Multiloop fRG flow equations in Keldysh formalism

General properties of the four-point vertex

$$\Gamma = \Gamma_{1'2'|12} = \Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma_1'\sigma_2'|\sigma_1\sigma_2}\,(q_1'q_2'|q_1q_2\,,\,\omega_1'\omega_2'|\omega_1\omega_2) \tag{1}$$

(I) frequency conservation: $\omega_1' + \omega_2' = \omega_1 + \omega_2$
$\Rightarrow$ introduce new frequencies

$$A = \omega_2' - \omega_1 = \omega_2 - \omega_1' \tag{2}$$
$$\Pi = \omega_1 + \omega_2 = \omega_1' + \omega_2' \tag{3}$$
$$T = \omega_1' - \omega_1 = \omega_2 - \omega_2' \tag{4}$$

or reversely

$$\omega_1 = \tfrac{1}{2}(-A + \Pi - T) \tag{5}$$
$$\omega_2 = \tfrac{1}{2}(A + \Pi + T) \tag{6}$$
$$\omega_1' = \tfrac{1}{2}(-A + \Pi + T) \tag{7}$$
$$\omega_2' = \tfrac{1}{2}(A + \Pi - T) \tag{8}$$

(II) spin conservation: $\sigma_1' + \sigma_2' = \sigma_1 + \sigma_2$

1) all spins equal: $\Gamma_{\sigma\sigma|\sigma\sigma}$
2) incoming spins reversed: $\Gamma_{\sigma\bar{\sigma}|\sigma\bar{\sigma}}$ or $\Gamma_{\sigma\bar{\sigma}|\bar{\sigma}\sigma}$

(III) particle exchange:

$$\Gamma^{\alpha_2'\alpha_1'|\alpha_1\alpha_2}_{\sigma_2'\sigma_1'|\sigma_1\sigma_2}\,(q_2'q_1'|q_1q_2\,,\,\omega_2'\omega_1'|\omega_1\omega_2) \quad | \quad \Gamma^{\alpha_2'\alpha_1'|\alpha_1\alpha_2}_{\sigma_2'\sigma_1'|\sigma_1\sigma_2}\,(q_2',q_1'|q_1q_2\,,\,T,\Pi,A) \tag{9}$$
$$= \Gamma^{\alpha_1'\alpha_2'|\alpha_2\alpha_1}_{\sigma_1'\sigma_2'|\sigma_2\sigma_1}\,(q_1'q_2'|q_2q_1\,,\,\omega_1'\omega_2'|\omega_2\omega_1) \quad | \quad \Gamma^{\alpha_2'\alpha_1'|\alpha_1\alpha_2}_{\sigma_2'\sigma_1'|\sigma_1\sigma_2}\,(q_1',q_2'|q_2q_1\,,\,-T,\Pi,-A) \tag{10}$$
$$= -\Gamma^{\alpha_2'\alpha_1'|\alpha_2\alpha_1}_{\sigma_2'\sigma_1'|\sigma_2\sigma_1}\,(q_2'q_1'|q_2q_1\,,\,\omega_2'\omega_1'|\omega_2\omega_1) \quad | \quad \Gamma^{\alpha_2'\alpha_1'|\alpha_1\alpha_2}_{\sigma_2'\sigma_1'|\sigma_1\sigma_2}\,(q_2',q_1'|q_2q_1\,,\,-A,\Pi,-T) \tag{11}$$
$$= -\Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma_1'\sigma_2'|\sigma_1\sigma_2}\,(q_1'q_2'|q_1q_2\,,\,\omega_1'\omega_2'|\omega_1\omega_2) \quad | \quad \Gamma^{\alpha_2'\alpha_1'|\alpha_1\alpha_2}_{\sigma_2'\sigma_1'|\sigma_1\sigma_2}\,(q_1',q_2'|q_1q_2\,,\,A,\Pi,T) \tag{12}$$

(IV) complex conjugation:

$$\Big[\Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma_1'\sigma_2'|\sigma_1\sigma_2}\,(q_1'q_2'|q_1q_2\,,\,\omega_1'\omega_2'|\omega_1\omega_2)\Big]^* = (-1)^{1+\alpha_1+\alpha_2+\alpha_1'+\alpha_2'}\,\Gamma^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}_{\sigma_1\sigma_2|\sigma_1'\sigma_2'}\,(q_1q_2|q_1'q_2'\,,\,\omega_1\omega_2|\omega_1'\omega_2') \tag{13}$$
$$\Big[\Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma_1'\sigma_2'|\sigma_1\sigma_2}\,(q_1'q_2'|q_1q_2\,,\,A,\Pi,T)\Big]^* = (-1)^{1+\alpha_1+\alpha_2+\alpha_1'+\alpha_2'}\,\Gamma^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}_{\sigma_1\sigma_2|\sigma_1'\sigma_2'}\,(q_1q_2|q_1'q_2'\,,\,A,\Pi,-T) \tag{14}$$

(V) causality: $\Gamma^{cc|cc} = 0$

Define transformations based on these properties:

- spin flip:

$$T_S\Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma_1'\sigma_2'|\sigma_1\sigma_2}\,(q_1'q_2'|q_1q_2\,,\,\omega_1'\omega_2'|\omega_1\omega_2) = \Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\bar{\sigma}_1'\bar{\sigma}_2'|\bar{\sigma}_1'\bar{\sigma}_2'}\,(q_1'q_2'|q_1q_2\,,\,\omega_1'\omega_2'|\omega_1\omega_2) \tag{15}$$

- exchange Keldysh and spin indices of the incoming (T_1), outgoing (T_2) or both incoming and outgoing

(T_3) legs:

$$\begin{aligned}
T_1\Gamma^{\alpha_2'\alpha_1'|\alpha_1\alpha_2}_{\sigma_2'\sigma_1'|\sigma_1\sigma_2}(q_1'q_2'|q_1q_2\,,\,\omega_1'\omega_2'|\omega_1\omega_2) &= \Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma_1'\sigma_2'|\sigma_1\sigma_2}(q_1'q_2'|q_1q_2\,,\,\omega_1'\omega_2'|\omega_1\omega_2) \\
&= -\Gamma^{\alpha_2'\alpha_1'|\alpha_1\alpha_2}_{\sigma_2'\sigma_1'|\sigma_1\sigma_2}(q_2'q_1'|q_1q_2\,,\,\omega_2'\omega_1'|\omega_1\omega_2) \qquad (16)\\
T_1\Gamma^{\alpha_2'\alpha_1'|\alpha_1\alpha_2}_{\sigma_2'\sigma_1'|\sigma_1\sigma_2}(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) &= \Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma_1'\sigma_2'|\sigma_1\sigma_2}(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) \\
&= -\Gamma^{\alpha_2'\alpha_1'|\alpha_1\alpha_2}_{\sigma_2'\sigma_1'|\sigma_1\sigma_2}(q_2'q_1'|q_1q_2\,,\,T,\Pi,A) \qquad (17)\\
T_2\Gamma^{\alpha_1'\alpha_2'|\alpha_2\alpha_1}_{\sigma_1'\sigma_2'|\sigma_2\sigma_1}(q_1'q_2'|q_1q_2\,,\,\omega_1'\omega_2'|\omega_1\omega_2) &= \Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma_1'\sigma_2'|\sigma_1\sigma_2}(q_1'q_2'|q_1q_2\,,\,\omega_1'\omega_2'|\omega_1\omega_2) \\
&= -\Gamma^{\alpha_1'\alpha_2'|\alpha_2\alpha_1}_{\sigma_1'\sigma_2'|\sigma_2\sigma_1}(q_1'q_2'|q_2q_1\,,\,\omega_1'\omega_2'|\omega_2\omega_1) \qquad (18)\\
T_2\Gamma^{\alpha_1'\alpha_2'|\alpha_2\alpha_1}_{\sigma_1'\sigma_2'|\sigma_2\sigma_1}(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) &= \Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma_1'\sigma_2'|\sigma_1\sigma_2}(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) \\
&= -\Gamma^{\alpha_1'\alpha_2'|\alpha_2\alpha_1}_{\sigma_1'\sigma_2'|\sigma_2\sigma_1}(q_1'q_2'|q_2q_1\,,\,-T,\Pi,-A) \qquad (19)\\
T_3\Gamma^{\alpha_2'\alpha_1'|\alpha_2\alpha_1}_{\sigma_2'\sigma_1'|\sigma_2\sigma_1}(q_1'q_2'|q_1q_2\,,\,\omega_1'\omega_2'|\omega_1\omega_2) &= \Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma_1'\sigma_2'|\sigma_1\sigma_2}(q_1'q_2'|q_1q_2\,,\,\omega_1'\omega_2'|\omega_1\omega_2) \\
&= \Gamma^{\alpha_2'\alpha_1'|\alpha_2\alpha_1}_{\sigma_2'\sigma_1'|\sigma_2\sigma_1}(q_2'q_1'|q_2q_1\,,\,\omega_2'\omega_1'|\omega_2\omega_1) \qquad (20)\\
T_3\Gamma^{\alpha_2'\alpha_1'|\alpha_2\alpha_1}_{\sigma_2'\sigma_1'|\sigma_2\sigma_1}(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) &= \Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma_1'\sigma_2'|\sigma_1\sigma_2}(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) \\
&= \Gamma^{\alpha_2'\alpha_1'|\alpha_2\alpha_1}_{\sigma_2'\sigma_1'|\sigma_2\sigma_1}(q_2'q_1'|q_2q_1\,,\,-A,\Pi,-T) \qquad (21)
\end{aligned}$$

- complex conjugation (exchange incoming and outgoing legs):

$$\begin{aligned}
T_C\Gamma^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}_{\sigma_1\sigma_2|\sigma_1'\sigma_2'}(q_1'q_2'|q_1q_2\,,\,\omega_1'\omega_2'|\omega_1\omega_2) &= \Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma_1'\sigma_2'|\sigma_1\sigma_2}(q_1'q_2'|q_1q_2\,,\,\omega_1'\omega_2'|\omega_1\omega_2) \\
&= (-1)^{1+\alpha_1+\alpha_2+\alpha_1'+\alpha_2'}\left[\Gamma^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}_{\sigma_1\sigma_2|\sigma_1'\sigma_2'}(q_1q_2|q_1'q_2'\,,\,\omega_1\omega_2|\omega_1'\omega_2')\right]^* \qquad (22)\\
T_C\Gamma^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}_{\sigma_1\sigma_2|\sigma_1'\sigma_2'}(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) &= \Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma_1'\sigma_2'|\sigma_1\sigma_2}(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) \\
&= (-1)^{1+\alpha_1+\alpha_2+\alpha_1'+\alpha_2'}\left[\Gamma^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}_{\sigma_1\sigma_2|\sigma_1'\sigma_2'}(q_1q_2|q_1'q_2'\,,\,A,\Pi,-T)\right]^* \qquad (23)
\end{aligned}$$

Independent components of the four-point vertex

$$
\begin{aligned}
\Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} &= \begin{pmatrix} \Gamma^{qq|qq} & \Gamma^{qq|cq} & \Gamma^{qq|qc} & \Gamma^{qq|cc} \\ \Gamma^{cq|qq} & \Gamma^{cq|cq} & \Gamma^{cq|qc} & \Gamma^{cq|cc} \\ \Gamma^{qc|qq} & \Gamma^{qc|cq} & \Gamma^{qc|qc} & \Gamma^{qc|cc} \\ \Gamma^{cc|qq} & \Gamma^{cc|cq} & \Gamma^{cc|qc} & \Gamma^{cc|cc} \end{pmatrix}_{\sigma\sigma|\sigma\sigma} \\
&= \begin{pmatrix} \Gamma^{qq|qq} & T_C\Gamma^{cq|qq} & T_2T_C\Gamma^{cq|qq} & \Gamma^{qq|cc} \\ \Gamma^{cq|qq} & \Gamma^{cq|cq} & T_2\Gamma^{cq|cq} & \Gamma^{cq|cc} \\ T_1\Gamma^{cq|qq} & T_1\Gamma^{cq|cq} & T_3\Gamma^{cq|cq} & T_1\Gamma^{cq|cc} \\ T_C\Gamma^{qq|cc} & T_C\Gamma^{cq|cc} & T_2T_C\Gamma^{cq|cc} & 0 \end{pmatrix}_{\sigma\sigma|\sigma\sigma} \\
&=: \begin{pmatrix} \Psi^A & T_C\Phi^A & T_2T_C\Phi^A & \Psi^B \\ \Phi^A & \Phi^B & T_2\Phi^B & \Phi^D \\ T_1\Phi^A & T_1\Phi^B & T_3\Phi^B & T_1\Phi^D \\ T_C\Psi^B & T_C\Phi^D & T_2T_C\Phi^D & 0 \end{pmatrix}_{\sigma\sigma} \\
\Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma\bar\sigma|\sigma\bar\sigma} &= \begin{pmatrix} \Gamma^{qq|qq} & T_C\Gamma^{cq|qq} & T_3T_CT_S\Gamma^{cq|qq} & \Gamma^{qq|cc} \\ \Gamma^{cq|qq} & \Gamma^{cq|cq} & \Gamma^{cq|qc} & \Gamma^{cq|cc} \\ T_3T_S\Gamma^{cq|qq} & T_3\Gamma^{cq|qc} & T_3T_S\Gamma^{cq|cq} & T_3T_S\Gamma^{cq|cc} \\ T_C\Gamma^{qq|cc} & T_C\Gamma^{cq|cc} & T_3T_CT_S\Gamma^{cq|cc} & 0 \end{pmatrix}_{\sigma\bar\sigma|\sigma\bar\sigma} \\
&=: \begin{pmatrix} \Psi^A & T_C\Phi^A & T_3T_CT_S\Phi^A & \Psi^B \\ \Phi^A & \Phi^B & \Phi^C & \Phi^D \\ T_3T_S\Phi^A & T_3\Phi^C & T_3T_S\Phi^B & T_3T_S\Phi^D \\ T_C\Psi^B & T_C\Phi^D & T_3T_CT_S\Phi^D & 0 \end{pmatrix}_{\sigma\bar\sigma} \\
\Gamma^{\alpha_1'\alpha_2'|\alpha_1\alpha_2}_{\sigma\bar\sigma|\bar\sigma\sigma} &= \begin{pmatrix} T_2\Gamma^{qq|qq} & T_1T_CT_S\Gamma^{cq|qq} & T_2T_C\Gamma^{cq|qq} & T_2\Gamma^{qq|cc} \\ T_2\Gamma^{cq|qq} & T_2\Gamma^{cq|qc} & T_2\Gamma^{cq|cq} & T_2\Gamma^{cq|cc} \\ T_1T_S\Gamma^{cq|qq} & T_1T_S\Gamma^{cq|cq} & T_1T_S\Gamma^{cq|qc} & T_1T_S\Gamma^{cq|cc} \\ T_1T_CT_S\Gamma^{qq|cc} & T_1T_CT_S\Gamma^{cq|cc} & T_2T_C\Gamma^{cq|cc} & 0 \end{pmatrix}_{\sigma\bar\sigma|\sigma\bar\sigma} \\
&= \begin{pmatrix} T_2\Psi^A & T_1T_CT_S\Phi^A & T_2T_C\Phi^A & T_2\Psi^B \\ T_2\Phi^A & T_2\Phi^C & T_2\Phi^B & T_2\Phi^D \\ T_1T_S\Phi^A & T_1T_S\Phi^B & T_1T_S\Phi^C & T_1T_S\Phi^D \\ T_1T_CT_S\Psi^B & T_1T_CT_S\Phi^D & T_2T_C\Phi^D & 0 \end{pmatrix}_{\sigma\bar\sigma}
\end{aligned} \tag{24}
$$

General properties of the single-particle Green's function

$$
G = G_{1|1'} = G^{\alpha_1|\alpha_1'}_{\sigma_1|\sigma_1'}(q_1|q_1'\,,\,\omega_1|\omega_1') \tag{25}
$$

