Impacts of K-12 Schooling on COVID-19 Transmission

Lawrence, Practicallearning.org

1 Background

Under the shadow of the current COVID-19 pandemic, schools are confronted with a dilemma, should classes continue to be taught in person, or should schools be closed and classes moved online? This dilemma has agonized school district officials, parents, and students since the start of the pandemic. As CDC Principal Deputy Director, Dr Anne Schuchat, noted during a U.S. Senate Committee hearing, "You have this balance between the earlier you act, the more impact it can have in slowing the spread, and the enormous disruption we see with school closures."[1]

This study is conducted to investigate whether K-12 schools teaching students online can significantly reduce COVID-19 transmission by comparing the R_0, the basic reproduction number, in communities with different in-person-teaching ratios in their K-12 schools.

2 The Data

2.1 School year 2019 -2020

As the first COVID-19 cases began to emerge in America, school districts began to take action starting with deep cleaning schools in early February 2020 to full closures starting in March 2020. Below is a brief timeline from Education Week.[2]

- 1/29/2020: First U.S. cases emerge.
- 2/27/2020: First School shut down. Bothell High School in Washington State closed for 2 days.
- 3/5/2020: Shift to distance learning begins. First school to do so was the 24,000-student Northshore School district in Washington State, which shifted to online learning for up to 14 days.
- 3/16/2020: 27 states and territories close their schools. By this time, more than half of all students in the U.S. have been impacted by school closures.
- 3/17/2020: Kansas State became the first state to close for the rest of the academic year.
- 3/25/2020: All U.S. public school buildings are closed.
- 5/6/2020: Nearly all states closed schools for the rest of the 2019-2020 academic year. The only two states that did not were Wyoming and Montana.

The magnitude and speed of school closures was unprecedented, and the impact on students' learning was tremendous.[2]

2.2 School year 2020-2021

As the 2020 summer ended, with no COVID-19 containment in sight, parents, students, educators, health professionals and policymakers faced a daunting dilemma of what to do with the new 2020-2021 school year.

There are 13,000+ public K-12 school districts, in 3000+ counties, each school district has its own teaching plan: on-line, in-person, or hybrid combinations of the two.

As half of the school year has passed by, studying the COVID-19 daily cases of counties with different in-person teaching ratios, will shed insights of whether teaching in-person causes a significant increase in COVID-19 transmissions.

Since the Johns Hopkins' Coronavirus Resource Center data tallies COVID-19 cases of each county, this work calculated each county's in-person teaching ratio by calculating the mean of in-person-teaching ratio of all the school districts inside that county.

For county i with n school districts, the in person-teaching ratio m_i at day t can be calculated as:

$$m_i(t) = \frac{\sum_{j=1}^{n} E_{i,j} h_{i,j}(t)}{\sum_{j=1}^{n} E_{i,j}} \tag{1}$$

where $E_{i,j}$ is the enrollment in the j^{th} school district in county i and $h_{i,j}(t)$ is a numeric representation of this school district's teaching method on day t,

$$h_{i,j}(t) = \begin{cases} 1, & \text{if the school district is teaching classes solely in person} \\ 0, & \text{if the school district is teaching classes solely online} \\ 0.5, & \text{if the school district is teaching classes through a hybrid method} \end{cases} \tag{2}$$

To simplify the following discussion, a few variables are defined as below:

- R_0 - basic reproduction number
- β - transmission rate
- T_{Delay} - the time delay from a person being exposed to a virus to being infected.
- T - infectious period, the time a person remains infectious after being infected.
- $m_i(t)$ the i^{th} county's in-person teaching ratio on day t.
- $\sigma_{m_i}^2$ - variance of $m_i(t)$ across time for the i^{th} county.
- $\bar{m}_i$ - average in-person teaching ratio for the i_{th} county(some schools changed their in-person teaching ratio from time to time)
- C - numbers of counties
- P - population
- $N(t)$ - total new cases on day t

- $n(t)$ - daily new case on day t

2.2.1 Distribution of School Teaching Methods

Figure 1 and 2 show the number of counties and population as a function of in-person teaching ratio $m(t)$ on date 9/20/2020, 12/17/2020, and 1/24/2021. These plots show that over the time, some counties did change their in-person teaching ratio, many shifting toward hybrid teaching.

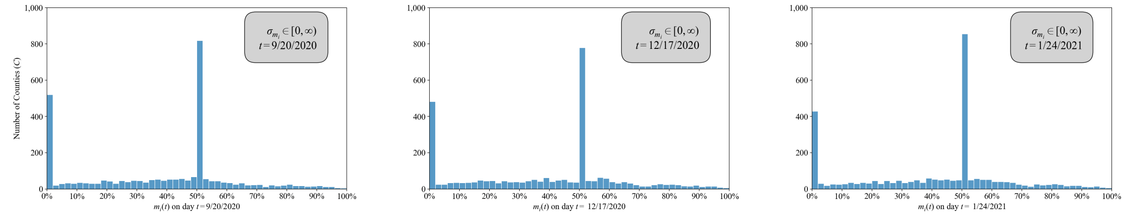

Figure 1: *Tally of counties as a function of in-person teaching ratio* $m(t)$*, at* t *=9/20/2020, 12/17/2020, and 1/24/2021*

