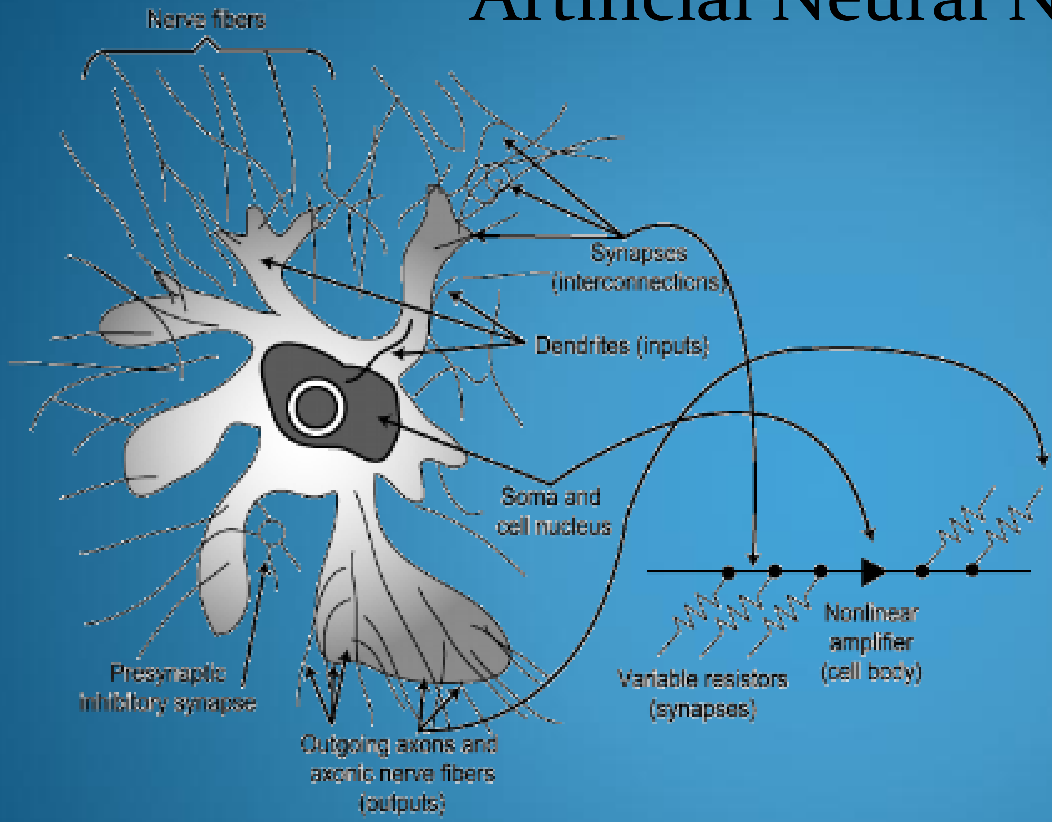
Artificial Neural Networks



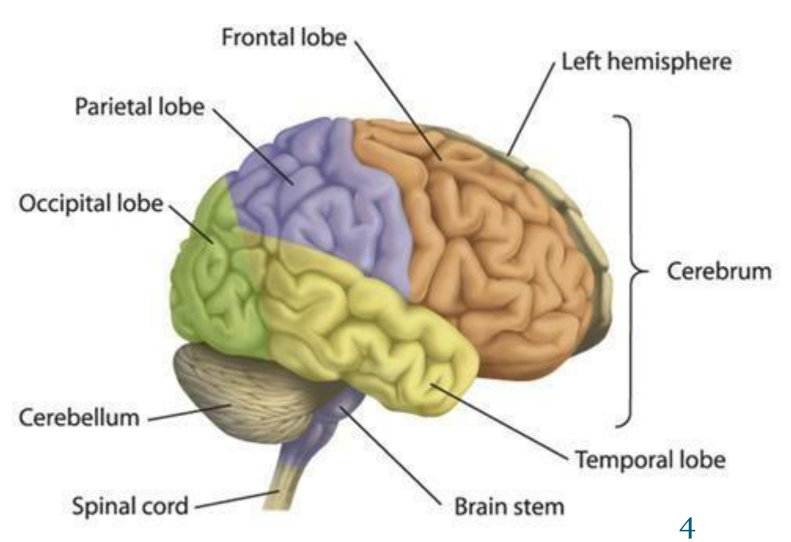
BIOLOGICAL INSPIRATION OF NN

Neural Networks

- Analogy to biological neural systems,
- Attempt to understand natural biological systems through computational modeling.
- Massive parallelism allows for computational efficiency.
- Intelligent behavior as an “emergent” property of large number of simple units rather than from explicitly encoded symbolic rules and algorithms.

Biological Inspiration

- The brain has been extensively studied by scientists.
- Vast complexity prevents all but rudimentary understanding.
- Even the behaviour of an individual neuron is extremely complex
- Engineers modified the neural models to make them more useful
  - less like biology
  - kept much of the terminology



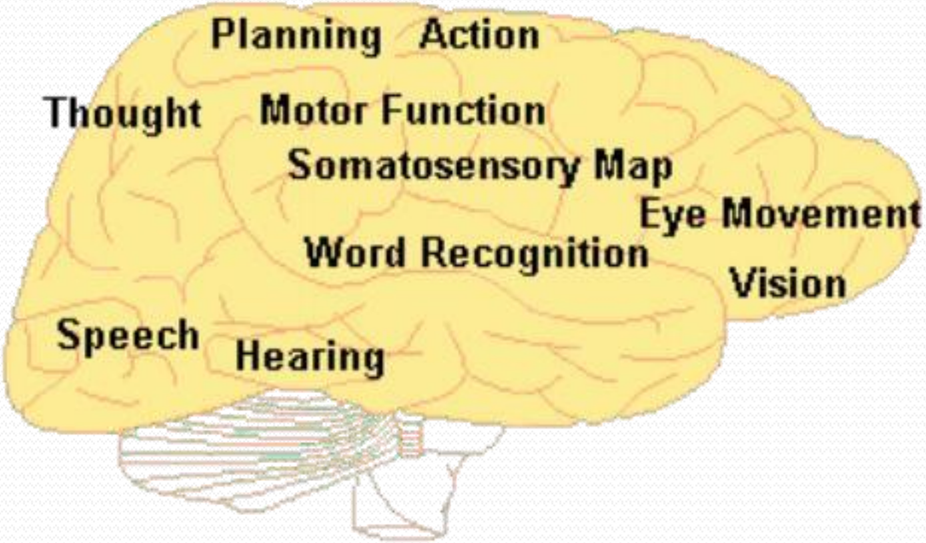
The Biological Neural Network

Characteristics of Human Brain

- Ability to learn from experience
- Ability to generalize the knowledge it possess
- Ability to perform abstraction
- To make errors

Objective

- To emulate or simulate the human brain.

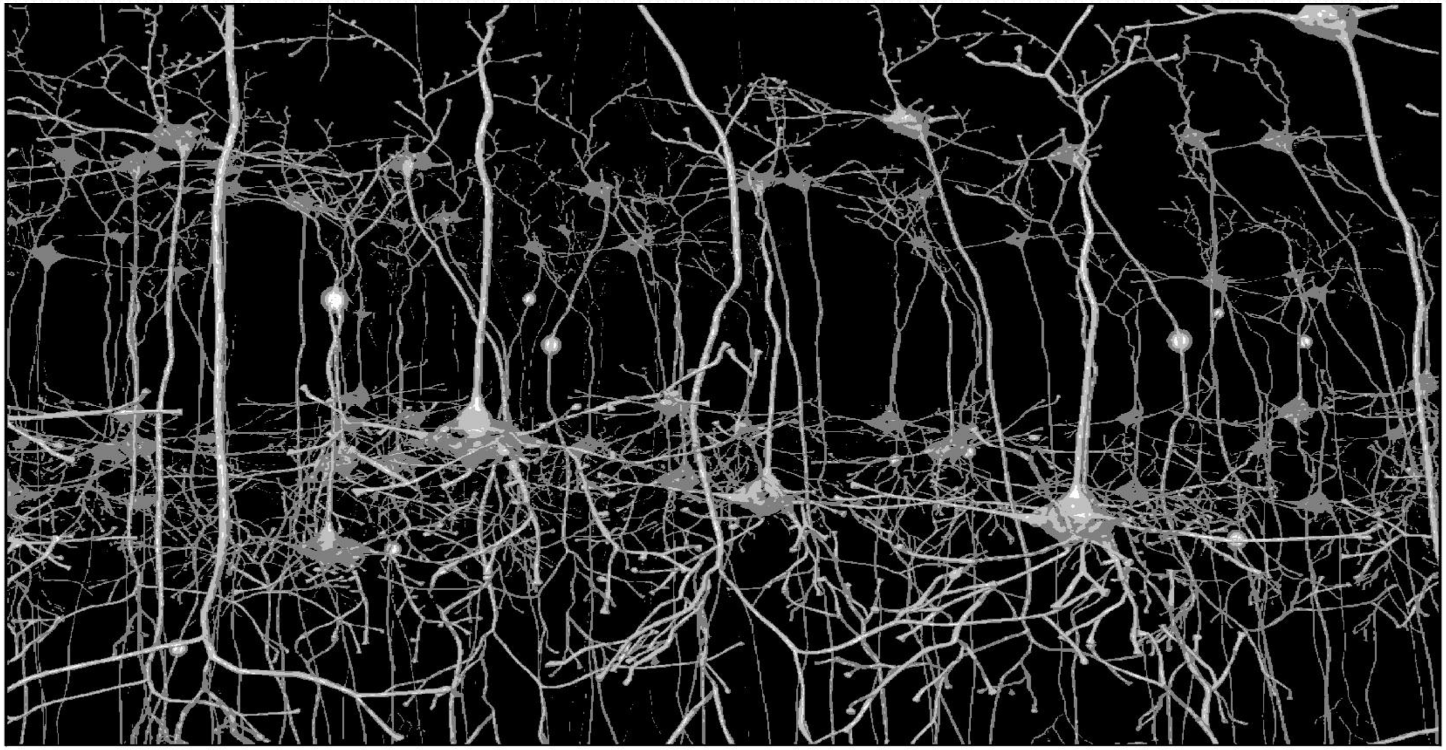


Organization of Human Brain

Stimulus $\longrightarrow$ Receptors $\rightleftarrows$ Neural net $\rightleftarrows$ Effectors $\longrightarrow$ Response

- Over one hundred billion neurons.
- Over one hundred trillion connections called synapses.
- Neurons are responsible for thought emotion, cognition etc.
- Consists of a dense network blood vessels.