(I) frequency conservation (time translational invariance $G(t,t') = G(t-t')$):

$$
G^{\alpha_1|\alpha_1'}_{\sigma_1|\sigma_1'}(q_1|q_1'\,,\,\omega_1|\omega_1') = 2\pi\,\delta(\omega_1-\omega_1')G^{\alpha_1|\alpha_1'}_{\sigma_1|\sigma_1'}(q_1|q_1'\,,\,\omega_1) \tag{26}
$$

(II) spin conservation:

$$
G_{\sigma_1|\sigma_1'} = \delta_{\sigma_1'\sigma_1}G_{\sigma_1} \tag{27}
$$

(III) —

(IV) complex conjugation:

$$
\left[G^{\alpha_1|\alpha_1'}_{\sigma_1}(q_1|q_1'\,,\,\omega_1)\right]^* = (-1)^{1+\alpha_1+\alpha_1'}\;G^{\alpha_1'|\alpha_1}_{\sigma_1}(q_1'|q_1\,,\,\omega_1) \tag{28}
$$

(V) causality: $G^{q|q} = 0$

Independent components of the single-particle Green's function

retarded, advanced and Keldysh Green's functions: $G^R = G^{c|q}$, $G^A = G^{q|c}$, $G^K = G^{c|c}$
complex conjugation: $\left[G^{c|q}_{\sigma_1}(q_1|q_1'\,,\,\omega_1)\right]^* = G^{q|c}_{\sigma_1}(q_1'|q_1\,,\,\omega_1) \quad\Rightarrow\quad (G^R)^* = G^A$

$$
G^{\alpha_1|\alpha_1'}_{\sigma_1} = \begin{pmatrix} G^{q|q} & G^{q|c} \\ G^{c|q} & G^{c|c} \end{pmatrix}_{\sigma_1|\sigma_1} = \begin{pmatrix} 0 & G^A \\ G^R & G^K \end{pmatrix}_{\sigma_1|\sigma_1} \tag{29}
$$

Green's function loops and differentiated loops

Single scale propagator: $S^{\Lambda}_{1|1'} = \partial_\Lambda G^{\Lambda}_{1|1'}$

Loop:

$$\begin{aligned}
&L^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}_{\sigma_1\sigma_2|\sigma_1'\sigma_2'}(q_1q_2|q_1'q_2'\,,\,\omega_1\omega_2|\omega_1'\omega_2') \\
&= G^{\alpha_1|\alpha_1'}_{\sigma_1|\sigma_1'}(q_1|q_1'\,,\,\omega_1|\omega_1')\,G^{\alpha_2|\alpha_2'}_{\sigma_2|\sigma_2'}(q_2|q_2'\,,\,\omega_2|\omega_2') \\
&= (2\pi)^2\,\delta(\omega_1-\omega_1')\,\delta(\omega_2-\omega_2')\,\delta_{\sigma_1'\sigma_1}\,\delta_{\sigma_2'\sigma_2}\,\underbrace{G^{\alpha_1|\alpha_1'}_{\sigma_1}(q_1|q_1'\,,\,\omega_1)\,G^{\alpha_2|\alpha_2'}_{\sigma_2}(q_2|q_2'\,,\,\omega_2)}_{\tilde{L}^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}_{\sigma_1\sigma_2|\sigma_1\sigma_2}}
\end{aligned} \tag{30}$$

Differentiated loop:

$$\begin{aligned}
&\dot{L}^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}_{\sigma_1\sigma_2|\sigma_1'\sigma_2'}(q_1q_2|q_1'q_2'\,,\,\omega_1\omega_2|\omega_1'\omega_2') \\
&= S^{\alpha_1|\alpha_1'}_{\sigma_1|\sigma_1'}(q_1|q_1'\,,\,\omega_1|\omega_1')\,G^{\alpha_2|\alpha_2'}_{\sigma_2|\sigma_2'}(q_2|q_2'\,,\,\omega_2|\omega_2') + G^{\alpha_1|\alpha_1'}_{\sigma_1|\sigma_1'}(q_1|q_1'\,,\,\omega_1|\omega_1')\,S^{\alpha_2|\alpha_2'}_{\sigma_2|\sigma_2'}(q_2|q_2'\,,\,\omega_2|\omega_2') \\
&= (2\pi)^2\,\delta(\omega_1-\omega_1')\,\delta(\omega_2-\omega_2')\,\delta_{\sigma_1'\sigma_1}\,\delta_{\sigma_2'\sigma_2} \\
&\quad\times\underbrace{S^{\alpha_1|\alpha_1'}_{\sigma_1}(q_1|q_1'\,,\,\omega_1)\,G^{\alpha_2|\alpha_2'}_{\sigma_2}(q_2|q_2'\,,\,\omega_2) + G^{\alpha_1|\alpha_1'}_{\sigma_1}(q_1|q_1'\,,\,\omega_1)\,S^{\alpha_2|\alpha_2'}_{\sigma_2}(q_2|q_2'\,,\,\omega_2)}_{\dot{\tilde{L}}^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}_{\sigma_1\sigma_2|\sigma_1\sigma_2}}
\end{aligned} \tag{31}$$

Differentiated loop in matrix form in Keldysh space:

$$\begin{aligned}
\dot{\tilde{L}}^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}_{\sigma_1\sigma_2|\sigma_1\sigma_2} &= \begin{pmatrix} 0 & 0 & 0 & S^AG^A+G^AS^A \\ 0 & 0 & S^RG^A+G^RS^A & S^KG^A+G^KS^A \\ 0 & S^AG^R+G^AS^R & 0 & S^AG^K+G^AS^K \\ S^RG^R+G^RS^R & S^KG^R+G^KS^R & S^RG^K+G^RS^K & S^KG^K+G^KS^K \end{pmatrix}_{\sigma_1\sigma_2|\sigma_1\sigma_2} \\
&=: \begin{pmatrix} 0 & 0 & 0 & \dot{L}^{AA} \\ 0 & 0 & \dot{L}^{RA} & \dot{L}^{KA} \\ 0 & \dot{L}^{AR} & 0 & \dot{L}^{AK} \\ \dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK} \end{pmatrix}_{\sigma_1\sigma_2}
\end{aligned} \tag{32}$$

and similarly defining $\tilde{L}^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}_{\sigma_1\sigma_2|\sigma_1\sigma_2}$ and $L^{IJ} := G^IG^J$

General multiloop flow equations

$$\partial_\Lambda\Gamma = \partial_\Lambda\gamma_a + \partial_\Lambda\gamma_p + \partial_\Lambda\gamma_t \tag{33}$$
$$\partial_\Lambda\gamma_r = \dot{\gamma}_r^{(1)} + \dot{\gamma}_r^{(2)} + \dot{\gamma}_r^{(3)} + \dots \tag{34}$$
$$\dot{\gamma}_r^{(1)} = \dot{B}_r(\Gamma,\Gamma) \tag{35}$$
$$\dot{\gamma}_r^{(2)} = B_r(\dot{\gamma}_{\bar{r}}^{(1)},\Gamma) + B_r(\Gamma,\dot{\gamma}_{\bar{r}}^{(1)}) \tag{36}$$
$$\dot{\gamma}_r^{(l+2)} = B_r(\dot{\gamma}_{\bar{r}}^{(l+1)},\Gamma) + \dot{\gamma}_{r,C}^{l+2} + B_r(\Gamma,\dot{\gamma}_{\bar{r}}^{(l+1)}) \tag{37}$$
$$\dot{\gamma}_{r,C}^{(l+2)} = B_r(\Gamma,B_r(\dot{\gamma}_{\bar{r}}^{(l)},\Gamma)) = B_r(B_r(\Gamma,\dot{\gamma}_{\bar{r}}^{(1)}),\Gamma) \tag{38}$$
$$\dot{\gamma}_{\bar{r}}^{(l)} = \sum_{r'\neq r}\dot{\gamma}_{r'}^{(l)} \tag{39}$$

Bubbles:

$$B_a(X,Y)_{1'2'|12} = X_{1'4'|32}\,L_{34|3'4'}\,Y_{3'2'|14} \tag{40}$$
$$B_p(X,Y)_{1'2'|12} = \tfrac{1}{2}X_{1'2'|34}\,L_{34|3'4'}\,Y_{3'4'|12} \tag{41}$$
$$B_t(X,Y)_{1'2'|12} = -X_{3'2'|42}\,L_{34|3'4'}\,Y_{1'4'|13} \tag{42}$$
$$\dot{B}_a(X,Y)_{1'2'|12} = X_{1'4'|32}\,\dot{L}_{34|3'4'}\,Y_{3'2'|14} \tag{43}$$
$$\dot{B}_p(X,Y)_{1'2'|12} = \tfrac{1}{2}X_{1'2'|34}\,\dot{L}_{34|3'4'}\,Y_{3'4'|12} \tag{44}$$
$$\dot{B}_t(X,Y)_{1'2'|12} = -X_{3'2'|42}\,\dot{L}_{34|3'4'}\,Y_{1'4'|13} \tag{45}$$
$$L_{34|3'4'} = G_{3|3'}G_{4|4'} \tag{46}$$
$$\dot{L}_{34|3'4'} = \partial_\Lambda G_{3|3'}G_{4|4'} = S_{3|3'}G_{4|4'} + G_{3|3'}S_{4|4'} \tag{47}$$

Derivation of the Keldysh multiloop flow equations

One-loop equations

$$
\begin{aligned}
\dot{\gamma}_p^{(1)} &= \dot{B}_p(\Gamma,\Gamma)_{1'2'|12} = \tfrac{1}{2}\,\Gamma_{1'2'|34}\dot{L}_{34|3'4'}\Gamma_{3'4'|12} \\
&= \tfrac{1}{2} \sum_{\substack{\alpha_3\alpha_4 \\ \alpha_3'\alpha_4'}} \sum_{\substack{\sigma_3\sigma_4 \\ \sigma_3'\sigma_4'}} \sum_{\substack{q_3q_4 \\ q_3'q_4'}} \int \mathrm{d}\omega_3 \int \mathrm{d}\omega_4 \int \mathrm{d}\omega_3' \int \mathrm{d}\omega_4' \; \Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma_1'\sigma_2'|\sigma_3\sigma_4}\,(q_1'q_2'|q_3q_4\,,\,\omega_1'\omega_2'|\omega_3\omega_4) \\
&\qquad \times \dot{\tilde{L}}{}^{\,\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\,\sigma_3\sigma_4|\sigma_3\sigma_4}\,(q_3q_4|q_3'q_4'\,,\,\omega_3\omega_4|\omega_3\omega_4)\;\; \Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma_3'\sigma_4'|\sigma_1\sigma_2}\,(q_3'q_4'|q_1q_2\,,\,\omega_3'\omega_4'|\omega_1\omega_2) \\
&\qquad \times (2\pi)^2\,\delta(\omega_3-\omega_3')\,\delta(\omega_4-\omega_4')\,\delta_{\sigma_3'\sigma_3}\,\delta_{\sigma_4'\sigma_4} \\
&= \tfrac{1}{2} \sum_{\substack{\alpha_3\alpha_4 \\ \alpha_3'\alpha_4'}} \sum_{\sigma} \sum_{\sigma_3\sigma_4} \sum_{\substack{q_3q_4 \\ q_3'q_4'}} \int \mathrm{d}\omega_3 \int \mathrm{d}\omega_4 \int \mathrm{d}\omega_3' \int \mathrm{d}\omega_4' \; (2\pi)^2\,\delta(\omega_3-\omega_3')\,\delta(\omega_4-\omega_4') \\
&\qquad \times \big(\,\delta_{\sigma\sigma_1'}\,\delta_{\sigma\sigma_2'}\,\delta_{\sigma\sigma_1}\,\delta_{\sigma\sigma_2} + \,\delta_{\sigma\sigma_1'}\,\delta_{\sigma\bar{\sigma}_2'}\,\delta_{\sigma\sigma_1}\,\delta_{\sigma\bar{\sigma}_2} + \,\delta_{\sigma\sigma_1'}\,\delta_{\sigma\bar{\sigma}_2'}\,\delta_{\sigma\bar{\sigma}_1}\,\delta_{\sigma\sigma_2}\big)\, \Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma_1'\sigma_2'|\sigma_3\sigma_4}\;\dot{\tilde{L}}{}^{\,\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\,\sigma_3\sigma_4|\sigma_3\sigma_4}\;\Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma_3\sigma_4|\sigma_1\sigma_2} \\
&= \tfrac{1}{2} \sum_{\substack{\alpha_3\alpha_4 \\ \alpha_3'\alpha_4'}} \sum_{\sigma} \sum_{\substack{q_3q_4 \\ q_3'q_4'}} \int \mathrm{d}\omega_3 \int \mathrm{d}\omega_4 \int \mathrm{d}\omega_3' \int \mathrm{d}\omega_4' \; (2\pi)^2\,\delta(\omega_3-\omega_3')\,\delta(\omega_4-\omega_4') \\
&\qquad \times \Big\{\,\delta_{\sigma\sigma_1'}\,\delta_{\sigma\sigma_2'}\,\delta_{\sigma\sigma_1}\,\delta_{\sigma\sigma_2}\;\Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma\sigma|\sigma\sigma}\;\dot{\tilde{L}}{}^{\,\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\,\sigma\sigma|\sigma\sigma}\;\Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} \\
&\qquad\quad + \,\delta_{\sigma\sigma_1'}\,\delta_{\sigma\bar{\sigma}_2'}\,\delta_{\sigma\sigma_1}\,\delta_{\sigma\bar{\sigma}_2}\Big(\Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma\bar{\sigma}|\sigma\bar{\sigma}}\;\dot{\tilde{L}}{}^{\,\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\,\sigma\bar{\sigma}|\sigma\bar{\sigma}}\;\Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma\bar{\sigma}|\sigma\bar{\sigma}} + \Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma\bar{\sigma}|\bar{\sigma}\sigma}\;\dot{\tilde{L}}{}^{\,\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\,\bar{\sigma}\sigma|\bar{\sigma}\sigma}\;\Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\bar{\sigma}\sigma|\sigma\bar{\sigma}}\Big) \\
&\qquad\quad + \,\delta_{\sigma\sigma_1'}\,\delta_{\sigma\bar{\sigma}_2'}\,\delta_{\sigma\bar{\sigma}_1}\,\delta_{\sigma\sigma_2}\Big(\Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma\bar{\sigma}|\sigma\bar{\sigma}}\;\dot{\tilde{L}}{}^{\,\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\,\sigma\bar{\sigma}|\sigma\bar{\sigma}}\;\Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma\bar{\sigma}|\bar{\sigma}\sigma} + \Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma\bar{\sigma}|\bar{\sigma}\sigma}\;\dot{\tilde{L}}{}^{\,\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\,\bar{\sigma}\sigma|\bar{\sigma}\sigma}\;\Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\bar{\sigma}\sigma|\bar{\sigma}\sigma}\Big)\Big\}
\end{aligned} \tag{48}
$$

