Caley Hamilton Theorem:-

Every square matrix satisfies its characteristic equation.

E.X.

1. Verify caley Hamilton theorem for matrix A and find A^{-1}, A^4.

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

Soln cht. eq of A is

$$|A - \lambda I| = 0$$

$$\Rightarrow \lambda^3 - (1+3-4)\lambda^2 + (2-24+2)\lambda - |A| = 0$$

$$\Rightarrow \lambda^3 - 20\lambda + 8 = 0$$

$\therefore$ By caley Hamilton theorem

$$A^3 - 20A + 8I = 0$$

$$A^2 = A \cdot A = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -8 & -12 \\ 10 & 22 & 6 \\ 2 & 2 & 22 \end{bmatrix}$$

$$A^3 = A \cdot A^2 = \begin{bmatrix} & & \end{bmatrix}\begin{bmatrix} & & \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 20 & 60 \\ 20 & 52 & -60 \\ -40 & -80 & -88 \end{bmatrix}$$

$\therefore$ $A^3 - 20A + 8I$

$$= \begin{bmatrix} & & \end{bmatrix} - 20\begin{bmatrix} & & \end{bmatrix} + 8\begin{bmatrix} & & \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence caley Hamilton theorem is verified.

(ii) We have,

$$A^3 - 20A + 8I = 0$$

multiplying by A,

$$A^4 - 20A^2 + 8A = 0$$

$$\Rightarrow A^4 = 20A^2 - 8A$$

$$= 20\begin{bmatrix} & & \end{bmatrix} - 8\begin{bmatrix} & & \end{bmatrix}$$

$$\Rightarrow \quad A^4 = \begin{bmatrix} \quad \end{bmatrix}$$

(ii) $A^3 - 20A + 8I = 0$

multiplying A^{-1},

$$A^2 - 20I + 8A^{-1} = 0$$

$$\Rightarrow \quad 8A^{-1} = -A^2 + 20I$$

$$= -\begin{bmatrix} \quad \end{bmatrix} + 20 \begin{bmatrix} \quad \end{bmatrix}$$

$$= \begin{bmatrix} \quad \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{8} \begin{bmatrix} \quad \end{bmatrix}$$

(2) Find the matrix represented by

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$

where
$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

Soln Let $f(A) = A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

We have cht. Eq

$$|A - \lambda I| = 0$$

$$\Rightarrow \lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$$

$\Rightarrow$ by C.H.T.

$$A^3 - 5A^2 + 7A - 3I = 0$$

$$\begin{array}{r|l}
 & A^5 + A \\ \hline
A^3 - 5A^2 + 7A - 3I & A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I \\
 & A^8 - 5A^7 + A^6 - 3A^5 \\
 & - \quad + \quad - \quad + \\ \hline
 & 0 + 0 + 0 + 0 + A^4 - 5A^3 + 8A^2 - 2A + I \\
 & A^4 - 5A^3 + 7A^2 - 3A \\
 & - \quad + \quad - \quad + \\ \hline
 & 0 + 0 + A^2 + A + I
\end{array}$$

$$\therefore f(A) = (A^3 - 5A^2 + 7A - 3I)(A^5 + A) + A^2 + A + I$$

$$= 0 + A^2 + A + I$$

$$\therefore A^2 = \overset{A}{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}} \overset{A}{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

$$\therefore f(A) = \overset{A^2}{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}} + \overset{A}{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}} + \overset{I}{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

Functions of a square matrix :-

To find a function of a matrix A,

Let A be a 3X3 matrix, then any function f(A) is

$$f(A) = a A^2 + b A + c I \qquad \text{—— (1)}$$

Consider the Auxilliary Eq. (A.E.)

$$a \lambda^2 + b\lambda + c = f(\lambda) \qquad \text{—— (2)}$$

Eq. (2) is satisfied by the eigen values of matrix A.

Let $\lambda_1, \lambda_2, \lambda_3$ are eigen values of A, then

Case (1) If $\lambda_1, \lambda_2, \lambda_3$ are all distinct,

$$a \lambda_1^2 + b\lambda_1 + c = f(\lambda_1)$$

$$a \lambda_2^2 + b\lambda_2 + c = f(\lambda_2)$$

$$a \lambda_3^2 + b\lambda_3 + c = f(\lambda_3)$$

Case ② If λ_1 is repeated twice and λ_2 is repeated once,

$$a\lambda_1^2 + b\lambda_1 + c = f(\lambda_1)$$

$$\frac{d}{d\lambda}(a\lambda^2 + b\lambda + c) = f'(\lambda) \Big|_{\lambda=\lambda_1}$$

$$a\lambda_2^2 + b\lambda_2 + c = f(\lambda_2)$$

Case ③ If λ_1 is repeated thrice,

$$a\lambda_1^2 + b\lambda_1 + c = f(\lambda_1)$$

$$\frac{d}{d\lambda}(a\lambda^2 + b\lambda + c) = f'(\lambda) \Big|_{\lambda=\lambda_1}$$

$$\frac{d^2}{d\lambda^2}(a\lambda^2 + b\lambda + c) = f''(\lambda) \Big|_{\lambda=\lambda_1}$$

E.x.

