chapter = 11

Quantity of Heat

Good
06/07

Note
Principle of calorimetry:
Heat lost = Heat gain

Calorimetry:
Calorimetry is a experimental technique to measure heat exchange.
or
The branch of physics which deals with the measurement of quantities of heat when two bodies share the heat is known as calorimetry.

Calorimeter:
It is a device that isolates object to measure temperature changes due to heat flow.

NOTE
1 calorie heat = 4.2 Joule

Heat capacity or Thermal capacity:
The amount of heat required to raise the temp of substance by 1 degree is called thermal capacity or heat capacity.

$$Q = ms\Delta\theta$$

here $\Delta\theta = 1^\circ$

$$Q = ms$$

$\therefore s = \frac{Q}{m}$

-K represents inverse rela...

K depend on nature of body & exposed area to surrounding

SI unit is $\frac{J}{kg}$ or $\frac{cal}{gm}$.

Imp

Newton's law of cooling

Newton's law of cooling states that, "the rate of loss of heat by a body is directly proportional to the temp. difference between body and the surrounding".

Consider, a body at temp 'θ' is kept at surrounding of temp θ_s. If dQ amount of heat is lost at time dt. Then according to Newton's law of cooling

$$\frac{dQ}{dt} \propto \theta - \theta_s$$

or, $$\frac{dQ}{dt} = -K(\theta - \theta_s) \quad \text{(i)}$$

-ve sign indicates that heat is lost with time. K is constant which is depend upon nature of body & exposed surface area to the surrounding.

The amount of heat required to raise the temp of body of mass 'm' by $\Delta\theta$ is,

$$Q = ms\Delta\theta$$

Differentiating both side with respect to 't'.

$$\frac{dQ}{dt} = ms\frac{d\theta}{dt} \quad \text{(ii)}$$

From eq^n (i) & (ii)

$$ms\frac{d\theta}{dt} = -K(\theta - \theta_s)$$

or, $$\frac{d\theta}{\theta - \theta_s} = -\frac{K}{ms}dt$$

$\int \frac{1}{x} dx = \log x + c$

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Integrating both side

$$\int \frac{d\theta}{\theta - \theta_s} = -K_1 \int dt$$

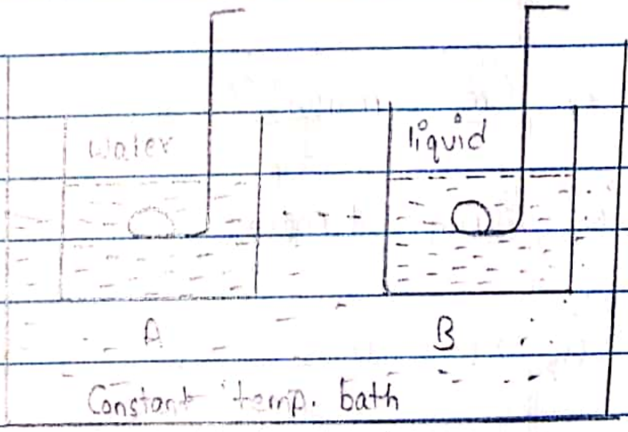
where $K_1 = \frac{K}{ms}$

$\log(\theta - \theta_s) = -K_1 t + C \quad \cdots$ (ii)

which represents the equation of straight line at 't', which verifies Newton's law of cooling.

Sp. heat capacity of liquid by method of cooling

Newton's law of cooling can be used to determine the sp. heat capacity of liquid.



This experiment is based on the principle that, if two liquids are cooled under identical condition, the rate of loss of heat of two liquids are same.

Take two calorimeters A & B of same material having mass m_1 and m_2 respectively. Let s be the sp. heat capacity of calorimeters. first calorimeter A is filled with water of mass m_w and second calorimeter B is filled with same volume of liquid m_l. S_w and S_l are specific heat capacity of water and liquid respectively. Both calorimeters are kept in constant temp bath as shown in figure.

let t_1 and t_2 be the time taken by water & liquid to cool down from θ_1°C to θ_2°C.