Sum over Keldysh indices:

$$
\begin{aligned}
&\sum_{\substack{\alpha_3\alpha_4 \\ \alpha_3'\alpha_4'}} \Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma\sigma|\sigma\sigma}\;\dot{\tilde{L}}{}^{\,\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\,\sigma\sigma|\sigma\sigma}\;\Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} \\
&= \Gamma^{\alpha_1'\alpha_2'|qq}_{\sigma\sigma|\sigma\sigma}\;\dot{\tilde{L}}{}^{\,qq|cc}_{\,\sigma\sigma|\sigma\sigma}\;\Gamma^{cc|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} \\
&+ \Gamma^{\alpha_1'\alpha_2'|cq}_{\sigma\sigma|\sigma\sigma}\;\dot{\tilde{L}}{}^{\,cq|qc}_{\,\sigma\sigma|\sigma\sigma}\;\Gamma^{qc|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} + \Gamma^{\alpha_1'\alpha_2'|cq}_{\sigma\sigma|\sigma\sigma}\;\dot{\tilde{L}}{}^{\,cq|cc}_{\,\sigma\sigma|\sigma\sigma}\;\Gamma^{cc|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} \\
&+ \Gamma^{\alpha_1'\alpha_2'|qc}_{\sigma\sigma|\sigma\sigma}\;\dot{\tilde{L}}{}^{\,qc|cq}_{\,\sigma\sigma|\sigma\sigma}\;\Gamma^{cq|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} + \Gamma^{\alpha_1'\alpha_2'|qc}_{\sigma\sigma|\sigma\sigma}\;\dot{\tilde{L}}{}^{\,qc|cc}_{\,\sigma\sigma|\sigma\sigma}\;\Gamma^{cc|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} \\
&+ \Gamma^{\alpha_1'\alpha_2'|cc}_{\sigma\sigma|\sigma\sigma}\;\dot{\tilde{L}}{}^{\,cc|qq}_{\,\sigma\sigma|\sigma\sigma}\;\Gamma^{qq|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} + \Gamma^{\alpha_1'\alpha_2'|cc}_{\sigma\sigma|\sigma\sigma}\;\dot{\tilde{L}}{}^{\,cc|cq}_{\,\sigma\sigma|\sigma\sigma}\;\Gamma^{cq|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} + \Gamma^{\alpha_1'\alpha_2'|cc}_{\sigma\sigma|\sigma\sigma}\;\dot{\tilde{L}}{}^{\,cc|qc}_{\,\sigma\sigma|\sigma\sigma}\;\Gamma^{qc|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} + \Gamma^{\alpha_1'\alpha_2'|cc}_{\sigma\sigma|\sigma\sigma}\;\dot{\tilde{L}}{}^{\,cc|cc}_{\,\sigma\sigma|\sigma\sigma}\;\Gamma^{cc|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} \\
&= \Gamma^{\alpha_1'\alpha_2'|qq}_{\sigma\sigma|\sigma\sigma}\;\dot{L}^{AA}_{\sigma\sigma|\sigma\sigma}\Gamma^{cc|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} \\
&+ \Gamma^{\alpha_1'\alpha_2'|cq}_{\sigma\sigma|\sigma\sigma}\;\dot{L}^{RA}_{\sigma\sigma|\sigma\sigma}\Gamma^{qc|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} + \Gamma^{\alpha_1'\alpha_2'|cq}_{\sigma\sigma|\sigma\sigma}\;\dot{L}^{KA}_{\sigma\sigma|\sigma\sigma}\Gamma^{cc|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} \\
&+ \Gamma^{\alpha_1'\alpha_2'|qc}_{\sigma\sigma|\sigma\sigma}\;\dot{L}^{AR}_{\sigma\sigma|\sigma\sigma}\Gamma^{cq|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} + \Gamma^{\alpha_1'\alpha_2'|qc}_{\sigma\sigma|\sigma\sigma}\;\dot{L}^{AK}_{\sigma\sigma|\sigma\sigma}\Gamma^{cc|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} \\
&+ \Gamma^{\alpha_1'\alpha_2'|cc}_{\sigma\sigma|\sigma\sigma}\;\dot{L}^{RR}_{\sigma\sigma|\sigma\sigma}\Gamma^{qq|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} + \Gamma^{\alpha_1'\alpha_2'|cc}_{\sigma\sigma|\sigma\sigma}\;\dot{L}^{KR}_{\sigma\sigma|\sigma\sigma}\Gamma^{cq|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} + \Gamma^{\alpha_1'\alpha_2'|cc}_{\sigma\sigma|\sigma\sigma}\;\dot{L}^{RK}_{\sigma\sigma|\sigma\sigma}\Gamma^{qc|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} + \Gamma^{\alpha_1'\alpha_2'|cc}_{\sigma\sigma|\sigma\sigma}\;\dot{L}^{KK}_{\sigma\sigma|\sigma\sigma}\Gamma^{cc|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma}
\end{aligned} \tag{49}
$$

(similarly for other spin configurations (adjust/remove spin labels here?))

Frequency integrals: assume frequency conservation, for p bubble: $\omega_1'+\omega_2' = \omega_3+\omega_4 = \omega_1+\omega_2$
$\Rightarrow$ new frequencies A, Π, T, and $\omega_3 =: \Omega$, $\omega_4 = \Pi - \Omega$

$$
\begin{aligned}
&\int \mathrm{d}\omega_3 \int \mathrm{d}\omega_4 \int \mathrm{d}\omega_3' \int \mathrm{d}\omega_4'\; \delta(\omega_3-\omega_3')\,\delta(\omega_4-\omega_4')\;\Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma\sigma|\sigma\sigma}\,(q_1'q_2'|q_3q_4\,,\,\omega_1'\omega_2'|\omega_3\omega_4) \\
&\qquad \times \dot{\tilde{L}}{}^{\,\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\,\sigma\sigma|\sigma\sigma}\,(q_3q_4|q_3'q_4'\,,\,\omega_3\omega_4|\omega_3\omega_4)\,\Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma}\,(q_3'q_4'|q_1q_2\,,\,\omega_3'\omega_4'|\omega_1\omega_2) \\
&= \int \mathrm{d}\omega_3 \int \mathrm{d}\omega_4\; \Gamma(\omega_2'-\omega_3,\omega_1'+\omega_2',\omega_1'-\omega_3)\,\dot{\tilde{L}}(\omega_3,\omega_4)\,\Gamma(\omega_2-\omega_3,\omega_1+\omega_2,\omega_2-\omega_4)\,\delta(\omega_1+\omega_2-\omega_3-\omega_4) \\
&= \int \mathrm{d}\Omega\;\Gamma\left(\tfrac{A+\Pi-T}{2}-\Omega,\Pi,\tfrac{-A+\Pi+T}{2}-\Omega)\right)\dot{\tilde{L}}(\Omega,\Pi-\Omega)\Gamma\left(\tfrac{A+\Pi+T}{2}-\Omega,\Pi,\tfrac{A-\Pi+T}{2}+\Omega\right)
\end{aligned} \tag{50}
$$

Notation for frequency dependence:

$$\begin{aligned}
\Psi_{p_1}\left(q_1'q_2'|q_3q_4\,,\ A,\Pi,T\right) &= \Psi\Big(q_1'q_2'|q_3q_4\,,\ \tfrac{A+\Pi-T}{2}-\Omega,\Pi,\tfrac{-A+\Pi+T}{2}-\Omega)\Big) &(51)\\
L_p\left(q_3q_4|q_3'q_4'\,,\ \omega_3\omega_4|\omega_3\omega_4\right) &= L\left(q_3q_4|q_3'q_4'\,,\ \Omega,\Pi-\Omega\right) &(52)\\
\Psi_{p_2}\left(q_3'q_4'|q_1q_2\,,\ A,\Pi,T\right) &= \Psi\Big(q_3'q_4'|q_1q_2\,,\ \tfrac{A+\Pi+T}{2}-\Omega,\Pi,\tfrac{A-\Pi+T}{2}+\Omega\Big) &(53)
\end{aligned}$$

One-loop equations for all components of the p channel:

$$\begin{aligned}
\dot\psi^{A(1)}_{p,\sigma\sigma} &= (\dot\gamma_p^{(1)})^{qq|qq}_{\sigma\sigma|\sigma\sigma}\left(q_1'q_2'|q_1q_2\,,\ A,\Pi,T\right)\\
&= \frac{1}{2}(2\pi)^2\sum_{\substack{q_3q_4\\q_3'q_4'}}\int \mathrm{d}\Omega\ \begin{pmatrix}\Psi^A\\ T_C\Phi^A\\ T_2T_C\Phi^A\\ \Psi^B\end{pmatrix}^T_{p_1,\sigma\sigma}\cdot\begin{pmatrix}0&0&0&\dot L^{AA}\\0&0&\dot L^{RA}&\dot L^{KA}\\0&\dot L^{AR}&0&\dot L^{AK}\\\dot L^{RR}&\dot L^{KR}&\dot L^{RK}&\dot L^{KK}\end{pmatrix}_{p,\sigma\sigma}\cdot\begin{pmatrix}\Psi^A\\ \Phi^A\\ T_1\Phi^A\\ T_C\Psi^B\end{pmatrix}_{p_2,\sigma\sigma} \qquad (54)
\end{aligned}$$

$$\begin{aligned}
\dot\psi^{B(1)}_{p,\sigma\sigma} &= (\dot\gamma_p^{(1)})^{qq|cc}_{\sigma\sigma|\sigma\sigma}\left(q_1'q_2'|q_1q_2\,,\ A,\Pi,T\right)\\
&= \frac{1}{2}(2\pi)^2\sum_{\substack{q_3q_4\\q_3'q_4'}}\int \mathrm{d}\Omega\ \begin{pmatrix}\Psi^A\\ T_C\Phi^A\\ T_2T_C\Phi^A\\ \Psi^B\end{pmatrix}^T_{p_1,\sigma\sigma}\cdot\begin{pmatrix}0&0&0&\dot L^{AA}\\0&0&\dot L^{RA}&\dot L^{KA}\\0&\dot L^{AR}&0&\dot L^{AK}\\\dot L^{RR}&\dot L^{KR}&\dot L^{RK}&\dot L^{KK}\end{pmatrix}_{p,\sigma\sigma}\cdot\begin{pmatrix}\Psi^B\\ \Phi^D\\ T_1\Phi^D\\ 0\end{pmatrix}_{p_2,\sigma\sigma} \qquad (55)
\end{aligned}$$

$$\begin{aligned}
\dot\phi^{A(1)}_{p,\sigma\sigma} &= (\dot\gamma_p^{(1)})^{cq|qq}_{\sigma\sigma|\sigma\sigma}\left(q_1'q_2'|q_1q_2\,,\ A,\Pi,T\right)\\
&= \frac{1}{2}(2\pi)^2\sum_{\substack{q_3q_4\\q_3'q_4'}}\int \mathrm{d}\Omega\ \begin{pmatrix}\Phi^A\\ \Phi^B\\ T_2\Phi^B\\ \Phi^D\end{pmatrix}^T_{p_1,\sigma\sigma}\cdot\begin{pmatrix}0&0&0&\dot L^{AA}\\0&0&\dot L^{RA}&\dot L^{KA}\\0&\dot L^{AR}&0&\dot L^{AK}\\\dot L^{RR}&\dot L^{KR}&\dot L^{RK}&\dot L^{KK}\end{pmatrix}_{p,\sigma\sigma}\cdot\begin{pmatrix}\Psi^A\\ \Phi^A\\ T_1\Phi^A\\ T_C\Psi^B\end{pmatrix}_{p_2,\sigma\sigma} \qquad (56)
\end{aligned}$$

$$\begin{aligned}
\dot\phi^{B(1)}_{p,\sigma\sigma} &= (\dot\gamma_p^{(1)})^{cq|cq}_{\sigma\sigma|\sigma\sigma}\left(q_1'q_2'|q_1q_2\,,\ A,\Pi,T\right)\\
&= \frac{1}{2}(2\pi)^2\sum_{\substack{q_3q_4\\q_3'q_4'}}\int \mathrm{d}\Omega\ \begin{pmatrix}\Phi^A\\ \Phi^B\\ T_2\Phi^B\\ \Phi^D\end{pmatrix}^T_{p_1,\sigma\sigma}\cdot\begin{pmatrix}0&0&0&\dot L^{AA}\\0&0&\dot L^{RA}&\dot L^{KA}\\0&\dot L^{AR}&0&\dot L^{AK}\\\dot L^{RR}&\dot L^{KR}&\dot L^{RK}&\dot L^{KK}\end{pmatrix}_{p,\sigma\sigma}\cdot\begin{pmatrix}T_C\Phi^A\\ \Phi^B\\ T_1\Phi^B\\ T_C\Phi^D\end{pmatrix}_{p_2,\sigma\sigma} \qquad (57)
\end{aligned}$$

$$\begin{aligned}
\dot\phi^{D(1)}_{p,\sigma\sigma} &= (\dot\gamma_p^{(1)})^{cq|cc}_{\sigma\sigma|\sigma\sigma}\left(q_1'q_2'|q_1q_2\,,\ A,\Pi,T\right)\\
&= \frac{1}{2}(2\pi)^2\sum_{\substack{q_3q_4\\q_3'q_4'}}\int \mathrm{d}\Omega\ \begin{pmatrix}\Phi^A\\ \Phi^B\\ T_2\Phi^B\\ \Phi^D\end{pmatrix}^T_{p_1,\sigma\sigma}\cdot\begin{pmatrix}0&0&0&\dot L^{AA}\\0&0&\dot L^{RA}&\dot L^{KA}\\0&\dot L^{AR}&0&\dot L^{AK}\\\dot L^{RR}&\dot L^{KR}&\dot L^{RK}&\dot L^{KK}\end{pmatrix}_{p,\sigma\sigma}\cdot\begin{pmatrix}\Psi^B\\ \Phi^D\\ T_1\Phi^D\\ 0\end{pmatrix}_{p_2,\sigma\sigma} \qquad (58)
\end{aligned}$$

$$\begin{aligned}
\dot\psi^{A(1)}_{p,\sigma\bar\sigma} &= (\dot\gamma_p^{(1)})^{qq|qq}_{\sigma\sigma|\sigma\sigma}\left(q_1'q_2'|q_1q_2\,,\ A,\Pi,T\right)\\
&= \frac{1}{2}(2\pi)^2\sum_{\substack{q_3q_4\\q_3'q_4'}}\int \mathrm{d}\Omega\ \left\{\begin{pmatrix}\Psi^A\\ T_C\Phi^A\\ T_3T_CT_S\Phi^A\\ \Psi^B\end{pmatrix}^T_{p_1,\sigma\bar\sigma}\cdot\begin{pmatrix}0&0&0&\dot L^{AA}\\0&0&\dot L^{RA}&\dot L^{KA}\\0&\dot L^{AR}&0&\dot L^{AK}\\\dot L^{RR}&\dot L^{KR}&\dot L^{RK}&\dot L^{KK}\end{pmatrix}_{p,\sigma\bar\sigma}\cdot\begin{pmatrix}\Psi^A\\ \Phi^A\\ T_3T_S\Phi^A\\ T_C\Psi^B\end{pmatrix}_{p_2,\sigma\bar\sigma}\right.\\
&\qquad\left.+\begin{pmatrix}T_2\Psi^A\\ T_1T_CT_S\Phi^A\\ T_2T_C\Phi^A\\ T_2\Psi^B\end{pmatrix}^T_{p_1,\sigma\bar\sigma}\cdot\begin{pmatrix}0&0&0&\dot L^{AA}\\0&0&\dot L^{RA}&\dot L^{KA}\\0&\dot L^{AR}&0&\dot L^{AK}\\\dot L^{RR}&\dot L^{KR}&\dot L^{RK}&\dot L^{KK}\end{pmatrix}_{p,\bar\sigma\sigma}\cdot T_S\begin{pmatrix}T_2\Psi^A\\ T_2\Phi^A\\ T_1T_S\Phi^A\\ T_1T_CT_S\Psi^B\end{pmatrix}_{p_2,\sigma\bar\sigma}\right\} \qquad (59)
\end{aligned}$$

