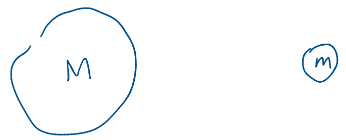
L22 Energy Momentum Method

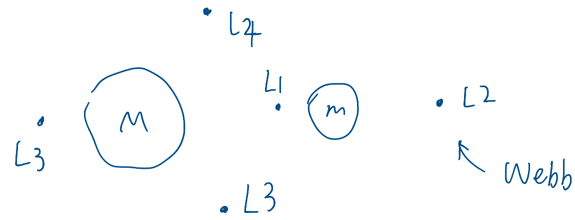
2024年4月8日 星期一 10:16

Restricted 3 Body problem.

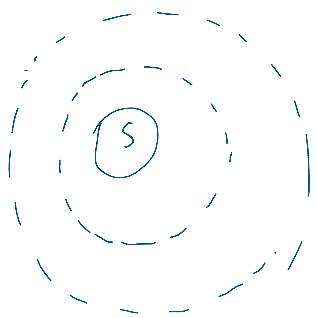
Stationary pts



$m = \varepsilon \ll m, M.$



Hirarachical model



Darwin.

Energy-Casimir Energy-Momentum method for providing non-lin stability

① $\dot{u} = X(u)$ Poisson Sys / Hamotonian. 找到一个守恒量 H

② 找到另一族 C 是守恒量 (称为 Casimirs) $\{C, G\} = 0, \forall G$.

③ $H_C = H + \phi(C)$. 选择一个 ϕ 使 $\delta H_C(u_e) = 0$.

④ 检查 ϕ 是否满足 $\delta^2 H_C(u_e) > 0$ 或 < 0. - formal stable

Lyapnov 稳定

例如对于刚体而言 $H = \frac{1}{2}\Pi \cdot \omega$. $C = \Pi \cdot \Pi$ $\quad \phi(C) + H$.

Controlled Rigid Body.

$$\begin{cases} I_1 \dot{w}_1 = (I_2 - I_3) w_2 w_3 + u_1 \\ I_2 \dot{w}_2 = (I_3 - I_1) w_3 w_1 + u_2 \end{cases}$$

Check: 只需要2个扭矩就可控

用 Lie-algebra rank condition

$$\begin{cases} I_1 \dot{w}_1 = (I_2 - I_3) w_2 w_3 + u_1 \\ I_2 \dot{w}_2 = (I_3 - I_1) w_3 w_1 + u_2 \\ I_3 \dot{w}_3 = (I_1 - I_2) w_1 w_2 + u_3 \end{cases}$$

Check: 只需要2个控制的 [illegible]

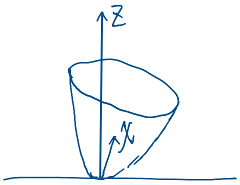
用 Lie-algebra rank condition

只需要一个扭矩就可以沿一个轴平衡

例. intermediate axis 不稳定. $\Rightarrow u_3 = -K w_1 w_2$

用一个 Control 就可以实现该轴平衡

Heavy top (陀螺)



$$\begin{cases} \dfrac{d\Pi}{dt} = \Pi \times w + Mgl\, \nu \times \chi \\ \dfrac{d\nu}{dt} = \nu \times w . \end{cases}$$

ν 向量代表着 z 轴方向　　　χ 代表从相交点到质心的向量

$$H(\Pi, \nu) = \frac{1}{2} \Pi w + Mgl\, \nu \cdot \chi$$

Π, K, $\|\nu\|^2$ 守恒

找到 $\{F, G\}(\Pi) = -\Pi \cdot (\nabla_\Pi F \times \nabla_\Pi G) - \nu (\nabla_\Pi F \times \nabla_\nu G + \nabla_\nu F \times \nabla_\nu G)$.