Now, Heat lost by water and calorimeter =

$Q = m_1 s(\theta_1 - \theta_2) + m_w s_w(\theta_1 - \theta_2)$

$= (m_1 s + m_3 s_w)(\theta_1 - \theta_2)$

Here, Rate of heat lost by water & calorimeter

$\frac{Q}{t_1} = \frac{(m_1 s + m_w s_w)(\theta_1 - \theta_2)}{t_1}$ ---- eqⁿ (i)

Ily, Rate of heat lost by liquid & calorimeter

$\frac{Q}{t_2} = \frac{(m_2 s + m_l s_l)(\theta_1 - \theta_2)}{t_2}$ --- eqⁿ (ii)

Now, two liquids are cooled under identical condition then rate of loss of heat should be equal

$\frac{Q}{t_1} = \frac{Q}{t_2}$

or, $\frac{(m_1 s + m_w s_w)(\theta_1 - \theta_2)}{t_1} = \frac{(m_2 s + m_l s_l)(\theta_1 - \theta_2)}{t_2}$

or, $(m_2 s + m_l s_l) = (m_1 s + m_w s_w) \times \frac{t_2}{t_1}$

or, $s_l = \frac{(m_1 s + m_w s_w)}{m_l} \times \frac{t_2}{t_1} - \frac{m_2 s}{m_l}$

Specific heat capacity of solid by method of mixture

Draw from book

Mass of solid $= m_1$
Mass of calorimeter with stirrer $= m_2$
Mass of water $= m_3$
Initial temp of solid $= \theta_1$
Initial temp of water & calorimeter $= \theta_2$
final temp of mixture $= \theta_3$
sp. heat capacity of solid $= s_1$
sp heat capacity of calorimeter $= s_2$
Sp heat capacity of water $= s_3$

Heat lost by solid $= m_1 s_1 (\theta_1 - \theta_3)$
Heat gain by water & calorimeter $= m_2 s_2 (\theta_3 - \theta_2) + m_3 s_3 (\theta_3 - \theta_2)$
$= (m_2 s_2 + m_3 s_3)(\theta_3 - \theta_2)$

Now, using principle of calorimetry
Heat lost = Heat gain
$m_1 s_1 (\theta_1 - \theta_3) = (m_2 s_2 + m_3 s_3)(\theta_3 - \theta_2)$

$\therefore s_1 = \dfrac{(m_2 s_2 + m_3 s_3)(\theta_3 - \theta_2)}{m_1 (\theta_1 - \theta_3)}$

Thus, the specific heat capacity of a solid can be calculated by using the method of mixture.

Ques 1) In an experiment on the sp. heat of a solid, a 200gm block of metal at 150°C is dropped in a copper calorimeter of mass 270gm containing 150cm³ of water at 27°C. The final temp is 40°C. Calculate sp heat of metal. ($S_w = 4200 J/Kg\ K$, $S_C = 390 J/Kg\ K$)

Soln:

Given,

Mass of block of metal (M_m) = 200gm

$= \left(\frac{200}{1000}\right) Kg = 0.2 Kg$

Initial temp of metal (θ_1) = 150°C

Mass of copper calorimeter (M_c) = 270gm

$= \left(\frac{270}{1000}\right) Kg = 0.27 Kg$

Volume of water = 150cm³

$M = D \times V$

$= 1 \times 150$ [∵ Density of water = 1gm/cm³)

= 150gm

$= \left(\frac{150}{1000}\right) Kg = 0.15 Kg$

Initial temp of calorimeter (θ_2) = 27°C

final temp (θ_3) = 40°C

sp heat of water (s_w) = 4200 J/Kg K

sp heat of copper calorimeter (s_c) = 390 J/Kg K

sp heat of metal (s_m) = ?

We know that, By using principle of calorimetry,

Heat lost = Heat gained

$m_m s_m(\theta_1 - \theta_3) = m_c s_c(\theta_3 - \theta_2) + m_w s_w(\theta_3 - \theta_2)$

or, $0.2 \times s_m(150 - 40) = 0.27 \times 390(40 - 27) + 0.15 \times 4200(40 - 27)$

or, $22 s_m = 9558.9$

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or, $sm = \frac{9558.9}{22}$

$\therefore$ $sm = 434.495$ J/Kg K

Ques 2.) A copper pot with mass 0.5 Kg contains 0.17 Kg of water at a temp of 20°C. A 0.25 Kg block of iron at 85°C is dropped into the pot. Find the final temp assuming no heat loss to the surrounding. [$s_{cu} = 390$ J/Kg K, $s_i = 470$ J/KgK
$s_w = \cancel{4200}$ 4190 J/Kg K]

