Problem set 1 : A simple population growth model

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1 Introduction

The aim of this problem set is to simulate the growth of a population of animals over a hundred year period. The first assumption of this model is that the population grows by a factor of itself. The yearly growth of the population is modeled by the following equation (1) with p_n the population at year n and α the growth factor of the population considered.

(1) $$p_n = p_{n-1} + \alpha p_{n-1}$$

2 Population growth in the case of unlimited resources

2.1 Population growth model with a fixed coefficient $\alpha = 0.1$ and fixed initial population $N0 = 2$

Given an initial population N_0 such that $N_0 = 2$, which means that at year 0, the population $p = 2$, if we set the growth factor α in (1) at 0.1, we can observe that at year 1, $p_1 \approx 2.2$.
Over the course of a hundred of years, we can see in figure 1 that the population grows exponentially up to approximately 25000 individuals, which is to be expected since we are considering a population in a environment where resources are unlimited : there are no external constraints on the augmentation of the population.

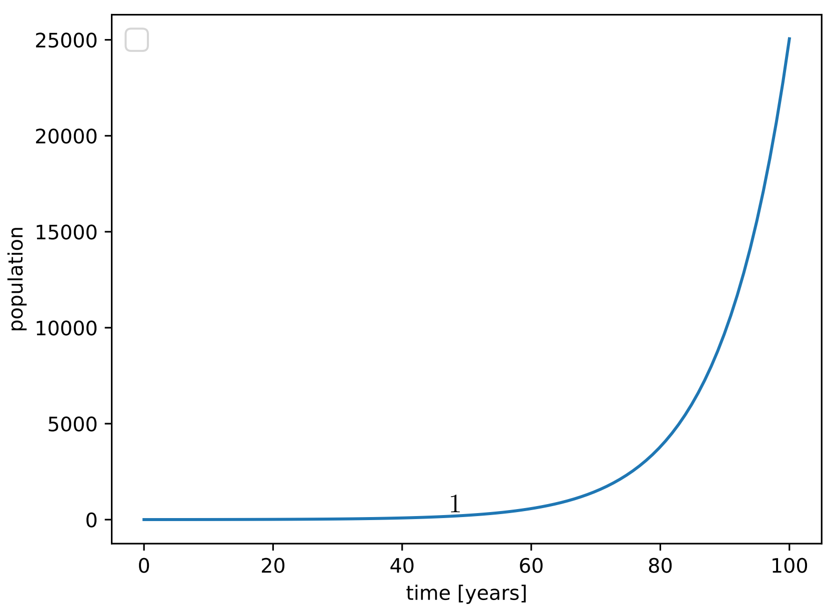

Figure 1: Population growth over 100 years with $\alpha = 0.1$

2.2 Population growth model with a varying coefficient α and fixed initial population $N0 = 2$

The more we increase the growth factor α, the faster the population grows as we can see in figure 2. For every α considered, the growth pattern is the same : the population grows exponentially, and the dependency on the growth factor is linear.

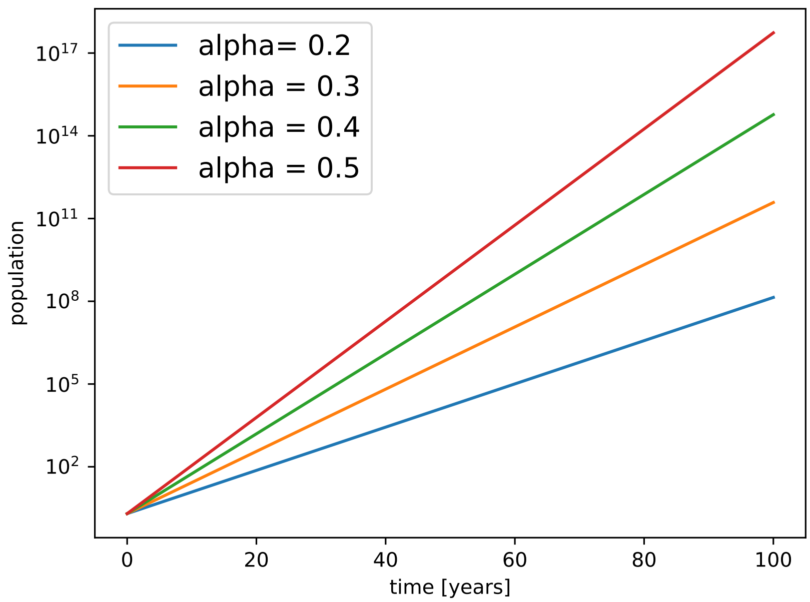

Figure 2: Population growth over 100 years with different α

Furthermore, We can see that the population reached when α increases is drastically higher than when it is set at its default value 0.1. Since the population reaches unrealistic levels, it may signify that population growth factors chosen here are not appropriate and are more likely to be smaller.

