# VE 230 Midterm RC

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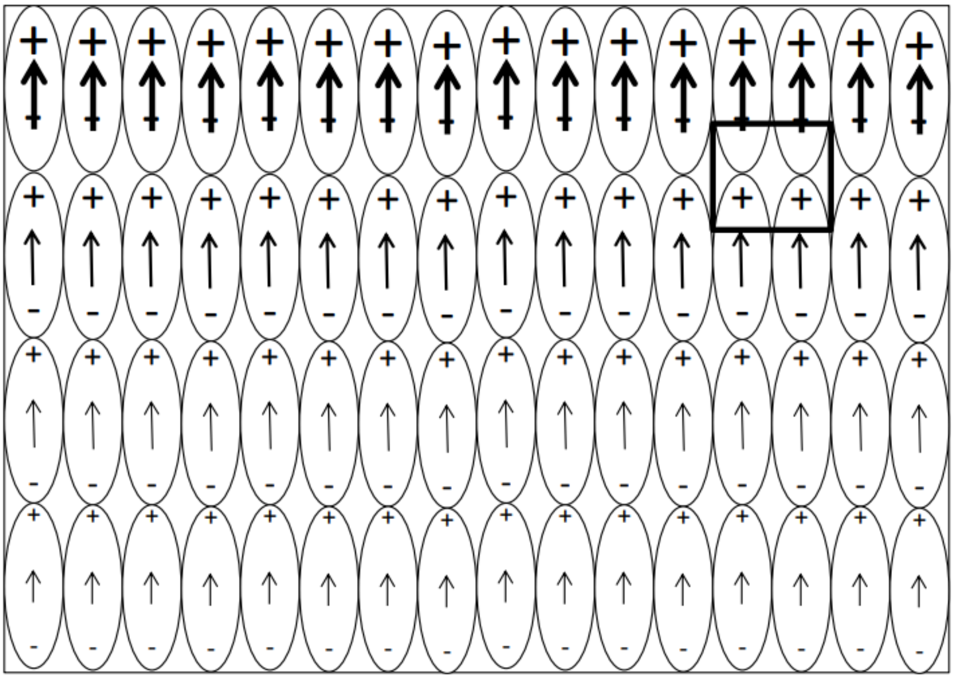
## Polarization



- Polarization vector, $\mathbf{P}$:

  $\mathbf{P} = \lim_{v\to 0} \frac{1}{v} \sum_{k=1}^{n} \mathbf{p}_k$

  where the numerator represents the vector sum of the induced dipole moment contained in a very small volume $v$.
- Charge distribution on surface density:

  $\rho_{ps} = \mathbf{P} \cdot \mathbf{a}_n$
- Volume charge distribution density:

  $\rho_p = -\nabla \cdot \mathbf{P}$

## Electric Flux Density and Dielectric Constant

Polarization is proportional to external electric field, which is an linear approximation.

Counter example: Iron.

- Electric flux density/electric displacement, $\mathbf{D}$:

  $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \ (C/m^2)$

  $\nabla \cdot \mathbf{D} = \rho \ (C/m^3)$

  where $\rho$ is the volume density of free charges.
- Another form of Gauss's law:

  $\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{\text{free}} \ (C)$

The total outward flux of the electric displacement (the total outward electric flux) over any closed surface is equal to the total free charge enclosed in the surface.

- If the dielectric of the medium is linear and isotropic,

  $$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$$

  $$\mathbf{D} = \epsilon_0 (1 + \chi_e)\mathbf{E} = \epsilon_0 \epsilon_r \mathbf{E} = \epsilon \mathbf{E}$$

  where $\chi_e$ is a dimensionless quantity called electric susceptibility, $\epsilon_r$ is a dimensionless quantity called the relative permittivity/electric constant of the medium, and $\epsilon$ is the absolute permittivity/permittivity of the medium (F/m).

- For anisotropic media,

  $$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

  For bi-axial,

  $$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

  For uni-axial, $\epsilon_1 = \epsilon_2$. For isotropic, $\epsilon_1 = \epsilon_2 = \epsilon_3$.

- Dielectric breakdown: Electric field is very strong, causing permanent dislocations and damage in the material.

- Dielectric strength: The maximum electric field intensity that a dielectric material can withstand without breakdown.

## Exercise:

Determine the electric field intensity at the center of a small spherical cavity cut out of a large block of dielectric in which a polarization P exists.

## Boundary Conditions for Electrostatic Fields

- The tangential component of an electric field is continuous across an interface.

  $E_{1t} = E_{2t}\ (V/m)$

  or

  $\frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}$

- The normal component of the displacement field $\mathbf{D}$ is discontinuous across an interface where a surface charge exists, with the amount of discontinuity being equal to the surface charge density.