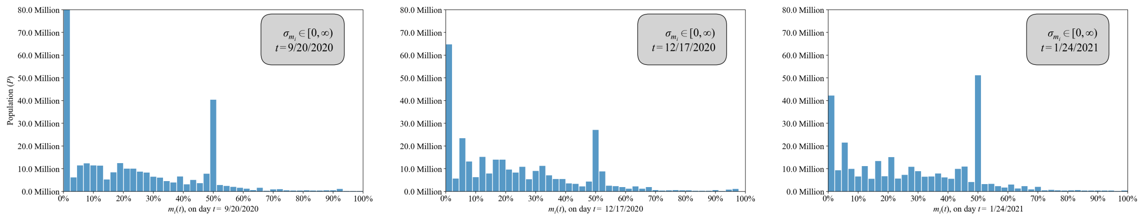

Figure 2: *Tally of population as a function of in-person teaching ratio* $m(t)$*, with* t = *9/20/2020, 12/17/2020, and 1/24/2021*

To characterize how many changes are made to each county's teaching plan, each county's in-person teaching ratio $m_i(t)$ was recorded for 9/20/2020, 12/17/2020, 1/14/2021, 1/24/2021, where i represents the i^{th} county. The mean of $m_i(t)$ over time, $\bar{m_i}$ and its variance σ_{m_i} is calculated as below,

$$\bar{m_i} = \mathbb{E}[m_i(t) \text{ for } t = 9/20/2020,\ 12/17/2020,\ 1/14/2021,\ 1/24/2021] \tag{3}$$

$$\sigma^2_{m_i} = \mathrm{Var}[m_i(t) \text{ for } t = 9/20/2020,\ 12/17/2020,\ 1/14/2021,\ 1/24/2021] \tag{4}$$

Figure 3 plots the number of counties and population as a function of the mean in-person teaching ratio ($\bar{m_i}$). It shows most of the U.S. population live in counties whose schools were doing in-person teaching $\bar{m} \leq 50\%$. Figure 4 plots the numbers of county and population as a function of the standard deviation σ_{m_i} of $m_i(t)$, and this plot shows not that much population have their schools that significantly changed their in-person teaching ratio, most kept their change well under 25%.

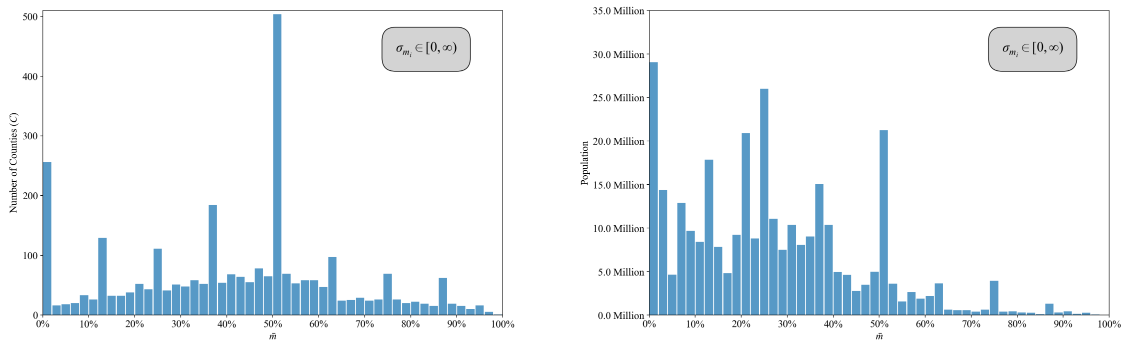

Figure 3: *Left: County number as a function of average in-person teaching ratio $\bar{m}_i$ right: population as a function of average in-person teaching ratio $\bar{m}_i$*

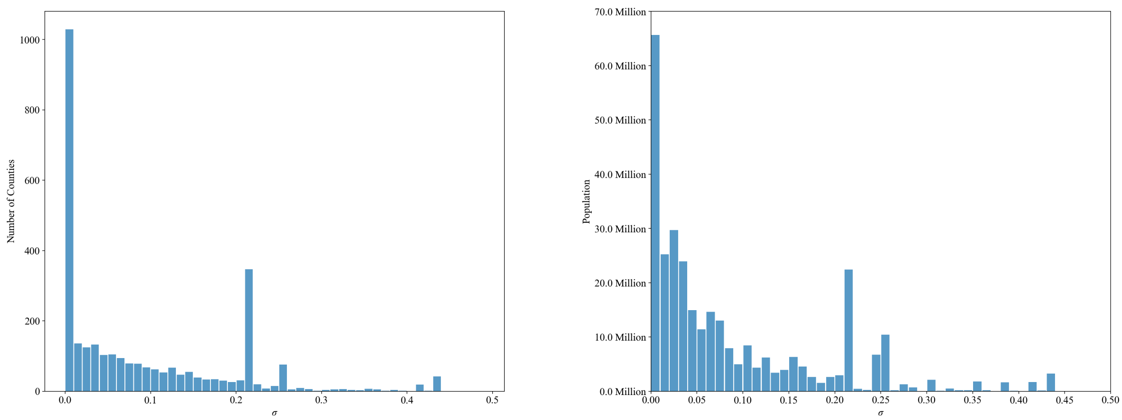

Figure 4: *Left: Number of counties as a function of σ_{m_i} right: Population as a function σ_{m_i}*

To reduce noise caused by uncertainties, this work focused on the data from the counties with little changes in $m_i(t)(\sigma_{m_i} < 0.01\%)$during the school year. Figure 5 plots the number of counties and population as a function of the in-person teaching ratio $m_i(t)$ for counties with $\sigma_{m_i} < 0.01\%$.

