# Assignment 01

### A posteriori probability

Let $P(X|v=u)$ denote the probability of drawing out $X$ blacks in a total of $N=10$ draws from urn $v$. Obviously, the probability of drawing out a black each time from urn $v$ is

$$p_v \equiv P(color = black|v=u) = \frac{u}{10}$$

Considering the process of drawing 10 balls to be a Bernoulli distribution, we have the probability

$$\begin{aligned} P(X=3|v=u) &= C_{10}^3 \times p_v^3 \times (1-p_v)^7 \\ &= 120 \times (\frac{u}{10})^3 \times (1-\frac{u}{10})^7 \end{aligned}$$

Applying Bayesian law we have

$$P(v=u|X=3) = \frac{P(X=3|v=u) \times P(v=u)}{P(X=3)}$$

where $P(v=u)=\frac{1}{11}$, and $P(X=3)$ can be calculated as

$$P(X=3) = \frac{1}{11}\sum_{i=0}^{10} P(X=3|v=i)$$

Applying the figures we have results as follows:

| $u$ | $P(v=u\|X=3)$ |
|---|---|
| 0 | 0 |
| 1 | 0.063073 |
| 2 | 0.221240 |
| 3 | **0.293220** |
| 4 | 0.236256 |
| 5 | 0.128779 |
| 6 | 0.046668 |
| 7 | 0.009892 |
| 8 | 0.000864 |
| 9 | 0.000010 |
| 10 | 0 |

where maximum probability occurs when $u=3$.

### Computer Generation of Random Variables

*(a)*

Since $Y=G(X)$, we have $P(Y \le y) = P(G(X) \le y)$.

According to the question, $F(t)$ must be monotone increasing and differentiable. Applying $F(\cdot)$ to both sides we have

