Recitation Class for Curves

Provided by Yang Mo

Something More about Continuity

Review: Suppose $lim_{x \to x_0, y \to y_0} f(x,y) = f(x_0, y_0)$, then the function $f(x,y)$ is said to be continuous at point (x_0, y_0).

Example 1

Discuss the continuity of

$$f(x,y) = \begin{cases} ln(1+xy)^{\frac{1}{xy}}, xy \neq 0 \\ 1, \text{otherwise} \end{cases} \tag{1}$$

Discuss the continuity of

$$f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2}, xy \neq 0 \\ 0, \text{otherwise} \end{cases} \tag{2}$$

at the point where $(x,y) = (0,0)$.

What is Curve?

Definition

Let $(V; \| \cdot \|)$ be a normed vector space and $I \subset R$ an interval.

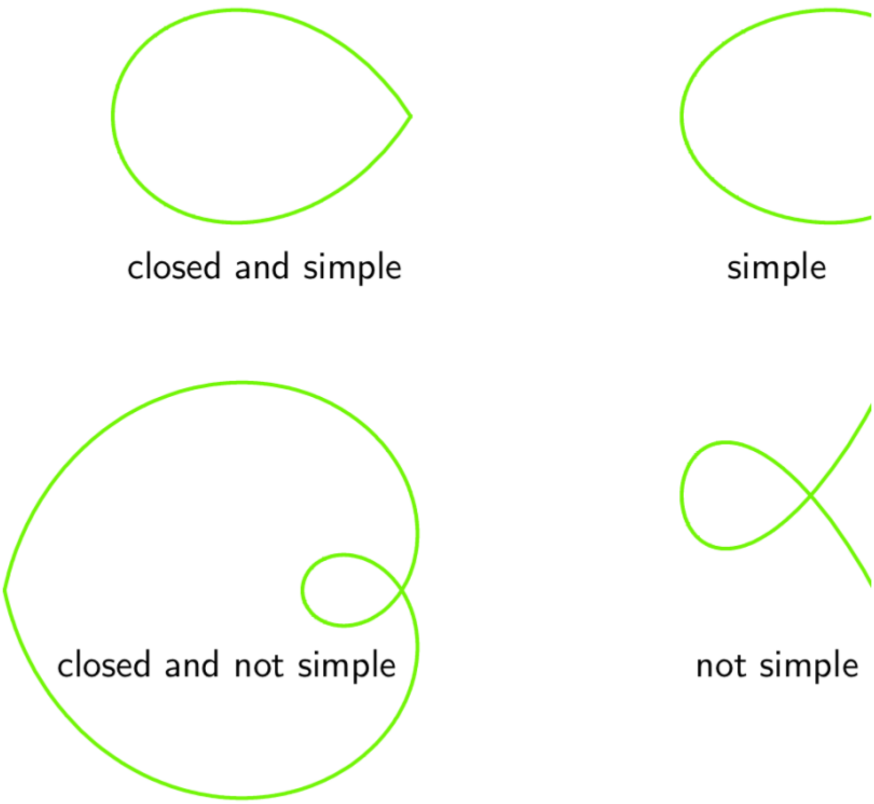
A set $C \subset V$ for which there exists a continuous, surjective and locally injective map γ: $I \to C$ is called a curve.

The map γ, is called a parametrization of C.

A curve C together with a parametrization γ,, i.e., the pair $(C; \gamma)$, is called a parametrized curve.

(From Horst's Slides, Page 352)

Simple, Open and Closed Curves



(From Horst's Slides, Page 358)

Some Characters to describe Curve

Tangent Vector

Tangent line at $t_0 : T_pC = x \in V : x = \gamma(t_0) + \gamma\prime(t_0)t, t \in R$

It is an approximation of the curve!

Unit tangent vector: $T \circ \gamma(t) := \frac{\gamma\prime(t)}{\|\gamma\prime(t)\|}$

Curve's Length

Suppose C is a curve in V and $\gamma : [b, s] \to C$ the parameterization.

$$L = \int_b^s ||\gamma'(t)||dt \tag{1}$$

This is actually induced with the help of Lagrange's Mean Value Theory:

$$L = \lim_{n \to \infty} \sum_{i=1}^{n} |\gamma(t_i) - \gamma(t_{i-1})| \tag{2}$$

Curvature

$$\kappa \circ \gamma(t) = \frac{\| \ (T \circ \gamma)\prime(t) \ \|}{\| \ \gamma\prime(t) \ \|} \tag{3}$$

Example 2

Calculate the curvature of the curve with parameterization
$f : [0, 1] \to R, f(x) = (cos(2\pi x), sin(2\pi x))$

(From Chen YuXiang and Zhang Leyang)

Example 3

Calculate the curvature of the curve $y = f(x)$ (Consider the curve consist of all (x,y))

Line Integral and Potential Fields

Line Integral

line integral of a potential function: the line integral of f along C^* by

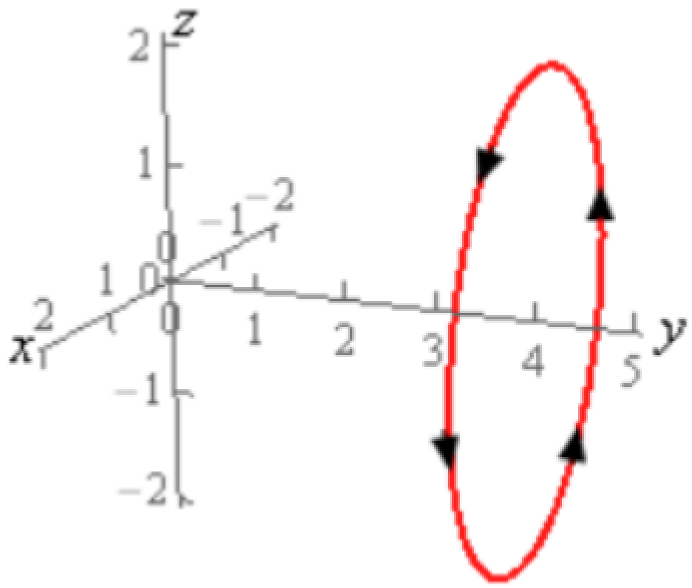
$$\int_{C^*} f\ dl := \int_I (f \circ \gamma)(t) \cdot \parallel \gamma'(t) \parallel dt \tag{4}$$

If we calculate the line integral using a concrete parametrization $\gamma : I \to C$, we obtain

$$\int_{C^*} F d\vec{l} = \int_{C^*} \langle F, T \rangle dl = \int_I \langle F \circ \gamma(t), T \circ \gamma(t) \rangle \parallel \gamma'(t) \parallel dt \tag{5}$$

Example 4

Evaluate $\int_{C^*} x^2 y^2 dl$ where C is the circle centered at (0, 4, 0) with radius 2. The orientation is shown in following figure.



Potential Fields

Condition: There exist U(x) such that $F(x) = \nabla U(x)$

In physics there is a negative sign

Conservative Field:

$$\oint_{\mathcal{C}} F\, d\vec{\ell} = 0.$$

Potential Fields are Conservative!

Example 5

Prove the gravity field is conservative.

References

- VV285, slide. Horst Hohberger
- RC, slides. TA-Pingbang Hu
- RC, slides. TA-Chen Yuxiang

For further questions, you can contact me through wechat.

Fragments

芬兰

扫一扫上面的二维码图案，加我为朋友。

The end.