VESIT, CHEMBUR, MUMBAI

Tutorial No. 3

Subject: Engineering Mathematics - III **Class: SE**

1. In a particular pain clinic, 10% of patients are prescribed narcotic pain killers. Overall, five percent of the clinic's patients are addicted to narcotics (including pain killers and illegal substances). Out of all the people prescribed pain pills, 8% are addicts. If a patient is an addict, what is the probability that they will be prescribed pain pills?

2. At a certain university, 4% of men are over 6 feet tall and 1% of women are over 6 feet tall. The total student population is divided in the ratio 3:2 in favour of women. If a student is selected at random from among all those over six feet tall, what is the probability that the student is a woman?

3. A factory production line is manufacturing bolts using three machines, A, B and C. Of the total output, machine A is responsible for 25%, machine B for 35% and machine C for the rest. It is known from previous experience with the machines that 5% of the output from machine A is defective, 4% from machine B and 2% from machine C. A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from
(a) machine A (b) machine B (c) machine C?

4. A r.v. X has the distribution

X :	0	1	2	3	4	5	6
$p(x)$:	k	3k	5k	7k	9k	11k	13k

Find i) k ii) $P(X<4)$, iii) $P(3<X\le 6)$

5. The probability mass function of a random variable zero except at $x=0,1,2$. At these points it has values $3c^3,\ 4c-10c^2,\ 5c-1$. Determine c, $p(x<1)$, $p(1<x\le 2)$, $p(0<x\le 2)$.

6. A r.v. assumes the values $-3,-2,-1,0,1,2,3$ such that

$$P(X=-3)=P(X=-2)=P(X=-1),\ P(X=1)=P(X=2)=P(X=3)$$

and $P(X=0)=P(X>0)=P(X<0)$ i) find distribution of X ii) find the distribution

of $2X^2+3X+4$.

7. A random variable X has the following probability distribution

x :	-2	-1	0	1	2
$p(x)$:	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{15}$	$\frac{1}{15}$

Find the probability distribution of i) $V=X^2+1$ ii) $W=X^2+2X+3$

8. Suppose that the life in hours of a certain part of radio tube is a continuous random variable X with pdf

$$f(x)=\begin{cases}\dfrac{100}{x^2} & x\ge 100\\ 0 & \text{elsewhere}\end{cases}$$

i) what is the probability that all of three such tubes in a given radio set will have to be replaced during the first 150 hours of operation ?

ii) what is the probability that none of three of the original tubes will have to be replaced during that first 150 hrs of operation

9. Prove that $\sigma_{x-y}^2=\sigma_x^2+\sigma_y^2-2\,\gamma_{xy}\,\sigma_x\,\sigma_y$ in usual notation.

10. A random variable X has pdf

X :	-2	3	1
$f(x)$:	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

Find the mean and variance of X.

11. If $f(x) = k\, x^2\, e^{-x}, \quad x \geq 0$ is a pdf, find k, mean and variance.

12. A random variable X has the probability function $f(x) = k\, x\left(9 - x^2\right), \; 0 \leq x \leq 3$.
Find k, mean and variance of the distribution.

13. If X_1 has mean 5 and variance 5, X_2 has mean -2 and variance 3, X_1 & X_2 are independent random variables, find $E(X_1 + X_2)$, $E(X_1 - X_2)$, $E(2X_1 + 3X_2 - 5)$, $Var(X_1 + X_2)$, $Var(X_1 - X_2)$, $Var(2X_1 + 3X_2 - 5)$.

14. A coin is tossed until a head appears. Find the expectation of the number of tosses required.

15. Suppose that a game is played with a single die. In this game a player win Rs 20 if 2 or 3 turns up, Rs 40 if 4 turns up, losses Rs 30 if 6 turns up, while the player neither wins nor looses if any other face turns up. Find the expected sum of money to be won.

16. A manufacturer of paper pins knows that 5% of his product is defective. If he sells paper pins in boxes of 100 and guarantees that not more than 10 pins will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality?

17. If the probability that an individual suffers a bad reaction from a certain injection is 0.001. Determine the probability that out of 2000 individuals i) exactly 3 ii) more than 2 will suffer a bad reaction.

18. If X is a Poisson variates with $P\{X=1\} = P\{X=2\}$, find $E\left(X^2\right)$.

19. Fit a Poisson distribution to the following data.

x :	0	1	2	3	4	5	6	7	8
f :	56	156	132	92	37	22	4	0	1

20. If 2% bulbs are known to be defective bulbs, find the probability that in a lot of 300 bulbs, there will be 2 or 3 defective bulbs using Poisson distribution.

21. Let X is normally distributed with mean 12 and s.d. 4. Find i) $P\{X \leq 20\}$ ii) $P\{3 \leq X \leq 12\}$

22. A manufacturer wishes to give a safe guarantee for his product against manufacturing defects. He proposes to replace a product if it fails to work any time within the period of guarantee. He considered that a guarantee is safe he is required to replace not more than 6% of his product. If the life time of his product is normally distributed with mean life 2 years and s.d. 4 months, then what should be the maximum period of guarantee in terms of whole month, so that the guarantee is safe for him ?

23. In a distribution exactly normal, 7% of the item are under 35 and 89% are under 63. What are mean and s.d. of the distribution.

24. In an examination, suppose there are 100 questions of 1 marks each. Each question has three choices of answers of which one is correct. One passes the examination if he|she scores at least 40. Find the probability that a candidate who chooses the answer to each question randomly will pass the examination.

25. In an examination, the marks obtained by students in Mathematics, Physics and Chemistry are normally distributed with means 40, 46, 44 and with standard deviation 15, 12, 16 respectively. Find the probability of a student securing total marks i) 180 or above ii) 90 or above

26. If the heights of 500 students is normally distributed with mean 68 inches and standard deviation 4 inches, estimate the number of students having heights i) greater than 72 inches ii) between 65 and 71 inches.

27. The marks of 1000 students of an Engineering college are distributed normally with mean 70 and standard deviation 5 . Estimate the number of students whose marks will be i) between 60 and 75 ii) more than 75 iii) less than 68 .