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Tutorial 5.

Q1.

x	-3	6	9
$p(x)$	$1/6$	$1/2$	$1/3$

→ $E(x) = \sum p_i x_i = \frac{1}{6} \times (-3) + \frac{1}{2} \times 6 + \frac{1}{3} \times 9$

$= \frac{-1}{2} + 3 + 3$

$$\boxed{E(x) = 5.5}$$

$$\therefore E(x^2) = \sum P_i (x_i^2)$$

$$= \frac{1}{6} \times 9 + \frac{1}{2} \times 36 + \frac{1}{3} \times 81$$

$$= \frac{3}{2} + 18 + 27$$

$$\boxed{\therefore E(x^2) = 46.5}$$

$$\therefore E(x^2 - 12x + 5) = \sum P_i (x_i^2 - 12x_i + 5)$$

$$= \frac{1}{6} \times (9 + 12 \times 3 + 5) + \frac{1}{2}(36 - 72 + 5)$$

$$+ \frac{1}{3}(81 - 108 + 5)$$

$$= \frac{1}{6} \times 50 + \frac{1}{2}(-31) + \frac{1}{3}(-22) \quad \text{--- } ①$$

$$= 8.33... + (-15.5) + (-7.33)$$

$$= -14.5$$

$$\boxed{\therefore E(x^2 - 12x + 5) = -14.5} = \mu \quad \text{--- } ②$$

$$Var(x^2-12x+5) = \sum p_i(x_i^2-12x_i+5)^2 - \mu^2$$

$$= \frac{1}{6} \times (50)^2 + \frac{1}{2}(-31)^2 + \frac{1}{3}(-22)^2 - (-14.5)^2$$

$$= 416.67 + 480.5 + 161.33 - 210.25$$

$$= 848.25$$

$$\boxed{\therefore Var(x^2-12x+5) = 848.25}$$

$$S.D(x) = \sqrt{Var(x)}$$

$$= \sqrt{\sum x_i^2 p_i - \mu^2}$$

$$= \sqrt{\left(9 \times \frac{1}{6} + 36 \times \frac{1}{2} + 81 \times \frac{1}{3}\right) - (5.5)^2}$$

$$= \sqrt{(3/2 + 18 + 27) - 27.5}$$

$$= \sqrt{19}$$

$$S.D(x) = 4.36$$

Q2. a random variable assumes the values
$-3, -2, -1, 0, 1, 2, 3.$

$\rightarrow$ Given $P(x=-3) = P(x=-2) = P(x=-1).$
$P(x=1) = P(x=2) = P(x=3)$

$P(x=0) = P(x<0) = P(x>0)$ ______ ①

$\therefore$ Let & $P(x=-3) = P(x=-2) = P(x=-1) = a.$
$P(x=1) = P(x=2) = P(x=3) = b.$
$P(x=0) = c.$

from eqn ①, $3a = 3b = c$ ______ ②
$\therefore a = b$ ______ ③.

& $\sum p_i = 1.$

$\therefore 3a + c + 3b = 1$ $\quad \therefore 3c = 1$ $\quad \boxed{\therefore c = 1/3}$

$$\boxed{a = b = \frac{1}{9}.}$$ from (2)

$\therefore$

x	-3	-2	-1	0	1	2	3
$P(x)$	$1/9$	$1/9$	$1/9$	$1/3$	$1/9$	$1/9$	$1/9$

is the distribution of x.

$$\therefore f(x) = \begin{cases} 1/9 & -3 \le x < 0. \\ 1/3 & x = 0 \\ 1/9 & 0 > x \geq 3. \end{cases}$$

is the distribution function of x.

Q3.

$$f(x) = \begin{cases} ax & 0 \le x \le 1. \\ a & 1 \le x \le 2. \\ -ax+3a & 2 \le x \le 3. \end{cases}$$

$\sum P(x) = 1.$

$$\therefore \int_0^1 ax.dx + \int_1^2 a.dx + \int_2^3 (-ax+3a).dx = 1$$

$$\left[\frac{ax^2}{2}\right]_0^1 + [ax]_1^2 + \left[\frac{-ax^2}{2} + 3ax\right]_2^3 = 1$$

$$\frac{a}{2} + a + \left[\frac{-9a}{2} + 9a - \left(-\frac{4a}{2} + 6a\right)\right] = 1$$

$$a + (-4a + 9a + 2a - 6a) = 1$$

$$2a = 1$$

(i) $\boxed{\therefore a = 1/2}$

$\therefore$ Distribution function

(ii)

$$f(x) = \begin{cases} x/2 & 0 \le x \le 1. \\ 1/2 & 1 \le x \le 2. \\ -x/2 + 3/2 & 2 \le x \le 3. \end{cases}$$