$$
\begin{aligned}
\dot{\psi}^{B(1)}_{p,\sigma\bar{\sigma}} &= (\dot{\gamma}^{(1)}_{p})^{qq|cc}_{\sigma\sigma|\sigma\sigma}\,(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) \\
&= \tfrac{1}{2}(2\pi)^2 \sum_{\substack{q_3q_4\\ q_3'q_4'}} \int \mathrm{d}\Omega \;\left\{ \begin{pmatrix} \Psi^A \\ T_C\Phi^A \\ T_3T_CT_S\Phi^A \\ \Psi^B \end{pmatrix}^T_{p_1,\sigma\bar{\sigma}} \cdot \begin{pmatrix} 0 & 0 & 0 & \dot{L}^{AA} \\ 0 & 0 & \dot{L}^{RA} & \dot{L}^{KA} \\ 0 & \dot{L}^{AR} & 0 & \dot{L}^{AK} \\ \dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK} \end{pmatrix}_{p,\sigma\bar{\sigma}} \cdot \begin{pmatrix} \Psi^B \\ \Phi^D \\ T_3T_S\Phi^D \\ 0 \end{pmatrix}_{p_2,\sigma\bar{\sigma}} \right. \\
&\qquad \left. + \begin{pmatrix} T_2\Psi^A \\ T_1T_CT_S\Phi^A \\ T_2T_C\Phi^A \\ T_2\Psi^B \end{pmatrix}^T_{p_1,\sigma\bar{\sigma}} \cdot \begin{pmatrix} 0 & 0 & 0 & \dot{L}^{AA} \\ 0 & 0 & \dot{L}^{RA} & \dot{L}^{KA} \\ 0 & \dot{L}^{AR} & 0 & \dot{L}^{AK} \\ \dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK} \end{pmatrix}_{p,\bar{\sigma}\sigma} \cdot T_S \begin{pmatrix} T_2\Psi^B \\ T_2\Phi^D \\ T_1T_S\Phi^D \\ 0 \end{pmatrix}_{p_2,\sigma\bar{\sigma}} \right\}
\end{aligned} \tag{60}
$$

$$
\begin{aligned}
\dot{\phi}^{A(1)}_{p,\sigma\bar{\sigma}} &= (\dot{\gamma}^{(1)}_{p})^{cq|qq}_{\sigma\sigma|\sigma\sigma}\,(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) \\
&= \tfrac{1}{2}(2\pi)^2 \sum_{\substack{q_3q_4\\ q_3'q_4'}} \int \mathrm{d}\Omega \;\left\{ \begin{pmatrix} \Phi^A \\ \Phi^B \\ \Phi^C \\ \Phi^D \end{pmatrix}^T_{p_1,\sigma\bar{\sigma}} \cdot \begin{pmatrix} 0 & 0 & 0 & \dot{L}^{AA} \\ 0 & 0 & \dot{L}^{RA} & \dot{L}^{KA} \\ 0 & \dot{L}^{AR} & 0 & \dot{L}^{AK} \\ \dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK} \end{pmatrix}_{p,\sigma\bar{\sigma}} \cdot \begin{pmatrix} \Psi^A \\ \Phi^A \\ T_3T_S\Phi^A \\ T_C\Psi^B \end{pmatrix}_{p_2,\sigma\bar{\sigma}} \right. \\
&\qquad \left. + \begin{pmatrix} T_2\Phi^A \\ T_2\Phi^B \\ T_2\Phi^C \\ T_2\Phi^D \end{pmatrix}^T_{p_1,\sigma\bar{\sigma}} \cdot \begin{pmatrix} 0 & 0 & 0 & \dot{L}^{AA} \\ 0 & 0 & \dot{L}^{RA} & \dot{L}^{KA} \\ 0 & \dot{L}^{AR} & 0 & \dot{L}^{AK} \\ \dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK} \end{pmatrix}_{p,\bar{\sigma}\sigma} \cdot T_S \begin{pmatrix} T_2\Psi^A \\ T_2\Phi^A \\ T_1T_S\Phi^A \\ T_1T_CT_S\Psi^B \end{pmatrix}_{p_2,\sigma\bar{\sigma}} \right\}
\end{aligned} \tag{61}
$$

$$
\begin{aligned}
\dot{\phi}^{B(1)}_{p,\sigma\bar{\sigma}} &= (\dot{\gamma}^{(1)}_{p})^{cq|cq}_{\sigma\sigma|\sigma\sigma}\,(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) \\
&= \tfrac{1}{2}(2\pi)^2 \sum_{\substack{q_3q_4\\ q_3'q_4'}} \int \mathrm{d}\Omega \;\left\{ \begin{pmatrix} \Phi^A \\ \Phi^B \\ \Phi^C \\ \Phi^D \end{pmatrix}^T_{p_1,\sigma\bar{\sigma}} \cdot \begin{pmatrix} 0 & 0 & 0 & \dot{L}^{AA} \\ 0 & 0 & \dot{L}^{RA} & \dot{L}^{KA} \\ 0 & \dot{L}^{AR} & 0 & \dot{L}^{AK} \\ \dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK} \end{pmatrix}_{p,\sigma\bar{\sigma}} \cdot \begin{pmatrix} T_C\Phi^A \\ \Phi^B \\ T_3\Phi^C \\ T_C\Psi^D \end{pmatrix}_{p_2,\sigma\bar{\sigma}} \right. \\
&\qquad \left. + \begin{pmatrix} T_2\Phi^A \\ T_2\Phi^B \\ T_2\Phi^C \\ T_2\Phi^D \end{pmatrix}^T_{p_1,\sigma\bar{\sigma}} \cdot \begin{pmatrix} 0 & 0 & 0 & \dot{L}^{AA} \\ 0 & 0 & \dot{L}^{RA} & \dot{L}^{KA} \\ 0 & \dot{L}^{AR} & 0 & \dot{L}^{AK} \\ \dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK} \end{pmatrix}_{p,\bar{\sigma}\sigma} \cdot T_S \begin{pmatrix} T_1T_CT_S\Phi^A \\ T_2\Phi^C \\ T_1T_S\Phi^B \\ T_1T_CT_S\Phi^D \end{pmatrix}_{p_2,\sigma\bar{\sigma}} \right\}
\end{aligned} \tag{62}
$$

$$
\begin{aligned}
\dot{\phi}^{C(1)}_{p,\sigma\bar{\sigma}} &= (\dot{\gamma}^{(1)}_{p})^{cq|qc}_{\sigma\sigma|\sigma\sigma}\,(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) \\
&= \tfrac{1}{2}(2\pi)^2 \sum_{\substack{q_3q_4\\ q_3'q_4'}} \int \mathrm{d}\Omega \;\left\{ \begin{pmatrix} \Phi^A \\ \Phi^B \\ \Phi^C \\ \Phi^D \end{pmatrix}^T_{p_1,\sigma\bar{\sigma}} \cdot \begin{pmatrix} 0 & 0 & 0 & \dot{L}^{AA} \\ 0 & 0 & \dot{L}^{RA} & \dot{L}^{KA} \\ 0 & \dot{L}^{AR} & 0 & \dot{L}^{AK} \\ \dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK} \end{pmatrix}_{p,\sigma\bar{\sigma}} \cdot \begin{pmatrix} T_3T_CT_S\Phi^A \\ \Phi^C \\ T_3T_S\Phi^B \\ T_3T_CT_S\Psi^D \end{pmatrix}_{p_2,\sigma\bar{\sigma}} \right. \\
&\qquad \left. + \begin{pmatrix} T_2\Phi^A \\ T_2\Phi^B \\ T_2\Phi^C \\ T_2\Phi^D \end{pmatrix}^T_{p_1,\sigma\bar{\sigma}} \cdot \begin{pmatrix} 0 & 0 & 0 & \dot{L}^{AA} \\ 0 & 0 & \dot{L}^{RA} & \dot{L}^{KA} \\ 0 & \dot{L}^{AR} & 0 & \dot{L}^{AK} \\ \dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK} \end{pmatrix}_{p,\bar{\sigma}\sigma} \cdot T_S \begin{pmatrix} T_2T_C\Phi^A \\ T_2\Phi^B \\ T_1T_S\Phi^C \\ T_2T_C\Psi^D \end{pmatrix}_{p_2,\sigma\bar{\sigma}} \right\}
\end{aligned} \tag{63}
$$

$$
\begin{aligned}
\dot{\phi}^{D(1)}_{p,\sigma\bar{\sigma}} &= (\dot{\gamma}^{(1)}_p)^{cq|cc}_{\sigma\sigma|\sigma\sigma}\,(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) \\
&= \tfrac{1}{2}(2\pi)^2 \sum_{\substack{q_3q_4\\q_3'q_4'}} \int \mathrm{d}\Omega\, \left\{ \begin{pmatrix} \Phi^A \\ \Phi^B \\ \Phi^C \\ \Phi^D \end{pmatrix}^T_{p_1,\sigma\bar{\sigma}} \cdot \begin{pmatrix} 0 & 0 & 0 & \dot{L}^{AA} \\ 0 & 0 & \dot{L}^{RA} & \dot{L}^{KA} \\ 0 & \dot{L}^{AR} & 0 & \dot{L}^{AK} \\ \dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK} \end{pmatrix}_{p,\sigma\bar{\sigma}} \cdot \begin{pmatrix} \Psi^B \\ \Phi^D \\ T_3T_S\Phi^D \\ 0 \end{pmatrix}_{p_2,\sigma\bar{\sigma}} \right. \\
&\qquad \left. + \begin{pmatrix} T_2\Phi^A \\ T_2\Phi^B \\ T_2\Phi^C \\ T_2\Phi^D \end{pmatrix}^T_{p_1,\sigma\bar{\sigma}} \cdot \begin{pmatrix} 0 & 0 & 0 & \dot{L}^{AA} \\ 0 & 0 & \dot{L}^{RA} & \dot{L}^{KA} \\ 0 & \dot{L}^{AR} & 0 & \dot{L}^{AK} \\ \dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK} \end{pmatrix}_{p,\bar{\sigma}\sigma} \cdot T_S \begin{pmatrix} T_2\Psi^B \\ T_2\Phi^D \\ T_1T_S\Phi^D \\ 0 \end{pmatrix}_{p_2,\sigma\bar{\sigma}} \right\}
\end{aligned} \tag{64}
$$

$$
\begin{pmatrix}
\Gamma^{qq|qq} & \Gamma^{qq|cq} & \Gamma^{qq|qc} & \Gamma^{qq|cc} \\
\Gamma^{cq|qq} & \Gamma^{cq|cq} & \Gamma^{cq|qc} & \Gamma^{cq|cc} \\
\Gamma^{qc|qq} & \Gamma^{qc|cq} & \Gamma^{qc|qc} & \Gamma^{qc|cc} \\
\Gamma^{cc|qq} & \Gamma^{cc|cq} & \Gamma^{cc|qc} & \Gamma^{cc|cc}
\end{pmatrix}_{\sigma\bar{\sigma}}
$$
$$
\begin{pmatrix}
\Psi^{A} & T_C\Phi^{A} & T_2T_C\Phi^{A} & \Psi^{B} \\
\Phi^{A} & \Phi^{B} & T_2\Phi^{B} & \Phi^{D} \\
T_1\Phi^{A} & T_1\Phi^{B} & T_3\Phi^{B} & T_1\Phi^{D} \\
T_C\Psi^{B} & T_C\Phi^{D} & T_2T_C\Phi^{D} & 0
\end{pmatrix}_{\sigma\sigma}
$$
$$
\begin{pmatrix}
\Psi^{A} & T_C\Phi^{A} & T_3T_CT_S\Phi^{A} & \Psi^{B} \\
\Phi^{A} & \Phi^{B} & \Phi^{C} & \Phi^{D} \\
T_3T_S\Phi^{A} & T_3\Phi^{C} & T_3T_S\Phi^{B} & T_3T_S\Phi^{D} \\
T_C\Psi^{B} & T_C\Phi^{D} & T_3T_CT_S\Phi^{D} & 0
\end{pmatrix}_{\sigma\bar{\sigma}}
$$
$$
\begin{pmatrix}
T_2\Psi^{A} & T_1T_CT_S\Phi^{A} & T_2T_C\Phi^{A} & T_2\Psi^{B} \\
T_2\Phi^{A} & T_2\Phi^{C} & T_2\Phi^{B} & T_2\Phi^{D} \\
T_1T_S\Phi^{A} & T_1T_S\Phi^{B} & T_1T_S\Phi^{C} & T_1T_S\Phi^{D} \\
T_1T_CT_S\Psi^{B} & T_1T_CT_S\Phi^{D} & T_2T_C\Phi^{D} & 0
\end{pmatrix}_{\sigma\bar{\sigma}}
\tag{65}
$$

$$
\begin{aligned}
\dot{\psi}^{A(1)}_{p,\sigma\sigma} &= (\dot{\gamma}^{(1)}_p)^{qq|qq}_{\sigma\sigma|\sigma\sigma}\,(A,\Pi,T) \\
&= \tfrac{1}{2}(2\pi)^2 \sum_{\substack{\alpha_3\alpha_4\\ \alpha_3'\alpha_4'}} \sum_{\substack{q_3q_4\\ q_3'q_4'}} \int \mathrm{d}\Omega\; \Gamma^{qq|\alpha_3\alpha_4}_{\sigma\sigma|\sigma\sigma}\, \dot{\tilde{L}}^{\,\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\sigma\sigma|\sigma\sigma}\, \Gamma^{\alpha_3'\alpha_4'|qq}_{\sigma\sigma|\sigma\sigma} \\
&= \tfrac{1}{2}(2\pi)^2 \sum_{\substack{q_3q_4\\ q_3'q_4'}} \int \mathrm{d}\Omega \\
&\quad\times \Big\{ \Gamma^{qq|qq}_{\sigma\sigma|\sigma\sigma}\, \dot{L}^{AA}_{\sigma\sigma|\sigma\sigma} \Gamma^{cc|qq}_{\sigma\sigma|\sigma\sigma} \\
&\qquad + \Gamma^{qq|cq}_{\sigma\sigma|\sigma\sigma}\, \dot{L}^{RA}_{\sigma\sigma|\sigma\sigma} \Gamma^{qc|qq}_{\sigma\sigma|\sigma\sigma} + \Gamma^{qq|cq}_{\sigma\sigma|\sigma\sigma}\, \dot{L}^{KA}_{\sigma\sigma|\sigma\sigma} \Gamma^{cc|qq}_{\sigma\sigma|\sigma\sigma} \\
&\qquad + \Gamma^{qq|qc}_{\sigma\sigma|\sigma\sigma}\, \dot{L}^{AR}_{\sigma\sigma|\sigma\sigma} \Gamma^{cq|qq}_{\sigma\sigma|\sigma\sigma} + \Gamma^{qq|qc}_{\sigma\sigma|\sigma\sigma}\, \dot{L}^{AK}_{\sigma\sigma|\sigma\sigma} \Gamma^{cc|qq}_{\sigma\sigma|\sigma\sigma} \\
&\qquad + \Gamma^{qq|cc}_{\sigma\sigma|\sigma\sigma}\, \dot{L}^{RR}_{\sigma\sigma|\sigma\sigma} \Gamma^{qq|qq}_{\sigma\sigma|\sigma\sigma} + \Gamma^{qq|cc}_{\sigma\sigma|\sigma\sigma}\, \dot{L}^{KR}_{\sigma\sigma|\sigma\sigma} \Gamma^{cq|qq}_{\sigma\sigma|\sigma\sigma} + \Gamma^{qq|cc}_{\sigma\sigma|\sigma\sigma}\, \dot{L}^{RK}_{\sigma\sigma|\sigma\sigma} \Gamma^{qc|qq}_{\sigma\sigma|\sigma\sigma} + \Gamma^{qq|cc}_{\sigma\sigma|\sigma\sigma}\, \dot{L}^{KK}_{\sigma\sigma|\sigma\sigma} \Gamma^{cc|qq}_{\sigma\sigma|\sigma\sigma} \Big\} \\
&= \tfrac{1}{2}(2\pi)^2 \sum_{\substack{q_3q_4\\ q_3'q_4'}} \int \mathrm{d}\Omega \\
&\quad\times \Big\{ \Psi^A\, \dot{L}^{AA}\,(T_C\Psi^B) \\
&\qquad + (T_C\Phi^A)\, \dot{L}^{RA}\,(T_1\Phi^A) + (T_C\Phi^A)\, \dot{L}^{KA}\,(T_C\Psi^B) \\
&\qquad + (T_2T_C\Phi^A)\, \dot{L}^{AR}\,\Phi^A + (T_2T_C\Phi^A)\, \dot{L}^{AK}\,(T_C\Psi^B) \\
&\qquad + \Psi^B\, \dot{L}^{RR}\,\Psi^A + \Psi^B\, \dot{L}^{KR}\,\Phi^A + \Psi^B\, \dot{L}^{RK}\,(T_1\Phi^A) + \Psi^B\, \dot{L}^{KK}\,(T_C\Psi^B) \Big\}_{\sigma\sigma|\sigma\sigma} \\
&= \tfrac{1}{2}(2\pi)^2 \sum_{\substack{q_3q_4\\ q_3'q_4'}} \int \mathrm{d}\Omega \\
&\quad\times \Big\{ \Psi^A\,\big(q_1'q_2'|q_3q_4\,,\ \tfrac{A+\Pi-T}{2}-\Omega,\Pi,\tfrac{-A+\Pi+T}{2}-\Omega\big)\big)\ \dot{L}^{AA}(q_3q_4|q_3'q_4'\,,\ \Omega,\Pi-\Omega)\ \big(T_C\Psi^B\,\big(q_3'q_4'|q_1q_2\,,\ \tfrac{A+\Pi+T}{2}-\Omega,\Pi,\ \ldots \\
&\qquad + (T_C\Phi^A)\, \dot{L}^{RA}\,(T_1\Phi^A) + (T_C\Phi^A)\, \dot{L}^{KA}\,(T_C\Psi^B) \\
&\qquad + (T_2T_C\Phi^A)\, \dot{L}^{AR}\,\Phi^A + (T_2T_C\Phi^A)\, \dot{L}^{AK}\,(T_C\Psi^B) \\
&\qquad + \Psi^B\, \dot{L}^{RR}\,\Psi^A + \Psi^B\, \dot{L}^{KR}\,\Phi^A + \Psi^B\, \dot{L}^{RK}\,(T_1\Phi^A) + \Psi^B\, \dot{L}^{KK}\,(T_C\Psi^B) \Big\}_{\sigma\sigma|\sigma\sigma} \\
&\Gamma\big(\tfrac{A+\Pi-T}{2}-\Omega,\Pi,\tfrac{-A+\Pi+T}{2}-\Omega\big)\big)\,\dot{\tilde{L}}(\Omega,\Pi-\Omega)\Gamma\big(\tfrac{A+\Pi+T}{2}-\Omega,\Pi,\tfrac{A-\Pi+T}{2}+\Omega\big)
\end{aligned}
\tag{66}
$$