Real Neurons



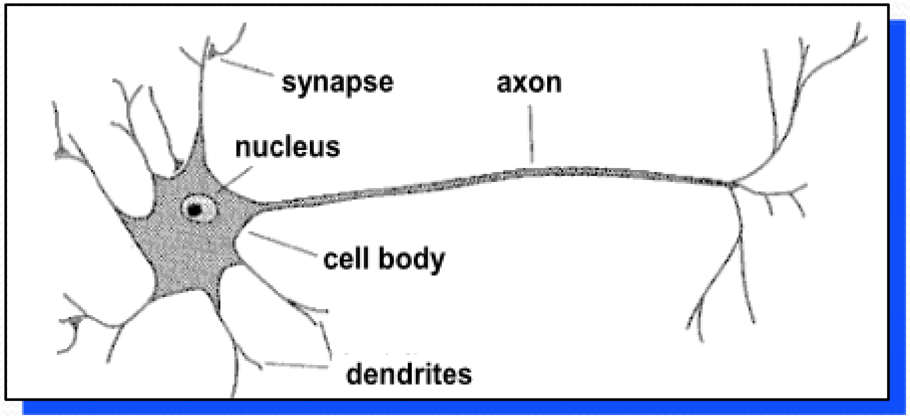
DIGITAL STUDIO SA

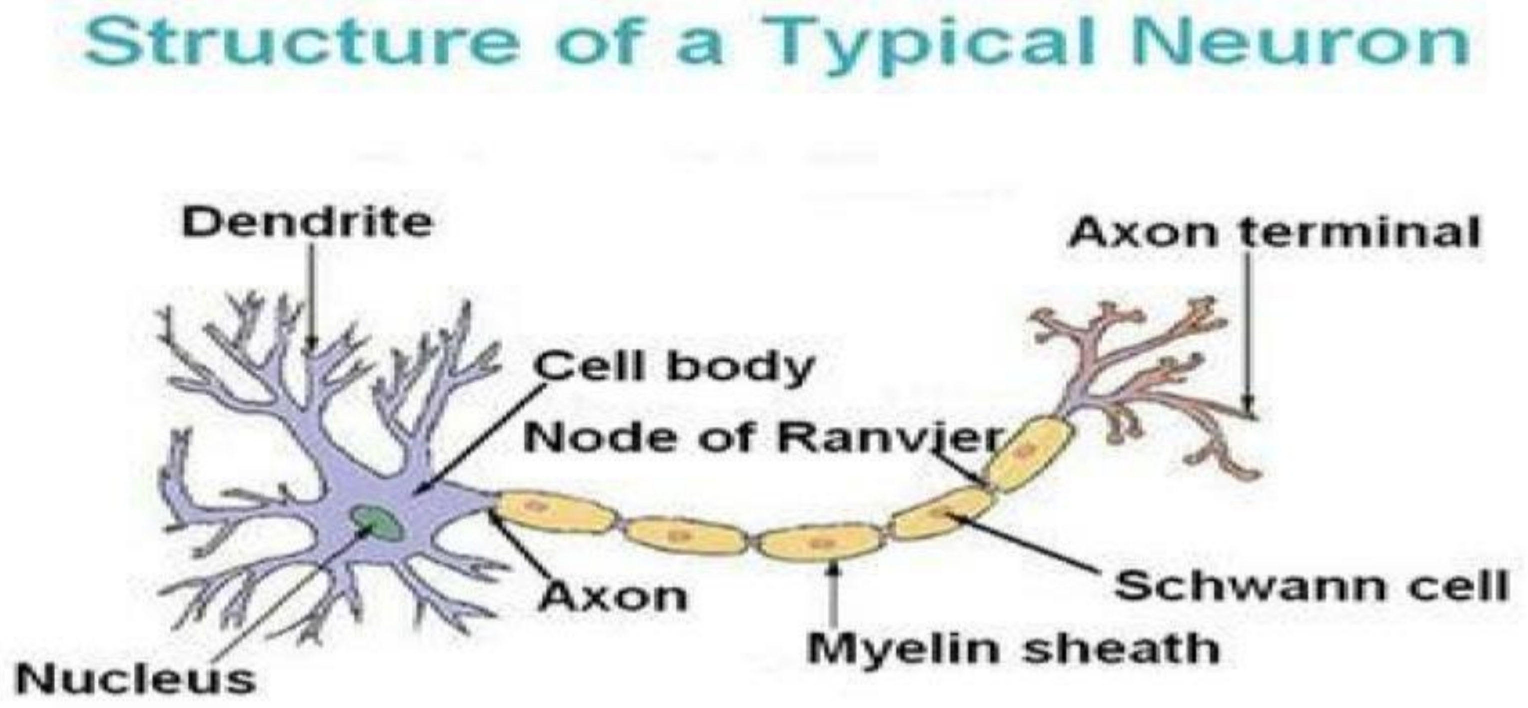
CG image of the vertical organization of neurons in the primary visual cortex (V1).
Smooth stellate and spiny stellate cells relay visual information coming out from the retina to pyramidal cells, themselves doing a first basic computation of visual motion perception.
version of July 2000

- The brain is a collection of about 10 billion interconnected neurons
- Each neuron is a cell that uses biochemical reactions to receive, process and transmit information

The Neuron

- Fundamental building block of the nervous system
- Performs all the computational and communication functions within the brain
- A many inputs/ one output unit





The biological neuron has four main regions to its structure

1. The cell body, or soma
2. The axon
3. The dendrites
4. Synapse

Cell body

- It is the heart of the cell. It contains the nucleolus and maintains protein synthesis
- manufactures a wide variety of complex molecules, to keep it renewed for a life time
- manages the energy economy of the neuron
- the outer membrane of the cell body generates nerve impulses.
- Cell body is 5 to 100 microns in dia

The Axon

- May be as short as 0.1 mm or it is 1 m in length.

- Has multiple branches each terminating in a *synapse.*

- The axon main purpose is to conduct electrical signals generated at the axon down its length. These signals are called *action potentials*

- The other end of the axon may split into several branches, which end in a pre-synaptic terminal.

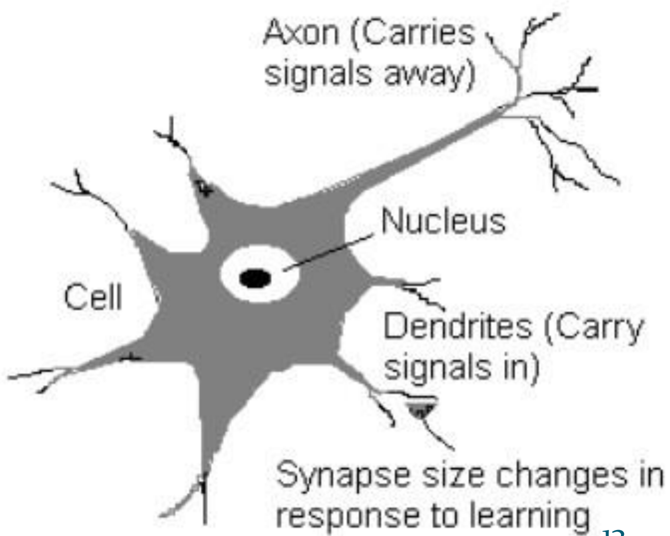
- The myelin is a fatty issue that insulates the axon. The non-insulated parts of the axon area are called Nodes of Ranvier.

- At these nodes, the signal traveling down the axon is regenerated. This ensures that the signal travel down the axon to be fast and constant.