2.3 Population growth model with a fixed coefficient $\alpha = 0.1$ and varying initial population $N0$

As we can see in figure 3, the level of the initial population does not affect the growth rate : if the initial population is doubled, the population at all time points is doubled too, but the growth rates remains the same.

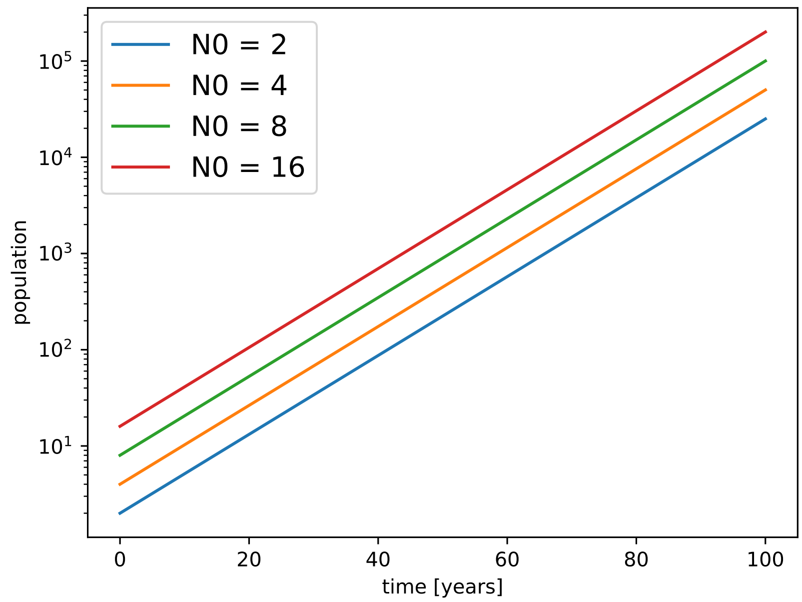

Figure 3: Population growth over 100 years with different initial populations (N0)

3 Population growth in the case of limited resources

3.1 Population growth model with parametric $\alpha = 200 - p$ and fixed initial population $N0 = 2$

Let's now consider a more refined model of population growth taking into considerations the problem of resources. In that case, the population lives in an environment in which resources are limited, which can be modeled modulating the growth factor of the population : we replace α in (1) by $\alpha = 200 - p$ with p the population. On figure 4, we can see the evolution of the population rates between 1 to 500 individuals.
As we can observe on figure 4, the growth factor decreases linearly as the population increases. Furthermore, when the population is over 200 individuals, the growth factor becomes negative, which means that the population decreases when it is over 200 individuals. The model thus predicts that the population should stabilize itself at around 200 individuals since 200 appears to be a stable point.

