Dynamics

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Force is the central concept of mechanics. Force is applied in term of a pull or a push.

When force is applied on a body either it comes on a rotional motion or a translation motion.

Inertia: Inability of a body to change its body from motion to rest or from rest to motion is known as inertia.

① Inertia of rest
② Inertia of motion
③ Inertia of direction

① Inertia of rest: Inability of a body to change its position from rest to motion is known as inertia of rest. for eg: when a carpet is hit by the stick then dust particles flies away due to inertia of rest.

ii) Leaves & fruits are fall off from the tree when branch is shake due to inertia of rest.

② Inertia of motion: Inability of a body to change its position from motion to rest is known as inertia of motion. for eg:

i) Fan keeps on rotating for some times when switched off due to inertia of motion.

ii) An athelete takes a long run before taking a long jump due to inertia of motion.

3) Inertia of direction. Inability of a body to change its direction is known as inertia of direction. For eg:

i) Dust particles flies off from the wheel of a vehicle due to inertia of direction.

ii) A stone tied on a string & whirled in a circular path. When the string breaks, stone flies off due to inertia of direction.

* Newton's first law of motion: According to Newton's first law of motion, everybody in universe either remains in rest or moves with a uniform motion until it is compelled by an external force. i.e. $u = v = constant$.

Momentum (P): Momentum is defined as the product of mass times velocity.

Mathematically,

Momentum (P) = mass × velocity

$$P = mv$$

It's unit in SI = $kgms^{-1}$ = NS

* Newton's second law of motion: According to Newton's second law of motion, the magnitude of net force applied is directly proportional to rate of change of momentum with respect to time.

Mathematically,

$$F \propto \frac{dP}{dt}$$

$$\text{or, } F = k\frac{dP}{dt} \qquad \because k = 1$$

$$or,\ F = \frac{dp}{dt}$$

$$or,\ F = \frac{d(mv)}{dt}$$

$$or,\ F = m\frac{dv}{dt} \qquad \because a = \frac{dv}{dt}$$

$$\therefore F = ma$$

It's unit in SI = $kgms^{-2}$ = Newton.

1 Newton of force: It is defined as when a unit mass is accelerated by 1 m per second square ($1 m/s^2$).

Newton's third law of motion: According to Newton's third law of motion, for every action there will be equal & opposite reaction

for eg: i) when a person walks, pushes the ground backward as a result person moves forwards.

ii) Swimmer pulls the water backward as the result swimmer moves forward.

Newton's IInd law is a real law:

Newton's IInd law is known as a real law because Newton's 1st law & the 3rd law can be derived from second law.

For Newton's 1st law of motion:

We know, $F = ma$ — ①

If there is no external force applied Then $F = 0$, so eqn ① becomes

$F = ma$

$\therefore m \neq 0$

or, $0 = ma$

or, $a = 0$

or, $\frac{dv}{dt} = 0$

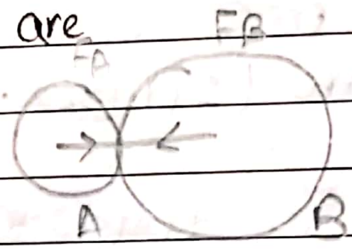
Now, Integrating above eq^n, we get:

$\int dv = 0$

or, $v =$ constant

Newton's IIIrd law of motion:

Consider two obj A & B are moving on a st. line & they collide for a certain time.



Change in Momentum of A: $\Delta P_A = F_A \times t$

Similarly

Change in momentum of B: $\Delta P_B = F_B \times t$

So, Total change in Momentum $\Delta P = \Delta P_A + \Delta P_B$

If there is no external force acted then $\Delta P = 0$.

So, $\Delta P = \Delta P_A + \Delta P_B$

or, $0 = F_A \cdot t + F_B \cdot t$

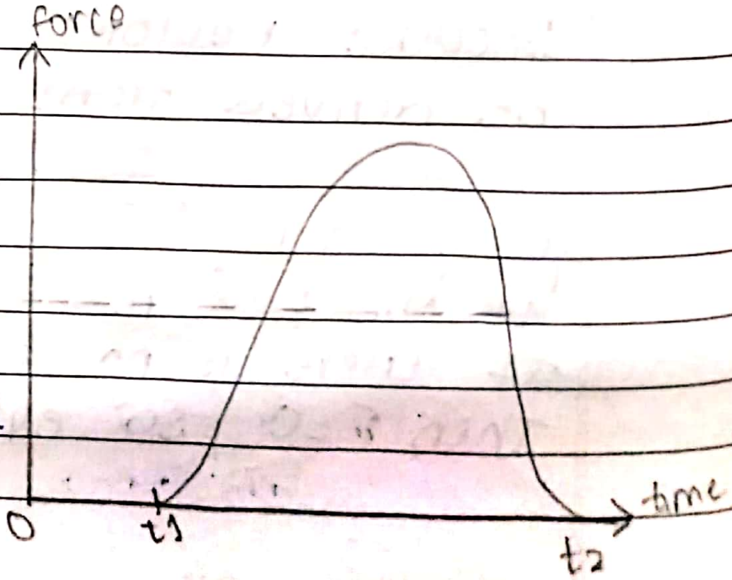
or, $0 = (F_A + F_B)$

or, $F_A = -F_B$

Impulse:

Sometimes, large amount of force acts between a short ~~period~~ interval of time. Fav

And such force increases from 0 ~~and~~ to maximum & again becomes zero.



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Impulsive force is defined as the product of average force & time of impact.

Impulse = Average force x time of impact

$I = F_{av} \Delta t$

We know, from Newton's 2nd law of motion.

$$F = \frac{dp}{dt}$$

or, $dp = F dt$

Integrating above eqn we get

or, $\int_{P_1}^{P_2} dp = \int_0^t F\, dt$

or, $(P)_{P_1}^{P_2} = F(t)_0^t$

or, $P_2 - P_1 = F(t-0)$

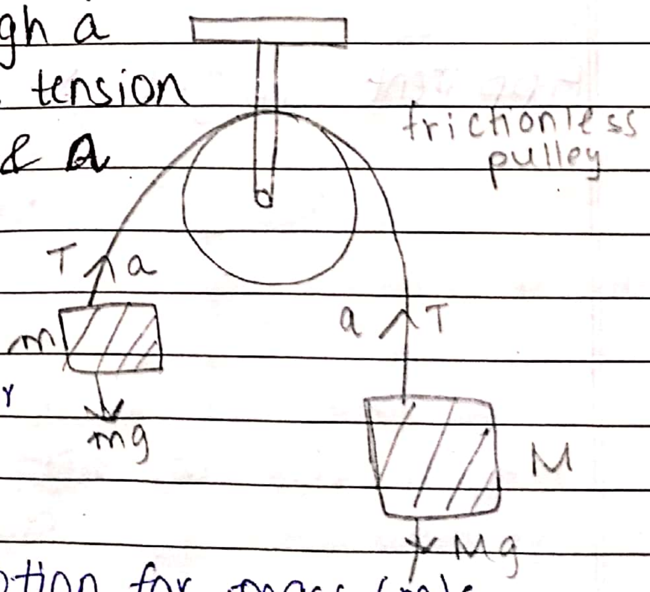
or, $P_2 - P_1 = F.t$

change in momentum = Impulse

Application of laws of motion:

① Mass-pulley system:

Consider a frictionless pulley where two masses M & m passes through a string. If t be the tension acting on the string & a be the acceleration produced by mass.



Equation of motion for mass (M):

$Mg - T = Ma$ — (i)

Similarly eqn of motion for mass (m):

$T - mg = ma$ — (ii)

$T = ma + mg$

Now, solving eq^n (i) & (ii), we get.

or, $Mg - T = Ma$

or, $Mg - (ma + mg) = Ma$

or, $Mg - ma - mg = Ma$

or, $Mg - mg = Ma + ma$

or, $(M - m)g = (M + m)a$

or, $\frac{(M-m)}{(M+m)} g = a$

Substituting value of 'a' in eq^n (ii), we get

$Mg - T = Ma$

or, $Mg - Ma = T$

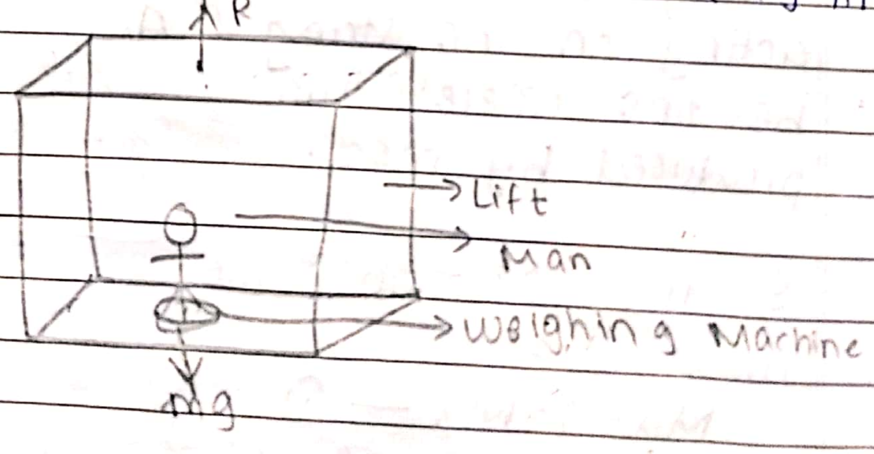
or, $Mg - M\left\{\frac{M-m}{M+m}\right\}g = T$

or, $\frac{Mg(M+m) - M(M-m)g}{(M+m)} = T$

or, $\frac{\cancel{M^2g} + Mmg - \cancel{M^2g} + Mmg}{(M+m)} = T$

or, $\frac{2Mmg}{M+m} = T$

Apparent weight of a person in a ~~##~~ lift:



Consider is a man having mass 'm' is standing on a weighing machine inside the

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lift. 'R' be the normal reaction acting upward whereas 'mg' be the weight acting downward.

1) When lift is at rest:
2) When lift is uniformly moving upward:
3) When lift is uniformly moving downward:

Since, there is no acceleration produced in the lift i.e $a=0$.

We have,

$$R - mg = m\cancel{a}^{0}$$
$$\text{or, } R - mg = 0$$
$$\text{or, } R = mg$$

Or,
$$\text{or, } mg - R = m\cancel{a}^{0}$$
$$\text{or, } mg - R = 0$$
$$\text{or, } mg = R$$

Hence, apparent weight is equals to real weight.

