Appendix W6

The Frequency-Response Design Method

W.6 Time Delay

Example W.1 ***Nyquist Plot for System with Time Delay*** *Consider the system with*

$$KG(s) = \frac{Ke^{-T_d s}}{s},$$

where $T_d = 1$ sec. Determine the range of K for which the system is stable.

SOLUTION Because the Bode plotting rules do not apply for the phase of a time-delay term, we will use an analytical approach to determine the key features of the frequency response plot. As just discussed, the magnitude of the frequency response of the delay term is unity and its phase is $-\omega$ radians. The magnitude of the frequency response of the pure integrator is $1/\omega$, with a constant phase of $-\pi/2$. Therefore,

$$\begin{aligned} G(j\omega) &= \frac{1}{\omega}e^{-j(\omega+\pi/2)} \\ &= \frac{1}{\omega}(-\sin\omega - j\cos\omega). \end{aligned} \tag{W.1}$$

Using Eq. (W.1) and substituting in different values of ω, we can make the Nyquist plot, which is the spiral shown in Fig. W.1.

Let us examine the shape of the spiral in more detail. We pick a Nyquist path with a small detour to the right of the origin. The effect of the pole at the origin is the large arc at infinity with a 180° sweep, as shown in Fig. W.1. From Eq. (W.1), for small values of $\omega > 0$, the real part of the frequency response is close to -1 because $\sin\omega \cong \omega$ and $\Re[G(j\omega)] \cong -1$. Similarly, for small values of $\omega > 0$, $\cos\omega \cong 1$ and $\Im[G(j\omega)] \cong -1/\omega$—that is, very large negative values, as shown in Fig. W.1. To obtain the crossover points on the real axis, we set the imaginary part equal to zero:

$$\frac{\cos\omega}{\omega} = 0. \tag{W.2}$$

The solution is then

$$\omega_0 = \frac{(2n+1)\pi}{2}, \quad n = 0, 1, 2, \ldots. \tag{W.3}$$

After substituting Eq. (W.3) back into Eq. (W.1), we find that

$$G(j\omega_0) = \frac{(-1)^n}{(2n+1)}\left(\frac{2}{\pi}\right), \quad n = 0, 1, 2, \ldots.$$

So the first crossover of the negative real axis is at $-2/\pi$, corresponding to $n = 0$. The first crossover of the positive real axis occurs for $n = 1$ and is located at $2/3\pi$. As we can infer from Fig. W.1, there are an infinite number of other crossings of the real axis. Finally, for $\omega = \infty$, the Nyquist

