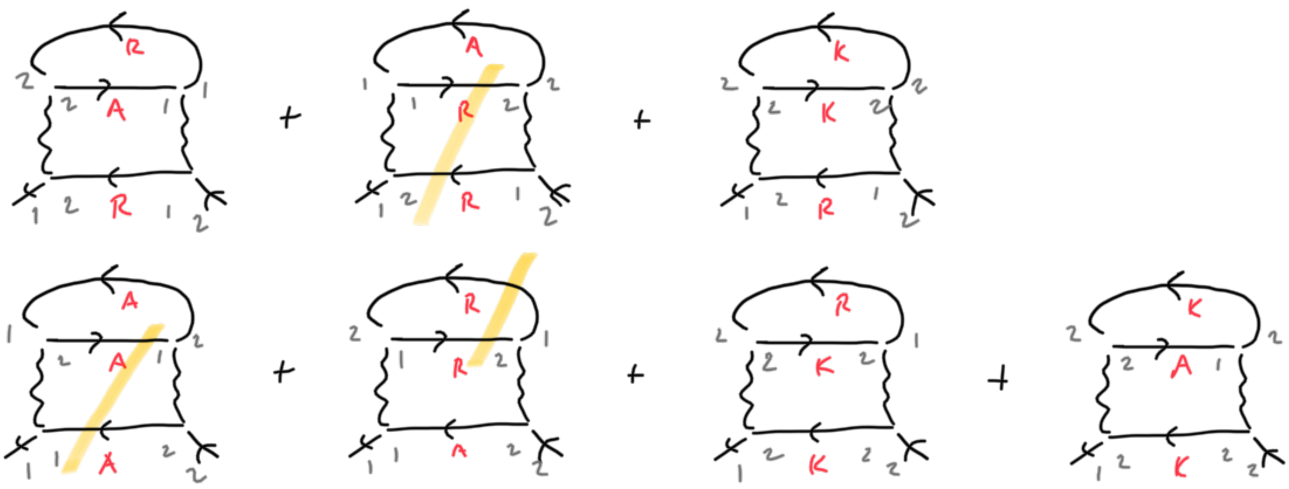
Keldysh diagrammatics

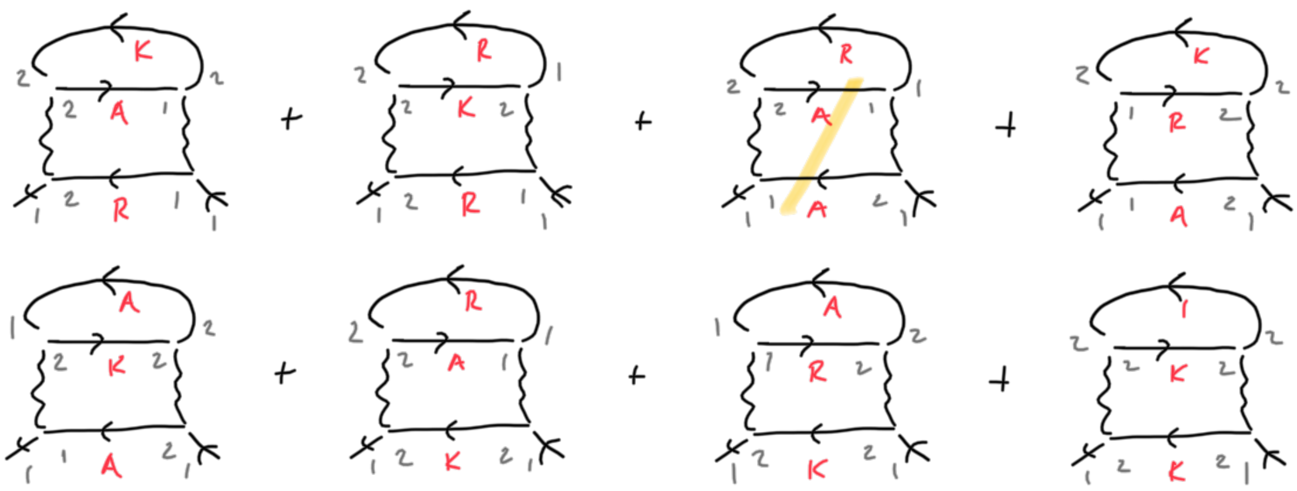
$$0 = G^{1|1}, \quad G^A = G^{1|2}, \quad G^R = G^{2|1}, \quad G^K = G^{2|2}$$
$$\Sigma^K = \Sigma^{1|1}, \quad \Sigma^R = \Sigma^{1|2}, \quad \Sigma^A = \Sigma^{2|1}, \quad 0 = \Sigma^{2|2}$$

only nonzero if $\alpha_1 + \alpha_2 + \alpha_1' + \alpha_2'$ odd

$\Sigma^R =$



$\Sigma^K =$

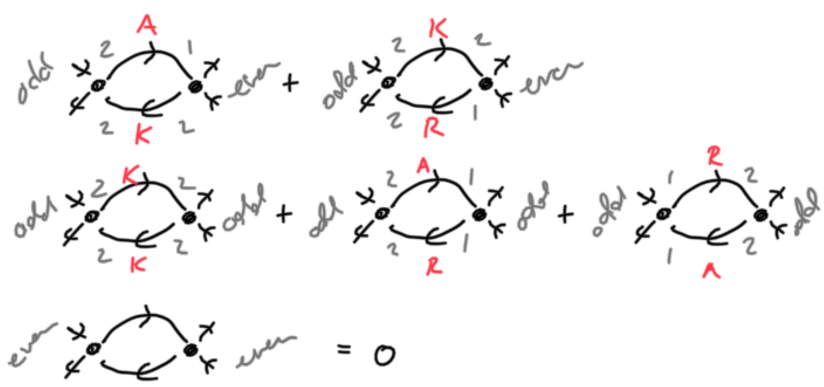


via bubble:

$$\Sigma^{1|2} = G^{2|2}\chi^{1|2} + G^{2|1}\chi^{1|1}$$

$$\chi^{1|2} = G^{1|2}G^{2|2} + G^{2|2}G^{2|1}$$

$$\chi^{1|1} = G^{2|2}G^{2|2} + G^{2|1}G^{1|2} + G^{1|2}G^{2|1}$$



examples:

$$G^R(t) = -i\theta(t)\, e^{-\Delta t} \quad \Rightarrow \quad \Sigma_1(t) = G^R(t)^2 G^A(-t) = -i\,\theta(t)\, e^{-3\Delta t}$$

$$\Rightarrow \quad \chi_1(\omega) = \int dt\, e^{i\omega t} G^R(t) G^A(-t) = \int_0^\infty dt\, e^{i\omega t} e^{-2\Delta t} = \frac{i}{\omega + 2i\Delta}$$

$$G^R(\nu) = \frac{1}{\nu + i\Delta} \quad \Rightarrow \quad G^K_\nu = \left[1 - 2n_F(\nu)\right]\left(G^R_\nu - G^A_\nu\right) = \left[1 - 2n_F(\nu)\right] \frac{-2i\Delta}{\nu^2 + \Delta^2}$$

$$\Delta^K(\nu) = -2i\Delta\left[1 - 2n_F(\nu)\right] \quad \Rightarrow \quad G^K = G^R\left(\Sigma^K + \Delta^K\right) G^A \overset{\Sigma^K = 0}{=} -2i\Delta \frac{1}{\nu + i\Delta}\left(1 - 2n_F(\nu)\right)\frac{1}{\nu - i\Delta} \quad ✓$$