- The brain analyzes all patterns of signals sent, and from that information it interprets the type of information received

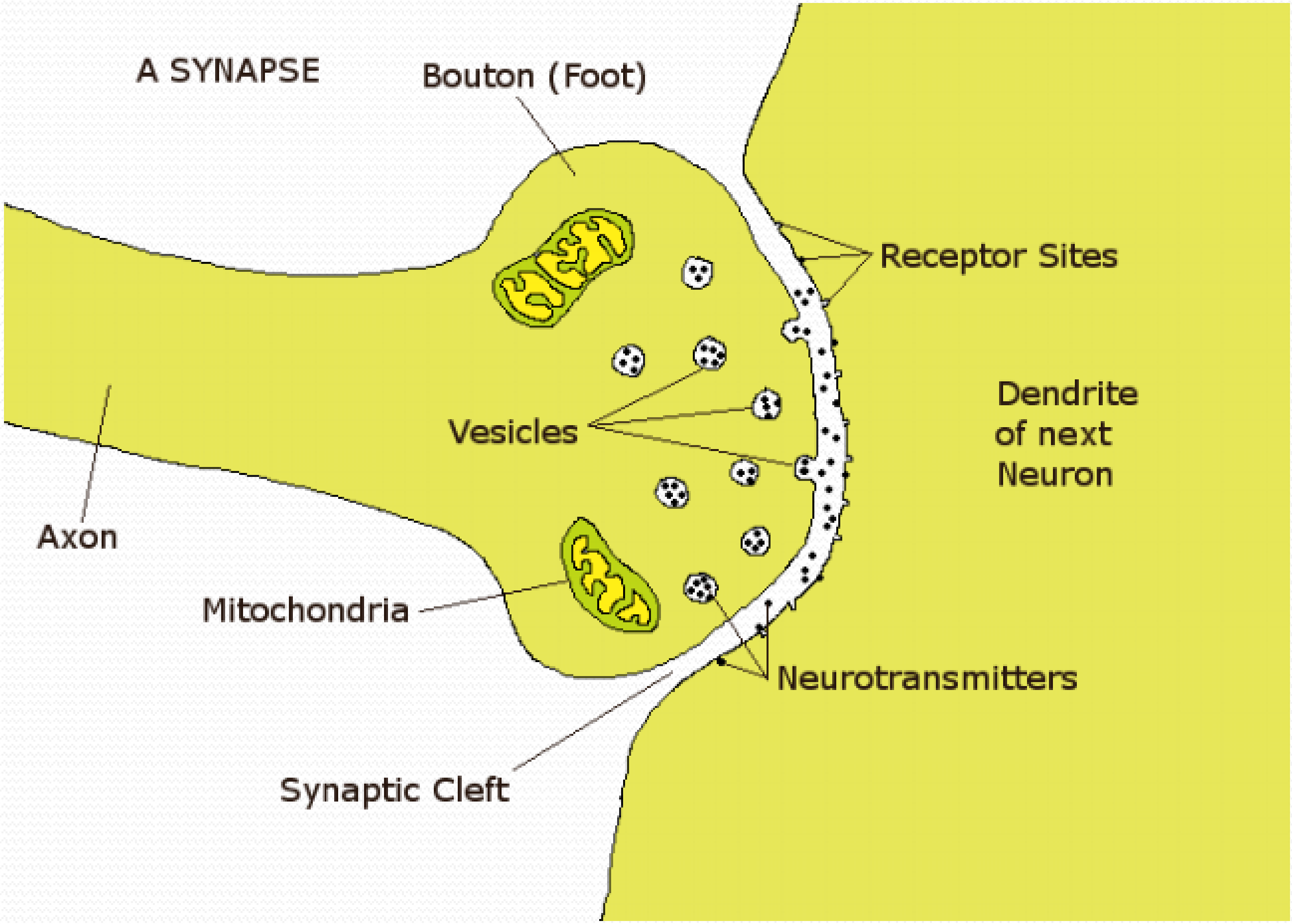
Dendrites

- bushy branching structure emanating from the cell body.
- Receive the signals from other cells at connection points called synapses.
- Usually no physical or electrical connection made at the synapse
- A neuron's dendritic tree is connected to a thousand neighbouring neurons (10,000)



Synapse

- The synapse is the area of contact between two neurons.
- They do not physically touch because they are separated by a cleft.
- The electric signals are sent through chemical interaction.
- The neuron sending the signal is called *pre-synaptic cell* and the neuron receiving the electrical signal is called *postsynaptic cell.*


A SYNAPSE
Bouton (Foot)
Receptor Sites
Vesicles
Dendrite of next Neuron
Axon
Mitochondria
Neurotransmitters
Synaptic Cleft

The Bio Neuron

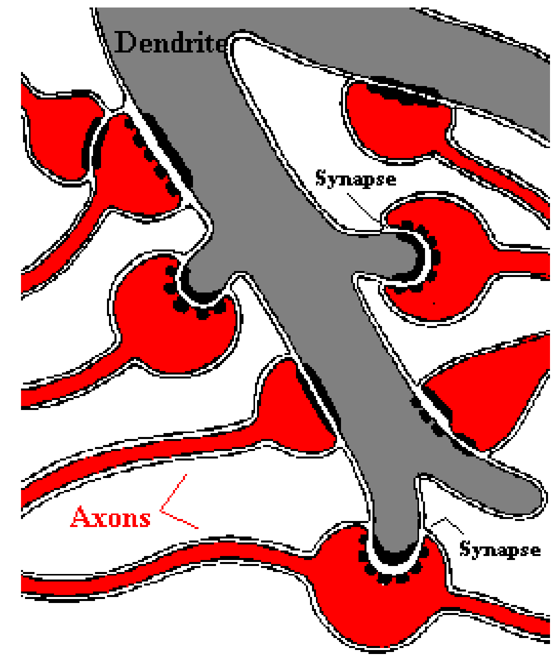
- Neurotransmitters which are specialized chemicals are released by the axon, into the synaptic cleft, diffuse across to the dendrite.

- When one of those neurons fire, a positive or negative charge is received by one of the dendrites. The strengths of all the received charges are added together through the processes of spatial and temporal summation.

- A neuron only fires if its input signal exceeds a certain amount (threshold) in a short time period.

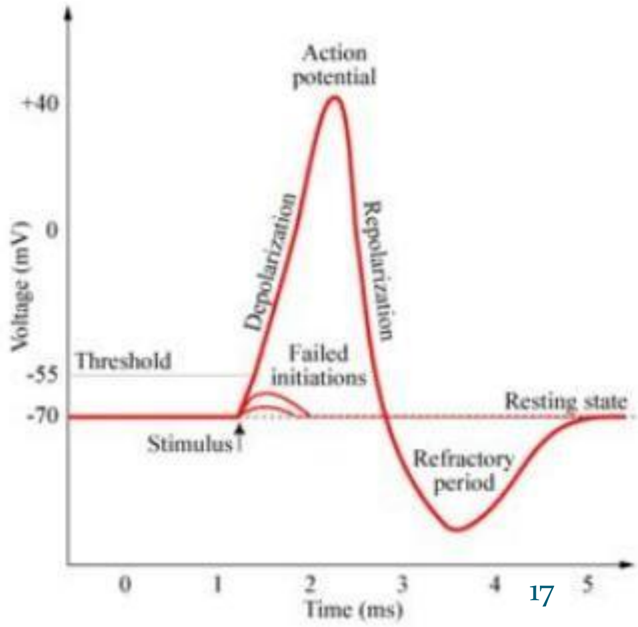
- Neurotransmitters are excitatory, which tend to produce an output pulse.

- Some are inhibitory, which tend to suppress such a pulse



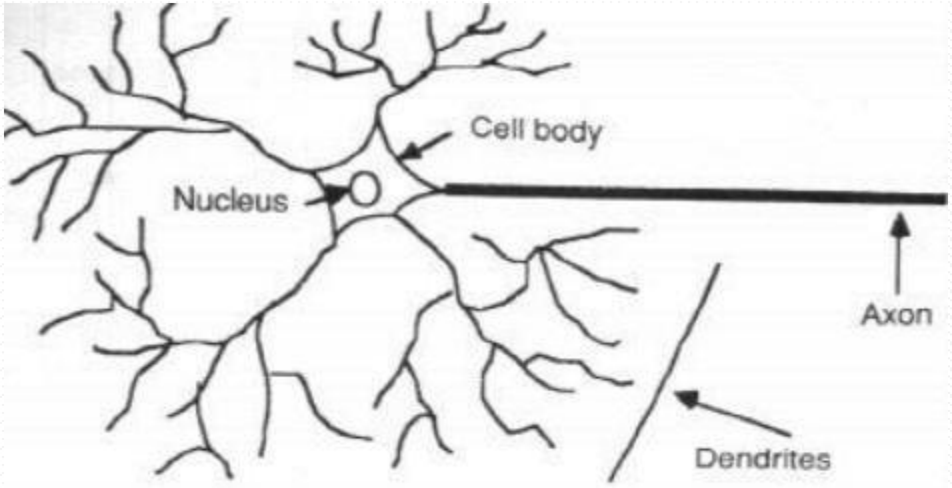
Neural Communication

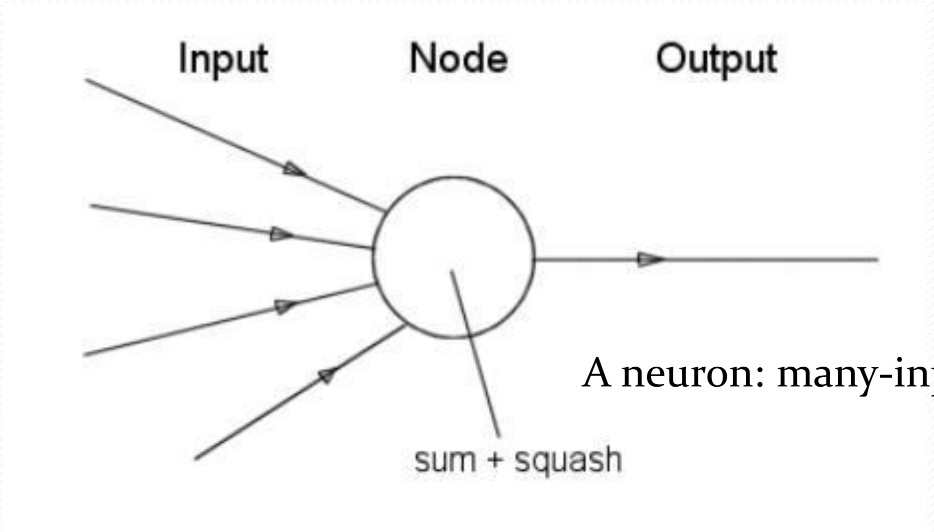
- Electrical potential across cell membrane exhibits spikes called action potentials.
- Spike originates in cell body, travels down axon, and causes synaptic terminals to release neurotransmitters.
- Chemical diffuses across synapse to dendrites of other neurons.
- If net input of neurotransmitters to a neuron from other neurons is excitatory and exceeds some threshold, it fires an action potential.



