Application of TDMI on Analyzing Neural Data(II)

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Review about last talk

Definition of entropy and (time-delayed) mutual information

$$I(X;Y,\tau) = I(X_t;Y_{t+\tau}) = \sum_{x\in X_t}\sum_{y\in Y_{t+\tau}} p(x,y)\log\frac{p(x,y)}{p(x)p(y)}$$

TDMI between Gaussian random variables

$$\begin{cases} X_n = \alpha X_{n-1} + \varepsilon_n \\ Y_n = \beta Y_{n-1} + \xi X_{n-1} + \eta_n \end{cases}$$

$$I(X,Y) = -\frac{1}{2}log(1-\rho^2)$$

$for\ \alpha < 1, \beta < 1\ and\ n \gg 1:$

$$\rho(\xi)^2 = \begin{cases} \dfrac{\xi^2}{(1-\alpha^2)^2+\xi^2(1+\alpha^2)} & \alpha = \beta \\ \dfrac{\xi^2(1-\beta^2)}{(1-\alpha\beta)^2(1-\alpha^2)+\xi^2(1-(\alpha\beta)^2)} & \alpha \neq \beta \end{cases}$$

TDMI analysis in two-neuron system

- Mutual information estimation scheme
- Fake signal in in-directed connected neural pairs

Outline

Mutual information estimation

Deviation of numerically calculated MI value away from true (theoretical) value

TDMI implies neuronal connecting pattern in networks :

- The difference of magnitude of TDMI signal between different neuronal pairs
- Fake signal in in-directed connected neural pairs

Numerical demo of TDMI analysis between spike train and local field potential

Mutual Information Estimation

$$I_{true}(X;Y) = \iint_{x,y} f(x,y)\log\frac{f(x,y)}{f(x)f(y)}dxdy$$

$$I_{est}(X;Y) = \sum_{x_i\in X}\sum_{y_j\in Y} P(x_i,y_j)log\frac{P(x_i,y_j)}{P(x_i)p(y_j)}$$

Approximation

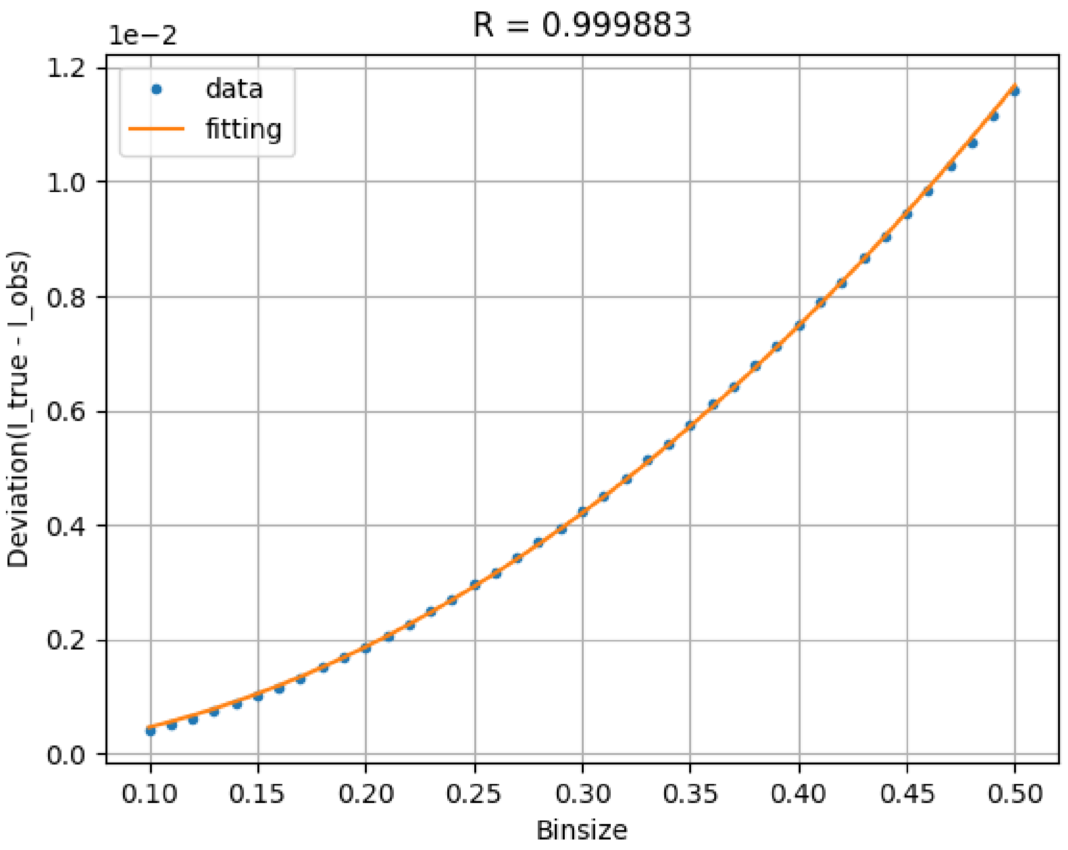
$$I_{true}(X;Y) = \sum_{x_i}\sum_{y_j}\iint_{x_i,y_j}^{x_i+\Delta x,y_j+\Delta y} f(x,y)\log\frac{f(x,y)}{f(x)f(y)}dxdy$$

$$I_{true}(X;Y) = \sum_{x_i}\sum_{y_j} f(x_i^c,y_j^c)\log\frac{f(x_i^c,y_j^c)}{f(x_i^c)f(y_j^c)}\Delta x\Delta y$$

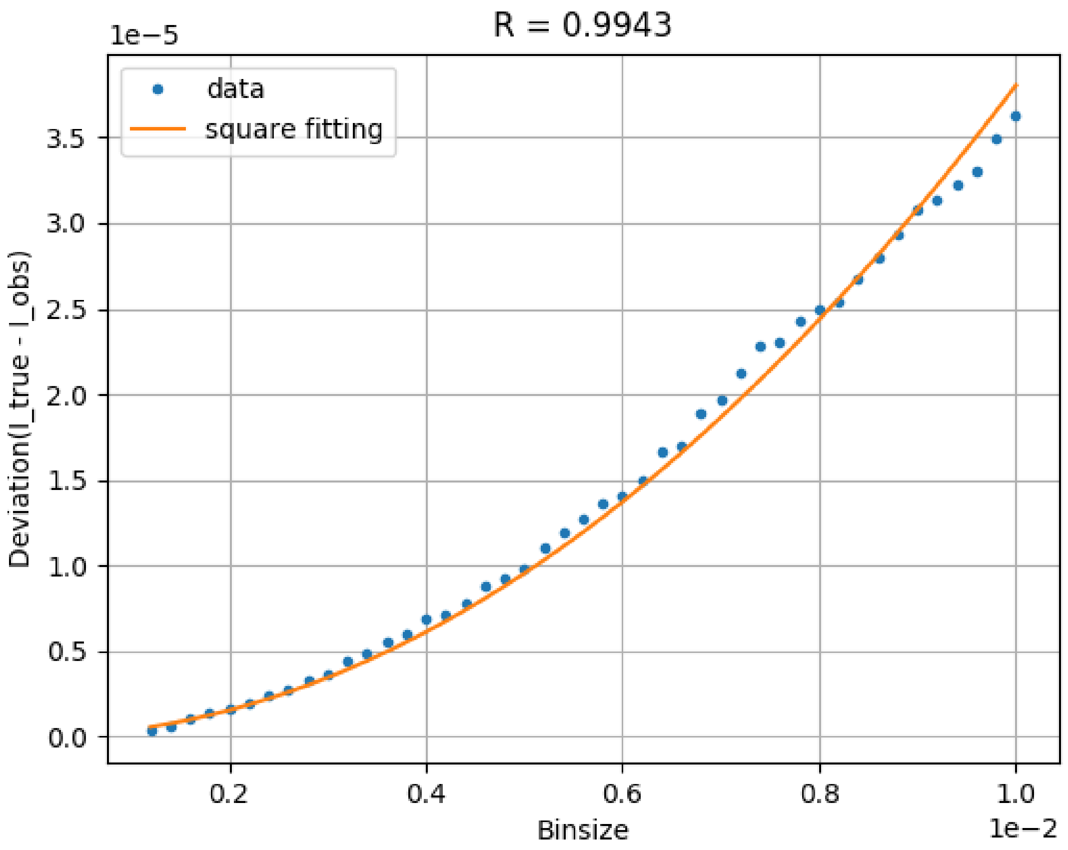
For Gaussian case: $\Delta I = I_{true} - I_{est} = \boxed{h^2}\sum_{x_i}\sum_{y_j} P(x_i,y_j)(\frac{df_x}{dx}c_i + \frac{df_y}{dy}c_j) + O(h^3)$

For spike-LFP case: $\Delta I = \boxed{h^2}\left[\sum_{x_i}^{\{0,1\}}\sum_{y_j}\frac{\partial f_{xy}}{\partial y}\Big|_{x=x_i} c_{ij}log\frac{P(x_i,y_j)}{P(x_i)P(y_j)} + \sum_{x_i}^{\{0,1\}} P(x_i,y_j)\left(\frac{\partial f_{xy}}{\partial y}\Big|_{x=x_i} c_{ij} - \frac{df_y}{dy}\hat{c}_{ij}\right)\right] + O(h^3)$