Given,
Mass of copper pot $(M_{cu}) = 0.5$ Kg
Water contained in calorimeter $(M_w) = 0.17$ Kg
Initial temp of pot & water $(\theta_1) = 20°C$
Mass of block of iron = 0.25 Kg
Initial temp of iron $(\theta_2) = 85°C$
$s_{cu} = 390$ J/Kg K $s_i = 470$ J/Kg K $s_w = 4190$ J/Kg K
Final temp $(\theta_3) = ?$

Now,
Heat lost = Heat gained

or, $m_i s_i (\theta_2 - \theta_3) = (m_{cu} s_{cu} + m_w s_w)(\theta_3 - \theta_1)$

or, $0.25 \times 470 (85 - \theta_3) = (0.5 \times 390 + 0.17 \times 4190)(\theta_3 - 20)$

or, $117.5 (85 - \theta_3) = 907.3 (\theta_3 - 20)$

or, $85 - \theta_3 = \frac{907.3}{117.5} (\theta_3 - 20)$

or, $239.434 = 8.7217 \theta_3$

or, $\theta_3 = \frac{239.434}{8.7217}$

$\therefore$ $\theta_3 = 27.45°C$

Ques 3.) A ball of copper weighing 400gm is transferred from a furnish to a copper calorimeter of mass 300gm and

containing 1kg of water at 20°C, Temp of water rises to 50°C. What is the original temp of the ball?
[S_{cu} = 400 J/Kg K], S_w = 4200 J/Kg K]
Ans: 860 °C

Ques 4.) A copper calorimeter of mass 300gm contains 500gm of water at 15°C. A 560gm of aluminium ball at temp of 100°C is dropped in the calorimeter and the temp is increased to 25°C. Find the specific heat capacity of aluminium.
[S_{cu} = 400 J/Kg K, S_w = 4200 J/Kg K]
Ans: 528.5 J/Kg K

3) Soln:-
Mass of copper ball (M_c) = 400 gm = $\frac{400}{1000}$ Kg = 0.4 Kg
Mass of copper calorimeter (M_{ca}) = 300 gm
$= \frac{300}{1000}$
= 0.3 Kg

Mass of water in calorimeter (M_w) = 1 kg
Initial temp of calorimeter & water (θ_1) = 200°C
Final temp (θ_2) = 50 °C
Original temp of ball (θ_3) = ?
S_{cu} = 400 J/Kg K S_w = 4200 J/Kg K
Now,
Using principle of calorimetry,
Heat lost by ball = Heat gained by calorimeter & water
or, $m_c S_{cu}(\theta_3 - \theta_2) = (m_{ca} S_{cu} + m_w S_w)(\theta_2 - \theta_1)$
or, $0.4 \times 400(\theta_3 - 50) = (0.3 \times 400 + 1 \times 4200)(50 - 20)$
or, $160(\theta_3 - 50) = 129600$
or, $\theta_3 - 50 = \frac{12960\cancel{0}}{16\cancel{0}}$

Latent = hidden

or, $\theta_3 - 50 = 810$

or, $\theta_3 = 810 + 50$

$\therefore$ $\theta_3 = 860^\circ C$

1) Given

Mass of copper calorimeter $(M_{cu}) = 300gm = \frac{300}{1000} Kg = 0.3 Kg$

Mass of water $(M_w) = 500gm = \frac{500}{1000} Kg = 0.5 Kg$

Initial temperature of calorimeter & water $(\theta_1) = 15^\circ C$

Mass of aluminium ball $(M_{al}) = 560gm$

$= \frac{560}{1000} Kg = 0.56 Kg$

Initial temp of ball $(\theta_2) = 100^\circ C$

final temperature $(\theta_3) = 25^\circ C$

$s_{cu} = 400 J/Kg\ K$ $\quad s_w = 4200 J/Kg\ K$ $\quad s_{al} = ?$

We know

Heat lost = Heat gained

or, $m_{al} s_{al} (\theta_2 - \theta_3) = (m_{cu} s_{cu} + m_w s_w)(\theta_3 - \theta_1)$

or, $0.56 \times s_{al} (100 - 25) = (0.3 \times 400 + 0.5 \times 4200)(25 - 15)$