Figure 5 shows two spikes, one at $\bar{m}_i = 0\%$, one at $\bar{m}_i = 50\%$. This means the majority counties that have not change their in-person teaching ratio $m_i(t)$ up till 1/24/2021 are those whose school districts were teaching classes entirely online, i.e. $\bar{m}_i = 0\%$, or teaching classes entirely through a hybrid method, i.e. $\bar{m}_i = 50\%$.

The analysis will focus on these two groups of counties. All the counties whose $\bar{m}_i = 0\%$ and $\sigma_{m_i} < 0.01\%$ are grouped together into *group 0%*, their new cases and death counts

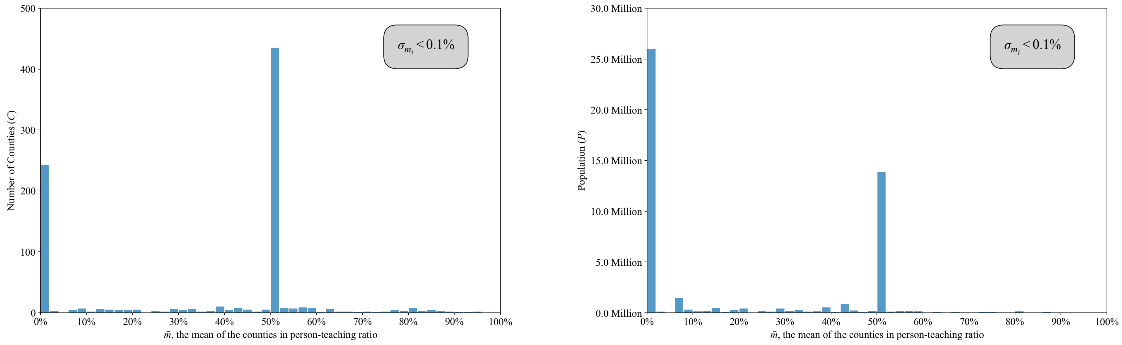

Figure 5: *Number of counties and population as a function of average in-person teaching ratio $\bar{m}$ for all the counties that did not change teaching plans, i.e. $\sigma_{m_i} < 0.01\%$, up till 1/24/2021.*

per million is denoted as $C_{0\%}$ and $D_{0\%}$. Likewise, all the counties whose $\bar{m}_i = 50\%$ and $\sigma_{m_i} < 0.01\%$ are grouped together into *group 50%*, their new cases and death counts per million is denoted as $C_{50\%}$ and $D_{50\%}$.

2.2.2 Daily new cases

Figure 6 shows that at the beginning of May, $C_{50\%}(t)$ was less than $C_{0\%}(t)$, This trend reversed around after the July 4^{th} holiday, when $C_{50\%}(t)$ became greater than $C_{0\%}(t)$ and stayed like that till the beginning of January 2021, when the trend reversed again and $C_{0\%}(t)$ became greater than $C_{50\%}(t)$.

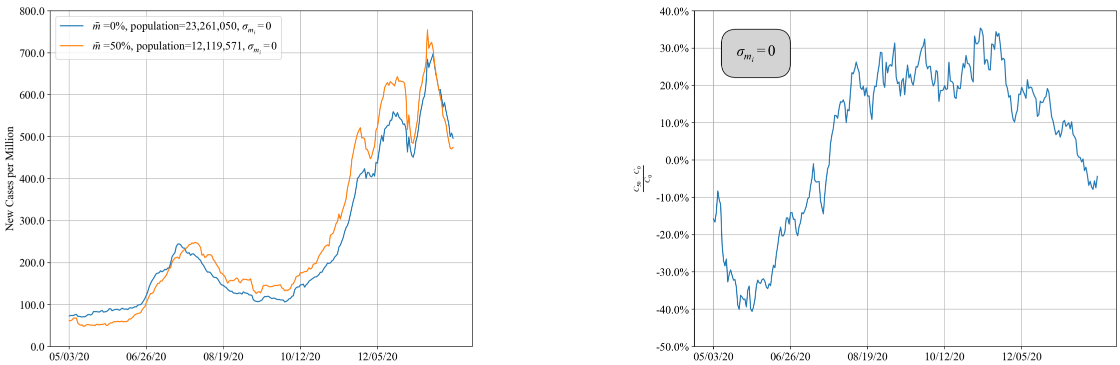

Figure 6: *Left:* $C_{0\%}(t)$ *and* $C_{50\%}(t)$. *Right: percentage difference between* $C_{0\%}(t)$ *and* $C_{50\%}(t)$.