Mutual Information Estimation



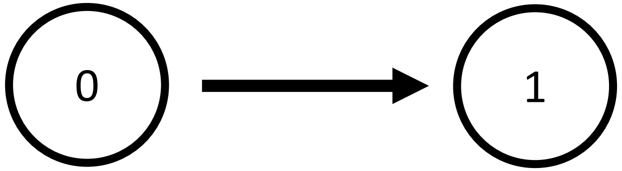
Gaussian case

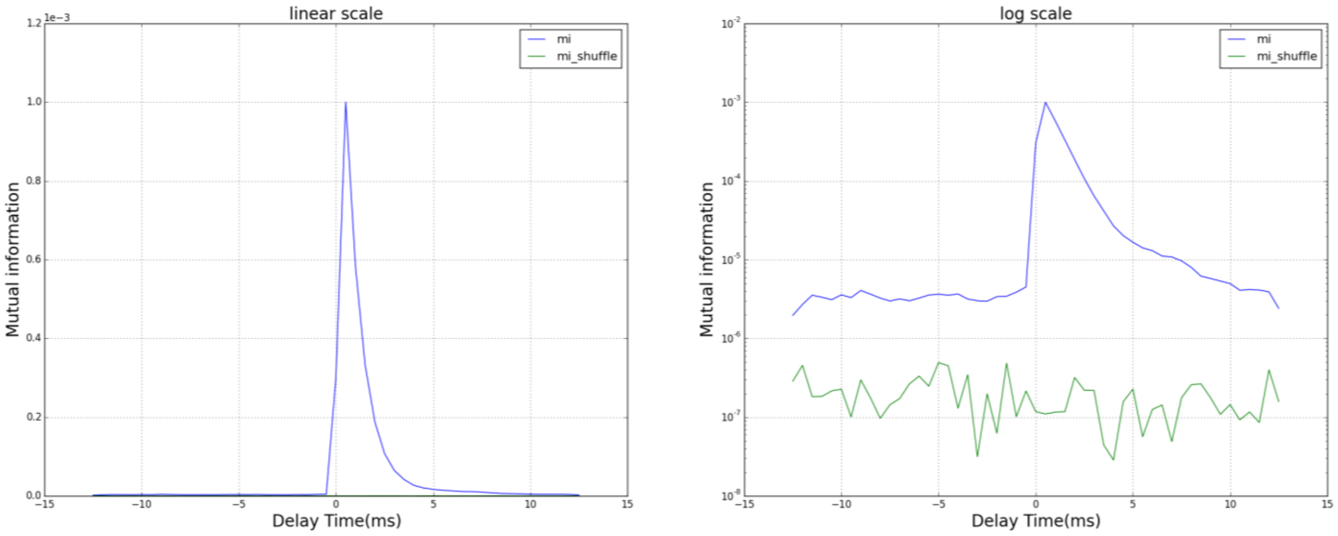


Spike-LFP case

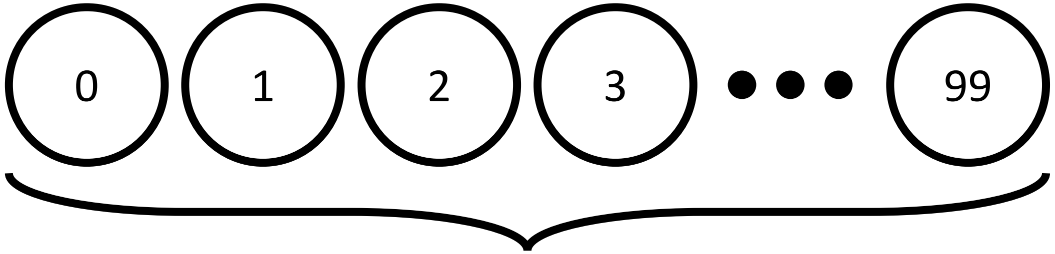
TDMI implies neuronal connecting pattern in networks

Previously: two neuron





Now: 100 neuron in a network

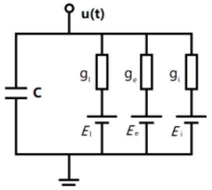


Network consisting 100 I&F neurons

Conductance-based Integrate-and-fire model:

$$C\frac{dv}{dt} = -g_l(v-\epsilon_l) - g_Q\left(v-\epsilon_Q\right) \qquad Q \in \{e,i\}$$

$$g_Q = S_Q \sum_{j,t\geq t_j} \exp\left(-\frac{t-t_j}{\tau_Q}\right)$$



Network Setups

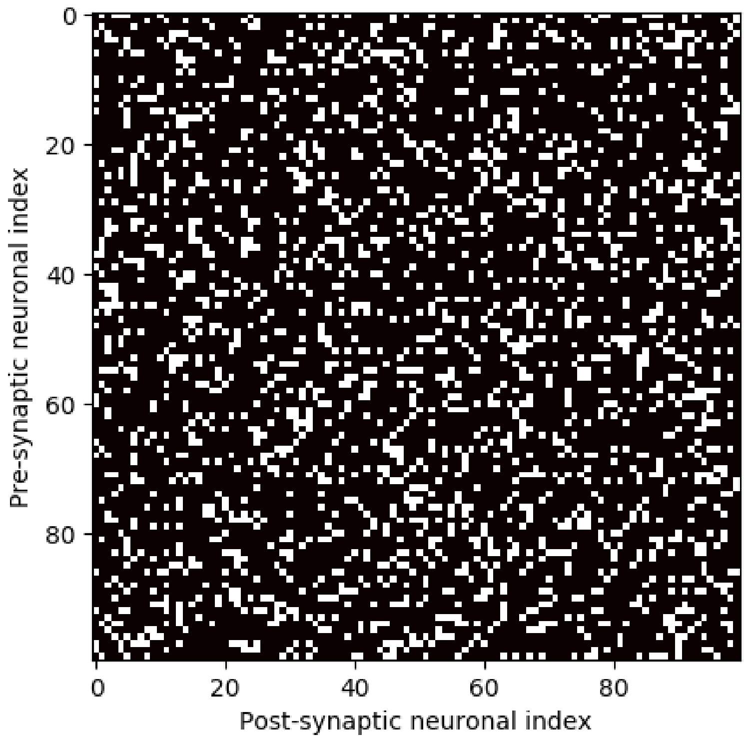
- 100 excitatory neurons
- Connectivity probability(p):

  Randomly connected by 20%
- Homogeneous Poisson input rate(ν):

  1.3 kHz
- Poisson strength(f):

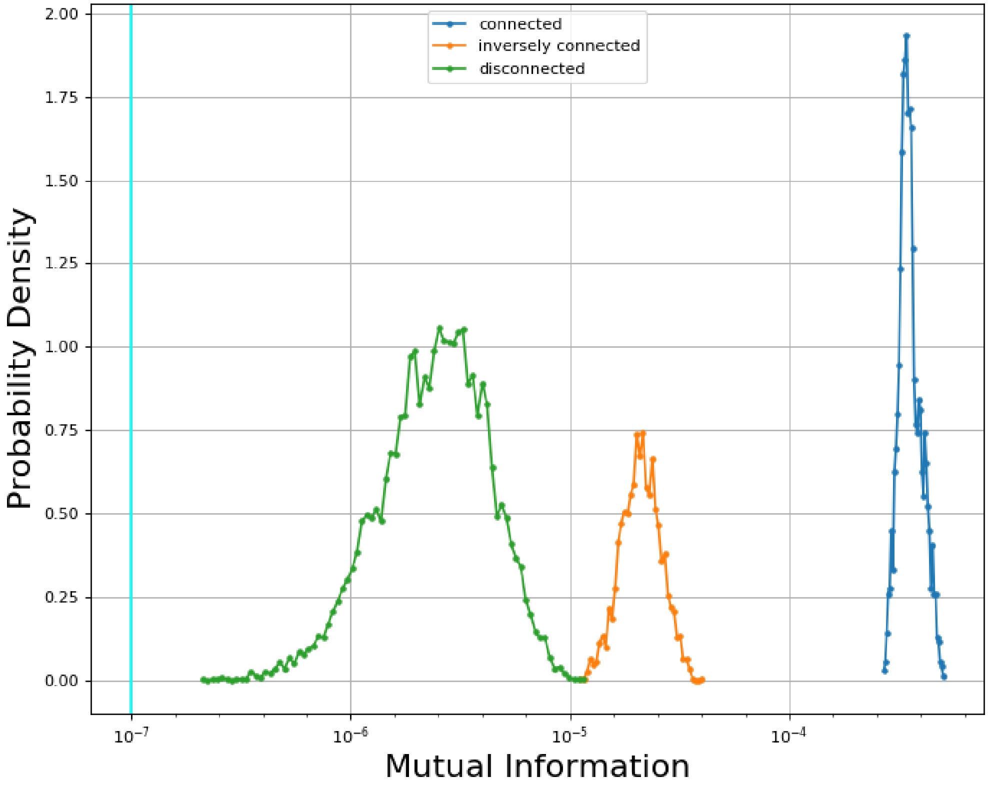
  5e-3 (Roughly 0.5mV EPSP for 15mV subthreshold range of voltage)

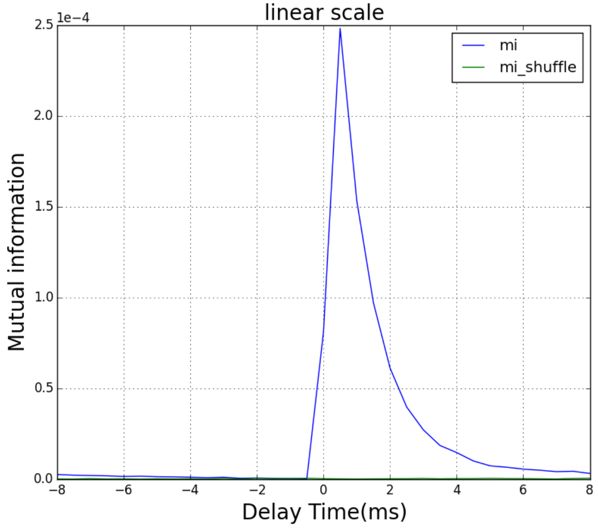
Refractory period = 2ms
Time constant of g = 2ms
Time delay of synaptic interactions = 0 ms