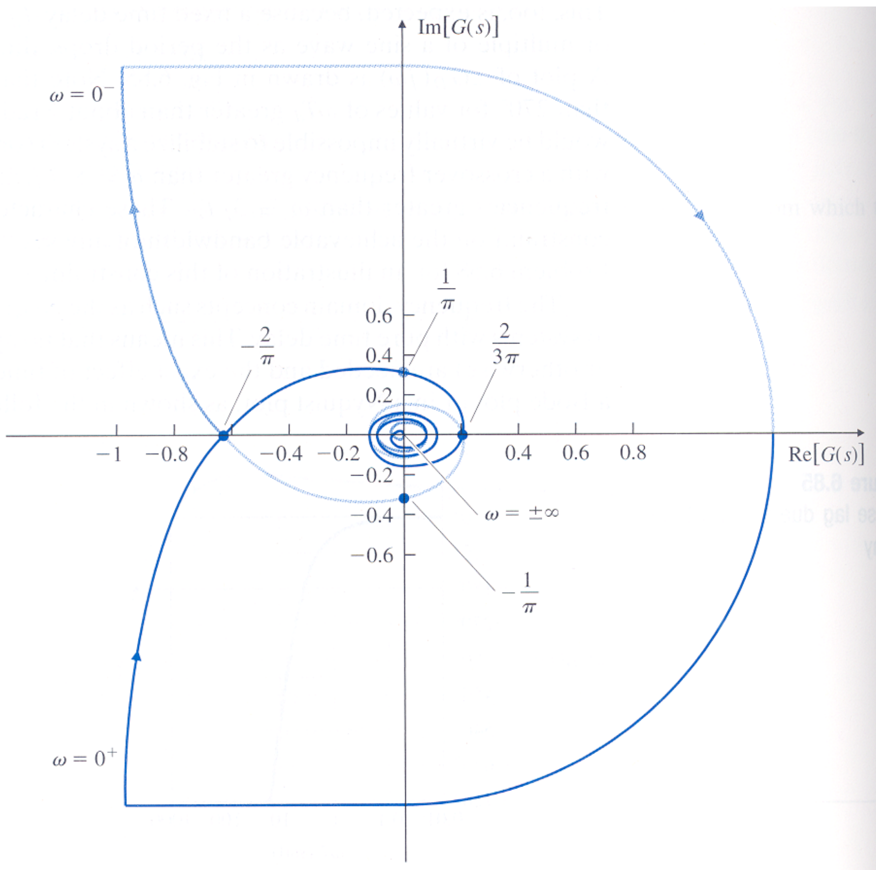

Figure W.1: Nyquist plot for Example W.1

plot converges to the origin. Note that the Nyquist plot for $\omega < 0$ is the mirror image of the one for $\omega > 0$.

The number of poles in the RHP is zero ($P = 0$), so for closed-loop stability, we need $Z = N = 0$. Therefore, the Nyquist plot cannot be allowed to encircle the $-1/K$ point. It will not do so as long as

$$-\frac{1}{K} < -\frac{2}{\pi}, \tag{W.4}$$

which means that, for stability, we must have $0 < K < \pi/2$.

W.7 Alternate Presentation of Data

W.7.1 Nichols Chart

Example W.2 ***Stability Margins from Nichols Chart*** *For the system of Example 6.13, whose Nyquist plot is shown in Fig. 6.42, determine the PM and GM using the Nichols plot.*

SOLUTION Figure W.2 shows a Nichols chart with the data from the same system shown in Fig. 6.42. Note that the PM for the magnitude 1 crossover frequency is $36°$ and the GM is 1.25 $(= 1/0.8)$. It is clear from this presentation of the data that the most critical portion of the curve is where it crosses the $-180°$ line; hence the GM is the most relevant stability margin in this example.

W.7.2 Inverse Nyquist

The **inverse Nyquist plot** is simply the reciprocal of the Nyquist plot described in Section 6.3 and used in Section 6.4 for the definition and discussion of stability margins. It is obtained most easily by computing the inverse of the magnitude from the Bode plot and plotting that quantity at an angle in the complex plane, as indicated by the phase from the Bode plot. It can be used to find the PM and GM in the same way that the Nyquist plot was used. When $|G(j\omega)| = 1$, $|G^{-1}(j\omega)| = 1$ also, so the definition of PM is identical on the two plots. However, when the phase is $-180°$ or $+180°$, the value of $|G^{-}1(j\omega)|$ is the GM directly; no calculation of an inverse is required, as was the case for the Nyquist plot.

The inverse Nyquist plot for the system in Fig. 6.24 (Example 6.9) is shown in Fig. W.3 for the case where $K = 1$ and the system is stable. Note that GM = 2 and PM $\cong 20°$. As an example of a more complex case, Fig. W.4 shows an inverse Nyquist plot for the sixth-order case whose Nyquist plot was shown in Fig. 6.42 and whose Nichols chart was shown in Fig. W.2. Note here that GM = 1.2 and PM $\cong 35°$. Had the two crossings of the unit circle not occurred at the same point, the crossing with the smallest PM would have been the appropriate one to use.