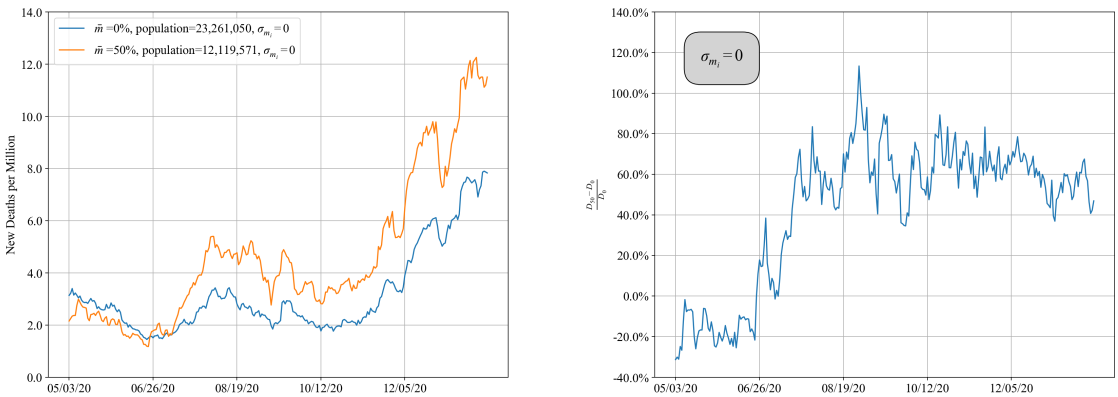

Figure 7: *Left:* $D_{0\%}(t)$ *and* $D_{50\%}(t)$*. Right: percentage difference between* $D_{0\%}(t)$ *and* $D_{50\%}(t)$*.*

Figure 7, the daily new death per million, shows $D_{50\%}$ stayed above $D_{0\%}$ since late June 2020, and the pecentage difference between $D_{0\%}(t)$ and $D_{50\%}(t)$ is much bigger than that between $C_{0\%}(t)$ and $C_{50\%}(t)$.

Does the increased daily cases in *group 50%* mean that in-person teaching caused a noticeable increase in the COVID-19 transmission? A close observation of the above charts shows that the new cases for *group 50%* already started to increase above that of *group 0%* at the beginning July 2020, when students are still on summer break. When most schools started at the end of August and beginning of September 2020, the new cases ratio plotted in figure 6 (right) stopped rising in mid August and stayed about the same level afterward, and then came down at around December. How to interpret these data to derive the degree of impact on the COVID-19 transmission from school teaching students in-person?

Lets refer to epidemiology and how the transmissibility of an infectious disease is defined.

3 Method

In epidemiology, the *basic reproduction number*, R_0, a measure of disease transmission, is defined as the number of cases one case generates on average over the course of their infectious period T in a population where all individuals are susceptible to infection.[4]

In the early stages of an epidemic, the increase of cases during an infinitesimal time dt is proportional to the existing case $N(t)$,

$$\frac{dN(t)}{dt} = N(t)K \tag{5}$$

where K is a constant. The cases number by time t is

$$N(t) = N(0)e^{Kt} \tag{6}$$

Therefore R_0 is:

$$R_0 = \frac{N(T)}{N(0)} = e^{KT} \tag{7}$$

R_0 is affected by numerous biological, sociobehavioral, and environmental factors that govern pathogen transmission.[5] It is not a biological constant for a pathogen, and may change with time. For example, if there are mitigation measures. then R_0 is changing due to the effectiveness of mitigation. Therefore, R_0 and K are functions of t, when sociobehavioral, and environmentalfactor changes, and is denoted with $R_0(t)$ and $K(t)$, where

$$R_0(t) = e^{K(t)T} \tag{8}$$

With a time varying $R_0(t)$, the cases number on day t becomes

$$N(t) = N(t-1)e^{K(t)} \tag{9}$$

The first order approximation is:

$$N(t) - N(t-1) = N(t-1)(e^{K(t)} - 1) \approx N(t-1)K(t) \tag{10}$$

Therefore,

$$K(t) = (N(t) - N(t-1))/N(t-1) \tag{11}$$

Assuming the existing cases are only infectious for T days, then the above equation will become

$$K(t) = \frac{\sum_{x=t-T+1}^{t} n(x) - \sum_{x=t-T}^{t-1} n(x)}{\sum_{x=t-T}^{t-1} n(x)} \tag{12}$$

$$k(t) = \frac{n(t) - n(t-T)}{\sum_{x=t-T}^{t-1} n(x)} \tag{13}$$

Equation 8 and 13 can be used to estimate the instantaneous $R_0(t)$. Comparing $R_{0,x\%}(t)$ for schools using $x\%$ in-person teaching ratio, with the $R_{0,0\%}(t)$ for schools using 0% in-person teaching can give us indication of whether in-person teaching increases the COVID-19 transmission.

What is the infectious period T of COVID-19? According to the Harvard Medical School article “If you’ve been exposed to the coronavirus”, by the 10th day after COVID symptoms begin, most people will no longer be contagious, as long as their symptoms have continued to improve and their fever has resolved.[3] Therefore, in our model let us set the infectious period T as

$$T = 10 \text{ days} \tag{14}$$

The $R_0(t)$ of group 50% and 0% are plotted in Figure 8. It shows before the 2020-2021 school year, group 50% has a slightly higher $R_{0,50\%}(t)$ than the $R_{0,0\%}$ of group 0%. After school opened in the fall of 2020, $R_{0,50\%}(t)$ is similar to $R_{0,0\%}(t)$. The plot does not show a significant transmission increase because of the higher in-person teaching ratio in the fall of 2020.