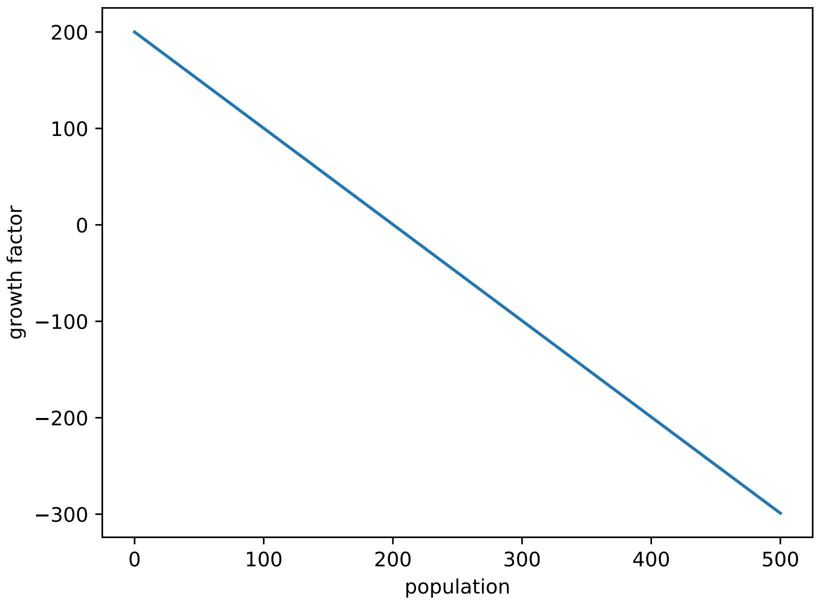

Figure 4: Population rate evolution between 1 to 500 individuals

3.2 Population growth model with fixed parameter $coeff = 0.001$ in δ_n and fixed initial population $N0 = 2$

If we take into consideration the preceding modulation of the growth factor into the previous model of population growth, we have that the population evolution can be modeled by the following equation : $p_n = p_{n-1} + \delta_n$ with p_n the population at year n and $\delta_n = 0.001 p_{n-1}(200 - p_{n-1})$.

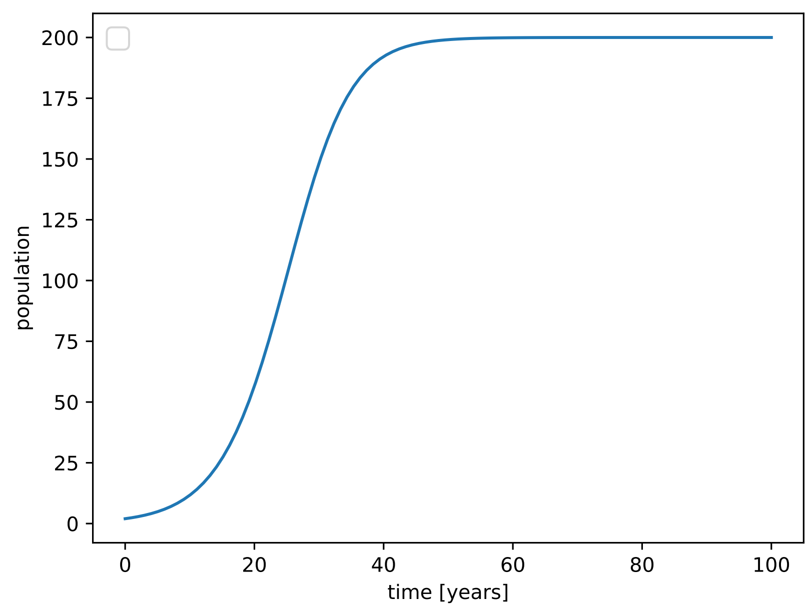

Figure 5: Population growth with limited resources over 100 years

As predicted earlier, we can see on figure 5 that the population grows exponentially until it reaches the stable system size of 200 individuals.

3.3 Population growth model with varying parameter $coeff$ in δ_n and fixed initial population $N0 = 2$

As we can see in figure 6, if we manipulate the additional parameter coeff = 0.001 in δ_n, we can see that when the coeff increases, the population reaches the stable point of 200 individuals increasingly fast and for coeff big (in this case, for coeff = 0.01), a chaos regime appears and the population oscillates around 200 individuals.

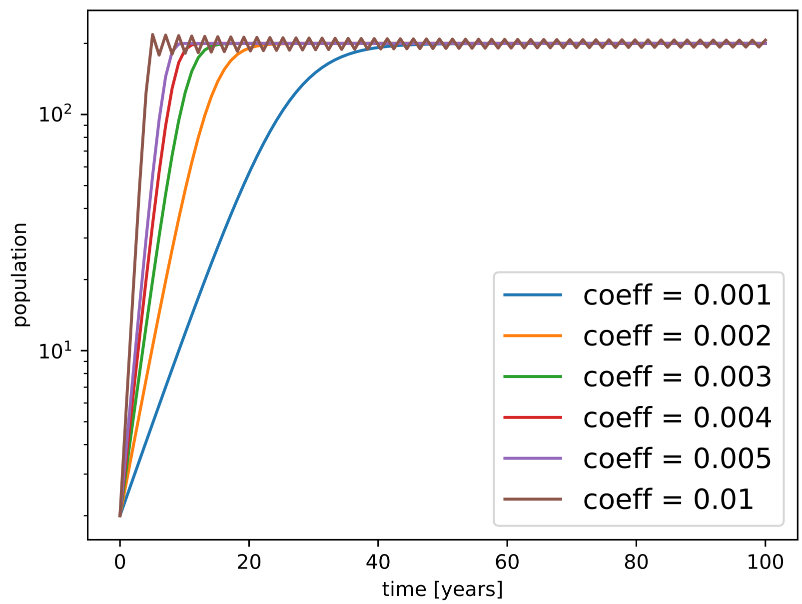

Figure 6: Population growth for varying values of coeff

3.4 Population growth model with fixed parameter $coeff = 0.001$ in δ_n and varying initial population

On the other hand, if we change the initial number of individuals, we can see on figure 7 that the greater the initial number of individual is, the faster the population reaches the stable point of 200, although the growth pattern in itself remains the same (exponential growth) which means that the in this model, the dependence on the initial number of individuals is linear.