BIO NEURON NETWORK TO ARTIFICIAL NEURON NETWORK

Neuron vs. Node

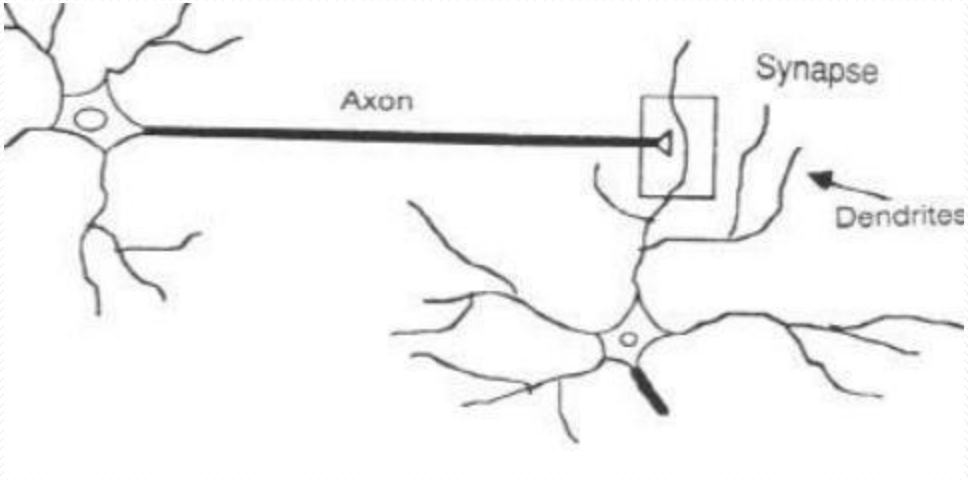


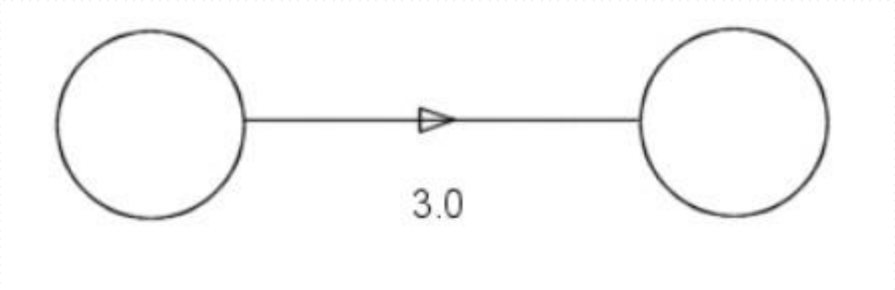


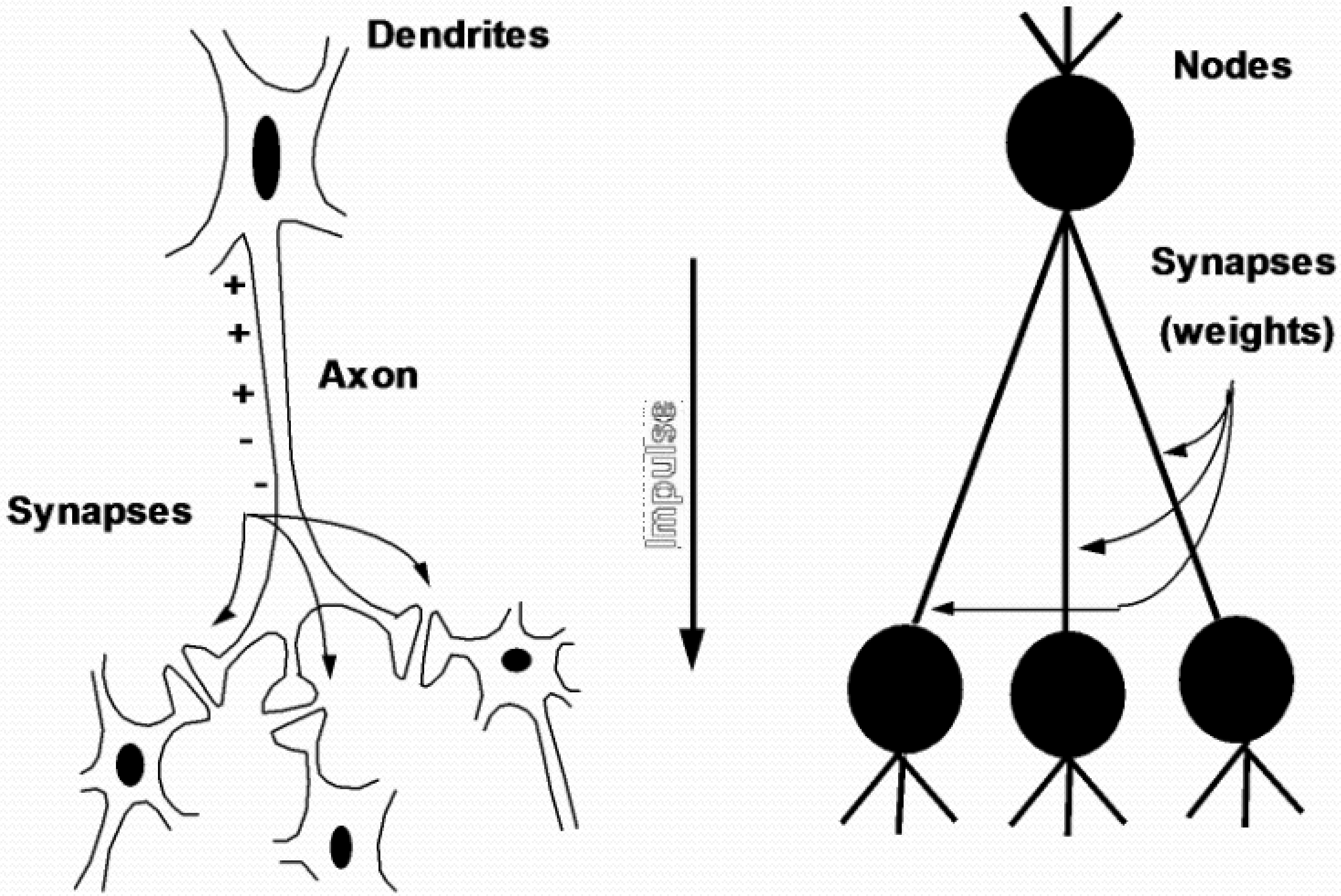
A neuron: many-inputs / one-output unit

Synapse vs weight

--Axon turn the processed inputs to outputs. --- Synapses are the electrochemical contact between neurons.






Dendrites
Axon
Synapses
Impulse
Nodes
Synapses
(weights)

- Each neuron receives inputs from other neurons
  - A few neurons also connect to receptors.
  - Cortical neurons use spikes to communicate.

- The effect of each input line on the neuron is controlled by a synaptic weight
  - *The weights can be positive or negative.*
- The synaptic weights adapt so that the whole network learns to perform useful computations
  - *Recognizing objects, understanding language, making plans, controlling the body.*
- *We have about 10^{11} neurons each with about 10^{4} weights.*
  - *A huge number of weights can* affect the computation *in a very short time.*

Idealized neurons

- To model things we have to idealize them
  - Idealization removes complicated details that are not essential for understanding the main principles.
  - It allows us to apply mathematics and to make analogies to other, familiar systems.
  - Once we understand the basic principles, its easy to add complexity to make the model more faithful.