$$P(G(X) \leq y) = P(F(G(X)) \leq F(y)) = P(X \leq F(y))$$

Because $X \sim Uniform(0, 1)$, its CDF takes the form

$$P(X \leq x) = x = F(y), \quad \forall u \in [0, 1)$$

Thus

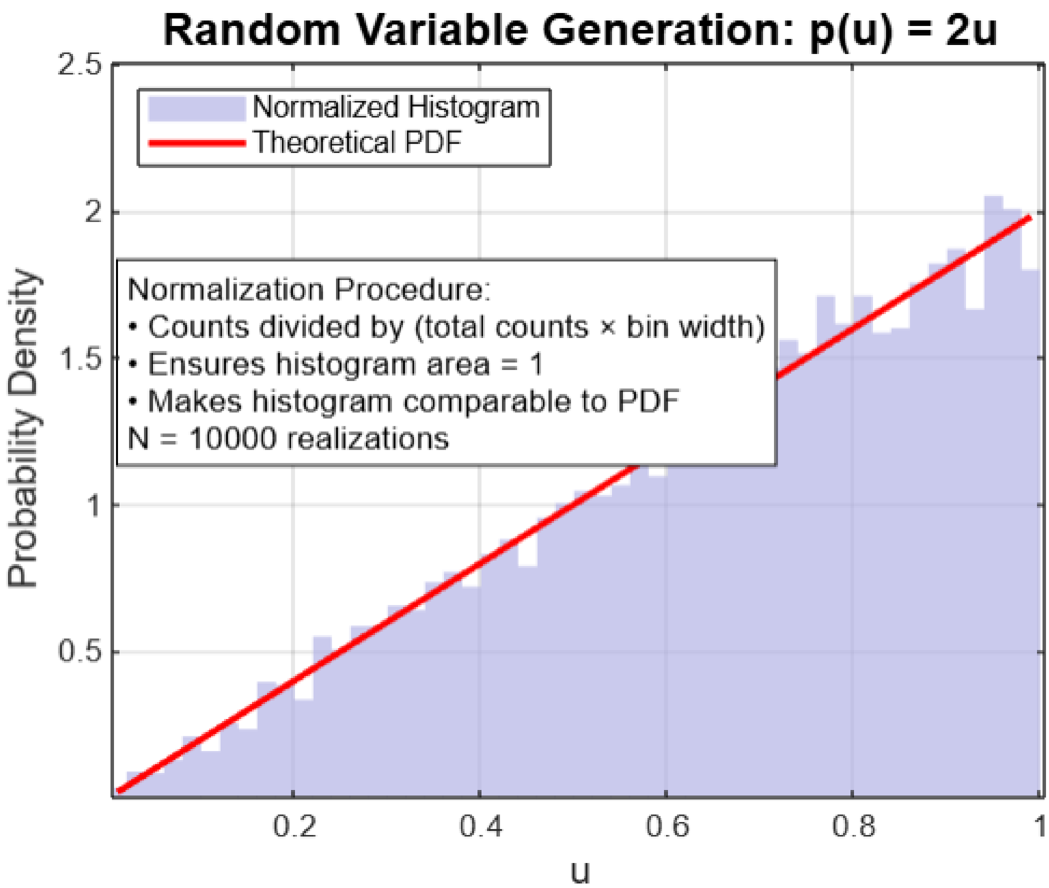
$$P(Y \leq y) = F(y)$$

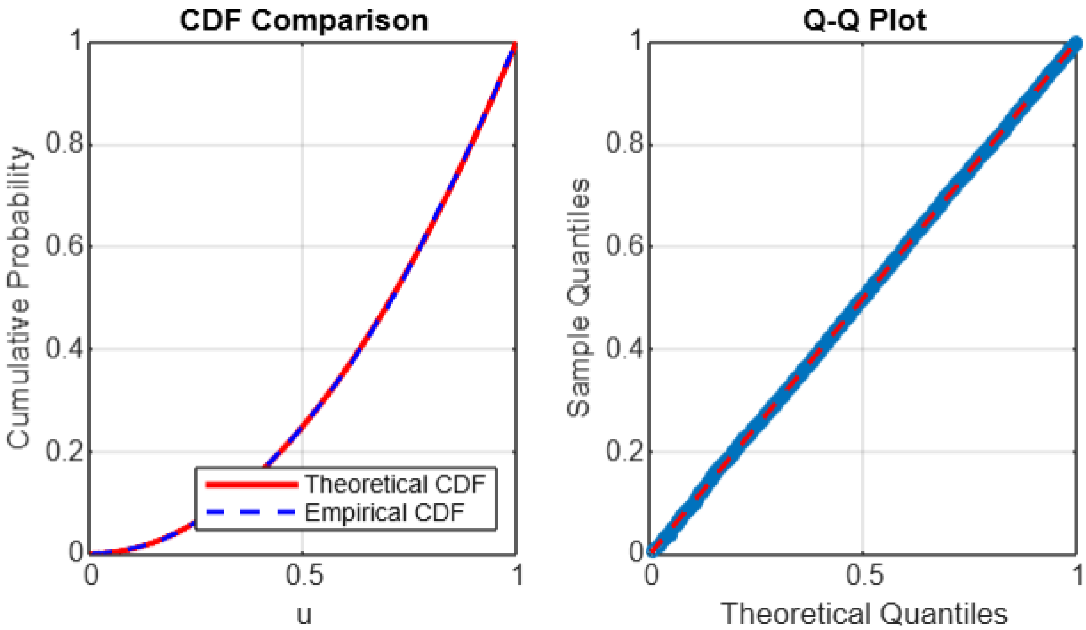
Applying the derivative to both side we have

