Chapter - 3

Kinematics

1.

Distance	Displacement
(i) The actual length of the path covered by a body is called distance.	(i) The actual length of the path covered by a body at particular time is called displacement
(ii) It is scalar quantity	(ii) It is vector quantity.
(iii) It has on a positive value.	(iii) It has positive, negative & zero value.
(iv) The distance travelled by the body depend upon the shape of the path followed by the body.	iv) The distance travelled by the body doesn't depend upon the shape of the path followed by the body.

2.

Speed	Velocity
The rate of change of distance with time is called speed.	(i) The rate of change of displacement with time is called velocity.
It is a scalar quantity.	(ii) It is a vector quantity.
It has only a positive value.	(iii) It may have positive, zero or negative values.
The speed of the body doesn't show any direction of motion of the body.	(iv) The velocity of the body shows the direction of motion.

3. Average velocity - It is the ratio of displacement with the time taken by an object to cover that displacement.

4. Uniform velocity - If a body travels equal displacement in equal intervals of time, the velocity of the body

said to be uniform velocity.

5. Non-uniform velocity - If an object doesn't travel equal distances in equal intervals of time or direction of motion changes, the body is said to be non-uniform velocity or variable velocity.

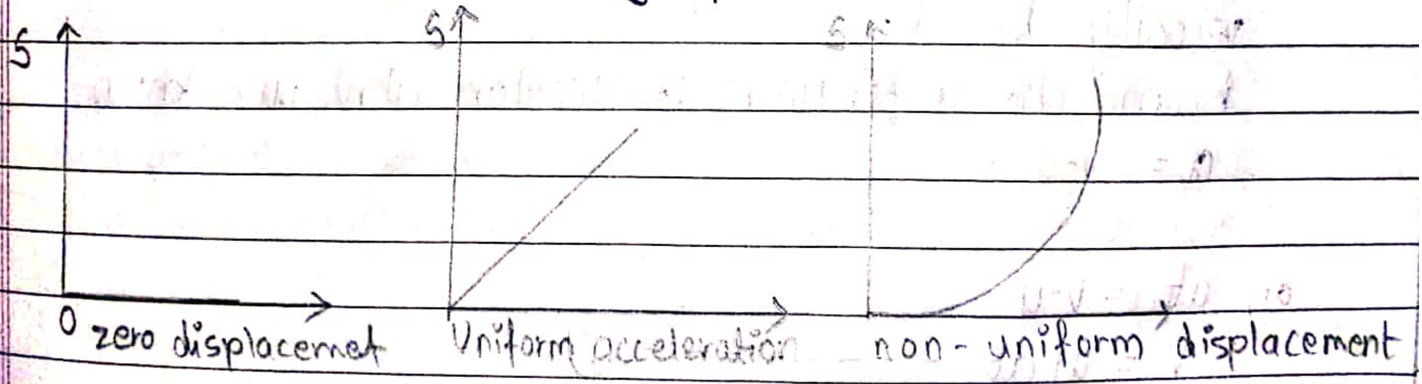
6. Instantaneous velocity - It is the velocity of an object at any given instant of time.

