

## DOT PRODUCT $(x, y) = \sum_{k=0}^n x_n y_n$ COMPUTATION

### Using SIMD ( SSE vector instructions)

Reference: *Implement a Horizontal Add/Subtract with SSE3 Instructions for dot product computation*  
<https://software.intel.com/en-us/articles/implement-a-horizontal-addsubtract-with-sse3-instructions>

#### Source code

```
int main(int argc, char* argv[])
{
    /*
        float a[N], b[N], x = 0.0;

        for (i = 0; i < N; i++)

            x = x + a[i]*b[i];

    */

    const int N = 8;
    static float a[N]={1.0,2.0,1.0,2.0,1.0,2.0,1.0,2.0},
                b[N]={2.0,1.0,2.0,1.0,2.0,1.0,2.0,1.0},
                x = 0.0;
    float *aPointer = a;
    float *bPointer = b;

    __asm
    {
        pxor    xmm0, xmm0    ;initialize xmm0 to 0 , xmm0 will serve as x
        mov     eax, dword ptr[aPointer]    ;eax points to a[]
        mov     ebx, dword ptr[bPointer]    ;ebx points to b[]
        mov     ecx, N          ;number of elements in arrays

myLOOP:
        movups  xmm1, [eax]      ;four values of a in xmm1
        movups  xmm2, [ebx]      ;four values of b in xmm2
        mulps   xmm1, xmm2       ;mulitply a[i]*b[i]
        addps   xmm0, xmm1 ;add x + a[i]*b[i]


        add eax, 16              ;increment pa by 4
        add ebx, 16              ;increment pb by 4
        sub ecx, 4               ;loop-4
        jnz myLOOP              ;loop if ecx not 0


        haddps  xmm0, xmm0        ;horizontal add
        haddps  xmm0, xmm0        ;horizontal add
        movss   dword ptr[x], xmm0 ;result goes to x

    }
    return 0;
}
```

## Disassembly window

```

14: static float a[N]={1.0,2.0,1.0,2.0,1.0,2.0,1.0,2.0},
15:          b[N]={2.0,1.0,2.0,1.0,2.0,1.0,2.0,1.0},
16:          x = 0.0;
20:          float *aPointer = a;
013913E5 C7 45 EC 00 80 39 01 mov          dword ptr [aPointer],1398000h
21:          float *bPointer = b;
013913EC C7 45 E0 20 80 39 01 mov          dword ptr [bPointer],1398020h
22:
23:          __asm
24:          {
26: pxor      xmm0, xmm0 ;initialize xmm0 to 0 , xmm0 will serve as x
013913F3 66 0F EF C0          pxor          xmm0,xmm0
27:          mov      eax, dword ptr[aPointer]      ;eax points to a[]
013913F7 8B 45 EC          mov          eax,dword ptr [aPointer]
28:          mov      ebx, dword ptr[bPointer]      ;ebx points to b[]
013913FA 8B 5D E0          mov          ebx,dword ptr [bPointer]
29:          mov      ecx, N          ;number of elements in arrays
013913FD 8B 4D F8          mov          ecx,dword ptr [N]
30:
31:          myLOOP:
32:          movups   xmm1, [eax]      ;four values of a in xmm1
01391400 0F 10 08          movups          xmm1,xmmword ptr [eax]
33:          movups   xmm2, [ebx] ;four values of b in xmm2
01391403 0F 10 13          movups          xmm2,xmmword ptr [ebx]
34:          mulps    xmm1, xmm2      ;mulitply a[i]*b[i]
01391406 0F 59 CA          mulps          xmm1,xmm2
35:          addps    xmm0, xmm1      ;add x + a[i]*b[i]
01391409 0F 58 C1          addps          xmm0,xmm1
36:

```

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```

38:      add eax, 16           ;increment pa by 4
0139140C 83 C0 10           add         eax,10h
39:      add ebx, 16           ;increment pb by 4
0139140F 83 C3 10           add         ebx,10h
40:      sub ecx, 4            ;loop-4
01391412 83 E9 04           sub         ecx,4
41:      jnz myLOOP            ;loop if ecx not 0
01391415 75 E9              jne         myLOOP (01391400h)
42:
43:      haddps xmm0, xmm0      ;horizontal add
01391417 F2 0F 7C C0         haddps      xmm0,xmm0
44:      haddps xmm0, xmm0      ;horizontal add
0139141B F2 0F 7C C0         haddps      xmm0,xmm0
45:      movss  dword ptr[x], xmm0 ;result goes to x
0139141F F3 0F 11 05 80 81 39 01 movss      dword ptr ds:[1398180h],xmm0
48:      }
49:
50:      return 0;
01391427 33 C0              xor         eax,eax
51: }

```

## Register file

```
Registers
EAX = CCCCCCCC EBX = 7EDC8000 ECX = 00000000 EDX = 00000001 ESI = 00000000 EDI = 002DF974 EIP = 013913E5
ESP = 002DF884 EBP = 002DF974 EFL = 00000206

MM0 = 0000000000000000 MM1 = 0000000000000000 MM2 = 0000000000000000 MM3 = 0000000000000000
MM4 = 0000000000000000 MM5 = 0000000000000000 MM6 = 0000000000000000 MM7 = 0000000000000000

XMM0 = 00000000000000000000000000000000
XMM1 = 00000000000000000000000000000000
XMM2 = 00000000000000000000000000000000
XMM3 = 00000000000000000000000000000000
XMM4 = 00000000000000000000000000000000
XMM5 = 00000000000000000000000000000000
XMM6 = 00000000000000000000000000000000
XMM7 = 00000000000000000000000000000000

XMM00 = +0.00000E+000 XMM01 = +0.00000E+000 XMM02 = +0.00000E+000 XMM03 = +0.00000E+000
XMM10 = +0.00000E+000 XMM11 = +0.00000E+000 XMM12 = +0.00000E+000 XMM13 = +0.00000E+000
XMM20 = +0.00000E+000 XMM21 = +0.00000E+000 XMM22 = +0.00000E+000 XMM23 = +0.00000E+000
XMM30 = +0.00000E+000 XMM31 = +0.00000E+000 XMM32 = +0.00000E+000 XMM33 = +0.00000E+000
XMM40 = +0.00000E+000 XMM41 = +0.00000E+000 XMM42 = +0.00000E+000 XMM43 = +0.00000E+000
XMM50 = +0.00000E+000 XMM51 = +0.00000E+000 XMM52 = +0.00000E+000 XMM53 = +0.00000E+000
XMM60 = +0.00000E+000 XMM61 = +0.00000E+000 XMM62 = +0.00000E+000 XMM63 = +0.00000E+000
XMM70 = +0.00000E+000 XMM71 = +0.00000E+000 XMM72 = +0.00000E+000 XMM73 = +0.00000E+000
MXCSR = 00001F80

0x002df960 = CCCCCCCC
```

## Memory stores array

At the address **0x01398000** is floating point representation

0x3f800000=

0011 1111 1000 0000 0000 0000 0000 0000

+  $2^{127-127}(1+0) = 1.0$

0x40000000=

0100 0000 0000 0000 0000 0000 0000 0000

+  $2^{128-127}(1+0) = 2.0$

Register file after loading first 4 elements of the array

EAX contains the address **0X01398000** of the first element of the array a[]

EBX contains the address **0X01398020** of the first element of the array b[]

ECX contains the number of elements in each array

```
Registers
EAX = 01398000 EBX = 01398020 ECX = 00000008 EDX = 00000001 ESI = 00000000 EDI = 002DF974 EIP = 01391403
ESP = 002DF884 EBP = 002DF974 EFL = 00000206