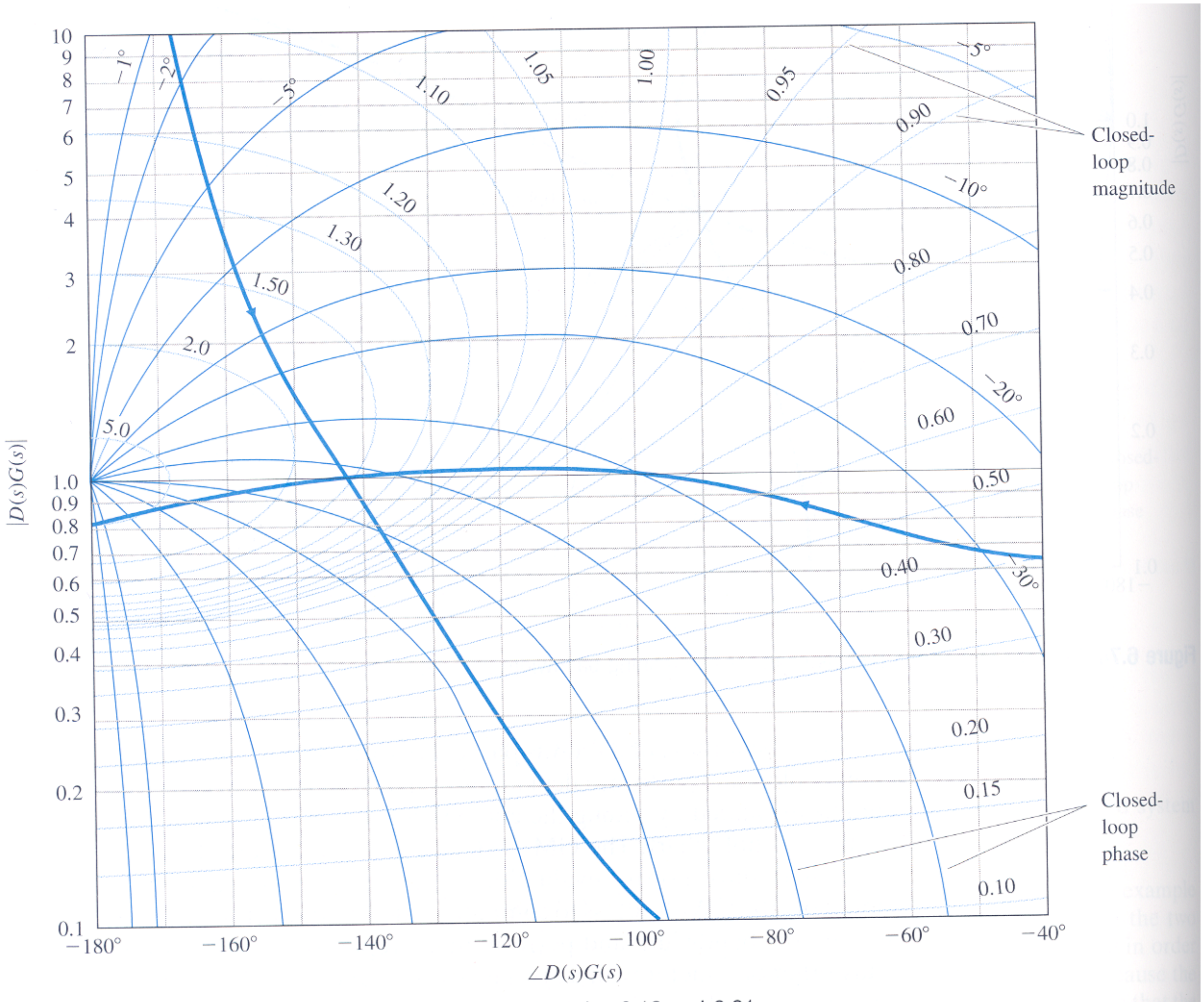

Figure W.2: Nichols chart of the complex system in Examples 6.13 and W6.2

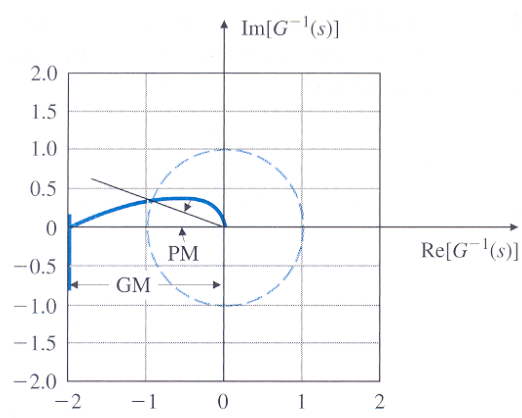

Figure W.3: Inverse Nyquist plot for Example 6.9

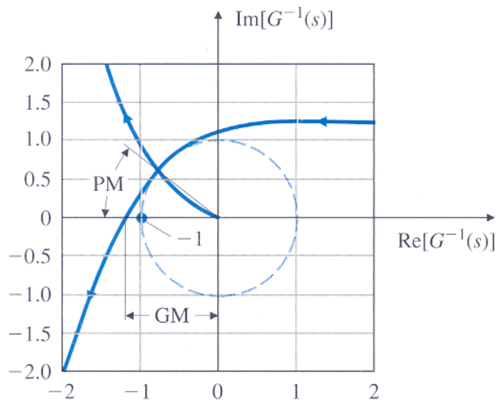

Figure W.4: Inverse Nyquist plot of the system whose Nyquist plot is in Fig. 6.26