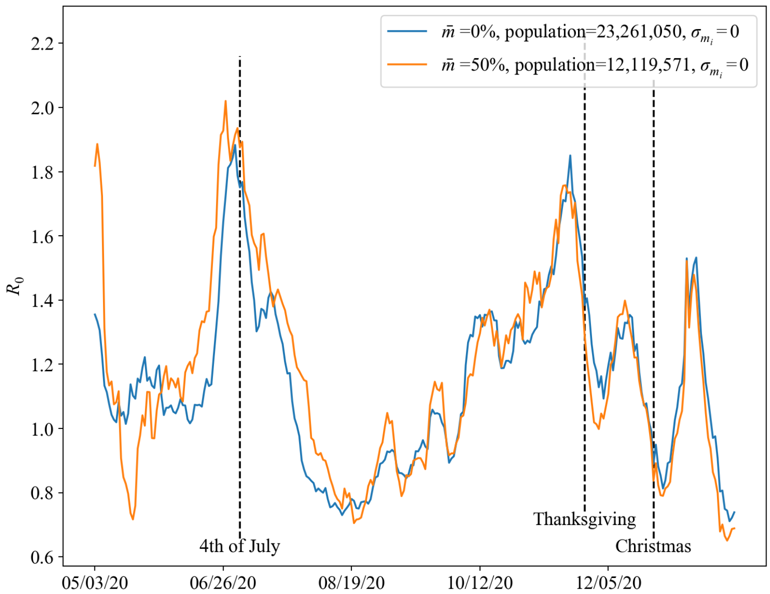

Figure 8: $R_0(t)$ *for group* 0% *and group* 50%

There are about 23 million people in group 0%, the control group, less than 10% of the total US population (328 Million). The rest of the 90%+ population has various degrees of in-person teaching ratio as shown in Figure 3. To further study the correlation between COVID-19 transmission and in-person teaching, data from 3 more groups of counties are studied. *Group* 25% − 28%, *group* 50% − 55%, *group* 0% − 100% are defined as the counties with in-person teaching ration $\bar{m}_i = 25\% - 28\%, 50\% - 55\%, 0\% - 100\%$, and the each groups' daily new case counts and the basic reproduction factor R_0 are calculated, and plot in Figure 9.

A few counties, such as those in Georgia, had very irregular new cases data due to various reasons, such as the accumulation of a few day's samples got tested or reported in one day. This caused a sudden burst in new cases data not due to a sudden burst of transmission. This study excluded those counties in the analysis.

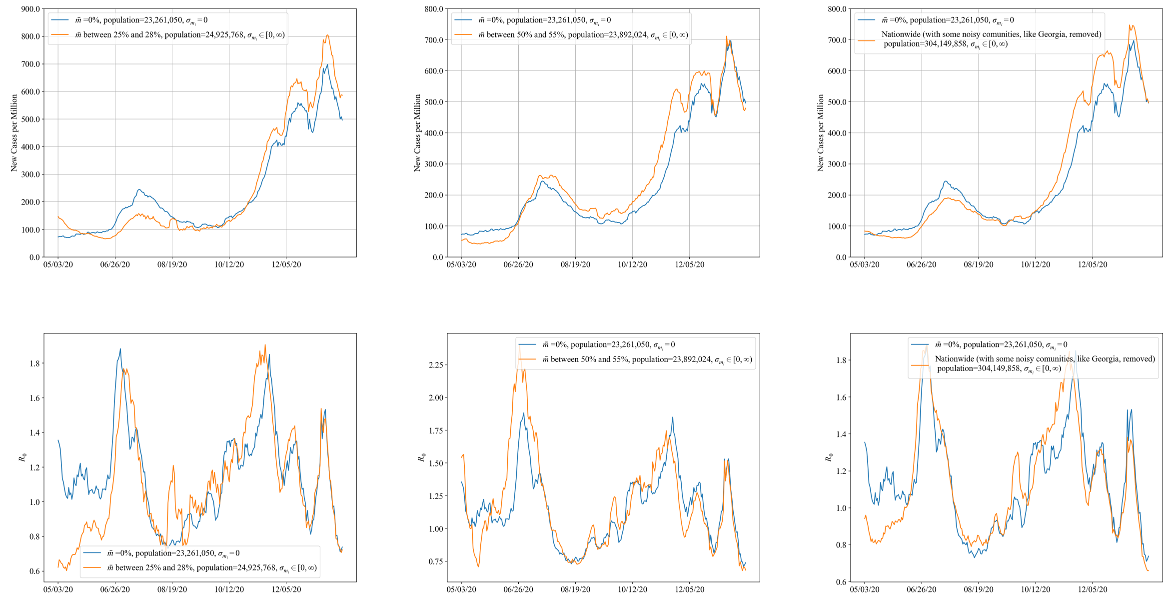

Figure 9: *New case per million and* R_0 *of the group* $(m = 0\%)$ *compared with the* $m = 25\% - 28\%$, $\sigma_{m_i} \in [0, \infty)$ *group (left), the* $m = 50\% - 55\%$, $\sigma_{m_i} \in [0, \infty)$ *group (middle), and* $m = 0\% - 100\%$, $\sigma_{m_i} \in [0, \infty)$ *group (right) i.e. all U.S. counties excluding several counties with irregular testing data, like the ones in Georgia.*

The above results show that the correlation between in-person teaching and the new COVID-19 case is not very obvious, and does not show a predominant reduction in COVID-19 transmission by keeping students on-line.

Why is this the case? Below a detailed study is carried out to shed some light of this.

3.1 A detailed example for School year 2020 - 2021

LA county in California and King county in Washington state serve as an detailed example of how in-person teaching does not dominantly affect the COVID-19 new cases under current overall mitigation measures.