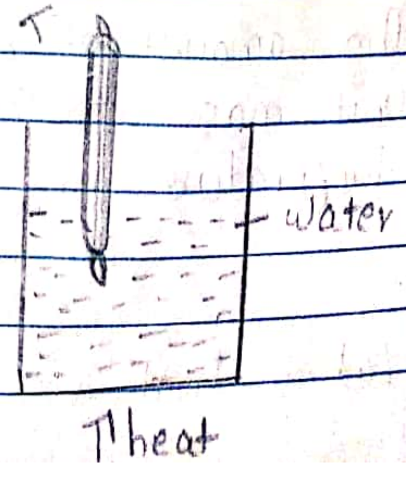
or, $0.56 s_{al} \times 75 = 22200$

or, $0.56 s_{al} = \frac{22200}{75}$

or, $s_{al} = \frac{296}{0.56}$

$\therefore$ $s_{al} = 528.57 J/Kg\ K$

Latent heat:



The amount of heat required to change the state of unit mass of substance from solid to liquid or liquid to gas is called latent heat. without changing temperature

Now, latent heat is directly proportional to mass of a body.

$$Q \propto m$$

$$Q = lm, \quad l = \text{Latent heat}$$

$$l = \frac{Q}{m}$$

$$\text{SI unit} = \frac{J}{Kg} \text{ or } \frac{Cal}{gm}$$

Latent heat of fusion: The amount of heat required to change the state of unit mass of substance from solid to liquid at constant temperature is latent heat of fusion.

Latent heat of fusion of ICE

The amount of heat required to change 1gm of ice at 0°C to water at same temperature is called latent heat of fusion of ice.

$$L_F = 3.36 \times 10^5 J/Kg \text{ or } 80 cal/gm$$

Latent heat of Vaporization:

The amount of heat required to change the state of unit mass of substance from liquid to gas at constant temperature is called latent heat of vaporization.

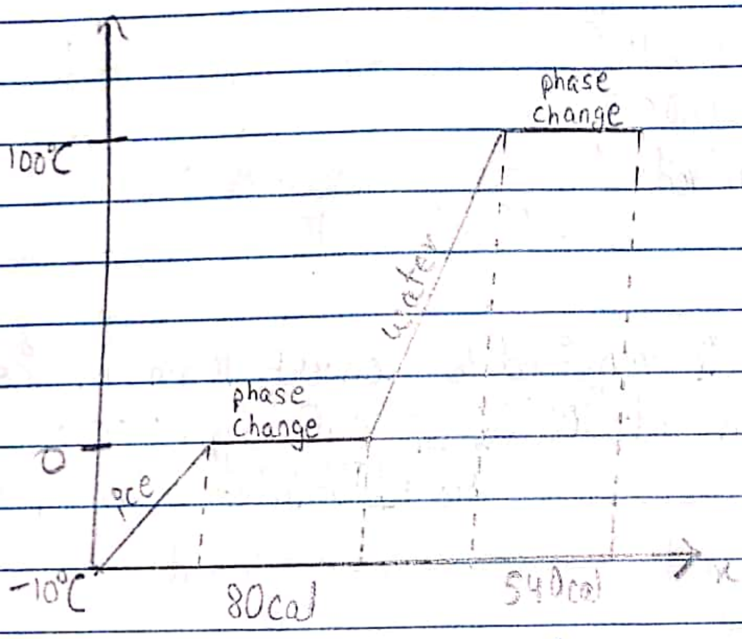
Latent heat of ~~fusi~~ Vaporization of Water:

The amount of heat required to change 1gm of water at 100°C to vapour at same temperature is called latent heat of vaporization of water.

$$L_v = 2.26 \times 10^6 \ J/Kg \quad \text{or} \quad 540 \ cal/gm$$

Note

$$\boxed{\text{ice } -10^\circ C} \xrightarrow[\Delta]{2100 \ Sp} \boxed{\text{ice } 0^\circ C} \xrightarrow[\Delta]{3.36\times10^5 \ Lf} \boxed{\text{water } 0^\circ C} \xrightarrow[\Delta]{4200 \ Sp} \boxed{\text{water } 100^\circ C} \xrightarrow[\Delta]{2.26\times10^6 \ Lf} \boxed{\text{vapour } 100^\circ C}$$



