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D6AD/47

Maths Tutorial 6

Q1) x: 65 / 63 / 67 / 64 / 68 / 62 / 70 / 66 / 68 / 67 / 69 / 71

y: 68 / 66 / 68 / 65 / 69 / 66 / 68 / 65 / 71 / 67 / 68 / 70

To find : Rank correlation co-efficient b/w x, y.

i.e $\rho_{x,y}$.

Formulae : $\rho_{xy} = \mathrm{Corr}(x,y) = \dfrac{\mathrm{Cov}(x,y)}{\sigma_x \, \sigma_y}$

Solution: $\mu_x = \dfrac{65+63+67+64+68+62+70+66+68+67+69+71}{12}$

$= \dfrac{65 \times \cancel{12}^{3} + \cancel{20}^{5}}{\cancel{12}\ 3} = \boxed{\dfrac{200}{3} = \mu_x}$

$\mu_y = 68+66+68+65+69+66+68+65+71+67+68+70$

$= \dfrac{68 \times 12 - 5}{12} = \dfrac{680+136-5}{12} = \boxed{\dfrac{811}{12} = \mu_y}$

$\mu_x = 66.67$, $\mu_y = 67.58$.

$\mu_x = 66.67$, $\mu_y = 67.58$

x	y	~~μ_x μ_y~~	$x-\mu_x$	$y-\mu_y$	$(x-\mu_x)(y-\mu_y)$
65	68		-1.67	0.42	-0.7014
63	66		-3.67	-1.58	5.7986
67	68		0.33	0.42	0.1386
64	65		-2.67	-2.58	6.8886
68	69		1.33	1.42	1.8886
62	66		-4.67	-1.58	7.3786
70	68		3.33	0.42	1.3986
66	65		-0.67	-2.58	1.7286
68	71		1.33	3.42	4.5486
67	67		0.33	-0.58	-0.1914
69	68		2.33	0.42	0.9786
71	70		4.33	2.42	10.4786

Covariance $= \therefore \dfrac{\sum (x-\mu_x)(y-\mu_y)}{n} = \dfrac{39.546}{12} = 3.2955.$

for $S.D_x = \sqrt{\dfrac{\sum (x-\mu_x)^2}{12}} = \sqrt{\dfrac{84.6668}{12}} = 2.66$

$\therefore \sigma_x = 2.66.$

for $S.D_y = \sigma_y = \sqrt{\frac{\Sigma(y-\mu_y)}{12}} = \sqrt{\frac{44.7732}{12}}$

$\sigma_y = 1.93.$

$\therefore$ Correlation $(x, y) = \frac{Cov(x, y)}{\sigma_x \cdot \sigma_y}$

$= \frac{3.2955}{2.66 \times 1.93}.$

$= 0.642$

$\boxed{\therefore \text{Correlation}(x, y) = r = 0.642}$

Q2.

x	y	xy	x^2	y^2
65	67	4355	4225	4489
66	68	4488	4356	4624
67	65	4355	4489	4225
67	68	4556	4489	4624
68	72	4846	4624	5184
69	72	4968	4761	5184
70	69	4830	4900	4761
72	71	5112	5184	5041

$\boxed{\Sigma x = 544}$ $\boxed{\Sigma xy = 37560}$ $\boxed{\Sigma y^2 = 38132}$

$\boxed{\Sigma y = 552}$ $\boxed{\Sigma x^2 = 37028}$

$$E(x) = \frac{544}{8} = 68, \quad E(y) = \frac{552}{8} = 69,$$

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \frac{8 \times 37560 - 544 \times 552}{8 \times 37028 - 544 \times 544}$$

$$= \frac{192}{288} = \frac{2}{3}$$

$\therefore$ Regression line of y on x is

$$y - \mu_y = b_{yx}(x - \mu_x)$$

$$y - 69 = \frac{2}{3}(x - 68)$$

$$2x - 3y - 136 + 207 = 0.$$

$\boxed{\therefore\ 2x - 3y + 71 = 0 \text{ is the regression line of } y \text{ on } x}$

$$\boxed{y = 0.666x + 23.667.}$$

$$b_{xy} = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2} = \frac{8 \times 37560 - 544 \times 552}{8 \times 38132 - 552 \times 552}$$

$$= \frac{192}{352} = \frac{6}{11}.$$

$\therefore$ Regression line of x on y is

$$x - \mu_x = b_{xy}(y - \mu_y)$$

$$x - 68 = \frac{6}{11}(y - 69)$$

$$\boxed{x = 0.545y + 30.36}$$

for $y = 70$,

using the regression line of x on y

$$x = 0.545y + 30.36.$$

$$x = 0.545(70) + 30.36.$$

$$\boxed{x = 68.51}$$

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Q2.

Given: $\sigma_x^2 = 9$.

$8x - 10y + 66 = 0$,

$40x - 18y - 214 = 0$.

Solution: $40x - 18y - 214 = 0$ —— ①

$5 \times 8x - 5 \times 10y + 66 \times 5 = 0$ —— ②

Subtracting ② from ①, we get

$-18y - (-50y) - 214 - 66 \times 5 = 0$.

$32y - 544 = 0$.

$y = \frac{544}{32}$.

$\therefore y = 17$ is the mean, $\boxed{\therefore \mu_y = 17}$ —— ③

from ③ and ①, we get

$40x - 18 \times 17 - 214 = 0$.

$x = \frac{52\cancel{0}}{4\cancel{0}}$

$\therefore x = 13$ is the mean, $\boxed{\therefore \mu_x = 13}$ —— ④.

(i) $\boxed{\therefore \text{The means } \bar{x} \text{ \& } \bar{y} \text{ are 13 \& 17.}}$

$$x = \frac{18}{40}y + \frac{214}{40}$$

$$r_x = \frac{18}{40} = 0.45.$$

$$\boxed{r_x = 0.45}$$

$$y = \frac{8x}{10} + \frac{66}{10}$$

$$\boxed{r_y = 0.8.}$$

$$r^2 = r_x \times r_y = 0.36.$$

$$\therefore r = \pm\sqrt{0.36}$$

$$r = \pm 0.6.$$

we know that if both regression coefficients are positive, r would be positive.

(ii) $\boxed{r = 0.6}$

$$\sigma_x^2 = 9.$$

Since standard deviation σ_x is never negative,

$$\sigma_x = \sqrt{9}$$

$$= 3.$$

(iii) $\boxed{\therefore \sigma_x = 3}$