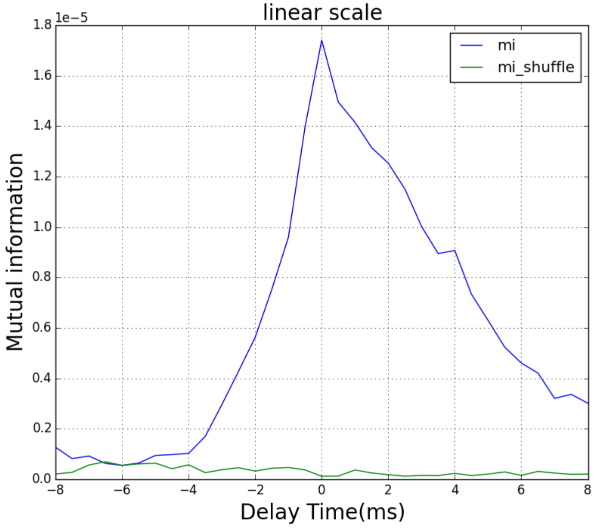


Maximum TDMI for neuronal pairs in network

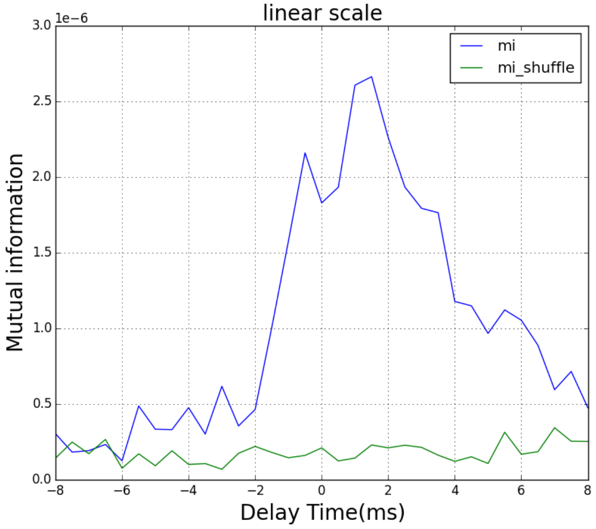
Normal case



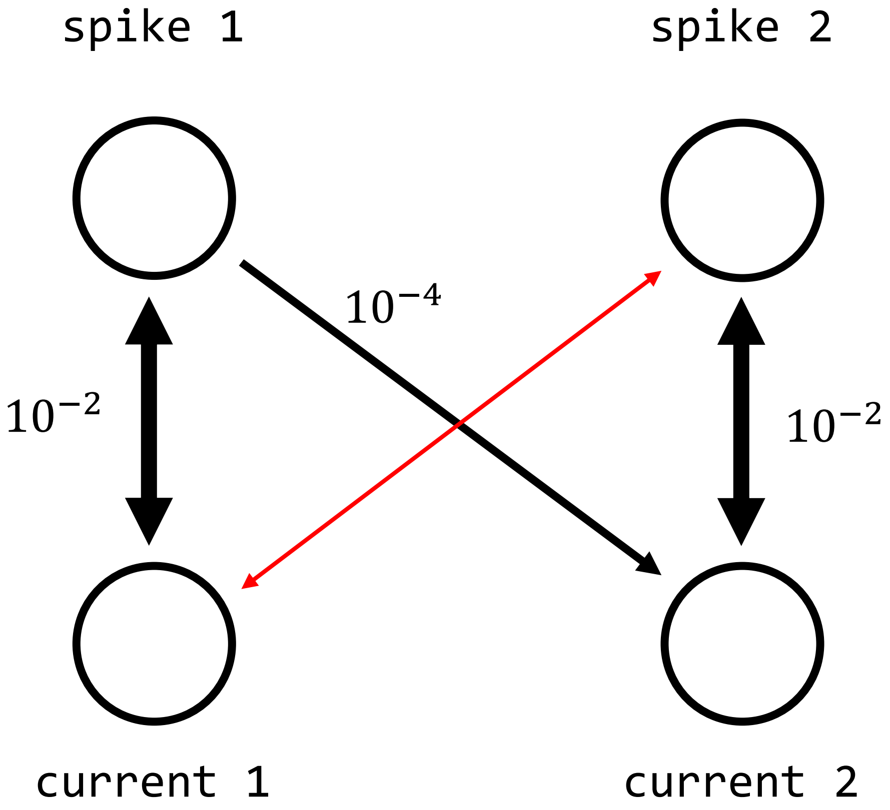


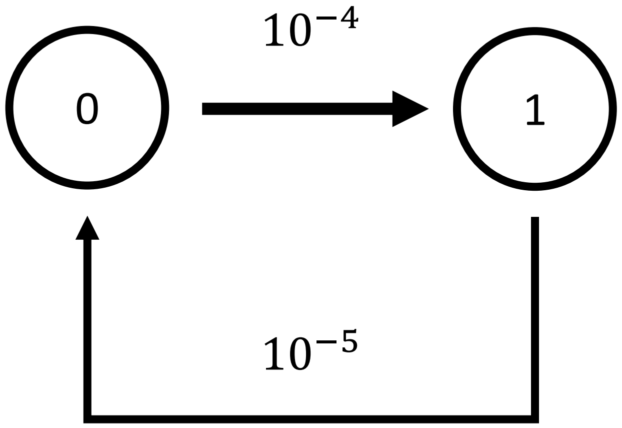


Parameters	Values
L	1e7
Recording rate	2 /ms
Binsize of LFP	0.03
Effective #bins	6-7
S	2e-3 (0.2mV)