Artificial neural networks

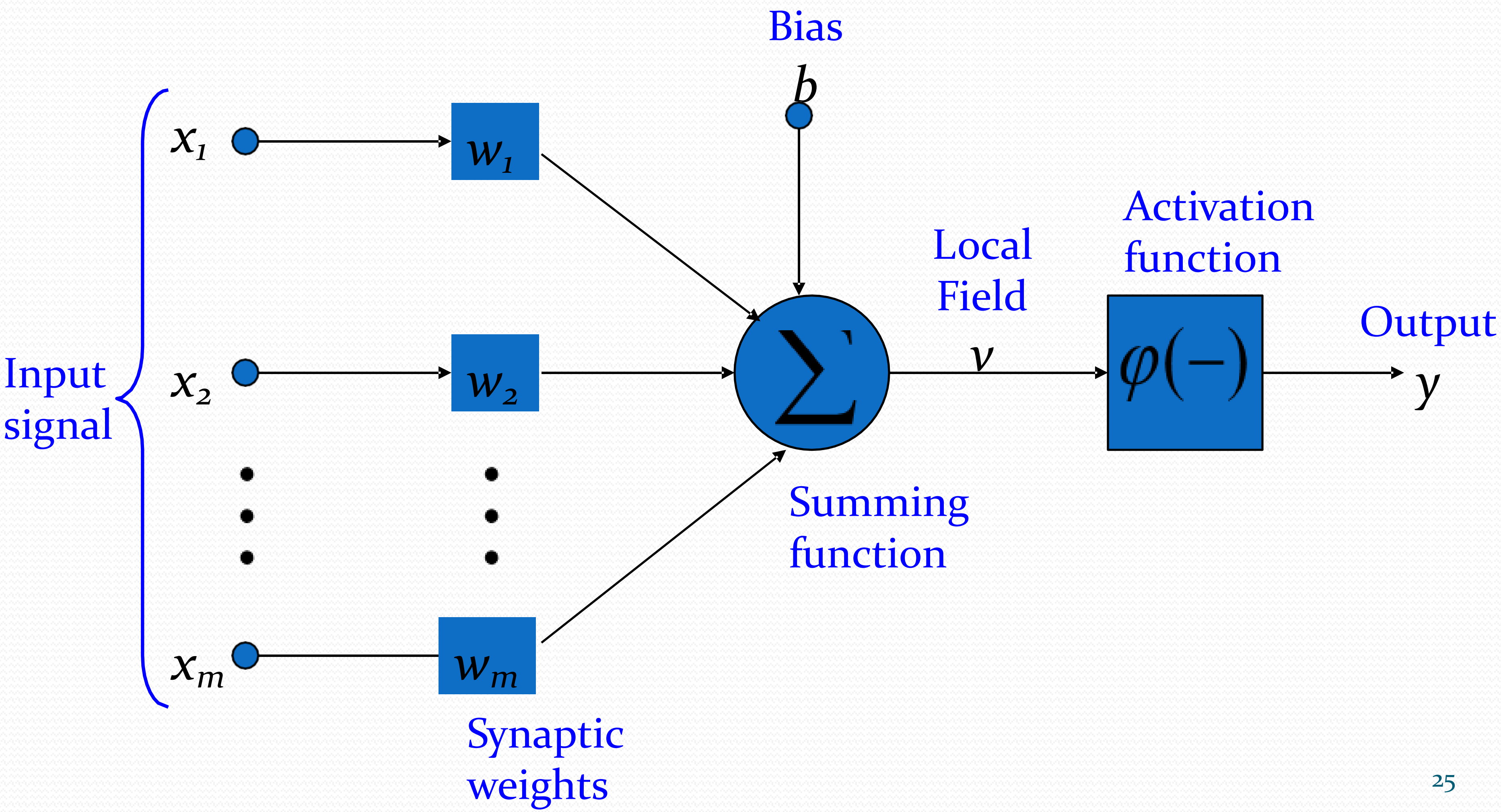
- An artificial neural network is composed of many artificial neurons that are linked together according to a specific network architecture.

- Signals (action potentials) appear at the node's inputs (synapses).

- The each input is multiplied by a certain weight, before being added together at the node (neuron) to produce an overall activation.

- If this exceeds a threshold, the node fires, sending signals to other nodes.

The Artificial Neuron



The Neuron

- The neuron is the basic information processing unit of a NN.

- It consists of:

1 A set of synapses or connecting links, each link characterized by a weight: (W kj)

$W_1, W_2, \ldots, W_m$

2 An adder function (linear combiner) which computes the weighted sum of the inputs:

$$u = \sum_{j=1}^{m} w_j x_j$$

3 Activation function (squashing function) for limiting the amplitude of the output of the neuron

Y = *activation potential/ induced local field*

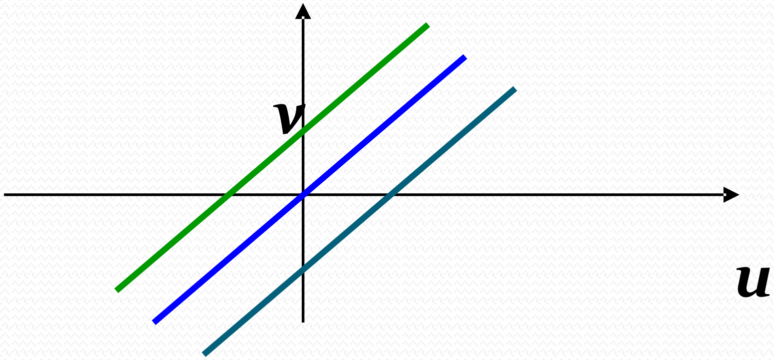
$$y = \varphi(u + b)$$

Bias of a Neuron

- Bias b has the effect of applying an affine transformation to u

$$v = u + b$$

- v is the **induced field** of the neuron



Linear neurons

- These are simple but computationally limited
  - If we can make them learn we may get insight into more complicated neurons.

bias

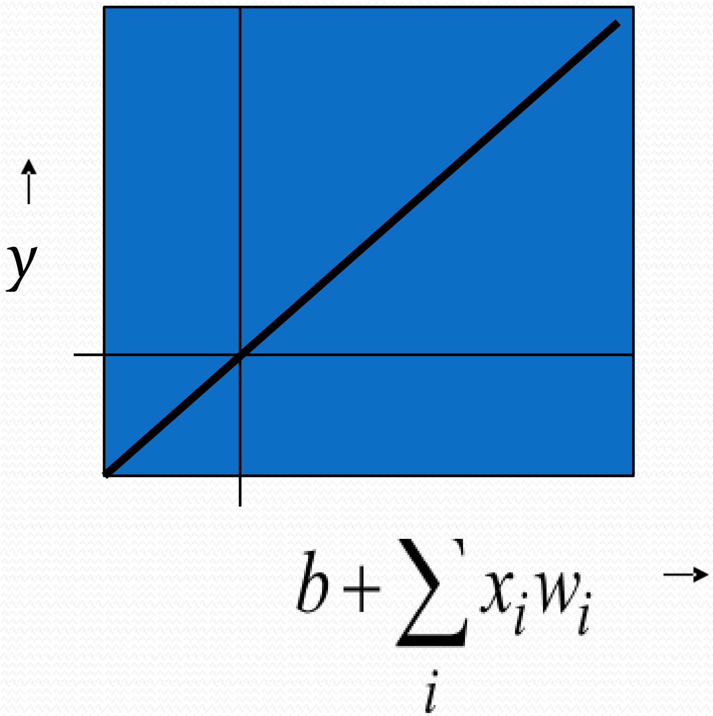
i^{th} input

$$y = b + \sum_i x_i w_i$$

output

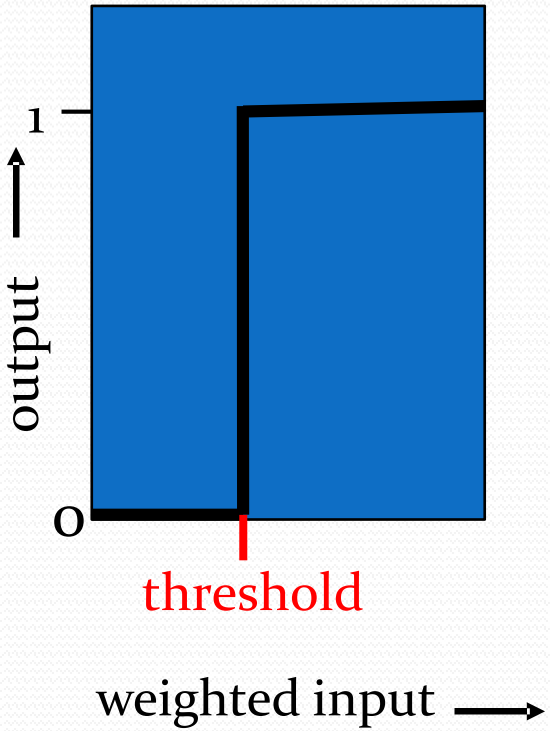
index over input connections

weight on i^{th} input



Binary threshold neurons

- McCulloch-Pitts (1943): inf luenced Von Neumann.
  - First compute a weighted sum of the inputs.
  - Then send out a fixed size spike of activity if the weighted sum exceeds a threshold.
  - McCulloch and Pitts thought that each spike is like the truth value of a proposition and each neuron combines truth values to compute the truth value of another proposition!



