

The Complete A-Level Guide to Logarithms

General Formula:

Any number, n , can be expressed as a power, p , of a base, b , thusly:

$$n = b^p$$

$$\text{where } b = \sqrt[p]{n}$$

$$\text{and } p = \log_b(n)$$

The logarithmic function, $\log_b(n) = p$, is the mathematically inverse function to exponentiation.

Identities:

The logarithmic function has a few established identities as follows:

| | |
|----------------|--|
| Product | $\log_b(xy) = \log_b(x) + \log_b(y)$ |
| Quotient | $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$ |
| Power | $\log_b(x^p) = p \log_b(x)$ |
| Root | $\log_b(\sqrt[p]{x}) = \frac{\log_b(x)}{p}$ |
| Change of Base | $\log_b(x) = \frac{\log_k(x)}{\log_k(b)}$ |

Particular Bases:

The logarithmic function is often used with particular bases which have specialised names and notation as follows:

| Base b | Name | Notation |
|----------|-------------------|----------------|
| 2 | Binary Logarithm | $\text{lb}(x)$ |
| e | Natural Logarithm | $\ln(x)$ |

| | | |
|-----|-----------------------|-------------------|
| 10 | Common Logarithm | $\lg(x), \log(x)$ |
| b | Logarithm to Base b | $\log_b(x)$ |

Examples of Logarithmic Scales and their Related Expressions:

Logarithmic scales are used within many contexts and logarithmic equations are used to calculate many quantities, below are just a few:

| | |
|---------------|---|
| Richter Scale | $M = \lg\left(\frac{I}{S}\right)$ |
| pH Scale | $pH \text{ of solution}$ $= -\lg(\text{Concentration of } H^+ \text{ ions})$ |
| Decibel Scale | $dB = 10 \lg\left(\frac{S_1}{S_2}\right)$ |
| Half-Life | $t_{1/2} = \frac{t}{\lg\left(\frac{N_0}{N(t)}\right)}$ |