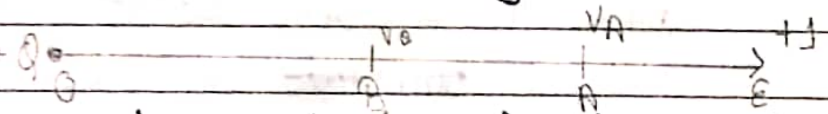
Chapter - 21

Electric Potential

Electrical potential at a point in electric field is define as work done by a unit positive test charge in moving from infenity to that point against electric field without acceleration.

Electrical potential due to point charge :



Consider a +q charge is placed at position O from where electric field is extended till infinity. A unit positive test charge is moving from infinity to +q charge.

Now,

The potential at position A is

$V_A = W_{\infty A}$ = Workdone by unit positive test charge in moving from ∞ to point A.

Similarly, the potential at position B is

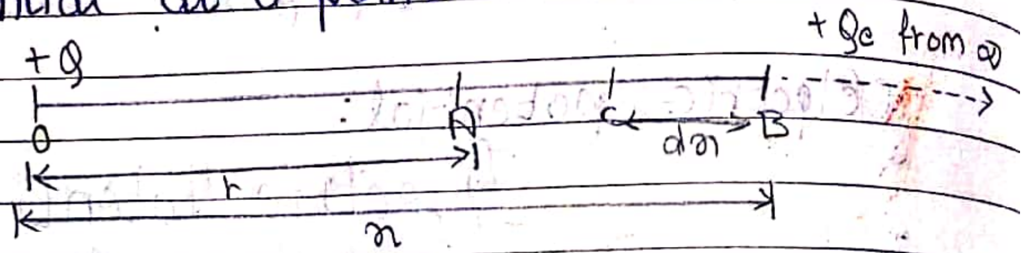
$V_B = W_{\infty B}$ = Workdone by unit positive test charge in moving from ∞ to point B.

Electric potential is also define as ratio of workdone done per unit test charge.

i.e. $V_A = \frac{W_{\infty A}}{Q_0}$

Also, $V_B = \frac{W_{\infty B}}{Q_0}$

Electric potential at a point :



Consider a +Q charge is placed at position O from where electric field is extended upto infinity. A unit positive test charge is ending from infinity towards source charge.

Now, the potential at position A is define as work done by a unit positive test charge in moving from infinity to that point. then,

$V_A = W_{\infty A}$ ------ (i)

Then,

Force experienced by the unit positive test charge at position B

$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{QQ_0}{x^2}$ ------- (ii)

If dw be the small amount of workdone by unit positive ~~heat~~ test charge in moving from position B to C.

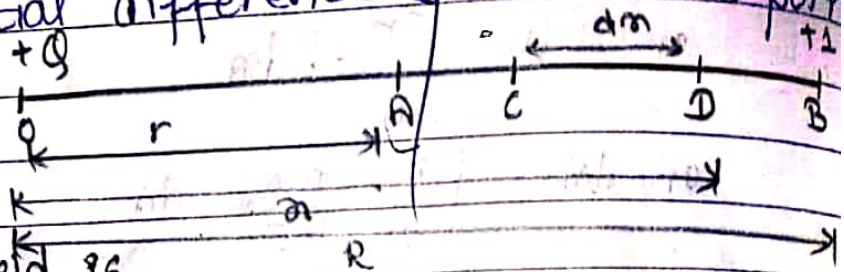
$dw = -Fdx$ ---- (iii)

Potential difference

Potential difference between two points on the electric field is define as work done by unit positive test charge in moving from on point to another against the electric force without acceleralating.

Unit of Potential = JC^{-1}

Expersion for potential difference between two points



Consider a +Q charge is placed at position O from where electric field is extended upto infinity. A unit positive test charge is ending from infinity towards source charge.

Now, the potential difference between Point A & B is define as work done by a unit positive test charge moving from A to point. B

$$V_A - V_B = W_{BA} \quad \text{----(i)}$$

then, Force experienced by the unit positive test charge at position B.

