Fuzzy Logic

Unit 5

Solved Example

1. Find the power set and cardinality of the given set $X = \{2, 4, 6\}$. Also find cardinality of power set.

Solution: Since set X contains three elements, so its cardinal number is

$$n_X = 3$$

The power set of X is given by

$$P(X) = \{\phi, \{2\}, \{4\}, \{6\}, \{2, 4\}, \{4, 6\}, \{2, 6\}, \{2, 4, 6\}\}$$

The cardinality of power set $P(X)$, denoted by $n_{P(X)}$, is found as

$$n_{P(X)} = 2^{n_X} = 2^3 = 8$$

2. Consider two given fuzzy sets

$$\underset{\sim}{A} = \left\{\frac{1}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0.2}{8}\right\}$$

$$\underset{\sim}{B} = \left\{\frac{0.5}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{1}{8}\right\}$$

Perform union, intersection, difference and complement over fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$.

Solution: For the given fuzzy sets we have the following

(a) Union

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\} = \left\{\frac{1}{2} + \frac{0.4}{4} + \frac{0.5}{6} + \frac{1}{8}\right\}$$

(b) Intersection

$$\underset{\sim}{A} \cap \underset{\sim}{B} = \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\} = \left\{\frac{0.5}{2} + \frac{0.3}{4} + \frac{0.1}{6} + \frac{0.2}{8}\right\}$$

(c) Complement

$$\overline{\underset{\sim}{A}} = 1 - \mu_{\underset{\sim}{A}}(x) = \left\{\frac{0}{2} + \frac{0.7}{4} + \frac{0.5}{6} + \frac{0.8}{8}\right\}$$

$$\overline{\underset{\sim}{B}} = 1 - \mu_{\underset{\sim}{B}}(x) = \left\{\frac{0.5}{2} + \frac{0.6}{4} + \frac{0.9}{6} + \frac{0}{8}\right\}$$

(d) Difference

$$\underset{\sim}{A} | \underset{\sim}{B} = \underset{\sim}{A} \cap \overline{\underset{\sim}{B}} = \left\{\frac{0.5}{2} + \frac{0.3}{4} + \frac{0.5}{6} + \frac{0}{8}\right\}$$

$$\underset{\sim}{B} | \underset{\sim}{A} = \underset{\sim}{B} \cap \overline{\underset{\sim}{A}} = \left\{\frac{0}{2} + \frac{0.4}{4} + \frac{0.1}{6} + \frac{0.8}{8}\right\}$$

3. Given the two fuzzy sets

$$\tilde{B}_1 = \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

$$\tilde{B}_2 = \left\{ \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0} \right\}$$

find the following:

(a) $\tilde{B}_1 \cup \tilde{B}_2$; (b) $\tilde{B}_1 \cap \tilde{B}_2$; (c) $\overline{\tilde{B}_1}$;
(d) $\overline{\tilde{B}_2}$; (e) $\tilde{B}_1 | \tilde{B}_2$; (f) $\overline{\tilde{B}_1 \cup \tilde{B}_2}$;
(g) $\overline{\tilde{B}_1 \cap \tilde{B}_2}$; (h) $\tilde{B}_1 \cap \overline{\tilde{B}_1}$; (i) $\tilde{B}_1 \cup \overline{\tilde{B}_1}$;
(j) $\tilde{B}_2 \cap \overline{\tilde{B}_2}$; (k) $\tilde{B}_2 \cup \overline{\tilde{B}_2}$

Solution: For the given fuzzy sets, we have the following:

(a) $$\tilde{B}_1 \cup \tilde{B}_2 = \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

(b) $$\tilde{B}_1 \cap \tilde{B}_2 = \left\{ \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0} \right\}$$

(c) $$\overline{\tilde{B}_1} = \left\{ \frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0} \right\}$$

(d) $$\overline{\tilde{B}_2} = \left\{ \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1}{3.0} \right\}$$

(e) $$\tilde{B}_1 | \tilde{B}_2 = \tilde{B}_1 \cap \overline{\tilde{B}_2} = \left\{ \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

(f) $$\overline{\tilde{B}_1 \cup \tilde{B}_2} = \left\{ \frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0} \right\}$$

(g) $$\overline{\tilde{B}_1 \cap \tilde{B}_2} = \left\{ \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1}{3.0} \right\}$$

(h) $$\tilde{B}_1 \cap \overline{\tilde{B}_1} = \left\{ \frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

(i) $$\tilde{B}_1 \cup \overline{\tilde{B}_1} = \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0} \right\}$$

(j) $$\tilde{B}_2 \cap \overline{\tilde{B}_2} = \left\{ \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0} \right\}$$

(k) $$\tilde{B}_2 \cup \overline{\tilde{B}_2} = \left\{ \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.8}{2.0} + \frac{0.9}{2.5} + \frac{1}{3.0} \right\}$$

4. It is necessary to compare two sensors based upon their detection levels and gain settings. The table of gain settings and sensor detection levels with a standard item being monitored providing typical membership values to represent the detection levels for each sensor is given in Table 1.

Table 1

Gain setting	Detection level of sensor 1	Detection level of sensor 2
0	0	0
10	0.2	0.35
20	0.35	0.25
30	0.65	0.8
40	0.85	0.95
50	1	1

Now given the universe of discourse $X = \{0, 10, 20, 30, 40, 50\}$ and the membership functions for the two sensors in discrete form as

$$\underset{\sim}{D}_1 = \left\{\frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50}\right\}$$

$$\underset{\sim}{D}_2 = \left\{\frac{0}{0} + \frac{0.35}{10} + \frac{0.25}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50}\right\}$$

find the following membership functions:

(a) $\mu_{\underset{\sim}{D}_1 \cup \underset{\sim}{D}_2}(x)$; (b) $\mu_{\underset{\sim}{D}_1 \cap \underset{\sim}{D}_2}(x)$; (c) $\mu_{\overline{\underset{\sim}{D}_1}}(x)$;
(d) $\mu_{\overline{\underset{\sim}{D}_2}}(x)$; (e) $\mu_{\underset{\sim}{D}_1 \cup \overline{\underset{\sim}{D}_1}}(x)$; (f) $\mu_{\underset{\sim}{D}_1 \cap \overline{\underset{\sim}{D}_1}}(x)$;
(g) $\mu_{\underset{\sim}{D}_2 \cup \overline{\underset{\sim}{D}_2}}$; (h) $\mu_{\underset{\sim}{D}_2 \cap \overline{\underset{\sim}{D}_2}}(x)$; (i) $\mu_{\underset{\sim}{D}_1 | \underset{\sim}{D}_2}(x)$;
(j) $\mu_{\underset{\sim}{D}_2 | \underset{\sim}{D}_1}(x)$

Solution: For the given fuzzy sets we have

(a) $\mu_{\underset{\sim}{D}_1 \cup \underset{\sim}{D}_2}(x)$

$$= \max\{\mu_{\underset{\sim}{D}_1}(x), \mu_{\underset{\sim}{D}_2}(x)\} = \left\{\frac{0}{0} + \frac{0.35}{10} + \frac{0.35}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50}\right\}$$

(b) $\mu_{\underset{\sim}{D}_1 \cap \underset{\sim}{D}_2}(x)$

$$= \min\{\mu_{\underset{\sim}{D}_1}(x), \mu_{\underset{\sim}{D}_2}(x)\} = \left\{\frac{0}{0} + \frac{0.2}{10} + \frac{0.25}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50}\right\}$$

(c) $\mu_{\overline{\underset{\sim}{D}_1}}(x)$

$$= 1 - \mu_{\underset{\sim}{D}_1}(x) = \left\{\frac{1}{0} + \frac{0.8}{10} + \frac{0.65}{20} + \frac{0.35}{30} + \frac{0.15}{40} + \frac{0}{50}\right\}$$

(d) $\mu_{\overline{\underset{\sim}{D}_2}}(x)$

$$= 1 - \mu_{\underset{\sim}{D}_2}(x) = \left\{\frac{1}{0} + \frac{0.65}{10} + \frac{0.75}{20} + \frac{0.2}{30} + \frac{0.05}{40} + \frac{0}{50}\right\}$$

(e) $\mu_{\underset{\sim}{D}_1 \cup \overline{\underset{\sim}{D}_1}}(x)$

$$= \max\{\mu_{\underset{\sim}{D}_1}(x), \mu_{\overline{\underset{\sim}{D}_1}}(x)\} = \left\{\frac{1}{0} + \frac{0.8}{10} + \frac{0.65}{20} + \frac{0.65}{30} + \frac{0.85}{40} + \frac{1}{50}\right\}$$

(f) $\mu_{\underset{\sim}{D}_1 \cap \overline{\underset{\sim}{D}_1}}(x)$

$$= \min\{\mu_{\underset{\sim}{D}_1}(x), \mu_{\overline{\underset{\sim}{D}_1}}(x)\} = \left\{\frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.35}{30} + \frac{0.15}{40} + \frac{0}{50}\right\}$$

(g) $\mu_{\underset{\sim}{D}_2 \cup \overline{\underset{\sim}{D}_2}}(x)$

$$= \max\{\mu_{\underset{\sim}{D}_2}(x), \mu_{\overline{\underset{\sim}{D}_2}}(x)\} = \left\{\frac{1}{0} + \frac{0.65}{10} + \frac{0.75}{20} + \frac{0.8}{30} + \frac{0.95}{40} + \frac{1}{50}\right\}$$

(h) $\mu_{\underset{\sim}{D}_2 \cap \overline{\underset{\sim}{D}_2}}(x)$

$$= \min\{\mu_{\underset{\sim}{D}_2}(x), \mu_{\overline{\underset{\sim}{D}_2}}(x)\} = \left\{\frac{0}{0} + \frac{0.35}{10} + \frac{0.25}{20} + \frac{0.2}{30} + \frac{0.05}{40} + \frac{0}{50}\right\}$$

(i) $\mu_{\underset{\sim}{D}_1 | \underset{\sim}{D}_2}(x)$

$$= \mu_{\underset{\sim}{D}_1 \cap \overline{\underset{\sim}{D}_2}}(x) = \min\{\mu_{\underset{\sim}{D}_1}(x), \mu_{\overline{\underset{\sim}{D}_2}}(x)\} = \left\{\frac{0}{0} + \frac{0.2}{10} + \frac{0.35}{20} + \frac{0.2}{30} + \frac{0.05}{40} + \frac{0}{50}\right\}$$

(j) $\mu_{\underset{\sim}{D}_2 | \underset{\sim}{D}_1}(x)$

$$= \mu_{\underset{\sim}{D}_2 \cap \overline{\underset{\sim}{D}_1}}(x) = \min\{\mu_{\underset{\sim}{D}_2}(x), \mu_{\overline{\underset{\sim}{D}_1}}(x)\} = \left\{\frac{0}{0} + \frac{0.35}{10} + \frac{0.25}{20} + \frac{0.35}{30} + \frac{0.15}{40} + \frac{0}{50}\right\}$$

5. Design a computer software to perform image processing to locate objects within a scene. The two fuzzy sets representing a plane and a train image are:

$$\text{Plane} = \left\{\frac{0.2}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.3}{\text{boat}} + \frac{0.8}{\text{plane}} + \frac{0.1}{\text{hou}}\right\}$$

$$\text{Train} = \left\{\frac{1}{\text{train}} + \frac{0.2}{\text{bike}} + \frac{0.4}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.2}{\text{house}}\right\}$$

Find the following:

(a) Plane $\cup$ Train; (b) Plane $\cap$ Train;
(c) $\overline{\text{Plane}}$; (d) $\overline{\text{Train}}$;
(e) Plane|Train; (f) $\overline{\text{Plane} \cup \text{Train}}$
(g) $\overline{\text{Plane} \cap \text{Train}}$; (h) Plane $\cup$ $\overline{\text{Plan}}$
(i) Plane $\cap$ $\overline{\text{Plane}}$; (j) Train $\cup$ $\overline{\text{Train}}$
(k) Train $\cup$ $\overline{\text{Train}}$

Solution: For the given fuzzy sets we have the following:

(a) Plane $\cup$ Train

$$= \max\{\mu_{\text{Plane}}(x), \mu_{\text{Train}}(x)\}$$
$$= \left\{\frac{1.0}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.4}{\text{boat}} + \frac{0.8}{\text{plane}} + \frac{0.2}{\text{house}}\right\}$$

(b) Plane $\cap$ Train

$$= \min\{\mu_{\text{Plane}}(x), \mu_{\text{Train}}(x)\}$$
$$= \left\{\frac{0.2}{\text{train}} + \frac{0.2}{\text{bike}} + \frac{0.3}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.1}{\text{house}}\right\}$$

(c) $\overline{\text{Plane}} = 1 - \mu_{\text{Plane}}(x)$
$$= \left\{\frac{0.8}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.7}{\text{boat}} + \frac{0.2}{\text{plane}} + \frac{0.9}{\text{house}}\right\}$$

(d) $\overline{\text{Train}} = 1 - \mu_{\text{Train}}(x)$
$$= \left\{\frac{0}{\text{train}} + \frac{0.8}{\text{bike}} + \frac{0.6}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.8}{\text{house}}\right\}$$

(e) Plane|Train

$$= \text{Plane} \cap \overline{\text{Train}}$$
$$= \min\{\mu_{\text{Plane}}(x), \mu_{\overline{\text{Train}}}(x)\}$$
$$= \left\{\frac{0}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.3}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.1}{\text{house}}\right\}$$

(f) $\overline{\text{Plane} \cup \text{Train}}$

$$= 1 - \max\{\mu_{\text{Plane}}(x), \mu_{\text{Train}}(x)\}$$
$$= \left\{\frac{0}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.6}{\text{boat}} + \frac{0.2}{\text{plane}} + \frac{0.8}{\text{house}}\right\}$$

(g) $\overline{\text{Plane} \cap \text{Train}}$

$$= 1 - \min\{\mu_{\text{Plane}}(x), \mu_{\text{Train}}(x)\}$$
$$= \left\{\frac{0.8}{\text{train}} + \frac{0.8}{\text{bike}} + \frac{0.7}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.9}{\text{house}}\right\}$$

(h) Plane $\cup$ $\overline{\text{Plane}}$

$$= \max\{\mu_{\text{Plane}}(x), \mu_{\overline{\text{Plane}}}(x)\}$$
$$= \left\{\frac{0.8}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.7}{\text{boat}} + \frac{0.8}{\text{plane}} + \frac{0.9}{\text{house}}\right\}$$

(i) Plane $\cap$ $\overline{\text{Plane}}$

$$= \min\{\mu_{\text{Plane}}(x), \mu_{\overline{\text{Plane}}}(x)\}$$
$$= \left\{\frac{0.2}{\text{train}} + \frac{0.5}{\text{bike}} + \frac{0.3}{\text{boat}} + \frac{0.2}{\text{plane}} + \frac{0.1}{\text{house}}\right\}$$

(j) Train $\cup$ $\overline{\text{Train}}$

$$= \max\{\mu_{\text{Train}}(x), \mu_{\overline{\text{Train}}}(x)\}$$
$$= \left\{\frac{1.0}{\text{train}} + \frac{0.8}{\text{bike}} + \frac{0.6}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.8}{\text{house}}\right\}$$

(k) Train $\cap$ $\overline{\text{Train}}$

$$= \min\{\mu_{\text{Train}}(x), \mu_{\overline{\text{Train}}}(x)\}$$
$$= \left\{\frac{0}{\text{train}} + \frac{0.2}{\text{bike}} + \frac{0.4}{\text{boat}} + \frac{0.5}{\text{plane}} + \frac{0.2}{\text{house}}\right\}$$

6. For aircraft simulator data the determination of certain changes in its operating conditions is made on the basis of hard break points in the mach region. We define two fuzzy sets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ representing the condition of "near" a mach number of 0.65 and "in the region" of a mach number of 0.65, respectively, as follows

$\underset{\sim}{A}$ = near mach 0.65

$$= \left\{\frac{0}{0.64} + \frac{0.75}{0.645} + \frac{1}{0.65} + \frac{0.5}{0.655} + \frac{0}{0.66}\right\}$$

$\underset{\sim}{B}$ = in the region of mach 0.65

$$= \left\{\frac{0}{0.64} + \frac{0.25}{0.645} + \frac{0.75}{0.65} + \frac{1}{0.655} + \frac{0.5}{0.66}\right\}$$

For these two sets find the following:

(a) $\underset{\sim}{A} \cup \underset{\sim}{B}$; (b) $\underset{\sim}{A} \cap \underset{\sim}{B}$; (c) $\overline{\underset{\sim}{A}}$;

(d) $\overline{\underset{\sim}{B}}$; (e) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}}$; (f) $\overline{\underset{\sim}{A} \cap \underset{\sim}{B}}$

Solution: For the two given fuzzy sets we have the following:

(a) $\underset{\sim}{A} \cup \underset{\sim}{B}$

$$= \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$$

$$= \left\{\frac{0}{0.64} + \frac{0.75}{0.645} + \frac{1}{0.65} + \frac{1}{0.655} + \frac{0.5}{0.66}\right\}$$

(b) $\underset{\sim}{A} \cap \underset{\sim}{B}$

$$= \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$$

$$= \left\{\frac{0}{0.64} + \frac{0.25}{0.645} + \frac{0.75}{0.65} + \frac{0.5}{0.655} + \frac{0}{0.66}\right\}$$

(c) $\overline{\underset{\sim}{A}} = 1 - \mu_{\underset{\sim}{A}}(x)$

$$= \left\{\frac{1}{0.64} + \frac{0.25}{0.645} + \frac{0}{0.65} + \frac{0.5}{0.655} + \frac{1}{0.66}\right\}$$

(d) $\overline{\underset{\sim}{B}} = 1 - \mu_{\underset{\sim}{B}}(x)$

$$= \left\{\frac{1}{0.64} + \frac{0.75}{0.645} + \frac{0.25}{0.65} + \frac{0}{0.655} + \frac{0.5}{0.66}\right\}$$