MM0 = 0000000000000000 MM1 = 0000000000000000 MM2 = 0000000000000000 MM3 = 0000000000000000
MM4 = 0000000000000000 MM5 = 0000000000000000 MM6 = 0000000000000000 MM7 = 0000000000000000

XMM0 = 00000000000000000000000000000000
XMM1 = 400000003F800000400000003F800000
XMM2 = 00000000000000000000000000000000
XMM3 = 00000000000000000000000000000000
XMM4 = 00000000000000000000000000000000
XMM5 = 00000000000000000000000000000000
XMM6 = 00000000000000000000000000000000
XMM7 = 00000000000000000000000000000000
XMM00 = +0.00000E+000 XMM01 = +0.00000E+000 XMM02 = +0.00000E+000 XMM03 = +0.00000E+000
XMM10 = +1.00000E+000 XMM11 = +2.00000E+000 XMM12 = +1.00000E+000 XMM13 = +2.00000E+000
XMM20 = +0.00000E+000 XMM21 = +0.00000E+000 XMM22 = +0.00000E+000 XMM23 = +0.00000E+000
XMM30 = +0.00000E+000 XMM31 = +0.00000E+000 XMM32 = +0.00000E+000 XMM33 = +0.00000E+000
XMM40 = +0.00000E+000 XMM41 = +0.00000E+000 XMM42 = +0.00000E+000 XMM43 = +0.00000E+000
XMM50 = +0.00000E+000 XMM51 = +0.00000E+000 XMM52 = +0.00000E+000 XMM53 = +0.00000E+000
XMM60 = +0.00000E+000 XMM61 = +0.00000E+000 XMM62 = +0.00000E+000 XMM63 = +0.00000E+000
XMM70 = +0.00000E+000 XMM71 = +0.00000E+000 XMM72 = +0.00000E+000 XMM73 = +0.00000E+000
MXCSR = 00001F80
```

## AFTER LOADING THE SECOND ARRAY TO XMM2

```
Registers
EAX = 01398000 EBX = 01398020 ECX = 00000008 EDX = 00000001 ESI = 00000000 EDI = 002DF974 EIP = 01391406
ESP = 002DF884 EBP = 002DF974 EFL = 00000206

MM0 = 0000000000000000 MM1 = 0000000000000000 MM2 = 0000000000000000 MM3 = 0000000000000000
MM4 = 0000000000000000 MM5 = 0000000000000000 MM6 = 0000000000000000 MM7 = 0000000000000000

XMM0 = 00000000000000000000000000000000
XMM1 = 400000003F800000400000003F800000
XMM2 = 3F800000400000003F80000040000000
XMM3 = 00000000000000000000000000000000
XMM4 = 00000000000000000000000000000000
XMM5 = 00000000000000000000000000000000
XMM6 = 00000000000000000000000000000000
XMM7 = 00000000000000000000000000000000
XMM00 = +0.00000E+000 XMM01 = +0.00000E+000 XMM02 = +0.00000E+000 XMM03 = +0.00000E+000
XMM10 = +1.00000E+000 XMM11 = +2.00000E+000 XMM12 = +1.00000E+000 XMM13 = +2.00000E+000
XMM20 = +2.00000E+000 XMM21 = +1.00000E+000 XMM22 = +2.00000E+000 XMM23 = +1.00000E+000
XMM30 = +0.00000E+000 XMM31 = +0.00000E+000 XMM32 = +0.00000E+000 XMM33 = +0.00000E+000
XMM40 = +0.00000E+000 XMM41 = +0.00000E+000 XMM42 = +0.00000E+000 XMM43 = +0.00000E+000
XMM50 = +0.00000E+000 XMM51 = +0.00000E+000 XMM52 = +0.00000E+000 XMM53 = +0.00000E+000
XMM60 = +0.00000E+000 XMM61 = +0.00000E+000 XMM62 = +0.00000E+000 XMM63 = +0.00000E+000
XMM70 = +0.00000E+000 XMM71 = +0.00000E+000 XMM72 = +0.00000E+000 XMM73 = +0.00000E+000
MXCSR = 00001F80
```

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Register file after **01391406 0F 59 CA** **mulps** **xmm1,xmm2**