7. Acceleration - The rate of change of velocity is called acceleration.

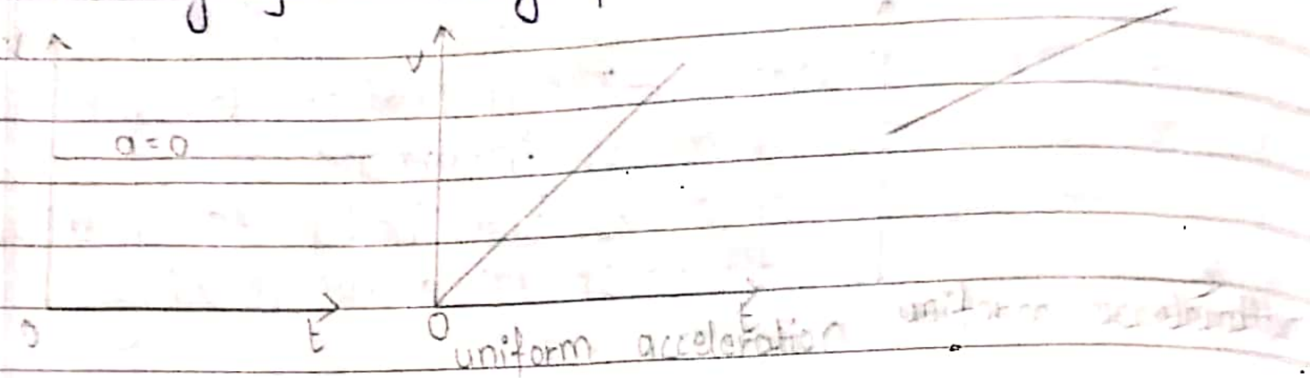
8. Average acceleration - It is the ratio of change in velocity of an object with a time interval when the object moves with variable velocity.

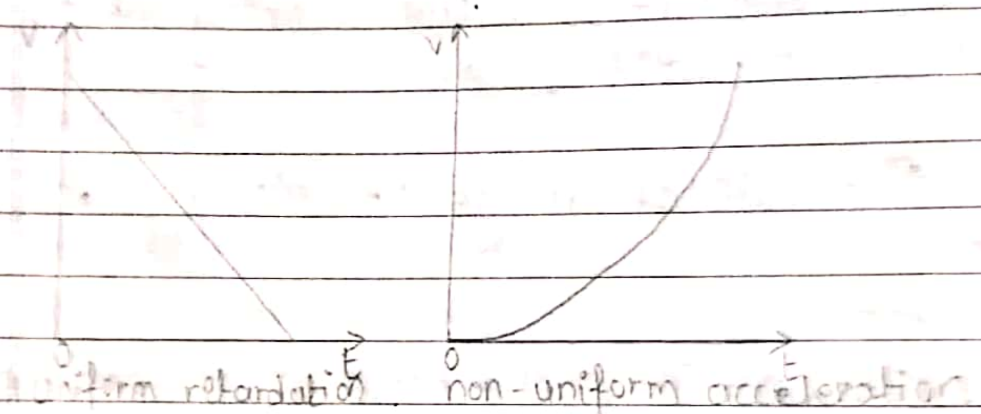
9. Instantaneous acceleration - It is the rate of change of velocity at any instant in time.

Displacement & Time graph:

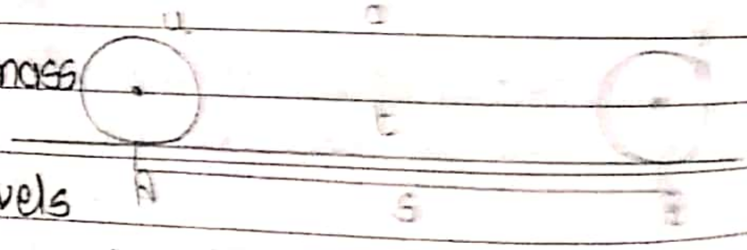


Velocity & Time graph :





Equation of Motion



Conside a body having mass m is moving with initial velocity. At time t it travels a distance s having a acceleration a whose final velocity be v.

From the definition of acceleration, we know.

$a = \frac{v-u}{t}$

or, $at = v-u$

$v = u+at$ - - - - - - - - (i)