The first COVID-19 community outbreak was found in King county in the early March 2020, and soon made King county the early epicenter of this pandemic in late March, 2020. At the beginning, the case count per million in King county $C_{\text{King County}}(t)$ lead every other county in the US. However, as shown in figure 10, $C_{\text{LA County}}(t)$ surpassed $C_{\text{King County}}(t)$ from April to early fall 2020, and then $C_{\text{LA County}}(t) \approx C_{\text{King County}}(t)$ until November. But after Thanksgiving, $C_{\text{LA County}}(t)$ skyrocketed to more than 3 times $C_{\text{King County}}(t)$, and remained like that until 1/24/2021, when this study collected the most recent data.

What caused this?

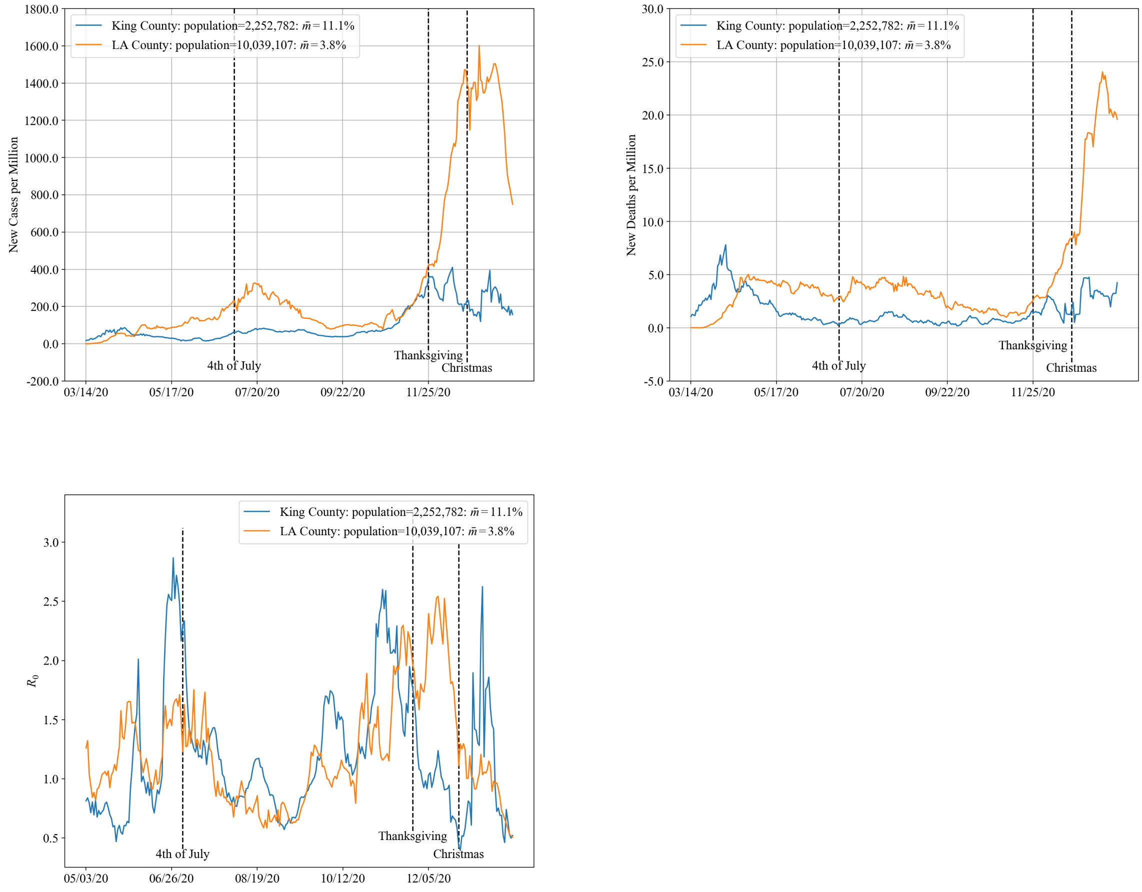

Figure 10: *New Cases, Death and R_0 of LA county and King county*

As the 2020 fall approached, Los Angeles County did not allow any K-12 schools to reopen until at least November, except small in-person classes for children with special needs at no more than 10% of capacity at one time.[8]

Since the beginning of November, schools in LA county planning to open in-person teaching have to apply for a waiver. Only 30 waiver applications per week could be received from the County's more than 2,000 public schools and 1,000 parochial and private schools.[6]

As the data collected about LA county's school in-person teaching ratio on 9/20/2020, 12/17/2020, 1/14/20201, 1/24/20201, LA county had more conservative in-person teaching ratio $\bar{m}_{LA\ county} = 3\%$, than that of King county $\bar{m}_{king\ county} = 12\%$.

The explosion of new cases in LA between Thanksgiving and Christmas is captured in the R_0 plot in Figure 10. While King county quickly brought R_0 to below 1, the R_0 in LA county remained between 1.5 to 2.5 for a long time.

Ryan Craig, Senior Contributor of Fobes.com, attributed LA becoming the epicenter of the COVID-19 epidemic on the failure to follow public health guidelines and the resulting busy beaches and shopping malls.[6]

Los Angeles County issues a new stay-at-home order that prohibited people from gathering with anyone outside of their household either in public or private, effective on November

30.[7]

Comparing the containment measures taken in these two counties might give some insight into the effectiveness of different measures. This could be the topic of some future study.

