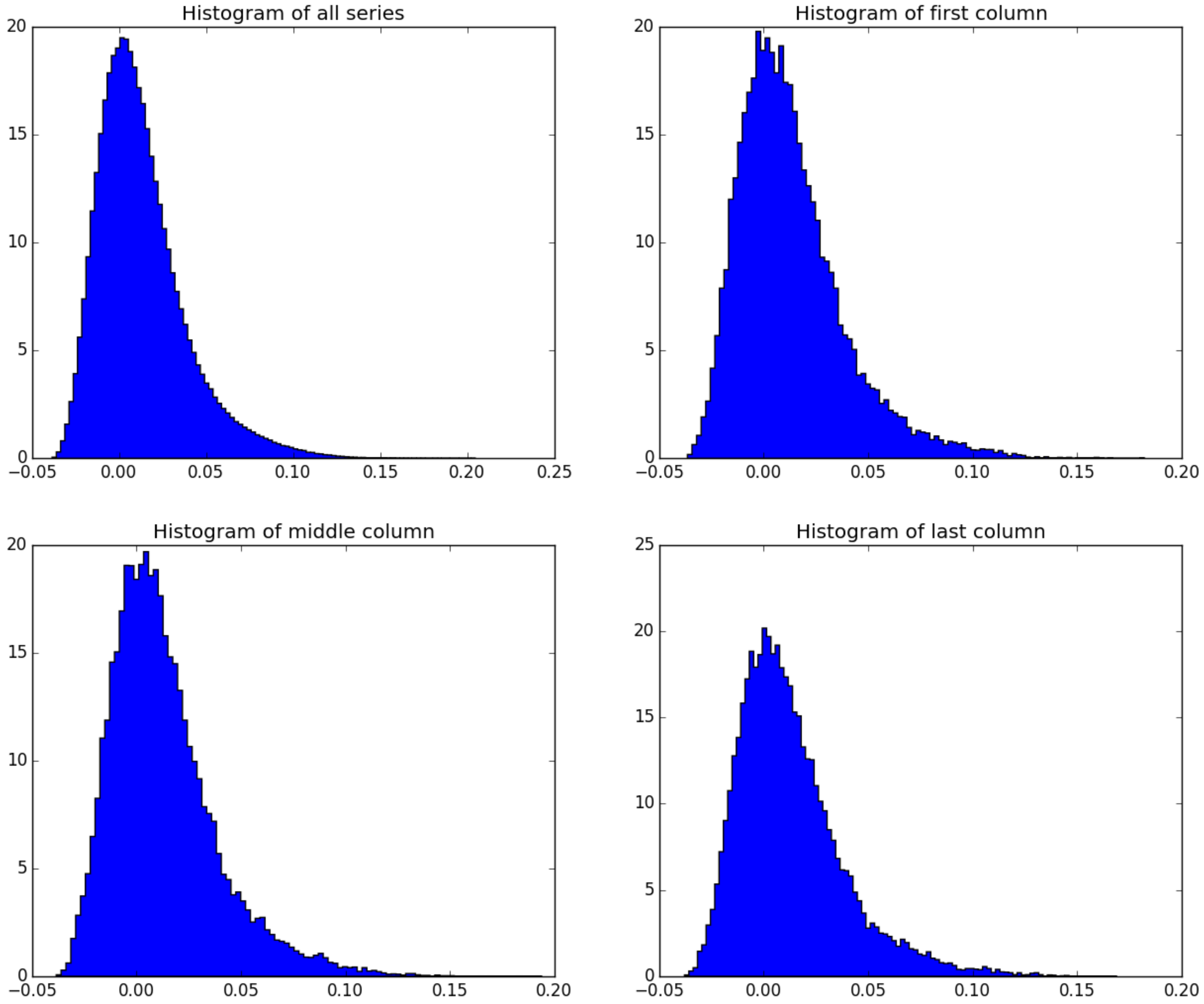
- Find the autocovariance length of LFP, which is 20 ms here.
- Break time series of LFP in pieces with length 20 ms, and treat all those pieces as a statistical ensemble.
- Calculate the PDF at every time point in such 20 ms.
- Do the same operates to the time series of binarized spike trains
- For each time delay, calculate the mutual information between any possible combinations between spike train and LFP, and then take averages.

dt = 0.5 ms



Histograms

Histograms of LFPs in different time point



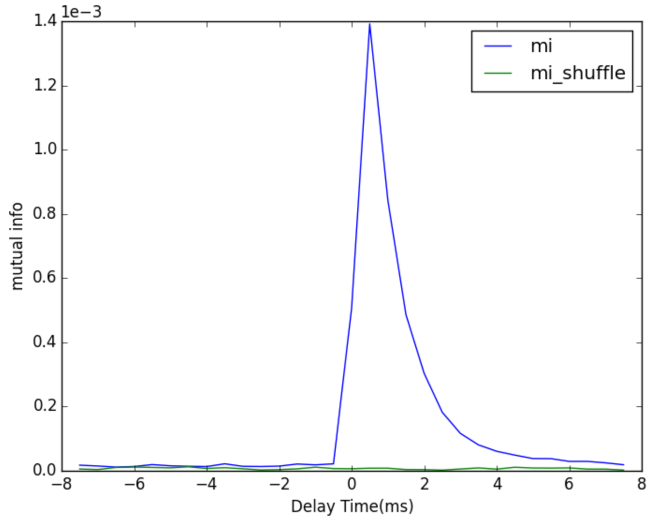
Mono-direction

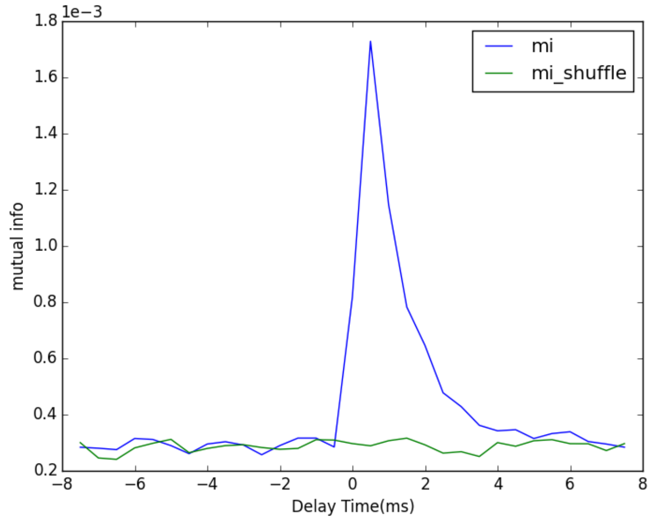
Poisson Rate	1.5 kHz
S	0.005
F	0.005
dt	0.5 ms
#bins	20
T	59.5 s