Vapourization curve:

Ques

Numericals

Ques How much heat is required to convert 5Kg of ice at -10°C into steam at 100°C? $S_i = 2100 \ J/Kg \ K$

Soln:-

$$\boxed{\text{ice } -10^\circ C} \xrightarrow[\Delta]{m, s, \Delta\theta \ (Q_1)} \boxed{\text{ice } 0^\circ C} \xrightarrow[\Delta]{m, Lf \ (Q_2)} \boxed{\text{water } 0^\circ C} \xrightarrow[\Delta]{m, S_w \Delta\theta \ (Q_3)} \boxed{\text{water } 100^\circ C} \xrightarrow[\Delta]{m, Lv \ (Q_4 =)} \boxed{\text{vapour } 100^\circ C}$$

Sol^n:

$Q_1 = m_i s_i \Delta\theta$

$= 5 \times 2100 \times 10$

$\therefore Q_1 = 105000 J$

Now, $Q_2 = m_i L_f$

$= 5 \times 3.36 \times 10^5$

$\therefore Q_2 = 1680000 J$

Again, $Q_3 = m_i s_w \Delta\theta$

$= 5 \times 4200 \times 100$

$\therefore Q_3 = 2100000 J$

And, $Q_4 = m_i L_v$

$= 5 \times 2.26 \times 10^6$

$\therefore Q_4 = 11300000 J$

Total heat required $(Q) = Q_1 + Q_2 + Q_3 + Q_4$

$= 15185000 J$

Ques How much heat is required to convert 10gm of ice at $-10°C$ into steam at $100°C$? $s_i = 0.5 \text{ cal gm}^{-1} °C^{-1}$

$s_w =$ ~~1J/Kg°C~~ 1 Cal/gm°C

Sol^n:

-10°C	$m_i s_i \Delta\theta$ 0.5 cal/gm°C	0°C	$m_i L_f$ 80 cal/gm	0°C	$m_w s_w \Delta\theta$ 1 cal/gm°C	100°C	$m_w L_v$ 540 cal/gm	100°C
ice	$Q_1 =$	ice	$Q_2 =$	water	$Q_3 =$	water	$Q_4 =$	vapour

$Q_1 = m_i s_i \Delta\theta$

$= 10 \times 0.5 \times 10°$

$= 50$ ~~J~~ Cal

$Q_2 = m_i L_f$

$= 10 \times 80$

$= 800$ Cal

$Q_3 = m_w s_w \Delta\theta$

$= 10 \times 1 \times 100$

$= 1000\,Cal$

$Q_4 = m_w\, l_v$

$= 10 \times 540$

$= 5400\,Cal$

Now, Total heat required $(Q) = Q_1 + Q_2 + Q_3 + Q_4$

$= (50 + 800 + 1000 + 5400)\,Cal$

$= 7250\,Cal$ //

Ques From what height should a block of ice be dropped in order that it may melt completely? [$L_f = 3.36 \times 10^5\,J/kg$, $g = 10\,m/s^2$]

→ Let 'm' be the mass of ice & 'h' be the height from which it should be dropped to melt completely.

P.E (Potential energy) of ice at 'h' = mgh

latent heat = mlf

Now, $\cancel{m}gh = \cancel{m}lf$

or, $10 \times h = 3.36 \times 10^5$

or, $h = \frac{3.36 \times 10^5}{10}$

$\therefore h =$

Ques From what height a block of ice should be dropped in order that it may melt completely. It is assumed that 20% of energy of fall is retained by ice. [$l = 3.36 \times 10^5\,J/k$,

→ Let 'm' be the mass of ice & h be the height from which it should be dropped to melt completely.

P.E = mgh

latent heat = mlf

By the question,

20% of potential energy = latent heat of fusion

20% of $mgh = mL_f$

or, $\frac{\cancel{20}}{\cancel{100}} \times \cancel{m} \times \cancel{10} \times h = \cancel{m} \times 3.36 \times 10^5$

or, $h = \frac{3.36 \times 10^5}{2}$

$\therefore h = 168000$

Question: Calculate the difference in temp. between the water at the top and bottom of a waterfall 200 m high, [Specific heat of water = 4200 J/Kg K^{-1}]

Here,
Let m be the mass of the water
& h be the height of waterfall
~~Now~~ $\Delta\theta$ be the temperature difference.

