Vertex Conventions

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1 Objects related to Green's function

one-particle Green's function:

G [1–19], g (local) [11–13, 20]

bare one-particle Green's function:

G_0 [3–7, 10, 16, 17], G^0 [1, 13, 19]

self-energy:

Σ [1, 3, 5–15, 17–22]

Hartree term of the self-energy:

Σ_H [6], Σ^HF [13], Σ_Hartree [19]

bubble, i.e., product of two Green's functions:

Π_r [3, 5, 10, 15, 16], $(GG)_r$ [17]

bubble integrated over momenta:

χ_0 [18], X^0 [11]

2 Vertex objects

full two-particle vertex:

Γ [1, 5–7, 18], $\Gamma^{(4)}$ [2, 4], Γ^* [19], f (local) [11, 12, 20, 21], F [13, 14, 17, 21, 22], V [10, 15, 16], L^X [16]

bare two-particle vertex:

U [2, 4, 5, 10–12, 14–18, 20–22], Γ_0 [6, 7], Γ^0 [1], V [8, 9, 19], f (bare vertex for dual fermions) [13]

two-particle correlator:

$G_r^{(4)}$ [5, 6], $g^{(4),\alpha}$ (local) [11, 12, 20], G_2 [17], $G^{(4),\alpha}$ [11]

2.1 Parquet formalism

channels of two-particle reducibility:

a, p, t [1–7], $\overline{\mathrm{ph}}, \mathrm{pp}, \mathrm{ph}$ [12, 19, 22], $\overline{ph}, pp, ph$ [10, 17]

two-particle reducible vertex:

γ_r [1–9], Φ^r [14, 17, 22], ϕ^X [15, 16], Φ_X [10]

two-particle irreducible vertex:

R [1–6, 8, 9], Λ [12, 14] $\tilde{\Lambda}$ [22], $\mathcal{I}$ [15], Λ_{2PI} [10, 17]

two-particle irreducible vertex in a specific channel:

I_r [1–6, 8, 9], Γ_r [17, 19], Γ^i [11], γ^α (local) [11, 21], Γ^α [14]

2.2 Asymptotic classes

first class: part of the two-particle reducible vertex depending on one bosonic frequency

$\mathcal{K}_1^r$ [5, 6], $K_{1,c}$ [7], $\mathcal{K}^{(1)X}$ [15], $\mathcal{K}_{1,c}$ [17]

second class: part of the two-particle reducible vertex depending on one bosonic frequency and one fermionic frequency

$\mathcal{K}_2^r, \mathcal{K}_{2'}^r$ [5, 6], $K_{2,c}, K_{2',c}$ [7], $\mathcal{K}_k^{(2)X}(Q), \mathcal{K}_{k'}^{(2)X}(Q)$ [15, 16], $\mathcal{K}_{2,c}, \bar{\mathcal{K}}_{2,c}$ [17]

sum of the first and second class:

$\Gamma_2^r, \Gamma_{2'}^r$ [5], $Q_{2,c}, Q_{2',c}$ [7]

part of the two-particle reducible vertex depending on one bosonic frequency and two fermionic frequencies

$\mathcal{K}_3^r$ [5, 6], $K_{3,c}$ [7], $\mathcal{R}^{\mathrm{asym},X}$ [16], $\mathcal{R}_{kk'}^X(Q)$ [15], $\mathcal{R}_c$ [17]

2.3 SBE formalism

U-reducible / single-boson exchange vertex:

∇_r [5, 12, 20, 21], Δ^α [13, 14, 21, 22], $\mathcal{M}, \mathcal{C}, \mathcal{S}$ [16]

multi-boson exchange vertex, i.e., two-particle reducible, but U-irreducible vertex

M_r [5, 13, 14, 22], $\mathcal{R}^X$ [16]

U-irreducible vertex in a specific channel

T_r [5], φ^α [12, 20], $\mathcal{I}^X$ [15], $T^{i,\alpha}$ [14, 21]

two-particle irreducible and U-irreducible vertex in a specific channel

$S^{i,\alpha}$ [14, 21]

fully U-irreducible vertex

$\varphi^{U_{\mathrm{irr}}}$ [5], φ^{firr} [12, 20, 21], Λ^{Uirr} [22], $\Lambda_{U\mathrm{irr}}$ [16], $\mathcal{I}_{U_{\mathrm{irr}}}$ [15], Φ^{Uirr} [13, 21], Λ^{Uirr} [14]

bosonic propagator:

w_r [5, 11, 12, 15, 21], W^α [11, 13, 14, 19–22], D^X [16]

polarization / bosonic self-energy:

P_r [5,19], π^α (local) [11,12,20], Π^α [11,14,21,22]

Hedin vertex, i.e., three-point correlator with amputated fermionic legs:

$\bar\lambda_r, \lambda_r$ [5,11,12,15,20,21], γ^α [14,22], $\bar h^X, h^X$ [16], $\bar\Lambda^{i,\alpha}, \Lambda^{i,\alpha}$ (lattice) [11,13,21]

three-point vertex with two amputated fermionic legs & one amputated bosonic leg:

$\bar\Gamma_r^{(3)}, \Gamma_r^{(3)}$ [5], Γ^{ijk} [4]

3 Susceptibilities

physical susceptibility:

χ_r [5–10,12,15–18,20,21], X^α (lattice) [11,21], Π [2,4]

three-point susceptibility:

$\bar\chi_r^{(3)}, \chi_r^{(3)}$ [5]

generalized (i.e., four-point) susceptibility:

$\chi_r^{(4)}$ [5], $\chi^\alpha_{\nu\nu'\tilde\omega}$ [12]

bosonic two-point correlator

D_r [5]

three-point correlator

$\bar G_r^{(3)}, G_r^{(3)}$ [5], $g^{(3),\alpha}$ (local) [11,12]

4 fRG related objects

single-scale propagator:

S [1–7,15,16]

scale parameter:

Λ [1–7,10]

regulator:

R [7]

5 Frequency/momentum parametrization

bosonic frequency:

ω [5–7,10–13,18,22], Ω [15], $\bar\omega, \bar\nu$ [2]

fermionic frequency:

ν [5–7,10–13,18,22], ω, ν [2]

bosonic momentum:

$\boldsymbol{q}$ [10,11,13,14,16–18,22], $\boldsymbol{Q}$ [15]

fermionic momentum:

$\boldsymbol{k}$ [10,11,13,14,16–18,22]

6 Spin channels

spin channels of the two-particle vertex

$\Gamma^{\sigma\bar{\sigma}}, \hat{\Gamma}^{\sigma\bar{\sigma}}, \Gamma^{\sigma\sigma}$ [5]

physical channels of the two-particle vertex

sp, ch, tr, si [5], m, d [6], sp, ch, t, s [11–14, 20, 22], M, D, SC [15], m, c, s [16]

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