Assignment 01

A posteriori probability

Let $P(X|v=u)$ denote the probability of drawing out X blacks in a total of $N=10$ draws from urn v. Obviously, the probability of drawing out a black each time from urn v is

$$p_v \equiv P(color = black|v=u) = \frac{u}{10}$$

Considering the process of drawing 10 balls to be a Bernoulli distribution, we have the probability

$$\begin{aligned} P(X=3|v=u) &= C_{10}^3 \times p_v^3 \times (1-p_v)^7 \\ &= 120 \times (\frac{u}{10})^3 \times (1-\frac{u}{10})^7 \end{aligned}$$

Applying Bayesian law we have

$$P(v=u|X=3) = \frac{P(X=3|v=u) \times P(v=u)}{P(X=3)}$$

where $P(v=u) = \frac{1}{11}$, and $P(X=3)$ can be calculated as

$$P(X=3) = \frac{1}{11}\sum_{i=0}^{10} P(X=3|v=i)$$

Applying the figures we have results as follows:

u	$P(v=u\|X=3)$
0	0
1	0.063073
2	0.221240
3	**0.293220**
4	0.236256
5	0.128779
6	0.046668
7	0.009892
8	0.000864
9	0.000010
10	0

where maximum probability occurs when $u=3$.

Computer Generation of Random Variables

(a)

Since $Y=G(X)$, we have $P(Y \leq y) = P(G(X) \leq y)$.

According to the question, $F(t)$ must be monotone increasing and differentiable. Applying $F(\cdot)$ to both sides we have

