When the neural network is ready in its final version, it can be used to determine the membership values of any input data [Figure 9-7(G)] in the different regions (classes) [Figure 9-7(I)]. A complete mapping of the membership of various data points in various fuzzy classes can be derived to determine the overlap of the different classes. The overlap of the three fuzzy classes is shown in hatched portion of Figure 9-7(C). In this manner, neural network is used to determine the fuzzy membership functions.

9.4.6 Genetic Algorithms

Genetic algorithm is based on the Darwin's theory of evolution; the basic rule is "survival of the fittest." The genetic algorithm is used here to determine the fuzzy membership functions. This can be done using the following steps:

1. For a particular functional mapping system, the same membership functions and shapes are assumed for various fuzzy variables to be defined.
2. These chosen membership functions are then coded into bit strings.
3. Then these bit strings are concatenated together.
4. The fitness function to be used here is noted. In genetic algorithm, fitness function plays a major role similar to that played by activation function in neural network.
5. The fitness function is used to evaluate the fitness of each set of membership functions.
6. These membership functions define the functional mapping of the system.

The process of generating and evaluating strings is carried out until we get a convergence to the solution within a generation, i.e., we obtain the membership functions with best fitness value. Thus, fuzzy membership functions can be obtained from genetic algorithm.

9.4.7 Induction Reasoning

Induction is used to deduce causes by means of backward inference. The characteristics of inductive reasoning can be used to generate membership functions. Induction employs entropy minimization principle, which clusters the parameters corresponding to the output classes. To perform inductive reasoning method, a well-defined database for the input–output relationship should exist. The inductive reasoning can be applied for complex systems where the data are abundant and static. For dynamic data sets, this method is not best suited, because the membership functions continually changes with time. There exist three laws of induction (Christeuseu, 1980):

1. Given a set of irreducible outcomes of an experiment, the induced probabilities are those probabilities consistent with all available information that maximize the entropy of the set.
2. The induced probability of a set of independent observations is proportional to the probability density of the induced probability of a single observation.
3. The induced rule is that rule consistent with all available information of that minimizes the entropy.

The third law stated above is widely used for the development of membership functions. The membership functions using inductive reasoning are generated as follows:

1. A fuzzy threshold is to be established between classes of data.

2. Using entropy minimization screening method, first determine the threshold line.
3. Then start the segmentation process.
4. The segmentation process results into two classes.
5. Again partitioning the first two classes one more time, we obtain three different classes.
6. The partitioning is repeated with threshold value calculations, which lead us to partition the data set into a number of classes or fuzzy sets.
7. Then on the basis of the shape, membership function is determined.

Thus the membership function is generated on the basis of the partitioning or analog screening concept. This draws a threshold line between two classes of sample data. The idea behind drawing the threshold line is to classify the samples when minimizing the entropy for optimum partitioning.

9.5 Summary

Membership functions and their features are discussed in this chapter. Also, the different methods of obtaining the membership functions are dealt with. The formation of the membership function is the core for the entire fuzzy system operation. The capability of human reasoning is very important for membership functions. The inference method is based on the geometrical shapes and geometry, whereas the angular fuzzy set is based on the angular features. Using neural networks and reasoning methods the memberships are tuned in a cyclic fashion and are based on rule structure. The improvements are carried out to achieve an optimum solution using genetic algorithms. Thus, the membership function can be formed using any one of the methods discussed.

9.6 Solved Problems

1. Using your own intuition and definitions of the universe of discourse, plot fuzzy membership functions for "weight of people."

Solution: The universe of discourse is weight of people. Let the weights be in kg, i.e., kilogram. Let the linguistic variables be the following:

Very thin (VT) : $W \leq 25$
Thin (T) : $25 < W \leq 45$
Average (AV) : $45 < W \leq 60$
Stout (S) : $60 < W \leq 75$
Very stout (VS) : $W > 75$

Now plotting the defined linguistic variables using triangular membership functions, we obtain Figure 1.