$$
\begin{pmatrix}
\Gamma^{qq|qq} & \Gamma^{qq|cq} & \Gamma^{qq|qc} & \Gamma^{qq|cc} \\
\Gamma^{cq|qq} & \Gamma^{cq|cq} & \Gamma^{cq|qc} & \Gamma^{cq|cc} \\
\Gamma^{qc|qq} & \Gamma^{qc|cq} & \Gamma^{qc|qc} & \Gamma^{qc|cc} \\
\Gamma^{cc|qq} & \Gamma^{cc|cq} & \Gamma^{cc|qc} & \Gamma^{cc|cc}
\end{pmatrix}
\begin{pmatrix}
\Psi^A & T_C\Phi^A & T_2T_C\Phi^A & \Psi^B \\
\Phi^A & \Phi^B & T_2\Phi^B & \Phi^D \\
T_1\Phi^A & T_1\Phi^B & T_3\Phi^B & T_1\Phi^D \\
T_C\Psi^B & T_C\Phi^D & T_2T_C\Phi^D & 0
\end{pmatrix}
\tag{67}
$$

$$
\begin{aligned}
\Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma\sigma|\sigma\sigma}\, \dot{\tilde{L}}^{\,\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\sigma\sigma|\sigma\sigma}\, \Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma\sigma|\sigma\sigma} &=
\begin{pmatrix}
\Psi^A & T_C\Phi^A & T_2T_C\Phi^A & \Psi^B \\
\Phi^A & \Phi^B & T_2\Phi^B & \Phi^D \\
T_1\Phi^A & T_1\Phi^B & T_3\Phi^B & T_1\Phi^D \\
T_C\Psi^B & T_C\Phi^D & T_2T_C\Phi^D & 0
\end{pmatrix}_{\sigma\sigma} \\
\times \begin{pmatrix}
0 & 0 & 0 & \dot{L}^{AA} \\
0 & 0 & \dot{L}^{RA} & \dot{L}^{KA} \\
0 & \dot{L}^{AR} & 0 & \dot{L}^{AK} \\
\dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK}
\end{pmatrix}_{\sigma\sigma}
&\cdot
\begin{pmatrix}
\Psi^A & T_C\Phi^A & T_2T_C\Phi^A & \Psi^B \\
\Phi^A & \Phi^B & T_2\Phi^B & \Phi^D \\
T_1\Phi^A & T_1\Phi^B & T_3\Phi^B & T_1\Phi^D \\
T_C\Psi^B & T_C\Phi^D & T_2T_C\Phi^D & 0
\end{pmatrix}_{\sigma\sigma}
\end{aligned}
\tag{68}
$$

Explicit expressions (one-loop p channel):

$$
\begin{aligned}
\dot\psi^{A(1)}_{p,\sigma\sigma} &= (\dot\gamma^{(1)}_p)^{qq|qq}_{\sigma\sigma|\sigma\sigma}\,(q_1'q_2'|q_1q_2\,,\,A,\Pi,T)\\
&= \tfrac12(2\pi)^2\sum_{\substack{q_3q_4\\q_3'q_4'}}\int\mathrm{d}\Omega\,\begin{pmatrix}\Psi^A\\ T_C\Phi^A\\ T_2T_C\Phi^A\\ \Psi^B\end{pmatrix}^T_{\sigma\sigma}\big(q_1'q_2'|q_3q_4\,,\,\tfrac{A+\Pi-T}{2}-\Omega,\Pi,\tfrac{-A+\Pi+T}{2}-\Omega)\big)\\
&\quad\times\begin{pmatrix}0&0&0&\dot L^{AA}\\0&0&\dot L^{RA}&\dot L^{KA}\\0&\dot L^{AR}&0&\dot L^{AK}\\ \dot L^{RR}&\dot L^{KR}&\dot L^{RK}&\dot L^{KK}\end{pmatrix}_{\sigma\sigma}(q_3q_4|q_3'q_4'\,,\,\Omega,\Pi-\Omega)\\
&\quad\times\begin{pmatrix}\Psi^A\\ \Phi^A\\ T_1\Phi^A\\ T_C\Psi^B\end{pmatrix}_{\sigma\sigma}\big(q_3'q_4'|q_1q_2\,,\,\tfrac{A+\Pi+T}{2}-\Omega,\Pi,\tfrac{A-\Pi+T}{2}+\Omega\big)\\
&= \tfrac12(2\pi)^2\sum_{\substack{q_3q_4\\q_3'q_4'}}\int\mathrm{d}\Omega\,\begin{pmatrix}\Psi^A\,\big(q_1'q_2'|q_3q_4\,,\,\tfrac{A+\Pi-T}{2}-\Omega,\Pi,\tfrac{-A+\Pi+T}{2}-\Omega\big)\\ (\Phi^A)^*\,\big(q_3q_4|q_1'q_2'\,,\,\tfrac{A+\Pi-T}{2}-\Omega,\Pi,\tfrac{A-\Pi-T}{2}+\Omega\big)\\ -(\Phi^A)^*\,\big(q_4q_3|q_1'q_2'\,,\,\tfrac{-A+\Pi+T}{2}-\Omega,\Pi,\tfrac{-A-\Pi+T}{2}+\Omega\big)\\ \Psi^B\,\big(q_1'q_2'|q_3q_4\,,\,\tfrac{A+\Pi-T}{2}-\Omega,\Pi,\tfrac{-A+\Pi+T}{2}-\Omega\big)\end{pmatrix}^T_{\sigma\sigma}\\
&\quad\times\begin{pmatrix}0&0&0&\dot L^{AA}\\0&0&\dot L^{RA}&\dot L^{KA}\\0&\dot L^{AR}&0&\dot L^{AK}\\ \dot L^{RR}&\dot L^{KR}&\dot L^{RK}&\dot L^{KK}\end{pmatrix}_{\sigma\sigma}(q_3q_4|q_3'q_4'\,,\,\Omega,\Pi-\Omega)\\
&\quad\times\begin{pmatrix}\Psi^A\,\big(q_3'q_4'|q_1q_2\,,\,\tfrac{A+\Pi+T}{2}-\Omega,\Pi,\tfrac{A-\Pi+T}{2}+\Omega\big)\\ \Phi^A\,\big(q_3'q_4'|q_1q_2\,,\,\tfrac{A+\Pi+T}{2}-\Omega,\Pi,\tfrac{A-\Pi+T}{2}+\Omega\big)\\ -\Phi^A\,\big(q_4'q_3'|q_1q_2\,,\,\tfrac{A-\Pi+T}{2}+\Omega,\Pi,\tfrac{A+\Pi+T}{2}-\Omega\big)\\ -(\Psi^B)^*\,\big(q_1q_2|q_3'q_4'\,,\,\tfrac{A+\Pi+T}{2}-\Omega,\Pi,\tfrac{-A+\Pi-T}{2}-\Omega\big)\end{pmatrix}_{\sigma\sigma}
\end{aligned}\tag{69}
$$

$$
\begin{aligned}
\dot\psi^{B(1)}_{p,\sigma\sigma} &= (\dot\gamma^{(1)}_p)^{qq|cc}_{\sigma\sigma|\sigma\sigma}\,(q_1'q_2'|q_1q_2\,,\,A,\Pi,T)\\
&= \tfrac12(2\pi)^2\sum_{\substack{q_3q_4\\q_3'q_4'}}\int\mathrm{d}\Omega\,\begin{pmatrix}\Psi^A\,\big(q_1'q_2'|q_3q_4\,,\,\tfrac{A+\Pi-T}{2}-\Omega,\Pi,\tfrac{-A+\Pi+T}{2}-\Omega\big)\\ (\Phi^A)^*\,\big(q_3q_4|q_1'q_2'\,,\,\tfrac{A+\Pi-T}{2}-\Omega,\Pi,\tfrac{A-\Pi-T}{2}+\Omega\big)\\ -(\Phi^A)^*\,\big(q_4q_3|q_1'q_2'\,,\,\tfrac{-A+\Pi+T}{2}-\Omega,\Pi,\tfrac{-A-\Pi+T}{2}+\Omega\big)\\ \Psi^B\,\big(q_1'q_2'|q_3q_4\,,\,\tfrac{A+\Pi-T}{2}-\Omega,\Pi,\tfrac{-A+\Pi+T}{2}-\Omega\big)\end{pmatrix}^T_{\sigma\sigma}\\
&\quad\times\begin{pmatrix}0&0&0&\dot L^{AA}\\0&0&\dot L^{RA}&\dot L^{KA}\\0&\dot L^{AR}&0&\dot L^{AK}\\ \dot L^{RR}&\dot L^{KR}&\dot L^{RK}&\dot L^{KK}\end{pmatrix}_{\sigma\sigma}(q_3q_4|q_3'q_4'\,,\,\Omega,\Pi-\Omega)\\
&\quad\times\begin{pmatrix}\Psi^B\,\big(q_3'q_4'|q_1q_2\,,\,\tfrac{A+\Pi+T}{2}-\Omega,\Pi,\tfrac{A-\Pi+T}{2}+\Omega\big)\\ \Phi^D\,\big(q_3'q_4'|q_1q_2\,,\,\tfrac{A+\Pi+T}{2}-\Omega,\Pi,\tfrac{A-\Pi+T}{2}+\Omega\big)\\ -\Phi^D\,\big(q_4'q_3'|q_1q_2\,,\,\tfrac{A-\Pi+T}{2}+\Omega,\Pi,\tfrac{A+\Pi+T}{2}-\Omega\big)\\ 0\end{pmatrix}_{\sigma\sigma}
\end{aligned}\tag{70}
$$

$$
\begin{aligned}
\dot{\phi}^{A(1)}_{p,\sigma\sigma} &= (\dot{\gamma}^{(1)}_p)^{cq|qq}_{\sigma\sigma|\sigma\sigma}\,(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) \\
&= \tfrac{1}{2}(2\pi)^2 \sum_{\substack{q_3q_4\\ q_3'q_4'}} \int \mathrm{d}\Omega
\begin{pmatrix}
\Phi^A\left(q_1'q_2'|q_3q_4\,,\,\frac{A+\Pi-T}{2}-\Omega,\Pi,\frac{-A+\Pi+T}{2}-\Omega\right)\\
\Phi^B\left(q_1'q_2'|q_3q_4\,,\,\frac{A+\Pi-T}{2}-\Omega,\Pi,\frac{-A+\Pi+T}{2}-\Omega\right)\\
-\Phi^B\left(q_1'q_2'|q_4q_3\,,\,\frac{A-\Pi-T}{2}+\Omega,\Pi,\frac{-A-\Pi+T}{2}+\Omega\right)\\
\Phi^D\left(q_1'q_2'|q_3q_4\,,\,\frac{A+\Pi-T}{2}-\Omega,\Pi,\frac{-A+\Pi+T}{2}-\Omega\right)
\end{pmatrix}^T_{\sigma\sigma} \\
&\quad\times \begin{pmatrix}
0 & 0 & 0 & \dot{L}^{AA}\\
0 & 0 & \dot{L}^{RA} & \dot{L}^{KA}\\
0 & \dot{L}^{AR} & 0 & \dot{L}^{AK}\\
\dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK}
\end{pmatrix}_{\sigma\sigma} (q_3q_4|q_3'q_4'\,,\,\Omega,\Pi-\Omega) \\
&\quad\times \begin{pmatrix}
\Psi^A\left(q_3'q_4'|q_1q_2\,,\,\frac{A+\Pi+T}{2}-\Omega,\Pi,\frac{A-\Pi+T}{2}+\Omega\right)\\
\Phi^A\left(q_3'q_4'|q_1q_2\,,\,\frac{A+\Pi+T}{2}-\Omega,\Pi,\frac{A-\Pi+T}{2}+\Omega\right)\\
-\Phi^A\left(q_4'q_3'|q_1q_2\,,\,\frac{A-\Pi+T}{2}+\Omega,\Pi,\frac{A+\Pi+T}{2}-\Omega\right)\\
-(\Psi^B)^*\left(q_1q_2|q_3'q_4'\,,\,\frac{A+\Pi+T}{2}-\Omega,\Pi,\frac{-A+\Pi-T}{2}-\Omega\right)
\end{pmatrix}_{\sigma\sigma}
\end{aligned}
\tag{71}
$$

$$
\begin{aligned}
\dot{\phi}^{B(1)}_{p,\sigma\sigma} &= (\dot{\gamma}^{(1)}_p)^{cq|cq}_{\sigma\sigma|\sigma\sigma}\,(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) \\
&= \tfrac{1}{2}(2\pi)^2 \sum_{\substack{q_3q_4\\ q_3'q_4'}} \int \mathrm{d}\Omega
\begin{pmatrix}
\Phi^A\left(q_1'q_2'|q_3q_4\,,\,\frac{A+\Pi-T}{2}-\Omega,\Pi,\frac{-A+\Pi+T}{2}-\Omega\right)\\
\Phi^B\left(q_1'q_2'|q_3q_4\,,\,\frac{A+\Pi-T}{2}-\Omega,\Pi,\frac{-A+\Pi+T}{2}-\Omega\right)\\
-\Phi^B\left(q_1'q_2'|q_4q_3\,,\,\frac{A-\Pi-T}{2}+\Omega,\Pi,\frac{-A-\Pi+T}{2}+\Omega\right)\\
\Phi^D\left(q_1'q_2'|q_3q_4\,,\,\frac{A+\Pi-T}{2}-\Omega,\Pi,\frac{-A+\Pi+T}{2}-\Omega\right)
\end{pmatrix}^T_{\sigma\sigma} \\
&\quad\times \begin{pmatrix}
0 & 0 & 0 & \dot{L}^{AA}\\
0 & 0 & \dot{L}^{RA} & \dot{L}^{KA}\\
0 & \dot{L}^{AR} & 0 & \dot{L}^{AK}\\
\dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK}
\end{pmatrix}_{\sigma\sigma} (q_3q_4|q_3'q_4'\,,\,\Omega,\Pi-\Omega) \\
&\quad\times \begin{pmatrix}
(\Phi^A)^*\left(q_1q_2|q_3'q_4'\,,\,\frac{A+\Pi+T}{2}-\Omega,\Pi,\frac{-A+\Pi-T}{2}-\Omega\right)\\
\Phi^B\left(q_3'q_4'|q_1q_2\,,\,\frac{A+\Pi+T}{2}-\Omega,\Pi,\frac{A-\Pi+T}{2}+\Omega\right)\\
-\Phi^B\left(q_4'q_3'|q_1q_2\,,\,\frac{A-\Pi+T}{2}+\Omega,\Pi,\frac{A+\Pi+T}{2}-\Omega\right)\\
(\Phi^D)^*\left(q_1q_2|q_3'q_4'\,,\,\frac{A+\Pi+T}{2}-\Omega,\Pi,\frac{-A+\Pi-T}{2}-\Omega\right)
\end{pmatrix}_{\sigma\sigma}
\end{aligned}
\tag{72}
$$