direct

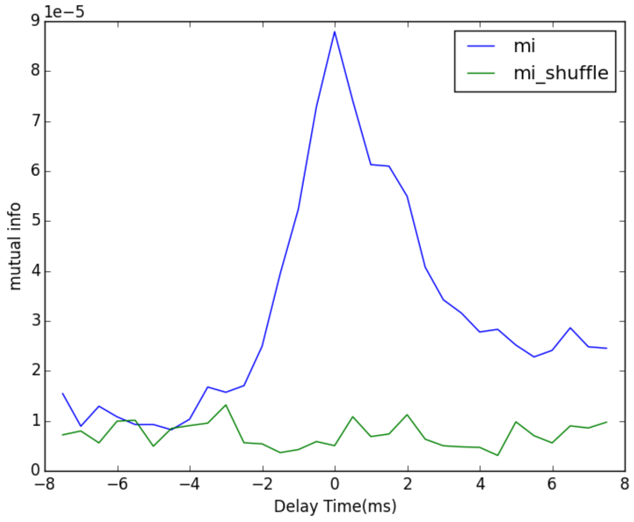
full

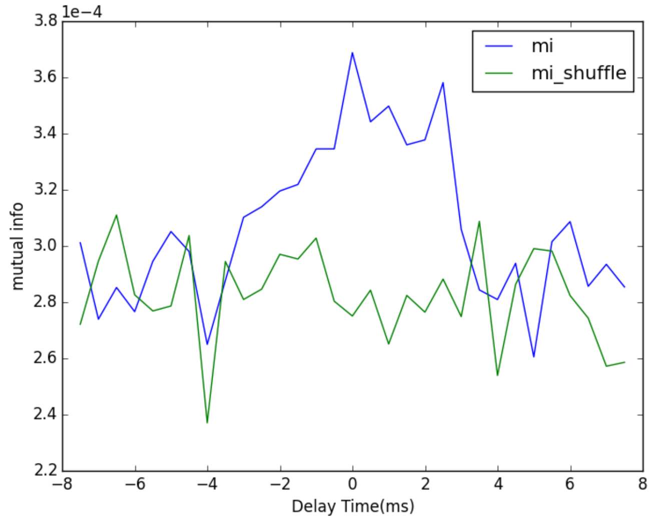
From 1 to 2



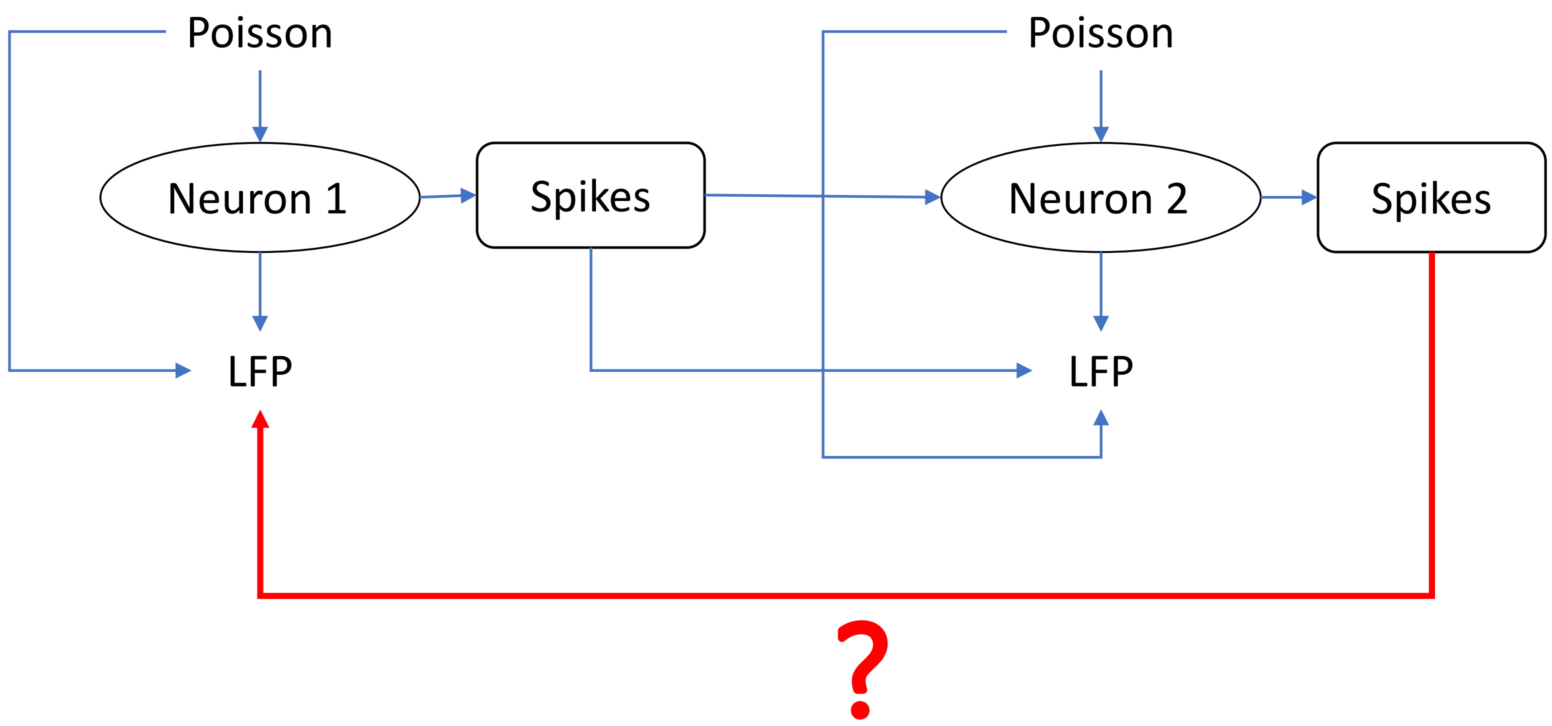


From 2 to 1



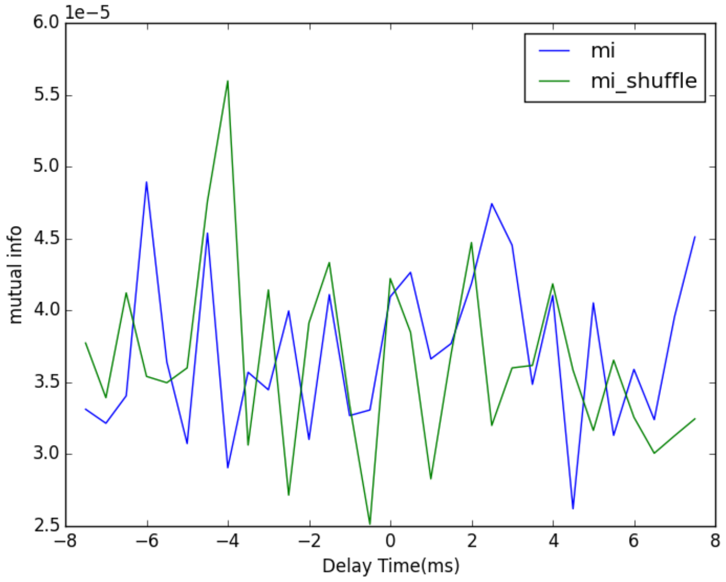


Neuronal interacting layout

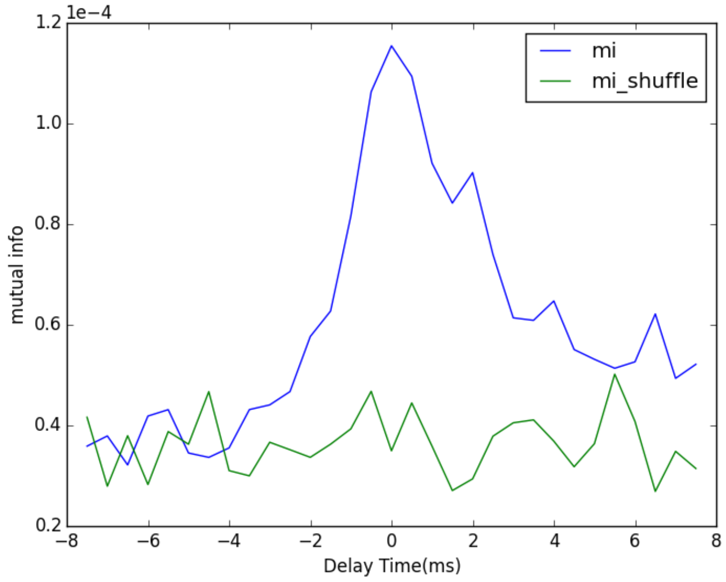