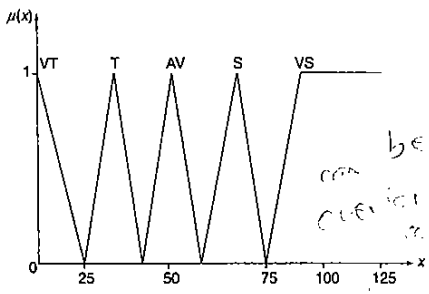

Figure 1 Membership function for weight of people.

2. Using your own intuition, plot the fuzzy membership function for the age of people.

Solution: The universe of discourse is age of people. Let "A" denote age of people in years. The linguistic variables are defined as follows:

Very young (VY) : $A < 12$
Young (Y) : $10 \leq A \leq 22$

Middle age (M) : $20 \le A \le 42$

Old (O) : $40 \le A \le 72$

Very old (VO) : $70 < A$

These variables are represented using triangular membership function in Figure 2.

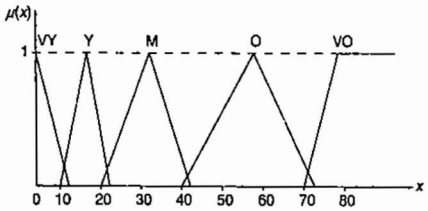

Figure 2 Membership function for age of people.

3. Compare "medium wave (MW)" and "short wave (SW)" receivers according to their frequency range. Plot the membership functions using intuition. The linguistic variables are defined based on the following:

   Medium wave receivers: frequency lesser than $\approx 10^6$ Hz

   Short wave receivers: frequency greater than $\approx 10^6$ Hz

Solution: Let the frequency range of receivers be universe of discourse. The linguistic variables are the following:

Medium wave receivers (MW): frequency lesser than $\approx 10^6$ Hz

Short wave receivers (SW): frequency greater than $\approx 10^6$ Hz

This is represented using Gaussian membership function in Figure 3.

4. Using the inference approach, find the membership values for the triangular shapes $\underset{\sim}{I}, \underset{\sim}{R}, \underset{\sim}{E}, \underset{\sim}{IR}$, and $\underset{\sim}{T}$ for a triangle with angles 45°, 55° and 80°.

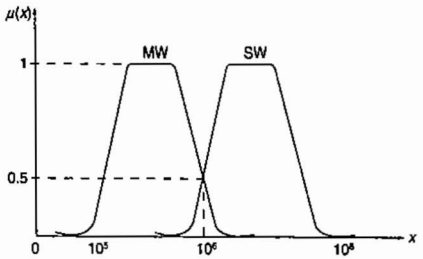

Figure 3 Membership function for frequency range of receivers.

Solution: Let the universe of discourse be

$$U = \{(X, Y, Z) : X = 80° \ge Y = 55° \ge Z = 45° \text{ and } X + Y + Z = 80° + 55° + 45° = 180°\}$$

- Membership value of isosceles triangle, $\underset{\sim}{I}$:

$$\mu_{\underset{\sim}{I}} = 1 - \frac{1}{60°}\min(X - Y, Y - Z)$$
$$= 1 - \frac{1}{60°}\min(80° - 55°, 55° - 45°)$$
$$= 1 - \frac{1}{60°}\min(25°, 10°)$$
$$= 1 - \frac{1}{60°} \times 10°$$
$$= 1 - 0.1667 = 0.833$$

- Membership value of right-angle triangle, $\underset{\sim}{R}$:

$$\mu_{\underset{\sim}{R}} = 1 - \frac{1}{90°}|X - 90°| = 1 - \frac{1}{90°}|80° - 90°|$$
$$= 1 - \frac{1}{90°} \times 10° = 0.889$$

- Membership value of equilateral triangle, $\underset{\sim}{E}$:

$$\mu_{\underset{\sim}{E}} = 1 - \frac{1}{180°}(X - Z) = 1 - \frac{1}{180°}(80° - 45°)$$
$$= 1 - \frac{1}{180°} \times 35° = 0.8056$$

- Membership value of isosceles and right-angle triangle, $\underset{\sim}{IR}$:

$$\mu_{\underset{\sim}{IR}} = \min[\mu_{\underset{\sim}{I}}, \mu_{\underset{\sim}{R}}] = \min[0.833, 0.889]$$
$$= 0.833$$

- Membership value of other triangles, $\underset{\sim}{T}$:

$$\mu_{\underset{\sim}{T}} = \min[1 - \mu_{\underset{\sim}{I}}, 1 - \mu_{\underset{\sim}{E}}, 1 - \mu_{\underset{\sim}{R}}]$$
$$= \min[0.167, 0.1944, 0.111] = 0.111$$

Thus the membership function is calculated for the triangular shapes.