Also, from average velocity,

$V_{av} = \frac{v+u}{2}$

We know,

$$S = V_{av} t$$

or, $S = \left(\frac{v+u}{2}\right) t$

or, $S = \frac{(u+at+u)t}{2}$

or, $S = \frac{(2u+at)t}{2}$

or, $S = \frac{2ut}{2} + \frac{at^2}{2}$

$$S = ut + \frac{1}{2}at^2 \text{ ------- (ii)}$$

Now, squaring both side, of eq^n (i),

$$v^2 = (u+at)^2$$

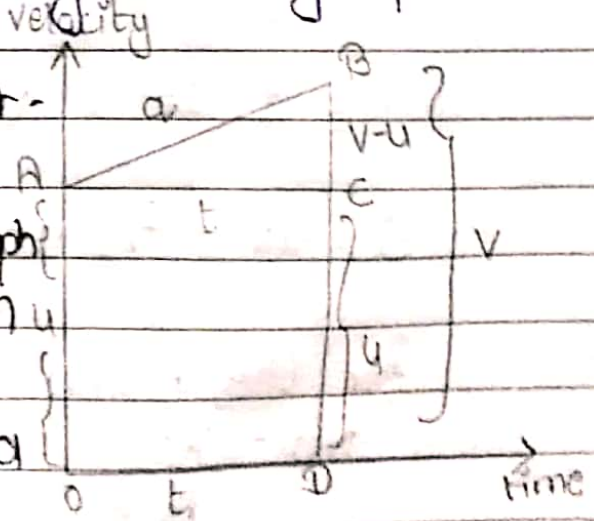
or, $v^2 = u^2 + 2uat + a^2t^2$

or, $v^2 = u^2 + 2a(ut + \frac{1}{2}at^2)$

$\therefore v^2 = u^2 + 2as$ #

Hence, it is the required equation of Motion.

Equation of motion from velocity-time graph:



Consider an object moving on a straight line with initial velocity u & uniform acceleration a. In graph AB represents a velocity-time graph for such motion. Here,
OA = DC = u, OB = v, OD = AC = t & AB = a
& CB = v - u.

For the first equation,
The slope between the velocity & time given by acceleration

We know,

$a = \frac{CB}{AC}$ or, $a = \frac{DB - DC}{AC}$

or, $a = \frac{v-u}{t}$

$\therefore v = u + at$ ------ (i)

For the second equation, we know,

$AB = \frac{BC}{AC}$

$a = \frac{BC}{t}$

$at = BC$

Area under the velocity time graph gives the displacement of a moving body.

Displacement (t) = Area of trapezium

s = Area of ΔABC + Area of trapezium

$s = \frac{1}{2} AC \times BC + BD \times CD$

$= \frac{1}{2} t \cdot at + t \cdot u$

$= \frac{1}{2} at^2 + ut$

$s = ut + \frac{1}{2} at^2$ --------- (ii)

For the third equation, we know,

$AB = \frac{BC}{AC}$

$AB = \frac{DB - DC}{AC}$ or, $AC = \frac{DB - DC}{a}$

$a = \frac{DB - DC}{AC} \Rightarrow$

Area under velocity time graph gives the displacement of the moving body.

$$s = \frac{1}{2}(DB + OA) \times AC$$

$$= \frac{1}{2}(DB + OA) \times \left(\frac{DB - DC}{a}\right)$$

$$= \frac{1}{2}(DB + OA) \times \left(\frac{DB - OA}{a}\right)$$

$$= \frac{1}{2}\left(\frac{DB^2 - OA^2}{a}\right)$$

$$s = \frac{1}{2}\left(\frac{v^2 - u^2}{a}\right)$$

$$2as = v^2 - u^2$$

$$\therefore v^2 = u^2 + 2as \text{ ------- (iii)}$$

Distance travelled in t^{th} second.