4) When lift is accelerating upward with acceleration a:

$$R - mg = ma \quad [ma = F]$$
$$\text{or, } R = ma + mg$$
$$\text{or, } R = m(a+g)$$

Hence apparent weight is greater than real weight.

5) When lift is accelerating downward:

When lift is accelerating downward with acceleration a.

$$mg - R = ma$$
$$\text{or, } mg - ma = R$$
$$\text{or, } m(g-a) = R$$

Hence apparent weight is less than real weight.

6) When lift is at free fall: $(a=g)$

$$mg - R = ma$$

$$\text{or, } mg - ma = R$$

$$\text{or, } m(g-g) = R$$

$$0 = R$$

Hence apparent weight will be zero.

7) When lift is accelerating greater than g: $(a>g)$

$$mg - R = ma$$

$$mg - ma = R$$

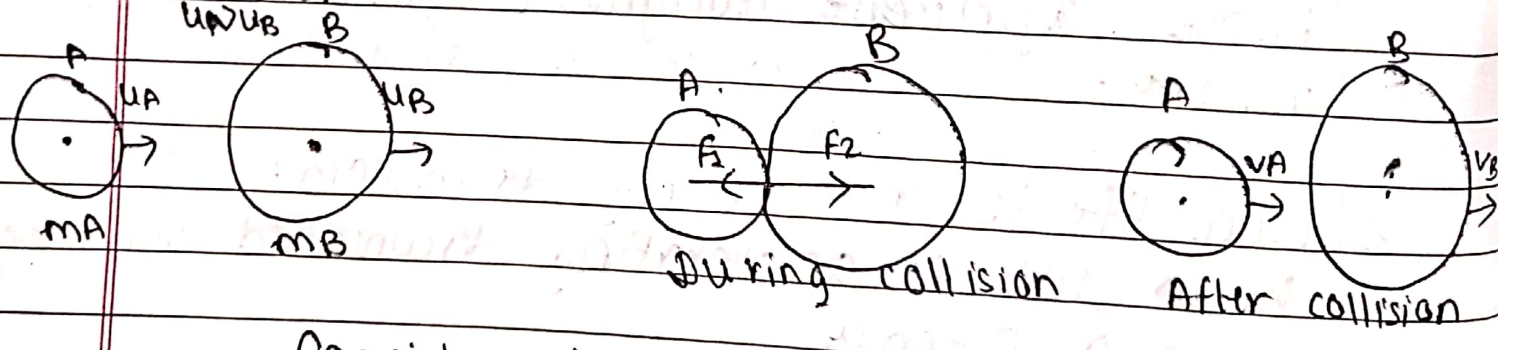
$$m(g-a) = R$$

$$-ve = R$$

Hence man will stick on the ceiling of a lift.

Principle of conservation of linear momentum:

According to principle of conservation of linear momentum, if there is no external force acting on the system, then momentum before collision is equals to momentum after collision.



Consider two object A & B having mass m_A & m_B moving with initial velocity u_A & u_B i.e $u_A > u_B$. They collide for a short interval of time & after collision they ~~collide for a~~ final velocity of m_A & m_B be V_A & V_B respectively.

Then, Force acting on A by B is given by

$$F_1 = \frac{m_A v_A - m_A u_A}{t}$$

Similarly, force acting on B by A is given by

$$F_2 = \frac{m_B v_B - m_B u_B}{t}$$

We know, from Newton's 3rd law of motion

$$F_1 = -F_2$$

or, $\frac{m_A v_A - m_A u_A}{\cancel{t}} = -\left\{\frac{m_B v_B - m_B u_B}{\cancel{t}}\right\}$

or, $m_A v_A - m_A u_A = -m_A v_B + m_B u_B$

or, $m_A v_A + m_B v_B = m_A u_A + m_B u_B$

or, $\boxed{m_A u_A + m_B u_B = m_A v_A + m_B v_B}$

Momentum before collision = Momentum after collision.

Sample Example. Pg: 101

A cricket ball of mass 0.2 kg moving with a velocity of 20 ms^{-1} is brought to rest by a player in 0.1 second. Find the impulse to the ball & the average force applied by the player.

Soln: Given,

Mass of cricket ball (m) = 0.2 kg

Initial velocity (u) = 20 m/s

Final velocity (v) = 0

Time taken (t) = 0.1 s

Impulse to the ball (I) = ? Average Force (F) = ?

Since the impulse to the ball is

$I = F.t = mu - mv = m(u-v) = 0.2(20-0) = 4\ kgms^{-1}$

Now, the average force applied by the player in

$$F = \frac{\text{change in linear momentum}}{\text{time taken}}$$

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$= \frac{m(u-v)}{t} = \frac{0.2(20-0)}{0.1} = \frac{4}{0.1} = 40\ N$

Thus, the impulse to the ball is $4\ kg\ ms^{-1}$ & the average force applied by the player is 40N.

Pg: 104

A ball A of mass 0.1 kg moving with a velocity of $6 ms^{-1}$ collides directly with a ball B of mass 0.2 kg. If A had rebounded with a velocity of $2 ms^{-1}$ in the opposite direction after the collision, what would be the new velocity of B?

Sol^n: Here,

Mass of ball A$(m_1) = 0.1$ kg

Mass of ball B$(m_2) = 0.2$ kg

Initial velocity of ball A$(u_1) = 6\ ms^{-1}$

Initial velocity of ball B $(u_2) = 0$

rebounded final velocity of ball A$(v_1) = -2\ ms^{-1}$

final velocity of ball B$(v_2) = ?$

If ball A is rebounded in opposite direction after the collision, then

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

or, $0.1 \times 6 + 0.1 \times 0 = 0.1 \times (-2) + 0.2 \times v_2$

or, $0.6 + 0 = -0.2 + 0.2\ v_2$

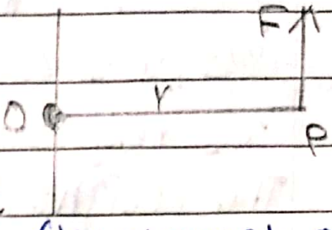
$\therefore v_2 = \frac{0.8}{0.2} = 4 ms^{-1}$

Thus, the new velocity of ball B is $4 ms^{-1}$.

~~Principle of~~ Moment of force

The moment of force is called torque & it is denoted by τ. So,

moment of force = force × perpendicular distance
or, τ = force × perpendicular distance of the pivot from the line of action of the force
= force × moment arm
$\therefore \tau = F \times r$



In the form of a vector, we can write
$\vec{\tau} = \vec{r} \times \vec{F}$

fig: Moment of force

Unit: The unit of moment of force (τ) is N-m in the SI system.

Principle of Moments

Statement: The principle of moment states that for any object in equilibrium, the sum of the clockwise moments about any point provided by the forces acting on the object equals the sum of the anti-clockwise moments about that same point. In other words, if a body is in equilibrium under the action of several forces at different points of the body, the algebraic sum of the moment of all the forces about any point is zero i.e.
sum of clockwise moment = sum of anticlockwise moments
This is known as the principle of the moment.

Verification: Suppose a one-meter wooden scale AB suspends with the help of a thread on a stand. Adjust this scale in such a way that it can be in equilibrium i.e it can stay horizontally. The position of thread O on the scale of this state is the position of the center of gravity. Suspend three different weights on a scale with the

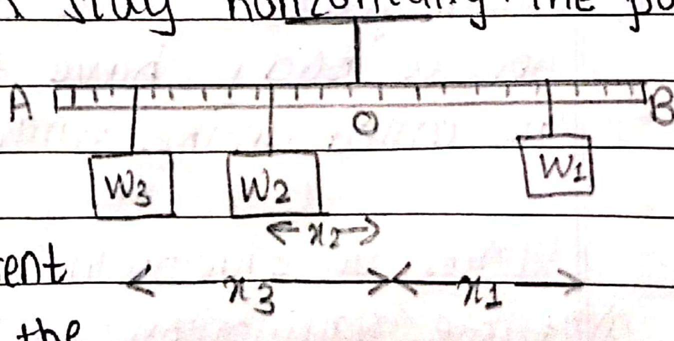


fig: Verification of moments of forces

help of a thread on both sides of the scale i.e. along with OA & OB.

Referring to the figure, three weights W_1, W_2 & W_3 are placed at distances x_1, x_2 & x_3 respectively from the center of gravity O. The weight W_1 produces a clockwise moment whereas weights W_2 & W_3 produce anticlockwise moments about the center of gravity O. According to the principle of the moment, the sum of the clockwise moment is equal to the sum of the anticlockwise moment in equilibrium condition.

$$W_1 x_1 = W_2 x_2 + W_3 x_3$$

This verifies the principle of moments. This experiment can be performed for different sets of weights also.

Parallel Forces

If several forces acting on a body are parallel, these forces are called parallel forces. There are two types of parallel forces:-

a) Like parallel force: If the parallel forces acting on a body have the same direction, the forces are called like parallel forces.

b) Unlike parallel forces: If the parallel forces acting on a body have the opposite direction, the forces are called unlike parallel forces.

States of Equilibrium

a) stable equilibrium: A body is in stable equilibrium

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if it returns to its equilibrium position after it has been displaced slightly. In this equilibrium, the C.G of the body lies low & when the body is displaced from its equilibrium position, the C.G at this position is higher than before. In a stable equilibrium, a body has the minimum potential energy.

b) Unstable equilibrium: A body is in unstable equilibrium if it does not return to its equilibrium position after it has been displaced slightly & does not remain in the displaced position. When the body is displaced from its equilibrium position, its C.G is lower than before. In an unstable equilibrium, a body has the maximum potential energy.

c) Neutral equilibrium: A body is in neutral equilibrium if it always stays in the displaced position after it has been displaced slightly. The kind of equilibrium of a body so placed that when moved slightly it neither tends to return to its former position nor depart more widely from it. In this equilibrium, the height of the C.G of the body does not change but remains at the same height from the base in all displaced positions & its potential energy remains constant.