(e) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}}$

$$= 1 - \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$$

$$= \left\{\frac{1}{0.64} + \frac{0.25}{0.645} + \frac{0}{0.65} + \frac{0}{0.655} + \frac{0.5}{0.66}\right\}$$

(f) $\overline{\underset{\sim}{A} \cap \underset{\sim}{B}}$

$$= 1 - \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$$

$$= \left\{\frac{1}{0.64} + \frac{0.75}{0.645} + \frac{0.25}{0.65} + \frac{0.5}{0.655} + \frac{1}{0.66}\right\}$$

7. For the two given fuzzy sets

$$\underset{\sim}{A} = \left\{ \frac{0.1}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{1}{4} \right\}$$

$$\underset{\sim}{B} = \left\{ \frac{1}{0} + \frac{0.5}{1} + \frac{0.7}{2} + \frac{0.3}{3} + \frac{0}{4} \right\}$$

find the following:

(a) $\underset{\sim}{A} \cup \underset{\sim}{B}$; (b) $\underset{\sim}{A} \cap \underset{\sim}{B}$; (c) $\overline{\underset{\sim}{A}}$;
(d) $\overline{\underset{\sim}{B}}$; (e) $\underset{\sim}{A} \cup \overline{\underset{\sim}{A}}$; (f) $\underset{\sim}{A} \cap \overline{\underset{\sim}{A}}$;
(g) $\underset{\sim}{B} \cup \overline{\underset{\sim}{B}}$; (h) $\underset{\sim}{B} \cap \overline{\underset{\sim}{B}}$; (i) $\underset{\sim}{A} \cap \overline{\underset{\sim}{B}}$;
(j) $\underset{\sim}{A} \cup \overline{\underset{\sim}{B}}$; (k) $\underset{\sim}{B} \cap \overline{\underset{\sim}{A}}$; (l) $\underset{\sim}{B} \cup \overline{\underset{\sim}{A}}$;
(m) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}}$; (n) $\overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{B}}$

Solution: For the given sets we have:

(a) $\underset{\sim}{A} \cup \underset{\sim}{B} = \max\{\mu_{A}(x), \mu_{B}(x)\}$

$$= \left\{ \frac{1}{0} + \frac{0.5}{1} + \frac{0.7}{2} + \frac{0.6}{3} + \frac{1}{4} \right\}$$

(b) $\underset{\sim}{A} \cap \underset{\sim}{B} = \min\{\mu_{A}(x), \mu_{B}(x)\}$

$$= \left\{ \frac{0.1}{0} + \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.3}{3} + \frac{0}{4} \right\}$$

(c) $\overline{\underset{\sim}{A}} = 1 - \mu_{A}(x)$

$$= \left\{ \frac{0.9}{0} + \frac{0.8}{1} + \frac{0.6}{2} + \frac{0.4}{3} + \frac{0}{4} \right\}$$

(d) $\overline{\underset{\sim}{B}} = 1 - \mu_{B}(x)$

$$= \left\{ \frac{0}{0} + \frac{0.5}{1} + \frac{0.3}{2} + \frac{0.7}{3} + \frac{1}{4} \right\}$$

(e) $\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} = \max\{\mu_{A}(x), \mu_{\overline{A}}(x)\}$

$$= \left\{ \frac{0.9}{0} + \frac{0.8}{1} + \frac{0.6}{2} + \frac{0.6}{3} + \frac{1}{4} \right\}$$

(f) $\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} = \min\{\mu_{A}(x), \mu_{\overline{A}}(x)\}$

(g) $\underset{\sim}{B} \cup \overline{\underset{\sim}{B}} = \max\{\mu_{B}(x), \mu_{\overline{B}}(x)\}$

$$= \left\{ \frac{1}{0} + \frac{0.5}{1} + \frac{0.7}{2} + \frac{0.7}{3} + \frac{1}{4} \right\}$$

(h) $\underset{\sim}{B} \cap \overline{\underset{\sim}{B}} = \min\{\mu_{B}(x), \mu_{\overline{B}}(x)\}$

$$= \left\{ \frac{0}{0} + \frac{0.5}{1} + \frac{0.3}{2} + \frac{0.3}{3} + \frac{0}{4} \right\}$$

(i) $\underset{\sim}{A} \cap \overline{\underset{\sim}{B}} = \min\{\mu_{A}(x), \mu_{\overline{B}}(x)\}$

$$= \left\{ \frac{0}{0} + \frac{0.2}{1} + \frac{0.3}{2} + \frac{0.6}{3} + \frac{1}{4} \right\}$$

(j) $\underset{\sim}{A} \cup \overline{\underset{\sim}{B}} = \max\{\mu_{A}(x), \mu_{\overline{B}}(x)\}$

$$= \left\{ \frac{0.1}{0} + \frac{0.5}{1} + \frac{0.4}{2} + \frac{0.7}{3} + \frac{1}{4} \right\}$$

(k) $\underset{\sim}{B} \cap \overline{\underset{\sim}{A}} = \min\{\mu_{B}(x), \mu_{\overline{A}}(x)\}$

$$= \left\{ \frac{0.9}{0} + \frac{0.5}{1} + \frac{0.6}{2} + \frac{0.3}{3} + \frac{0}{4} \right\}$$

(l) $\underset{\sim}{B} \cup \overline{\underset{\sim}{A}} = \max\{\mu_{B}(x), \mu_{\overline{A}}(x)\}$

$$= \left\{ \frac{1}{0} + \frac{0.8}{1} + \frac{0.7}{2} + \frac{0.4}{3} + \frac{0}{4} \right\}$$

(m) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}} = 1 - \max\{\mu_{A}(x), \mu_{B}(x)\}$

$$= \left\{ \frac{0}{0} + \frac{0.5}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0}{4} \right\}$$

(n) $\overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{B}} = \min\{\mu_{\overline{A}}(x), \mu_{\overline{B}}(x)\}$

$$= \left\{ \frac{0}{0} + \frac{0.5}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0}{4} \right\}$$