We know, distance travelled by a body in t second

$$S_t = ut + \frac{1}{2}at^2 \text{ ---- (i)}$$

Again,

distance travelled by a body in (t-1) second

$$S_{t-1} = u(t-1) + \frac{1}{2}a(t-1)^2 \text{ ---- (ii)}$$

Now, distance travelled in t^{th} second,

$$S_t = S_t - S_{(t-1)}$$

$$= ut + \frac{1}{2}at^2 - \left\{u(t-1) + \frac{1}{2}a(t-1)^2\right\}$$

$$= ut + \frac{1}{2}at^2 - \left\{ut + u + \frac{1}{2}a(t^2 - 2t - 1)\right\}$$

$$= ut + \frac{1}{2}at^2 - ut + u - \frac{1}{2}at^2 + \frac{1}{t}a \cdot 2t + \frac{1}{2}a$$

$$= u + at - \frac{1}{2}a$$

$$= u + a\left(t - \frac{1}{2}\right)$$

$$= u + a\left(\frac{2t-1}{2}\right)$$

$$= u + \frac{a}{2}(2t-1) \quad \#$$

Relative Velocity

Relative velocity is defined as change in position of one object with respect to another object with time.

* a) When two objects are moving in same direction.

let two object A & B is moving in same direction with velocity V_A & V_B. Then the resultant velocity is given by

$\xrightarrow{V_A}$

$\xrightarrow{V_B}$

$$V_{AB} = V_A - V_B$$

b) When two objects are moving in opposite direction.

let, two object A & B is moving with velocity V_A & V_B in opposite direction. Then the resultant velocity is given by :

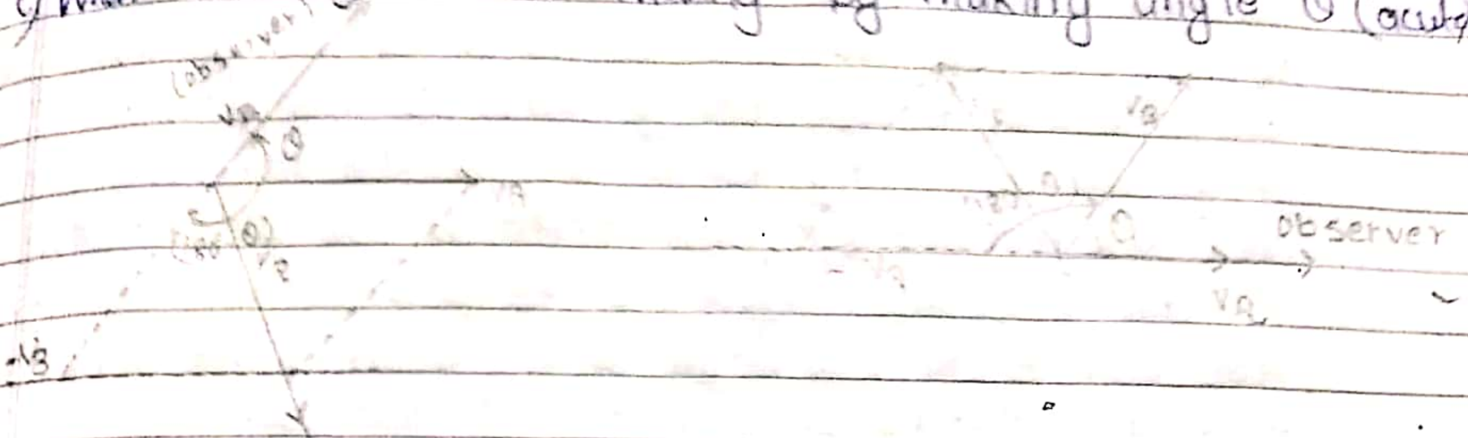
$\xleftarrow{V_A}$

$\xleftarrow{V_B}$

$$V_{AB} = V_A - V_B$$

$$= V_A + V_B$$

c) When two objects are moving by making angle θ (acute)



When two objects A & B are moving with velocity V_A & V_B by making angle θ (acute angle) then velocity of observer is always back produced & parallelogram law is applied.