Conditions for a body to be in stable equilibrium

For a body to be in equilibrium, the resultant force acting on the object must be zero and the resultant moment must be zero. The main conditions for the equilibrium of a rigid body are

a) For a body to be in translational equilibrium, the vector sum of all forces acting on it should be zero. Mathematically, $\Sigma F = 0$.

b) For a body to be in rotational equilibrium, the vector sum of moments of all forces acting on it should be zero. Mathematically, $\Sigma \tau = 0$.

c) For a body to be in mechanical equilibrium (both translational & rotational), the vector sum of all forces & moments of all forces should be zero. Mathematically, $\Sigma F = 0$ & $\Sigma \tau = 0$.

Q.1 A 15 kg load of bricks hangs from one end of a rope that passes on a small frictionless pulley. A 28 kg counter weight is suspended from other end of a rope. Find the magnitude of acceleration of a rope & tension in the rope while the load is moving.

Soln:

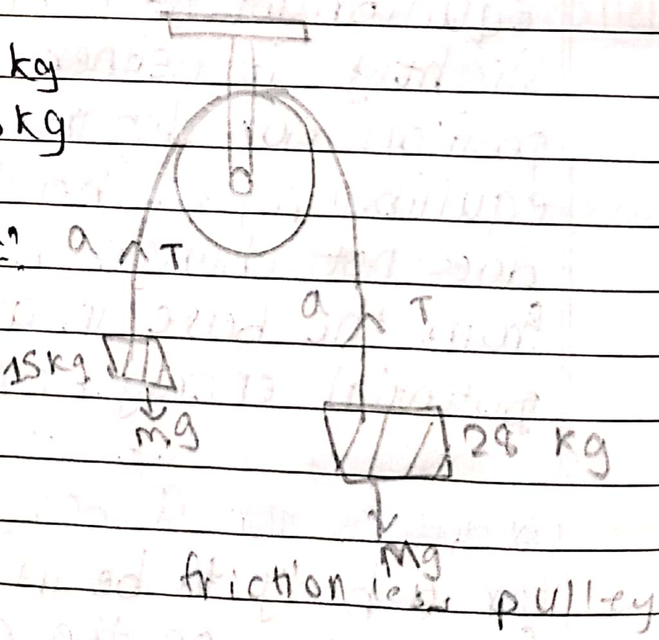
Mass of small body (m) = 15 kg

Mass of greater body (M) = 28 kg

accl^n due to Gravity (g) = 10

Magnitude of acceleration (a) = ?

Intension of load (T) = ?



Now,

$$a = \frac{(M-m)g}{(M+m)}$$

$$= \frac{(28-15) \times 10}{(28+15)}$$

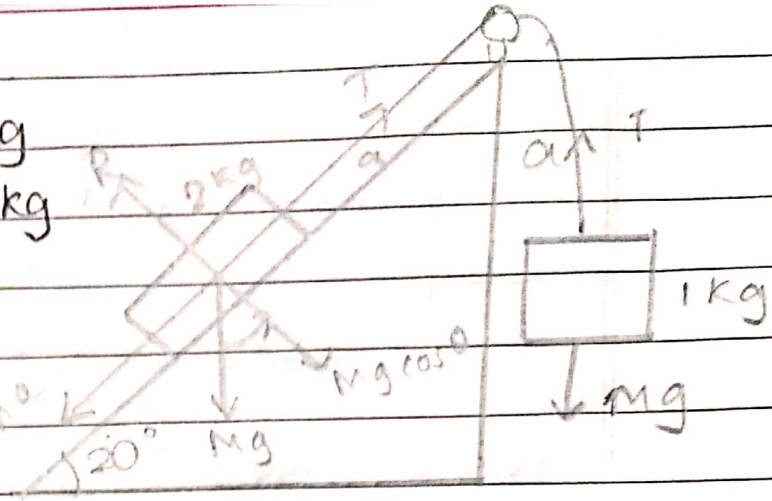
$$= 3.02 \, m/s^2$$

Again, $T = \frac{2Mmg}{M+m} = \frac{2 \times 28 \times 15 \times 10}{15+28} = 195.34$ N

2) Sol^n: Here,

Mass of small body(m) = 1 kg

Mass of greater body(M) = 2 kg



Now,

When $\theta = 20°$,

In 1 kg $= mg$

$= 1 \times 10 = 10\,N$

In 2 kg $= Mg \sin\theta$

$= 2 \times 10 \times \sin 20°$

$= 6.84\,N$

Now,

$mg - T = ma$

$mg - ma = T$ —— (i)

Again, $T - Mg\sin\theta = Ma$ —— (ii)

Putting the value of T in eq^n (ii),

or, $T - Mg\sin\theta = Ma$

or, $mg - ma - Mg\sin\theta = Ma$

or, $mg - Mg\sin\theta = Ma + ma$

or, $\dfrac{mg - Mg\sin\theta}{M+m} = a$

or, $\dfrac{10 - 6.84}{2+1} = a$

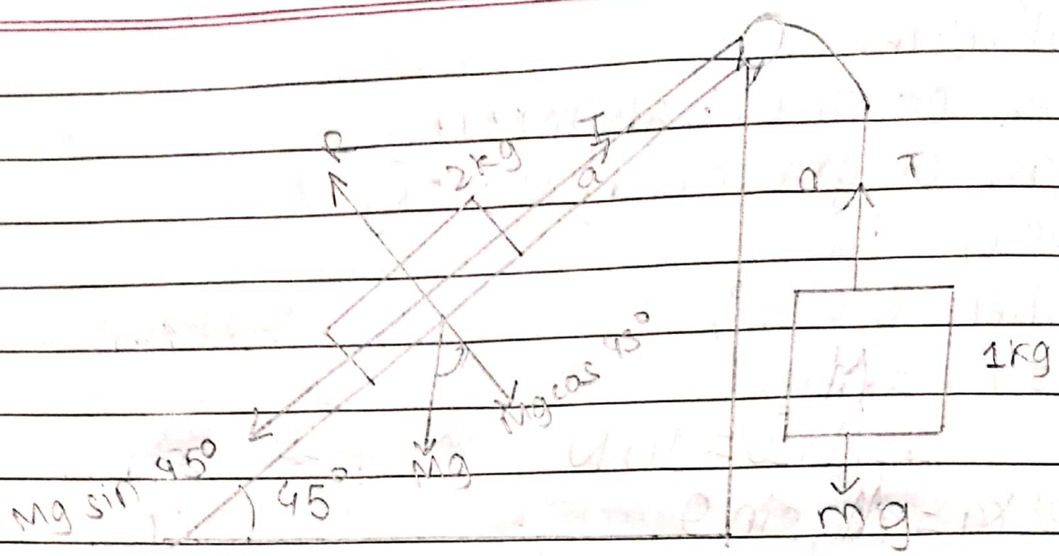
$\therefore a = 1.05\,m/s^2$

From eq^n (i),

$T = mg - ma$

$= 10 - 1 \times 1.05$

$= 8.95\,N$



Solⁿ:

$mg = 1 \times 10 = 10\,N$

$Mg\sin 45° = 2 \times 10 \times \frac{1}{\sqrt{2}} = 14.14\,N$

So,

$T - mg = ma$

$T = ma + mg$ — (i)

Again, $Mg\sin 45° - T = Ma$ — (ii)

Substituting the value of T in eqⁿ (ii),

$Mg\sin 45° - (ma + mg) = Ma$

or, $Mg\sin 45° - mg = Ma + ma$

or, $14.14 - 10 = a(M+m)$

or, $\frac{14.14 - 10}{2+1} = a$

$\therefore a = 1.38\,m/s^2$

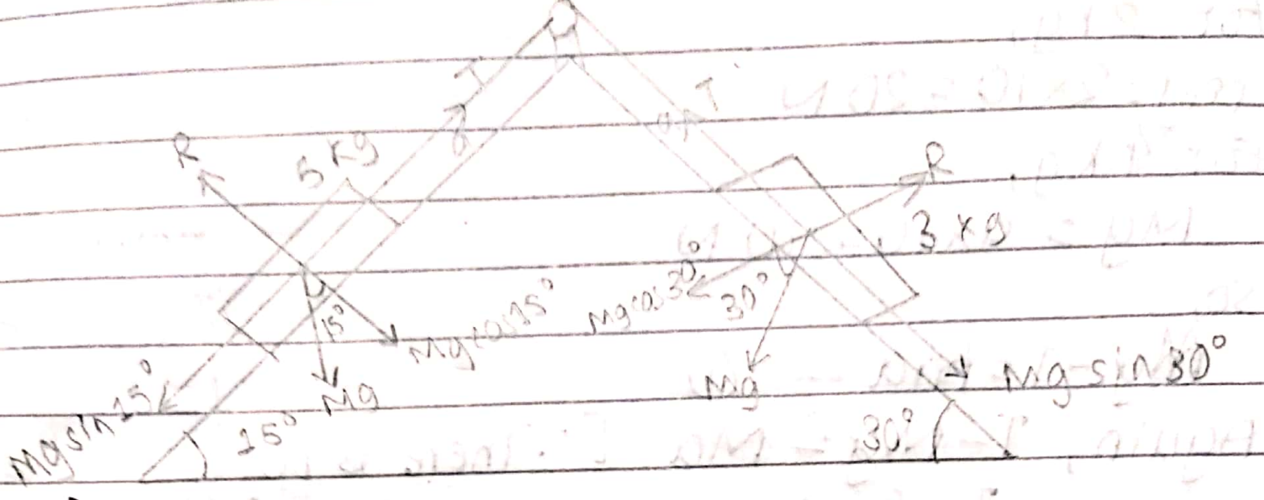
Now, from eqⁿ (i),

$T = ma + mg$

$= 1 \times 1.38 + 10$

$= 11.38$,,

3)



Sol^n: Here,

For $\theta = 15°$,

$Mg \sin 15°$

$= 5 \times 10 \times 0.25$

$= 12.94$ N

For $\theta = 30°$,

$mg \sin 30°$

$= 3 \times 10 \times 0.25$

$= 15$ N

So, $mg \sin 30° - T = ma$

$mg \sin 30° - ma = T$ — (i)

Again, $T - Mg \sin 15° = Ma$ — (ii)