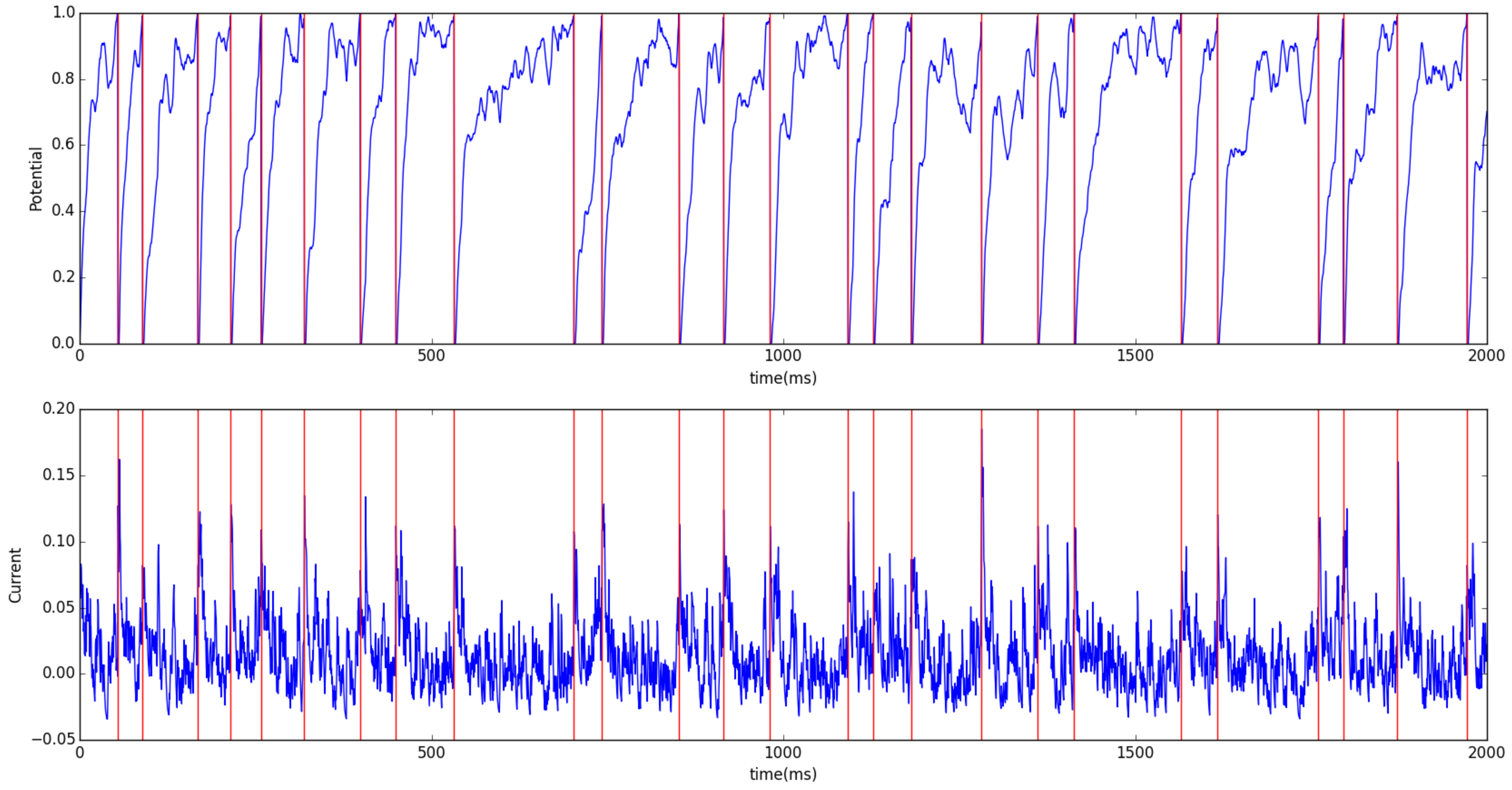


Neuronal Interaction layout



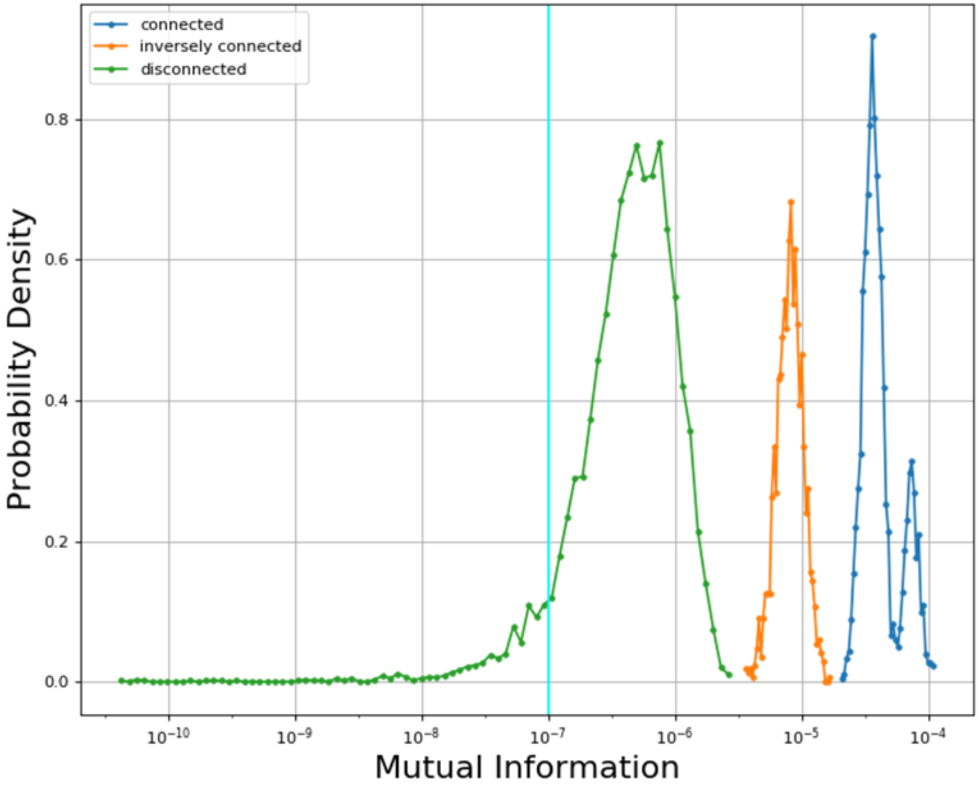


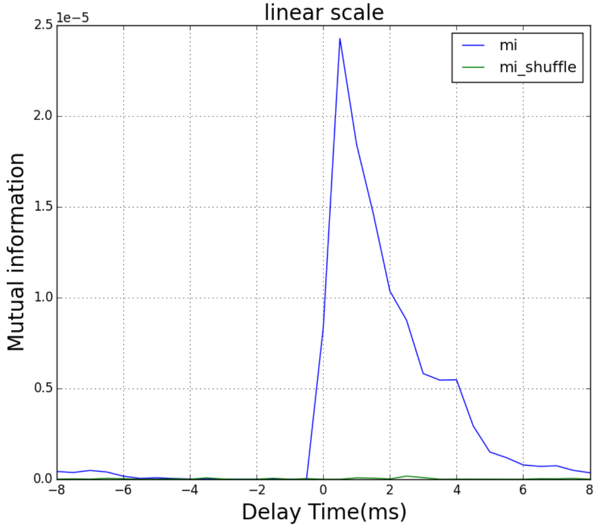
TDMI estimation with threshold scheme

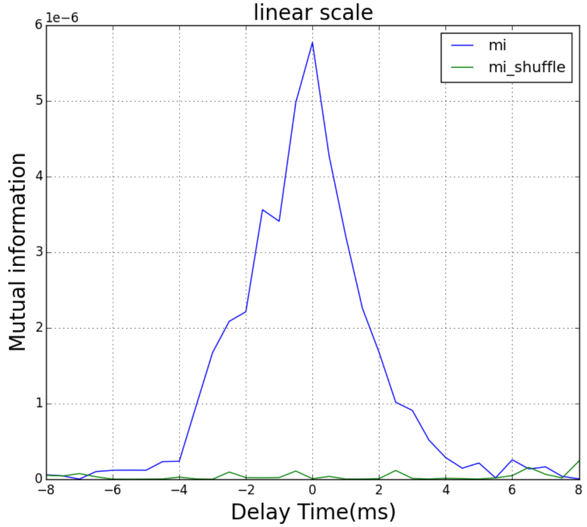


Maximum TDMI for neuronal pairs in network

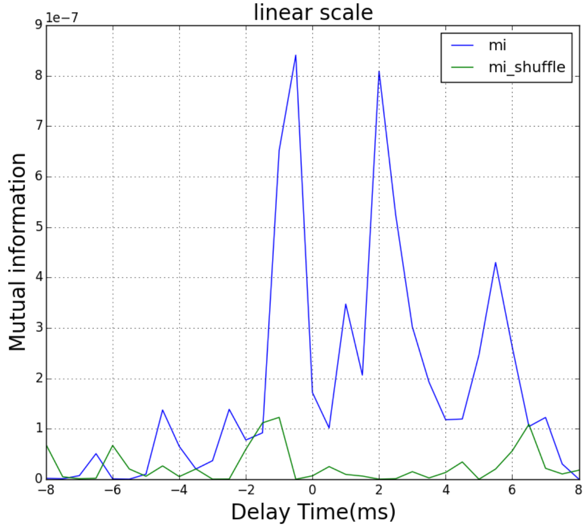
Strong interaction case





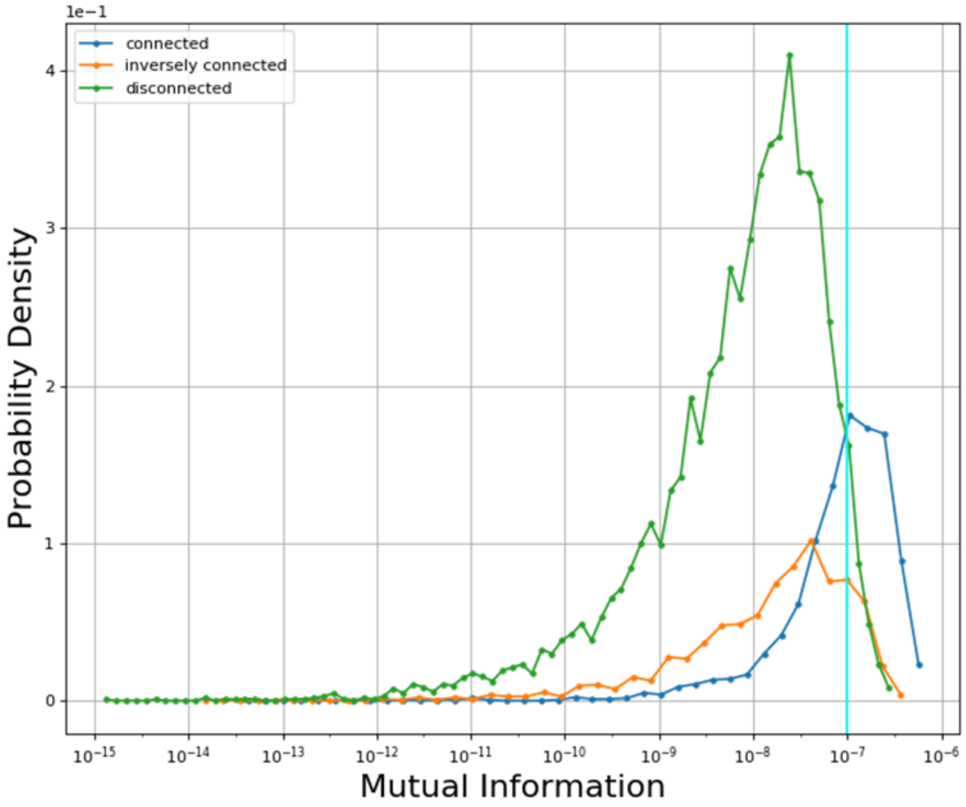


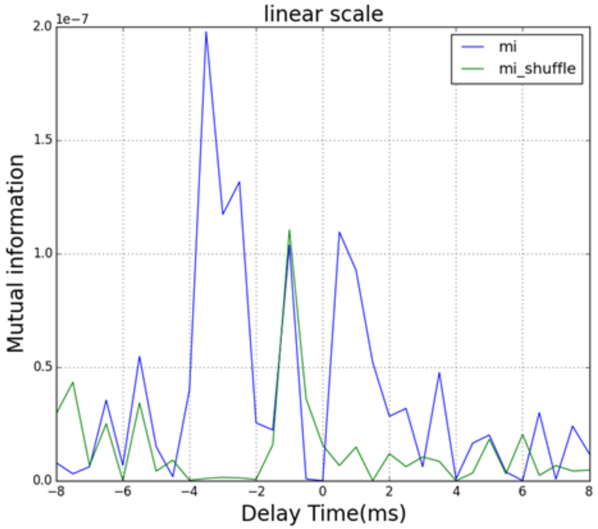
Parameters	Values
L	1e7
Recording rate	2 /ms
Threshold	0.1
Effective #bins	2
S	2e-3 (0.2mV)