$$V_{AB} = \sqrt{V_A^2 + V_B^2 + 2 \cdot V_A \cdot V_B \cos(180^\circ - \theta)}$$

$$= \sqrt{V_A^2 + V_B^2 - 2V_A \cdot V_B \cos 180^\circ}$$

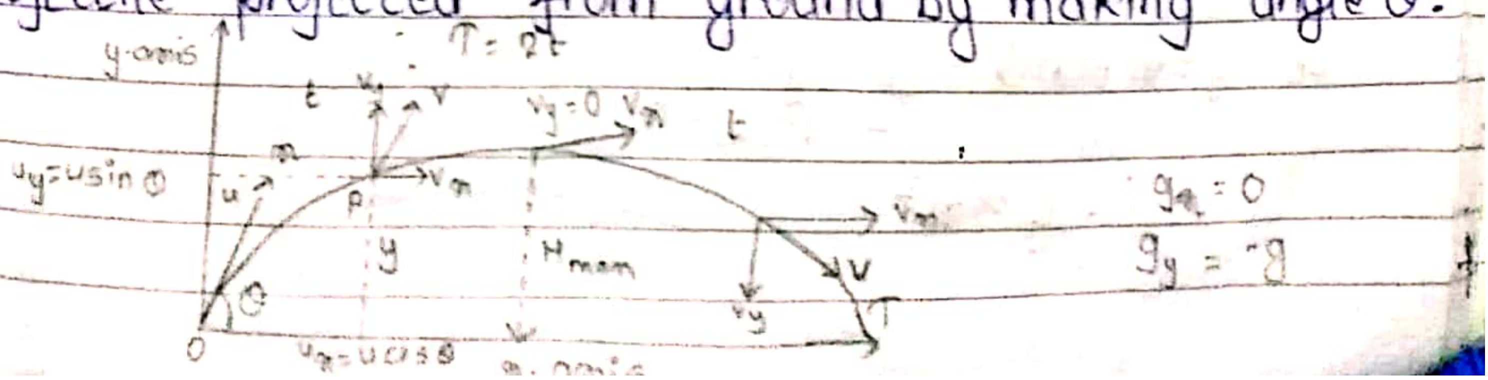
Projectile

A motion of a body under action of is known as projectile. The path followed by the body is known as tragectory.

Projectile can be categorized in two parts:

1. Projectile projected from ground.
2. Projectile projected horizontally (from tower)

Projectile projected from ground by making angle θ:



Consider a body having mass 'm' is projected with initial velocity 'v' by making angle θ resulting into two components horizontal & verticle u_x & u_y.
If the air resistance is negligible, the horizontal component u_x remain constant through out the motion whereas the vertical component u_y goes on decreasing, becomes zero at the top point of its path & finally, it increases downwards ground due to gravity.

$u_x = u\cos\theta$ & $u_y = u\sin\theta$

For part of projectile:

(i) For x-components:

$$x = u_x t + \frac{1}{2} g_x t^2$$

or, $x = u\cos\theta \cdot t + 0$ $\quad [\because g_x = 0]$

or, $x = u\cos\theta \cdot t$

or, $\boxed{\frac{x}{u\cos\theta} = t}$ --- (i)

(ii) For y-components:

$$y = u_y t + \frac{1}{2} g_y t^2$$

or, $y = u\sin\theta \cdot t + \frac{1}{2}(-g)t^2$

or, $y = u\sin\theta \cdot \frac{x}{u\cos\theta} - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$

or, $y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$

$\therefore y \propto x^2$

(iii) For maximum height

We know,

$$v_y^2 = u_y^2 + 2g_y h$$

$$0 = (u\sin\theta)^2 + 2(-g)\,H_{max}$$

$$-u^2\sin^2\theta = -2g\,H_{max}$$

$$\boxed{\frac{u^2\sin^2\theta}{2g} = H_{max}}$$

(iv) For Time of flight:

We know,

$$h = u_y t + \frac{1}{2} g_y t^2$$

or, $0 = u\sin\theta \cdot T - \frac{1}{2} g t^2$

or, $-u\sin\theta T = -\frac{1}{2} g T^2$

or, $\frac{2u\sin\theta}{g} = T$

(v) For Horizontal Range:

Horizontal Range = Horizontal velocity × time of flight

or, $R = u_x \cdot T$

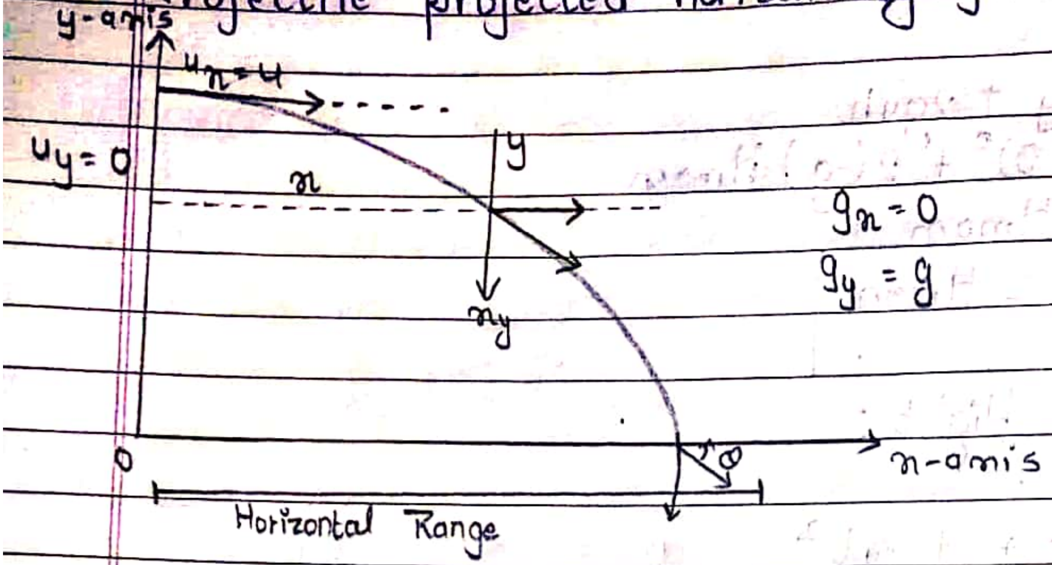
or, $R = u\cos\theta \cdot \frac{2u\sin\theta}{g}$

or, $R = u^2 \frac{2\sin\theta\cdot\cos\theta}{g}$

or, $R = \frac{u^2\sin 2\theta}{g}$

CamScanner

Projectile projected horizontally: from tower:



Consider a body is projected of mass (m) from a tower of height (h) with uniform velocity (u) so that the initial downward velocity is zero.
Suppose x & y are the horizontal & vertical distance travelled by a projectile in time t respectively. For horizontal motion, $u_x = u$ & $u_y = 0$.

For Path of Projectile.

1. For x- component

$$x = u_x t + \frac{1}{2} g_x t^2$$

$$= u \cdot t + \frac{1}{2} \cdot 0 t^2$$

$$x = ut$$

$$\frac{x}{u} = t \quad \text{--- (i)}$$

2. for y - component

or, $y = u_y t + \frac{1}{2} g_y t^2$

or, $y = 0 \cdot t + \frac{1}{2} g t^2$

or, $y = \frac{1}{2} \left(\frac{x}{u}\right)^2 \cdot g$

or, $y = \frac{1}{2} g \frac{x^2}{u^2}$ //

$\therefore y \propto x^2$

Hence, path of projectile ~~parallel~~ parabolic in

3. For time of flight

$h = u_y t + \frac{1}{2} g_y t^2$

or, $h = 0 + \frac{1}{2} g T^2$

or, $h = \frac{1}{2} g T^2$

$\therefore T = \sqrt{\frac{2h}{g}}$

4. For horizontal range

$R = u_x \cdot T$ [Horizontal range = Horizontal velocity x time of flight]

