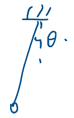
L1 Introduction

2024年1月10日 星期三 10:00

Dynamics - Application in Mechanism.

Pendulum.



Unstable

Cartpole

(Controlled)

Stablize

Kapitza (高速振动).

Topics:

① ODE 存在性和唯一性

② 稳定性 (线性 & 非线性) Bifurcations.

③ Mechanics : Lagrangian $(T-V)$.

Hamiltonian $(T+V+$ generalization$)$ Poisson

④ System on $\mathbb{R}^n$ 和流形 M 和群

例: $SO(3)$ 指 3×3 的正交矩阵群, 行列式为 1.

$O^TO = OO^T = I$. - 旋转群 (Rotation group)

例 $SO(2)$ $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \in SO(2)$.

⑤ 可积系统 (Solvable).

例: $\ddot{x} + x = 0$ 刚体动力学 -些振动 (coupled oscillation).

⑥ Chaotic Dynamics

i) 刚体 + 转子 (rotor)

ii) Lorentz System

⑦ Controlled Dynamics & Stability.

⑧ Quantum, PDE.

Examples:

1. 谐振子. Harmonic Oscillator.

$m\ddot{x} + kx = 0$. - linear 2^{nd} orders.

$x(t) = A\cos t + B\sin t$.

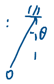
Linear 1^{st} order.

2. $\dot{x} = Ax$ (其 x 向量) $x(t) = \exp(At)\, x$

2. $\underset{\sim}{\dot{x}} = A\underset{\sim}{x}$ （指 $\underset{\sim}{x}$ 向量）. $\underset{\sim}{x}(t) = \exp(At)\,\underset{\sim}{x_0}$

Linear 1^{st} order.

而对非线性情况.

3. Pendulum:  $\ddot{\theta} + \sin\theta = 0.$

而对于小角 $\ddot{\theta} + \theta = 0$. 线性化.

4. Rigid Body Equations. (Euler Equations).

刚体3个方向的角速度为 w_1, w_2, w_3、转动惯量为 I_1, I_2, I_3.

$$\begin{cases} I_1 \dot{w}_1 = (I_2 - I_3)\, w_2 w_3 \\ I_2 \dot{w}_2 = (I_3 - I_1)\, w_3 w_1 \\ I_3 \dot{w}_3 = (I_1 - I_2)\, w_1 w_2 \end{cases}$$

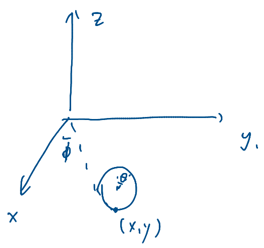
其中一个解: $\begin{cases} w_1 = const \\ w_2 = w_3 = 0 \end{cases}$

5. Hamaltonian.

$$H = \frac{1}{2} m \dot{x}^2 + V(x) = \frac{p^2}{2m} + V(x). \quad p = m\dot{x}$$

6. Non-Hamoltonian.

1) Nonholomic System - Penny.



Configuration Space: (x, y, θ, ϕ) （确认空间）

$$E = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} J \dot{\phi}^2$$

无滑滚动: $\begin{cases} \dot{x} = R\cos\phi\, \dot{\theta} \\ \dot{y} = R\sin\phi\, \dot{\theta} \end{cases} \Rightarrow \begin{cases} dx = R\cos\phi\, d\theta \\ dy = R\sin\phi\, d\theta. \end{cases}$

Non holonomic Constraint

$\begin{cases}\text{Nonholomic 约束: 对于速度的约束} \\ \text{Holonomic 约束: 对于位置的约束.}\end{cases}$

Control 了 θ 和 ϕ. Controllable: 可以从确认空间任意坐标移动到另一个坐标.

2) Coupled Oscillator.

Linear springs $k_i x$ $\quad k_i \frac{x^2}{2}$ - Solvable.

Non-Linear Spring. $V(1,2)$ $\quad e^{(x_1 - x_2)}$ - Solvable

(- FPU)

次回预告： Newton's Law. Least Action. Ham/Lag dynamics.