5. Using the inference approach, obtain the membership values for the triangular shapes $(\underset{\sim}{I}, \underset{\sim}{R}, \underset{\sim}{T})$ for a triangle with angles 40°, 60° and 80°.

Solution: Let the universe of discourse be

$$U = \{(X, Y, Z) : X = 80° \ge Y = 60° \ge Z = 40° \text{ and } X + Y + Z = 80° + 60° + 40° = 180°\}$$

- Membership value of isosceles triangle, $\underset{\sim}{I}$:

$$\mu_{\underset{\sim}{I}} = 1 - \frac{1}{60°}\min(X - Y, Y - Z)$$
$$= 1 - \frac{1}{60°}\min(80° - 60°, 60° - 40°)$$
$$= 1 - \frac{1}{60°}\min(20°, 20°)$$
$$= 1 - \frac{1}{60°} \times 20° = 0.667$$

- Membership value of right-angle triangle, $\underset{\sim}{R}$:

$$\mu_{\underset{\sim}{R}} = 1 - \frac{1}{90°}|X - 90°| = 1 - \frac{1}{90°}|80° - 90°|$$
$$= 1 - \frac{1}{90°} \times 10° = 0.889$$

- Membership value of other triangles, $\underset{\sim}{T}$:

$$\mu_{\underset{\sim}{T}} = \min[1 - \mu_{\underset{\sim}{I}}, 1 - \mu_{\underset{\sim}{R}}]$$
$$= \min[1 - 0.667, 1 - 0.889]$$
$$= \min[0.333, 0.111] = 0.111$$

Thus the membership values for isosceles, right-angle triangle and other triangles are calculated.

6. The energy E of a particle spinning in a magnetic field B is given by the equation

$$E = \mu B \sin\theta$$

where μ is magnetic moment of spinning particle and θ is complement angle of magnetic moment

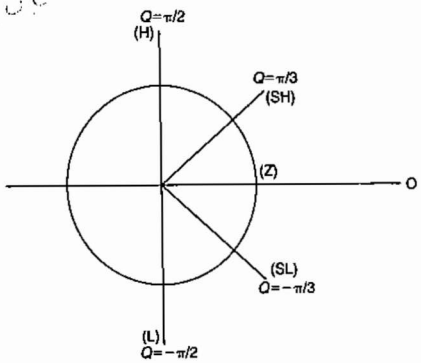

Figure 4 Angular fuzzy set.

with respect to the direction of the magnetic field.

Assume the magnetic field B and magnetic moment μ to be constant, and the linguistic terms for the complement angle of magnetic moment be given as

High moment (H): $\theta = \pi/2$

Slightly high moment (SH): $\theta = \pi/3$

No moment (Z): $\theta = 0$

Slightly low moment (SL): $\theta = -\pi/3$

Low moment (L): $\theta = -\pi/2$

Find the membership values using the angular fuzzy set approach for these linguistic labels and plot these values versus θ.

Solution: The angular fuzzy set is shown in Figure 4. Now calculate the angular fuzzy membership values as shown in the table below.

θ	$\tan\theta$	$z = \cos\theta$	$\mu = \lvert(z)\tan\theta\rvert$
$\pi/2$	∞	0	1
$\pi/3$	1.732	0.5	0.866
0	0	1	0
$-\pi/3$	−1.732	0.5	−0.866
$-\pi/2$	∞	0	1

The plot for the membership function shown in this table is given in Figure 5.

