TUTORIAL - 2

INVERSE LAPLACE TRANSFORM

Q1) $\dfrac{e^{4-3s}}{(s+4)^{3/2}}$

$$L^{-1}\left(\frac{e^{4-3s}}{(s+4)^{3/2}}\right) = L^{-1}\frac{(e^{4}e^{-3s})}{(s+4)^{3/2}} = e^{4}L^{-1}\left(\frac{e^{-3s}}{(s+4)^{3/2}}\right)$$

$$f(t) = L^{-1}\left(\frac{1}{(s+4)^{3/2}}\right) = e^{-4} + L^{-1}\frac{1}{s^{3/2}}$$

$$= e^{-4t}\frac{t^{3/2}}{\Gamma 3/2} = \frac{e^{-4t}\,t^{3/2}}{3/4\sqrt{\pi}}$$

$$L^{-1}\left(\frac{e^{-3s}\times e^{4}}{(s+4)^{3/2}}\right) = g(t) = \begin{cases} f(t-3) & t \geq 3 \\ 0 & t < 3 \end{cases}$$

$$= e^{4}e^{-4(t-3)} \times 4 \times \frac{(t-3)^{3/2}}{\sqrt{\pi}\times 3}$$

$t \geq 3$

$= 0 \quad t < 0$

2) $$\frac{1}{(s^2+2s+2)(s^2+2s+5)}$$

$$L^{-1}\left(\frac{1}{\left[(s+1)^2+(1)^2\right]\left[(s+1)^2+(2)^2\right]}\right)$$

$$= e^{-t}\left(\frac{1}{(s^2+1)(s^2+4)^2}\right)$$

let → $s^2 = x$

$$= \frac{1}{(x+1)(x+4)} = \frac{A}{x+1} + \frac{B}{x+4}$$

$$1 = A(x+4) + B(x+1)$$

$$= (A+B) + (4A+B)$$

$4A+B = 1$ $\quad A+B = 0$

$3A = 1$

$A = \frac{1}{3}$ $\quad B = -\frac{1}{3}$

$$= e^{-t} L^{-1}\left(\frac{1}{3} \times \frac{1}{s^2+1} - \frac{1}{3} \times \frac{1}{s^2+4}\right)$$

$$= \frac{e^{-t}}{3}\left[\sin t - \frac{\sin 2t}{2}\right]$$

3) $\frac{s}{(s^2+4)^2}$ Using Convolution thm

$$L^{-1}\left\{\frac{s}{(s^2+2^2)^2}\right\}$$

$$= L^{-1}\left[\frac{s}{(s^2+2^2)} \times \frac{1}{(s^2+2^2)}\right]$$

Let $\Rightarrow$ $\bar{f}(s) = \frac{s}{s^2+2^2}$ $\qquad \bar{g}(s) = \frac{1}{s^2+2^2}$

$$L^{-1}(\bar{f}(s)) = L^{-1}\left\{\frac{s}{s^2+2^2}\right\} = \cos 2t = f(t)$$

$$L^{-1}(\bar{g}(s)) = L^{-1}\left\{\frac{1}{s^2+2^2}\right\} = \frac{1}{2}\sin 2t = g(t)$$

By Convolution thm

$$L^{-1}\{f(t)\cdot g(t)\} = \int_0^t F(u)\cdot g(t-u)\,du$$

$$= \int_0^t \cos 2t \, \frac{1}{2}\sin(2t-2u)\,du$$

$$= \frac{1}{4}\int_0^t \{(\sin \cancel{au} + at - \cancel{au}) - \sin(au - at + au)\}\,du$$

$$= \frac{1}{4}\int_0^t \{(\sin at - \sin(2au - at)\}\,du$$

$$= \frac{1}{4}\left[\sin at\,[u]_0^t + \left[\frac{\cos(2au-at)}{4}\right]_0^t\right]$$

$$= \frac{1}{4}\left[t\sin at + \cancel{\frac{\cos at}{4}} - \cancel{\frac{\cos at}{4}}\right]$$

$$= \frac{t\sin 2t}{4}$$

5)

s^2+6

4) $\tan^{-1}\left(\frac{2}{s^2}\right)$

$$= L^{-1}\left[\tan^{-1}\left(\frac{2}{s^2}\right)\right]$$

$$= -\frac{1}{t} L^{-1}\left[\frac{d}{ds}\left(\tan^{-1}\frac{2}{s^2}\right)\right]$$

$$= -\frac{1}{t} L^{-1}\left(\frac{1}{1+(2/s^2)^2}\,\frac{d}{ds}\,\frac{2}{s^2}\right)$$

$$= -\frac{1}{t} L^{-1}\left[\frac{s^4}{s^4+4} \times \frac{-4}{s^3}\right]$$

$$= \frac{4}{t} L^{-1}\left[\frac{s}{(s^2+2)^2-(2s)^2}\right]$$

$$= \frac{4}{t} L^{-1}\left[\frac{1}{4}\left(\frac{1}{s^2+2-2s} - \frac{1}{s^2+2+2s}\right)\right]$$

$$= \frac{1}{t} L^{-1}\left[\frac{1}{(s-1)^2+1} - \frac{1}{(s+1)^2+1}\right]$$

$$= \frac{1}{t}\left[e^{t} L^{-1}\left\{\frac{1}{s^2+1}\right\} - e^{-t} L^{-1}\left[\frac{1}{s^2+1}\right]\right]$$

$$= \cancel{\frac{2}{t}}\ \frac{1}{t}\left[e^{t}\sin t - e^{-t}\sin t\right]$$

$$= \frac{2\sin t\,\sin t}{t}$$

5) $\dfrac{s^2+6}{(s^2+1)(s^2+4)}$

$$L^{-1}\left[\frac{s^2+6}{(s^2+1)(s^2+4)}\right]$$

$$= \frac{A}{s^2+1^2} + \frac{B}{s^2+2^2}$$

$$s^2+6 = (s^2+2^2)A + B(s^2+1^2)$$

$$A+B) = 1$$

$$4A+B = 6$$

$$A = \frac{5}{3} \qquad B = -\frac{2}{3}$$

$$L^{-1}\left[\frac{5}{3(s^2+1)} - \frac{2}{3(s^2+2^2)}\right]$$

$$\frac{5}{3}L^{-1}\left[\frac{1}{s^2+1}\right] - \frac{\cancel{2}}{3\times\cancel{2}}L^{-1}\left[\frac{2}{s^2+2^2}\right]$$

$$= \frac{5}{3}\sin t - \frac{1}{3}\sin 2t$$

Sundaram FOR EDUCATIONAL USE