Substituting the value of T in eq^n (ii),

$mg \sin 30° - ma - Mg \sin 15° = Ma$

~~15 − ma~~ or, $mg \sin 30° - Mg \sin 15° = Ma + ma$

or, $\frac{15 - 12.94}{5+3} = a$

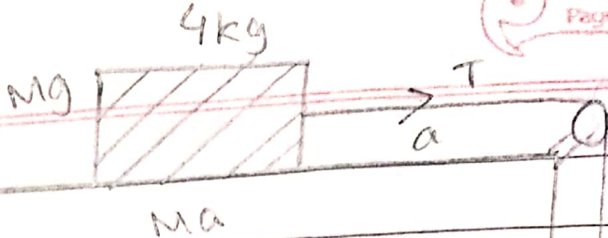
$\therefore a = 0.25 \text{ m/s}^2$

From eq^n (i),

$T = mg \sin 30° - ma$

$= 15 - 3 \times 0.25$

$= 14.25$



4) Soln: Here,

For 2 kg,

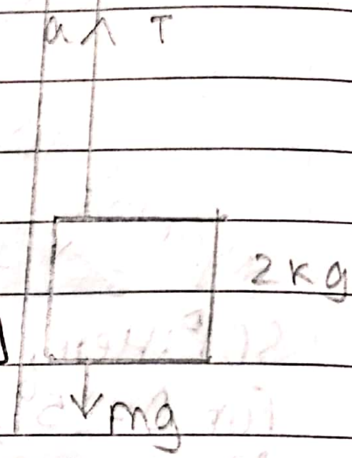
$mg = 2 \times 10 = 20\,N$

For 4 kg,

$Mg = 4 \times 10 = 40\,N$

So,

$Mg - T = Ma$ — (i)



Again, $T - \cancel{Mg}^{0} = Ma$ [∵ There is no effect of gravity]

$T - 0 = Ma$

$\therefore T = Ma$ — (ii)

Substituting the value of T in eqn (i),

$mg - T = ma$

or, $mg - Ma = ma$

or, $mg - \cancel{ma} = ma + Ma$

or, $\frac{20}{2+4} = a$

$\therefore a = 3.33\ m/s$

From eqn (ii),

$T = Ma$

$= 4 \times 3.33$

$= 13.32$

$Mg - Ma = ma$

$40 = Ma + ma$

$\frac{40}{4+2} = a$

$Mg - T = Ma$

$mg - ma = Ma$

$Mg = Ma + ma$

$\frac{40}{6} = a$

ii) On the horizontal surface

→ Suppose a body of mass m_2 rests on a horizontal surface. It is connected to another body of mass m_1 through a string, which passes over a frictionless pulley. Suppose the first body of mass m_1 falls downward with acceleration a, then another body of mass m_2 moves to the right on the horizontal surface with the same acceleration a.

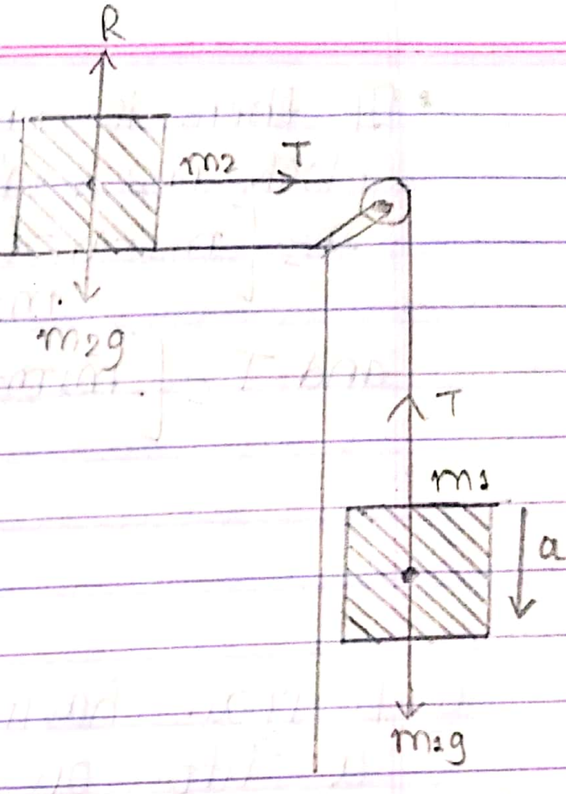


fig : Motion of two bodies on a table

- If there is no friction between block m_2 & table, then

$$a = \left(\frac{m_1}{m_1+m_2}\right)g \quad \& \quad T = \left(\frac{m_1 m_2}{m_1+m_2}\right)g$$

Reaction on the pulley is

$$T' = \sqrt{2}\,T = \left(\frac{\sqrt{2}\, m_1 m_2}{m_1+m_2}\right)g$$

- If there is friction between block m_2 & table:

$$a = \left(\frac{m_1 - \mu m_2}{m_1+m_2}\right)g \quad \text{and} \quad T = \left(\frac{m_1 m_2 (1+\mu)}{m_1+m_2}\right)g$$

iii) Two masses suspended over a pulley on an inclined plane:

- If there is no friction between block m_2 & table, then

$$a = \left(\frac{m_1 - m_2 \sin\theta}{m_1+m_2}\right)g$$

and $T = \left(\frac{m_1 m_2 (1+\sin\theta)}{m_1+m_2}\right)g$

- If there is friction between block m2 & table, then

$$a = \left[\frac{m_1 - m_2(\sin\theta + \mu\cos\theta)}{m_1+m_2}\right]g$$

and $$T = \left[\frac{m_1 m_2(1-\sin\theta+\mu\cos\theta)}{m_1+m_2}\right]g$$

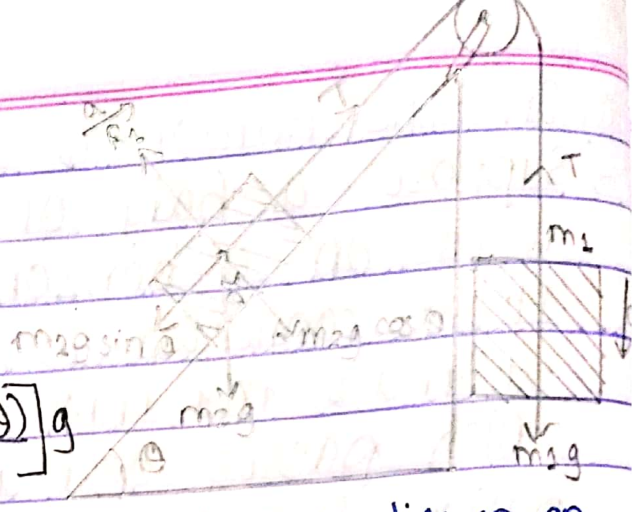


fig: Motion of two bodies on an inclined plane

Q. A man having mass 50 kg is standing inside a lift on a weighing machine. Calculate normal reaction when

a) lift is at rest

b) lift is uniformly moving upward with velocity 2m/s.

c) accelerating upward with velocity $2m/s^2$.

d) accelerating downward with velocity $2m/s^2$.

e) lift is in free fall.

Soln: Here,

$m = 50$ kg.

a) When lift is at rest,

$R = mg$ $[\because a = 0]$

$= 50 \times 10 = 500$ N

b) When lift is uniformly moving upward,

$R = m(a+g)$ $a = 0$ [$\because$ uniformly velocity]

$= 50(0+10) = 500$ N

c) accelerating upward,

$R = m(a+g) = 50(2+10) = 600$ N

d) accelerating downward,

$R = m(g-a) = 50(10-2) = 400$ N

e) In free fall, $R = 0$ ॥

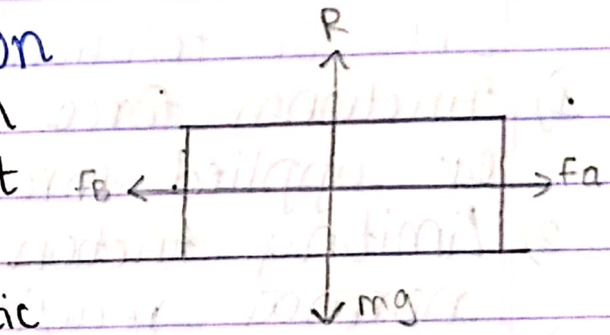
Friction:

When an object slides over a horizontal surface then opposing force acts at the point of contact which is known as friction. A force which opposes the relative motion of a body is known as a frictional force.

1) Static or limiting friction
2) Kinetic friction

1) Static or limiting friction

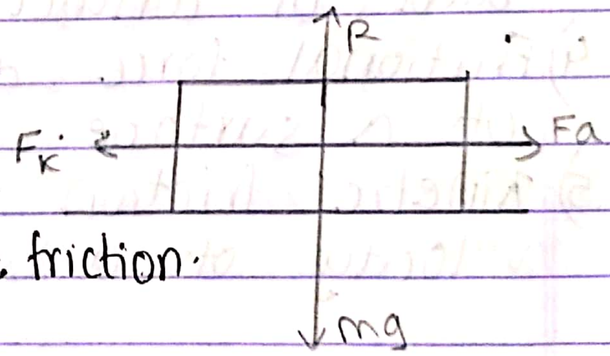
A force of friction acting on a body when it is at rest is called static friction.



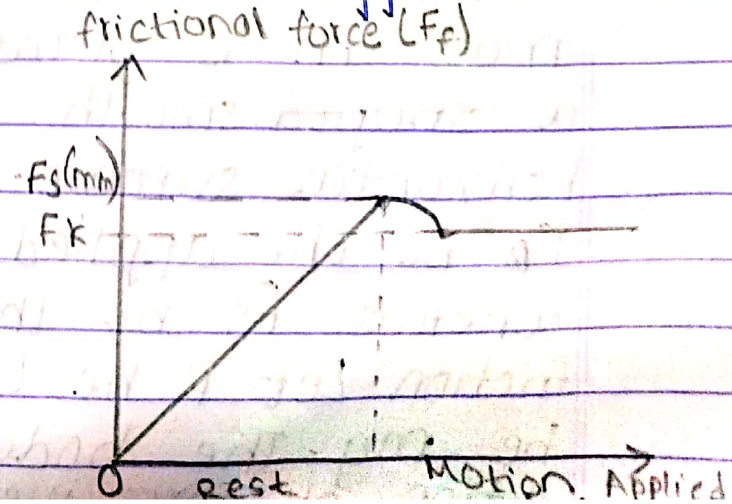
The maximum value of static friction is known as limiting friction.

2) Kinetic friction

A force of friction acting on a body when it is at motion is known as kinetic friction. It is denoted by 'fk'.