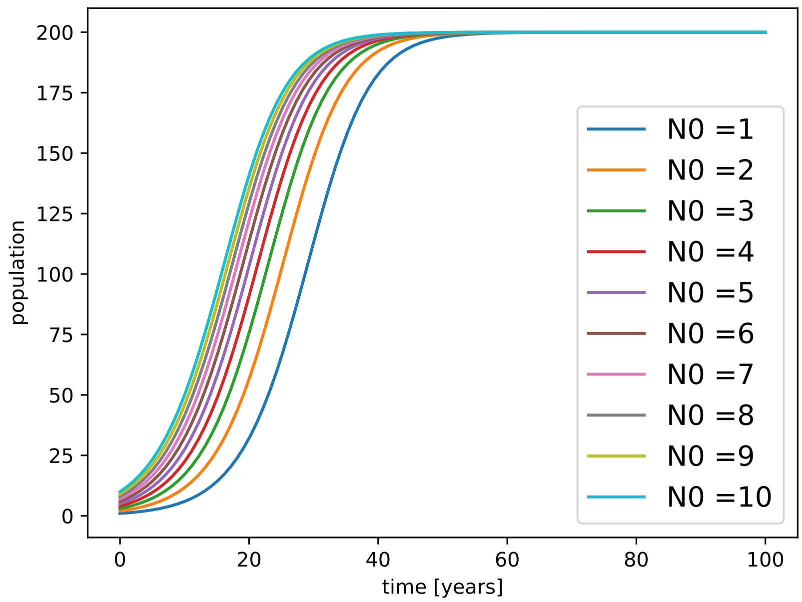

Figure 7: Population growth with varying initial populations

Furthermore, if the initial population is above the stable system size as it is the case in figure 8, then we can observe that the population exponentially decreases until it reaches 200 individuals at which point it stabilizes.

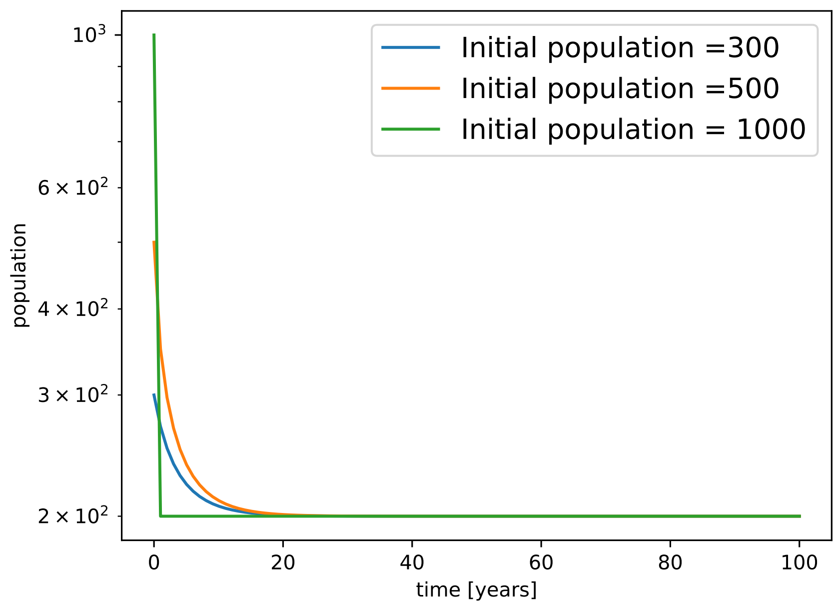

Figure 8: Population growth for an initial population above 200 individuals

4 Conclusion

To conclude, we saw in section 3 that the very simple model considered in section 2 could be refined to take into account the effect of the environment on the population growth and thus yield a more realistic modelisation of actual population evolution.
Based on both model considered, we saw that the population growth pattern is exponential and that the dependency of the growth on model parameters such as the growth rate and the initial population is linerar.
In the more refined model, in the case of an environment with limited resources, the population growth pattern is still exponential, but it converges toward the system stable size of 200 individuals, which is a more realistic prediction than in the case of the fist model (section 2) in which the population growth is unlimited.