3.2 Death Rate

Figure 7 shows that *group* 50% has a much higher death per million population than *group* 0%. What caused this increase?

It is reported that the COVID-19 fatality increases dramatically with age.[10] With the data gathered in this study, the population distribution as a function of the percentage of population with age 70+ years old are plotted in figure 11 for *group* 0%, *group* 50%, and *group* 0% − 100%, i.e. the whole U.S population. It shows that *group* 50% has 13.58% of population with age 70+, higher than *group* 0% (10.71%) or the whole U.S. (11.15%).

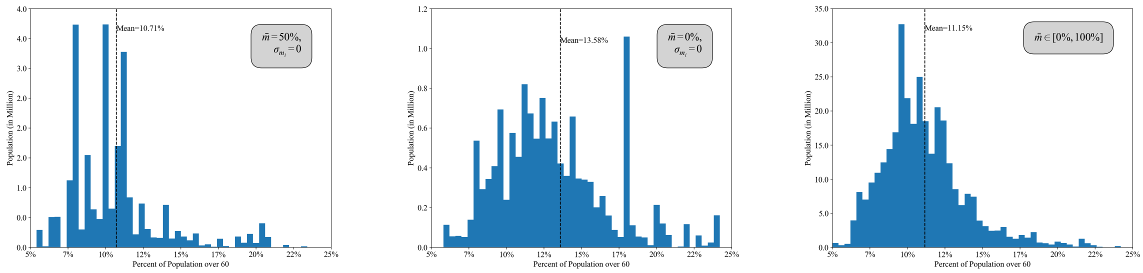

Figure 11: *Population distribution as a function of how much percentage of the population are 70+ year old.*

For counties with the same percentage of population age 70+, the average death rate is calculated, where the death rate on day t is defined as follows,

$$\text{death rate}_{x\%}(t) = \frac{\sum_{j=1}^{s}\sum_{\tau=1}^{t} d_{x\%,j}(\tau)}{\sum_{j=1}^{s}\sum_{\tau=1}^{t} n_{x\%,j}(\tau)} \tag{15}$$

with $d_{x\%,j}(\tau)$ and $n_{x\%,j}(\tau))$ being the new death and case counts on day τ for the j^{th} county with $x\%$ of population age 70+. The death rate as a function of percentage of population age 70+ is plotted using Kernel Regression in figure 12. It shows if the percentage of 70+ year old in a county population increase from 6% to 16%, the death rate of COIVD infection increases roughly by 1%.

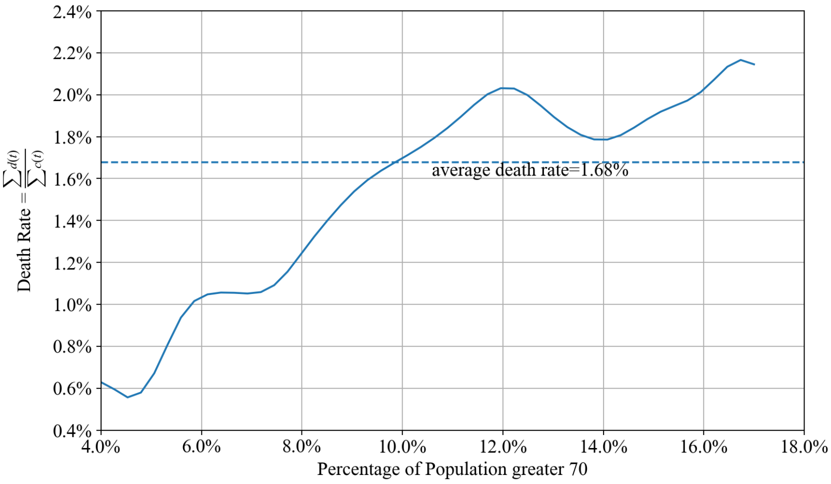

Figure 12: *Death rate as a function of percentage population who are 70+ years old*

Figure 13 plots the death rate as a function of time for a few groups under study. The left plot is for *group* 0% and *group* 50% vs *group* 0% − 100%, i.e. the national average. The right plot is for King county and LA county vs national average. The plot shows that with more testing available and improvement in treatment know-how, the death rate is going down with time, and currently settled at around 1.68. As show earlier in figure 7, *group* 50% has a higher death per million population than that of *group* 0%, the left plot in figure 13 shows that *group* 50% has a slightly higher death rate (1.85%) than *group* 0% (1.42%). This could be explained by that *Group* 50% has a higher percentage of 70+ population (13.58%) than that of the *group* 0% (10.71%).