Graphical representation between applied force & frictional force:-



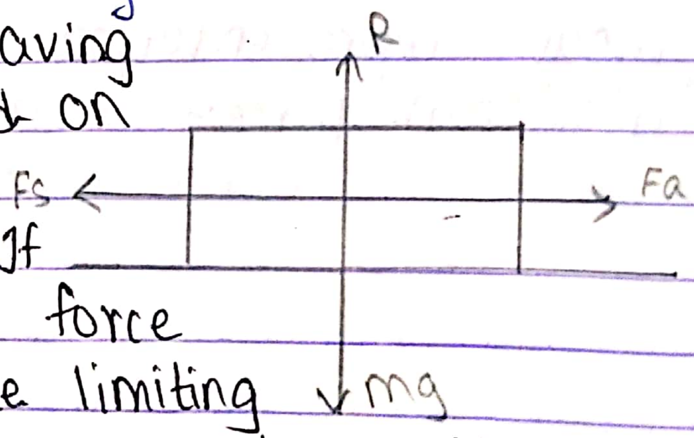
3) Rolling friction

A force of friction acting on a body when it is rolling is known as rolling friction. The magnitude of rolling friction will be less in comparision to sliding friction.

Laws of frictional force

1) Frictional force acts on opposite diraction of applied force.
2) Limiting friction is directly proportional to normal reaction.
3) Limting friction is independent of surface area of contact.
4) Frictional force depends upon the roughness of a surface.
5) Kinetic friction is independent of the velocity of a body.

1) Coefficient of limiting friction:



Consider a block having mass m is placed on a ~~sloped~~ smooth horizontal surface. If Fa be the applied force whereas Fs be the limiting friction. Let R be the normal reaction & weight be mg. The body is at rest under the action of force. We know, from the Law of friction, frictional force is directly proportional to normal reaction. i.e $F_s \propto R$

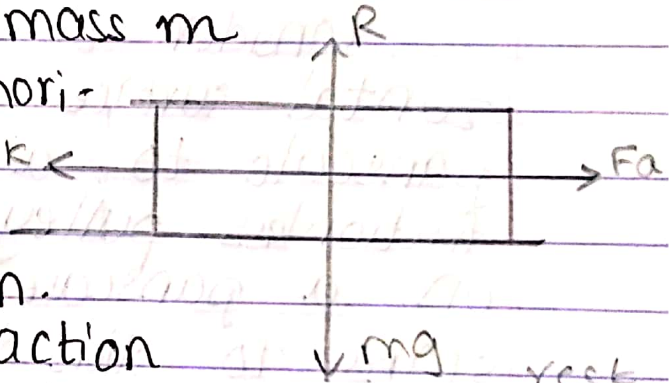
or, $F_s = \mu_s R$ where μ_s is a proportionality constant known as coefficient of limiting friction.

$$\frac{F_s}{R} = \mu_s$$

$$\boxed{\frac{F_s}{mg} = \mu_s}$$

2) Coefficient of kinetic friction:-

Consider a block having mass m is placed on a smooth horizontal surface. If Fa be the applied force whereas Fk be the kinetic friction.



Let R be the normal reaction & weight be mg. The body is at motion under the action of force. We know, from the Law of friction,

i.e. $F_K \propto R$

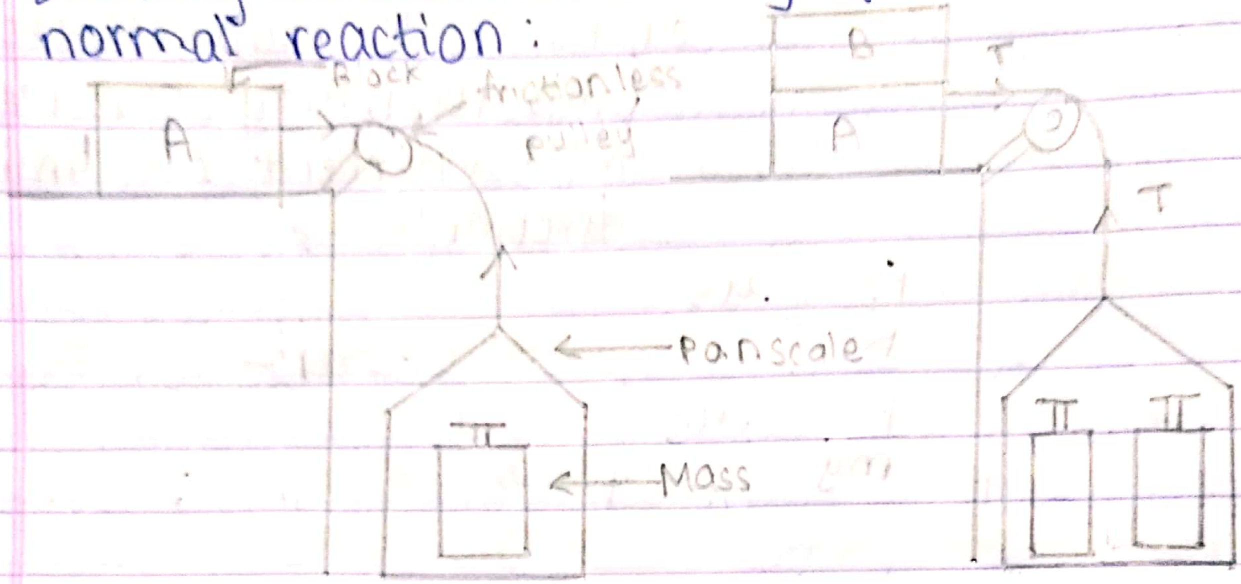
or, $F_K = \mu_K R$ where μ_K is a proportionality constant known as coefficient of kinetic friction.

$$\frac{F_K}{R} = \mu_K$$

$$\boxed{\frac{F_K}{mg} = \mu_K}$$

Experimental verification of limiting friction:

① Limiting friction is directly proportional to normal reaction:

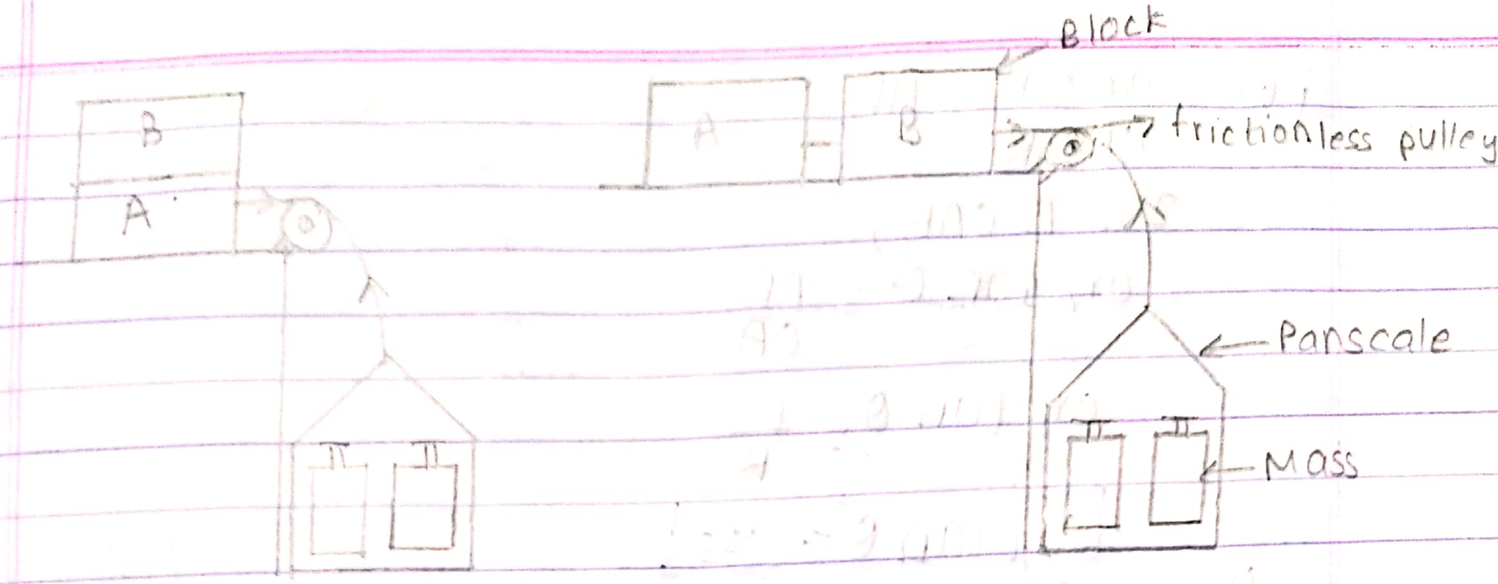


Consider a block A is placed on a horizontal surface which is attached to a panscale to a string passes through a frictionless pulley. Mass is gradually added on a panscale & for certain mass m, block starts to slide down.

Now, block B which is similar to block A is overlapped. Similarly, mass is gradually added on a panscale. For mass 2m, the block start to slide.

Hence, limiting friction is directly proportional to normal reaction.

② Limiting friction is independent of surface area of contact:

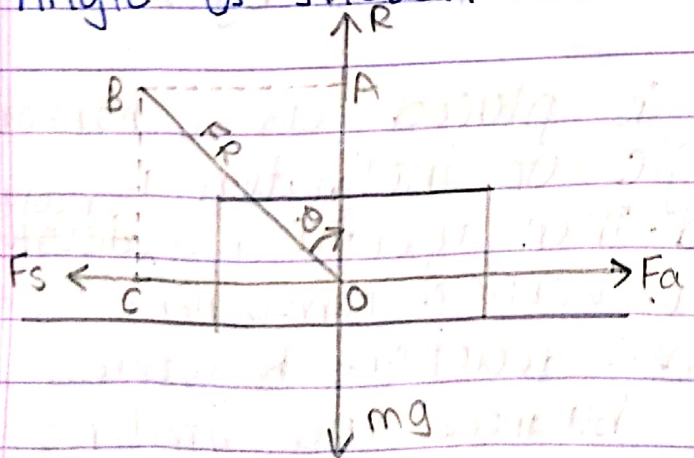


Two blocks A & B are overlapped & mass is gradually increased on a panscale. For mass 2m, the blocks start to slide.