Table 1

	Number who preferred							
	Maruti 800	Scorpio	Matiz	Santro	Octavia	Total	Percentage	Rank order
Maruti 800	—	192	246	592	621	1651	16.5	5
Scorpio	403	—	621	540	391	1955	19.6	2
Matiz	235	336	—	797	492	1860	18.6	4
Santro	523	364	417	—	608	1912	19.1	3
Octavia	616	534	746	726	—	2622	26.2	1
Total						10000		

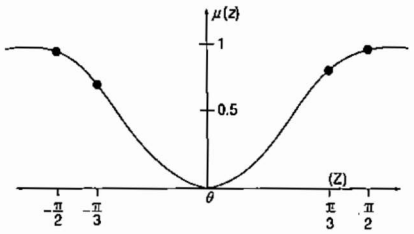

Figure 5 Plot of membership function.

7. Suppose 1000 people respond to a questionnaire about their pairwise preferences among five cars, X = {Maruti 800, Scorpio, Matiz, Santro, Octavia}. Define a fuzzy set $\underline{A}$ on the universe of cars "best car."

Solution: Table 1 shows the rank ordering for performance of cars is a summary of the opinion survey.

In Table 1, for example, out of 1000 people, 192 preferred Maruti 800 to the Scorpio, etc. The total number of responses is 10,000 (10 comparisons). On the basis of the preferences, the percentage is calculated. The ordering is then performed. It is found that Octavia is selected as the best car. Figure 6 shows the membership function for this example.

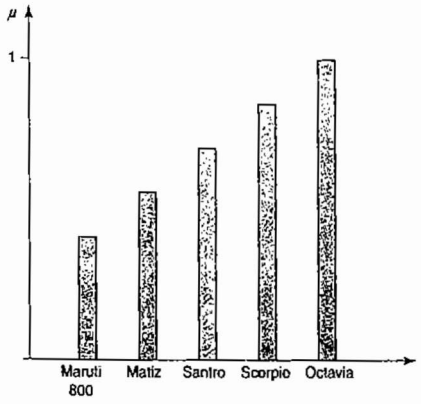

Figure 6 Membership function for best car.

9.7 Review Questions

1. Define membership function and state its importance in fuzzy logic.
2. Explain the features of membership functions.
3. Differentiate the following:
   - Convex and nonconvex fuzzy set.
   - Normal and subnormal fuzzy set.
4. What is meant by crossover point in a fuzzy set?
5. Define height of a fuzzy set.
6. Write short note on fuzzification.
7. List the various methods employed for the membership value assignment.
8. With suitable examples, explain how membership assignment is performed using intuition.
9. Define fuzzy number.
10. Explain in detail the inference method adopted for assigning membership values.
11. How is rank ordering used to define membership functions based on polling concept?
12. Discuss in detail the membership value assignments using angular fuzzy sets.
13. Describe how neural network is used to obtain fuzzy membership functions.
14. With suitable example, explain the method by which membership value assignments are performed using genetic algorithm.
15. Give details on membership value assignments using inductive reasoning.

9.8 Exercise Problems

1. Using intuition, assign the membership functions for (a) population of cars and (b) library usage.
2. Using your own intuition, develop fuzzy membership functions on the real line for the fuzzy number 5, using the following shapes:
   (a) Quadrilateral
   (b) Trapezoid
   (c) Gaussian function
   (d) Isosceles triangle
   (e) Symmetric triangle
3. Using intuition and your own definition of the universe of discourse, plot fuzzy membership functions to the following variables:
   (i) Liquid level in the tank
      (a) Very small
      (b) Small
      (c) Empty
      (d) Full
      (e) Very full
   (ii) Race of people
      (a) White
      (b) Moderate
      (c) Black
   (iii) Height of people
      (a) Very tall
      (b) Tall
      (c) Normal
      (d) Short
      (e) Very short
4. Using inference approach outlined in this chapter, find the membership values for each of the triangular shapes ($\underline{L}, \underline{R}, \underline{E}, \underline{IR}, \underline{T}$) for each of the following (all in degrees):
   (a) 20°, 40°, 120°
   (b) 90°, 45°, 45°
   (c) 35°, 75°, 70°
   (d) 10°, 60°, 110°
   (e) 50°, 75°, 55°