Chapter-20

Electric Field

<u>test charge</u> $(+q_2)$ → Small charge which detech the electric field.

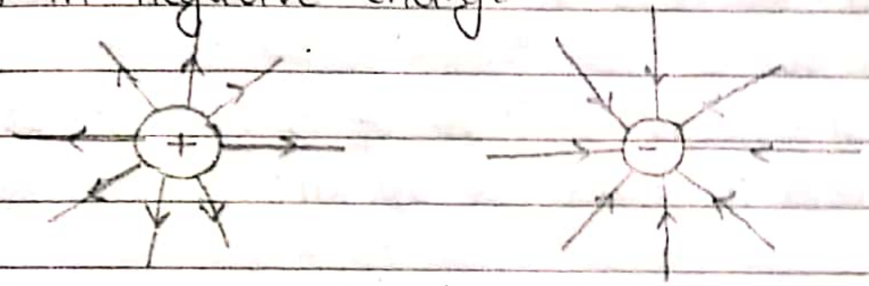
<u>Source charge</u> $(+Q)$ → which measure the electric field.

Electric Field :

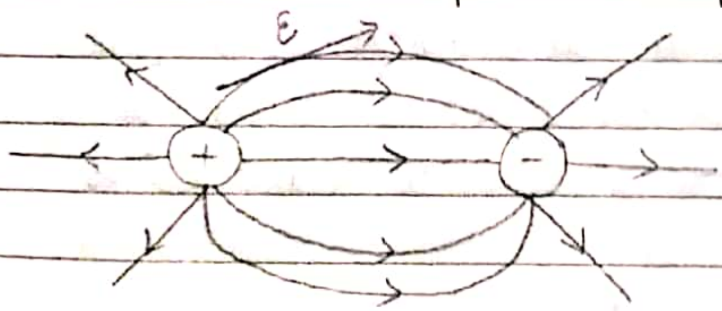
The space around the electric charge or source charge where the electric force of attraction or repaltion exists is called electric force.

Properties of electric lines of forces :

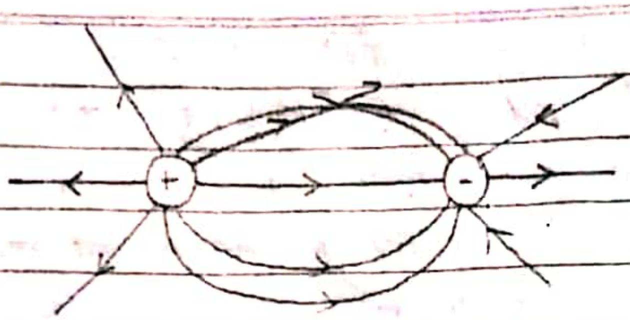
1) The electric lines of forces start from positive charge & ends in negative charge.



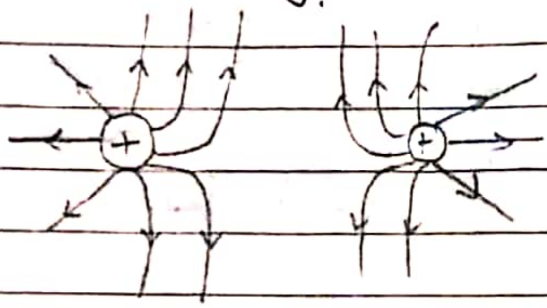
2) When the tangent is drawn on the lines of force it gives the direction of electric field



3) Two electric lines of force they never intersect each other. If they do so, then a tangent is drawn then their will be two dimension of electric field.



4) Electric lines of force exerts lateral pressure due to repulsion between two charges.



5) Electric lines are the continuous curves.
6) The number of lines of forces perunit area is directly proportional to Electric field strength.
i.e. Esq where the lines of force of $\mathcal{E}$ will be stronger & ~~whe~~ if the lines of force are appart $\mathcal{E}$ will be weaker

Electric field Intensity : (vector quantity)

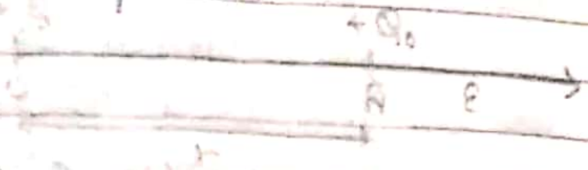
Electric field Intensity at a point in electric field is define as force experienced by a unit positive test charged ~~plased~~ placed at that point.
Mathematically,

$$\mathcal{E} = \frac{F}{Q_0}$$

It's units in SI, J/C

$$\mathcal{E} = \frac{F}{Q_0} = \frac{J}{C}.$$

Electric field Intensity at a place:



$\cancel{F = \frac{QQ_0}{4\pi\varepsilon_0 r^2}}$

Consider, a charge +Q which is placed at position O & a unit positive test charge $+Q_0$ is placed at position A. which is at distance r. i.e. OA = r

The force experienced by unit positive test charge due to ~~uni~~ +Q charge given as,

$$F = \frac{QQ_0}{4\pi\varepsilon_0 r^2} \quad --- (i)$$

We know from the electric field Intensity

$$E = \frac{F}{Q_0} \quad --- (ii)$$

Now, Solving eqn (i) & (ii) we get,

$$E = \frac{F}{Q_0}$$

or, $$E = \frac{QQ_0}{4\pi\varepsilon_0 r^2} \times \frac{1}{Q_0}$$

or, $$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

If the electric field intensity $E_1, E_2, E_3 ----- E_n$ is due to the point changes $Q_1, Q_2, Q_3 ----- Q_n$ then total electric field intensity is general,

$$E = E_1 + E_2 + E_3 + ----- E_n$$

Electric Flux : (Scalar quantity)

The number of lines of force passing through the surface area when held perpendicular to it is known as electric flux.

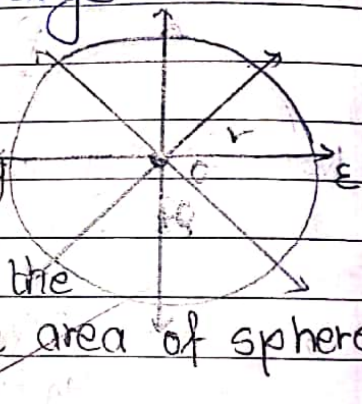
Electric flux is denoted by ϕ.

If the lines of forces are denser then the electric field intensity is stronger whereas if the lines of forces are less then the electric field intensity is weaker.

$$E = \frac{\phi}{A}$$

or, $\phi = E \cdot A$ (It's unit is Nm^2C^{-1} or Volt-meter (V-m))

Electric flux due to a point Charge



Consider a +Q charge is placed at position O which is inclosed by a spear having radius r. If E be the electric field intensity on the surface of sphere. then the surface area of sphere

$A = 4\pi r^2$ ---- (i)

We know, from electric field Intensity

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad \text{--- (ii)}$$

Also, from the definition of electric flux.

$$E = \frac{\phi}{A}$$

$$\phi = E \cdot A \quad \text{--- (iii)}$$

Now, Solving eq^n (i), (ii) & (iii), we get,

$\phi = E \cdot A$

or, $\phi = \frac{Q}{4\pi\epsilon_0 r^2} \times 4\pi r^2$

or, $\phi = \frac{Q}{\epsilon_0}$ which is the required equation of electric flux due to a point charge.

Gauss's Theorem

Gauss's Theorem state that the total electric flux passing through a close surface enclosing a charge is equal to $1/\epsilon_0$ times magnitude of net electric charge ~~closing in a surface~~ inclosed by a closed surface.

That is, $\phi = \frac{Q}{\epsilon_0}$.

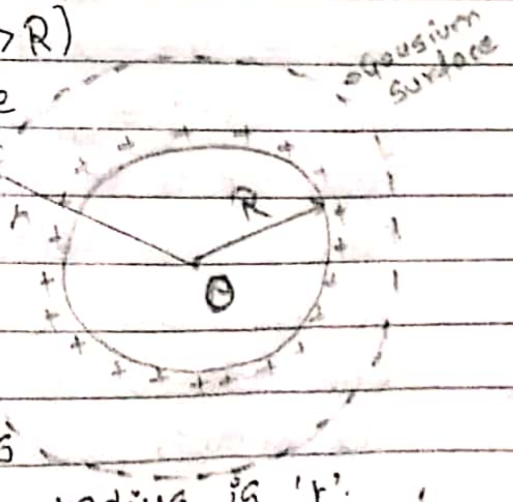
Application of Gauss's Theorem

Electric field Intensity can be calculated more appropratily from Gauss's law compartative to Coulumb's law.

1) Electric field intensity due to a hollow charge sphere.

Consider a hollow charge sphere having radius R & centre O. The charges are uniformly disturbuted on the surface of sphere.

a) When P lies outside the sphere. $(r > R)$



Consider a position P lies the outside the charge sphere an electric field intensity is to be determined. A concentric sphere is drawn from position P with respect to centre O. which encloses the charge. It is known as Gauseum surface whose radius is 'r'.