Mutual information from 2 to 1

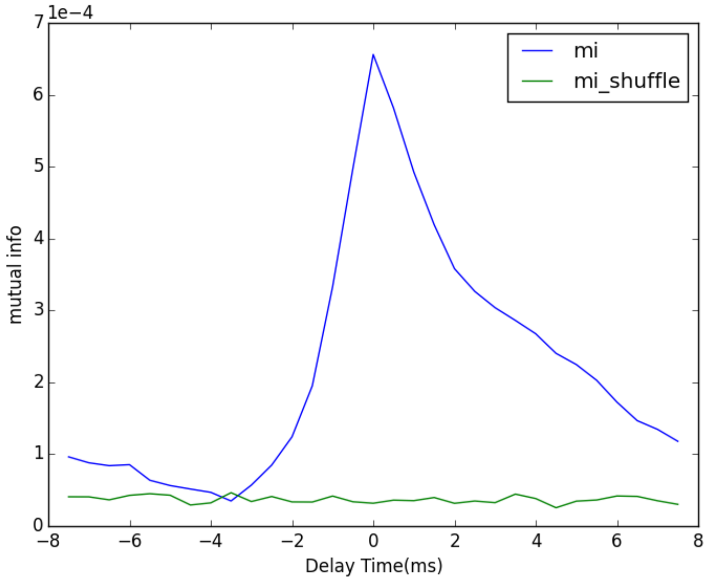
S = 0.001



S = 0.005



S = 0.0145



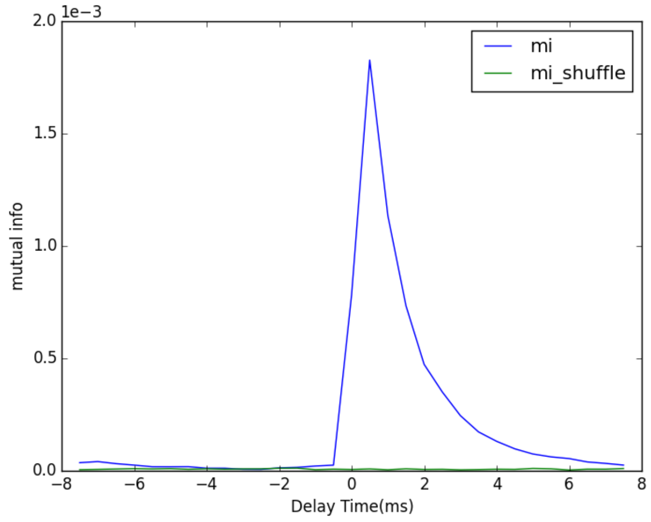
Bi-direction

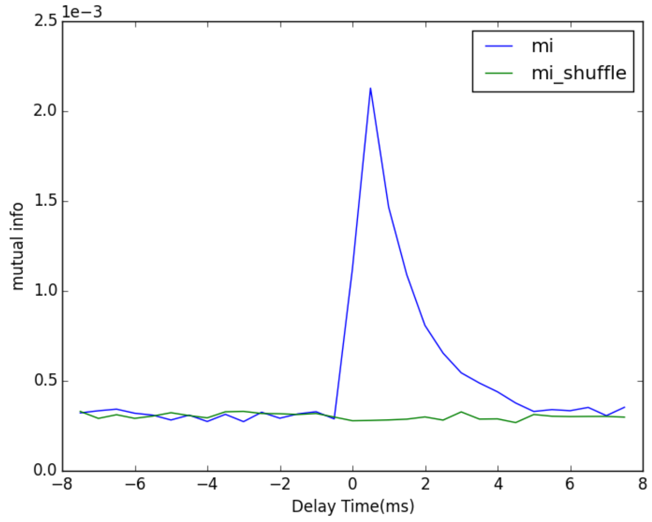
Poisson Driving Rate	1.5 kHz
Synaptic Strength	0.005
Feedforward Strength	0.005