8. Let U be the universe of military aircraft of interest' as defined below:

$$U = \{a10,\ b52,\ c130,\ f2,\ f9\}$$

Let $\underset{\sim}{A}$ be the fuzzy set of bomber class aircraft:

$$\underset{\sim}{A} = \left\{\frac{0.3}{a10} + \frac{0.4}{b52} + \frac{0.2}{c130} + \frac{0.1}{f2} + \frac{1}{f9}\right\}$$

Let $\underset{\sim}{B}$ be the fuzzy set of fighter class aircraft:

$$\underset{\sim}{B} = \left\{\frac{0.1}{a10} + \frac{0.2}{b52} + \frac{0.8}{c130} + \frac{0.7}{f2} + \frac{0}{f9}\right\}$$

Find the following:

(a) $\underset{\sim}{A} \cup \underset{\sim}{B}$; (b) $\underset{\sim}{A} \cap \underset{\sim}{B}$; (c) $\overline{\underset{\sim}{A}}$; (d) $\overline{\underset{\sim}{B}}$;
(e) $\underset{\sim}{A} | \underset{\sim}{B}$; (f) $\underset{\sim}{B} | \underset{\sim}{A}$; (g) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}}$;
(h) $\overline{\underset{\sim}{A} \cap \underset{\sim}{B}}$; (i) $\overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}}$; (j) $\overline{\underset{\sim}{B}} \cup \underset{\sim}{A}$

Solution: We have

(a) $\underset{\sim}{A} \cup \underset{\sim}{B} = \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0.3}{a10} + \frac{0.4}{b52} + \frac{0.8}{c130} + \frac{0.7}{f2} + \frac{1}{f9}\right\}$$

(b) $\underset{\sim}{A} \cap \underset{\sim}{B} = \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0.1}{a10} + \frac{0.2}{b52} + \frac{0.2}{c130} + \frac{0.1}{f2} + \frac{0}{f9}\right\}$$

(c) $\overline{\underset{\sim}{A}} = 1 - \mu_{\underset{\sim}{A}}(x)$

$$= \left\{\frac{0.7}{a10} + \frac{0.6}{b52} + \frac{0.8}{c130} + \frac{0.9}{f2} + \frac{0}{f9}\right\}$$

(d) $\overline{\underset{\sim}{B}} = 1 - \mu_{\underset{\sim}{B}}(x)$

$$= \left\{\frac{0.9}{a10} + \frac{0.8}{b52} + \frac{0.2}{c130} + \frac{0.3}{f2} + \frac{1}{f9}\right\}$$

(e) $\underset{\sim}{A} | \underset{\sim}{B} = \underset{\sim}{A} \cap \overline{\underset{\sim}{B}} = \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\overline{\underset{\sim}{B}}}(x)\}$

$$= \left\{\frac{0.3}{a10} + \frac{0.4}{b52} + \frac{0.2}{c130} + \frac{0.1}{f2} + \frac{1}{f9}\right\}$$

(f) $\underset{\sim}{B} | \underset{\sim}{A} = \underset{\sim}{B} \cap \overline{\underset{\sim}{A}} = \min\{\mu_{\underset{\sim}{B}}(x), \mu_{\overline{\underset{\sim}{A}}}(x)\}$

$$= \left\{\frac{0.1}{a10} + \frac{0.2}{b52} + \frac{0.8}{c130} + \frac{0.7}{f2} + \frac{0}{f9}\right\}$$

(g) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}} = 1 - \max\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0.7}{a10} + \frac{0.6}{b52} + \frac{0.2}{c130} + \frac{0.3}{f2} + \frac{0}{f9}\right\}$$

(h) $\overline{\underset{\sim}{A} \cap \underset{\sim}{B}} = 1 - \min\{\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{B}}(x)\}$

$$= \left\{\frac{0.9}{a10} + \frac{0.8}{b52} + \frac{0.8}{c130} + \frac{0.9}{f2} + \frac{1}{f9}\right\}$$

(i) $\overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}} = \max\{\mu_{\overline{\underset{\sim}{A}}}(x), \mu_{\overline{\underset{\sim}{B}}}(x)\}$

$$= \left\{\frac{0.9}{a10} + \frac{0.8}{b52} + \frac{0.8}{c130} + \frac{0.9}{f2} + \frac{1}{f9}\right\}$$

(j) $\overline{\underset{\sim}{B}} \cup \underset{\sim}{A} = \max\{\mu_{\overline{\underset{\sim}{B}}}(x), \mu_{\underset{\sim}{A}}(x)\}$

$$= \left\{\frac{0.9}{a10} + \frac{0.8}{b52} + \frac{0.2}{c130} + \frac{0.3}{f2} + \frac{1}{f9}\right\}$$

9. Consider two fuzzy sets

$$\underset{\sim}{A} = \left\{\frac{0.2}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0.5}{4}\right\}$$

$$\underset{\sim}{B} = \left\{\frac{0.1}{1} + \frac{0.2}{2} + \frac{0.2}{3} + \frac{1}{4}\right\}$$

Find the algebraic sum, algebraic product, bounded sum and bounded difference of the given fuzzy sets.

Solution: We have

(a) Algebraic sum

$$\mu_{\underset{\sim}{A}+\underset{\sim}{B}}(x) = [\mu_{\underset{\sim}{A}}(x)+\mu_{\underset{\sim}{B}}(x)] - [\mu_{\underset{\sim}{A}}(x)\cdot\mu_{\underset{\sim}{B}}(x)]$$

$$= \left\{\frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.5}{4}\right\}$$

$$- \left\{\frac{0.02}{1} + \frac{0.06}{2} + \frac{0.08}{3} + \frac{0.5}{4}\right\}$$

$$= \left\{\frac{0.28}{1} + \frac{0.44}{2} + \frac{0.52}{3} + \frac{0}{4}\right\}$$

(b) Algebraic product

$$\mu_{\underset{\sim}{A}\cdot\underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x)\cdot\mu_{\underset{\sim}{B}}(x)$$

$$= \left\{\frac{0.02}{1} + \frac{0.06}{2} + \frac{0.08}{3} + \frac{0.5}{4}\right\}$$