Now, the blocks are placed sidewise & similarly the mass is gradually increased on a panscale & for mass 2m, blocks start to slide.

Hence, limiting friction is independent of surface area of contact.

Angle of friction:



Angle of friction is defined as angle made by the resultant of normal reaction & limiting friction with normal reaction. It is denoted by θ.

Consider a block is placed on a horizontal surface. If Fa be the applied force & Fs be the limiting friction where R be the normal reaction & its weight be mg.

i.e. $OC = F_s = AB$

$OA = R$

In ΔOAB,

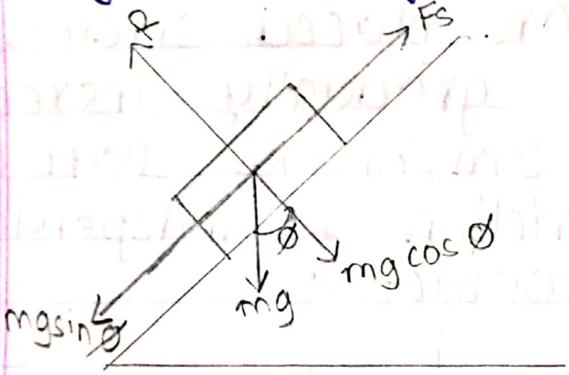
or, $\tan\theta = \frac{AB}{OA}$

or, $\tan\theta = \frac{F_s}{R}$

$\boxed{\text{or, } \tan\theta = \mu_s}$

Hence, tangent of angle of friction is equals to coefficient of limiting friction.

Angle of repose:-



Angle of repose is defined as angle made by horizontal plane with inclined plane when object placed on inclined plane just starts to slidedown. It is denoted by Ø.

Consider a block is placed on a horizontal surface. Let angle of inclination is gradually increased. Then weight mg resolve ~~dissolve~~ into two component. Vertical component mgcosØ balances the normal reaction R whereas horizontal component mgsinØ balances the limiting friction Fs. If Ø be the angle of repose & the obj on a inclined plane just starts to slide. Now,

$$mg\cos\phi = R \quad \text{--- (i)}$$
$$mg\sin\phi = F_s \quad \text{--- (ii)}$$

Also, dividing eqⁿ (ii) by (i), we get,

$$\frac{\cancel{mg}\sin\phi}{\cancel{mg}\cos\phi} = \frac{F_s}{R}$$

or, $\tan\phi = \frac{F_s}{R}$

$\boxed{\text{or, } \tan\phi = \mu_s}$

Hence, tangent of angle of repose is equals to coefficient of limiting friction.

Now,

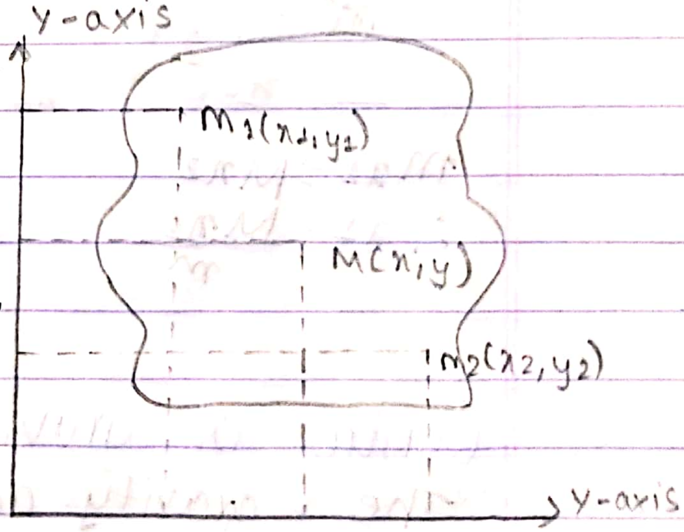
$\tan\theta = \mu_s = \tan\phi$

or, $\tan\theta = \tan\phi$

$\therefore \theta = \phi$

|Angle of friction = Angle of repose|

Centre of mass



Let us consider a body having large number of mass $m_1, m_2, m_3 \ldots$ where co-ordinates be $(x_1, y_1), (x_2, y_2), (x_3, y_3) \ldots$ then for mass m its co-ordinate be (x, y). Let the number of forces acting on the masses.

$\Sigma F = F_1 + F_2 + F_3 + \ldots + F_n$

or, $\Sigma F = m_1 a_1 + m_2 a_2 + m_3 a_3 + \ldots + m_n a_n$

or, $\Sigma F = m_1 \frac{d^2 x_1}{dt^2} + m_2 \frac{d^2 x_2}{dt^2} + m_3 \frac{d^2 x_3}{dt^2} + \ldots + m_n \frac{d^2 x_n}{dt^2}$

or, $\frac{M \times d^2 x}{M \times d^2} = \frac{d^2}{dt^2} \{ m_1 x_1 + m_2 x_2 + m_3 x_3 \ldots + m_n x_n \}$

or, $x = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 - \cdots + m_nx_n}{m_1+m_2+m_3+\cdots m_n}$

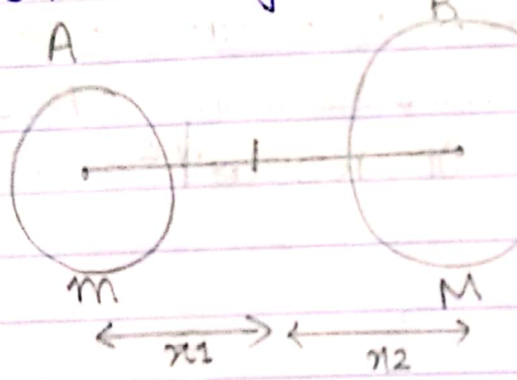
where, $M = m_1+m_2+m_3+\cdots m_n$

$y = \dfrac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots m_ny_n}{m_1+m_2+m_3+\cdots m_n}$

$= m_1y_1 + m_2y_2 + m_3$

centre of mass is defined as position where the entire mass of body is supposed to be concentrated.

Centre of ~~grat~~ mass of two bodies.

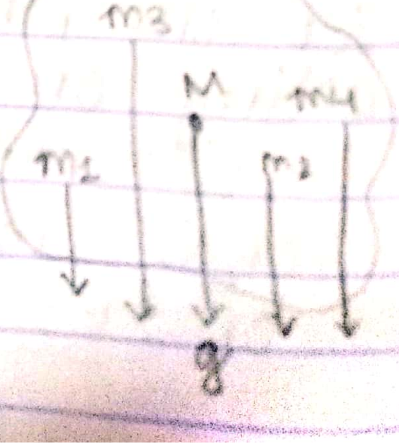


$mx_1 = Mx_2$

$\therefore x_1 = \dfrac{Mx_2}{m}$ or, $x_2 = \dfrac{mx_1}{M}$

Centre of gravity

The gravity acting on the centre of mass is defined as centre of gravity.



In a physics lab experiment, a 6 kg box is pushed across a flat table by a horizontal force F.

a) If the box is moving at a constant speed of 0.35 m/s & the coefficient of kinetic friction is 0.12, what is the magnitude of F?

b) If a box is speeding ~~a~~ up with the constant acceleration of 0.18 m/s^2, what will be the magnitude of F?

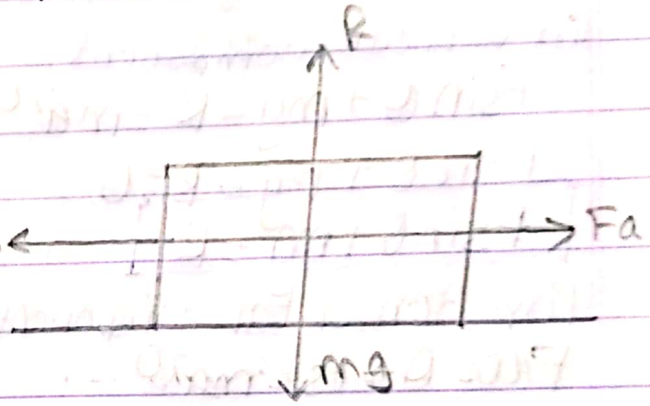
Sol^n:

Mass of a box = 6 kg

i) Speed (v) = 0.35 m/s

Kinetic fiction (u_k) = 0.12

f_a = ?



Now, $F_a - F_k = F_R$

$F_a - F_k = ma \quad [a=0]$

$F_a = F_k$

$F_a = u_k R$

$= u_k mg$

$= 0.12 \times 6 \times 10$

$= 7.2$ N

Also,

b) $F_a - F_k = F_R$

$F_a = u_k R + ma$

$F_a = u_k mg + ma$

$= 7.2 \text{ N} + 6 \times 0.18$

$= 8.28$ N ,,

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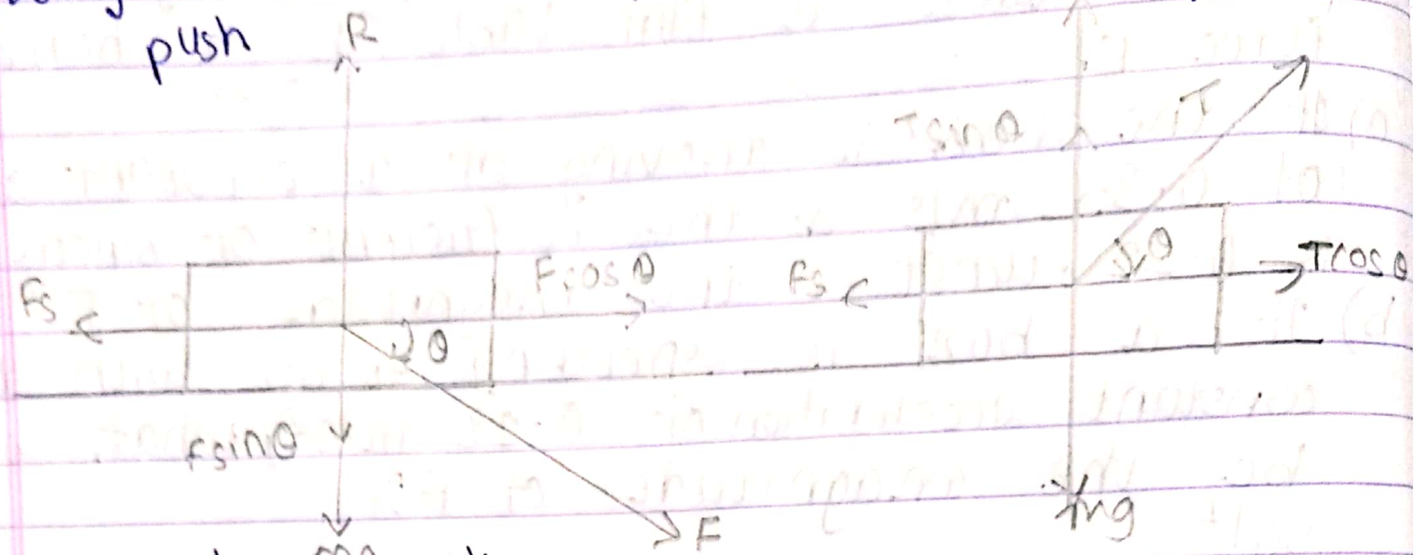
① Why is it easier to pull than push?