Binary threshold neurons

- There are two equivalent ways to write the equations for a binary threshold neuron:

$$u = \sum_i x_i w_i$$

$$v = b + \sum_i x_i w_i$$

$$\boxed{\theta = -b}$$

$$\varphi(v) = \begin{cases} 1 \text{ if } & v \geq \theta \\ 0 \text{ otherwise} \end{cases}$$

$$y = \begin{cases} 1 \text{ if } & v \geq 0 \\ 0 \text{ otherwise} \end{cases}$$

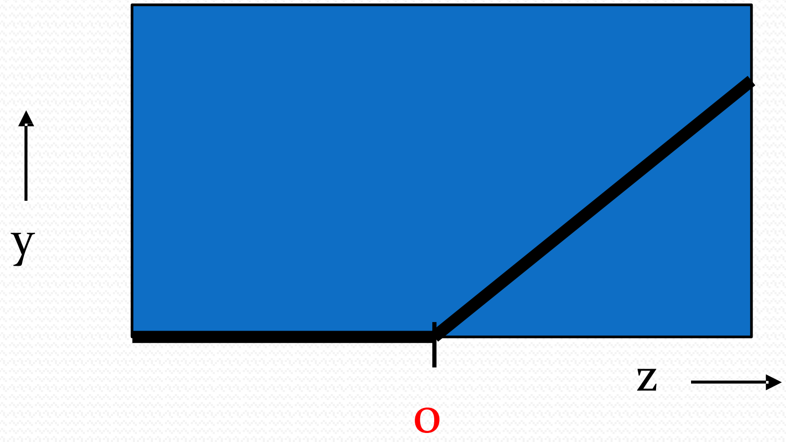
Rectified Linear Neurons

(sometimes called linear threshold neurons)

They compute a linear weighted sum of their inputs.
The output is a non-linear function of the total input.

$$v = b + \sum_i x_i w_i$$

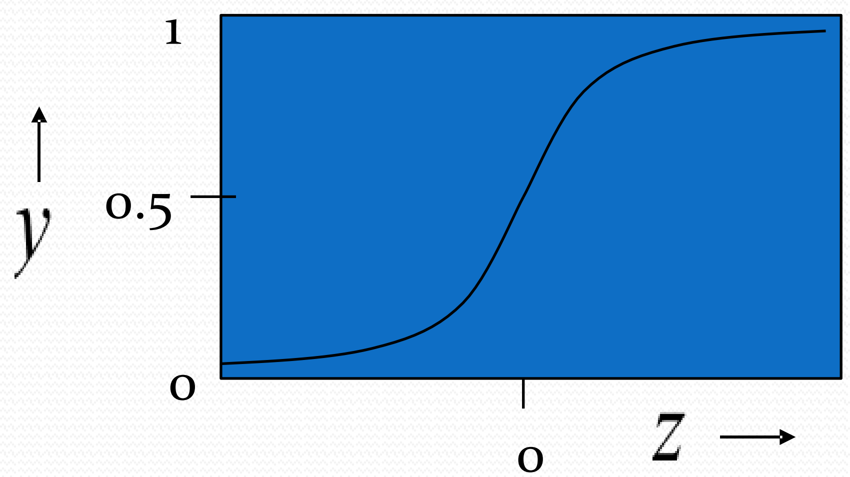
$$y = \begin{cases} v & \text{if } v > 0 \\ 0 & \text{otherwise} \end{cases}$$



Sigmoid neurons

- These give a real-valued output that is a smooth and bounded function of their total input.
  - Typically they use the logistic function
  - They have nice derivatives which make learning easy

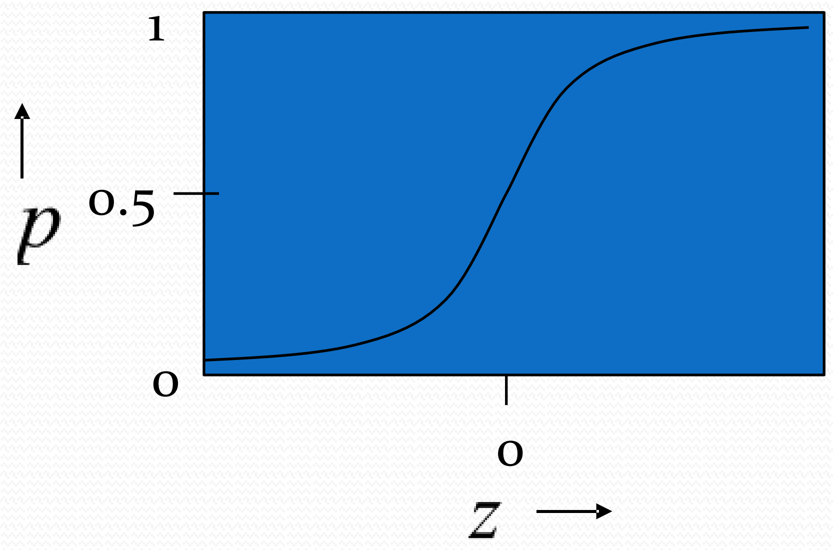
$$v = b + \sum_i x_i w_i \qquad y = \frac{1}{1 + e^{-av}}$$



Stochastic binary neurons

- These use the same equations as logistic units.
  - But they treat the output of the logistic as the probability of producing a spike in a short time window.
- We can do a similar trick for rectified linear units:
  - The output is treated as the Poisson rate for spikes.

$$z = b + \sum_i x_i w_i \qquad p(s=1) = \frac{1}{1+e^{-z}}$$



ACTIVATION FUNCTIONS

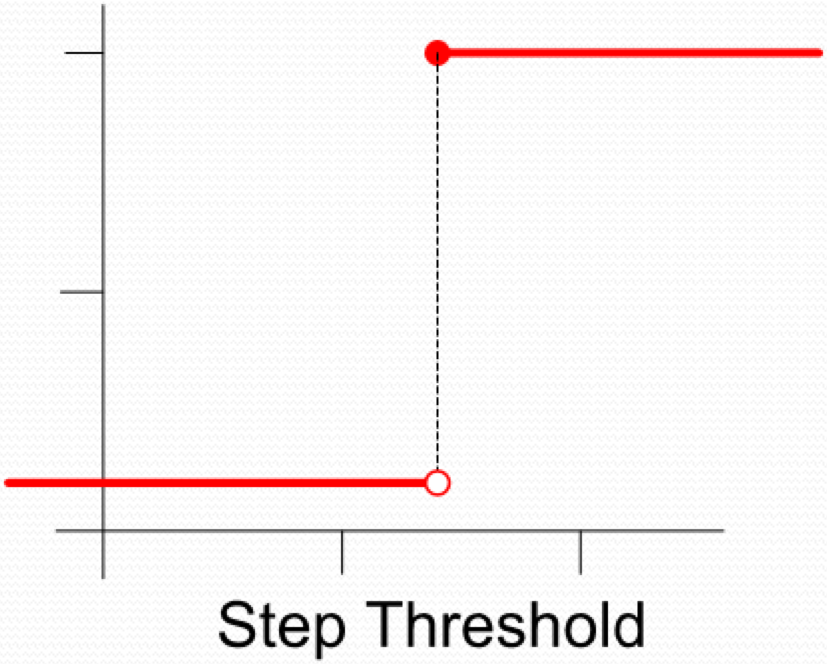
- To calculate the output response of a neuron

- Transforms neuron's input into output.
- Features of activation functions:
  - A squashing effect is required
    - Prevents accelerating growth of activation levels through the network.
  - Simple and easy to calculate

Threshold Activation Function

- Binary classifier functions

Heaveside function



Binary Threshold Signal Function

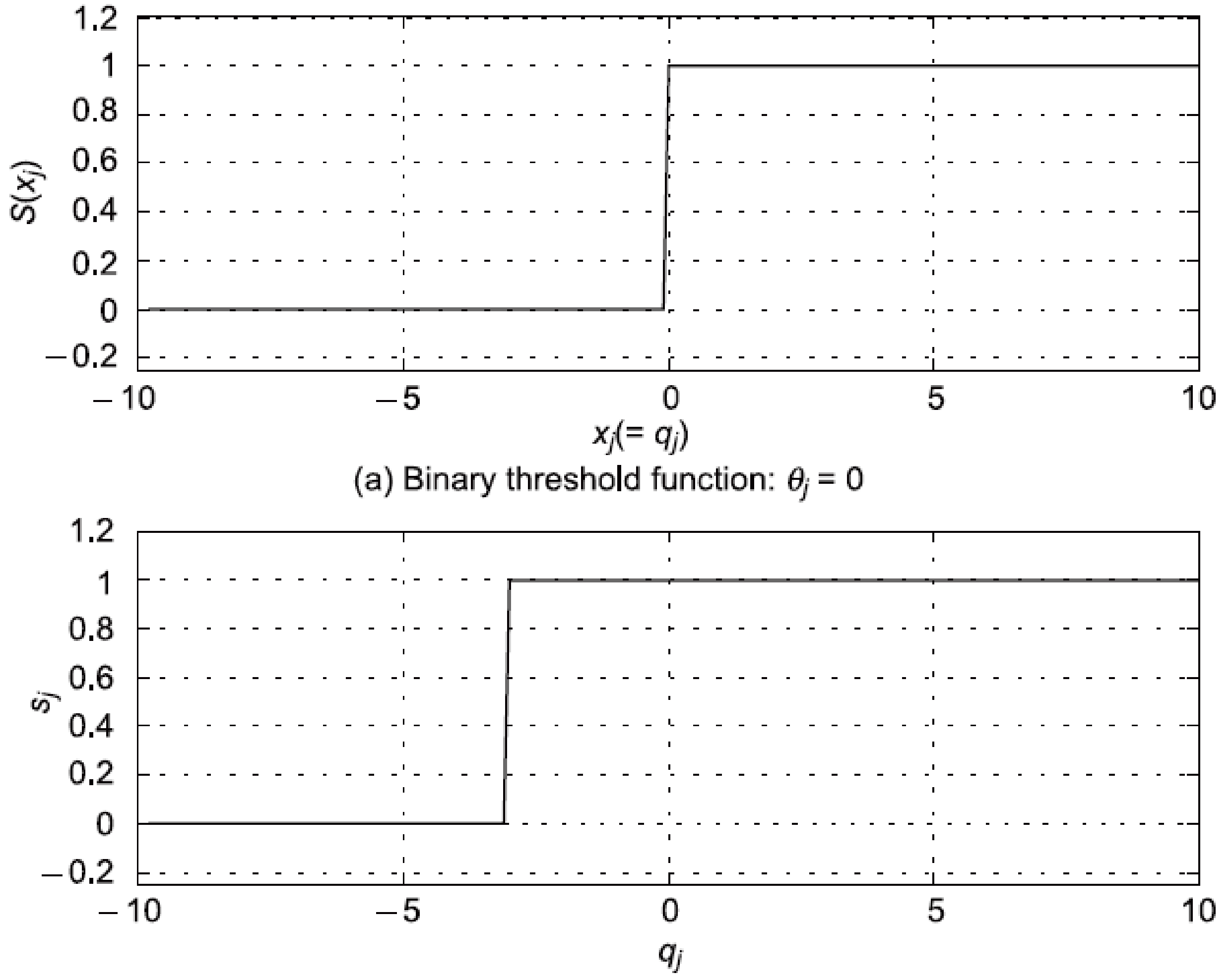
- The threshold logic neuron is a two state machine
  - $s_j = S(x_j) \in \{0, 1\}$

