RC 7： Steady Magnetic Fields

By Mo Yang

Magnetic Field Intensity, Relative Permeability and Magnetic circuit

Considering the effect of both the internal dipole moment and the induced magnetic moment in a magnetic material, the magnetic field intensity H is defined as:

$D = \varepsilon_0 E + P$

$$H = \frac{B}{\mu_0} - M \tag{1}$$

$B = \mu_0 H + M$

$$\nabla \times H = J \tag{2}$$

directly relates the magnetic field intensity with the density of free charge. The integral form of which is then, $\iint_S J\,dS = \iint_S \nabla \times H\,dS = \oint_{\partial S} H\,dl$, $\iint_S J\,dS = I$.

$$\oint_C H \cdot dl = I \tag{3}$$

It is another form of Ampere's circuital law: the circulation of the magnetic field intensity around any closed path is equal to the free current flowing through the surface bounded by the path.

Circuit

I

Φ Φ

If the closed path C is chosen to enclose N turns of a winding carrying a current I that excites a magnetic circuit, we have

$$\oint_C H \cdot dl = NI = V_m \tag{4}$$

V_m is analogous to electromotive force (emf) and is called magnetomotive force (mmf).

If we define: $D = \varepsilon_r \varepsilon_0 E \Rightarrow P = (\varepsilon_r - 1)\varepsilon_0 E$ $\quad \frac{\varepsilon_0 E}{P} = \varepsilon_r - 1$

Core: $P \propto E$

$$M = \chi_m H \tag{5}$$

Now we have $M \propto H$

where χ_m is magnetic susceptibility, Then

$$B = \mu_0(1+\chi_m)H = \boxed{\mu_0\mu_r H = \mu H} \tag{6}$$

M is a complex function of H → taylor expansion.

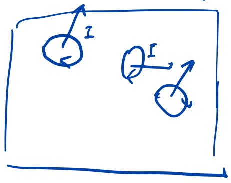
→ in a certain range: proportional

$$H = \frac{1}{\mu} B \tag{7}$$

$$\mu_r = 1 + \chi_m = \frac{\mu}{\mu_0} \tag{8}$$

where μ_r is the relative permeability of the medium. $\mu = \mu_r \mu_0$ is the absolute permeability/permeability.

$\rightarrow H$



Create Magnetic field M.

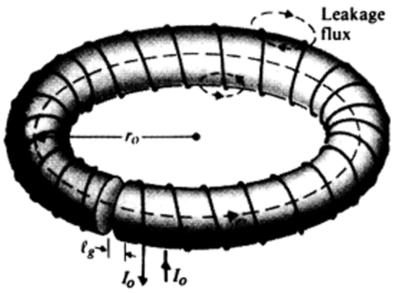
Exercise

Assume that N turns of wire are wound around a toroidal core of a ferromagnetic material with permeability μ. The core has a mean radius r_o, a circular cross section of radius a ($a \ll r_o$), and a narrow air gap of length l_g, as shown in the following figure. A steady current I_o flows in the wire. Determine:

(a) the magnetic flux density, B_f, in the ferromagnetic core;

(b) the magnetic field intensity, H_f, in the core;

(c) the magnetic field intensity, H_g, in the air gap.



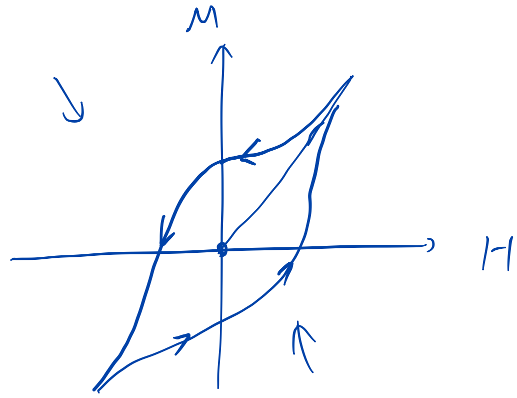
(a) B : continuity $\quad B_f = B_g = B$

$\mu H_f = \mu H_g = B. \quad \oint_c H\,dl = NI \quad \frac{B}{\mu}(2\pi r - l_g) + \frac{B}{\mu_0} l_g = NI$

$B_f = B_g = B = \dfrac{NI}{\frac{2\pi r - l_g}{\mu} + \frac{l_g}{\mu_0}}$

(b). $\quad H_f = \dfrac{B}{\mu} = \ldots$

(c) $\quad H_g = \dfrac{B}{\mu_0} = \ldots$

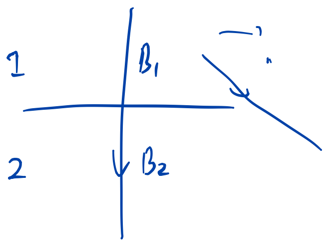


Behavior of Magnetic Materials

1. **Diamagnetic**, if $\mu_r \lesssim 1$ (χ_m is a very small negative number)
2. **Paramagnetic**, if $\mu_r \gtrsim 1$ (χ_m is a very small positive number)
3. **Ferromagnetic**, if $\mu_r \gg 1$ (χ_m is a large positive number)