$$p_Y(y) = p(y)$$

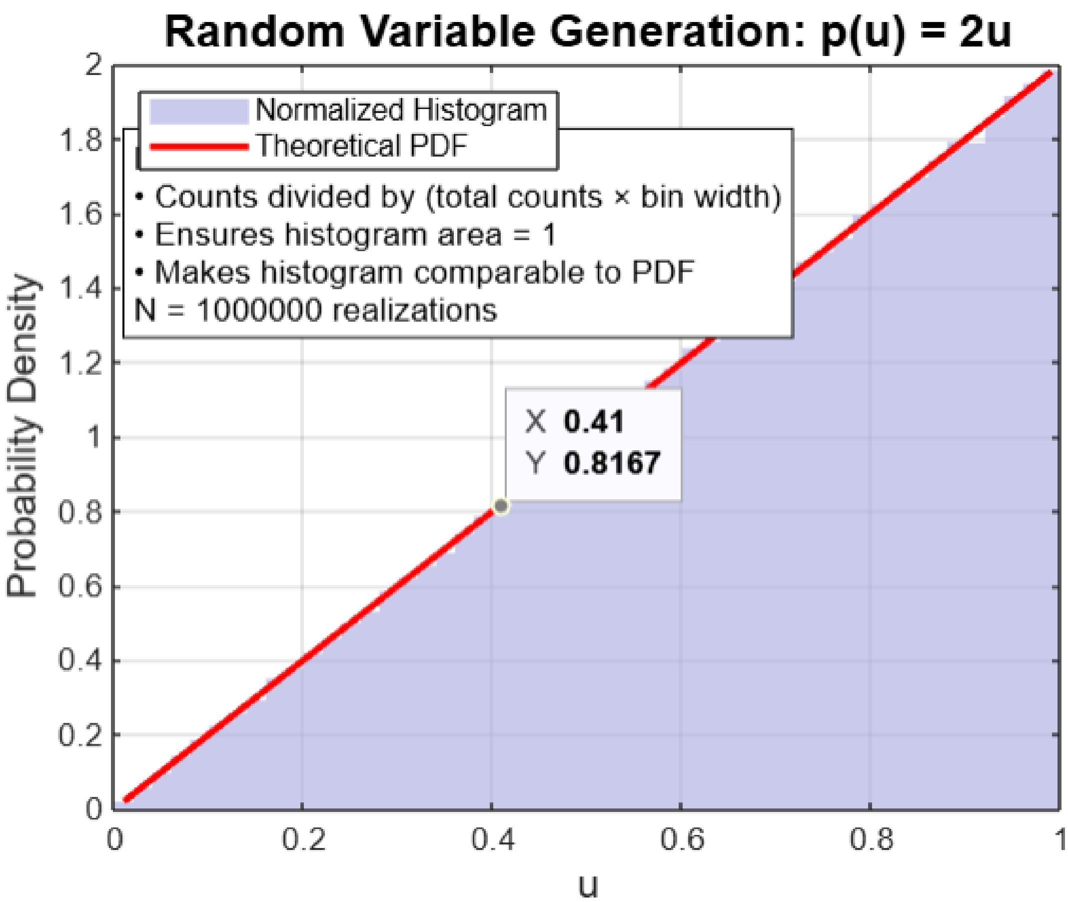
*(b)*

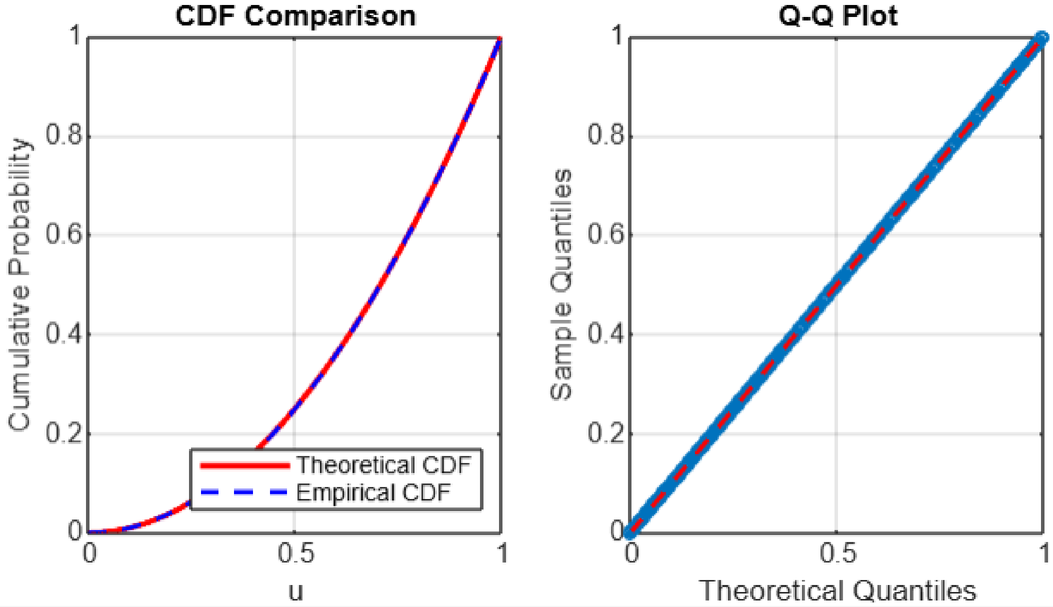
For $N = 10^4$, results are as follows:






For $N = 10^6$, we have:




CDF Comparison
Cumulative Probability
u
Theoretical CDF
Empirical CDF
0
0.2
0.4
0.6
0.8
1
0.5
Q-Q Plot
Sample Quantiles
Theoretical Quantiles