(c) Bounded sum

$$\mu_{\underset{\sim}{A}\oplus\underset{\sim}{B}}(x)$$

$$= \min[1, \mu_{\underset{\sim}{A}}(x)+\mu_{\underset{\sim}{B}}(x)]$$

$$= \min\left\{1, \left\{\frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.5}{4}\right\}\right\}$$

$$= \left\{\frac{0.3}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.5}{4}\right\}$$

(d) Bounded difference

$$\mu_{\underset{\sim}{A}\odot\underset{\sim}{B}}(x)$$

$$= \max[0, \mu_{\underset{\sim}{A}}(x)-\mu_{\underset{\sim}{B}}(x)]$$

$$= \max\left\{0, \left\{\frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} + \frac{0.5}{4}\right\}\right\}$$

$$= \left\{\frac{0.1}{1} + \frac{0.1}{2} + \frac{0.2}{3} + \frac{0.5}{4}\right\}$$

10. The discretized membership functions for a transistor and a resistor are given below:

$$\mu_{\tilde{T}} = \left\{\frac{0}{0} + \frac{0.2}{1} + \frac{0.7}{2} + \frac{0.8}{3} + \frac{0.9}{4} + \frac{1}{5}\right\}$$

$$\mu_{\tilde{g}} = \left\{\frac{0}{0} + \frac{0.1}{1} + \frac{0.3}{2} + \frac{0.2}{3} + \frac{0.4}{4} + \frac{0.5}{5}\right\}$$

Find the following: (a) Algebraic sum; (b) algebraic product; (c) bounded sum; (d) bounded difference.

Solution: We have

(a) Algebraic sum

$$\begin{aligned}
&\mu_{\tilde{T}+\tilde{g}}(x) \\
&= [\mu_{\tilde{T}}(x)+\mu_{\tilde{g}}(x)] - [\mu_{\tilde{T}}(x)\cdot\mu_{\tilde{g}}(x)] \\
&= \left\{\frac{0}{0} + \frac{0.3}{1} + \frac{1.0}{2} + \frac{1.0}{3} + \frac{1.3}{4} + \frac{1.5}{5}\right\} \\
&\quad - \left\{\frac{0}{0} + \frac{0.02}{1} + \frac{0.21}{2} + \frac{0.16}{3} + \frac{0.36}{4} + \frac{0.5}{5}\right\} \\
&= \left\{\frac{0}{0} + \frac{0.28}{1} + \frac{0.79}{2} + \frac{0.84}{3} + \frac{0.94}{4} + \frac{1}{5}\right\}
\end{aligned}$$

(b) Algebraic product

$$\begin{aligned}
&\mu_{\tilde{T}\cdot\tilde{g}}(x) \\
&= \mu_{\tilde{T}}(x)\cdot\mu_{\tilde{g}}(x) \\
&= \left\{\frac{0}{0} + \frac{0.02}{1} + \frac{0.21}{2} + \frac{0.16}{3} + \frac{0.36}{4} + \frac{0.5}{5}\right\}
\end{aligned}$$

(c) Bounded sum

$$\begin{aligned}
&\mu_{\tilde{T}\oplus\tilde{g}}(x) \\
&= \min\{1, \mu_{\tilde{T}}(x)+\mu_{\tilde{g}}(x)\} \\
&= \min\left\{1, \left\{\frac{0}{0} + \frac{0.3}{1} + \frac{1.0}{2} + \frac{1.0}{3} + \frac{1.3}{4} + \frac{1.5}{5}\right\}\right\} \\
&= \left\{\frac{0}{0} + \frac{0.3}{1} + \frac{1.0}{2} + \frac{1.0}{3} + \frac{1.0}{4} + \frac{1.0}{5}\right\}
\end{aligned}$$

(d) Bounded difference

$$\begin{aligned}
&\mu_{\tilde{T}\ominus\tilde{g}}(x) \\
&= \max\{0, \mu_{\tilde{T}}(x)-\mu_{\tilde{g}}(x)\} \\
&= \max\left\{0, \left\{\frac{0}{0} + \frac{0.1}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.5}{4} + \frac{0.5}{5}\right\}\right\} \\
&= \left\{\frac{0}{0} + \frac{0.1}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.5}{4} + \frac{0.5}{5}\right\}
\end{aligned}$$

Membership Functions

1. Using your own intuition and definitions of the universe of discourse, plot fuzzy membership functions for "weight of people."

Solution: The universe of discourse is weight of people. Let the weights be in kg, i.e., kilogram. Let the linguistic variables be the following:

$$\text{Very thin (VT)} : W \leq 25$$
$$\text{Thin (T)} : 25 < W \leq 45$$
$$\text{Average (AV)} : 45 < W \leq 60$$
$$\text{Stout (S)} : 60 < W \leq 75$$
$$\text{Very stout (VS)} : W > 75$$

Now plotting the defined linguistic variables using triangular membership functions, we obtain Figure 1.

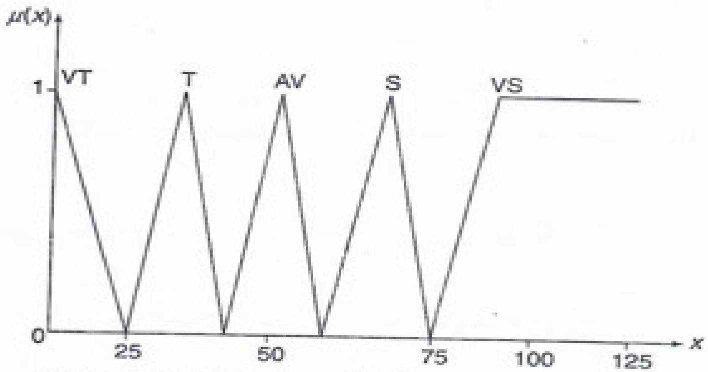

Figure 1 Membership function for weight of people.

2. Using your own intuition, plot the fuzzy membership function for the age of people.

Solution: The universe of discourse is age of people. Let "A" denote age of people in years. The linguistic variables are defined as follows:

$$\text{Very young (VY)} : A < 12$$
$$\text{Young (Y)} : 10 \le A \le 22$$

$$\text{Middle age (M)} : 20 \le A \le 42$$
$$\text{Old (O)} : 40 \le A \le 72$$
$$\text{Very old (VO)} : 70 < A$$

These variables are represented using triangular membership function in Figure 2.