Boundary Conditions for Magnetostatic Fields

1. The normal component of B is continuous across an interface,



$$\boxed{B_{1n} = B_{2n}} \tag{9}$$

which is

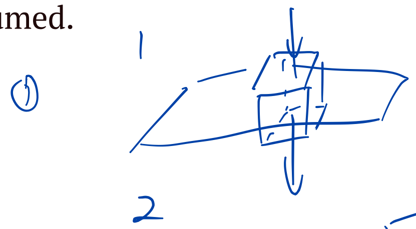
$a_n \times (H_1 - H_2) = J_S$ $\quad \nabla \cdot H = 0.$

$$\mu_1 H_{1n} = \mu_2 H_{2n} \tag{10}$$

2. The tangential component of the H field is discontinuous across an interface where a free surface current exists.

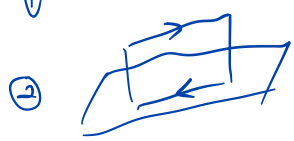
$$\mathbf{a_n} \times (H_1 - H_2) = J_s \tag{11}$$

When the conductivities of both media are finite, currents are defined by volume current densities and free surface currents do not exist on the interface. Hence $J_s = 0$, the tangential component of H is continuous across the boundary of almost all physical media; it is discontinuous only when an interface with an ideal perfect conductor or a superconductor is assumed.



$\nabla \cdot H = 0.$ no source.

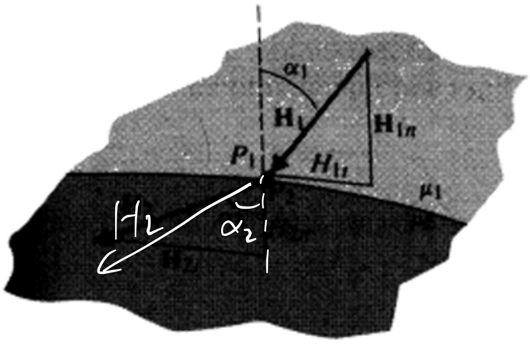
$\mu_1 H_{1n} = \mu_2 H_{2n}.$



$\nabla \times H = J \rightarrow a_n \times (H_1 - H_2) = J_S$

Exercise

Two magnetic media with permeabilities μ_1 and μ_2 have a common boundary, as shown in the following figure. The magnetic field intensity in medium 1 at the point P_1 has a magnitude H_1 and makes an angle α_1 with the normal. Determine the magnitude and the direction of the magnetic field intensity at point P_2 in medium 2.



① $\mu_1 H_1 \cos\alpha_1 = \mu_2 H_2 \cos\alpha_2$

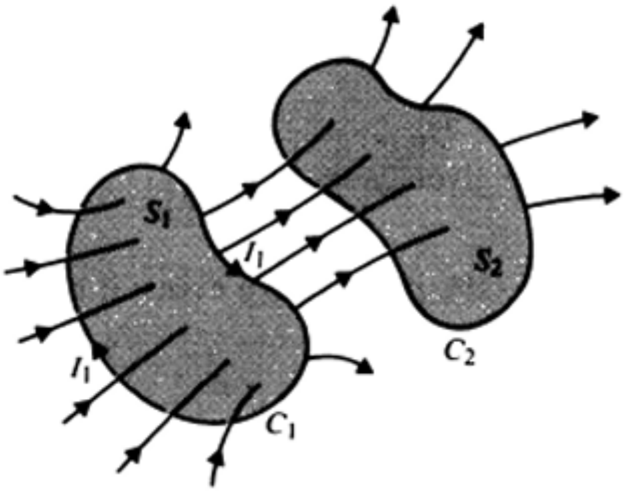
② 切: $H_1 \sin\alpha_1 = H_2 \sin\alpha_2$.

$$\alpha_2 = \arctan\left(\frac{\mu_2}{\mu_1}\tan\alpha_1\right).$$

$$|H_2| = H_1\sqrt{\sin^2\alpha_1 + \left(\frac{\mu_1}{\mu_2}\cos\alpha_1\right)^2}$$

Inductances and Inductors

For two loops C_1, C_2:



For linear media where permeability does not change with I_1,

$$L_{12} = \frac{\Lambda_{12}}{I_1} \tag{12}$$

where Λ_{12} is the flux linkage, L_{12} is the mutual inductance between loops C_1 and C_2. A more general definition for L_{12} is then:

$$\boxed{L_{12} = \frac{d\Lambda_{12}}{dI_1}} \tag{13}$$

The total flux linkage with C_1 caused by I_1 is calculated as:

$$\Lambda_{11} = N_1\Phi_{11} > N_1\Phi_{12} \tag{14}$$

The self-inductance of loop C_1 could be calculated as:

$$L_{11} = \frac{\Lambda_{11}}{I_1} \tag{15}$$

For linear medium, in general:

$$L_{11} = \frac{d\Lambda_{11}}{dI_1} \tag{16}$$

A conductor arranged in an appropriate shape (e.g., a conducting wire wound as a coil) to supply a certain amount of self-inductance is called an inductor. It can store magnetic energy.