(iii) $P\left(\frac{1}{2} \le x \le \frac{5}{2}\right)$

$$= \int_{1/2}^{1} x/2 \cdot dx + \int_{1}^{2} \frac{1}{2} \cdot dx + \int_{2}^{5/2} \left(\frac{-x}{2} + \frac{3}{2}\right) \cdot dx$$

$$= \left(\frac{x^2}{4}\right)_{1/2}^{1} + \left(\frac{x}{2}\right)_{1}^{2} + \left(\frac{-x^2}{4} + \frac{3x}{2}\right)_{2}^{5/2}$$

$$= \frac{1}{4} - \frac{1}{16} + 1 - \frac{1}{2} + \left(\frac{-25}{16} + \frac{15}{4}\right)$$

$$- \left(\frac{-4}{4} + \frac{6}{2}\right)$$

$$= 4 - \frac{\overset{13}{\cancel{26}}}{\cancel{16}\,8} + 1 - \frac{1}{2} + 1 - \frac{6}{2}$$

$$= 6 - \frac{7}{2} - \frac{13}{8}$$

$$= 6 - 3.5 - 1.625$$

$$= 0.875.$$

$$\boxed{\text{(iii)} \therefore P\left(\frac{1}{2} \le x \le \frac{5}{2}\right) = 0.875}$$

(Q4) $f(x) \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

→ for m.g.f

$$M_x(t) = E(e^{tx}) = \int_0^\infty e^{tx}\left(e^{-2x} + \frac{e^{-x}}{2}\right) . dx$$

$$= \left[\frac{1}{t-2} e^{(tx-2x)} + \frac{1}{2(t-1)} e^{tx-x}\right]_0^\infty$$

$$= 0 - \frac{1}{2-t} + 0 - \frac{1}{2(t-1)}$$

$$= \frac{1}{t-2} + \frac{1}{2(1-t)}.$$

$$= \frac{\cancel{2} - 2t + t \cancel{-2}}{(t-2)\,2(1-t)} = \frac{t}{2(1-t)(2-t)}.$$

$$\boxed{\therefore M_x(t) = \frac{t}{2(1-t)(2-t)}.}$$

$\therefore$ First central moment $= \dfrac{1}{(2-t)^2} + \dfrac{1}{2(1-t)^2}$

$\therefore$ First moment about origin $E(x) = m'_x(0) = \dfrac{3}{4}$
$(t=0)$

Similarly

Second central moment $m^2_x(t) = \dfrac{1}{(2-t)^3} + \dfrac{1}{2(1-t)^3}$

Second moment about origin $m^2_x(0) = \dfrac{5}{4}$

Similarly

Third central moment $m^3_x(t) = \dfrac{1}{(2-t)^4} + \dfrac{1}{2(1-t)^4}$.

Third moment about origin $m^4_x(0) = \dfrac{9}{16}$

Similarly

Fourth ~~moment~~ central moment $m^4_x(t) = \dfrac{1}{(2-t)^5} + \dfrac{1}{2(1-t)^5}$

fourth moment about origin $m^4_x(0) = \dfrac{17}{32}$

Q5)

Let x denote the number of trials.
(tails tossed before the heads appears).

Then, x is a geometric random variable.

Let $P(\text{heads}) = p$; where $0 < p \leq 1$,
because it is not given
if the coin is balanced.

The expectation of this geometric random variable x is

$$E(x) = 1/p.$$

If the coin is balanced, then $p = P(\text{heads}) = \frac{1}{2}$.

and the expectation would be

$$E(x) = \frac{1}{\left(\frac{1}{2}\right)} = 2.$$

$\therefore E(x) = 2$ would be the expectation of the number of tosses.

$\therefore E(x) = 5.5$

$E(x^2) = 46.5$

$E(x^2 - 12x + 5) = -14.5$

$Var(x^2 - 12x + 5) = 848.25$

$S.D(x) = 4.36$