2-D 不可压缩流体　Euler equations

$$\frac{\partial v}{\partial t} + (v \cdot \nabla) v = -\nabla p \qquad \operatorname{div} v = 0$$

$v(x, 0) = v_0(x)$. $\equiv (v \cdot \nabla) v + \nabla p$　　tangent to body ∂D of D.

div free.　　　- domain of fluid.

$$H(v) = \int_D \frac{1}{2} |v|^2 \, dx\, dy$$

$$\{F, G\} = -\int_D v \cdot \left[\frac{\delta F}{\delta v}, \frac{\delta G}{\delta v} \right] dx\, dy$$

$\dfrac{\delta F}{\delta v}$ = determined by $DF(v) \cdot \delta v = \displaystyle\int \frac{\delta F}{\delta v} \cdot \delta v \, dx\, dy$

$$\frac{\delta F}{\delta v} = \text{determined by } DF(v)\cdot\delta v = \int \frac{\delta F}{\delta v}\cdot \delta v \, dx\, dy$$

$$\left[\frac{\delta F}{\delta v}, \frac{\delta G}{\delta v}\right] = \left(\frac{\delta F}{\delta v}\cdot\nabla\right)\frac{\delta G}{\delta v} - \left(\frac{\delta G}{\delta v}\cdot\nabla\right)\frac{\delta F}{\delta v}$$

If $w = \tilde{z}$ $\quad C_{\Phi} = \int_D \Phi(w)\, dx\, dy.$

在无限维空间上面 H_c 需要是 topology 空间上的 norms 决定.

例：在 $\mathbb{L}^2$ 上定义函数 $(y_1, y_2, \cdots y_n, \cdots)$

$$H(y_1, \cdots y_n, \cdots) = \sum_{k=1}^{\infty}\left(\frac{y_k^2}{2k} - \frac{y_k^4}{4}\right) \qquad \text{在原点 } H''(0) = \frac{\partial^2 H}{\partial y_k \partial y_j} = \begin{bmatrix} 1 & & & \\ & \frac{1}{2} & & \\ & & \frac{1}{3} & \\ & & & \ddots \end{bmatrix} > 0$$

而可以选择一个 y_k 使得 H 是负的

<u>Non-linear Stability for Energy-Casimir</u>

Step 5: a) let $\Delta u = u - u_e$.

b). 寻找 Q_1, Q_2 的 quadratic functionals

$$Q_1(\Delta u) \leq H(u_e + \Delta u) - H(u_e) - \delta H(u_e)\cdot \Delta u$$

$$Q_2(\Delta u) \leq C(u_e + \Delta u) - C(u_e) - \delta C(u_e)\cdot \Delta u.$$

$H_c = H + C$

c) Require $Q_1(\Delta u) + Q_2(\Delta u) > 0$

d) Set $\|\Delta u\|^2 = Q_1(\Delta u) + Q_2(\Delta u)$ $\qquad$ 则 $d(u, u_e) = \|\Delta u\|^2$

e) Require
$$\begin{cases} |H(u_e + \Delta u) - H(u_e)| \leq C_1 \|\Delta u\|^{\alpha} \\ |C(u_e + \Delta u) - C(u_e)| \leq C_2 \|\Delta u\|^{\alpha} \end{cases}$$

$\Rightarrow \|\Delta u\|^2 \leq H_c(u_e + \Delta u) - H_c(u_e)$ 因为 $\delta H_c(u_e) = 0$.

H. C. const in time

$\|\Delta u_{\text{time } t}\|^2 \leq H_c(u_e + \Delta u) - H_c(u_e)\Big|_{t=0}$

$\|(\Delta u)_{\text{time } t}\|^2 \leq (C_1 + C_2)\, \|\Delta u|_{t=0}\|^{\alpha}$

Add dissipation

S^2 Symmetry.

$M\ddot{q} + Sq + Vq = 0.$ ($Dq > 0$ Damping)

M: postive definite. S: Skew V. potentical const.