① If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, Find A^{50}.

Soln

$$A^{50} = aA + bI$$

Consider A. E.

$$a\lambda + b = \lambda^{50}$$

cht. Eq of A is

$$\lambda^2 - (2+2)\lambda + |A| = 0$$

$$\Rightarrow \lambda^2 - 4\lambda + 3 = 0$$

$$\Rightarrow \lambda = 3, 1$$

$\therefore$ For $\lambda = 3$, $\quad 3a + b = 3^{50}$ ———— (i)

$\lambda = 1$, $\quad a + b = (1)^{50} = 1$ ———— (ii)

(i) − (ii) $\Rightarrow$ $2a = 3^{50} - 1 \Rightarrow a = \frac{1}{2}(3^{50} - 1)$

from (ii), $\quad b = 1 - a = 1 - \frac{1}{2}(3^{50} - 1) = \frac{1}{2}(3 - 3^{50})$

$$\therefore \quad A^{50} = \frac{1}{2}(3^{50} - 1)A + \frac{1}{2}(3 - 3^{50})I$$

$$= \frac{1}{2}\begin{bmatrix} 2\,3^{50} - 2 & 3^{50} - 1 \\ 3^{50} - 1 & 2\,3^{50} - 2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 3 - 3^{50} & 0 \\ 0 & 3 - 3^{50} \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 3^{50} + & 3^{50} - 1 \\ 3^{50} - 1 & 3^{50} + 1 \end{bmatrix}$$

(2) Find e^A and 4^A for $A = \begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$

Solⁿ $\quad e^A = aA + bI$

A. E.

$a\lambda + b = e^{\lambda}$

cht. Eq of A is

$|A - \lambda I| = 0$

$\Rightarrow \lambda^2 - (3/2 + 3/2)\lambda + |A| = 0$

$\Rightarrow \quad \lambda^2 - 3\lambda + 2 = 0$

$\Rightarrow \quad \lambda = 2, 1$

$\therefore \quad 2a + b = e^2$

$a + b = e^1$

$\Rightarrow \quad a = e^2 - e^1 , \quad b = 2e^1 - e^2$

$\therefore \quad e^A = (e^2 - e^1) A + (2e^1 - e^2) I$

$$= \begin{bmatrix} 3/2(e^2 - e^1) & \frac{1}{2}(e^2 - e^1) \\ \frac{1}{2}(e^2 - e^1) & \frac{3}{2}(e^2 - e^1) \end{bmatrix} + \begin{bmatrix} 2e^1 - e^2 & 0 \\ 0 & 2e^1 - e^2 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} e^2 + e^1 & e^2 - e^1 \\ e^2 - e^1 & e^2 + e^1 \end{bmatrix}$$

$$\therefore \quad 4^A = \frac{1}{2} \begin{bmatrix} 4^2 + 4 & 4^2 - 4 \\ 4^2 - 4 & 4^2 + 4 \end{bmatrix} = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix}$$

(3) Find e^{At} for $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and

hence find e^{-8A}.

Soln We know, $e^{At} = aA + bI$

A. E. is

$a\lambda + b = e^{\lambda t}$

cht Eq is

$|A - \lambda I| = 0$

$\Rightarrow \lambda^2 - 0\lambda + 1 = 0$

$\Rightarrow \lambda = \pm i$

$\therefore \quad ai + b = e^{it}$

$-ai + b = e^{-it}$

$\Rightarrow 2b = e^{it} + e^{-it}$

$\Rightarrow b = \frac{1}{2}(e^{it} + e^{-it}) = \cos t$

$2ai = e^{it} - e^{-it}$

$\Rightarrow a = \frac{1}{2i}(e^{it} - e^{-it}) = \sin t$

$\therefore \quad e^{At} = \sin t\, A + \cos t\, I$

$= \begin{bmatrix} 0 & \sin t \\ -\sin t & 0 \end{bmatrix} + \begin{bmatrix} \cos t & 0 \\ 0 & \cos t \end{bmatrix}$

$= \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$

$\therefore \quad e^{-8A} = \begin{bmatrix} \cos(-8) & \sin(-8) \\ -\sin(-8) & \cos(-8) \end{bmatrix} = \begin{bmatrix} \cos 8 & -\sin 8 \\ \sin 8 & \cos 8 \end{bmatrix}$

(4) Find $\cos A$ for $A = \begin{bmatrix} \pi & \pi/4 \\ 0 & \pi/2 \end{bmatrix}$

Soln

$\cos A = aA + bI$

A.E.

