L19 Controlability

2024年3月25日 星期一 10:01

<u>Control System</u> Kinematic control system $\sum_{i=1}^{m} u_i f_i(x)$ $x \in \mathbb{R}^m$, $x \in M^n$

Lie-Bracket $[f_1, f_2]$, $[X_1, X_2]$ 得到一个新的与两个向量场垂直的向量场

General: $\dot{x} = f(x,u)$. $x \in M^n$. $u \in \mathbb{R}^m$

Distribution: 定义：在一个流形M上的光滑分布是对于每个点 $x \in M$ 由 X的值扩展出的子空间

i.e. Smooth assigment of a subspace to the tangent space at each point : vector sub-bundle.

Denote Δ and the subspace at each x is denoted by $\Delta_x \subset T_x M$.

Distribution Δ 被称为 involutive 如果对任何两个 vector field X和Y on M with values in Δ

$[X, Y]$ is also vector field with values in Δ

一个 sub-bundle Δ 被称为 integrable (可积的) 如果对于每个点 $x \in M$, $\exists$ local sub-manifold on M. contain x s.t. its tangent bundle equals Δ resticted to this sub-manifold.

如果 Δ 是可积的, the local 积分流形 可以被扩展, through each $x \in M$ to a maximal 积分流形

Collection of all maximal integral sub-manifolds of M 被称为 Foliation.

Frobenius 定理: Involving of Δ is equivalent to the integrability of Δ, which is in turn equivalent to existence of a foliation on M whose tangent bundle equals Δ

给定 $X_1, \cdots X_d$ on M. denote dist Δ defined by their span $\Delta = \mathrm{span}\{X_1, \cdots X_d\}$

Δ 被称为 Regular 如果 各处的维度是一样的

e.g. $X_1 = \frac{\partial}{\partial x}$ $X_2 = \frac{\partial}{\partial y}$ on $\mathbb{R}^2$.

Non-linear Control

Defn: 有限维非线性控制问题，在一个光滑流形 M 上，是一个形式为 $\dot{x}=f(x,u)$ 的微分方程

where $x\in M$. $u(t)$ 是一个 time-dependent map 从 $\mathbb{R}^+$ 到 constraint set $\Omega\subset\mathbb{R}^m$

$f\subset \underset{\sim}{C^\infty}$ or $\underset{\sim}{C^\omega}$ 是 piecewise smooth or analytic. 这样的 u 是 admissable

一直可导　所有点可泰勒展开

Special Case : Affine Control (仿射控制). $\dot{X}=f(x)+\sum_{i=1}^{m} u_i(t)\, g_i(x)$

e.g. Heisenberg Sys. Affine with $f=0$.

称 f 为 drift vector field. g_i 为 control v.f.

e.g. $I_1\dot{w}_1=(I_2-I_3)\,w_2w_3 + \underset{\sim}{u_i}$ Control.

Defn $\dot{x}=f(x)+\sum u_i(t)\,g_i(x)$ ⊛ 是可控的. 若对任何两个点 $x_0, x_f\in M$. ∃ admissable control $u(t)$ defined on $[0,T]$

s.t. ⊛ with x_0 的初条件 可以最终经过时间 T 到达 x_f 点

Hard to prove in general. NP 难问题.

Defn : Given $x_0\in M$. define $R(x_0,t)$. to be the set of all $X\in M$. for which ∃ an admissable control u

s.t. ∃ a trajectory x. such $X(0)=x_0$. $X(t)$

Reachable set from x_0 at time T 定义为 $R_T(x_0)=\bigcup_{0\le t\le T} R(x_0,t)$.

Defn : Accessibility Algebra $\mathcal{C}$ of $(*)$ is the smallest Lie-algebra of M 上的向量场 contains $f, g_1, \cdots g_m$

Defined by $\{f, g_1, \cdots g_m\}_{L.A.}$

Defn : Accessibility Distribution C of ⊛ is the distribution generated by v.f. in $\mathcal{C}$.

Defn : System $(*)$ on M. 被称为 accessible for $p\in M$ 如果对 $\forall\, T>0$. $R_T(p)$ contains 一个 non-empty open set.

定理 : 考虑 x 并假定 向量场是 C^∞ 如果 $\dim C(X_0)=n$. then for $\forall\, T>0$. Set $R_T(x_0)$ 有 nonemptic interior. i.e. Sympletic

定理：如果 $f(x)$ is analytic. 如果 $\dim C(x) = n$ everywhere on M. 并且 either ① $f=0$ ② $\operatorname{div} f = 0$, M. compact. Rem 总可控的

情况 1：因为 $\dot{x} = \sum_{i=1}^{m} u_i(t)\, g_i(x)$. 而 $\{g_i(x)\}_{LA}$ 是 dim n 的. 所以可控.

情况 2：$\operatorname{div} f = 0$. volume conserve. Poincare Recurrence. + Compact

对于线性控制系统.

$\dot{x} = Ax + Bu$. $x \in \mathbb{R}^n$ $A^{n\times n}$ $B^{n\times m}$ Controllabilble iff $[B, AB, A^2B, A^{n-1}B]$ 有秩 n.

Use accessibility rank condition

$\dot{x} = Ax + \sum_{i=1}^{m} b_i u_i$ e spand smallest Lie-Algebra contains $Ax, b_1, b_2, \cdots b_m$.

Span of $[B, AB, A^2B, \cdots A^{n-1}B]$.

Kalman rank conditon Elmer Gilbert.

Hamotonian & Lagrangian Control Sys

Lagrangian $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}^i}(q, \dot{q}, u) - \frac{\partial L}{\partial q^i}(q, \dot{q}, u) = 0$

Hamotonian $\dot{q}^i = \frac{\partial H}{\partial p_i}(q, p, u)$ $\dot{p}_i = -\frac{\partial H}{\partial q^i}(q, p, u)$.

Affine Hamotonian control System

$$\dot{x} = X_{H_0}(x) + \sum_{j=1}^{m} X_{H_j}(x)\, u_j(t).$$

e.g. Quantum controls system . Spin control in NMR

Dual to observability. $x = f(x) + \sum_{i=1}^{n} g_i(x)$. $y_i = h_i(x)$. $i = 1, 2, \cdots k$

observability : Linear $y = Cx$

Hamotonian Sys. 我们能 rephrase 可控性 用 Poisson Bracket

$\{F, G\} = \omega(X_F, X_G)$

Lie - Algebra homomorphism between function & Poisson Bracket

如果 $\dot{x} = X_{\cdot}(x) + \sum^{m} u_i X_{\cdot}(x)$

Lie - Algebra homomorphism between function & Poisson Bracket

如果 $\dot{x} = X_H(x) + \sum_{i=1}^{m} u_i X_G(x).$

Lie - algebra rank condition

$$\{ G, \{H, G\}, \dots \{H, \{H, G\}\}, \dots \}$$