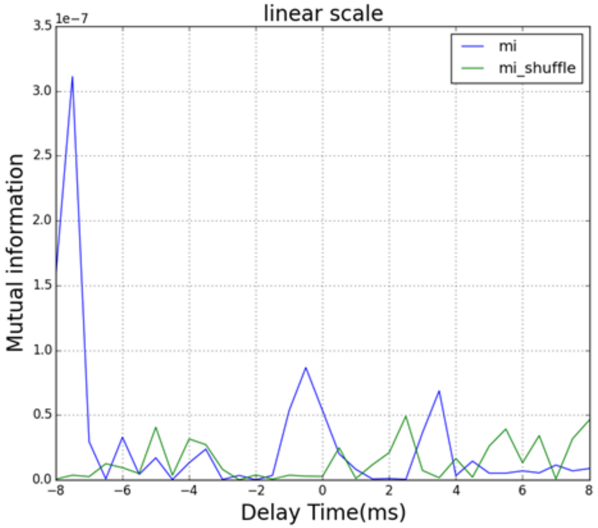


Maximum TDMI for neuronal pairs in network

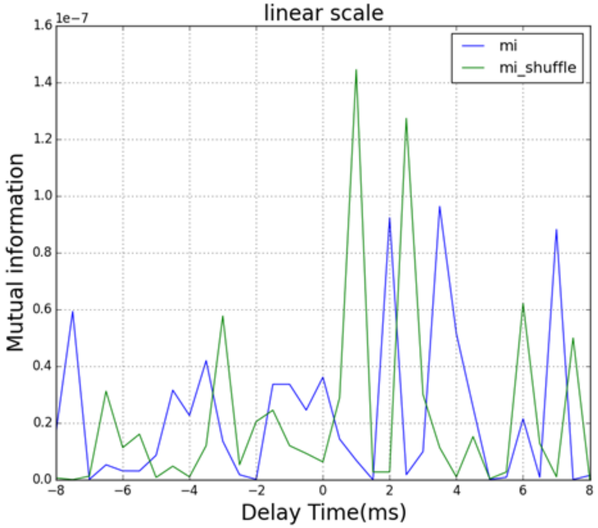
Weak interaction case



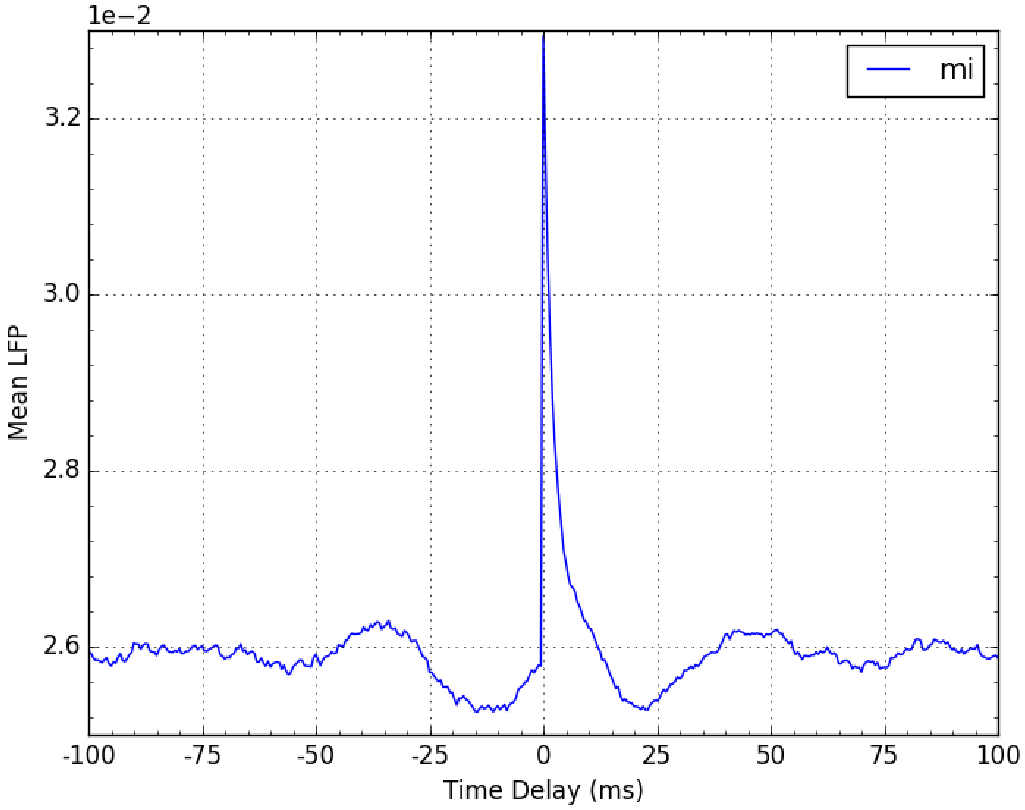




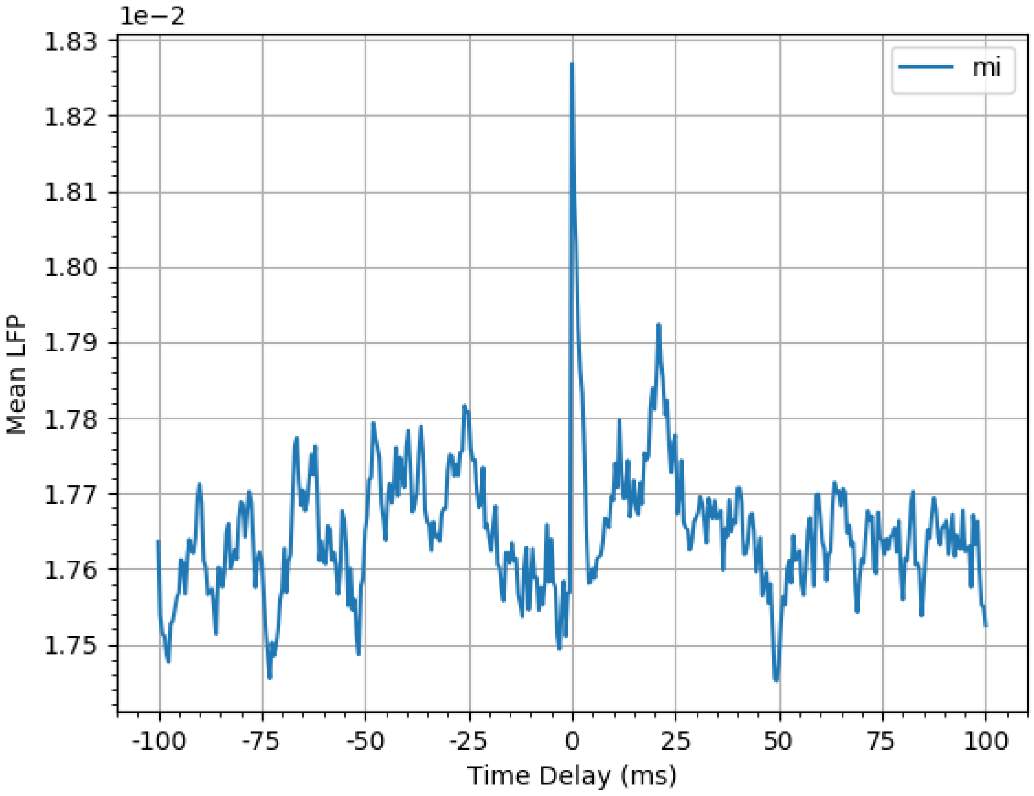
Parameters	Values
L	1e7
Recording rate	2 /ms
Threshold	0.1
Effective #bins	2
S	2e-4 (0.02mV)



Spike triggered average



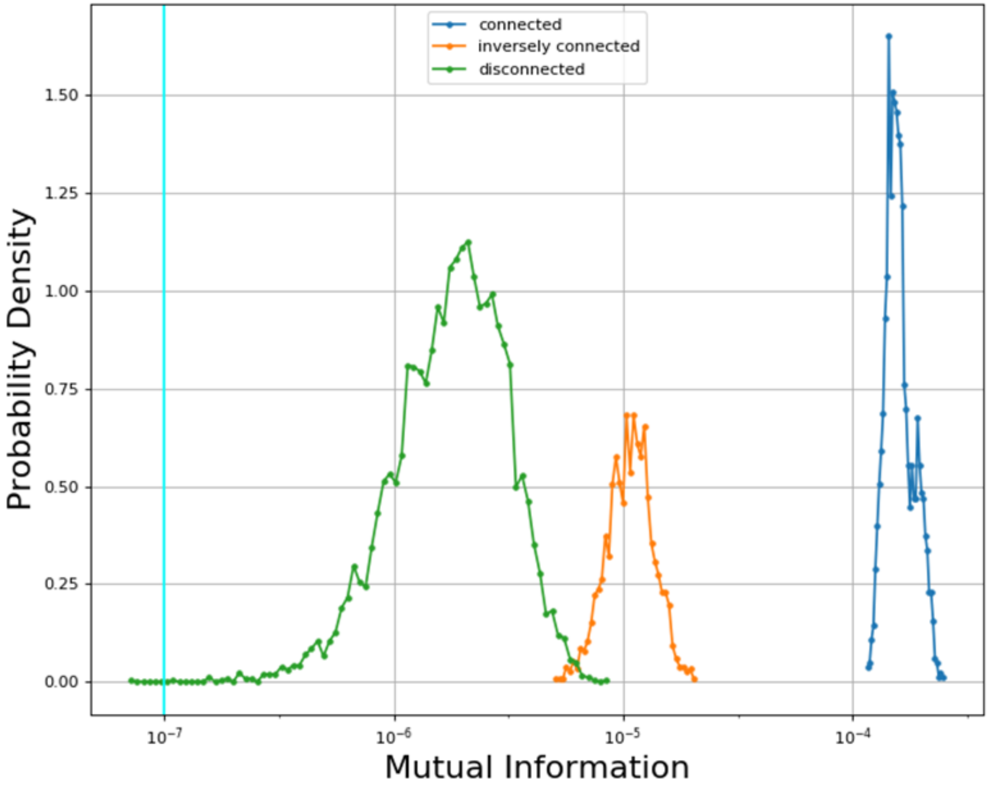
S = 2e-3 (0.2mV)

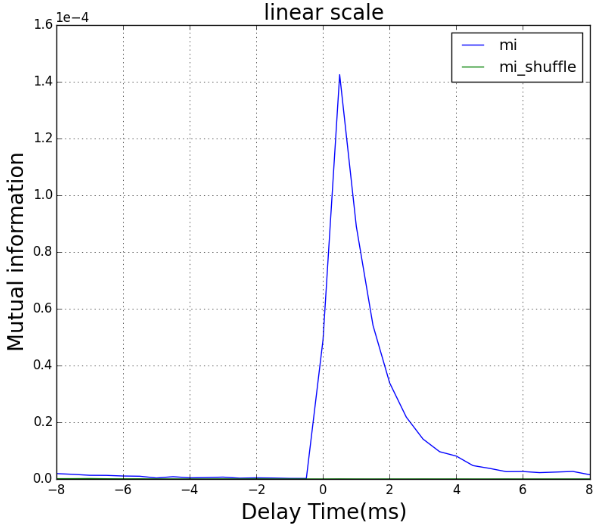


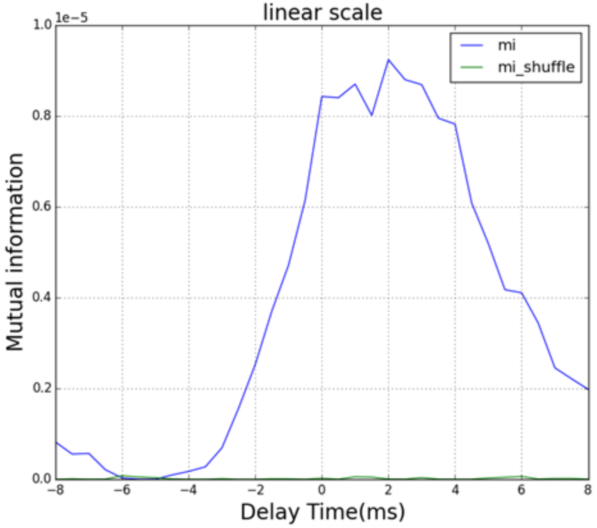
S = 2e-4 (0.02mV)