$$
\begin{aligned}
\dot{\phi}^{D(1)}_{p,\sigma\sigma} &= (\dot{\gamma}^{(1)}_p)^{cq|cc}_{\sigma\sigma|\sigma\sigma}\,(q_1'q_2'|q_1q_2\,,\,A,\Pi,T) \\
&= \tfrac{1}{2}(2\pi)^2 \sum_{\substack{q_3q_4\\ q_3'q_4'}} \int \mathrm{d}\Omega
\begin{pmatrix}
\Phi^A\left(q_1'q_2'|q_3q_4\,,\,\frac{A+\Pi-T}{2}-\Omega,\Pi,\frac{-A+\Pi+T}{2}-\Omega\right)\\
\Phi^B\left(q_1'q_2'|q_3q_4\,,\,\frac{A+\Pi-T}{2}-\Omega,\Pi,\frac{-A+\Pi+T}{2}-\Omega\right)\\
-\Phi^B\left(q_1'q_2'|q_4q_3\,,\,\frac{A-\Pi-T}{2}+\Omega,\Pi,\frac{-A-\Pi+T}{2}+\Omega\right)\\
\Phi^D\left(q_1'q_2'|q_3q_4\,,\,\frac{A+\Pi-T}{2}-\Omega,\Pi,\frac{-A+\Pi+T}{2}-\Omega\right)
\end{pmatrix}^T_{\sigma\sigma} \\
&\quad\times \begin{pmatrix}
0 & 0 & 0 & \dot{L}^{AA}\\
0 & 0 & \dot{L}^{RA} & \dot{L}^{KA}\\
0 & \dot{L}^{AR} & 0 & \dot{L}^{AK}\\
\dot{L}^{RR} & \dot{L}^{KR} & \dot{L}^{RK} & \dot{L}^{KK}
\end{pmatrix}_{\sigma\sigma} (q_3q_4|q_3'q_4'\,,\,\Omega,\Pi-\Omega) \\
&\quad\times \begin{pmatrix}
\Psi^B\left(q_3'q_4'|q_1q_2\,,\,\frac{A+\Pi+T}{2}-\Omega,\Pi,\frac{A-\Pi+T}{2}+\Omega\right)\\
\Phi^D\left(q_3'q_4'|q_1q_2\,,\,\frac{A+\Pi+T}{2}-\Omega,\Pi,\frac{A-\Pi+T}{2}+\Omega\right)\\
-\Phi^D\left(q_4'q_3'|q_1q_2\,,\,\frac{A-\Pi+T}{2}+\Omega,\Pi,\frac{A+\Pi+T}{2}-\Omega\right)\\
0
\end{pmatrix}_{\sigma\sigma}
\end{aligned}
\tag{73}
$$

$$
\begin{aligned}
\dot\psi^{A(1)}_{p,\sigma\bar\sigma} &= (\dot\gamma^{(1)}_p)^{qq|qq}_{\sigma\bar\sigma|\sigma\bar\sigma}\,(q_1'q_2'|q_1q_2\,,\,A,\Pi,T)\\
&= \tfrac{1}{2}(2\pi)^2\sum_{\substack{q_3q_4\\q_3'q_4'}}\int\mathrm{d}\Omega\,\left\{\begin{pmatrix}\Psi^A_{\sigma\bar\sigma}\big(q_1'q_2'|q_3q_4\,,\,\frac{A+\Pi-T}{2}-\Omega,\Pi,\frac{-A+\Pi+T}{2}-\Omega\big)\\(\Phi^A)^*_{\sigma\bar\sigma}\big(q_3q_4|q_1'q_2'\,,\,\frac{A+\Pi-T}{2}-\Omega,\Pi,\frac{A-\Pi-T}{2}+\Omega\big)\\-(\Phi^A)^*_{\bar\sigma\sigma}\big(q_4q_3|q_2'q_1'\,,\,\frac{-A-\Pi+T}{2}+\Omega,\Pi,\frac{-A+\Pi+T}{2}-\Omega\big)\\\Psi^B_{\sigma\bar\sigma}\big(q_1'q_2'|q_3q_4\,,\,\frac{A+\Pi-T}{2}-\Omega,\Pi,\frac{-A+\Pi+T}{2}-\Omega\big)\end{pmatrix}^T\right.\\
&\quad\times\begin{pmatrix}0&0&0&\dot L^{AA}\\0&0&\dot L^{RA}&\dot L^{KA}\\0&\dot L^{AR}&0&\dot L^{AK}\\\dot L^{RR}&\dot L^{KR}&\dot L^{RK}&\dot L^{KK}\end{pmatrix}_{\sigma\bar\sigma}(q_3q_4|q_3'q_4'\,,\,\Omega,\Pi-\Omega)\\
&\quad\times\begin{pmatrix}\Psi^A_{\sigma\bar\sigma}\big(q_3'q_4'|q_1q_2\,,\,\frac{A+\Pi+T}{2}-\Omega,\Pi,\frac{A-\Pi+T}{2}+\Omega\big)\\\Phi^A_{\sigma\bar\sigma}\big(q_3'q_4'|q_1q_2\,,\,\frac{A+\Pi+T}{2}-\Omega,\Pi,\frac{A-\Pi+T}{2}+\Omega\big)\\\Phi^A_{\bar\sigma\sigma}\big(q_2'q_1'|q_4q_3\,,\,\frac{-A-\Pi+T}{2}+\Omega,\Pi,\frac{A-\Pi-T}{2}+\Omega\big)\\-(\Psi^B)^*_{\sigma\bar\sigma}\big(q_1q_2|q_3'q_4'\,,\,\frac{A+\Pi+T}{2}-\Omega,\Pi,\frac{-A+\Pi-T}{2}-\Omega\big)\end{pmatrix}\\
&\quad+\begin{pmatrix}-\Psi^A_{\sigma\bar\sigma}\big(q_1'q_2'|q_4q_3\,,\,\frac{A-\Pi-T}{2}+\Omega,\Pi,\frac{-A-\Pi+T}{2}+\Omega\big)\\-(\Phi^A)^*_{\bar\sigma\sigma}\big(q_4q_3|q_1'q_2'\,,\,\frac{A-\Pi-T}{2}+\Omega,\Pi,\frac{A+\Pi-T}{2}-\Omega\big)\\(\Phi^A)^*_{\sigma\bar\sigma}\big(q_3q_4|q_2'q_1'\,,\,\frac{-A+\Pi+T}{2}-\Omega,\Pi,\frac{-A-\Pi+T}{2}+\Omega\big)\\-\Psi^B_{\sigma\bar\sigma}\big(q_1'q_2'|q_4q_3\,,\,\frac{A-\Pi-T}{2}+\Omega,\Pi,\frac{-A-\Pi+T}{2}+\Omega\big)\end{pmatrix}^T\\
&\quad\times\begin{pmatrix}0&0&0&\dot L^{AA}\\0&0&\dot L^{RA}&\dot L^{KA}\\0&\dot L^{AR}&0&\dot L^{AK}\\\dot L^{RR}&\dot L^{KR}&\dot L^{RK}&\dot L^{KK}\end{pmatrix}_{\bar\sigma\sigma}(q_3q_4|q_3'q_4'\,,\,\Omega,\Pi-\Omega)\\
&\quad\left.\times\begin{pmatrix}-\Psi^A_{\bar\sigma\sigma}\big(q_3'q_4'|q_2q_1\,,\,\frac{-A+\Pi-T}{2}-\Omega,\Pi,\frac{-A-\Pi-T}{2}+\Omega\big)\\-\Phi^A_{\bar\sigma\sigma}\big(q_3'q_4'|q_2q_1\,,\,\frac{-A+\Pi-T}{2}-\Omega,\Pi,\frac{-A-\Pi-T}{2}+\Omega\big)\\-\Phi^A_{\sigma\bar\sigma}\big(q_4'q_3'|q_1q_2\,,\,\frac{A-\Pi+T}{2}+\Omega,\Pi,\frac{A+\Pi+T}{2}-\Omega\big)\\(\Psi^B)^*_{\sigma\bar\sigma}\big(q_2q_1|q_3'q_4'\,,\,\frac{-A+\Pi-T}{2}-\Omega,\Pi,\frac{A+\Pi+T}{2}-\Omega\big)\end{pmatrix}\right\}
\end{aligned}
\tag{74}
$$

$$
\begin{gathered}
\begin{pmatrix}\Gamma^{qq|qq}&\Gamma^{qq|cq}&\Gamma^{qq|qc}&\Gamma^{qq|cc}\\\Gamma^{cq|qq}&\Gamma^{cq|cq}&\Gamma^{cq|qc}&\Gamma^{cq|cc}\\\Gamma^{qc|qq}&\Gamma^{qc|cq}&\Gamma^{qc|qc}&\Gamma^{qc|cc}\\\Gamma^{cc|qq}&\Gamma^{cc|cq}&\Gamma^{cc|qc}&\Gamma^{cc|cc}\end{pmatrix}_{\sigma\bar\sigma}\\
\begin{pmatrix}\Psi^A&T_C\Phi^A&T_2T_C\Phi^A&\Psi^B\\\Phi^A&\Phi^B&T_2\Phi^B&\Phi^D\\T_1\Phi^A&T_1\Phi^B&T_3\Phi^B&T_1\Phi^D\\T_C\Psi^B&T_C\Phi^D&T_2T_C\Phi^D&0\end{pmatrix}_{\sigma\sigma}\\
\begin{pmatrix}\Psi^A&T_C\Phi^A&T_3T_CT_S\Phi^A&\Psi^B\\\Phi^A&\Phi^B&\Phi^C&\Phi^D\\T_3T_S\Phi^A&T_3\Phi^C&T_3T_S\Phi^B&T_3T_S\Phi^D\\T_C\Psi^B&T_C\Phi^D&T_3T_CT_S\Phi^D&0\end{pmatrix}_{\sigma\bar\sigma}\\
\begin{pmatrix}T_2\Psi^A&T_1T_CT_S\Phi^A&T_2T_C\Phi^A&T_2\Psi^B\\T_2\Phi^A&T_2\Phi^C&T_2\Phi^B&T_2\Phi^D\\T_1T_S\Phi^A&T_1T_S\Phi^B&T_1T_S\Phi^C&T_1T_S\Phi^D\\T_1T_CT_S\Psi^B&T_1T_CT_S\Phi^D&T_2T_C\Phi^D&0\end{pmatrix}_{\sigma\bar\sigma}
\end{gathered}
\tag{75}
$$

$$
= \begin{pmatrix} \Psi^A & T_C\Phi^A & T_2T_C\Phi^A & \Psi^B \\ \Phi^A & \Phi^B & T_2\Phi^B & \Phi^D \\ T_1\Phi^A & T_1\Phi^B & T_3\Phi^B & T_1\Phi^D \\ T_C\Psi^B & T_C\Phi^D & T_2T_C\Phi^D & 0 \end{pmatrix}_{\sigma\sigma|\sigma\sigma}
$$

$$
\times \begin{pmatrix} \dot{L}^{AA\,T_C\Psi^B} & \dot{L}^{AA\,T_C\Phi^D} & \dot{L}^{AA\,T_2T_C\Phi^D} \\ \dot{L}^{RA\,T_1\Phi^A} + \dot{L}^{KA\,T_C\Psi^B} & \dot{L}^{RA\,T_1\Phi^B} + \dot{L}^{KA\,T_C\Phi^D} & \dot{L}^{RA\,T_3\Phi^B} + \dot{L}^{KA\,T_2T_C\Phi^D} \\ \dot{L}^{AR\,\Phi^A} + \dot{L}^{AK\,T_C\Psi_B} & \dot{L}^{AR\,\Phi^B} + \dot{L}^{AK\,T_C\Phi^D} & \dot{L}^{AR\,T_2\Phi^B} + \dot{L}^{AK\,T_2T_C\Phi^D} \\ \dot{L}^{RR\,\Psi^A} + \dot{L}^{KR\,\Phi^A} + \dot{L}^{RK\,T_1\Phi^A} + \dot{L}^{KK\,T_C\Phi^B} & \dot{L}^{RR\,T_C\Phi^A} + \dot{L}^{KR\,\Phi^B} + \dot{L}^{RK\,T_1\Phi^B} + \dot{L}^{KK\,T_C\Phi^D} & \dot{L}^{RR\,T_2T_C\Phi^A} + \dot{L}^{KR\,T_2\Phi^B} + \dot{L}^{RK\,T_3\Phi^B} + \dot{L}^{KK\,T_2} \end{pmatrix} \tag{76}
$$

$$
\begin{aligned}
&\times \left(\Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma_1'\sigma_1'|\sigma_1'\sigma_1'}\ \delta_{\sigma_1'\sigma_2'}\,\delta_{\sigma_1'\sigma_3}\,\delta_{\sigma_1'\sigma_4} + \Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma_1'\bar{\sigma}_1'|\sigma_1'\bar{\sigma}_1'}\ \delta_{\sigma_1'\bar{\sigma}_2'}\,\delta_{\sigma_1'\sigma_3}\,\delta_{\sigma_1'\bar{\sigma}_4} + \Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma_1'\bar{\sigma}_1'|\bar{\sigma}_1'\sigma_1'}\ \delta_{\sigma_1'\bar{\sigma}_2'}\,\delta_{\sigma_1'\bar{\sigma}_3}\,\delta_{\sigma_1'\sigma_4} \right) \\
&\times \dot{\tilde{L}}^{\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\sigma_3\sigma_4|\sigma_3\sigma_4} \\
&\times \left(\Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma_1\sigma_1|\sigma_1\sigma_1}\ \delta_{\sigma_1\sigma_2}\,\delta_{\sigma_1\sigma_3}\,\delta_{\sigma_1\sigma_4} + \Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma_1\bar{\sigma}_1|\sigma_1\bar{\sigma}_1}\ \delta_{\sigma_1\bar{\sigma}_2}\,\delta_{\sigma_1\sigma_3}\,\delta_{\sigma_1\bar{\sigma}_4} + \Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\bar{\sigma}_1\sigma_1|\sigma_1\bar{\sigma}_1}\ \delta_{\sigma_1\bar{\sigma}_2}\,\delta_{\sigma_1\bar{\sigma}_3}\,\delta_{\sigma_1\sigma_4} \right) \\
&\times \Bigg\{ \delta_{\sigma\sigma_1'}\,\delta_{\sigma\sigma_2'}\,\delta_{\sigma\sigma_1}\,\delta_{\sigma\sigma_2} \left(\Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma\sigma|\sigma\sigma} \right) + \delta_{\sigma\sigma_1'}\,\delta_{\sigma\bar{\sigma}_2'}\,\delta_{\sigma\sigma_1}\,\delta_{\sigma\bar{\sigma}_2}\{\} + \delta_{\sigma\sigma_1'}\,\delta_{\sigma\bar{\sigma}_2'}\,\delta_{\sigma\bar{\sigma}_1}\,\delta_{\sigma\sigma_2}\{\} \\
&\quad \Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma_1\sigma_2|\sigma_1\sigma_2}\,\dot{\tilde{L}}^{\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\sigma_1\sigma_2|\sigma_1\sigma_2}\,\Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma_1\sigma_2|\sigma_1\sigma_2}\ \delta_{\sigma_1'\sigma_1}\,\delta_{\sigma_2'\sigma_2} + \Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma_2\sigma_1|\sigma_2\sigma_1}\,\dot{\tilde{L}}^{\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\sigma_1\sigma_2|\sigma_1\sigma_2}\,\Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma_1\sigma_2|\sigma_1\sigma_2}\ \delta_{\sigma_1'\sigma_2}\,\delta_{\sigma_2'\sigma_1} \\
&\quad + \Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma_2\sigma_1|\sigma_2\sigma_1}\,\dot{\tilde{L}}^{\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\sigma_2\sigma_1|\sigma_2\sigma_1}\,\Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma_2\sigma_1|\sigma_1\sigma_2}\ \delta_{\sigma_1'\sigma_2}\,\delta_{\sigma_2'\sigma_1} + \Gamma^{\alpha_1'\alpha_2'|\alpha_3\alpha_4}_{\sigma_1\sigma_2|\sigma_1\sigma_2}\,\dot{\tilde{L}}^{\alpha_3\alpha_4|\alpha_3'\alpha_4'}_{\sigma_2\sigma_1|\sigma_2\sigma_1}\,\Gamma^{\alpha_3'\alpha_4'|\alpha_1\alpha_2}_{\sigma_2\sigma_1|\sigma_1\sigma_2}\ \delta_{\sigma_1'\sigma_1}\,\delta_{\sigma_2'\sigma_2} \Bigg\}
\end{aligned} \tag{77}
$$