$$P(G(X) \leq y) = P(F(G(X)) \leq F(y)) = P(X \leq F(y))$$

Because $X \sim Uniform(0, 1)$, its CDF takes the form

$$P(X \leq x) = x = F(y), \quad \forall u \in [0, 1)$$

Thus

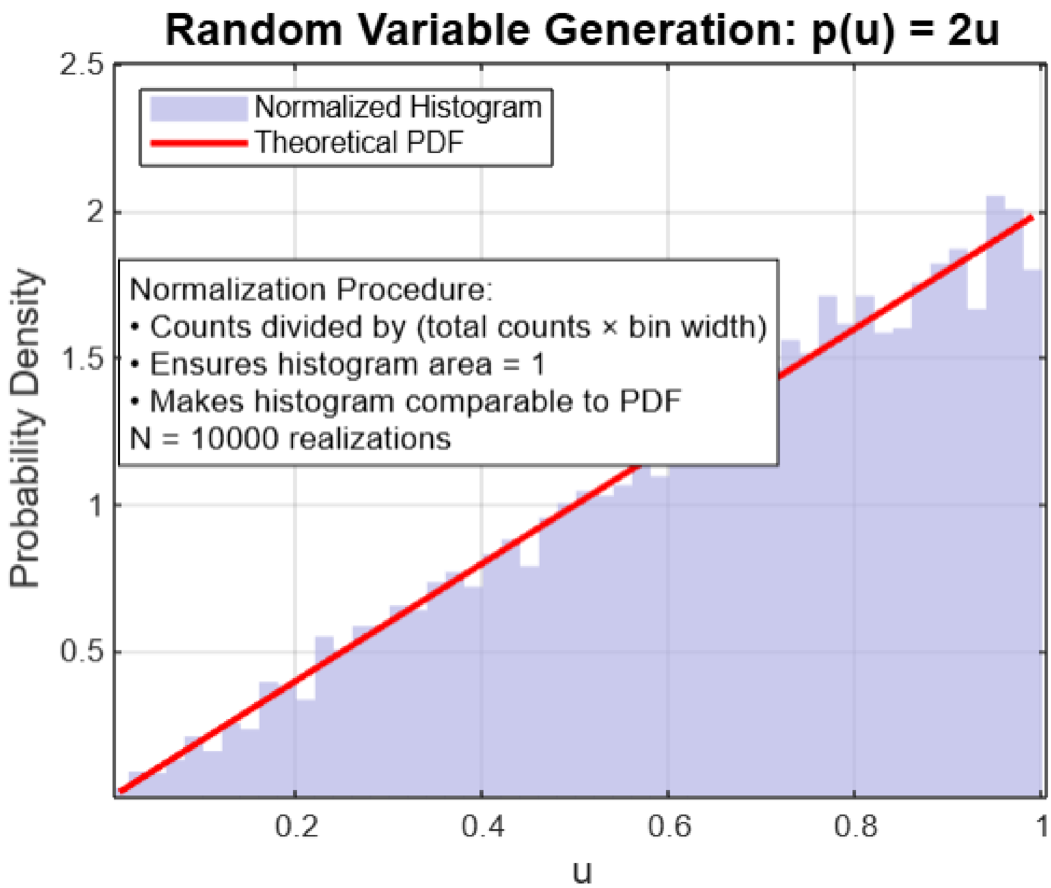
$$P(Y \leq y) = F(y)$$

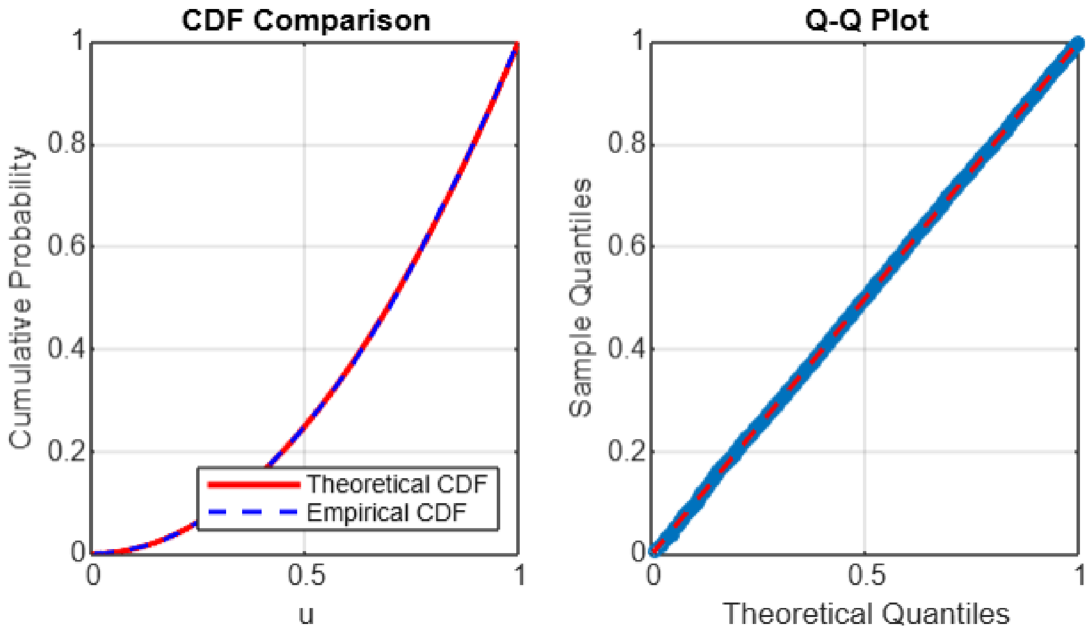
Applying the derivative to both side we have