- Net positive activations translate to a +1 signal value
- Net negative activations translate to a 0 signal value.

$$\Phi(v) = \begin{cases} 1 & v \geq 0 \\ 0 & v < 0 \end{cases}$$

$$Y_k = \begin{cases} 1 & v \geq 0 \\ 0 & v < 0 \end{cases}$$

Neuron Signal Functions: Binary Threshold Signal Function

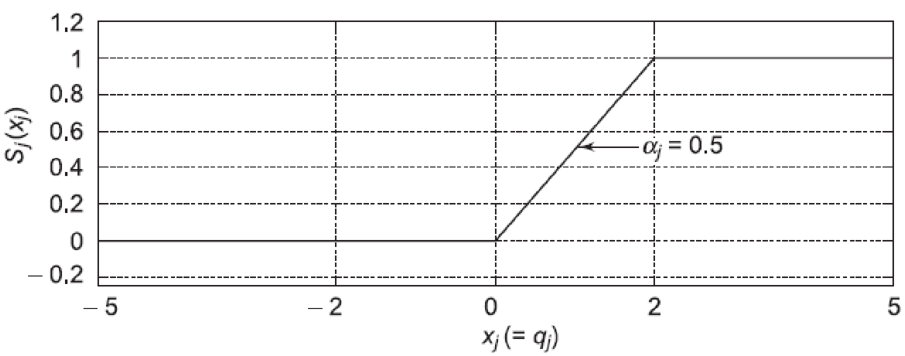


(b) Binary threshold function: $\theta_j = +3$

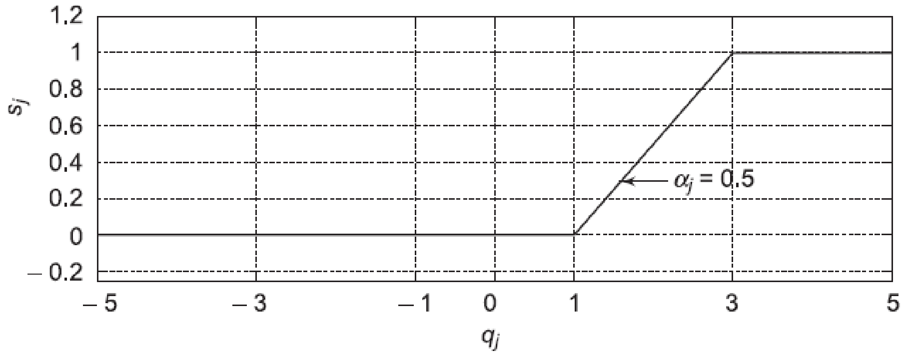
Linear Threshold Signal Function

$$S_j(x_j) = \begin{cases} 0 & x_j \le 0 \\ \alpha_j \, x_j & 0 < x_j < x_m \\ 1 & x_j \ge x_m \end{cases}$$

- $\alpha_j = 1/x_m$ is the slope parameter of the function
- Figure plotted for $x_m = 2$ and $\alpha_j = 0.5$. .



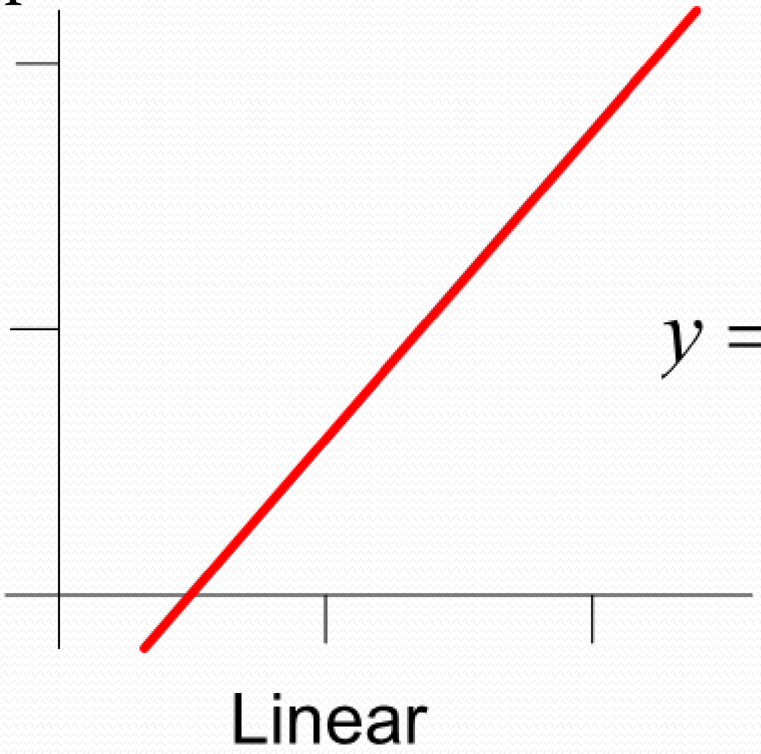
(a) Linear Threshold function: $\theta_j = 0$



(b) Shifted Linear Threshold function: $\theta_j = -1$

Linear Activation functions

- Output is scaled sum of inputs



$$y = u = \sum_{n=1}^{N} w_n x_n$$

Threshold Logic Neuron (TLN) in Discrete Time

- The updated signal value $S(x_j^{k+1})$ at time instant $k + 1$ is generated from the neuron activation x_i^{k+1} , sampled at time instant $k + 1$.
- The response of the threshold logic neuron as a two-state machine can be extended to the *bipolar* case where the signals are
  - $s_j \in \{-1, 1\}$

$$S(x_j^{k+1}) = \begin{cases} 1 & x_j^{k+1} > 0 \\ S(x_j^k) & x_j^{k+1} = 0 \\ 0 & x_j^{k+1} < 0 \end{cases}$$

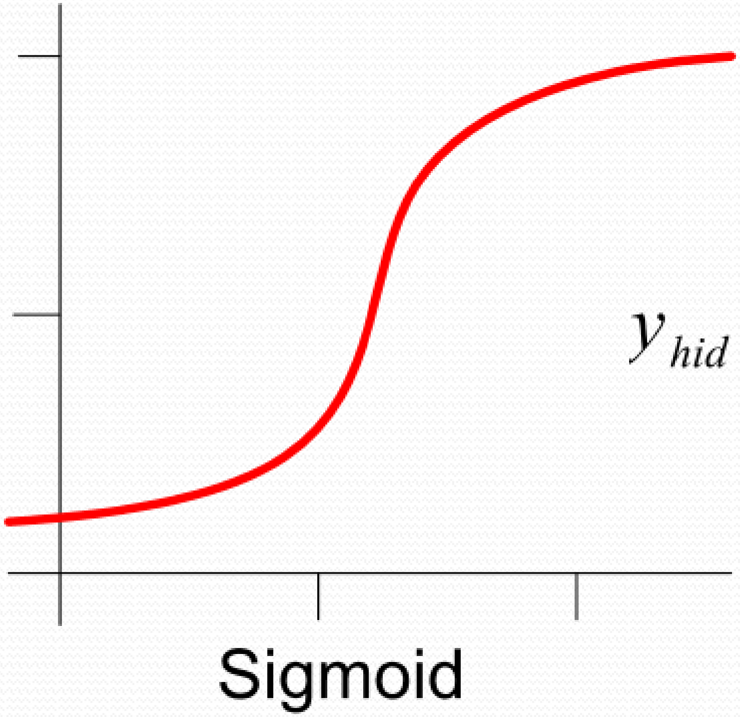
$$S(x_j) = \begin{cases} +1 & x_j > 0 \\ -1 & x_j < 0 \end{cases}$$

Threshold Logic Neuron (TLN) in Discrete Time

- The resulting signal function is then none other than the *signum function*, sign(x) commonly encountered in communication theory.