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{QQ_0}{x^2} \quad \text{-----(ii)}$$

If dw be the small amount of work done by unit positive test charge in moving from position D to C.

$$dw = -fdx \quad \text{-----(iii)} \quad = \frac{-QQ_0}{4\pi\epsilon_0 x^2}\,dx$$

Where -ve sign indicate that electric force & displacement are in opposite direction.

Now, Total work done by unit positive test charge in moving from B to positive A can be calculated by integration.

$$W = \int_B^A dw$$

$$= \int_R^r -\frac{QQ_0}{4\pi\epsilon_0} \cdot \frac{dx}{x^2}$$

$$= -\frac{QQ_0}{4\pi\epsilon_0} \int_R^r \frac{dx}{x^2}$$

$$= -\frac{QQ_0}{4\pi\epsilon_0} \int_R^r x^{-2}\, dx$$

$$= -\frac{QQ_0}{4\pi\epsilon_0} \left(\frac{x^{-2+1}}{-2+1} \right)_R^r$$

$$= -\frac{QQ_0}{4\pi\epsilon_0} \left(\frac{x^{-1}}{-1} \right)_R^r$$

$$= \frac{QQ_0}{4\pi\epsilon_0} \left(\frac{1}{x} \right)_R^r$$

$$= \frac{QQ_0}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{R} \right)$$

$$= \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{R} \right)$$

$\because Q_0 = +1$

So,

$$V_A - V_B = \frac{W_{BA}}{Q_0}$$

$$= \frac{Q}{4\pi\epsilon_0} \cdot \frac{\cancel{Q_0}}{\cancel{Q_0}} \left(\frac{1}{r} - \frac{1}{R} \right)$$

$$= \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{R} \right)$$

Unit of Potential:

We know,

$$V = \frac{W}{Q}$$

or, $\text{Volt} = \frac{\text{Joule}}{\text{Charge}}$

$\therefore V = JC^{-1}$

$$W = \int_B^A dw$$

$$= \int_R^r \frac{-QQ_0}{4\pi\epsilon_0} \cdot \frac{dx}{x^2}$$

$$= \frac{-QQ_0}{4\pi\epsilon_0} \int_R^r \frac{dx}{x^2}$$

$$= \frac{-QQ_0}{4\pi\epsilon_0} \int_R^r x^{-2}\, dx$$

$$= \frac{-QQ_0}{4\pi\epsilon_0} \left(\frac{x^{-2+1}}{-2+1}\right)_R^r$$

$$= \frac{-QQ_0}{4\pi\epsilon_0} \left(\frac{x^{-1}}{-1}\right)_R^r$$

$$= \frac{QQ_0}{4\pi\epsilon_0} \left(\frac{1}{x}\right)_R^r$$

$$= \frac{QQ_0}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{R}\right)$$

$$= \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{R}\right)$$

$\because Q_0 = +1$

So,

$$V_A - V_B = \frac{W_{BA}}{Q_0}$$

$$= \frac{Q}{4\pi\epsilon_0} \cdot \frac{\cancel{Q_0}}{\cancel{Q_0}} \left(\frac{1}{r} - \frac{1}{R}\right)$$

$$= \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{R}\right)$$

Unit of Potential:

We know,

$$V = \frac{W}{Q}$$

or, $\text{Volt} = \frac{\text{Joule}}{\text{Charge}}$

$\therefore V = JC^{-1}$

Electron - Volt

→ The energy gain by an electron when accelerated through the potential of 1V is known electron-volt.

i.e. $1 \cdot ev = 1.6 \times 10^{-19}$ J

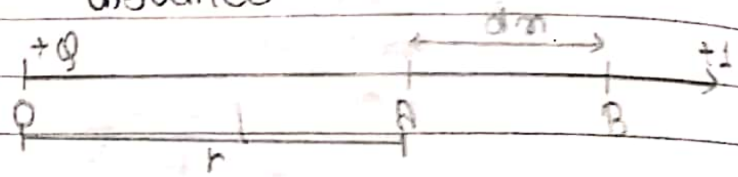
$\frac{1}{1.6 \times 10^{-19}} ev = 1$ J

Relation between electric field Intensity & electric gradients.