$\therefore R = u\sqrt{\frac{2h}{g}}$

5. For velocity of Projectile:

(i) Velocity for x-component

$V_x = U_x + g_x t$

$V_x = u + 0$

$\therefore V_x = u$

(ii) velocity for y-component

$V_y = U_y + g_y t$

$V_y = 0 + gt$

$\therefore V_y = gt$

So, Resultant velocity

$V = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{U^2 + (gt)^2}$

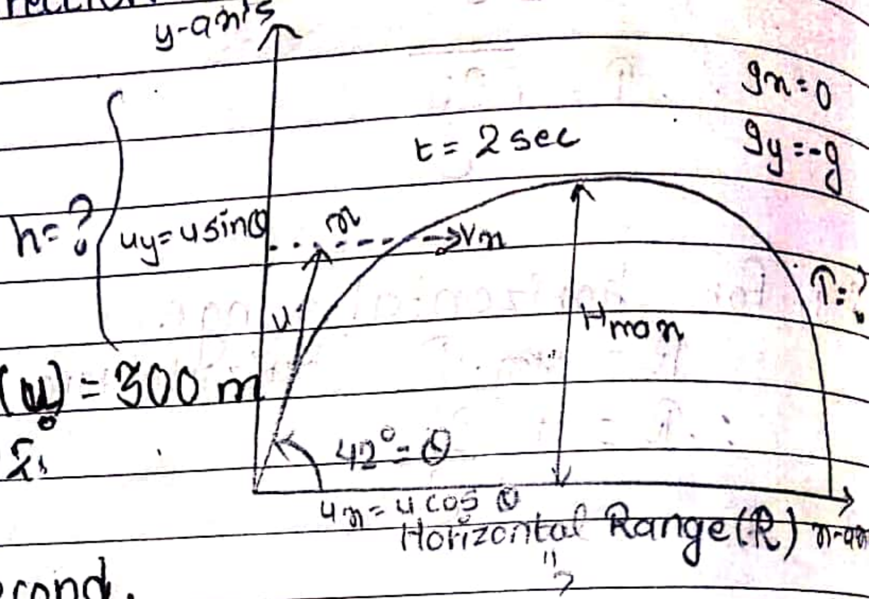
for direction

$\tan\theta = \frac{V_y}{V_x}$

or, $\tan\theta = \frac{gt}{U}$ $\quad \therefore \theta = \tan^{-1}\left(\frac{gt}{U}\right)$

CamScanner

Projectile is projected from ground with initial velocity 300m per second by making angle of 42°. Calculate
Position after 2 second.
Final velocity & direction after 2 sec.
Horizonal range
Maximum height
Time of flight.



Soln:

Here, Given,
Initial velocity (u) = 300 m
Angle made (θ) = 42°
Now,
Position after 2 second.
We have, t = 2 second & u = 300 m/s

$x = u_x t + \frac{1}{2} g_x t^2$

$= u\cos\theta t + \frac{1}{2} \times 0 \times (2)^2$

$= 300 \times \cos 42° \times 2$

$\therefore x = 222.94 \cancel{m/s} \times 2 = 445.88$ m

Also, $y = u_y t + \frac{1}{2} g_y t^2$

$= u\sin\theta t + \frac{-g}{2} \times 2^2$

$= 300 \times \sin 42° \times 2 - 2g$

$= 200.73 \times 2 - 2 \times 9.8$

$= 381.47$ m

Final velocity & direction after 2 sec
We know,

$V_x = u_x + g_x t$ & $V_y = u_y + g_y t$

$= u\cos\theta + 0$ $\qquad$ $= u\sin\theta + (-g)t$

$= 300 \times \cos 42°$ $\qquad$ $= 300 \times \sin 42° - 10 \times 2$

$= 222.94$ m $\qquad$ $= 180.73$ m

Now, final velocity $= \sqrt{(222.94)^2 + (180.73)^2} = 286.99 \ m/s$

Also,

Direction - $\tan\theta = \frac{V_y}{V_x}$

$$\theta = \tan^{-1}\left(\frac{180.73}{222.94}\right)$$

$$\therefore \theta = 39.03°$$

(iii) Horizontal range

$$R = \frac{u^2 \sin(2\theta)}{g}$$

$$= \frac{(300)^2 \times \sin 2\times42°}{10}$$

$$= 8950.69 \ m$$

(iv) Maximum height

$$H_{max} = \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{(300)^2 \times \sin^2 42°}{2\times10}$$

