Tutorial No. 2

Subject: Applied Mathematics - IV **Class: SE**

Find the all possible Laurent's series expansion and specify the domain of convergence.

1. $\dfrac{z^2-1}{z^2+5z+6}$ 2. $z\, e^{1/z^2}$ 3. $\dfrac{1}{z^2-1}$ about $z=-1$

4. $\dfrac{1}{z^2+4}$ about $z=-i$ 5. $\dfrac{4z^2+2z-4}{z^4-4z^2}$ about $z=2$

6. Find the Laurent's series expansion of $\dfrac{z+5}{(z+1)^2(z-2)}$ convergent in the region

i) $0<|z+1|<3$ ii) $|z+1|>3$ iii) $1<|z|<2$ iv) $|z-2|>3$ v) $1<|z-1|<2$

7. Find the type of singularity of $\dfrac{1}{z-\sin z}$ at $z=0$. Find the residue at this point.

Evaluate the Integrals using Residue Theory.

8. Evaluate $\int_C \dfrac{15z+9}{z^3-9z}\,dz$, $|z-1|=3$

9. Evaluate $\int_C \dfrac{(z+4)^2}{z^4+5z^3+6z^2}\,dz$, $|z|=\dfrac{5}{2}$

10. Evaluate $\int_C \dfrac{z-1}{z^2+2z+5}\,dz$, $|z|=\frac{3}{2}$

11. Evaluate $\int_C \dfrac{z^2}{z^4-1}\,dz$, where C is i) Rhombus formed by joining $\pm\dfrac{1}{2}, \pm 2i$

ii) $x^2+16y^2=4$ iii) $|z+i|=\sqrt{3}$

12. Evaluate $\int_C \dfrac{\sin \pi z^2+\cos \pi z^2}{(z-1)(z-2)^2}\,dz$, $C: |z|=3$

13. Evaluate $\int_C z^n\, e^{1/z}\,dz$, $|z|=1$, n is a positive integer.

14. Evaluate $\int_C \dfrac{1}{z^5} e^{z^2}\,dz$, $|z|=1$ 15. Evaluate $\int_C \dfrac{1}{z^3} e^{1-\cos z}\,dz$, $|z|=1$

16. Evaluate $\int_C z^2\, e^{-1/(z-1)}\,dz$, $|z-1|=1$ 17. Evaluate $\int_C \operatorname{cosec} z\,dz$, $|z|=1$

18. Evaluate $\int_C \dfrac{\tan \pi z}{z^4}\,dz$, $C: |z+3|=2$ 19. Evaluate $\int_C \operatorname{cosech} 2z\,dz$, $|z|=2$

20. Evaluate $\int_C \dfrac{1}{z-\sin z}\,dz$, $C: |z|=1$