The rate of change of electric potential with respect to distance along the lines of force is defined as potential gradient.

i.e.

Potential Gradient $= \frac{\text{electric potential}}{\text{distance}} = \frac{dv}{dr}$



Consider a +Q charge is placed at position from where electric field is extended upto infinity. A unit positive test charge is moving towards a source charge.

Workdone by a unit positive test charge from position B to A.

$$W = -f dx$$

i.e. $W = -E dx$ ----- (i)

Also,

The electric potential ~~of~~ experienced by test charge at position $P = dv$ ------ (ii)

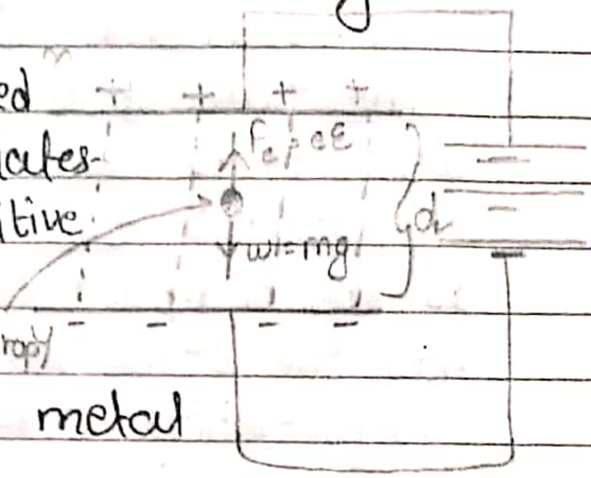
We know,

$$dv = W$$

or, $dv = -E\,dx$

$\therefore \frac{dv}{dx} = E$ # which is the relation between electric field Intensity & electric gradient

Action of an electric field on a Point charge:



Consider a point charge is placed between two parallel metal plates. A upper plate x is connected to Positive terminal & lower plate y is connected to negative terminal battery. If oil drop 'd' be distance between two metal plate.

Now, force acting upward due to electric field on a point charge:

$$F_e = e \cdot E \text{ -------- (i)}$$

Then, force acting downward due to weight

$$W = mg \text{ --------- (ii)}$$

Under this equilibrium condition,

force acting ~~down~~upward = force acting downward

or, $F_e = W$

or, $eE = mg$

or, $e\frac{V}{d} = mg$

or, $e\frac{V}{d} = \frac{4}{3}\pi r^3 \rho g$.

$\therefore \rho = \frac{m}{V}$

or, $V\rho = m$

$\frac{4}{3} \times \pi r^3 \rho = m$

... of potential gradient

... know,

$\frac{V}{d} = E$

$Vm^{-1} = JC^{-1}m^{-1}$

$Vm^{-1} = JC^{-1}m^{-1}$

$Vm^{-1} = NmC^{-1}m^{-1}$

$Vm^{-1} = NC^{-1}$

Equipotential Surface

The locus of all the points which are at the same potential is termed an equipotential surface. An equipotential surface in an electric field is defined as the surface over which the electric potential has the same value. If A & B are two points on an equipotential surface, then $V_A = V_B$.

$W_{AB} = q \cdot V_{AB} = q(V_A - V_B) = 0$

i.e. no work is done to move a unit positive charge on the surface of an equipotential surface.

Some examples of equipotential surface

... conductor placed inside the electric field behaves as an equipotential surface.

... concentric spheres having ... charge at the centre are equipotential surfaces.

Properties

a) The value of ... point of the ...

b) No work is ... equipotential ...

c) They are closer ... of force.

d) They indicate ... of electric ...

e) For a point charge ... a sphere ... the charge

Some Numerical

1. The potential ... at a distance ... is ? What ...

(i) At a point ...

(ii) Inside the ...

Sol:

Here, $Q = 1.6$ ...

(i) Electric field

$E = \frac{Q}{4\pi\varepsilon_0 r^2}$

$= \frac{1}{4\pi\varepsilon_0} \times$ ...

$= 9 \times 10^9 \times$ ...

$= 100000$ ...