For vertical component,

$F\sin\theta + mg - R = m\cancel{a}^{\to 0}$

$F\sin\theta + mg - R = 0$

$\boxed{F\sin\theta + mg = R}$

For horizontal component push

$F\cos\theta - F_s = m\cancel{a}^{\to 0}$

$F\cos\theta = F_s$

$F\cos\theta = \mu_s R$

$\frac{F\cos\theta}{\mu_s} = R$

$mg - R - T\sin\theta = \cancel{ma}^{\to 0}$

$mg = R + T\sin\theta$

$\boxed{mg - T\sin\theta = R}$

for pull-

$T\cos\theta - F_s = m\cancel{a}^{\to 0}$

$T\cos\theta = f_s$

$T\cos\theta = \mu_s R$

$\frac{T\cos\theta}{\mu_s} = R$

1. A man pulls a load of mass 75 kg along a horizontal surface at a constant velocity. The coefficient of kinetic friction is 0.15. the angle made by tension with horizontal component is 42.

a) What is the tension acting on the rope?

b) What is the normal force with which the floor ~~the~~ pushes vertically upward on the

Sol^n:

Mass (m) = 75 kg & Angle made (θ) = $42°$

Weight (w) = mg = $75 \times 10 = 750$ N

Coefficient of kinetic fiction (μk) = 0.15

Now,

$$mg - T\sin\theta = R$$

$$mg - T\sin\theta = \frac{T\cos\theta}{\mu k}$$

$$750 - T \times \sin 42° = \frac{T\cos 42°}{0.15}$$

$$or, 112.5 - 0.10T = 0.74T$$

$$or, 112.5 = 0.84T$$

$$\therefore T = 133.38 \text{ N}$$

Thus, tension acting on the row is 133.38 N.

Also, $R = \frac{T\cos\theta}{\mu k}$

$$R = \frac{133.38 \times \cos 42°}{0.15}$$

$$\therefore R = 660.8 \text{ N}$$

The normal reacting force is 660.67 N.

2. A block having mass 50 kg is placed on a horizontal floor. The coefficient of limiting friction is 0.35. The angle between the force & the horizontal component is 30°. Calculate the force applied on the block & normal reaction

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Sol^n: Here,

Mass $(m) = 50$ kg , Angle bet^n force & horizontal comp$(\theta) = 30^\circ$

Coefficient of limiting friction$(\mu s) = 0.35$, $W = mg$

Force applied $(F) = ?$ $= 50 \times 10$

Normal reaction$(R) = ?$ $= 500$ N

We know,

$F\sin\theta + mg = R$

$F\sin\theta + mg = \frac{F\cos\theta}{\mu s}$

$F\sin 30^\circ + 500 = \frac{F\cos 30^\circ}{0.35}$

or, $F \times 0.5 + 500 = \frac{F \times 0.866}{0.35}$

or, $0.175F + 175 = 0.866F$

or, $175 = 0.691F$

$\therefore F = 253.25$ N

Hence, the force applied on the block is 253.25 N.

Again,

$R = \frac{F\cos\theta}{\mu s}$

$= \frac{253.25 \times \cos 30^\circ}{0.35}$

$= 626.63$ N

Hence, the normal reaction is 626.63 N.

3) A ball A of mass 0.1 kg moving with velocity of 6 m/s collides directly with a ball B of mass 0.2 kg at rest. Calculate their common velocity of both balls moving off together. If A had rebounded with a velocity of 20 ms^{-1} in the

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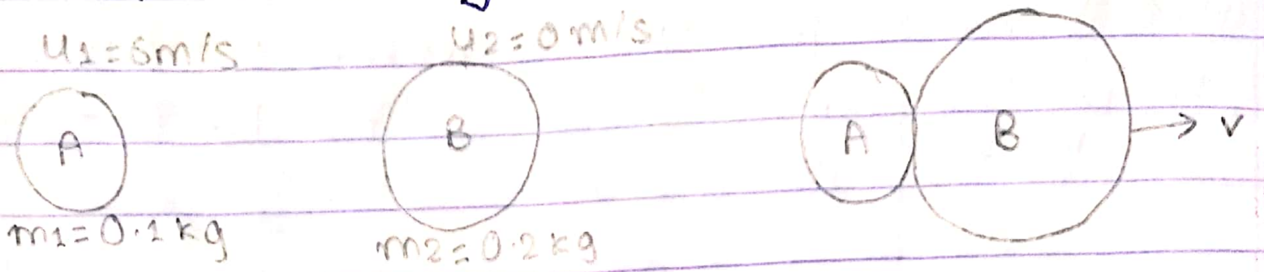
Opposite direction after the collision, what would be the new velocity of B?

Soln:



We know from conservation of momentum,

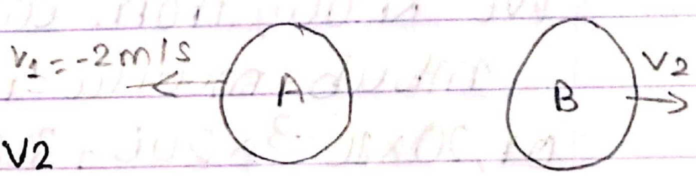
$m_1u_1+m_2u_2 = m_1v_1+m_2v_2$ $\quad \because v_1=v_2=v$

or, $0.1\times6+0.2\times0 = 0.1\times v+0.2\times v$

or, $0.6 = 0.3v$

$\therefore v = 2$ m/s

Again,



$m_1u_1+m_2u_2=m_1v_1+m_2v_2$

or, $0.1\times6+0.2\times0=0.1\times-2+0.2\times v_2$

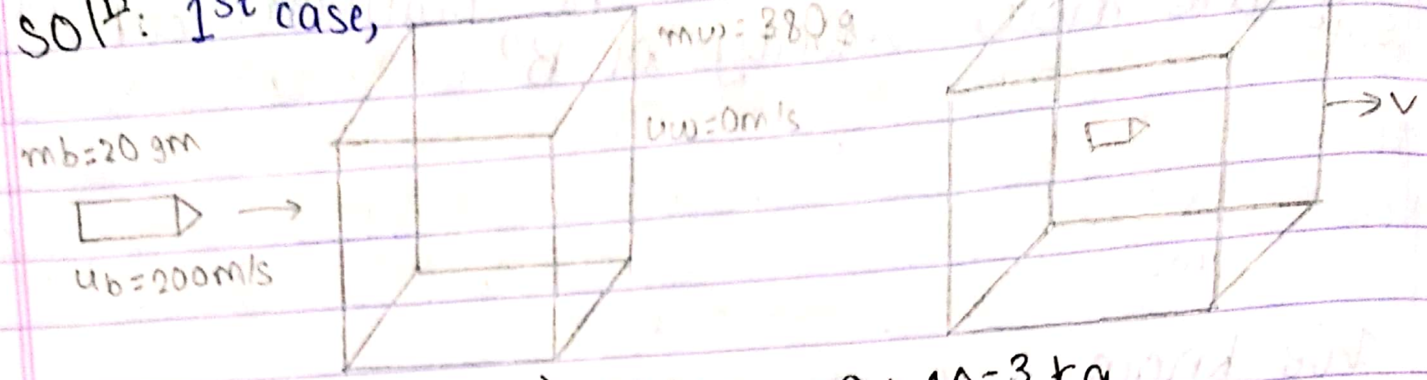
or, $0.6=-0.2+0.2v_2$ $\quad [\because v_1=-2]$

or, $0.8=0.2v_2$

$\therefore v_2=4$ m/s

4) A bullet of mass 20 g is fired horizontally into a suspended stationary wooden block of mass 380 g with a velocity of 200 ms^{-1}. What is the common velocity of the bullet & block of the bullet is embedded (stays inside) the block? If the block & bullet experience a constant opposing force of 2N, find the time taken by them to come to rest.

Solⁿ: 1st case,



Mass of bullet(mb) = 20gm = 20×10^{-3} kg
Mass of wooden block(mw) = 380 = 380×10^{-3} kg
Velocity of bullet (ub) = 200 m/s
Velocity of wooden block (uw) = 0 m/s
Common velocity (v) = ?

We know, from conservation of momentum,

$m_b u_b + m_w u_w = m_b v_b + m_w v_w$ $\quad \therefore v_b = v_w = v$

or, $20 \times 10^{-3} \times 200 + 380 \times 10^{-3} \times 0 = 20 \times 10^{-3} \times v + 380 \times 10^{-3} \times v$

or, $4 = 0.02v + 0.38v$

or, $4 = 0.4v$

$\therefore v = 10$ m/s

Hence, the common velocity of the bullet & block is 10 m/s.