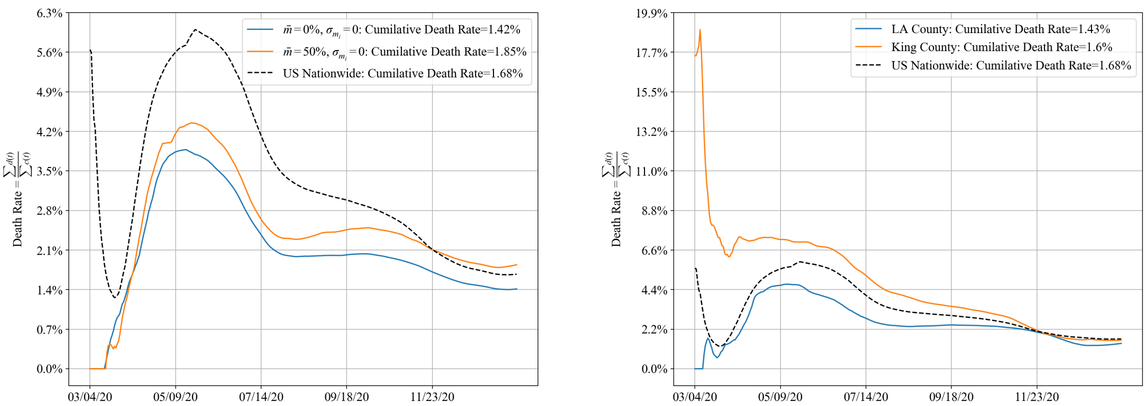

Figure 13: *Left: Death rate for group*0%*, group*50%*, and group*0 − 100% *or the whole US population. Right: Death rate for LA county, King County, and the whole US population.*

Even though LA has a much higher number of new cases than King county, the death rates in LA county (1.43%) is slightly lower than that of King county (1.60%), and both are lower than the national average (1.68%). They have a similar percentage of population age 70+ (8.4% in King county, and 8.3% in LA county), also lower than the national average of 11.15%. The initial outbreaks in nursing homes in King county in March 2020 significantly increased its death rate as can be seen in the daily death for King county in figure 10.

It would be helpful to get more detailed data such as the age distribution of new cases and death for each county. This could be a future study topic.

4 Conclusion

This study investigated the correlation of COVID-19 transmission with in-person K-12 teaching. The results did not show a statistically significant increase in new cases in counties with higher in-person teaching ratio. LA county compared with King county WA is used as a detailed example to illustrate this point. Even though LA county has the most stringent limitation on in-person teaching, it became the epicenter of COVID-19 during the 2020 fall semester. In contrast, King county had less strict limitations on in-person teaching, it was kept well below 1/3 of the daily new cases in LA during a big part of 2020 fall semester.

It worth emphasizing that even though current data did not show in-person teaching caused a significant increase in new COVID-19 cases, it does not mean that in-person teaching does not cause an increased risk of transmission. Instead, the data just showed that, under whatever social behaviors during the fall of 2020, in-person teaching did not make a significant difference in COVID-19 transmission. In other words, there were other social behaviors that caused more COVID-19 transmission than in-person teaching, and those behaviors are not well regulated.

Limiting social contact by keeping students on-line may reduce COVID-19 cases, but if other forms of social contact and "super spreader" events, such as large indoor social gathers, are unchecked, the effectiveness of closing in-person teaching to reduce COVID-19 transmission will be diminished. Containing COVID-19 and minimizing its disruption necessitates a balanced and scientific decisions making policy and the corporation of the whole public.

5 Discussion

Many factors contribute to the COVID-19 transmission, such as people congregating with limited social distancing. Many containment measures are taken, but the effectiveness of each measure was rarely quantified. If we limit school in-person teaching, close restaurants, hair salons and many small businesses while other social gatherings still go on unregulated, the effectiveness of the containment measure could be drowned out. It would be useful to systematically study the effectiveness of each measure and the combinations of them, and use science to help the policy makers to make the optimal decisions. This way, we could reduce the impact on people's lives from closed schools and businesses. This will not only serve the current pandemic but also provide some blueprint for future crises.

5.1 Limitation of this study

There are a few factors could significantly affect the accuracy of the about data:

- Not all people are tested for COVID-19, especially asymptomatic patients.
- Sick cases are not identified right after infection. The wait time from getting infected to be identified can vary widely, depending on when they get tested. Some cases will never be identified since they recovered on their own and never got tested.
- The time from exposure to symptom onset (known as the incubation period) is not a constant. It is thought to be 2 - 14 days, though symptoms typically appear within four or five days after exposure. A person with COVID-19 may be contagious 48 hours before starting to experience symptoms,[3]

$$T_{delay} = (4 \text{ or } 5) - 2 = 2 \text{ or } 3 \text{ days} \tag{16}$$

- The data are very noisy. Many factors contribute to the noise, such as reduced testing during weekends and holidays. Intermittent local business closures or stay home order at different locations and different time were hard to tally, thus became the background noise in the data.
- The test availability also causes a lot of noise in the data. There were many occasions that some counties had a large spike in new cases for certain days, not because of the sudden increase of transmission, but because of the accumulation of samples from a few days getting tested on that day or a sudden increase in test availability. If the test positivity data were available for each county, the estimation of R0 could be more accurate.
- The availability of vaccines since December 2020. With more people getting vaccination, the R_0 is no longer a good indicator of the severity of the transmission, rather, as discussed below, the SIR epidemic model is a better, but more complicated, model to study the spread of COVID-19.[11]

5.2 Possible further study for the future projects

At the onset of the pandemic, nearly the whole population is susceptible to the infection. The basic reproduction number, R_0 and equation (1) is a simple model to characterize the spread of the infection. At this moment, the total infection up today is 26 Million, less than 10% of the total U.S. population. This model is a quite good and simple way to characterize the spread of infection.

With the progress of the current COVID-19 pandemic, more and more people will be infected, combined with the roll out of vaccination, more and more people will have immunity to the COVID-19 virus. A better, but more complicated measure of the transmission would be β, transmission rate, in the SIR model,[11] which takes recovered population into account. However, this needs the data of recovered people per county, something that is difficult to obtain. This could be the topic of a future study.

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