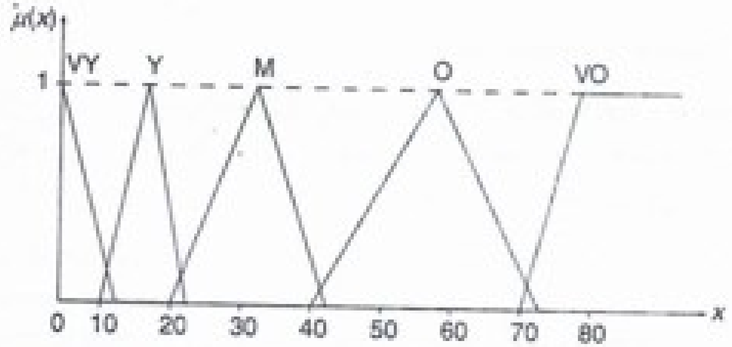

Figure 2 Membership function for age of people.

3. Compare "medium wave (MW)" and "short wave (SW)" receivers according to their frequency range. Plot the membership functions using intuition. The linguistic variables are defined based on the following:

Medium wave receivers: frequency lesser than $\approx 10^6$ Hz

Short wave receivers: frequency greater than $\approx 10^6$ Hz

Solution: Let the frequency range of receivers be universe of discourse. The linguistic variables are the following:

Medium wave receivers (MW): frequency lesser than $\approx 10^6$ Hz

Short wave receivers (SW): frequency greater than $\approx 10^6$ Hz

This is represented using Gaussian membership function in Figure 3.

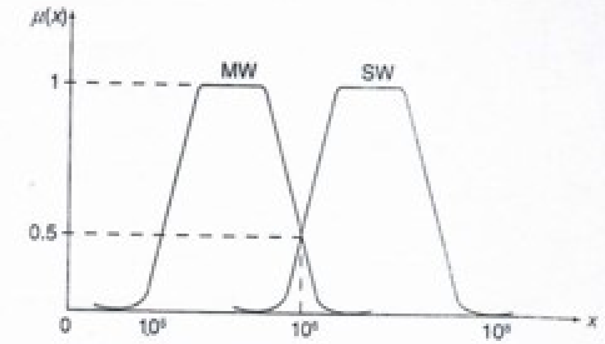

Figure 3 Membership function for frequency range of receivers.

4. Using the inference approach, find the membership values for the triangular shapes $\underset{\sim}{I}, \underset{\sim}{R}, \underset{\sim}{E}, \underset{\sim}{IR}$, and $\underset{\sim}{T}$ for a triangle with angles 45°, 55° and 80°.

Solution: Let the universe of discourse be

$$U = \{(X, Y, Z) : X = 80° \geq Y = 55° \geq Z = 45° \text{ and } X + Y + Z = 80° + 55° + 45° = 180°\}$$

- Membership value of isosceles triangle, $\underset{\sim}{I}$:

$$\begin{aligned}
\mu_{\underset{\sim}{I}} &= 1 - \frac{1}{60°}\min(X - Y, Y - Z) \\
&= 1 - \frac{1}{60°}\min(80° - 55°, 55° - 45°) \\
&= 1 - \frac{1}{60°}\min(25°, 10°) \\
&= 1 - \frac{1}{60°} \times 10° \\
&= 1 - 0.1667 = 0.833
\end{aligned}$$

- Membership value of right-angle triangle, $\underset{\sim}{R}$:

$$\begin{aligned}
\mu_{\underset{\sim}{R}} &= 1 - \frac{1}{90°}|X - 90°| = 1 - \frac{1}{90°}|80° - 90°| \\
&= 1 - \frac{1}{90°} \times 10° = 0.889
\end{aligned}$$

- Membership value of equilateral triangle, $\underset{\sim}{E}$:

$$\begin{aligned}
\mu_{\underset{\sim}{E}} &= 1 - \frac{1}{180°}(X - Z) = 1 - \frac{1}{180°}(80° - 45°) \\
&= 1 - \frac{1}{180°} \times 35° = 0.8056
\end{aligned}$$

- Membership value of isosceles and right-angle triangle, $\underset{\sim}{IR}$:

$$\begin{aligned}
\mu_{\underset{\sim}{IR}} &= \min[\mu_{\underset{\sim}{I}}, \mu_{\underset{\sim}{R}}] = \min[0.833, 0.889] \\
&= 0.833
\end{aligned}$$

- Membership value of other triangles, $\underset{\sim}{T}$:

$$\begin{aligned}
\mu_{\underset{\sim}{T}} &= \min[1 - \mu_{\underset{\sim}{I}}, 1 - \mu_{\underset{\sim}{E}}, 1 - \mu_{\underset{\sim}{R}}] \\
&= \min[0.167, 0.1944, 0.111] = 0.111
\end{aligned}$$

Thus the membership function is calculated for the triangular shapes.

5. Using the inference approach, obtain the membership values for the triangular shapes ($\underset{\sim}{I}, \underset{\sim}{R}, \underset{\sim}{T}$) for a triangle with angles 40°, 60° and 80°.