$$p_Y(y) = p(y)$$

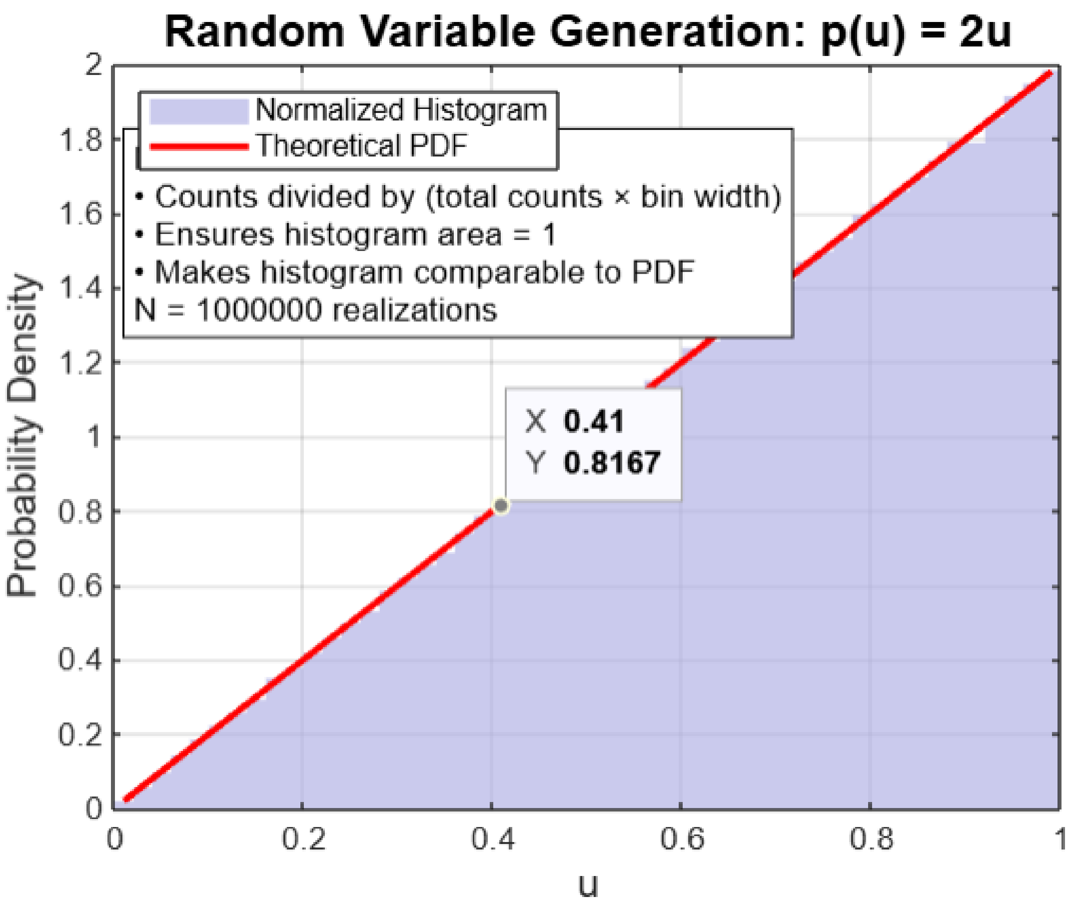
(b)

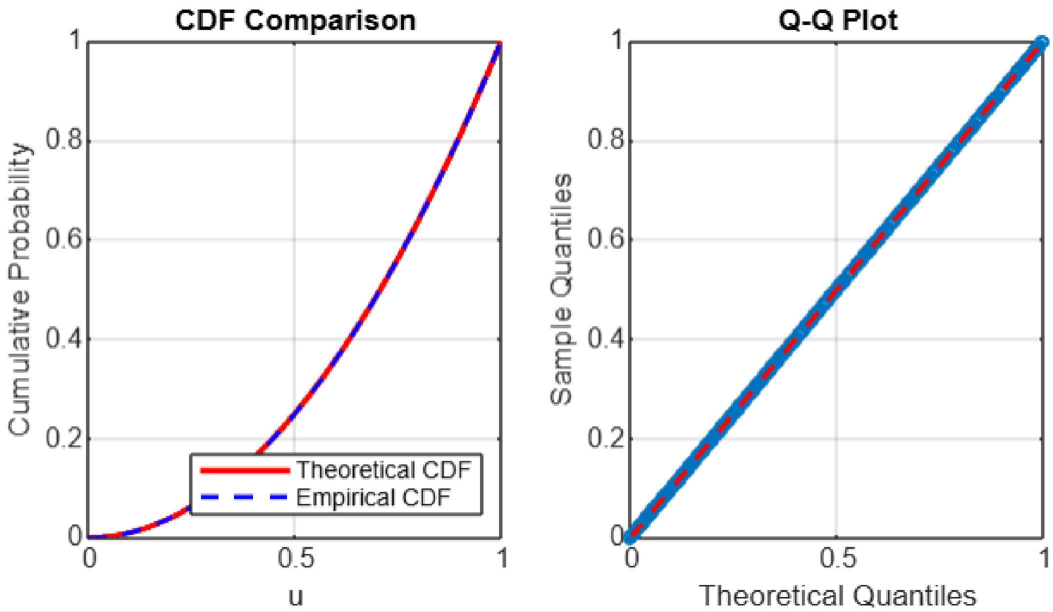
For $N = 10^4$, results are as follows:





For $N = 10^6$, we have:




CDF Comparison
Cumulative Probability
u
Theoretical CDF
Empirical CDF
0
0.2
0.4
0.6
0.8
1
0.5
Q-Q Plot
Sample Quantiles
Theoretical Quantiles