1) Sol

$$f(z) = 1 \log (x^{2} + y^{2}) + i \tan^{-1} kx$$

$$u(x,y) + iv(x,y) = 1 \log (x^{2} + y^{2}) + i \tan^{-1} kx$$

$$u(x,y) + iv(x,y) = 1 \log (x^{2} + y^{2}) + i \tan^{-1} kx$$

$$\vdots \quad u = 1 \log (x^{2} + y^{2}) \quad V = \tan^{-1} kx$$

$$y$$

$$f(z) \quad io \quad analytic$$

$$\vdots \quad u_{2} = v_{y} \quad f \quad t \quad v_{y} = -v_{x}$$

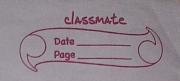
$$i \cdot e \quad c \quad r \quad eqn \quad ave \quad Satisfied$$

$$u_{2} = 1 \quad 1 \quad (2x) = x$$

$$u_{3} = -\frac{x^{2} + y^{2}}{x^{2} + y^{2}}$$

$$v_{4} = -\frac{x^{2} + y^{2}}{x^{2} + y^{2}}$$

$$v_{5} = \frac{1}{1 + 1c^{2}x^{2}} \quad \frac{1}{1 + 1c^{2}x^{2}$$



92)
$$f(z) = 1$$
; At origin $(z=0)$, $f(z)$ is not defined

 z

i.e not analytic

 $f(z) = 1 = e^{-i\theta}$
 $re^{i\theta}$
 $re^{i\theta}$

$$u = 1 \cos \theta$$

$$V = -1 \sin \theta$$

$$V = -2 \sin \theta$$

$$V = -2 \sin \theta$$

$$V = -1 \cos \theta$$

$$\frac{1}{y} = \frac{1}{y^2} = \frac{1}{y^2}$$

$$\frac{1}{y^2} = \frac{1}{y^2} = \frac{1}{y$$

And
$$-rV_{\gamma} = -y\sin\theta = -\sin\theta = U_{\theta}$$

 y^{2}
 y^{2}
 y^{2}

93)

$$U = y^3 - 3x^2y$$
 $4x = -6xy$
 $4y = 3y^2 - 3x^2$
 $4x = -6y$
 $4y = 6y$

Uxx + Uyy = - 64 + 64

... $u = yz^3 - 3z^2y$ is a harmonic fr we have v which is a harmonic Conjugate at a such that f(z) = u + iv is analytic. ... ux = vy uy = -vz... $\partial v = -6zy$ & ... $\partial v = 3z^2 - 3y^2$ ∂y

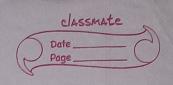
i. f(z)=1 dv + idv dy dx f'(z)=-6zy+i(3z²-3y²): Using Mike Thomson method

 $f'(z) = i (3z^2)$ Integrating w.r.t z $f(z) = i (3z^2)$ $2 \cdot f(z) = i (3z^2)$ $3 \cdot f(z) = i (3z^2)$

:. $f(z) = i (x+iy)^3 + C$ = $i (x^3 + 3x^2iy + 3x^2iy^2 + i^3y^3) + C$ = $i ((x^2 - 3xy^2) + i (3x^2y - y^3)^3 + C$:. $f(z) = (y^3 - 3x^2y) + i (x^3 - 3x^2y^2) ... (for C=0)$

Harmonic Conjugate (V) = 23-3xy2

Analytic function f(z) = (y3-3xy) + i(x3-3xy2)



Yet the analytic function be f(z) = u + i v $x^2 + y^2$ $\partial v = (x^2 + y^2) - x(2x) - y^2 - x^2$ $\partial x = (x^2 + y^2)^2 - (x^2 + y^2)^2$ (9.4)

 $\frac{\partial V}{\partial y} = \frac{1-2xy}{(x^2+ye)^2}$

F'(z)= dy + idv = dy + idv

 $f'(z) = -2xy + i(y^2 - x^2)$ $(x^2 + y^2)^2$ $(x^2 + y^2)^2$

Using Milne-Thomsons method

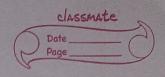
integration wort z

f(z) = -i(-1) + c = i + c = i + c (x+iy)

f(z) = ix + y + c $x^2 + y^2$

or $f(z) = \left(\frac{y}{x^2 + y^2} + c\right) + i\left(\frac{x'}{x^2 + y^2}\right) = Analytic$

.. Real part = 4 + c



of a sample of

Jan 2885 00

(4) = 8 10 1 CH + 10 17

95)

 $x^{2}-y^{2}+x=C$ $\therefore u = x^{2}-y^{2}+xe$ ux = 2x+1 uy = -2y

 $f'(z) = 4x + i \sqrt{x} = 4x - i u y$ f'(z) = 2x + 1 + 2i y= 2z + 1

:. $f(z) = \int 2z+1$ = $z^2 + z + c$ = $(x+iy)^2 + (x+iy) + c$ = $(x^2 - y^2 + 2xiy) + (x+iy) + c$ = $x^2 - y^2 + x + 2ixy + iy + c$

Fraginary part = 224+4

Reg orthogonal trajectory -> 2xy+y=C

(2) = 2 2 (626 + 12 2 6 1026 + 6

9251028=6

$$= e^{-i\theta} \left(ur - i u\theta \right)$$

=
$$e^{-i\theta}$$
 (2 r (0520 - 2 i r S i n 20)
: $f(z) = 2re^{-i\theta}$ (c 0520 - i S i n 20)

Using Milne Thomson method,

$$f(z) = x^2 + y^2 + 2ixy + c$$

f(z) = 200620 - 25ing + 2120050 25in 0 + C

.. Reg orthogonal trajectory

r= h cosec 0 -> r sin 0 = b r= a sec 0 -> r cos 0 = 0 (36)

let u=rcoso f V=rsino we form An f(z)= u+iv ... f(z)= r(os 0 + irsino

 $ur = \cos\theta \qquad \forall r = \sin\theta$ $ue = -r\sin\theta \qquad \forall e = r\cos\theta$ $ur = \int Ve \qquad \text{for } ue = -r\nabla r$

creque st satisfied :. f(z) is analytic

:. vsind=b f & vcost=a forms orthogonal
trajectories or v-b oseco f v= aseco forms
orthogonal trajectories