Properties:

a) The value of potential is always constant on each point of the equipotential surface.

b) No work is to be done to displace a charge ~~at the~~ on the equipotential surface.

c) They are always perpendicular to the electric lines of force.

d) They indicate regions of weak & strong values of electric flux.

e) For a point charge, equipotential surfaces will be a series of concentric spherical shells with the charge at the center.

Some Numericals:

1. The spherical conductor of radius 0.12 m as a charge of 1.6×10^{-7} C distributed uniformly on its surface. (i) What is the electric field just outside the sphere (ii) At a point 0.18 m from the centre of sphere. (iii) Inside the sphere at point 0.9 m

Soln:

Here, $Q = 1.6 \times 10^{-7}$ C & $r = 0.12$ m

Now

(i) Electric field just outside the sphere,

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \times \frac{Q}{r^2}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-7})}{(0.12)^2} \qquad \left[\because \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N}\right]$$

$$= 100000 \text{ NC}^{-1}$$

(ii) At a point 0.18 m from the centre of sphere.
Here, R = 0.18 m

$$\varepsilon = \frac{Q}{4\pi\varepsilon_0 R^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$
$$= \frac{9\times10^9\times(1.6\times10^{-7})}{(0.18)^2}$$
$$= 44444.44\ NC^{-1}$$

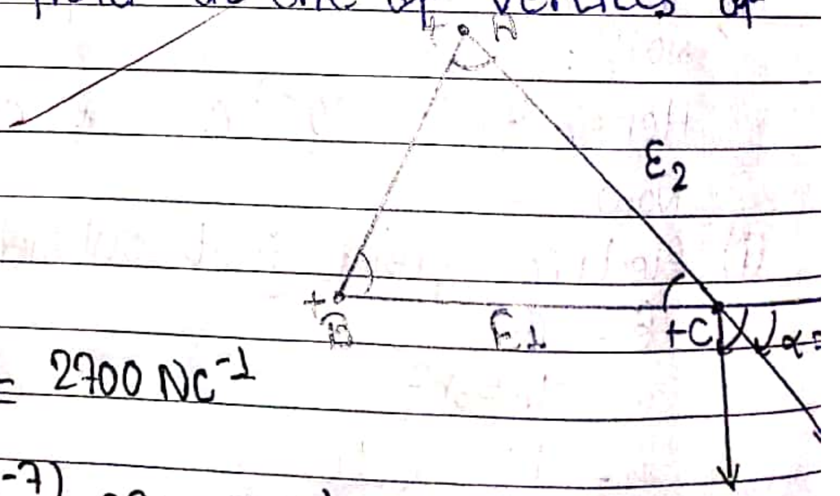
(iii) Inside the sphere at point 0.9 m
Here, Q = 0

$$\varepsilon = \frac{Q}{4\pi\varepsilon_0 R} = \frac{1}{4\pi\varepsilon_0}\times\frac{Q}{R} = 0\ NC^{-1}$$

$\therefore$ $\varepsilon = 0$ because it doesn't inclose any charge.

2. The 3 point charges 3×10^{-7}C are placed at the corner of an equilatoral triangle whose charge is 1m. What is the electric field at one of vertices of this charges.

Soln:



$Q = 3\times10^{-7}$C
r = 1m
Now,

$$\varepsilon_1 = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{9\times10^9(3\times10^{-7})}{1^2} = 2700\ NC^{-1}$$

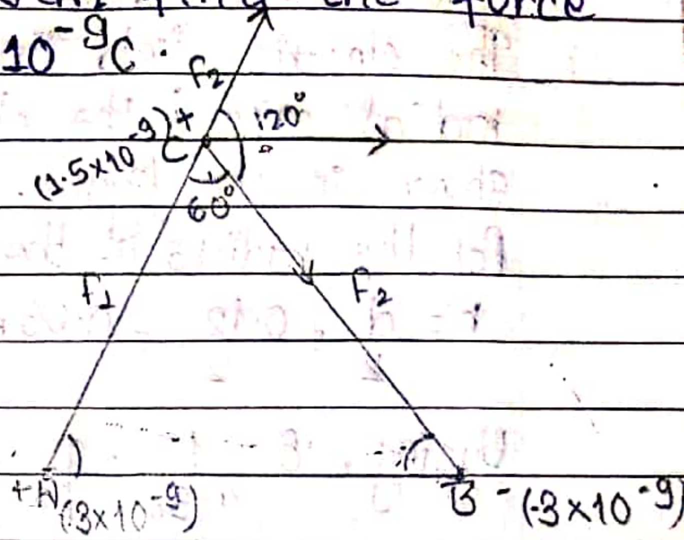
$$\varepsilon_2 = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{9\times10^9(3\times10^{-7})}{1^2} = 2700\ NC^{-1}$$