direct

full

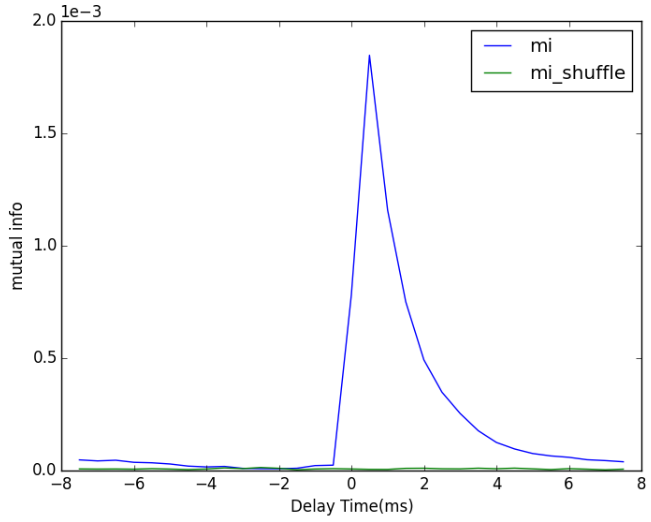
From 1 to 2

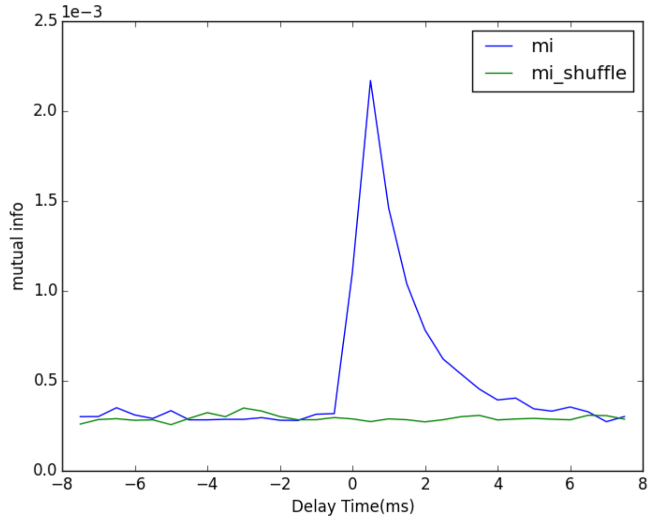




dt = 0.5 ms
#bins = 20
T = 59.5 s

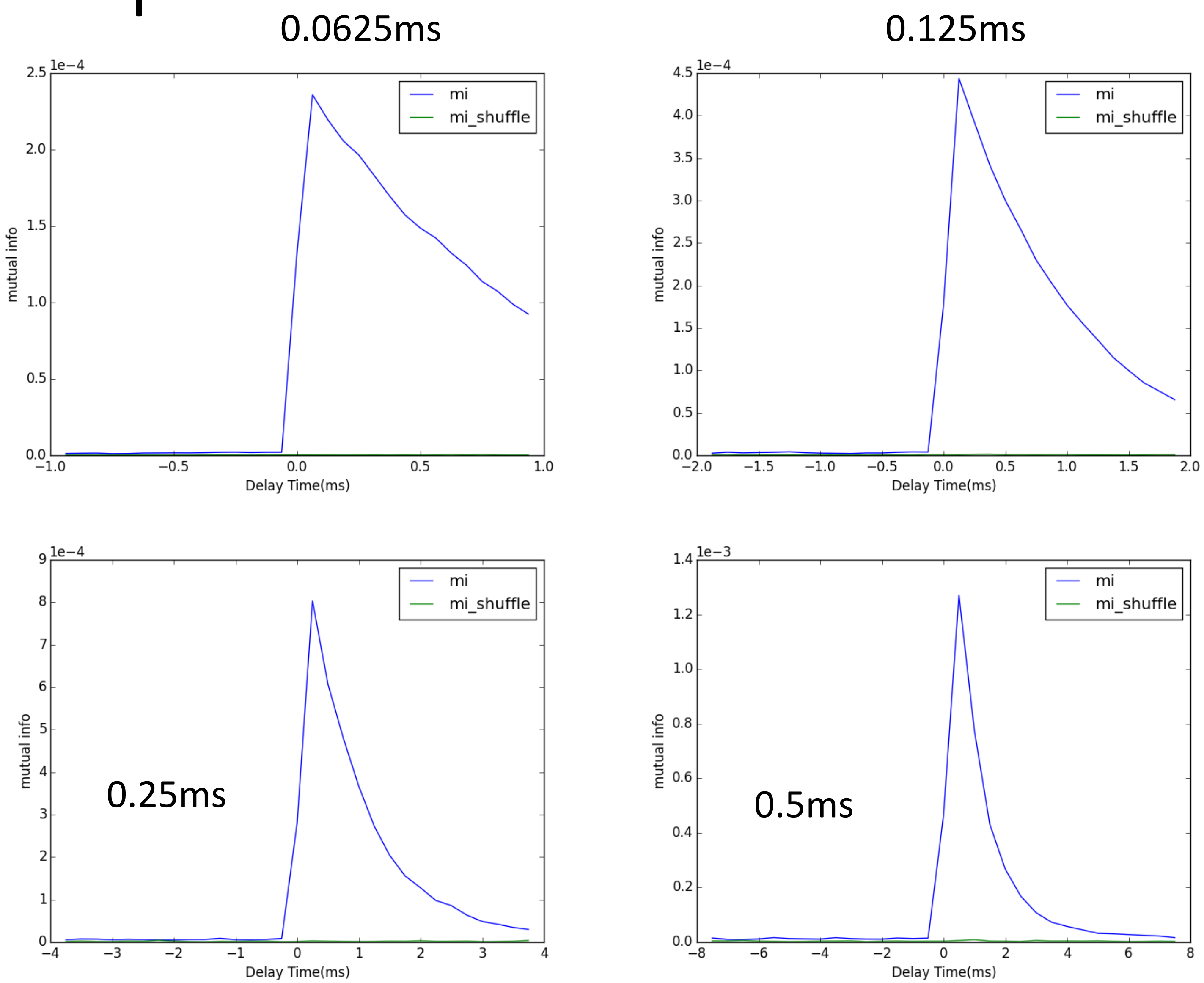
From 2 to 1





Different Timing Steps

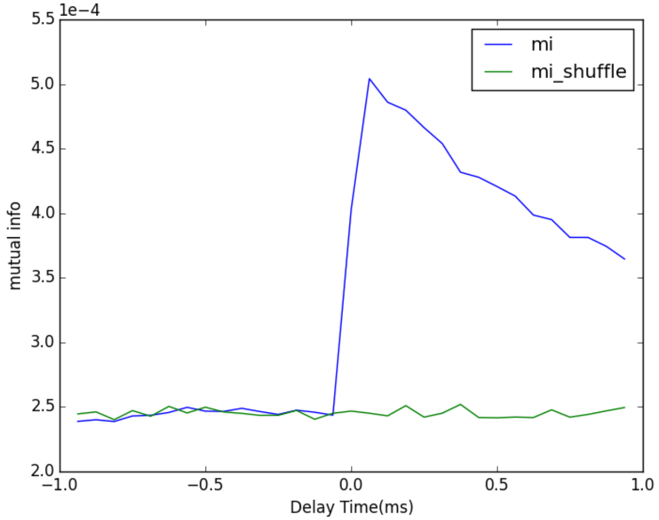
Poisson Rate	1.5 kHz
S	0.005
F	0.005
#bins	20
T	59.5 s
Direct Method	



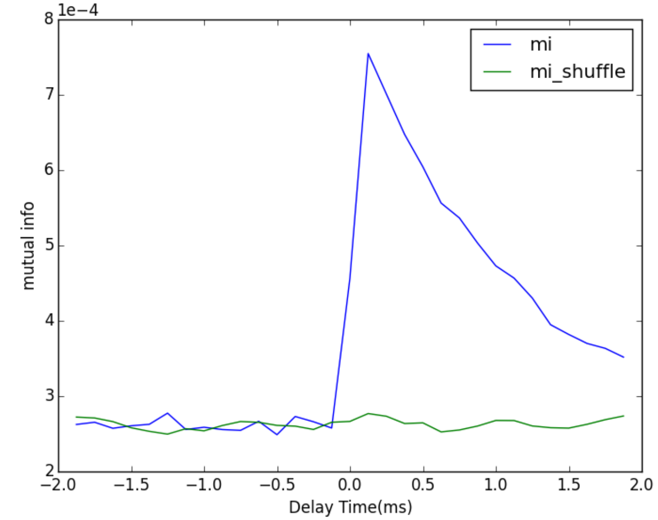
Different Timing Steps

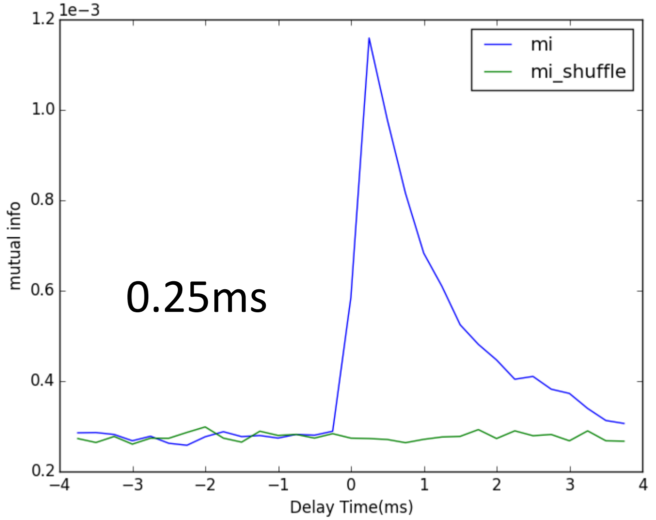
Poisson Rate	1.5 kHz
S	0.005
F	0.005
#bins	20
T	59.5 s
Ensemble Method with average	