Maximum TDMI for neuronal pairs in network

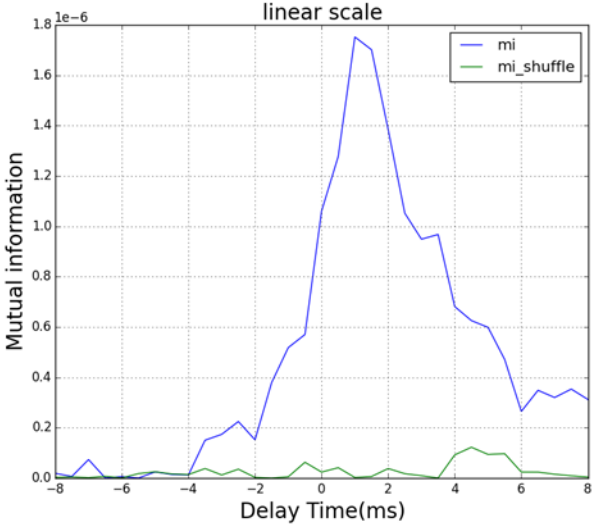
Strong interaction case





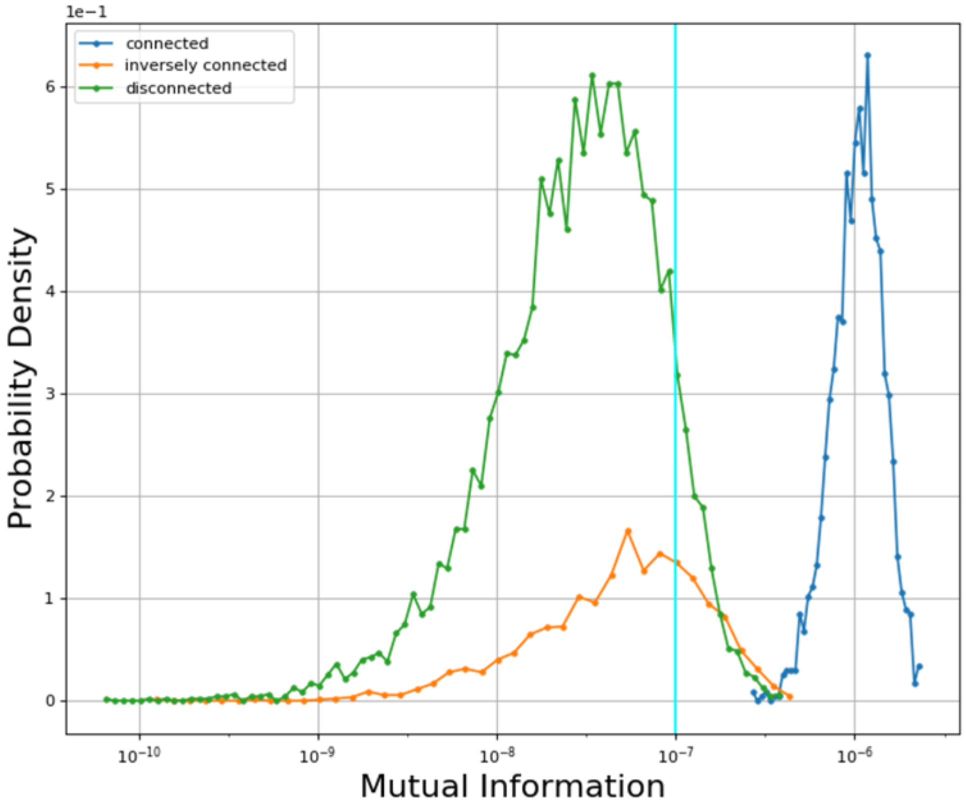


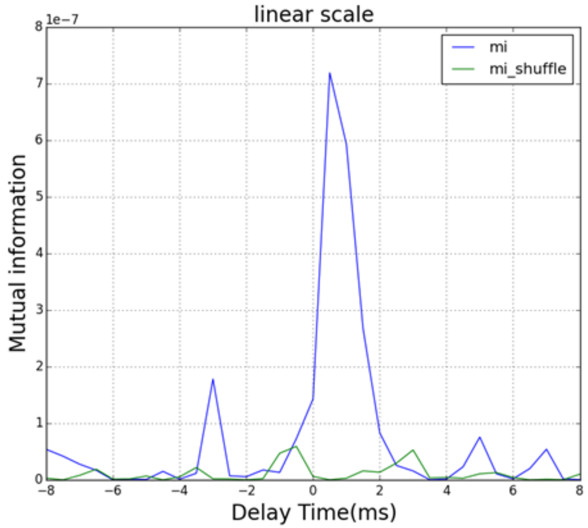
Parameters	Values
L	1e7
Recording rate	2 /ms
Threshold	0.0264
Effective #bins	2
S	2e-3 (0.2mV)

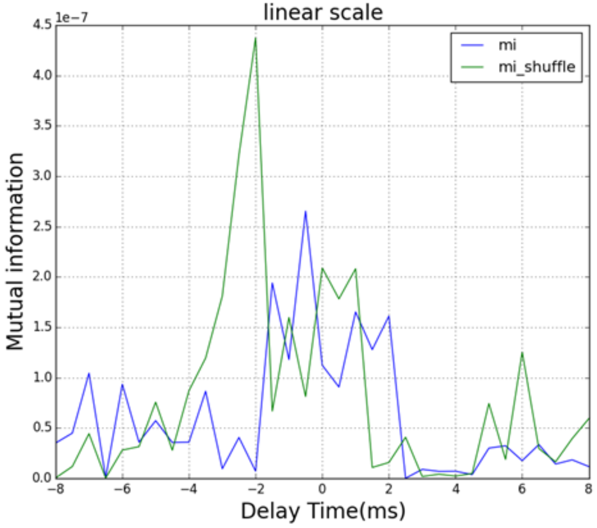


Maximum TDMI for neuronal pairs in network

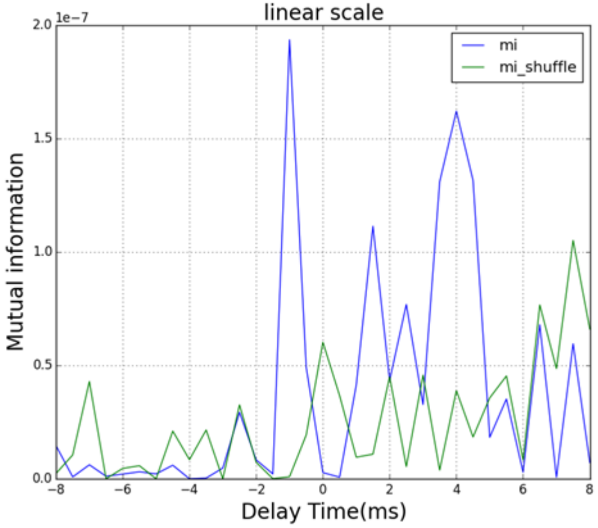
Weak interaction case



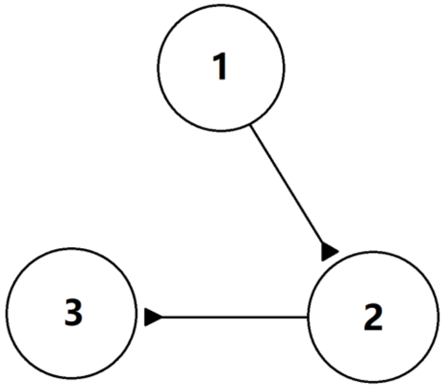




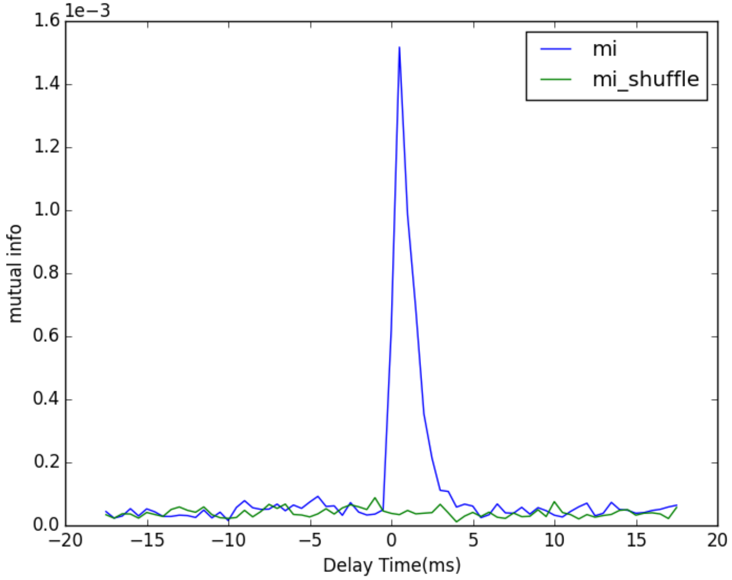
Parameters	Values
L	1e7
Recording rate	2 /ms
Threshold	0.018
Effective #bins	2
S	2e-4 (0.02mV)