Independent spin and Keldysh components

Keldysh components which are equal due to diagrammatic structure of $\mathcal{K}_1$, $\mathcal{K}_2$, $\mathcal{K}_3$
class $\mathcal{K}_1$:

$$
\begin{aligned}
(\mathcal{K}_1^a)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} &= (\mathcal{K}_1^a)^{\bar{\alpha}_1'\alpha_2'|\alpha_1\bar{\alpha}_2} = (\mathcal{K}_1^a)^{\alpha_1'\bar{\alpha}_2'|\bar{\alpha}_1\alpha_2} = (\mathcal{K}_1^a)^{\bar{\alpha}_1'\bar{\alpha}_2'|\bar{\alpha}_1\bar{\alpha}_2} \\
(\mathcal{K}_1^p)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} &= (\mathcal{K}_1^p)^{\bar{\alpha}_1'\bar{\alpha}_2'|\alpha_1\alpha_2} = (\mathcal{K}_1^p)^{\alpha_1'\alpha_2'|\bar{\alpha}_1\bar{\alpha}_2} = (\mathcal{K}_1^p)^{\bar{\alpha}_1'\bar{\alpha}_2'|\bar{\alpha}_1\bar{\alpha}_2} \\
(\mathcal{K}_1^t)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} &= (\mathcal{K}_1^t)^{\bar{\alpha}_1'\alpha_2'|\bar{\alpha}_1\alpha_2} = (\mathcal{K}_1^t)^{\alpha_1'\bar{\alpha}_2'|\alpha_1\bar{\alpha}_2} = (\mathcal{K}_1^t)^{\bar{\alpha}_1'\bar{\alpha}_2'|\bar{\alpha}_1\bar{\alpha}_2}
\end{aligned}
$$

class $\mathcal{K}_2$, $\bar{\mathcal{K}}_2$:

$$
\begin{aligned}
(\mathcal{K}_2^a)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} &= (\mathcal{K}_2^a)^{\alpha_1'\bar{\alpha}_2'|\bar{\alpha}_1\alpha_2}\,, \quad (\bar{\mathcal{K}}_2^a)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = (\bar{\mathcal{K}}_2^a)^{\bar{\alpha}_1'\alpha_2'|\alpha_1\bar{\alpha}_2} \\
(\mathcal{K}_1^p)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} &= (\mathcal{K}_1^p)^{\alpha_1'\alpha_2'|\bar{\alpha}_1\bar{\alpha}_2}\,, \quad (\bar{\mathcal{K}}_1^p)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = (\bar{\mathcal{K}}_1^p)^{\bar{\alpha}_1'\bar{\alpha}_2'|\alpha_1\alpha_2} \\
(\mathcal{K}_2^t)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} &= (\mathcal{K}_2^t)^{\bar{\alpha}_1'\alpha_2'|\bar{\alpha}_1\alpha_2}\,, \quad (\bar{\mathcal{K}}_2^t)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = (\bar{\mathcal{K}}_2^t)^{\alpha_1'\bar{\alpha}_2'|\alpha_1\bar{\alpha}_2}
\end{aligned}
$$

equal spins $\sigma\sigma|\sigma\sigma$

Symmetries under particle exchange, complex conjugation
class $\mathcal{K}_1$:

- particle exchange:
  - channels a, t:

$$
\begin{aligned}
\mathcal{K}_1^a(1'2'|12) &= -\mathcal{K}_1^t(1'2'|21) \quad \Rightarrow \quad (\mathcal{K}_1^a)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_1(\mathcal{K}_1^t)^{\alpha_1'\alpha_2'|\alpha_2\alpha_1} \\
&= -\mathcal{K}_1^t(2'1'|12) \quad \Rightarrow \quad (\mathcal{K}_1^a)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_2(\mathcal{K}_1^t)^{\alpha_2'\alpha_1'|\alpha_1\alpha_2} \\
\mathcal{K}_1^a(1'2'|12) &= \mathcal{K}_1^a(2'1'|21) \quad \Rightarrow \quad (\mathcal{K}_1^a)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_3(\mathcal{K}_1^a)^{\alpha_2'\alpha_1'|\alpha_2\alpha_1} \\
\mathcal{K}_1^t(1'2'|12) &= \mathcal{K}_1^t(2'1'|21) \quad \Rightarrow \quad (\mathcal{K}_1^t)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_3(\mathcal{K}_1^t)^{\alpha_2'\alpha_1'|\alpha_2\alpha_1}
\end{aligned}
$$

  - channel p:

$$
\begin{aligned}
\mathcal{K}_1^p(1'2'|12) &= -\mathcal{K}_1^p(1'2'|21) \quad \Rightarrow \quad (\mathcal{K}_1^p)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_1(\mathcal{K}_1^p)^{\alpha_1'\alpha_2'|\alpha_2\alpha_1} \\
&= -\mathcal{K}_1^p(2'1'|12) \quad \Rightarrow \quad (\mathcal{K}_1^p)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_2(\mathcal{K}_1^p)^{\alpha_2'\alpha_1'|\alpha_1\alpha_2} \\
&= \mathcal{K}_1^p(2'1'|21) \quad \Rightarrow \quad (\mathcal{K}_1^p)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_3(\mathcal{K}_1^p)^{\alpha_2'\alpha_1'|\alpha_2\alpha_1}
\end{aligned}
$$

- complex conjugation:

$$
\mathcal{K}_1^r(1'2'|12) = -(-1)^{\sum_j(\alpha_j+\bar{\alpha}_j)}(\mathcal{K}_1^r(12|1'2'))^* \quad \Rightarrow \quad (\mathcal{K}_1^r)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_C(\mathcal{K}_1^r)^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}
$$

class $\mathcal{K}_2$, $\bar{\mathcal{K}}_2$:

- particle exchange:
  - channels a, t:

$$
\begin{aligned}
\mathcal{K}_2^a(1'2'|12) &= -\bar{\mathcal{K}}_2^t(1'2'|21) \quad \Rightarrow \quad (\mathcal{K}_2^a)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_1(\bar{\mathcal{K}}_2^t)^{\alpha_1'\alpha_2'|\alpha_2\alpha_1} \\
&= -\mathcal{K}_2^t(2'1'|12) \quad \Rightarrow \quad (\mathcal{K}_2^a)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_2(\mathcal{K}_2^t)^{\alpha_2'\alpha_1'|\alpha_1\alpha_2} \\
\mathcal{K}_2^a(1'2'|12) &= \bar{\mathcal{K}}_2^a(2'1'|21) \quad \Rightarrow \quad (\mathcal{K}_2^a)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_3(\bar{\mathcal{K}}_2^a)^{\alpha_2'\alpha_1'|\alpha_2\alpha_1} \\
\mathcal{K}_2^t(1'2'|12) &= \bar{\mathcal{K}}_2^t(2'1'|21) \quad \Rightarrow \quad (\mathcal{K}_2^t)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_3(\bar{\mathcal{K}}_2^t)^{\alpha_2'\alpha_1'|\alpha_2\alpha_1}
\end{aligned}
$$

  - channel p:

$$
\begin{aligned}
\mathcal{K}_2^p(1'2'|12) &= -\mathcal{K}_2^p(1'2'|21) \quad \Rightarrow \quad (\mathcal{K}_2^p)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_1(\mathcal{K}_2^p)^{\alpha_1'\alpha_2'|\alpha_2\alpha_1} \\
&= -\mathcal{K}_2^p(2'1'|12) \quad \Rightarrow \quad (\mathcal{K}_2^p)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_2(\mathcal{K}_2^p)^{\alpha_2'\alpha_1'|\alpha_1\alpha_2} \\
&= \mathcal{K}_2^p(2'1'|21) \quad \Rightarrow \quad (\mathcal{K}_2^p)^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_3(\mathcal{K}_2^p)^{\alpha_2'\alpha_1'|\alpha_2\alpha_1}
\end{aligned}
$$

(similarly for $\bar{\mathcal{K}}_2^p$)

- complex conjugation:

$$\mathcal{K}_2^{a,p}(1'2'|12) = -(-1)^{\sum_j(\alpha_j+\bar{\alpha}_j)}(\bar{\mathcal{K}}_2^{a,p}(12|1'2'))^* \quad\Rightarrow\quad (\mathcal{K}_2^{a,p})^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_C(\bar{\mathcal{K}}_2^{a,p})^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}$$

$$\mathcal{K}_2^{t}(1'2'|12) = -(-1)^{\sum_j(\alpha_j+\bar{\alpha}_j)}(\mathcal{K}_2^{t}(12|1'2'))^* \quad\Rightarrow\quad (\mathcal{K}_2^{t})^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_C(\mathcal{K}_2^{t})^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}$$

$$\bar{\mathcal{K}}_2^{t}(1'2'|12) = -(-1)^{\sum_j(\alpha_j+\bar{\alpha}_j)}(\bar{\mathcal{K}}_2^{t}(12|1'2'))^* \quad\Rightarrow\quad (\bar{\mathcal{K}}_2^{t})^{\alpha_1'\alpha_2'|\alpha_1\alpha_2} = T_C(\bar{\mathcal{K}}_2^{t})^{\alpha_1\alpha_2|\alpha_1'\alpha_2'}$$

		$\sigma\sigma\|\sigma\sigma$			$\sigma\bar\sigma\|\sigma\bar\sigma$			$\sigma\bar\sigma\|\bar\sigma\sigma$		
		$\mathcal{K}_1^a$	$\mathcal{K}_1^p$	$\mathcal{K}_1^t$	$\mathcal{K}_1^a$	$\mathcal{K}_1^p$	$\mathcal{K}_1^t$	$\mathcal{K}_1^a$	$\mathcal{K}_1^p$	$\mathcal{K}_1^t$
1111	0	0	0	0	0	0	0	0	0	0
1112	1	B_1^a	B_1^p	B_1^t	$\bar B_1^a$	$\bar B_1^p$	$\bar B_1^t$	$T_ST_2\bar B_1^t$	$T_1\bar B_1^p$	$T_ST_2\bar B_1^a$
1121	2	$T_3B_1^a$	B_1^p	$T_3B_1^t$	$T_ST_3\bar B_1^a$	$\bar B_1^p$	$T_ST_3\bar B_1^t$	$T_1\bar B_1^t$	$T_1\bar B_1^p$	$T_1\bar B_1^a$
1122	3	C_1^a	0	C_1^t	$\bar C_1^a$	0	$\bar C_1^t$	$T_1\bar C_1^t$	0	$T_1\bar C_1^a$
1211	4	$T_3B_1^a$	$T_CB_1^p$	B_1^t	$T_ST_3\bar B_1^a$	$T_C\bar B_1^p$	$\bar B_1^t$	$T_1\bar B_1^t$	$T_1T_C\bar B_1^p$	$T_ST_2\bar B_1^a$
1212	5	C_1^a	D_1^p	0	$\bar C_1^a$	$\bar D_1^p$	0	$T_1\bar C_1^t$	$T_1\bar D_1^p$	0
1221	6	0	D_1^p	C_1^t	0	$\bar D_1^p$	$\bar C_1^t$	0	$T_1\bar D_1^p$	$T_1\bar C_1^a$
1222	7	B_1^a	$T_CB_1^p$	$T_3B_1^t$	$\bar B_1^a$	$T_C\bar B_1^p$	$T_ST_3\bar B_1^t$	$T_ST_2\bar B_1^t$	$T_1T_C\bar B_1^p$	$T_1\bar B_1^a$
2111	8	B_1^a	$T_CB_1^p$	$T_3B_1^t$	$\bar B_1^a$	$T_C\bar B_1^p$	$T_ST_3\bar B_1^t$	$T_ST_2\bar B_1^t$	$T_1T_C\bar B_1^p$	$T_1\bar B_1^a$
2112	9	0	D_1^p	C_1^t	0	$\bar D_1^p$	$\bar C_1^t$	0	$T_1\bar D_1^p$	$T_1\bar C_1^a$
2121	10	C_1^a	D_1^p	0	$\bar C_1^a$	$\bar D_1^p$	0	$T_1\bar C_1^t$	$T_1\bar D_1^p$	0
2122	11	$T_3B_1^a$	$T_CB_1^p$	B_1^t	$T_ST_3\bar B_1^a$	$T_C\bar B_1^p$	$\bar B_1^t$	$T_1\bar B_1^t$	$T_1T_C\bar B_1^p$	$T_ST_2\bar B_1^a$
2211	12	C_1^a	0	C_1^t	$\bar C_1^a$	0	$\bar C_1^t$	$T_1\bar C_1^t$	0	$T_1\bar C_1^a$
2212	13	$T_3B_1^a$	B_1^p	$T_3B_1^t$	$T_ST_3\bar B_1^a$	$\bar B_1^p$	$T_ST_3\bar B_1^t$	$T_1\bar B_1^t$	$T_1\bar B_1^p$	$T_1\bar B_1^a$
2221	14	B_1^a	B_1^p	B_1^t	$\bar B_1^a$	$\bar B_1^p$	$\bar B_1^t$	$T_ST_2\bar B_1^t$	$T_1\bar B_1^p$	$T_ST_2\bar B_1^a$
2222	15	0	0	0	0	0	0	0	0	0