The electric field intensity is same througout the gauseum surface.
The surface area of Gauseum surface = $4\pi r^2$
~~We know,~~ If E be the electric field intensity at position ~~P~~.
We know, $\phi = E \cdot A$ --- (i)
We know from Gauss's theorem,

$$\phi = \frac{Q}{\epsilon_0} \quad \text{---- (ii)}$$

Combining eq^n (i) & (ii), we get,

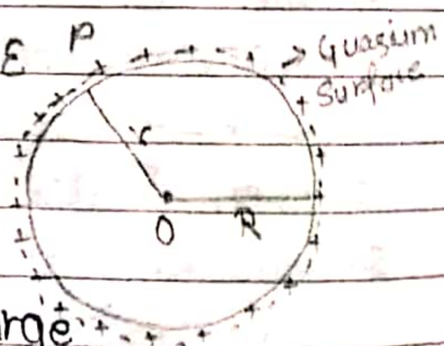
$$E \cdot A = \frac{Q}{\epsilon_0}$$

$$\text{or, } E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$\therefore E = \frac{Q}{4\pi r^2 \epsilon_0} \quad \#$$

is the required equation of electric field ~~when~~ intensity.

b) When P lies on the surface (r = R):
~~Consider a position lies a hollow charge sphere having radius R & centre O. The charge are uniformly disturbuted on the surface of sphere.~~



Consider a position P lies the outside the charge sphere on electric field intensity is to be determined. A concentric circle is drawn ~~the~~ from position P with respect to centre O which encloses the charges. It is known as Guasium surface with radius r.

The electric field intensity is same througout the guasium surface.

The surface area of Guasium surface = $4\pi r^2$ --- (i)

If $\mathcal{E}$ be the electric field intensity position P.

$\phi = \mathcal{E} \cdot A$ ---- (ii)

We know from Gauss's theorem,

$$\phi = \frac{Q}{\epsilon_0} \text{ --- (iii)}$$

Now, solving eq^n (i) & (ii) & (iii) we get,

$$\mathcal{E} \cdot A = \frac{Q}{\epsilon_0}$$

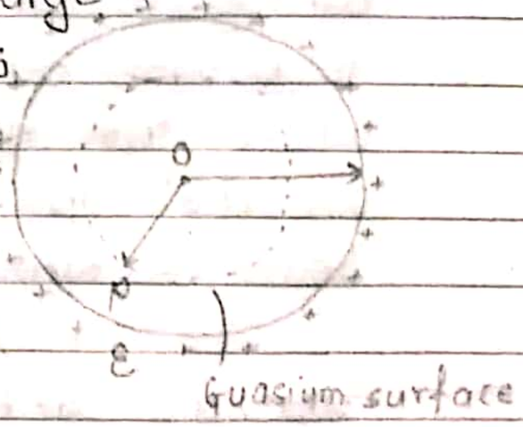
$$\text{or, } \mathcal{E} \cdot 4\pi R^2 = \frac{Q}{\epsilon_0}$$

$$\therefore \mathcal{E} = \frac{Q}{4\pi\epsilon_0 R^2}$$

which is the required equation for electric field intensity where P lies on the surface.

c) When P lies inside the sphere ($r<R$):

Consider a Position P lies inside the charge sphere where electric field intensity is to be determined. A Guasium surface is drawn from position P with rest to centre; which doesn't inclose the charge $Q = 0$.



The Surface area of Guasium surface is $4\pi r^2$ --- (i)

If $\mathcal{E}$ be the electric field intensity of position P.

We know, $\phi = \mathcal{E} \cdot A$ --- (i)

We know from Gauss's theorem,

$$\phi = \frac{Q}{\epsilon_0} \text{ ---- (ii)}$$

Combining eq^n (i) & (ii), we get,

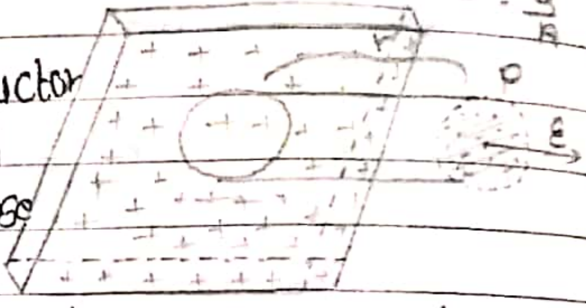
$$E \cdot A = \frac{Q}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0} \quad [\because Q = 0]$$

$\therefore E = 0$ # which is the required equation for electric field insensity where p lies inside the sphere & charge is 0.

2. Electric field Intensity due to plane charge sheet.

$\sigma = \frac{Q}{A}$



Consider a plane charge conductor where charges are distributed uniformly on the surface. Whose surface charge density be σ.