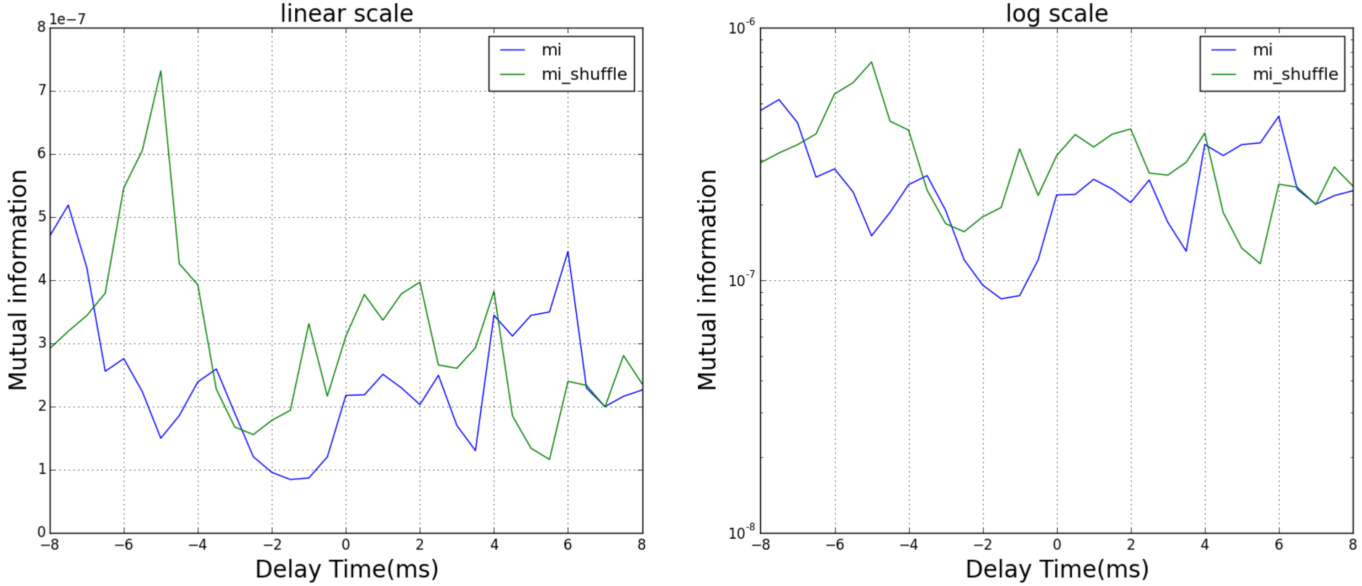
Three-neuron system



Poisson Rate	1.5 kHz
S	0.005
F	0.005
Binsize	0.03
L	1e7



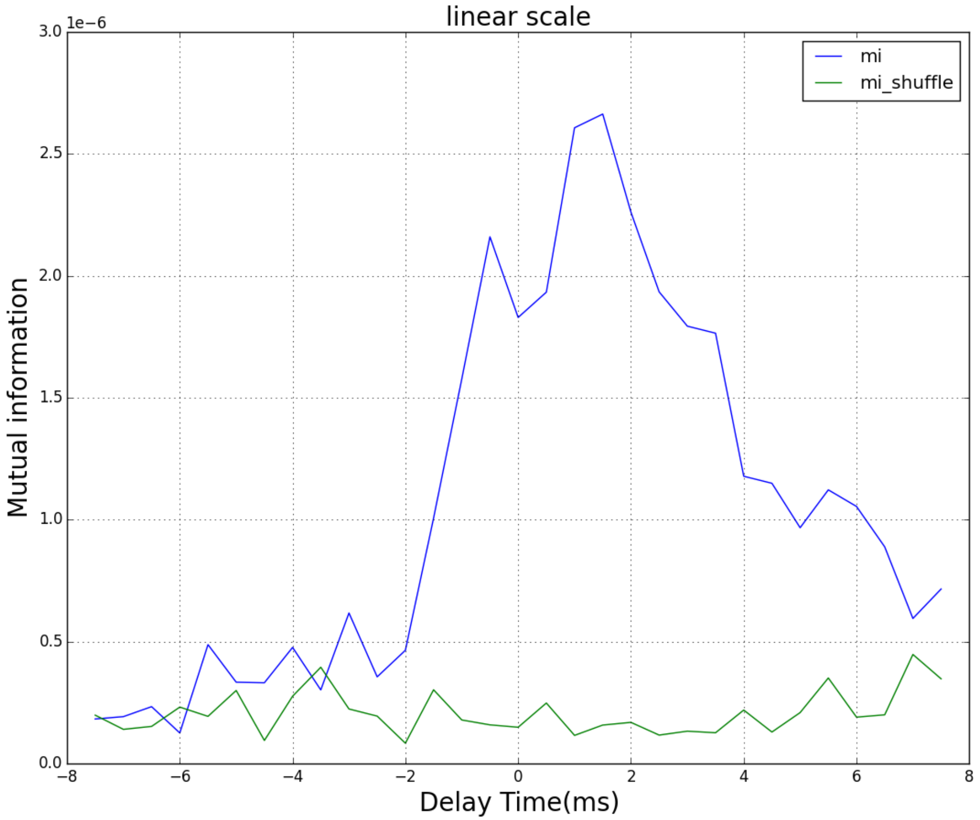
1to2



1to3

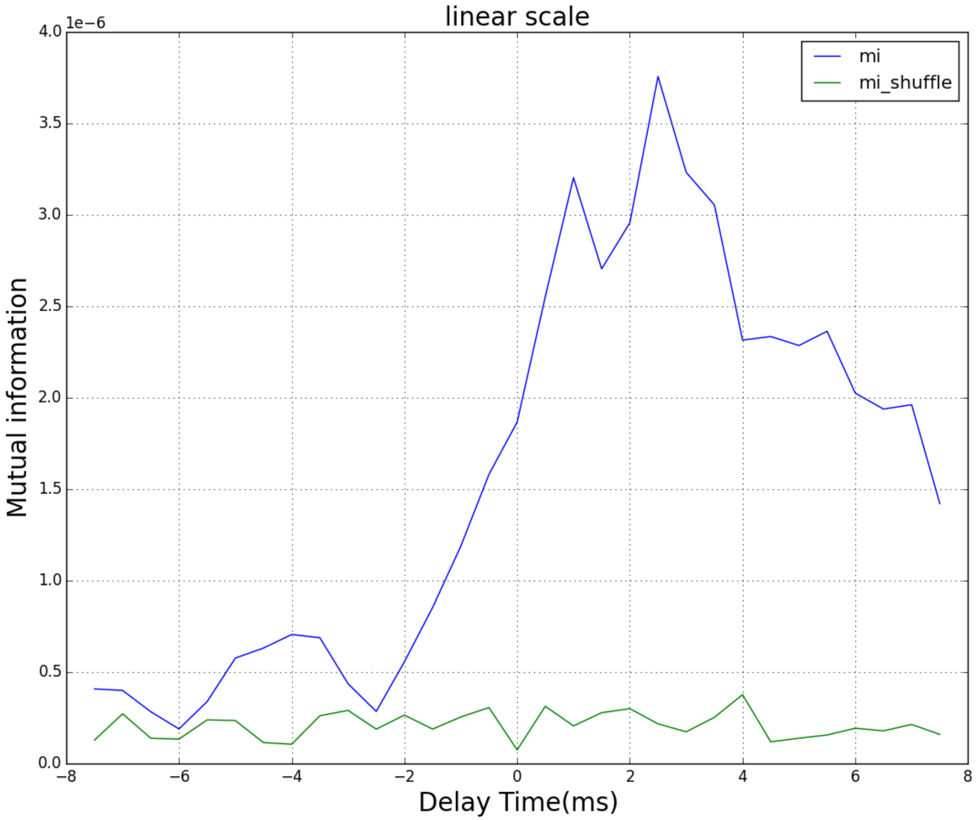
High-order neuronal pathway

#0 to #34



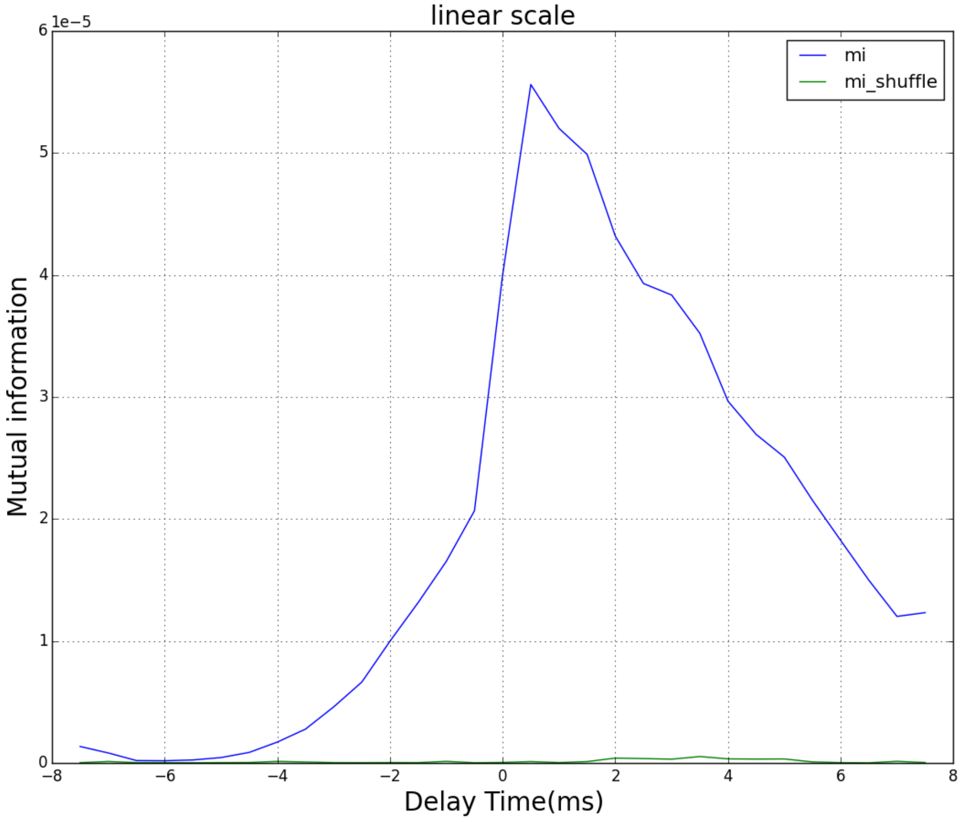
2^{nd} pathways = 4
3^{rd} pathways = 55