```
Registers
EAX = 01398000 EBX = 01398020 ECX = 00000008 EDX = 00000001 ESI = 00000000 EDI = 002DF974 EIP = 01391409
ESP = 002DF884 EBP = 002DF974 EFL = 00000206

MM0 = 0000000000000000 MM1 = 0000000000000000 MM2 = 0000000000000000 MM3 = 0000000000000000
MM4 = 0000000000000000 MM5 = 0000000000000000 MM6 = 0000000000000000 MM7 = 0000000000000000

XMM0 = 00000000000000000000000000000000
XMM1 = 40000000400000004000000040000000
XMM2 = 3F800000400000003F80000040000000
XMM3 = 00000000000000000000000000000000
XMM4 = 00000000000000000000000000000000
XMM5 = 00000000000000000000000000000000
XMM6 = 00000000000000000000000000000000
XMM7 = 00000000000000000000000000000000
XMM00 = +0.00000E+000 XMM01 = +0.00000E+000 XMM02 = +0.00000E+000 XMM03 = +0.00000E+000
XMM10 = +2.00000E+000 XMM11 = +2.00000E+000 XMM12 = +2.00000E+000 XMM13 = +2.00000E+000
XMM20 = +2.00000E+000 XMM21 = +1.00000E+000 XMM22 = +2.00000E+000 XMM23 = +1.00000E+000
XMM30 = +0.00000E+000 XMM31 = +0.00000E+000 XMM32 = +0.00000E+000 XMM33 = +0.00000E+000
XMM40 = +0.00000E+000 XMM41 = +0.00000E+000 XMM42 = +0.00000E+000 XMM43 = +0.00000E+000
XMM50 = +0.00000E+000 XMM51 = +0.00000E+000 XMM52 = +0.00000E+000 XMM53 = +0.00000E+000
XMM60 = +0.00000E+000 XMM61 = +0.00000E+000 XMM62 = +0.00000E+000 XMM63 = +0.00000E+000
XMM70 = +0.00000E+000 XMM71 = +0.00000E+000 XMM72 = +0.00000E+000 XMM73 = +0.00000E+000
MXCSR = 00001F80
```

After **01391409 0F 58 C1** **addps** **xmm0,xmm1 ;xmm1+xmm0->xmm0**

```
Registers
EAX = 01398000 EBX = 01398020 ECX = 00000008 EDX = 00000001 ESI = 00000000 EDI = 002DF974 EIP = 0139140C
ESP = 002DF884 EBP = 002DF974 EFL = 00000206

MM0 = 0000000000000000 MM1 = 0000000000000000 MM2 = 0000000000000000 MM3 = 0000000000000000
MM4 = 0000000000000000 MM5 = 0000000000000000 MM6 = 0000000000000000 MM7 = 0000000000000000