Now, the cylindrical Guosium surface is placed on the plane charge conductor. If r be the distance from surface to position p. The electric lines of forces are parallel to the curve surface of the cylindre whereas, it passes perpendicularly to the caps of the cylinder where the electric field intensity is same through out the cap.

Now, we know, from electric field intensity

$$\phi = E \cdot A \quad \text{--- (i)}$$

then, the surface charge density,

$$\sigma = \frac{Q}{A} \quad \text{--- (ii)} \quad \text{or,} \quad Q = \sigma \cdot A \quad \text{--- (ii)}$$

Now, from Guass's theorem,

$$\phi = \frac{Q}{\epsilon_0} \quad \text{--- (iii)}$$

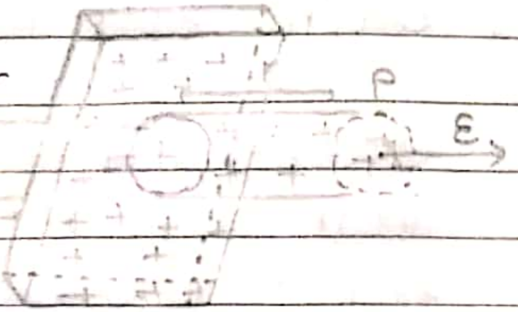
Now, solving eq^n (i), (ii) & (iii), we get,

$$E \cdot A = \frac{Q}{\varepsilon_0}$$

$$E \cdot A = \frac{\sigma \cdot A}{\varepsilon_0}$$

$$\therefore E = \frac{\sigma}{\varepsilon_0}$$

7. Electric field intensity due to infinite plane sheet..



Consider a plane charge conductor where charges are distributed uniformly on the surface. whose surface charge density be σ (sigma). Now, the cylindrical Guasium surface is placed on the plane charge conductor. If r be the distance from surface to position p. The electric lines of forces are parallel to the curve surface of the cylider whereas, it passes perpendicularly to the caps of the cylinder where the electric field intensity is same throughout the cap.

Now, we know, from electric field intensity

$$\phi = E \cdot 2A \quad \text{- - - - (i)}$$

then, the surface charge density,

$$\sigma = \frac{Q}{A} \quad \text{or, } Q = \sigma \cdot A \quad \text{- - - - (ii)}$$

Now, from Guass's theorem,

$$\phi = Q/\varepsilon_0 \quad \text{- - - (iii)}$$

Now, solving eq^n (i), (ii) & (iii), we get,

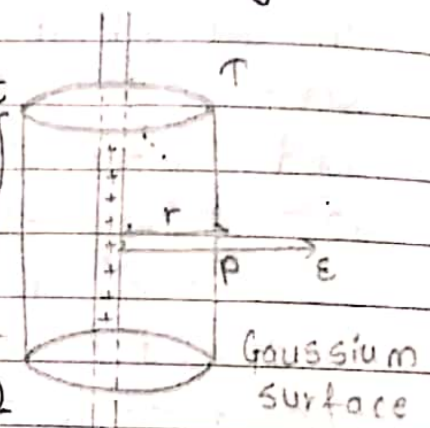
$$E \cdot 2A = \frac{q}{\epsilon_0}$$

$$\text{or, } E \cdot 2A = \frac{\sigma \cdot A}{\epsilon_0}$$

$$\text{or, } E \cdot 2 = \frac{\sigma}{\epsilon_0}$$

$$\therefore E = \frac{\sigma}{2\epsilon_0}$$

4. Electric field intensity due to linear charge density.



Consider a ~~plane~~ section of an infinite line of charge of constant linear charge density λ. We have to find an expression for electric field intensity at a perpendicular distance r from the line. To apply Gauss's theorem, a ciralar cylinder of radius r & height h is chosen at each end by plane caps normal to the axis. The electric field is constant over the cylindrical surface.

Now, we know, from electric field intensity

$$\phi = E \cdot A \quad \text{-------- (i)}$$

~~then, the surface charge density,~~

~~$\sigma =$~~

From the Gauss theorem, we have

$$\phi = \frac{1}{\epsilon_0} \times q \quad \text{----- (ii)}$$

from equation eq^n (i) & (ii), we get

or, $\mathcal{E} \cdot 2\pi r h h = \frac{1}{\epsilon_0} q \quad (\because A = 2\pi r h)$

or, $\mathcal{E} = \frac{q/h}{2\pi \epsilon_0 r}$

$\therefore \mathcal{E} = \frac{\lambda}{2\pi \epsilon_0 r} \quad \left(\because \lambda = \frac{q}{h}\right)$ ---- (iii)

The direction of the electric field is radially outward from a line of positive charge.