Q1)

1) p: It is below freezing.
q: It is snowing

a) $p \wedge q$

b) $p \wedge \neg q$

c) $\neg p \wedge \neg q$

d) $p \vee q$

e) $p \rightarrow q$

f) $(p \vee q) \wedge (p \rightarrow \neg q)$

g) $p \leftrightarrow q$

Q2)

2) p: You have flu.
q: You miss final exam
r: You pass the course.

a) If you have the flu, you will miss the final exam
↳ $(p \rightarrow q)$

b) If and only if you do not miss the final exam, you will pass the course.
↳ $(\neg q \leftrightarrow r)$

c) If you miss the final exam, you will not pass the course.
↳ $(q \rightarrow \neg r)$

d) You have the flu or You miss the final exam or You pass the course.
↳ $(p \vee q \vee r)$

e) If you have the flu, you will not pass the course or If you miss the final exam, you will not pass the course.
↳ $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

f) You have the flu and miss the final exam or You do not miss the final exam and pass the course.
↳ $(p \wedge q) \vee (\neg q \wedge r)$

Q3)

a) $[\neg p \wedge (p \vee q)] \rightarrow q$

$\propto$

p	q	$p \vee q$	$[\neg p \wedge (p \vee q)]$	$\propto \rightarrow q$
T	T	T	F	T
T	F	T	F	T
F	T	T	T	T
F	F	F	F	T

$\propto$: $-\propto \vee q$

$\therefore [\neg p \wedge (p \vee q)] \rightarrow q$ is a tautology

b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

$\propto$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$\propto \rightarrow (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$\therefore [(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology

c) $[p \wedge (p \to q)] \to q$

p	q	$p \to q$	$p \wedge (p \to q)$ (α)	$\alpha \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$\therefore [p \wedge (p \to q)] \to q$ is a tautology

d) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$ (α)	$\alpha \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

$\therefore [(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$ is a tautology.

Q4) $Q(x,y)$ "x has sent an email to y"

a) $\exists x \exists y\, Q(x,y)$ - Some students have sent email messages to some other students of the class.

b) $\exists x \forall y\, Q(x,y)$ - Some students have sent email messages to all students of the class

c) $\forall x \exists y\, Q(x,y)$ - Every student has sent email messages to some other students of the class

d) $\exists y \forall x\, Q(x,y)$ - Every student has sent email messages to some other students of the class

e) $\forall y \exists x\, Q(x,y)$ - Every student has sent email messages to some other students of the class.

f) $\forall x \forall y\, Q(x,y)$ - Every student has sent email messages to every other students in the class.

Q5)

a) $S(x)$: x is a student of the class.
$C(x)$: x owns a red convertible.
$T(x)$: x gets a speeding ticket.

1) $S(\text{Linda}) \wedge C(\text{Linda})$
2) $\forall x \, [C(x) \rightarrow T(x)]$
3) $C(\text{Linda}) \rightarrow T(\text{Linda})$
4) $C(\text{Linda})$
5) $S(\text{Linda})$
6) $T(\text{Linda})$
7) $S(\text{Linda}) \wedge T(\text{Linda})$

$\therefore \exists x [S(x) \wedge T(x)]$

Universal Instantiation
① Simplification.
② Simplification
③ & ④ Modus tonens
⑤ & ⑥ Conjuction.
⑦ Existential Generalization.

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