  $\mathbf{a}_n \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$

  or

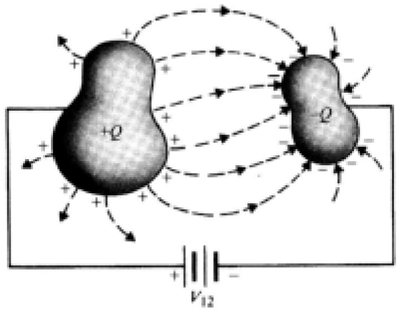
  $D_{1n} - D_{2n} = \rho_s\ (C/m^2)$

## Capacitors

Definition: The capacitance of isolated conducting body is the electric charge that must be added to the body per unit increase in its electric potential.

$$C = \frac{Q}{V} \tag{1}$$

Components: Two conductors with arbitrary shapes are separated by free space or dielectric medium.




**Components**: Two conductors with arbitrary shapes separated by free space or a dielectric medium.

- $C = \frac{Q}{V_{12}}$

Refer to the textbook for the standard.

- Its capacitance is independent of $V$ and $Q$, meaning a capacitor has capacitance even when no voltage is applied and no free charges exist on its conductors.

### How to Calculate Capacitance:

1. Choose a proper coordinate system.
2. Assume $+Q$ and $-Q$ on the conductors.
3. Find $E$ from $Q$ (e.g., Gauss's law, $D_n = \epsilon E_n = \rho_s$).
4. Find $V_{12} = \int_1^2 E \cdot dl$.
5. $C = \frac{Q}{V_{12}}$.

Specifically, for the connections of different capacitors, we can have

Series:

$$\frac{1}{C_{sr}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n} \tag{2}$$

Parallel:

$$C_{pr} = C_1 + C_2 + \ldots + C_n \tag{3}$$

## Electrostatic Energy and Forces

- **Potential difference between $P_1$ and $P_2$**:

$$W_{12} q = V_{21} = V_2 - V_1 = \int_{P_1}^{P_2} E \cdot dl \tag{4}$$

- **Self Energy**: Work done to bring a charge $Q_2$ from infinitely far away to distance $R_{12}$ with $Q_1$ (initially, $Q_1$ is in space):

$$W = Q_2 V_2 = Q_2 \frac{Q_1}{4\pi\epsilon_0 R_{12}} \tag{5}$$

- **Mutual Energy**: Potential energy of a group of $N$ discrete point charges at rest:

$$W_e = \frac{1}{2} \sum_{k=1}^{N} Q_k V_k \tag{6}$$

where $V_k = \frac{1}{4\pi\epsilon_0} \sum_{j=1, j\neq k}^{N} \frac{Q_j}{R_{jk}}$. Note that $W_e$ can be negative, e.g., in a 2-point charge system with one positive and one negative charge.

Electrostatic Energy (Volume) Density $w_e$ defined as:

$$W_e = \int_{v_0} w_e dv \tag{7}$$

Electrostatic Energy in terms of Field Quantities:

A continuous charge distribution of density $\rho$:

$$W_e = \frac{1}{2}\int_v \rho V dv = \frac{1}{2}\int_{v_0} (\nabla \cdot D) V dv \tag{8}$$

Another expression:

$$W_e = \frac{1}{2}\int_{v_0} D \cdot E dv \tag{9}$$

If it is a simple dielectric:

$$W_e = \frac{1}{2}\int_{v_0} \epsilon E^2 dv = \frac{1}{2}\int_{v_0} \frac{D^2}{\epsilon} dv \tag{10}$$

## Electrostatic Forces

Using the principle of virtual displacement to calculate force in two situations:

1. **System of bodies with fixed charges**:
   - Mechanical work is from the reduced stored electrostatic energy:

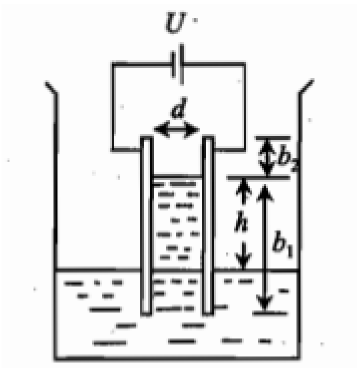
   $F_Q = \nabla W_e (N)$

   - Electric torque rotates one of the bodies by $d\theta$ (a virtual rotation) about an axis:

   $T_Q = \frac{\partial W_e}{\partial \theta}\ (N \cdot m)$
2. **System of conducting bodies with fixed potentials**:
   - The fixed potential can be retained by connecting with an external source.
   - $F_v = \nabla W_e$
   - $T_v = \frac{\partial W_e}{\partial \theta}$

## Exercise:

A parallel plate air capacitor is vertically inserted into a liquid dielectric with relative permittivity $\epsilon_r$ and density $\rho$. The capacitor plates have an area $S$ (where $S = ab$), and a separation distance $d$. The voltage $U$ between the two plates is kept constant. Find the height $h$ that the liquid level rises in the capacitor.



## Thanks for coming!