L2 Newton's Laws

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Newton Law:
① 物体在不受力时为静止或匀速直线运动.
② 速度的变化与力成正比.
③ 力有等大的反作用力.

Newton's Law 处理的是 Particle Mechanics. 而对于刚体运动, (Euler).

$$I\dot{\omega} = \underbrace{T}_{\text{Torch, 扭矩}}(t).$$

Variational Principals

最小作用原则: Principal of Least Action. 对于 particles 和 light 都用.

光: Fermat Principle of Least time. 粒子: Lagrange (1834) / Hamilton (1835).

Lagrangian & Hamiltonian 概览

- General Theory of ODE $\dot{x} = f(x)$, $x \in \mathbb{R}^n$ n维一阶系统.

Lagrangian (Mechanics ~1750) ⇒ Newton定律的替代.

首先, 选一个确认空间 (Configuration Space) Q (可以是 $\mathbb{R}^n$, 流形 M, ...)

- local coordinates: q^i, $i=1,2,\dots n$. $(q^1, q^2, \dots q^n)$ - vector. $(\dot{q}^1, \dot{q}^2 \dots \dot{q}^n)$ - velocity.

Lagrangian 为一个 Smooth Function.

$$L(q^i, \dot{q}^i, t) = \underbrace{T}_{\text{动能}} - \underbrace{V}_{\text{势能}} \qquad \begin{cases} T(KE) = \sum_{i,j=1}^{n} g_{ij} \cdot \dot{q}^i \cdot \dot{q}^j \\ V = V(q). \end{cases}$$

Variational Principal

$$\delta \int_a^b L(q^i, \dot{q}^i, t)\, dt = 0.$$ 其中 a, b 为两个 fixed point. time interval

- finding critical points of $\int L\,dt$ over a class of curves. L 是 functional (泛函).

δq 为 variation of the curve. 因为我们要找 ($\int L\,dt$ 的最小值)

$$\delta \int L\,dt = \int_a^b \left(\frac{\partial L}{\partial q^i} \delta q^i + \frac{\partial L}{\partial \dot{q}^i} \delta \dot{q}^i \right) dt = 0.$$ 其中 $\delta \dot{q}^i = \frac{d}{dt}(\delta q^i)$

分部积分. $$\int_a^b \left(\frac{\partial L}{\partial q^i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^i} \right) \right) \delta q^i\, dt = 0.$$ 而由于 δq^i 是任意取的.

那么有 $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^i} \right) - \frac{\partial L}{\partial q^i} = 0. \quad i = 1, 2, \dots n$$ 最小作用原则. Critical Action.

Newton

$L(q, \dot{q}, t) = \frac{1}{2} \sum_{i=1}^{m} m_i \|\dot{q}^i\|^2 - V(q^i) \quad \Rightarrow \frac{d}{dt}(m_i \dot{q}^i) = -\frac{\partial V}{\partial q^i}$

更广泛的. 给定 $g_{ij}(q)$. $KE = T = \frac{1}{2} \sum_{i,j=1}^{n} g_{ij}(q)\, \dot{q}^i \dot{q}^j$

其中 $g_{ij}(q)$ 是正定的.

Principal of Least Action (Hamilton's Principal)

拉格朗日一开始是从力的平衡推导出来的.

Variations $\delta \int_a^b L(q, \dot{q})\, dt = 0.$ $(t, \varepsilon) \to q(t, \varepsilon).$

(i). $q(t, 0) = q(t)$ $t \in [a, b]$.

(ii) $q(a, \varepsilon) = q_0$. $q(b, \varepsilon) = q_b$ δq: 虚位移.

$\delta q(t) = \frac{d}{d\varepsilon} q(t, \varepsilon) \Big|_{\varepsilon = 0}.$

例: $q(t, \alpha) = q(t, 0) + \alpha \eta(t)$ 是另外一个曲线起点 a 终点 b.

我们去优化方程 $J(\alpha) = \int_{t_1}^{t_2} L(q(t,\alpha), \dot{q}(t,\alpha))\, dt.$