2nd case,

$u = v = 10$ m/s

$v = 0$ m/s

$F = -2N$

or, $ma = -2N$

or, $(380 + 20) \times a = -2$

or, $400 \times 10^{-3} \times a = -2$

$\therefore a = -5$ m/s²

Now, $v = u + at$

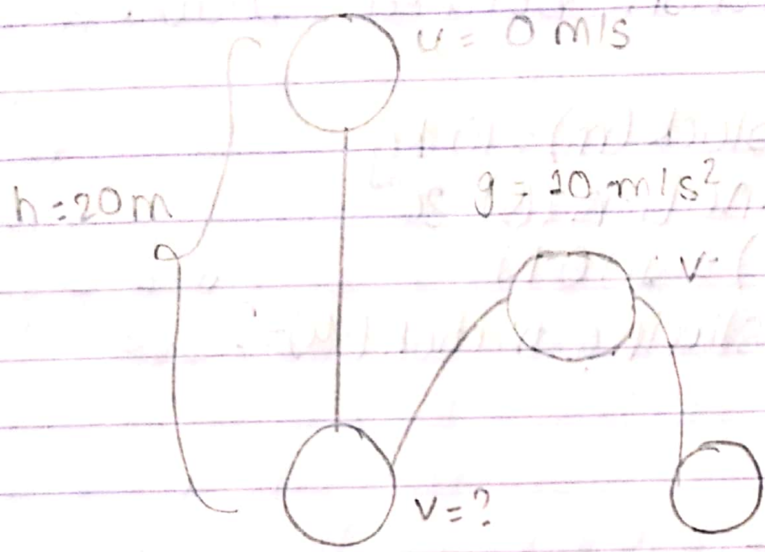
$0 = 10 + (-5)t$

$\cancel{-}10 = \cancel{-}5t$

$\therefore t = 2s$

Hence, the time taken by the block & bullet to come to rest is 2 sec.

5) A ball is dropped from a height of 20m & rebounds with a velocity which is (3/4)th of the velocity with which it hits the ground. What is the time interval between the first & second bounces?



Soln:

Height(h) = 20 m

Initial velocity (u) = 0 m/s , $g = 10\ m/s^2$,

Final velocity (v) = ?

Rebounds velocity (v') = 3/4 × v

Now,

or, $v^2 = u^2 + 2gh$

or, $v^2 = 2gh$

or, $v^2 = 2 \times 10 \times 20$

$\therefore v = 20\ m/s$

$v' = \frac{3}{4} \times v = \frac{3}{4} \times 20 = 15\ m/s$

$v = u + gt_1$

$20 = 0 + 10t_1$

$\therefore t_1 = 2$ sec

2nd case, u = 20 m/s , v = 15 m/s , g = −10 m/s

$v = u + gt$

$15 = 20 + 10t_2$

$\therefore t_2 = 0.5$ sec

Now,

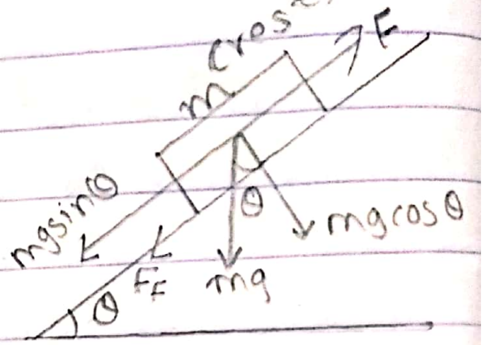
Total time $= t_1 + 2t_2$

$= 2 + 2 \times 0.5$

$= 3$ sec

6) An iron block of mass 10 kg rest on a wooden plate inclined at 30° to the horizontal. It is found that the least force parallel to the plane which causes the block to slide up the plane is 100 N. Calculate the coefficient of sliding friction between wood & iron.

Solⁿ:



Mass of iron block (m) = 10 kg

Angle of inclination (θ) = 30°

Least force (F) = 100 N

Coefficient of sliding friction (μ) = ?

$g = 9.8\ m/s^2$

We know,

$F = mg\sin\theta + F_F$

or, $100 = mg\sin\theta + \mu R$ [∵ $\mu = \frac{F_F}{R}$, where F_F is frictional force]

or, $100 = mg\sin\theta + \mu . mg\cos\theta$ [∵ $R = mg\cos\theta$]

or, $100 = mg(\sin\theta + \mu\cos\theta)$

or, $100 = 10\times 9.8(\sin 30° + \mu\cos 30°)$

or, $\frac{100}{10\times 9.8} = \frac{1}{2} + \mu\frac{\sqrt{3}}{2}$

or, $1.02 = \frac{1+\mu\sqrt{3}}{2}$

or, $2.04 - 1 = \mu\sqrt{3}$

or, $\frac{1.04}{\sqrt{3}} = \mu$

$\therefore \mu = 0.6$

Hence, the coefficient of sliding friction between wood & iron is 0.6.

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Very Short Q/A

8) If you drop a large stone and a small stone from the top of a tall building, which one will reach the ground first. Explain your answer.

→ The larger one; its weight is greater, so it reaches a greater speed before air resistance is sufficient to equal its weight.

15) Why do cricket players prefer heavy bat but not light bat?

→ Larger the mass of the body more is the momentum caused by it. When a heavy bat strikes the cricket ball, it transfers greater momentum to the ball & hence ball moves far. Hence, cricket player prefers heavy bat but not light bat.

17) The swimmer pushes the water in a backward direction while swimming, why?

→ When a swimmer pushes the water in the backward direction with a certain force, the water pushes the man forward with an equal & opposite force. Hence, it follows Newton's third law & conserve the linear momentum of the system.

22) How does friction help with walking? Explain.

→ When we walk, we push the ground in a backward direction our feet will get equal & opposite reactions from the ground. The forward horizontal component of reactional force helps to move forward & friction protects from slipping.

25) What is the direction of the force friction in the front wheel & the back wheel?

→ The direction of friction in the front wheel is in the backward direction as it rolls only. On the other hand, the direction of friction is in the forward direction in the back wheel.

Short Q/A

1) The mud from the rim of the bicycle flies off tangentially when the wheel is moving, why?

→ The mud from the rim of the bicyle flies off tangentially, when the wheel is moving, due to directional inertia. When the wheel of the bicyle is rotating, the direction of the velocity of mud attached to the rim of it is along the tange to the point. Hence, mud from the rim of the bicyle flies off tangentially when the wheel is rotating.

2) Athletes run some distance before taking a long jump, why? Explain.

→ When an athletes runs some distance, the velocit acquired due to inertia is added to the velocity of the athlete at the time of the jump. The distance covered by an athlete in a long jump depends on the velocity acquired just before the jump. Hence, athletes run some distance to cover more distance before jumping.

3) Why are shockers used in scooters & cars? Explain.

→ When a scooter or a car moves on a rough road, it receives an impulse due to jerky motion. In case, the shockers are used in the vehicle, the time of impact increases. Since impulse is a product of force & time of impact, due to the increased value of the impact time, the force of impact is reduced. It saves the vehicles from severe jerks.

10) Rocket works on the principle of conservation of momentum. Explain.

→ A rocket contains fuel. When fuel is burnt in the combustion chamber; it comes out in the form of gases. The gas moves downwards at high speed, & the rocket recoils & moves upward. Hence, the momentum of the rocket moving upward balances the linear momentum of gas moving downward. That's how, the rocket works on the principle of conservation of momentum.

15) Can a sailboat be propelled by air blown at the sails from a fan attached to the boat? Give a reason for your answer.

→ The sail & the fan both are attached to the boat. Force due to the air blown on the sail is internal force & the internal forces cannot cause any change in momentum. Hence the sailboat cannot be propelled by air blown at the sails from a fan attached to the boat.

19) When a horse pulls a cart, the cart pulls the horse backward. Explain how the motion takes place?

→ According to Newton's third law of motion, the action & reaction are equal but opposite. The action acts on a body whereas the reaction acts on another body. When the horse pulls the cart, the force exerted by the horse, ~~whereas~~ is inclined. The vertical component of reactional force balances the weight of the cart & the horse, whereas the horizontal component provides horizontal motion to the cart & horse. Hence, the cart pulls the horse backward, when a horse pulls a cart.

20) Air is blown on a sail attached to a boat from an electric fan placed on the boat. Will the boat start moving?

→ No, from Newton's second law of motion, a body comes into acceleration when some external force is applied to the body. When the fan pushes (as action) the sail by the air, then air also pushes (as a reaction) the fan in opposite direction. The fan is also a part of the boat that cannot be propelled. To move the boat, the action & reaction should act on different bodies.

6) Why is it difficult to move a bicycle with its brake on?

→ Between the tyres of the cycle & the road, there is rolling friction. But when the cycle is moved with brakess on, the wheels cannot revolve but can only skid. Due to this, sliding friction comes into play. As the sliding friction is greater than rolling friction, it becomes difficult to move the cycle with brakes on.

34) A ball having a momentum p hits a bat & its final momentum becomes −p. What is the change in momentum of the ball?

→ According to the question, we have

Initial momentum of the ball = p

Final momentum of the ball = −p

Thus the change in momentum of the ball is

Δp = final momentum − initial momentum

$\therefore \Delta p = -p - p = -2p = 2p$ (in magnitude)

Thus, the change in momentum of the ball after hitting by the bat will be 2p (in magnitude).

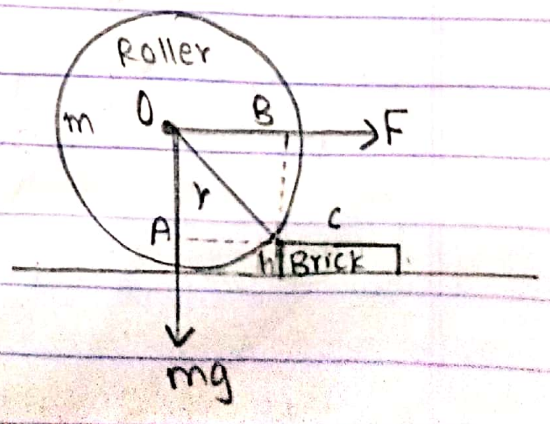
Q. A roller whose diameter is 1m weights 360 N. What horizontal force is necessary to pull a roller over a brick 0.1 m high where the force is applied at the centre?

Soln: Given,

Diameter of roller (d) = 1 m

Weight of roller (mg) = 360 N

Radius (r) $= \frac{d}{2} = \frac{1}{2} = 0.5$ m



Height of brick (h) = 0.1 m

Horizontal force (F) = ?

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Referring to the figure, we have

$BC = OA = r - h = 0.5 - 0.1 = 0.4\,m$

So, $AC = \sqrt{OC^2 - OA^2}$

$= \sqrt{0.5^2 - 0.4^2}$

$= \sqrt{0.09}$

$= 0.3\,m$

From the principle of the moment, we can write

$F \times BC = mg \times AC$

$\therefore F = \frac{AC}{BC} mg = \frac{0.3}{0.4} \times 360 = 270\,N$

Thus the necessary horizontal force to pull the roller over the brick is 270 N.