		$\sigma\sigma\|\sigma\sigma$					
		$\mathcal{K}_2^a$	$\bar{\mathcal{K}}_2^a$	$\mathcal{K}_2^p$	$\bar{\mathcal{K}}_2^p$	$\mathcal{K}_2^t$	$\bar{\mathcal{K}}_2^t$
1111	0	A_2^a	$T_3A_2^a$	A_2^p	$T_CA_2^p$	$T_2A_2^a$	$T_1A_2^a$
1112	1	B_2^a	$T_3C_2^a$	B_2^p	$T_CC_2^p$	$T_2B_2^a$	$T_1C_2^a$
1121	2	C_2^a	$T_3B_2^a$	B_2^p	$T_CT_3C_2^p$	$T_2C_2^a$	$T_1B_2^a$
1122	3	D_2^a	$T_3D_2^a$	A_2^p	0	$T_2D_2^a$	$T_1D_2^a$
1211	4	C_2^a	$T_CB_2^a$	C_2^p	$T_CB_2^p$	$T_CT_2B_2^a$	$T_1C_2^a$
1212	5	D_2^a	$T_CD_2^a$	D_2^p	$T_CD_2^p$	0	$T_1A_2^a$
1221	6	A_2^a	0	D_2^p	$T_CT_3D_2^p$	$T_CT_2D_2^a$	$T_1D_2^a$
1222	7	B_2^a	$T_3F_2^a$	C_2^p	$T_CF_2^p$	$T_2F_2^a$	$T_1B_2^a$
2111	8	$T_CT_3B_2^a$	$T_3C_2^a$	$T_3C_2^p$	$T_CB_2^p$	$T_2C_2^a$	$T_CT_1B_2^a$
2112	9	0	$T_3A_2^a$	$T_3D_2^p$	$T_CD_2^p$	$T_2D_2^a$	$T_CT_1D_2^a$
2121	10	$T_CT_3D_2^a$	$T_3D_2^a$	$T_3D_2^p$	$T_CT_3D_2^p$	$T_2A_2^a$	0
2122	11	F_2^a	$T_3B_2^a$	$T_3C_2^p$	$T_CF_2^p$	$T_2B_2^a$	$T_1F_2^a$
2211	12	$T_CT_3D_2^a$	$T_CD_2^a$	0	$T_CA_2^p$	$T_CT_2D_2^a$	$T_CT_1D_2^a$
2212	13	F_2^a	$T_CB_2^a$	F_2^p	$T_CC_2^p$	$T_2F_2^a$	$T_CT_1B_2^a$
2221	14	$T_CT_3B_2^a$	$T_3F_2^a$	F_2^p	$T_CT_3C_2^p$	$T_CT_2B_2^a$	$T_1F_2^a$
2222	15	0	0	0	0	0	0

		$\sigma\bar{\sigma}\lvert\sigma\bar{\sigma}$					
		$\mathcal{K}_2^a$	$\bar{\mathcal{K}}_2^a$	$\mathcal{K}_2^p$	$\bar{\mathcal{K}}_2^p$	$\mathcal{K}_2^t$	$\bar{\mathcal{K}}_2^t$
1111	0	$\bar{A}_2^a$	$T_ST_3\bar{A}_2^a$	$\bar{A}_2^p$	$T_C\bar{A}_2^p$	$\bar{A}_2^t$	$T_ST_3\bar{A}_2^t$
1112	1	$\bar{B}_2^a$	$T_ST_3\bar{C}_2^a$	$\bar{B}_2^p$	$T_C\bar{C}_2^p$	$\bar{B}_2^t$	$T_ST_3\bar{C}_2^t$
1121	2	$\bar{C}_2^a$	$T_ST_3\bar{B}_2^a$	$\bar{B}_2^p$	$T_ST_CT_3\bar{C}_2^p$	$\bar{C}_2^t$	$T_ST_3\bar{B}_2^t$
1122	3	$\bar{D}_2^a$	$T_ST_3\bar{D}_2^a$	$\bar{A}_2^p$	0	$\bar{D}_2^t$	$T_ST_3\bar{D}_2^t$
1211	4	$\bar{C}_2^a$	$T_C\bar{B}_2^a$	$\bar{C}_2^p$	$T_C\bar{B}_2^p$	$T_C\bar{B}_2^t$	$T_ST_3\bar{C}_2^t$
1212	5	$\bar{D}_2^a$	$T_C\bar{D}_2^a$	$\bar{D}_2^p$	$T_C\bar{D}_2^p$	0	$T_ST_3\bar{A}_2^t$
1221	6	$\bar{A}_2^a$	0	$\bar{D}_2^p$	$T_ST_CT_3\bar{D}_2^p$	$T_C\bar{D}_2^t$	$T_ST_3\bar{D}_2^t$
1222	7	$\bar{B}_2^a$	$T_ST_3\bar{F}_2^a$	$\bar{C}_2^p$	$T_C\bar{F}_2^p$	$\bar{F}_2^t$	$T_ST_3\bar{B}_2^t$
2111	8	$T_ST_CT_3\bar{B}_2^a$	$T_ST_3\bar{C}_2^a$	$T_ST_3\bar{C}_2^p$	$T_C\bar{B}_2^p$	$\bar{C}_2^t$	$T_ST_CT_3\bar{B}_2^t$
2112	9	0	$T_ST_3\bar{A}_2^a$	$T_ST_3\bar{D}_2^p$	$T_C\bar{D}_2^p$	$\bar{D}_2^t$	$T_ST_CT_3\bar{D}_2^t$
2121	10	$T_ST_CT_3\bar{D}_2^a$	$T_ST_3\bar{D}_2^a$	$T_ST_3\bar{D}_2^p$	$T_ST_CT_3\bar{D}_2^p$	$\bar{A}_2^t$	0
2122	11	$\bar{F}_2^a$	$T_ST_3\bar{B}_2^a$	$T_ST_3\bar{C}_2^p$	$T_C\bar{F}_2^p$	$\bar{B}_2^t$	$T_ST_3\bar{F}_2^t$
2211	12	$T_ST_CT_3\bar{D}_2^a$	$T_C\bar{D}_2^a$	0	$T_C\bar{A}_2^p$	$T_C\bar{D}_2^t$	$T_ST_CT_3\bar{D}_2^t$
2212	13	$\bar{F}_2^a$	$T_C\bar{B}_2^a$	$\bar{F}_2^p$	$T_C\bar{C}_2^p$	$\bar{F}_2^t$	$T_ST_CT_3\bar{B}_2^t$
2221	14	$T_ST_CT_3\bar{B}_2^a$	$T_ST_3\bar{F}_2^a$	$\bar{F}_2^p$	$T_ST_CT_3\bar{C}_2^p$	$T_C\bar{B}_2^t$	$T_ST_3\bar{F}_2^t$
2222	15	0	0	0	0	0	0

		$\sigma\bar{\sigma}\lvert\bar{\sigma}\sigma$					
		$\mathcal{K}_2^a$	$\bar{\mathcal{K}}_2^a$	$\mathcal{K}_2^p$	$\bar{\mathcal{K}}_2^p$	$\mathcal{K}_2^t$	$\bar{\mathcal{K}}_2^t$
1111	0	$T_ST_2\bar{A}_2^t$	$T_1\bar{A}_2^t$	$T_1\bar{A}_2^p$	$T_ST_CT_1\bar{A}_2^p$	$T_ST_2\bar{A}_2^a$	$T_1\bar{A}_2^a$
1112	1	$T_ST_2\bar{B}_2^t$	$T_1\bar{C}_2^t$	$T_1\bar{B}_2^p$	$T_ST_CT_1\bar{C}_2^p$	$T_ST_2\bar{B}_2^a$	$T_1\bar{C}_2^a$
1121	2	$T_ST_2\bar{C}_2^t$	$T_1\bar{B}_2^t$	$T_1\bar{B}_2^p$	$T_1T_C\bar{C}_2^p$	$T_ST_2\bar{C}_2^a$	$T_1\bar{B}_2^a$
1122	3	$T_ST_2\bar{D}_2^t$	$T_1\bar{D}_2^t$	$T_1\bar{A}_2^p$	0	$T_ST_2\bar{D}_2^a$	$T_1\bar{D}_2^a$
1211	4	$T_ST_2\bar{C}_2^t$	$T_1T_C\bar{B}_2^t$	$T_1\bar{C}_2^p$	$T_1T_C\bar{B}_2^p$	$T_1T_C\bar{B}_2^a$	$T_1\bar{C}_2^a$
1212	5	$T_ST_2\bar{D}_2^t$	$T_1T_C\bar{D}_2^t$	$T_1\bar{D}_2^p$	$T_ST_CT_1\bar{D}_2^p$	0	$T_1\bar{A}_2^a$
1221	6	$T_ST_2\bar{A}_2^t$	0	$T_1\bar{D}_2^p$	$T_1T_C\bar{D}_2^p$	$T_1T_C\bar{D}_2^a$	$T_1\bar{D}_2^a$
1222	7	$T_ST_2\bar{B}_2^t$	$T_1\bar{F}_2^t$	$T_1\bar{C}_2^p$	$T_ST_CT_1\bar{F}_2^p$	$T_ST_2\bar{F}_2^a$	$T_1\bar{B}_2^a$
2111	8	$T_ST_CT_1\bar{B}_2^t$	$T_1\bar{C}_2^t$	$T_ST_2\bar{C}_2^p$	$T_1T_C\bar{B}_2^p$	$T_ST_2\bar{C}_2^a$	$T_ST_CT_1\bar{B}_2^a$
2112	9	0	$T_1\bar{A}_2^t$	$T_ST_2\bar{D}_2^p$	$T_ST_CT_1\bar{D}_2^p$	$T_ST_2\bar{D}_2^a$	$T_ST_CT_1\bar{D}_2^a$
2121	10	$T_ST_CT_1\bar{D}_2^t$	$T_1\bar{D}_2^t$	$T_ST_2\bar{D}_2^p$	$T_1T_C\bar{D}_2^p$	$T_ST_2\bar{A}_2^a$	0
2122	11	$T_ST_2\bar{F}_2^t$	$T_1\bar{B}_2^t$	$T_ST_2\bar{C}_2^p$	$T_ST_CT_1\bar{F}_2^p$	$T_ST_2\bar{B}_2^a$	$T_1\bar{F}_2^a$
2211	12	$T_ST_CT_1\bar{D}_2^t$	$T_1T_C\bar{D}_2^t$	0	$T_ST_CT_1\bar{A}_2^p$	$T_1T_C\bar{D}_2^a$	$T_ST_CT_1\bar{D}_2^a$
2212	13	$T_ST_2\bar{F}_2^t$	$T_1T_C\bar{B}_2^t$	$T_1\bar{F}_2^p$	$T_ST_CT_1\bar{C}_2^p$	$T_ST_2\bar{F}_2^a$	$T_ST_CT_1\bar{B}_2^a$
2221	14	$T_ST_CT_1\bar{B}_2^t$	$T_1\bar{F}_2^t$	$T_1\bar{F}_2^p$	$T_1T_C\bar{C}_2^p$	$T_1T_C\bar{B}_2^a$	$T_1\bar{F}_2^a$
2222	15	0	0	0	0	0	0

		$\sigma\sigma\|\sigma\sigma$			$\sigma\bar{\sigma}\|\sigma\bar{\sigma}$			$\sigma\bar{\sigma}\|\bar{\sigma}\sigma$		
		$\mathcal{K}_3^a$	$\mathcal{K}_3^p$	$\mathcal{K}_3^t$	$\mathcal{K}_3^a$	$\mathcal{K}_3^p$	$\mathcal{K}_3^t$	$\mathcal{K}_3^a$	$\mathcal{K}_3^p$	$\mathcal{K}_3^t$
1111	0	A_3^a	A_3^p	$T_2A_3^a$	$\bar{A}_3^a$	$\bar{A}_3^p$	$\bar{A}_3^t$	$T_1\bar{A}_3^t$	$T_1\bar{A}_3^p$	$T_1\bar{A}_3^a$
1112	1	B_3^a	B_3^p	$T_2B_3^a$	$\bar{B}_3^a$	$\bar{B}_3^p$	$\bar{B}_3^t$	$T_ST_2\bar{B}_3^t$	$T_ST_2\bar{B}_3^p$	$T_ST_2\bar{B}_3^a$
1121	2	$T_3B_3^a$	$T_3B_3^p$	$T_1B_3^a$	$T_ST_3\bar{B}_3^a$	$T_ST_3\bar{B}_3^p$	$T_ST_3\bar{B}_3^t$	$T_1\bar{B}_3^t$	$T_1\bar{B}_3^p$	$T_1\bar{B}_3^a$
1122	3	C_3^a	C_3^p	$T_2C_3^a$	$\bar{C}_3^a$	$\bar{C}_3^p$	$\bar{C}_3^t$	$T_1\bar{C}_3^t$	$T_1\bar{C}_3^p$	$T_1\bar{C}_3^a$
1211	4	$T_CB_3^a$	$T_CB_3^p$	$T_1T_CB_3^a$	$T_C\bar{B}_3^a$	$T_C\bar{B}_3^p$	$T_C\bar{B}_3^t$	$T_1T_C\bar{B}_3^t$	$T_1T_C\bar{B}_3^p$	$T_1T_C\bar{B}_3^a$
1212	5	D_3^a	D_3^p	D_3^t	$\bar{D}_3^a$	$\bar{D}_3^p$	$\bar{D}_3^t$	$T_1\bar{E}_3^t$	$T_1\bar{E}_3^p$	$T_1\bar{E}_3^a$
1221	6	$T_1D_3^t$	$T_1D_3^p$	$T_1D_3^a$	$\bar{E}_3^a$	$\bar{E}_3^p$	$\bar{E}_3^t$	$T_1\bar{D}_3^t$	$T_1\bar{D}_3^p$	$T_1\bar{D}_3^a$
1222	7	F_3^a	F_3^p	$T_1F_3^a$	$\bar{F}_3^a$	$\bar{F}_3^p$	$\bar{F}_3^t$	$T_1\bar{F}_3^t$	$T_1\bar{F}_3^p$	$T_1\bar{F}_3^a$
2111	8	$T_CT_3B_3^a$	$T_CT_3B_3^p$	$T_CT_1B_3^a$	$T_ST_CT_3\bar{B}_3^a$	$T_ST_CT_3\bar{B}_3^p$	$T_ST_CT_3\bar{B}_3^t$	$T_ST_CT_1\bar{B}_3^t$	$T_ST_CT_1\bar{B}_3^p$	$T_ST_CT_1\bar{B}_3^a$
2112	9	$T_2D_3^t$	$T_2D_3^p$	$T_2D_3^a$	$T_ST_3\bar{E}_3^a$	$T_ST_3\bar{E}_3^p$	$T_ST_3\bar{E}_3^t$	$T_ST_2\bar{D}_3^t$	$T_ST_2\bar{D}_3^p$	$T_ST_2\bar{D}_3^a$
2121	10	$T_3D_3^a$	$T_3D_3^p$	$T_3D_3^t$	$T_ST_3\bar{D}_3^a$	$T_ST_3\bar{D}_3^p$	$T_ST_3\bar{D}_3^t$	$T_ST_2\bar{E}_3^t$	$T_ST_2\bar{E}_3^p$	$T_ST_2\bar{E}_3^a$
2122	11	$T_3F_3^a$	$T_3F_3^p$	$T_2F_3^a$	$T_ST_3\bar{F}_3^a$	$T_ST_3\bar{F}_3^p$	$T_ST_3\bar{F}_3^t$	$T_ST_2\bar{F}_3^t$	$T_ST_2\bar{F}_3^p$	$T_ST_2\bar{F}_3^a$
2211	12	$T_CC_3^a$	$T_CC_3^p$	$T_1T_CC_3^a$	$T_C\bar{C}_3^a$	$T_C\bar{C}_3^p$	$T_C\bar{C}_3^t$	$T_1T_C\bar{C}_3^t$	$T_1T_C\bar{C}_3^p$	$T_1T_C\bar{C}_3^a$
2212	13	$T_CF_3^a$	$T_CF_3^p$	$T_2T_CF_3^a$	$T_C\bar{F}_3^a$	$T_C\bar{F}_3^p$	$T_C\bar{F}_3^t$	$T_ST_CT_1\bar{F}_3^t$	$T_ST_CT_1\bar{F}_3^p$	$T_ST_CT_1\bar{F}_3^a$
2221	14	$T_CT_3F_3^a$	$T_CT_3F_3^p$	$T_CT_2F_3^a$	$T_ST_CT_3\bar{F}_3^a$	$T_ST_CT_3\bar{F}_3^p$	$T_ST_CT_3\bar{F}_3^t$	$T_1T_C\bar{F}_3^t$	$T_1T_C\bar{F}_3^p$	$T_1T_C\bar{F}_3^a$
2222	15	0	0	0	0	0	0	0	0	0