Nonlinear Activation Functions

- Sigmoid Neuron unit function

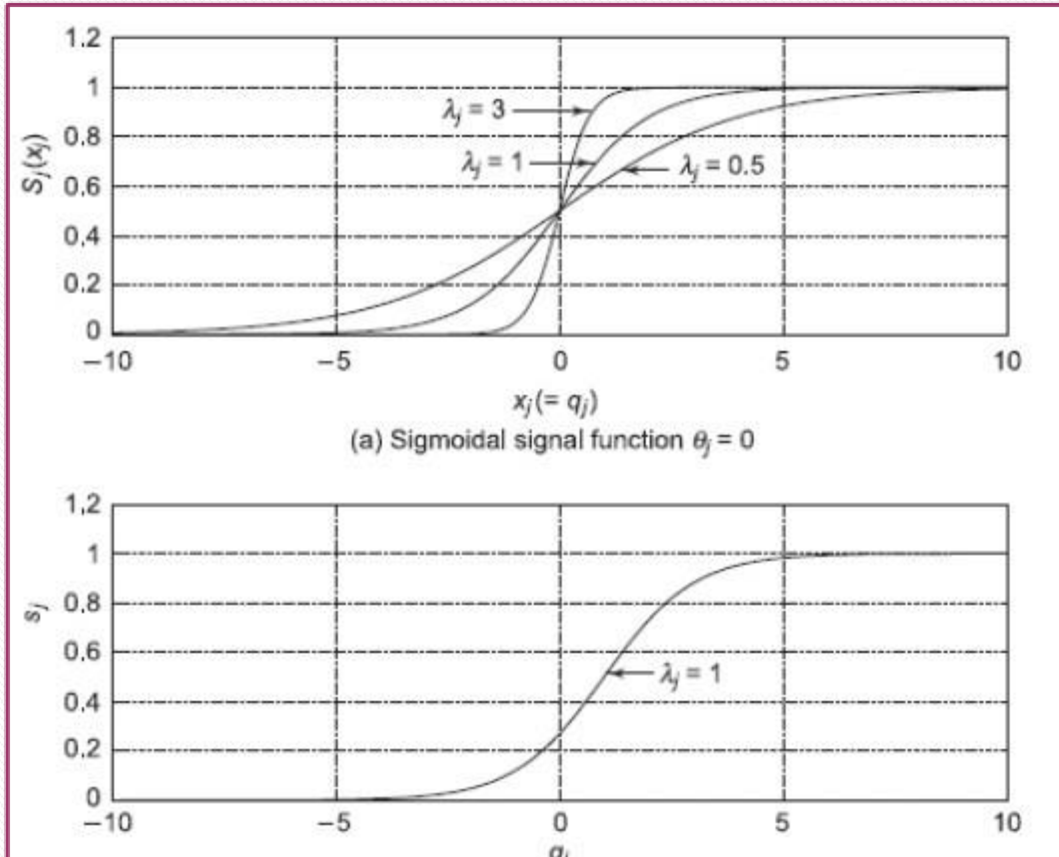


$$y_{hid}(u) = \frac{1}{1+e^{-u}}$$

Sigmoidal Signal Function

$$S_j(x_j) = \frac{1}{1 + e^{-\lambda_j x_j}}$$

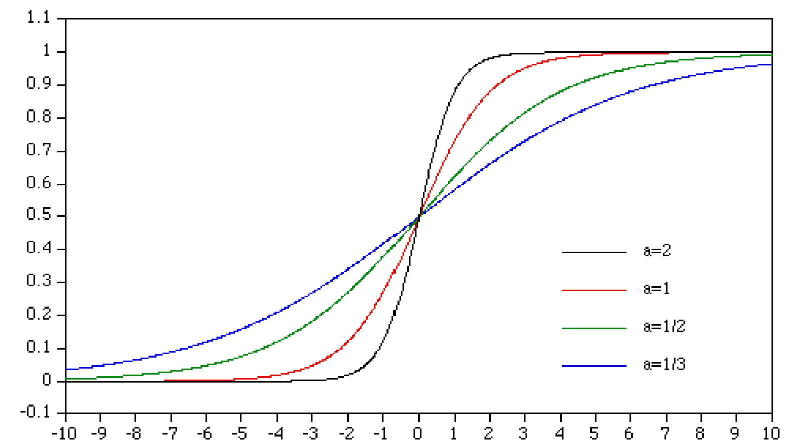
- λ_j is a gain scale factor
- In the limit, as $\lambda_j \rightarrow \infty$ the smooth logistic function approaches the non-smooth binary threshold function.



(a) Sigmoidal signal function $\theta_j = 0$

(b) Shifted sigmoidal signal function $\theta_j = -1$

Activation Function

- **Squashing Function or Logistic Function or Sigmoid Function.**
- 

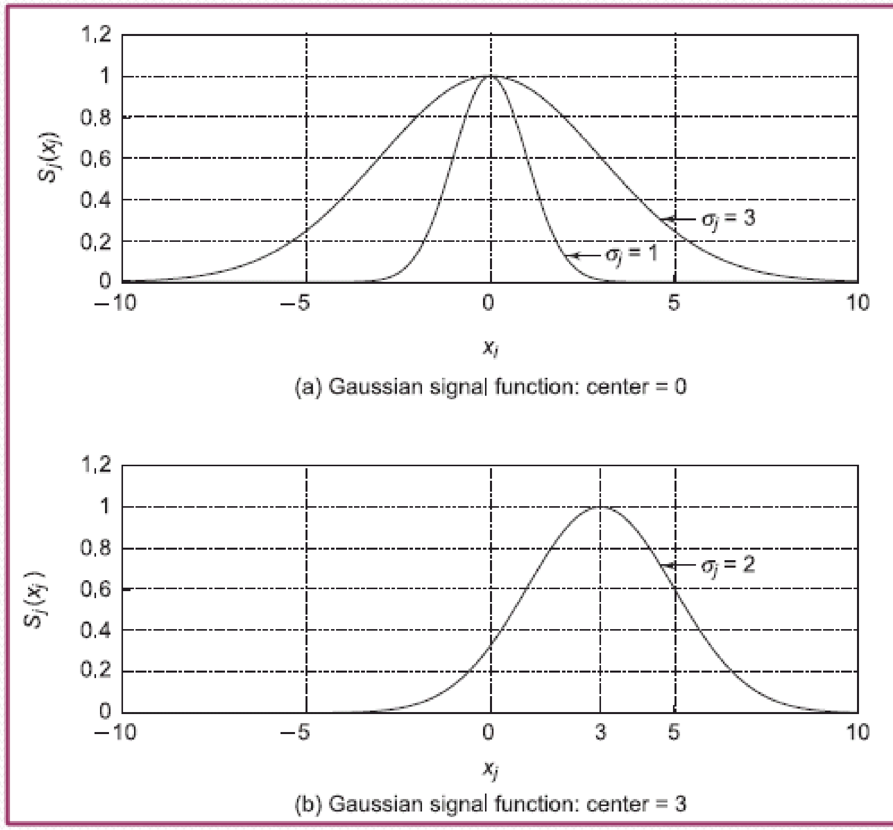
$Y = 1/1+e^{-f}$

f = 0	**Y = 0.5**
f > 0	**Y = 1**
f < 0	**Y = 0**

Gaussian Signal Function

$$S_j(x_j) = \exp\left(-\frac{(x_j - c_j)^2}{2\sigma_j^2}\right)$$

- σ_j is the Gaussian spread factor and c_j is the center.
- Varying the spread makes the function sharper or more diffuse.



Stochastic Neurons

- The signal is assumed to be two state
  - $s_j \in \{0, 1\}$ or $\{-1, 1\}$
- Neuron switches into these states depending upon a *probabilistic function of its activation*, $P(x_j)$.

$$P(x_j) = \frac{1}{1 + e^{-x_j/T}}$$

Summary of Signal Functions

Name	*Function*	*Characteristics*
Binary threshold	$\mathcal{S}(x_j) = \begin{cases} 1 & x_j \geq 0 \\ 0 & x_j < 0 \end{cases}$	Non-differentiable, step-like, $s_j \in \{0, 1\}$
Bipolar threshold	$\mathcal{S}(x_j) = \begin{cases} 1 & x_j \geq 0 \\ -1 & x_j < 0 \end{cases}$	Non-differentiable, step-like, $s_j \in \{-1, 1\}$
Linear	$\mathcal{S}_j(x_j) = \alpha_j x_j$	Differentiable, unbounded, $s_j \in (-\infty, \infty)$
Linear threshold	$\mathcal{S}_j(x_j) = \begin{cases} 0 & x_j \leq 0 \\ \alpha_j x_j & 0 < x_j < x_m \\ 1 & x_j \geq x_m \end{cases}$	Differentiable, piece-wise linear, $s_j \in [0, 1]$
Sigmoid	$\mathcal{S}_j(x_j) = \dfrac{1}{1 + e^{-\lambda_j x_j}}$	Differentiable, monotonic, smooth, $s_j \in (0, 1)$
Hyperbolic tangent	$\mathcal{S}_j(x_j) = \tanh(\lambda_j x_j)$	Differentiable, monotonic, smooth, $s_j \in (-1, 1)$
Gaussian	$e^{-(x_j - c_j)^2/2\sigma_j^2}$	Differentiable, non-monotonic, smooth, $s_j \in (0, 1)$
Stochastic	$\mathcal{S}_j(x_j) = \begin{cases} +1 & \text{with probability } P(x_j) \\ -1 & \text{with probability } 1 - P(x_j) \end{cases}$	Non-deterministic step-like, $s_j \in \{0, 1\}$ or $\{-1, 1\}$