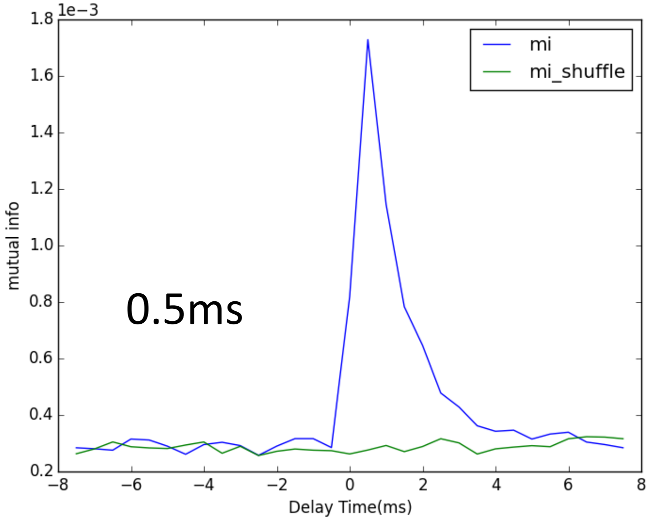
0.0625ms



0.125ms



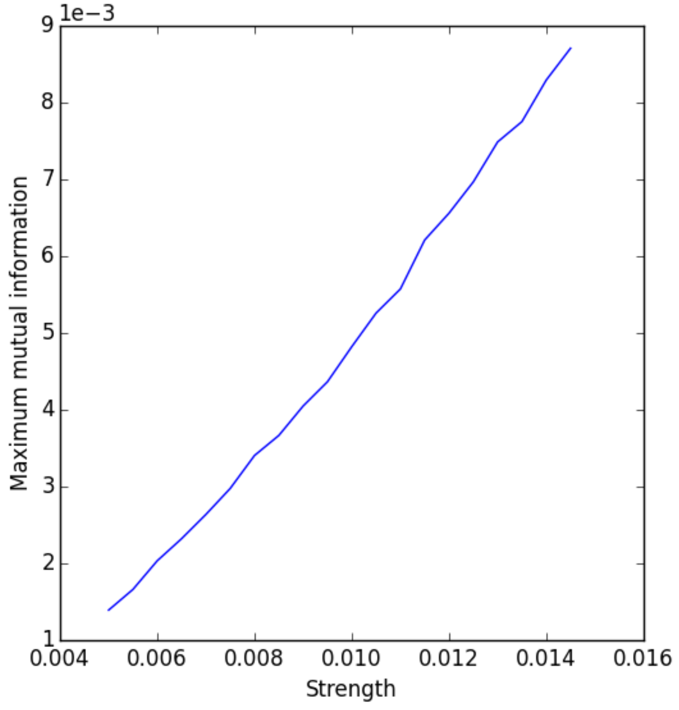


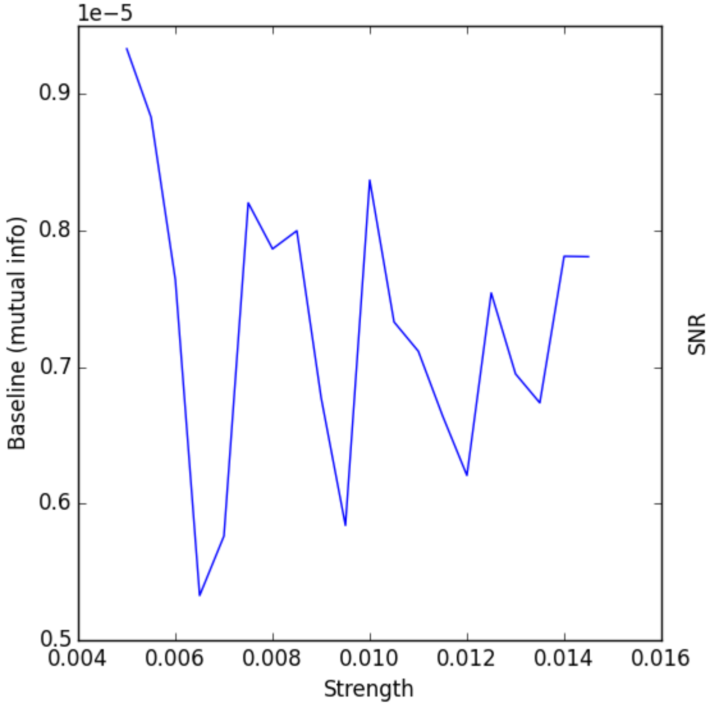


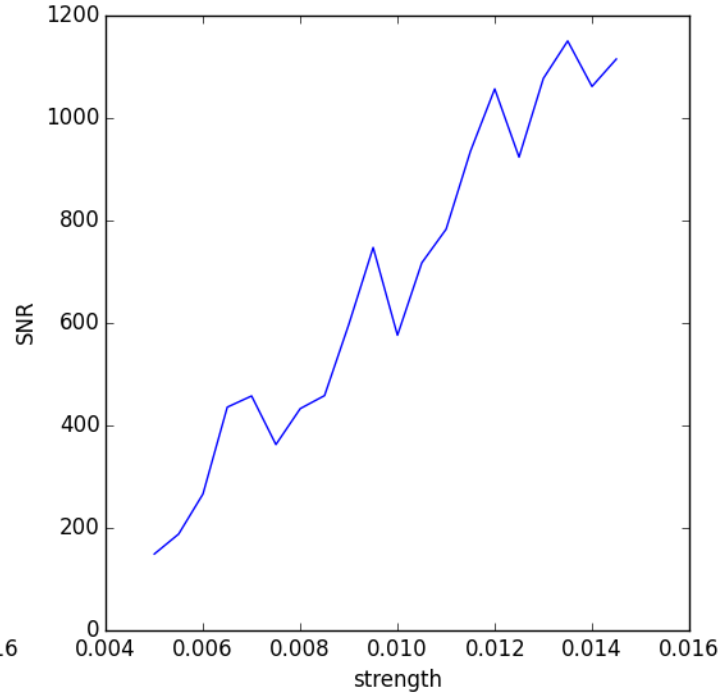
Direct calculation

dt = 0.5 ms
#bins = 20
T = 59.5 s

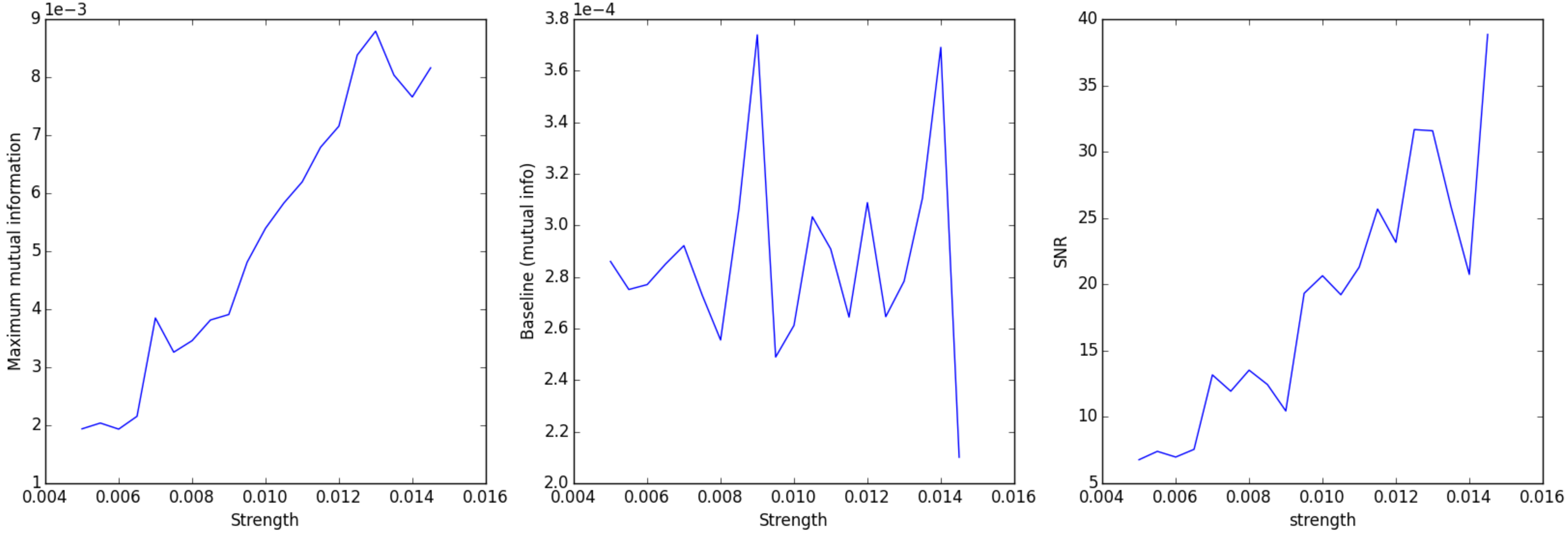
- Relation between synaptic strength and values of maximum mutual information
  - Synaptic strength ranging from 0.005 to 0.0145
  - Mean firing rate for presynaptic neuron is 10.5 Hz
  - Mean firing rate for postsynaptic neuron is ranging from 11Hz to 12.3 Hz