Method to determine the self-inductance of an inductor:

1. Choose an appropriate coordinate system for the given geometry.
2. Assume a current I in the conducting wire.
3. Find B from I by Ampere's circuital law,

$$\oint_C B \cdot dl = \mu_0 I \tag{17}$$

if symmetry exists; if not, Biot-Savart law

$$B = \frac{\mu_0 I}{4\pi} \oint_{C'} \frac{dl' \times \mathbf{a}_R}{R^2} \tag{18}$$

must be used.

4. Find the flux linking with each turn, Φ, from B by integration:

$$\Phi = \int_S B \cdot ds \tag{19}$$

get the flux

where S is the area over which B exists and links with the assumed current.

5. Find the flux linkage Λ by multiplying Φ by the number of turns.
6. Find L by taking the ratio $L = \frac{\Lambda}{I}$.

To determine the mutual inductance L_{12} between two circuits, after choosing an appropriate coordinate system, assume $I_1 \rightarrow$ Find $B_1 \rightarrow$ Find Φ_{12} by integrating B_1 over surface $S_2 \rightarrow$ Find flux linkage $\Lambda_{12} = N_2\Phi_{12} \rightarrow$ Find $L_{12} = \frac{\Lambda_{12}}{I_1}$.

In high-frequency applications, current tends to concentrate in the "skin" of the inner conductor as a surface current, internal self-inductance tends to zero. Neumann formula for mutual inductance:

$$L_{12} = L_{21} = \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{dl_1 \cdot dl_2}{R} \tag{20}$$

Magnetic Energy

For a system of N loops carrying currents,

$$W_m = \frac{1}{2} \sum_{j=1}^{N} \sum_{k=1}^{N} L_{jk} I_j I_k \tag{21}$$

For a single inductor,

$$W_m = \frac{1}{2} L I^2 \tag{22}$$

$$W_m = \frac{1}{2} \sum_{k=1}^{N} I_k \Phi_k \tag{23}$$

Magnetic Energy In Terms of Field Quantities

Generally,

$$W_m = \frac{1}{2} \int_{V'} \mathbf{A} \cdot \mathbf{J} \, dv' = \frac{1}{2} \int_{V'} \mathbf{H} \cdot \mathbf{B} \, dv' = \frac{1}{2} \int_{V'} \frac{B^2}{\mu} \, dv' = \frac{1}{2} \int_{V'} \mu H^2 \, dv' \tag{24}$$

If we define a magnetic energy density w_m such that

$$W_m = \int_{V'} w_m \, dv' \tag{25}$$

then

$$w_m = \frac{1}{2} \mathbf{H} \cdot \mathbf{B} = \frac{B^2}{2\mu} = \frac{1}{2} \mu H^2 \tag{26}$$

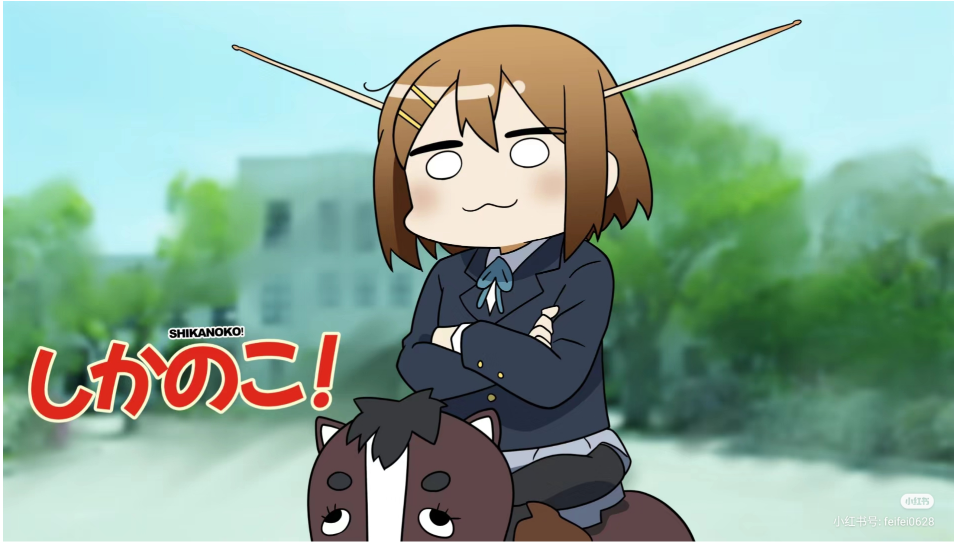
And we have

$$L = \frac{2 W_m}{I^2} \tag{27}$$

Reference

- Fan Hu, RC Slides, VE230
- Nana Liu, Slides, VE230

Thanks!



SHIKANOKO!
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小红书号: feifei0628