$$= 2014.81 \ m$$

(v) Time of flight

$$T = \frac{2u\sin\theta}{g}$$

$$= \frac{2\times300\times\sin 42°}{10}$$

$$= 40.14 \ s$$

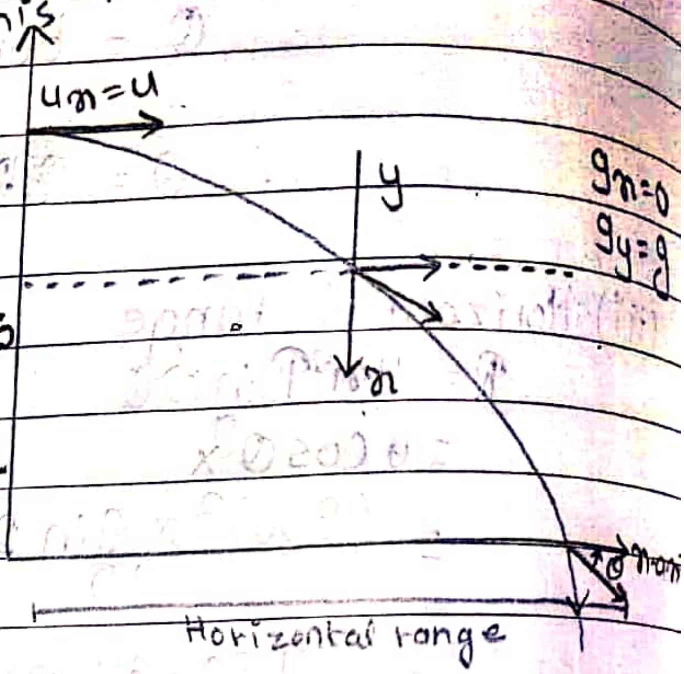
A body is projected horizontally with intial velocity 50 m/s from height 200 m. Calculate.

Find the position after 3 second.

Final velocity & direction after 3 sec.

Horizonatal range

Time of flight



Solⁿ:

Given,

Initial velocity (u) = 50 m/s

height (h) = 200 m

Position after 3 second.

t = 3 second.

$x = u_x t + \frac{1}{2} g_x t^2$

$= u \times 3 + \frac{1}{2} \cdot 0 \cdot t^2$

$x = 3 . u$

$\therefore x = 3 \times 50 = 150\ m$

Also,

$y = u_y t + \frac{1}{2} g_y t^2$

$= 0 \times t + \frac{1}{2} g t^2$

$= \frac{1}{2} \times 9.8 \times (3)^2$

$y = 44.1\ m$

Final velocity & direction after 3 sec

$V_x = u_x + g_x t$ & $V_y = u_y + g_y t$

$= u + 0 \cdot t$ $\quad = 0 + g \times t$

$= 50\ m/s$ $\quad = 9.8 \times 3$

$\quad = 29.4\ m/s$

$V = \sqrt{50^2 + 29.4^2} = 58\ m/s$

for direction,

$$\tan\theta = \frac{V_y}{V_x}$$

$$\theta = \tan^{-1}\left(\frac{29.4}{50}\right)$$

$$= 30.45^\circ$$

(iii) Horizontal range

$$R = u\sqrt{\frac{2h}{g}}$$

$$= 50\sqrt{\frac{2\times 200}{10}}$$

$$= 316.22\ m$$

(iv) Time of flight

$$T = \sqrt{\frac{2h}{g}}$$

$$= \sqrt{\frac{2\times 200}{10}}$$

$$= 6.32\ sec$$