Potential energy = mgh
specific heat capacity of water = $ms\Delta\theta$

Now,
loss in P.E. = ~~specific heat of water~~ Heat gain of water

or, $\cancel{m}gh = \cancel{m}s\Delta\theta$

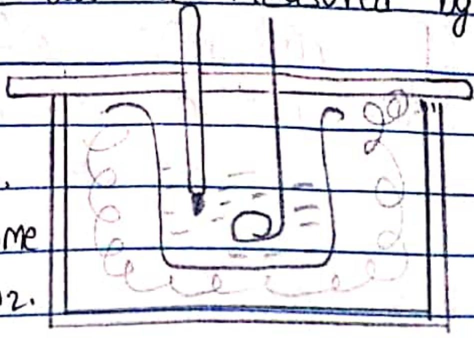
or, $10 \times 200 = 4200 \times \Delta\theta$

or, $\frac{2000}{4200} = \Delta\theta$

$\therefore \Delta\theta = 0.47\,°C$

Determination of latent heat of fusion of ice by the method of mixture

The latent heat of fusion of ice can be measured by using the method of mixture.



Let us consider a dry & clean calorimeter with stirrer of mass m_1. It is filled one third of its volume with warm water whose mass is m_2.

Let the initial temp of caloriemeter and its content be θ_1°C. Some small ice pieces are taken and water is soaked with paper. Then the ice pieces are dropped into water and mixture is then stirred gently until the temperature of calorimeter and mixture decreases to lower value. Let the final temperature of the mixture be θ. Finally, the calorimeter and the mixture is weighted to find the mass of ice. Let m_1 be mass of ice.

Now,

From the principle of caloriemetry,

Heat gained by ice in two steps:

1. When ice at 0°C change to water at 0°C =

l = latent heat of fusion of ice

2.) When the temp of water from formed ice increases from 0°C to θ°C $= m_3 s_2(\theta - 0)$

$= m_3 s_2 \theta$

∴ total amount of heat gained by ice $= m_3 l + m_3 s_2 \theta$

Heat gained = Heat lost

$m_3 l + m_3 s_2 \theta =$ Heat lost $(m_1 s_1 + m_2 s_2)(\theta_1 - \theta)$

or, $m_3 l + m_3 s_2 \theta = (m_1 s_1 + m_2 s_2)(\theta_1 - \theta)$

or, $l = \frac{(m_1 s_1 + m_2 s_2)(\theta_1 - \theta)}{m_3} - s_2\theta$

$\therefore$ This relation gives the latent heat of fusion of ice

Determination of latent heat of steam by the method of mixture

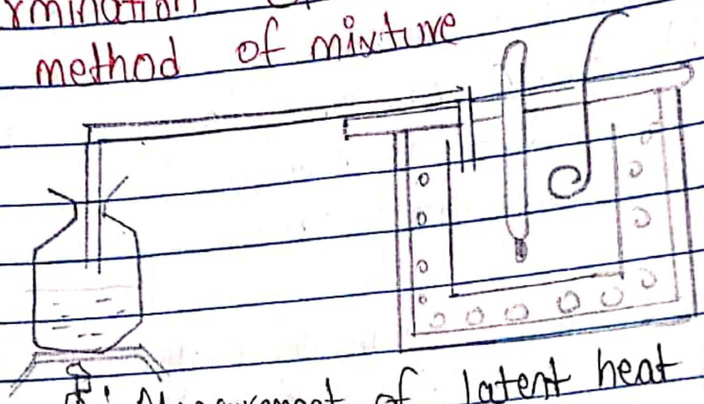


fig: Measurement of latent heat of vaporization of water

Let us consider a clean and dry caloriemeter of mass m_1 and fill it with one third of its volume with water of mass 'm_2' of water. Let θ_1°C be the initial temp of caloriemeter & its content. Let s_1 and s_2 be the sp heat capacity of caloriemeter and water heat steam is passed into the caloriemeter from the boiler. let m_3 be mass of steam, Suppose the final temp of mixture is θ_2°C.