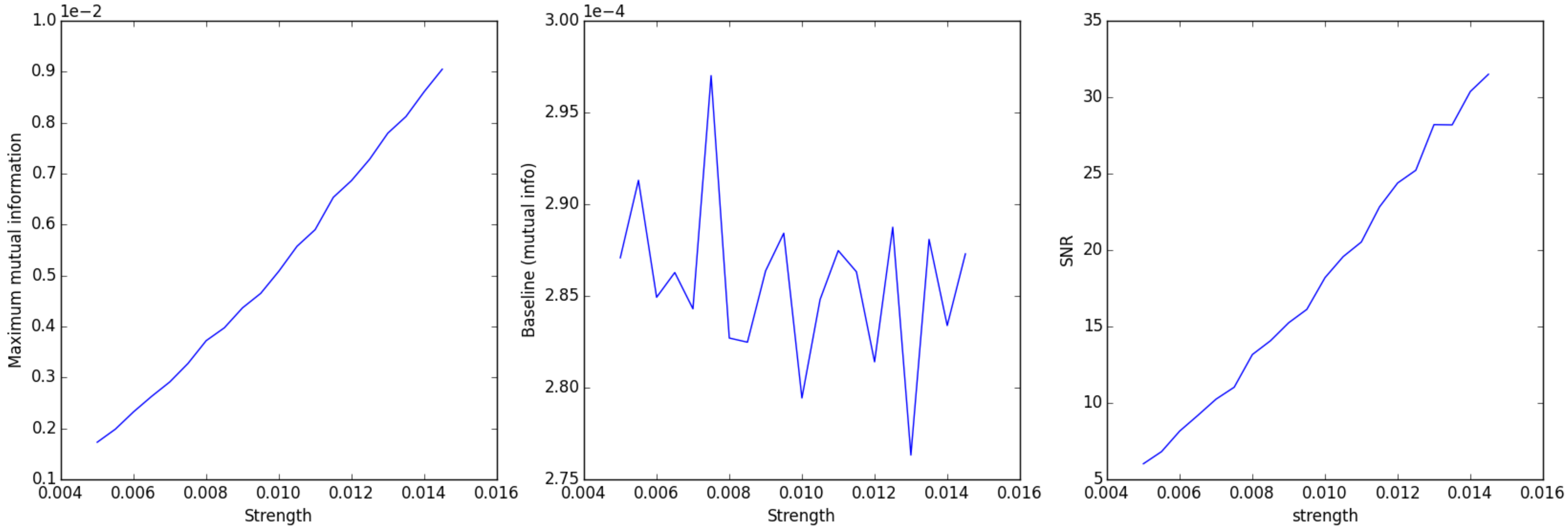


Autocovariance-based calculation without average



Cannot effectively calculate relationship between them even with small number of bins in histogram calculation

Autocovariance-based calculation with average



Mutual information of Gaussian random variables

$$X_n = \alpha X_{n-1} + \varepsilon_n$$
$$Y_n = \beta Y_{n-1} + \xi X_{n-1} + \eta_n$$

if $|\alpha| = |\beta| \ll 1$ $\quad$ $I(X,Y) = \frac{1}{2}\log(1 + \xi^2)$

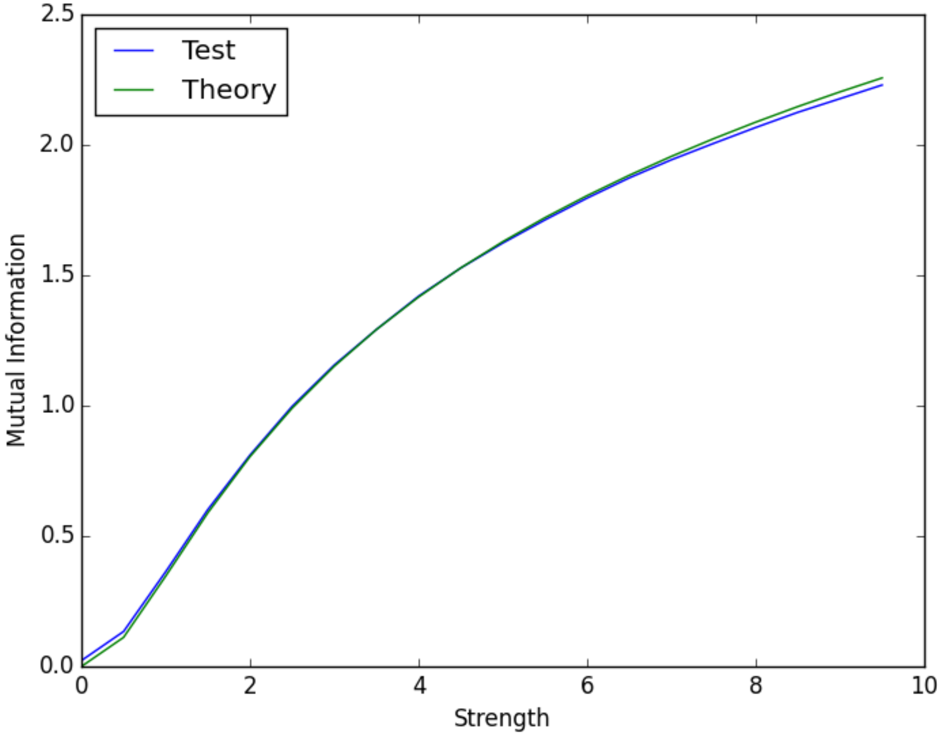
$$I(X,Y) = H(X) + H(Y) - H(X,Y)$$

$$I(X,Y) = -\frac{1}{2}\log(1-\rho^2)$$

$$= -\frac{1}{2}\log\left(\frac{(1-\alpha^2)(1-\beta^{2n+2})}{(1-\alpha^2)(1-\beta^{2n+2}) + \xi^2(1-\beta^2)(1-\alpha^{2n+2})}\right)$$

Since α and β is smaller than 1, when n is large enough:

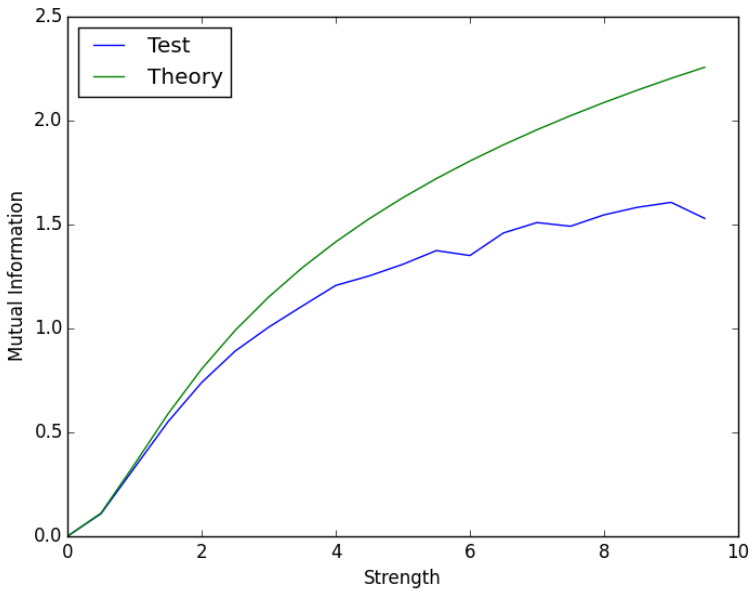
$$I(X,Y) = -\frac{1}{2}\log\left(\frac{1-\alpha^2}{1-\alpha^2+\xi^2(1-\beta^2)}\right)$$



