

Midterm Review

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强马尔科夫性. Hitting time之后的分布也和前面的独立

transient 定义: $S_{xy} > 0 \quad S_{yy} < 1 \quad \text{且 } x \neq \text{transient}$.

recurrent 定义: $S_{xy} > 0$ 就有 $\{y \in z\} \Rightarrow x \text{ recurrent}$

Decomposition 定理: 状态空间 S 有限 $\Rightarrow S = T \cup R_1 \cup \dots \cup R_n$.

T 为 transient state 的集合. 而 R_i 为 recurrent closed irreducible State.

$$N_y = \sum_{n=0}^{\infty} \mathbb{1}_{\{X_n=y\}} \quad \{N_y > k\} = \{T_y^k < \infty\}. \quad \mathbb{E}_x N_y = \sum_{n=0}^{\infty} p^n(x, y) = \frac{S_{xy}}{1 - S_{yy}}$$

稳态分布的存在性 & 唯一性

$$\mu_x(y) \text{ 一直存在} \quad \mu_x(y) = \sum_{n \geq 1} P_x(X_n=y, T_x \geq n) \quad \Rightarrow \bar{\pi}(y) = \frac{\mu_x(y)}{\mathbb{E}_x T_x}$$

$$\bar{\pi}(x) = \sum_y \bar{\pi}(y) p(y, x).$$

详细平衡条件

$$\begin{pmatrix} \text{一生死链} \\ \text{一网上随机游走} \end{pmatrix}$$

$$\bar{\pi}(x) p(x, y) = \bar{\pi}(y) p(y, x)$$

Reversibility: Kolmogorov Condition 用途不连比较快

收敛性

$$\lim_{n \rightarrow \infty} p^n(x, y) = \bar{\pi}(y) \quad (\text{即非aperiodic 条件}).$$

退出时间和退出分布

$$P_x(V_A < V_B) = g(x). \quad g(x) = \sum_y p(x, y) g(y) \quad g(a) = 1. \quad g(b) = 0.$$

$$E_A V_A = h(x) \quad h(x) = 1 + \sum_y p(x, y) h(y). \quad h(a) = 0.$$

Ergodic Theorem -

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=0}^{n-1} f(x_k) = \bar{\int}_y \pi(y) f(y).$$

Asymptotic Frequency $\lim_{n \rightarrow \infty} \frac{N_n(x)}{n} = \frac{1}{E_x T_x}$

MDP (S, A, P_a, R)

$$V(x) = \sup_a \left[R(x, a) + \lambda \sum_y P_a(x, y) V(y) \right]$$