$a\lambda + b = \cos\lambda$

Since A is a triangular matrix,

$\lambda = \pi, \pi/2$

$\therefore \quad a\pi + b = \cos\pi = -1$

$a\frac{\pi}{2} + b = \cos\pi/2 = 0$

$\Rightarrow \quad \frac{\pi}{2} a = -1 \Rightarrow a = \frac{-2}{\pi}$

$\therefore \quad b = -a\frac{\pi}{2} = 1$

$\therefore \quad \cos A = -\frac{2}{\pi} A + I$

$= \begin{bmatrix} -2 & -1/2 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 & -1/2 \\ 0 & 0 \end{bmatrix}$

5 If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, show that

$A^n = A^{n-2} + A^2 - I$ for $n \geq 3$ and hence find A^{50}.

Sol

$$A^n = A^{n-2} + A^2 - I$$

$$\Leftrightarrow A^n - A^{n-2} = A^2 - I$$

We have,

$$A^n - A^{n-2} = aA^2 + bA + cI$$

A.E. is

$$a\lambda^2 + b\lambda + c = \lambda^n - \lambda^{n-2}$$

cht. eq of A is

$$|A - \lambda I| = 0$$

$$\Rightarrow \lambda^3 - \lambda^2 - \lambda + 1 = 0$$

$$\Rightarrow \lambda = -1, 1, 1$$

For $\lambda = -1$, $a - b + c = (-1)^n - (-1)^{n-2} = (-1)^n - (-1)^n(-1)^{-2}$

$$= (-1)^n - (-1)^n = 0$$

$$\Rightarrow a - b + c = 0 \quad \text{—— (i)}$$

$\lambda = 1$, $a + b + c = (1)^n - (1)^{n-2} = 0$ —— (ii)

$$2a\lambda + b = n\lambda^{n-1} - (n-2)\lambda^{n-3}\Big|_{\lambda=1}$$

$$\Rightarrow 2a + b = n - (n-2) = 2 \quad \text{—— (iii)}$$

$\Rightarrow \quad a=1, \quad b=0, \quad c=-1$

$\Rightarrow \quad A^n - A^{n-2} = A^2 - I$

$\Rightarrow \quad A^n = A^{n-2} + A^2 - I$

Now, $\quad A^{50} = A^{48} + A^2 - I$

$= A^{46} + 2A^2 - 2I$

$\vdots$

$= A^0 + 25A^2 - 25I$

$= I + 25A^2 - 25I$

$= 25A^2 - 24I$

$= 25 \begin{bmatrix} & \\ & \end{bmatrix} - 24 \begin{bmatrix} & \\ & \end{bmatrix}$

<u>or</u>

$A^{50} = aA^2 + bA + cI$

A. E.

$a\lambda^2 + b\lambda + c = \lambda^{50}$

for $\lambda = -1, \quad a - b + c = (-1)^{50} = 1$

$\lambda = 1, \quad a + b + c = (1)^{50} = 1$

$2a\lambda + b = 50\lambda^{49} \quad |_{\lambda=1}$

$\Rightarrow 2a + b = 50$

$\Rightarrow \quad a = 25, \quad b = 0, \quad c = -24$

$\Rightarrow A^{50} = 25A^2 - 24I$

Digondlization of Matrix :-

Def:- Two matrices A and B are called <u>similar</u> to each other iff there exist a non-singular matrix M such that

$$M^{-1} A M = B$$

Note:- If A and B are similar and B & C are similar then A and C are also similar.

Def:- A matrix A is said to be ~~Diagonalizable~~ <u>Diagonalizable</u> if A is similar to a diagonal matrix i.e. there exist a matrix M such that

$$M^{-1} A M = D \text{ where D is a diagonal matrix.}$$

Note:- If A and B are diagonalizable and have same eigen values then A and B are similar.

Theorem:- Let $\lambda_1, \lambda_2, \dots, \lambda_n$ are eigen values of $n \times n$ matrix A and $X_1, X_2, \dots, X_n$ are ~~$X_1, X_2, \dots, X_n$ are~~ corresponding n linearly independent eigen vectors, then for $M = [X_1 \; X_2 \dots X_n]$,

$$M^{-1} A M = D \text{ where } D = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{bmatrix}$$

The matrix M is called Modal matrix or diagonalising matrix of A.

The matrix D is called the spectral matrix or the diagonal matrix of A.

Note :-

① A matrix $A = (a_{ij})_n$ is diagonalizable iff A has n linearly independent eigen vectors.