Also, $\theta = 60°$

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1 E_2 \times \cos\theta}$$
$$= \sqrt{2700^2 + 2700^2 + 2 \times 2700 \times 2700 \times \cos 60^\circ}$$
$$= 4676.53 \ NC^{-1}$$

3. 3 charges 3×10^{-9} C, -3×10^{-9} C, 1.5×10^{-9} C are placed in A, B, & C at corner of equilatoral triangle having each side 5 cm find the force acting on the charge 1.5×10^{-9} C.

Soln:



Here,

$$F_1 = \frac{Q \cdot Q_0}{4\pi\epsilon_0 r^2}$$
$$= \frac{9\times10^9 \times 3\times10^{-9} \times 1.5\times10^{-9}}{(5\times10^{-2})^2}$$
$$= 1.62\times10^{-5} \text{ N}$$

Again,

$$F_2 = \frac{Q \cdot Q_0}{4\pi\epsilon_0 r^2} = \frac{9\times10^9 \times -3\times10^{-9} \times 1.5\times10^{-9}}{(0.05)^2}$$
$$= -1.62\times10^{-5} \text{ N}$$

Now,

Total force on charge 1.5×10^{-9} C.

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1 \cdot F_2 \times \cos\theta} \qquad \therefore \theta = 120^\circ$$
$$= \sqrt{(1.62\times10^{-5})^2 + (-1.62\times10^{-5})^2 \times 2 \times 1.62\times10^{-5} \times -1.62\times10^{-5} \times -1/2}$$
$$= 1.62\times10^{-5} \text{ N}$$

Thus, the force acting on the charge 1.5×10^{-9} C is 1.62×10^{-5} N

08/25

① A metal sphere of diameter 12 cm is positively charged. The electric field strength at the surface of the sphere is 10^5 Vm^{-1}. Draw the electric field pattern for the sphere & determine the total surface charge.

Soln. Given,

Diameter $(d) = 12\,cm = 0.12\,m$ Electric field strength $(E) = 10^5\,Vm^{-1}$

Total surface charge $(q) = ?$

The electric field lines must be normal to the surface & radial around the charged sphere as shown in the figure,

For the radius of the metal sphere, we have,

$$r = \frac{d}{2} = \frac{0.12}{2} = 0.06\,m$$

Using, $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$

$$\therefore q = 4\pi\epsilon_0 \times r^2 = 4\pi \times 8.85 \times 10^{-12} \times 0.06^2 = 1.6 \times 10^{-7}\,C$$

Hence, the value of the total surface charge on the given metal dome is $1.6 \times 10^{-7}\,C$.

② Two charges $+1 \times 10^{-6}C$ & $-4 \times 10^{-6}C$ are separated by a distance of 2m. Determine the position of the null point.

Soln. Given,

First charge $(q_1) = 1 \times 10^{-6}C$

Second charge $(q_2) = -4 \times 10^{-6}C$

Separation of two charge $(r) = 2m$

Position of null point $(x) = ?$

Suppose a null point P lies at distance x from the first charge q_1 on the straight line joining them. To exist the null point, it is required that,

$E_1 = E_2$

or, $\frac{1}{4\pi\epsilon_0} \frac{q_2}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{(r+r)^2}$

or, $\frac{1\times10^{-6}}{r^2} = \frac{4\times10^{-6}}{(2+r)^2}$

or, $\frac{1}{r} = \frac{2}{2+r}$

$r = 2m$ from charge q_1

Thus, the position of the null point lies in between two charges is 2m from charge q.

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