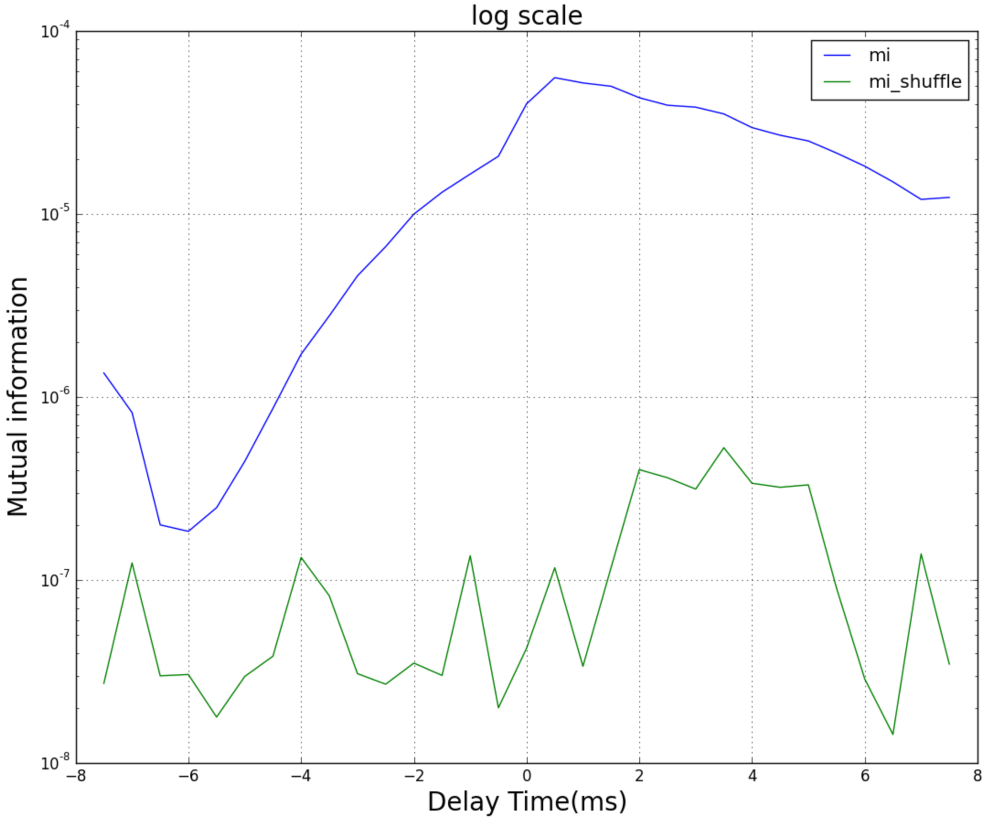
#0 to #89



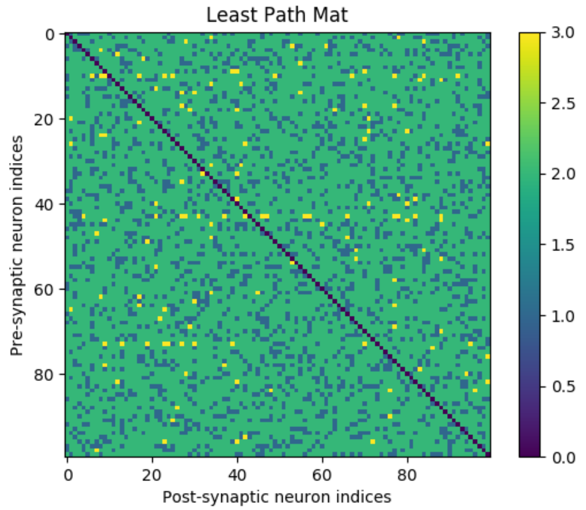
2^{nd} pathways = 6
3^{rd} pathways = 74

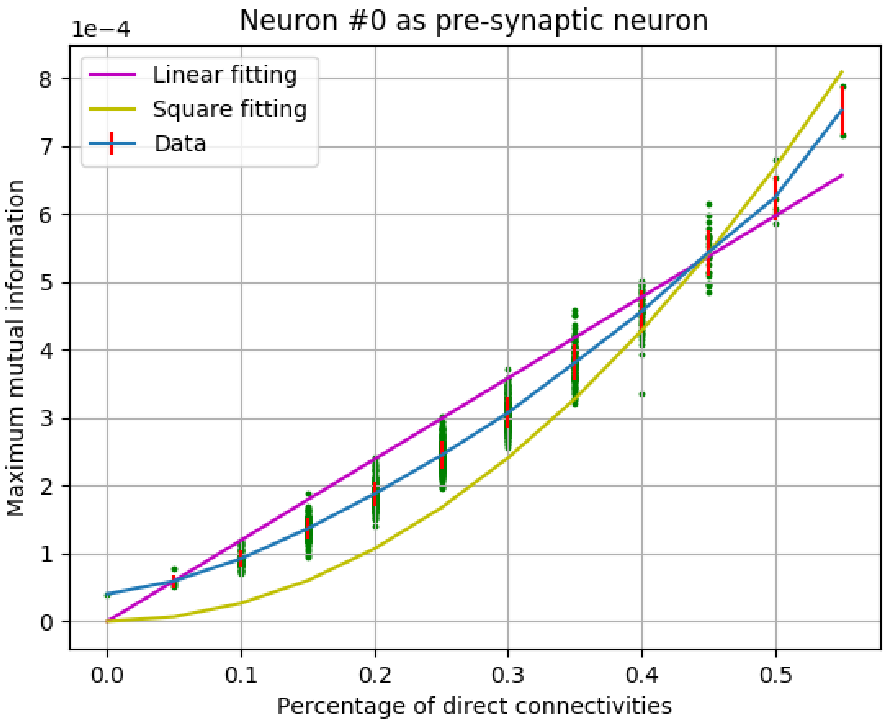
Numerical demo of TDMI analysis between spike train and local field potential

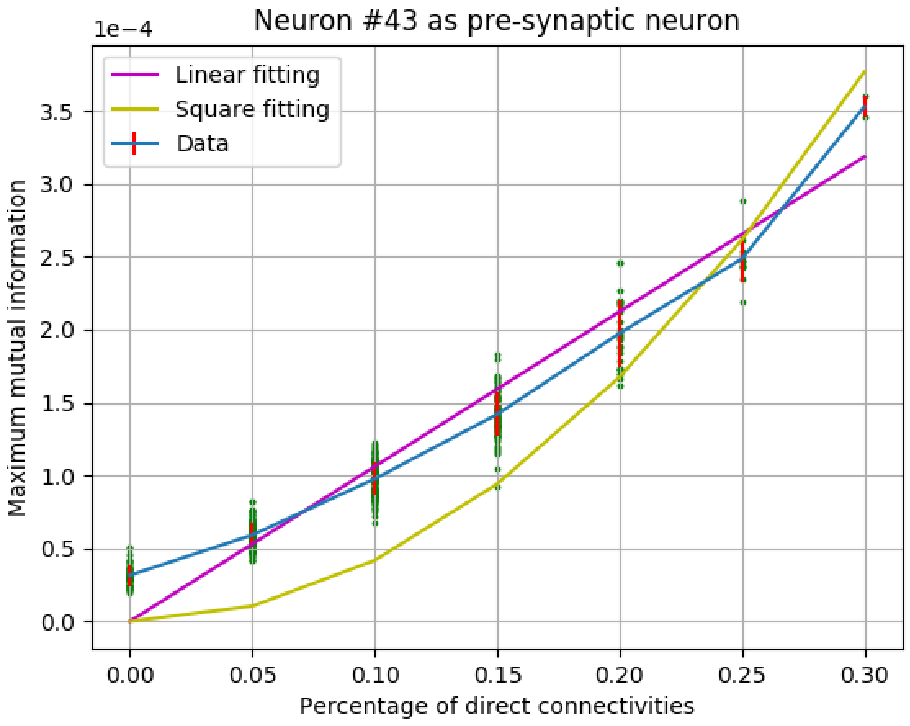




Numerical demo of TDMI analysis between spike train and local field potential

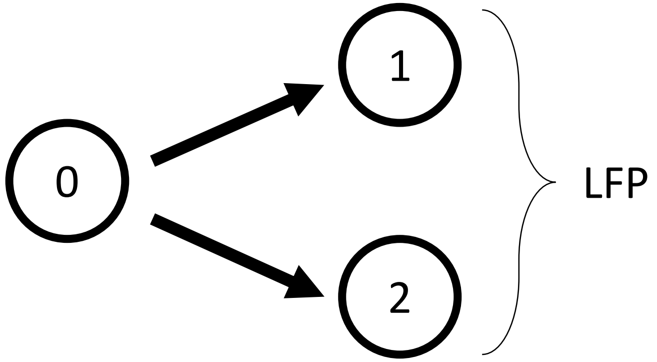




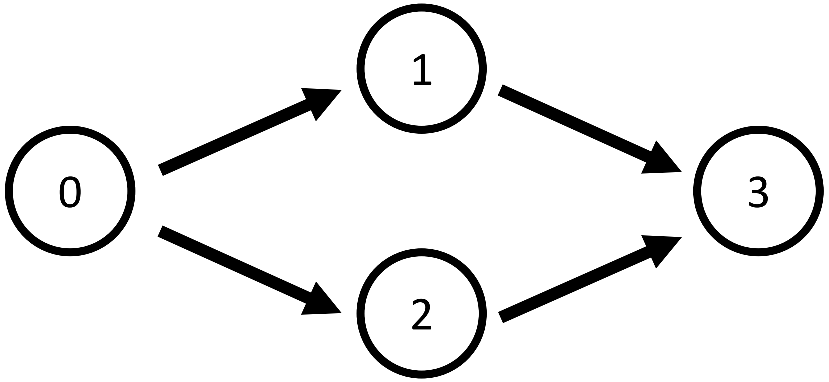


Questions to be answered

$$I(X; Y_1 + Y_2) \longleftrightarrow I(X; Y_1) + I(X; Y_2)$$



$$I(X; Y) \longleftrightarrow I(X; F(Y) + G(Y))$$



Summary

Mutual information estimation

The error in MI estimation is quadratically proportional to the binning size *h* in LFP’s histogram.

TDMI implies neuronal connecting pattern in networks :

- Direct connection can be inferred by the order of magnitude of maximum MI between neuronal pairs
- Inversely direct connection can be inferred if the amount of data is sufficiently large.

Numerical demo of TDMI analysis between spike train and local field potential