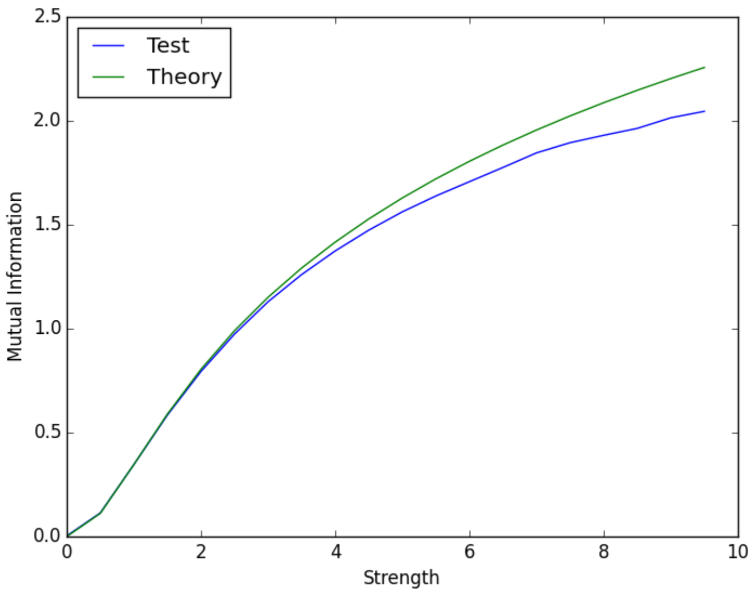
#bins = 150
300000 trials

Mutual info
VS
Bin numbers

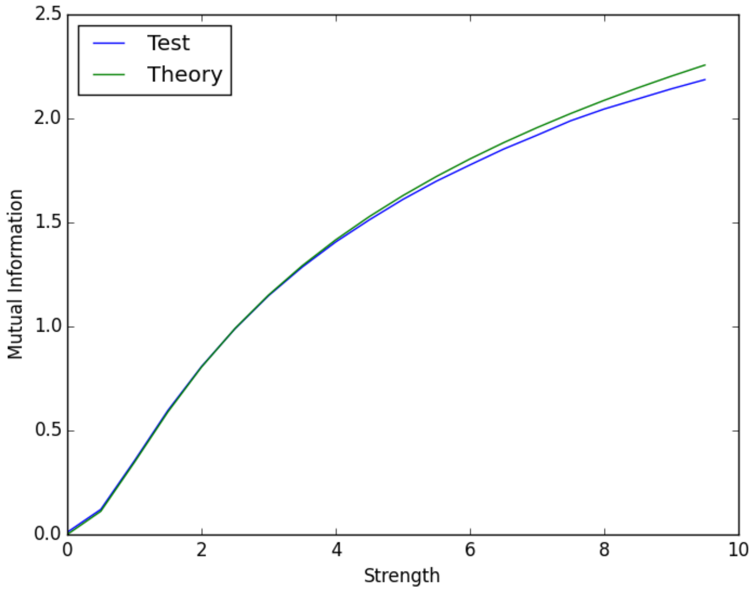
#bins = 20



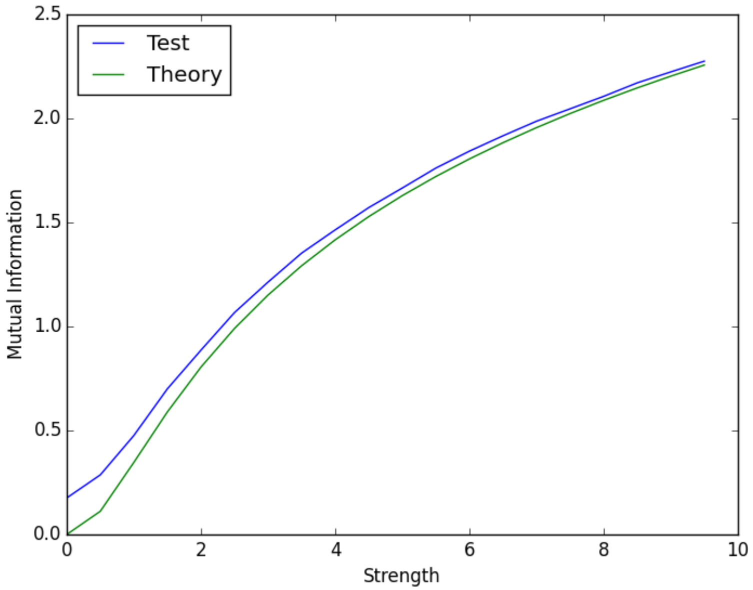
#bins = 50



#bins = 100



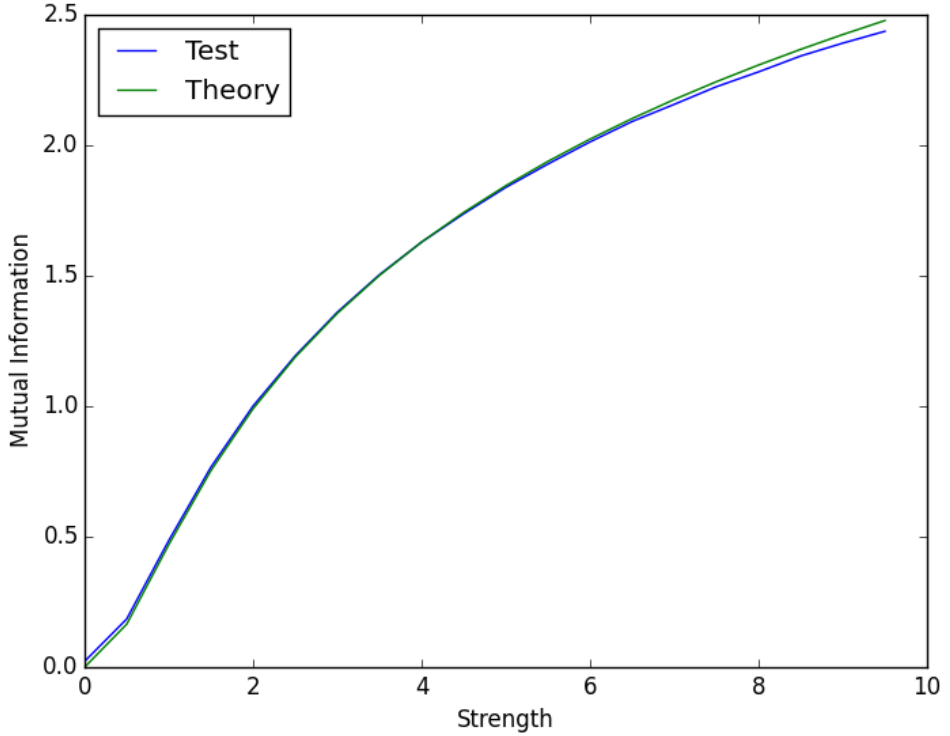
#bins = 500

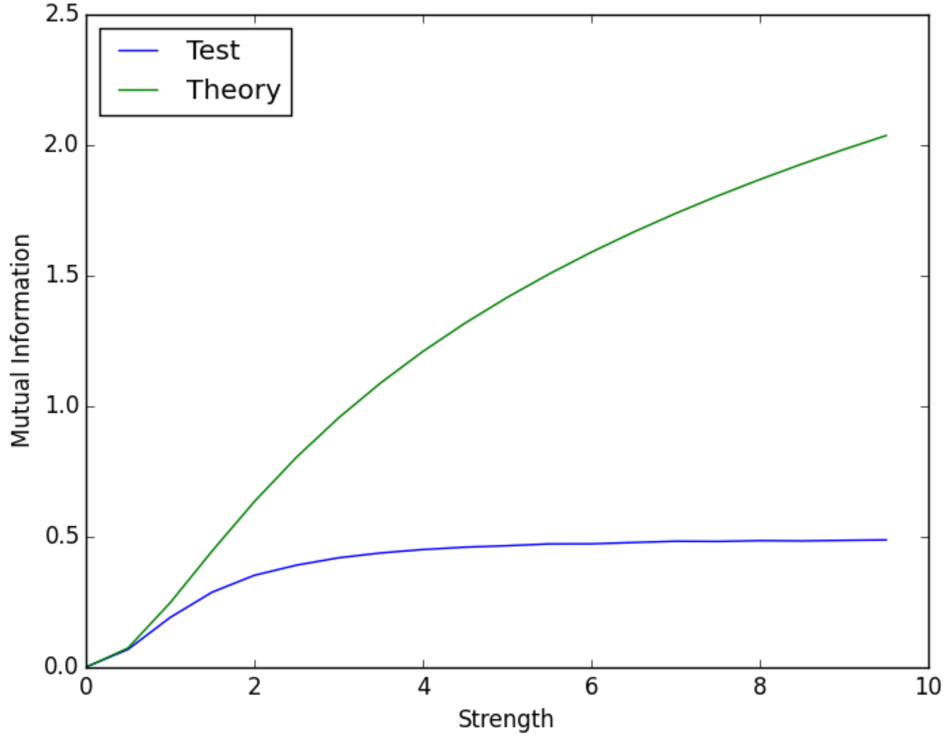


However, if $|\alpha| \ll |\beta| < 1$,

If $|\beta| \ll |\alpha| < 1$, $\quad I(X,Y) = -\frac{1}{2}\log\left(\frac{1-\alpha^2}{1-\alpha^2+\xi^2}\right)$

$$I(X,Y) = \frac{1}{2}\log(1 + (1-\beta^2)\xi^2)$$

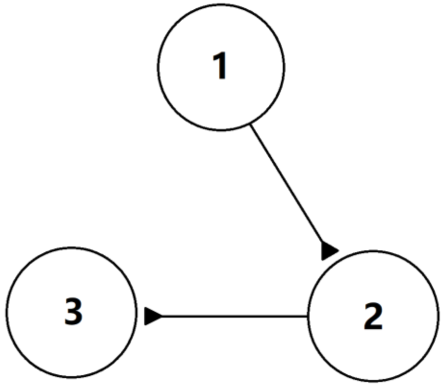




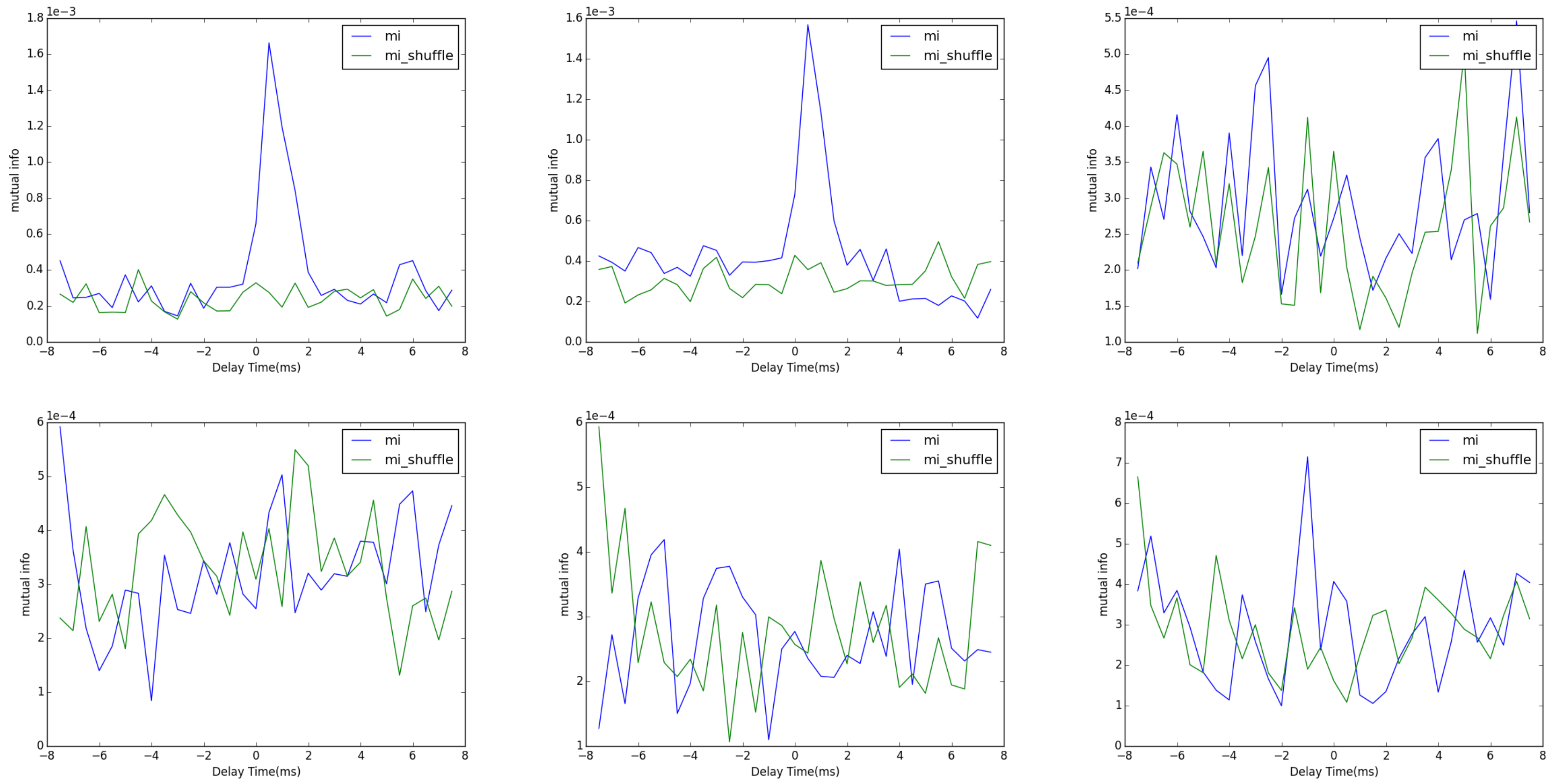
#bins = 150
T = 300000

Triple-neuron system

1to2	2to3	1to3
2to1	3to2	3to1



Poisson Rate	1.5 kHz
S	0.005
F	0.005
#bins	20
T	59.5 s
Ensemble Method	



Coming Questions:

- Why are the peaks of maximum for inversely directed pairs at zero? [It depends on the algorithm of mutual information. For direct method, it disappears when weakly interacted.]
- Maximum mutual information seems independent towards the dynamical regime. [need further tests]
- What is the relation between interacting strength and mutual information in the spike-LFP calculation?
- For Gaussian random variable analysis, what if a or b is no far less than one?
- Why does the curve of experiments change like that as the number of bins in histogram increases? [over fitting, larger amount of information.]
- Why does the curve of mutual information between spike train and voltage look like that?