② If a matrix A has all distinct eigen values then it is always ~~di~~ diagonalizable.

E.x.

Is the given matrix diagonalizable? Find the diagonalising and the diagonal matrix.

① $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

Solⁿ chti. Eq of A is

$$|A - \lambda I| = 0$$

$$\Rightarrow \lambda^3 - (8-3+1)\lambda^2 + (8-11+14)\lambda - |A| = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\Rightarrow \lambda = 1, 2, 3$$

Since all eigen values are distinct

$\Rightarrow$ A is Diagonalizable.

For $\lambda = 1$, $(A - I)X = 0$

$$\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 4x_1 - 4x_2 - 2x_3 = 0, \quad 3x_1 - 4x_2 = 0$

$\Rightarrow x_2 = 3/4\, x_1, \quad x_3 = 1/2\, x_1$

$$X_1 = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

For $\lambda = 2$, $(A - 2I)X = 0$

$$\begin{bmatrix} 6 & -8 & -2 \\ 4 & -5 & -2 \\ 3 & -4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1 - R_2 \quad \begin{bmatrix} 2 & -3 & 0 \\ 4 & -5 & -2 \\ 3 & -4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 2x_1 - 3x_2 = 0, \quad 4x_1 - 5x_2 - 2x_3 = 0$

$\Rightarrow x_1 = 3/2\, x_2, \quad x_3 = 1/2\, x_2$

$$X_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

For $\lambda = 3$, $(A - 3I)X = 0$

$$\begin{bmatrix} 5 & -8 & -2 \\ 4 & -6 & -2 \\ 3 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1 - R_2 \quad \begin{bmatrix} 1 & -2 & 0 \\ 4 & -6 & -2 \\ 3 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_1 - 2x_2 = 0, \quad 4x_1 - 6x_2 - 2x_3 = 0$$

$$\Rightarrow x_1 = 2x_2, \quad x_3 = x_2$$

$$X_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore$ Modal matrix or diagonalising matrix is $M = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$

and spectral or diagonal matrix is

$$M^{-1} A M = D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Note:- If any matrix A is diagonalizable then we can utilize the Modal matrix and spectral matrix to find a function $f(A)$ of the matrix A.

We have $M^{-1} A M = D$

$$\Rightarrow A = M D M^{-1}$$

$$\therefore f(A) = M f(D) M^{-1}$$

and for any diagonal matrix

$$D = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix},$$

$$f(D) = \begin{bmatrix} f(a) & 0 & 0 \\ 0 & f(b) & 0 \\ 0 & 0 & f(c) \end{bmatrix}$$

For example, in the previous problem,

To find e^A,

Find M^{-1} and then

$$e^D = \begin{bmatrix} e^1 & 0 & 0 \\ 0 & e^2 & 0 \\ 0 & 0 & e^3 \end{bmatrix}$$

$$\therefore e^A = M e^D M^{-1}$$

Note that, this method is lengthy and and applicable to only those matrices which are diagonalizable.

(2) $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$

cht. eq of A is

$|A - \lambda I| = 0$

$\Rightarrow \lambda^3 - (1+4-3)\lambda^2 + (4+0-3)\lambda - |A| = 0$

$\Rightarrow \lambda^3 - 2\lambda^2 + \lambda = 0$

$\Rightarrow \lambda = 0, 1, 1$

For $\lambda = 1$, $(A - I)X = 0$

$$\begin{bmatrix} 0 & -6 & -4 \\ 0 & 3 & 2 \\ 0 & -6 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow 3x_2 + 2x_3 = 0$

$\Rightarrow x_3 = -3/2\, x_2$

$\Rightarrow$ There are two L.I. eigen vectors

$\Rightarrow$ A is diagonalizable.

$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $X_2 = \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$

For $\lambda = 0$, $(A - 0I)X = 0$

$$\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 - 6x_2 - 4x_3 = 0, \quad 4x_2 + 2x_3 = 0$

$\Rightarrow x_3 = -2x_2, \quad x_1 = -2x_2$

$\therefore \quad X_3 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

$\therefore$ Modal matrix is $M = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & -1 \\ 0 & -3 & 2 \end{bmatrix}$

and spectral matrix is $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(3) $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

Since A is a triangular matrix,

$\lambda = 2, 2, 1$

For $\lambda = 2$, $(A - 2I)X = 0$

$\begin{bmatrix} 0 & 3 & 4 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow 3x_2 + 4x_3 = 0, \quad -x_3 = 0$

$\Rightarrow x_3 = 0, \quad x_2 = 0$

$\Rightarrow$ There is only one L.I. eigen vector

$\Rightarrow$ A is not diagonalizable.