So, the amount of heat gained by caloriemeter & water from θ_1°C to θ_2°C

$$m_1 s_1 (\theta_2 - \theta_1) + m_2 s_2 (\theta_2 - \theta_1)$$

$$(m_1 s_1 + m_2 s_2)(\theta_2 - \theta_1) \quad \text{--- (i)}$$

Steam looses heat in two steps:

1. The amount of heat lost by steam from 100°C steam to water at 100°C $= m_3 l$
2. The amount of heat lost by steam from 100°C water form by steam to θ_2°C water

$= m_3 s_2 (100-\theta_2)$

Total heat lost by steam $= m_3 l + m_3 s_2 (100-\theta_2)$

from principle of calorimetry,

Heat lost = Heat gain

$m_3 l + m_3 s_2 (100-\theta_2) = (m_1 s_1 + m_2 s_2)(\theta_2 - \theta_1)$

or, $m_3 l = (m_1 s_1 + m_2 s_2)(\theta_2 - \theta_1) - m_3 s_2 (100-\theta_2)$

or, $L_v = \frac{m_1 s_1 + m_2 s_2 (\theta_2 - \theta_1)}{m_3} - s_2 (100-\theta_2)$

from this we can determine the latent heat of steam by the method of mixture.

Hard Question

Ques find the result of mixing 0.8Kg ice at -10°C with 0.8Kg of water at 80°C? $S_i = 2100 J/Kg\ K$ $L_f = 3.36 \times 10^5 J/Kg$

Given,

Mass of ice $(m_i) = 0.8$ Kg, $S_i = 2100 J/Kg\ K$, $L_f = 3.36 \times 10^5$

Mass of water $(m_w) = 0.8$ Kg $S_w = 4200 J/Kg\ K$

Amount of heat required by ice to melt from -10°C to 0°C water.

ice -10°C → ice 0°C → water 0°C

$Q_1 = m_i s_i \Delta\theta + m_i L_f$

$= 0.8 \times 2100 (\cancel{0} - \cancel{-10}) + 0.8 \times 3.36 \times 10^5$

$= 0.8 \times 2100 \times 10 + 0.8 \times 3.36 \times 10^5$

$= 16800 + 268800$

$\therefore Q_1 = 285600$

Again, Amount of heat available from water at 80°C

$Q_2 = m_w s_w \Delta\theta$

$= 0.8 \times 4200 \times 80$

$\therefore Q_2 = 268800$

Since, $Q_1 > Q_2$ all ice does not melt,

let m be the mass of ice that melts,

$268800 = 16800 + mLf$

$268800 = 16800 + m \times 3.36 \times 10^5$

$\frac{268800 - 16800}{3.36 \times 10^5} = m$

$m = 0.75\ Kg$

Now,

Result of mixing is $(0.8 + 0.75)$ Kg of water

$= 1.55$ Kg water

and $0.8 - 0.75 = 0.05$ Kg ice

What is the result of mixing 100gm of ice at 0°C into 100gm of water at 20°C in an iron vessel of mass 100g? $L_F = 80$ cal/gm $S_w = 1$ cal/gm/°C $S_{iron} = 0.1$ Cal/gm/°C

$S_i =$

Given,

Mass of ice $(m_i) = 100gm$, $L_F = 80$ cal/gm

Mass of water $(m_w) = 100gm$, $S_w = 1$ cal/gm/°C

Mass of iron vessel $(m_i) = 100g$, $S_i = 0.1$ cal/gm/°C

Now,

the amount of heat required for ice to melt,

$Q_1 = m_i L_f$

$= 100 \times 80$

$= 8000$ Cal

Amt of heat provided by water & calorimetry,

$Q_2 = m_w s_w \Delta\theta + m_i s_i \Delta\theta$

$= 100 \times 1 \times 20 + 100 \times 0.1 \times 20$

$= 2000 + 200$

$= 2200$ Cal

Hence, $Q_1 > Q_2$

let m be mass of ice that melts into water,

$2200 = m \times Lf$

or, $2200 = m \times 80$

or, $m = \frac{2200}{80}$

$\therefore$ $m = 27.5\,gm$

Now, Result of mixing is $(100 + 27.5)gm$ of water

$= 127.5gm$

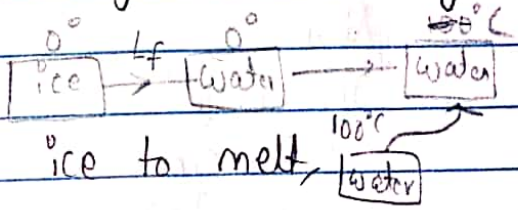
and $100 - 27.5gm = 72.5gm$ ice.