XMM0 = 40000000400000004000000040000000
XMM1 = 40000000400000004000000040000000
XMM2 = 3F800000400000003F80000040000000
XMM3 = 00000000000000000000000000000000
XMM4 = 00000000000000000000000000000000
XMM5 = 00000000000000000000000000000000
XMM6 = 00000000000000000000000000000000
XMM7 = 00000000000000000000000000000000
XMM00 = +2.00000E+000 XMM01 = +2.00000E+000 XMM02 = +2.00000E+000 XMM03 = +2.00000E+000
XMM10 = +2.00000E+000 XMM11 = +2.00000E+000 XMM12 = +2.00000E+000 XMM13 = +2.00000E+000
XMM20 = +2.00000E+000 XMM21 = +1.00000E+000 XMM22 = +2.00000E+000 XMM23 = +1.00000E+000
XMM30 = +0.00000E+000 XMM31 = +0.00000E+000 XMM32 = +0.00000E+000 XMM33 = +0.00000E+000
XMM40 = +0.00000E+000 XMM41 = +0.00000E+000 XMM42 = +0.00000E+000 XMM43 = +0.00000E+000
XMM50 = +0.00000E+000 XMM51 = +0.00000E+000 XMM52 = +0.00000E+000 XMM53 = +0.00000E+000
XMM60 = +0.00000E+000 XMM61 = +0.00000E+000 XMM62 = +0.00000E+000 XMM63 = +0.00000E+000
XMM70 = +0.00000E+000 XMM71 = +0.00000E+000 XMM72 = +0.00000E+000 XMM73 = +0.00000E+000
MXCSR = 00001F80
```

### The Dot product is stored in xmm0

Using the HADDPS instruction, as shown in the debug window lines 43 and 44. HADDPS performs a single-precision addition on contiguous data elements. The first data element of the result is obtained by adding the first and second elements of the first operand. The second element is obtained by adding the third and fourth elements of the first operand. The third element is obtained by adding the first and second elements of the second operand. The fourth element is obtained by adding the third and fourth elements of the second operand

```
Registers
EAX = 01398020 EBX = 01398040 ECX = 00000000 EDX = 00000001 ESI = 00000000 EDI = 002DF974 EIP = 0139141F
ESP = 002DF884 EBP = 002DF974 EFL = 00000246

MM0 = 0000000000000000 MM1 = 0000000000000000 MM2 = 0000000000000000 MM3 = 0000000000000000
MM4 = 0000000000000000 MM5 = 0000000000000000 MM6 = 0000000000000000 MM7 = 0000000000000000

XMM0 = 41800000418000004180000041800000
XMM1 = 40000000400000004000000040000000
XMM2 = 3F800000400000003F80000040000000
XMM3 = 00000000000000000000000000000000
XMM4 = 00000000000000000000000000000000
XMM5 = 00000000000000000000000000000000
XMM6 = 00000000000000000000000000000000
XMM7 = 00000000000000000000000000000000
XMM00 = +1.60000E+001 XMM01 = +1.60000E+001 XMM02 = +1.60000E+001 XMM03 = +1.60000E+001
XMM10 = +2.00000E+000 XMM11 = +2.00000E+000 XMM12 = +2.00000E+000 XMM13 = +2.00000E+000
XMM20 = +2.00000E+000 XMM21 = +1.00000E+000 XMM22 = +2.00000E+000 XMM23 = +1.00000E+000
XMM30 = +0.00000E+000 XMM31 = +0.00000E+000 XMM32 = +0.00000E+000 XMM33 = +0.00000E+000
XMM40 = +0.00000E+000 XMM41 = +0.00000E+000 XMM42 = +0.00000E+000 XMM43 = +0.00000E+000
XMM50 = +0.00000E+000 XMM51 = +0.00000E+000 XMM52 = +0.00000E+000 XMM53 = +0.00000E+000
XMM60 = +0.00000E+000 XMM61 = +0.00000E+000 XMM62 = +0.00000E+000 XMM63 = +0.00000E+000
XMM70 = +0.00000E+000 XMM71 = +0.00000E+000 XMM72 = +0.00000E+000 XMM73 = +0.00000E+000
MXCSR = 00001F80

0x01398180 = 00000000
```

Finally at the Register XMM0 4 copies of the result (dot product) is in floating point representation

$$\begin{aligned}
 &0 \times 41800000 = \\
 &0100 \ 0001 \ 1000 \ 0000 \ 0000 \ 0000 \ 0000 \ 0000 \\
 &+ 2^{131-127} (1+0) = 16.0 \\
 &0 \times 40000000 = \\
 &0100 \ 0000 \ 0000 \ 0000 \ 0000 \ 0000 \ 0000 \ 0000 \\
 &+ 2^{128-127} (1+0) = 2.0
 \end{aligned}$$