

# Homework 1

August 24, 2020

## 1 $k$ -connectivity

Let  $G$  be a 4-connected graph and let  $v_1, \dots, v_4$  be four vertices of  $G$ . Let  $H_0$  be formed from  $G$  by adding a new vertex  $u$  and four edges  $(u, v_1), \dots, (u, v_4)$ .

**1.1. Prove that  $H_0$  is 4-connected.**

Let  $H_1$  be formed from  $G$  by adding a new vertex  $u$  and the  $|V_G|$  edges  $\{(u, v) \mid v \in V_G\}$ . **1.2. Prove that  $H_1$  is 5-connected.**

## 2 Hamiltonian Paths

Recall that an Eulerian path in  $G$  is (the image of) a homomorphism  $f : P_m \rightarrow G$  such that  $f_E$  is bijective. Similarly, a Hamiltonian path in  $G$  is (the image of) a homomorphism  $f : P_m \rightarrow G$  such that  $f_V$  is bijective. Let  $G$  be a graph with a vertex  $u$  such that  $G \setminus u$  has three connected components. **Prove that  $G$  does not have a Hamiltonian path.**

## 3 Graph Coloring

Let  $f : G \rightarrow H$  be a graph homomorphism. **Prove that  $G$  is  $|V_H|$ -colorable.**