Ques What is the result of mixing 100gm of ice at 0°C and 100gm of water at 100°C. $L_f = 3.36 \times 10^5\, J/Kg$, $S_w = 4200\, J/Kg\,K$

Given,

Mass of ice $(M_i) = 100gm = \frac{100}{1000} = 0.1Kg$, $L_f = 3.36 \times 10^5 J/Kg$

mass of water $(M_w) = 100gm = \frac{100}{1000} = 0.1Kg$ $S_w = 4200J/Kg\,K$



We know,

The amount of heat required for ice to melt,

$Q_1 = L_f \times m_i$

$= 3.36 \times 10^5 \times 0.1$

$= \cancel{3.36 \times 10^6 J}$ $33600J$

The amount of heat provided by water,

$Q_2 = m_w S_w \times (\Delta\theta)$

$= \cancel{100\times}\, 0.1 \times 4200 \times 100$

$= 42000J$

Since, $Q_2 > Q_1$ all ice melts and remaining heat energy increases the temp of mixture

Let, θ be the final temp of mixture

Heat loss = Heat gain

or, $m_w s_w \Delta\theta = m_i L_f + m_i s_w \Delta\theta$

or, $0.1 \times 4200 \times (100 - \theta) = 0.1 \times 3.36 \times 10^5 + 0.1 \times 4200 \times \theta$

or, $420(100 - \theta) = 33600 + 420\theta$

or, $42000 - 420\theta = 33600 + 420\theta$

or, $42000 - 33600 = 420\theta + 420\theta$

or, $8400 = 840\theta$

or, $\theta = \frac{8400}{840}$

$\therefore$ $\theta = 10^\circ C$

Ques 25 gm of water at 100°C is mixed with 25 gm of ice at 0°C. Find the result of mixing. $L_f = 80 cal/gm$, $s_w = 1 cal/gm^\circ C^{-1}$

Given,
Mass of water = 25gm $s_w = 1 cal/gm^\circ C^{-1}$
Mass of ice = 25gm $L_f = 80 cal/gm$

Now,
The amount of heat required for ice to melt

$Q_1 = L_f m_i$
$= 80 \times 25$
$= 2000 \text{ Cal}$

Heat provided by water,

$Q_2 = m_w s_w \Delta\theta$
$= 25 \times 1 \times 100$
$= 2500 \text{ Cal}$

Since $Q_2 > Q_1$ all ice melts and remaining heat energy raises the temp of a mixture.

Let θ be final temp of mixture,

Heat lost = Heat gain

or, $m_w s_w \Delta\theta = L_f m_i + m_i s_w \Delta\theta$

or, $25 \times 1 (100 - \theta) = 80 \times 25 + 25 \times 1 \times \theta$

or, $2500 - \theta = 2000 + 25\theta$

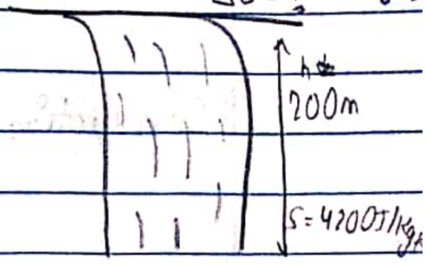
or, $500 = 50\theta$

$\therefore$ $\theta = 10^\circ C$

HW

Ques Calculate the difference in temp bet^n water at the top and bottom of waterfall 200m high. (sp heat capacity of water = 4200 J/kg K)

$\Delta\theta = ?$



Given,

Height (h) = 200m

$S_w = 4200 \, J/kg \, K$

Now,

Potential energy (P.E) of water = mgh

~~latent~~ specific heat capacity of water = $m S_w \Delta\theta$

Now,

$mgh = m S_w \Delta\theta$

or, $\cancel{m} \times 10 \times 200 = \cancel{m} \times 4200 \times \Delta\theta$

or, $\Delta\theta = \frac{2000}{4200}$

$\therefore \Delta\theta = 0.48°C$