47 D6AD Yash Sarang CG Assignment 1. 051

Q1) Translate a polygon with co-ordinates A(2,5) B(7,10) C(10,2) by 3 units in x direction, 4 units in y directions.

→

Translation ⇒ $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

$\therefore \; x' = x + t_x \quad , \quad y' = y + t_y .$

$t_x = 3 , \; t_y = 4$ & A(2,5) ; B(7,10) ; C(10,2).

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \\ 1 \end{bmatrix}$$

$\therefore$ A' = (5,9) Similarly B' = (10,14) , C' = (13,6)

Q2) Give 3x3 homogenous transformation matrix for each (ii following transformation.

(i) Shift image to the right 3 units.
(ii) Shift image to the up 2 units.
(iii) Move down 1/2 units & right 1 unit.
(iv) Move down 2/3 units & left 4 unit.

→

Translation matrix is given by $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$

(i) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1/2 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & -2/3 & 1 \end{bmatrix}$

Q3) Find the transformation of A(1,0), B(0,1), C(1,1) by

i) Rotating 45° about the origin and translating 1 unit in x and y direction.

ii) Translating 1 unit in x and y directions and then rotating 45° about the origin.

→ $[ABC] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

R_{45° (counter-clockwise) $= \begin{bmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Translation matrix $\rightarrow$ $1I + 1J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ tx & ty & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

(i) Rotation followed by translation

$$I_1 = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$[A'B'C'] = [ABC][I_1] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2}+1 & 1/\sqrt{2}+1 & 1 \\ -1/\sqrt{2}+1 & 1/\sqrt{2}+1 & 1 \\ 1 & \sqrt{2}+1 & 1 \end{bmatrix}$$

$\therefore$ New coordinates are $A\left(\frac{1}{\sqrt{2}}+1, \frac{1}{\sqrt{2}}+1\right)$,

$B\left(\frac{-1}{\sqrt{2}}+1, \frac{1}{\sqrt{2}}+1\right)$ & $C\left(1, \sqrt{2}+1\right)$.

Q4) Show that transformation matrix for reflection about a line $y=x$ is equivalent to reflection to x-axis followed by counter-clockwise rotation of 90°.

→ Transformation matrix for reflection about $y=x$ line

$$Ref_{(x,y)} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Counter clockwise rotation of 90° $M_1 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos 90^\circ & -\sin 90^\circ & 0 \\ \sin 90^\circ & \cos 90^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reflection about x axis $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = M_2$

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ $Ref_{(x,y)} = M_1 . M_2$.