Solution: Let the universe of discourse be

$$U = \{(X, Y, Z) : X = 80^\circ \geq Y = 60^\circ \geq Z = 40^\circ \text{ and } X + Y + Z = 80^\circ + 60^\circ + 40^\circ = 180^\circ\}$$

- Membership value of isosceles triangle, $\underset{\sim}{I}$:

$$\begin{aligned}
\mu_{\underset{\sim}{I}} &= 1 - \frac{1}{60^\circ}\min(X - Y, Y - Z) \\
&= 1 - \frac{1}{60^\circ}\min(80^\circ - 60^\circ, 60^\circ - 40^\circ) \\
&= 1 - \frac{1}{60^\circ}\min(20^\circ, 20^\circ) \\
&= 1 - \frac{1}{60^\circ} \times 20^\circ = 0.667
\end{aligned}$$

- Membership value of right-angle triangle, $\underset{\sim}{R}$:

$$\begin{aligned}
\mu_{\underset{\sim}{R}} &= 1 - \frac{1}{90^\circ}|X - 90^\circ| = 1 - \frac{1}{90^\circ}|80^\circ - 90^\circ| \\
&= 1 - \frac{1}{90^\circ} \times 10^\circ = 0.889
\end{aligned}$$

- Membership value of other triangles, $\underset{\sim}{T}$:

$$\begin{aligned}
\mu_{\underset{\sim}{T}} &= \min[1 - \mu_{\underset{\sim}{I}}, 1 - \mu_{\underset{\sim}{R}}] \\
&= \min[1 - 0.667, 1 - 0.889] \\
&= \min[0.333, 0.111] = 0.111
\end{aligned}$$

Thus the membership values for isosceles, right-angle triangle and other triangles are calculated.

6. The energy E of a particle spinning in a magnetic field **B** is given by the equation

$$\mathbf{E} = \mu \mathbf{B} \sin \theta$$

where μ is magnetic moment of spinning particle and θ is complement angle of magnetic moment

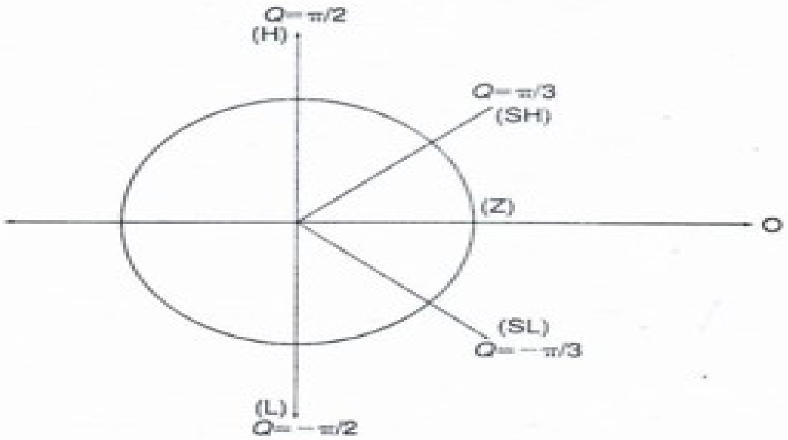

Figure 4 Angular fuzzy set.

with respect to the direction of the magnetic field.

Assume the magnetic field **B** and magnetic moment μ to be constant, and the linguistic terms for the complement angle of magnetic moment be given as

High moment (H): $\theta = \pi/2$

Slightly high moment (SH): $\theta = \pi/3$

No moment (Z): $\theta = 0$

Slightly low moment (SL): $\theta = -\pi/3$

Low moment (L): $\theta = -\pi/2$

Find the membership values using the angular fuzzy set approach for these linguistic labels and plot these values versus θ.

Solution: The angular fuzzy set is shown in Figure 4. Now calculate the angular fuzzy membership values as shown in the table below.

θ	$\tan\theta$	$z = \cos\theta$	$\mu = \|(z)\tan\theta\|$
$\pi/2$	∞	0	1
$\pi/3$	1.732	0.5	0.866
0	0	1	0
$-\pi/3$	-1.732	0.5	-0.866
$-\pi/2$	∞	0	1

The plot for the membership function shown in this table is given in Figure 5.

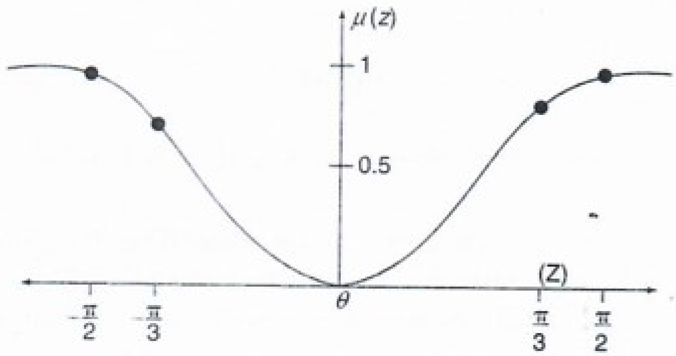

Figure 5 Plot of membership function.

7. Suppose 1000 people respond to a questionnaire about their pairwise preferences among five cars, X = {Maruti 800, Scorpio, Matiz, Santro, Octavia}. Define a fuzzy set $\underset{\sim}{A}$ on the universe of cars "best car."

Solution: Table 1 shows the rank ordering for performance of cars is a summary of the opinion survey.

In Table 1, for example, out of 1000 people, 192 preferred Maruti 800 to the Scorpio, etc. The total number of responses is 10,000 (10 comparisons). On the basis of the preferences, the percentage is calculated. The ordering is then performed. It is found that Octavia is selected as the best car. Figure 6 shows the membership function for this example.

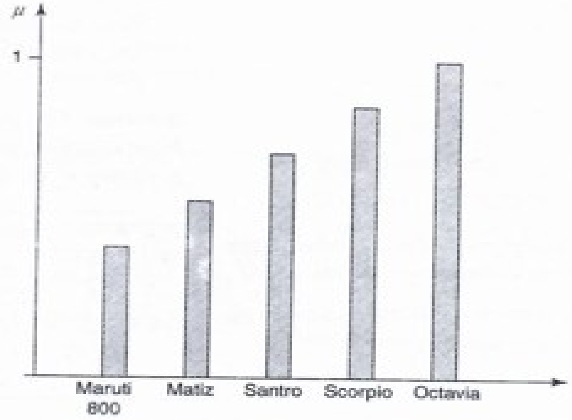

Figure 6 Membership function for best car.

Table 1

	Number who preferred							
	Maruti 800	Scorpio	Matiz	Santro	Octavia	Total	Percentage	Rank order
Maruti 800	—	192	246	592	621	1651	16.5	5
Scorpio	403	—	621	540	391	1955	19.6	2
Matiz	235	336	—	797	492	1860	18.6	4
Santro	523	364	417	—	608	1912	19.1	3
Octavia	616	534	746	726	—	2622	26.2	1
Total						10000		